CS4780 Final

December 2015

NAME:	
Net ID:	
Email:	

GML	
CART	
Bias Variance	
Kernel Machines	
Neural Networks	
TOTAL	

1 [??] General Machine Learning

Please identify if these statements are either True or False. Please justify your answer **if false**. Correct "True" questions yield 1 point. Correct "False" questions yield two points, one for the answer and one for the justification.

T/F In Adaboost, in each iteration weights of misclassified examples go up by the same multiplicative factor.

T/F When a CART tree is built, you stop when a new split does not resolve in a reduction of impurity.

T/F The impurity of a CART node is the sum of the two impurities of its children

$$G(S) = w_L G(S_L) + w_R G(S_R), \tag{1}$$

where S is the data set of the parent node, S_L and S_R are the two subsets that fall into the left and right child respectively. The weights w_L, w_R correspond to how many nodes the left and right sub-trees have respectively.

T/F When built to full depth, CART trees tend to suffer from high bias.

T/F Boosting reduces bias.

T/F In Boosting you stop when the training error is zero.

T/F Bagging introduces a new hyper-parameter (m), which specifies how many classifiers are averaged. This has to be set very carefully. If m is too high, the classifier is prone to overfitting.

T/F In Bagging, the samples for each individual classifier are drawn from the same distribution as the original data set and the samples are independent and identically distributed (i.i.d.).

T/F A *weak learner* is a classifier that obtains worse than 50% error on binary classification tasks.

T/F Kernel machines project the data into potentially very high dimensional spaces. This can make the algorithm very slow and it an be very time consuming to store these very high dimensional vectors.

T/F A kernel function $k(\mathbf{x}, \mathbf{z})$ is well defined if and only if it corresponds to some function ϕ such that $k(\mathbf{x}, \mathbf{z}) = \phi(\mathbf{x})^\top \phi(\mathbf{z})$. (Although this function may be intractable to compute.)

T/F Kernel ridge regression is a universal approximator.

T/F Neural Networks are universal approximators.

T/F The Perceptron is a universal approximators.

T/F Kernel matrices have to be positive semi-definite.

T/F The SVM optimization problem has a dual formulation. This dual has the same solution as the original (primal) formulation.

T/F For SVMs, *Support Vectors* are those training inputs that are on the correct side of the decision boundary.

T/F The polynomial kernel with degree 2 is equivalent to computing an inner-product after a mapping into an infinitely dimensional feature space.

T/F In order to *kernelize* an algorithm it has to be stated such that inputs are only accessed in terms pair-wise squared distances.

T/F Gaussian Processes and kernel-SVMs have in common that they both access the training inputs only through a kernel function.

T/F One advantage of Gaussian Processes is that they return distributions, which gives a natural measure of “uncertainty”.

2 [20] CART

Assume you are provided with the following data:

$$
\begin{aligned}
\mathbf{x}_1 &= [0,1,1]^\top & y_1 &= -1\\
\mathbf{x}_2 &= [0,0,1]^\top & y_2 &= -1\\
\mathbf{x}_3 &= [0,1,0]^\top & y_3 &= -1\\
\mathbf{x}_4 &= [1,1,1]^\top & y_4 &= -1\\
\mathbf{x}_5 &= [1,0,1]^\top & y_5 &= -1\\
\mathbf{x}_6 &= [1,1,1]^\top & y_6 &= -1\\
\mathbf{x}_7 &= [1,0,1]^\top & y_7 &= +1\\
\mathbf{x}_8 &= [1,0,1]^\top & y_8 &= +1\\
\mathbf{x}_9 &= [1,1,0]^\top & y_9 &= +1
\end{aligned}
$$

1. (1) State the base cases (terminating criteria) of the ID3 algorithm

2. (2) Build a tree that splits on feature one on the root node, then on feature 2 on the next level (all nodes in that level), and feature 3 on the next level etc. until the termination criteria is reached. Draw the tree below.

3. (2) What is the Gini impurity at the root of your tree?

4. (3) Assume you have the following validation set, and prune the tree obtained in part 2. Draw the pruned tree. What is the validation error before pruning? What is the validation error after pruning?

$$\mathbf{v_1} = [0, 1, 0]^\top \qquad y_{v,1} = -1$$
$$\mathbf{v_2} = [1, 0, 1]^\top \qquad y_{v,2} = +1$$
$$\mathbf{v_3} = [1, 1, 0]^\top \qquad y_{v,3} = -1$$

5. (2) Add a training sample $\mathbf{x}_{10}$ with label y_{10} such that the data set cannot be classified correctly with a single decision tree.

6. (4) One could argue that the $k-$nearest neighbor classifier with $KD-$Trees is similar to an ID3 tree classifier. Name one advantage of each over the other.

7. (6) Suppose we define the loss function at a leaf of a regression tree as

$$L(S, h(S)) = \frac{1}{|S|} \prod_{(x,y)\in S} \exp((y - h(x))^2))$$

Given this loss function, what is the prediction value at a leaf?

3 [25] Bias Variance

1. (2) Your machine learning algorithm's validation error is too high. Your data is pretty clean (i.e. little noise). How can you identify if you suffer from high variance or high bias?

2. (6) Name three actions to counter *high Bias* and three actions to counter *high Variance*.

3. (5) You use 1-nearest neighbor (1-NN) classification on a data set, with test error ϵ. You know that 1-NN suffers from high variance, so you apply Bagging. However, as the number of averaged classifiers m becomes very large, you observe that the test error of the bagged classifier matches exactly ϵ. Can you explain this phenomenon?

4. (2) In **AdaBoost** let the error of the first weak learner be $\epsilon > 0$. Remember from class that $\alpha = \frac{1}{2}\log(\frac{1-\epsilon}{\epsilon})$. State the weight update for correctly classified inputs and misclassified inputs (without normalization) in terms of e^{α}.

5. (5) Show that the normalization constant Z (the sum of all updated weights) is $Z = 2n\sqrt{\epsilon(1-\epsilon)}$, where n is the number of training inputs.

6. (5) Finally, show that after the weight updates and normalizing all weights, the re-weighted error of the first weak learner is exactly 0.5.

4 [12] Kernel Machines

1. (2) Show that a well-defined kernel function $k(\mathbf{x}, \mathbf{z})$ is symmetric. (i.e. $k(\mathbf{x}, \mathbf{z}) = k(\mathbf{z}, \mathbf{x})$)

2. (4) Show that $k(\mathbf{x}, \mathbf{z}) = \frac{\mathbf{x}^\top \mathbf{z}}{\|\mathbf{x}\| \|\mathbf{z}\|}$ is a well-defined kernel.

3. (2) Can you kernelize ID3 decision trees with the entropy splitting rule? If so, state the algorithm, if not, explain why it is impossible.

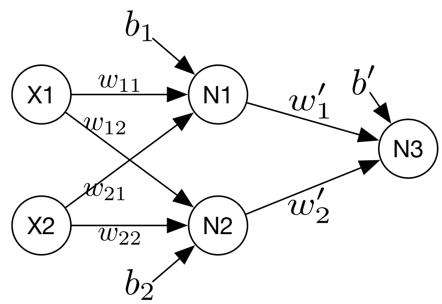
4. (2) One of the most popular kernels is the RBF kernel, $k(\mathbf{x}, \mathbf{z}) = \exp(-\gamma\|\mathbf{x} - \mathbf{z}\|^2)$. Assume we have two training points $\mathbf{x}_1, \mathbf{x}_2$ and a test-point $\mathbf{z}$ (for some $\gamma > 0$). In addition, you know that $\mathbf{z}$ is very close to $\mathbf{x}_1$, but very far away from $\mathbf{x}_2$. What statements are true about the kernel entries (multiple are possible):

   a) $k(\mathbf{x}_1, \mathbf{z}) \geq 0$ and $k(\mathbf{x}_2, \mathbf{z}) \geq 0$
   b) $k(\mathbf{x}_1, \mathbf{z})$ is close to 0, whereas $k(\mathbf{x}_2, \mathbf{z})$ is very large.
   c) $k(\mathbf{x}_1, \mathbf{z})$ is close to 1, whereas $k(\mathbf{x}_2, \mathbf{z})$ is very close to 0.
   d) $k(\mathbf{x}_1, \mathbf{z})k(\mathbf{x}_2, \mathbf{z})$ is close to 0.

5. (2) Explain the effects on the classifier if the constant γ is moved from a very large value to a very small value.

5 [11] Neural Network

Consider the following network with two inputs x_1, x_2, one output node N_3 and two hidden node N_1,N_2.

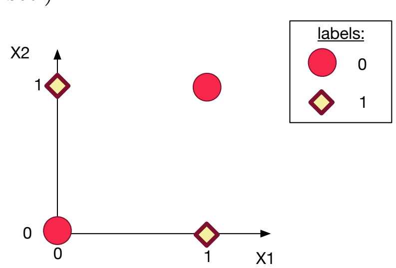


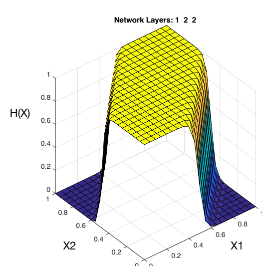
The weight w_{ij} connects X_i to N_j. Weight w'_j connects N_j to the output N_3.

1. Suppose N_1,N_2, N_3 all use **Sigmoid** transition functions, that is, their output is computed using the equation $\sigma(z) = \frac{1}{1+e^{-z}}$. Each node also has a bias term (b_1, b_2, b' respectively.)

   a) (2) A common approach to change the sigmoidal shape is to introduce a constant $a \geq 0$ and define the sigmoid function as $\sigma(z; a) = \frac{1}{1+e^{-az}}$. Sketch the sigmoid function $\sigma(z; a)$ as a function of z for $a = 1$ and for a very large ($a > 1000$).

   b) (5) Assume we use a sigmoid with a very large value of a. Assign weights such that this network **classifies** the XOR data set correctly. (Hint: The right figure shows the prediction of a neural net trained on the XOR data set.)





Please fill in your weights below:

w_{11}	w_{12}
w_{21}	w_{22}

w'_1
w'_2

b_1
b_2

b'

2. (2) Could you still classify the data correctly if node N_1 is removed from the network? If yes, assign the weights. If no, explain why not.

3. (2) Name an advantage of Rectified Linear Units over Sigmoidal transition functions.

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