Concept check - Week 1

Ashwin Bhola

CDS, NYU

Concept check Questions

1. Describe a subspace S of each vector space V and then a subspace SS of S:
   - $V_1 = all\ combinations\ of\ (1,1,0,0), (1,0,1,0)\ and\ (1,1,1,1)$
   - $V_2 = all\ symmetric\ 2\ by\ 2\ matrices$
   - $V_3 = all\ solutions\ to\ the\ equation\ \frac{d^4y}{dx^4} = 0$
2. Start with vectors $v_1 = (1,2,0)\ and\ v_2 = (2,3,0)$
   - Are $v_1\ and\ v_2$ linearly independent?
   - Are they a basis for any space?
   - What space V do they span?
   - What is the dimension of V?
   - Describe all vectors v_3 such that v_1, v_2, v_3 completes a basis of $\mathbb{R}^3$
3. Let w_1, w_2, w_3 be independent vectors. What can you say about the independence of $w_1 - w_2$, $w_1 - w_3, w_2 - w_3$? What about $w_1 + w_2, w_1 + w_3, w_2 + w_3$?

Concept check Questions

4. Find the basis for each of these subspaces in $\mathbb{R}^4$
   - All vectors whose components are equal
   - All vectors that are perpendicular to (1, 1, 0, 0) and (1, 0, 1, 1)
5. Choose $x = (x_1, x_2, x_3, x_4)\ in\ \mathbb{R}^4$. It has 24 rearrangements like (x_2, x_1, x_3, x_4) and (x_4, x_2, x_1, x_3). These 24 vectors span a subspace S. Find the specific vector x such that the dimension of S is (a) zero, (b) one, (c) three, (d) four
6. Consider the stacked vectors $v_1 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}, \ldots., v_k = \begin{pmatrix} a_k \\ b_k \end{pmatrix}$ where $a_1, \ldots, a_k$ are n vectors and $b_1, \ldots, b_k$ are m vectors.
   - Suppose $a_1, \ldots, a_k$ are linearly independent vector. Can we conclude that the stacked vectors $v_1, \ldots, v_k$ are linearly independent?
   - Suppose $a_1, \ldots, a_k$ are linearly independent vector. Can we conclude that the stacked vectors $v_1, \ldots, v_k$ are linearly dependent?