Bayesian Statistics

Quan Zhang

Bayes Theorem

$$p(H \mid E) = \frac{p(H) \times p(E \mid H)}{p(E)}$$

Influence of prior distributions

Suppose a person voted for Trump in 2020. Where is he/she most likely from?

A. CA B. NY C. NE

Strong Evidence

3% of a country's population has a particular disease.

A test has 98% true positive rate, i.e., p(+|disease)=0.98, and 4% false positive rate, i.e., p(+|no disease)=0.04.

If a person simultaneously take the test twice and have both positive result. What is the probability that he/she indeed has the disease?

(Assume the two simultaneous tests are independent)

• In frequentist statistics, a parameter is an unknown constant number that needs to be estimated.

• In Bayesian statistics, an unknown parameter or or a parameter of interest is a random variable and thus has a distribution. We estimate the distribution of the parameter. By assuming some prior distribution (hypothesis) and observing some data (evidence), we find the posterior distribution by Bayes' theorem.