Survival Analysis with competing Events. 10/21.

(Not required).

Competing events (risks): mutually exclusive.

Data structure : X : predictors.

$\begin{cases} t: & \text{time to event.} \\ y: & \text{event type.} \end{cases}$

Eg. Patient with cancer and diabetes.

cancer ———————— 5 yr.

diabete ———————————— 7 yr

t: 5 y: cancer.

Suppose we have J competing events, and latent survival times under each event. say

$t_1, t_2, t_3 \cdots t_J$.

$$\begin{cases} t = \min_{j=1 \cdots J} t_j \\ y = \operatorname{argmin}_j t_j \end{cases}$$

1. Cause-specific PH model

Cancer 5 yr.

Diabete 7 yr.

A Cox model for cancer.

$t = 5.$ $\quad c = 1.$ (exact event time).

Another Cox mode for diabetes.

$t = 5$ $\quad c = 0$ (right censored).

Suppose we have J competing events, we run J Cox models each of which for an event j.

1. { Cancer 5 yr

Diabetes 7 yr. Cox for Diabetes $t=5, c=0$.

2. { Cancer $t=5.\ c=0$

Diabetes

Censored at 5 yr

Cause-specific Survival models violate

Independent & non-informative censoring assumption

Joint analysis of all J events.

Exponential race.

If $t_1 \sim Exp(\lambda_1)$, $\cdots$ $t_J \sim Exp(\lambda_J)$.

$t_1, t_2 \cdots t_J$ are independent.

$$\min_j t_j \sim Exp\left(\sum_j \lambda_j\right)$$

Exponential race for competing events.

$$t_1 \sim Exp(\lambda_1) \quad \lambda_1 = e^{x'\beta_1}$$

$$\vdots$$

$$t_J \sim Exp(\lambda_J) \quad \lambda_J = e^{x'\beta_J}$$

We observe $\begin{cases} t = \min_j t_j \sim Exp\left(\sum_j \lambda_j\right) \\ y = \operatorname{argmin} t_j \sim Categorical\left(\left\{\frac{\lambda_1}{\Sigma\lambda_j}, \frac{\lambda_2}{\Sigma\lambda_j} \cdots \frac{\lambda_J}{\Sigma\lambda_j}\right\}\right) \end{cases}$

hazard function

$$h(t) = \frac{f(t)}{S(t)} = \frac{(\sum_j \lambda_j) e^{-(\sum_j \lambda_j) t}}{e^{-(\sum_j \lambda_j) t}}$$

$$= \sum_j \lambda_j$$

$$= \sum_j e^{x' \beta_j}$$

constant hazard.

Weibull race.

$$w \sim \text{Weibull}(\text{shape} = a, \text{"rate"} = \lambda)$$

with $f(w) = a \lambda w^{a-1} e^{-\lambda w^a}$

$$F(w) = 1 - e^{-\lambda w^a}$$

$$h(w) = f(w) / 1 - F(w) = a \lambda w^{a-1}$$

Property: $t_j \overset{ind}{\sim} \text{Weibull}(a, \lambda_j)$

$$\min_j t_j \sim \text{Weibull}(a, \sum_j \lambda_j)$$

$$\text{argmin}_j\, t_j \sim \text{Categorical}\left(\left\{ \frac{\lambda_1}{\Sigma \lambda_j}, \frac{\lambda_2}{\Sigma \lambda_j} \cdots \frac{\lambda_J}{\Sigma \lambda_j} \right\}\right)$$

Weibull race for competing events:

$$\lambda_j = e^{x'\beta_j} \quad - - - .$$