

Recitation – Week 2

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Sept 11th, 2019

Announcements

Office hour timings: Thursday, 1-2 PM, CDS Room 650

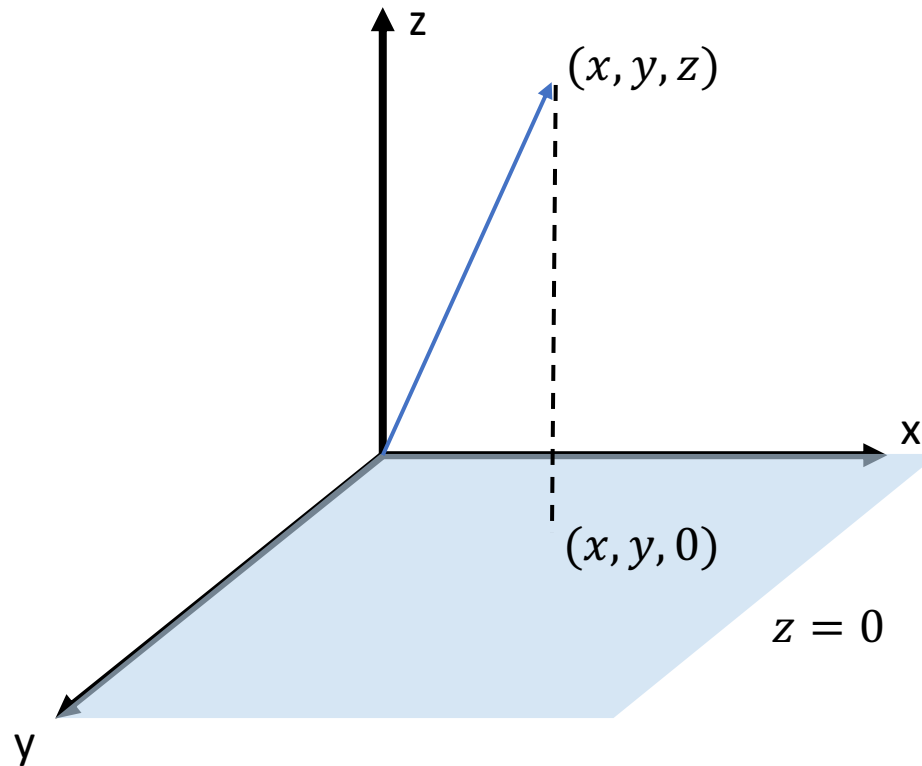
HW 2 due: 17th Sept 2019

Linear transformations

1. Project every vector $v \in \mathbb{R}^3$ onto the plane $z = 0$. How is this transformation defined? Is this a linear transformation? If yes, what's the matrix corresponding to this transformation? Also, what is the kernel and image of this transformation?
2. Which of the following functions are linear?
 - a) $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ such that $T(v_1, v_2) = (v_2, 4v_1 + v_2, 0)$
 - b) $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ such that $T(v_1, v_2) = v_1 - v_2 + 5$
 - c) $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ such that $T(v_1, v_2) = \sqrt{v_1^2 + v_2^2}$
3. Given a linear transformation $L: \mathbb{R}^m \rightarrow \mathbb{R}^n$, show that $\ker(L)$ is a subspace of \mathbb{R}^m and $\text{Im}(L)$ is a subspace of \mathbb{R}^n

Linear transformations

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Matrix multiplication (Method 1)

- Let $A \in \mathbb{R}^{m \times n}$ with columns a_1, \dots, a_n and let $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$

$$Ax = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ a_1 & a_2 & \cdots & a_n \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1 \begin{bmatrix} \vdots \\ a_1 \\ \vdots \end{bmatrix} +$$

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$$Ax = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ a_1 & a_2 & \cdots & a_n \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1 \begin{bmatrix} \vdots \\ a_1 \\ \vdots \end{bmatrix} + x_2 \begin{bmatrix} \vdots \\ a_2 \\ \vdots \end{bmatrix} +$$

Matrix multiplication (Method 1)

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Matrix multiplication (Method 2)

- Let $A \in \mathbb{R}^{m \times n}$ with rows a_1^T, \dots, a_m^T and let $x = [x_1, \dots, x_m] \in \mathbb{R}^m$

$$xA = [x_1 \ x_2 \ \dots \ x_m] \begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ & \vdots & \\ - & a_m^T & - \end{bmatrix} = x_1 [\dots a_1^T \dots] +$$

Matrix multiplication (Method 2)

- Let $A \in \mathbb{R}^{m \times n}$ with rows a_1^T, \dots, a_m^T and let $x = [x_1, \dots, x_m] \in \mathbb{R}^m$

$$xA = [x_1 \textcolor{blue}{x_2} \dots x_m] \begin{bmatrix} - & a_1^T & - \\ \textcolor{brown}{-} & \textcolor{brown}{a_2^T} & \textcolor{brown}{-} \\ & \vdots & \\ - & a_m^T & - \end{bmatrix} = x_1 [\dots a_1^T \dots] + \textcolor{green}{x_2 [\dots a_2^T \dots]} +$$

Matrix multiplication (2 new ways)

- Let $A \in \mathbb{R}^{m \times n}$ with rows a_1^T, \dots, a_m^T and let $x = [x_1, \dots, x_m] \in \mathbb{R}^m$

$$xA = [x_1 x_2 \dots x_m] \begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ & \vdots & \\ - & a_m^T & - \end{bmatrix} = x_1 [\dots a_1^T \dots] + x_2 [\dots a_2^T \dots] + \dots + x_m [\dots a_m^T \dots]$$

Matrix multiplication (2 new ways)

- Let $A \in \mathbb{R}^{m \times n}$ with columns a_1, \dots, a_n and let $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$

$$Ax = \begin{bmatrix} \vdots & \vdots & \vdots \\ a_1 & \cdots & a_n \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 \begin{bmatrix} \vdots \\ a_1 \\ \vdots \end{bmatrix} + \cdots + x_n \begin{bmatrix} \vdots \\ a_n \\ \vdots \end{bmatrix}$$

- Let $A \in \mathbb{R}^{m \times n}$ with rows a_1^T, \dots, a_m^T and let $x = [x_1, \dots, x_m] \in \mathbb{R}^m$

$$xA = [x_1 \dots x_m] \begin{bmatrix} - & a_1^T & - \\ \vdots & & \\ - & a_m^T & - \end{bmatrix} = x_1 [\cdots a_1^T \cdots] + \cdots + x_m [\cdots a_m^T \cdots]$$

Both these methods can be easily extended for cases where x is a matrix

Matrix multiplication

1. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 4 \\ 5 & 2 & 1 \end{bmatrix}$

- a) How can you swap the the first and third column of A via matrix multiplication?
- b) How can you replace the second row with twice the first row added to the second row of A and then swap the obtained second row with the third row via matrix multiplication?

2. Fix $A \in \mathbb{R}^{4 \times 5}$. Describe the following set:

$$\left\{ Ax : x = \begin{bmatrix} a \\ b \\ 0 \\ 0 \\ c \end{bmatrix}, a, b, c \in \mathbb{R} \right\}$$

Linear transformations

1. If $T: \mathbb{R}^m \rightarrow \mathbb{R}^n$ is linear transformation such that $\ker(T) = \{0\}$, and v_1, \dots, v_k is a list of vectors in \mathbb{R}^m , then prove that v_1, \dots, v_k is linearly independent iff Tv_1, \dots, Tv_k is linearly independent
2. If $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is a linear transformation such:

$$L(1,2) = (1,3,0)$$

$$L(2,3) = (0,1,1)$$

Write the matrix representation of L.

Revisiting Basis

1. Prove that any basis for \mathbb{R}^n has length n

Lemma 3.1: Let v_1, \dots, v_m span \mathbb{R}^n and suppose $w_1, \dots, w_p \in \mathbb{R}^n$ with $p > m$. Then w_1, \dots, w_p are linearly dependent