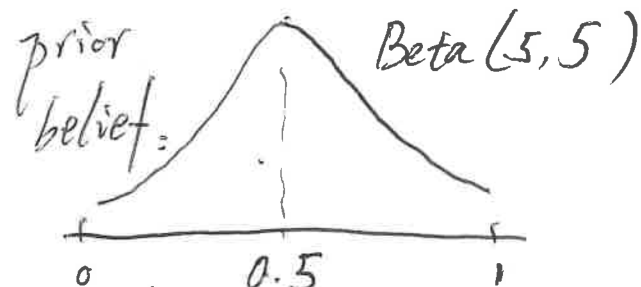
Bayes' Thm

$$p(H|E) = \frac{p(E,H)}{p(E)} = \frac{p(H)p(E|H)}{p(E)}.$$

$$= \frac{p(H)p(E|H)}{\sum_{i=1}^{\infty} p(E|H_i)p(H_i)}$$

$$\propto p(H)p(E|H).$$

H: A fair coin. $P_t \in [0,1] \sim Beta(a,b)$ with $a=b=5$.



Note: In Bayesian stats,

a parameter is a r.v. with some distribution.

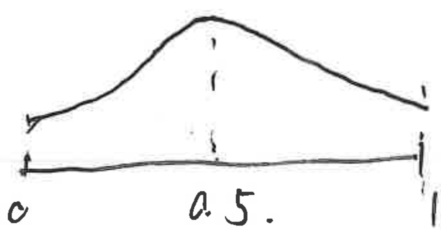
Before we know evidence, Beta(5,5) is called the prior distn of P_t.

In Bayesian stats, we estimate the distn $p(H|E)$ (this is called the posterior of P_t).

Prior: A coin is fair.

P_{tail} : is a r.v.

$P_{tail} \sim \text{Beta}(a=5, \ b=5)$.



prior ~~dist~~ density:

$$p(P_{tail}) = f_{Beta(5,5)}(P_{tail})$$

$$= \frac{P_{tail}^{a-1} (1-P_{tail})^{b-1}}{B(5,5)}, \quad a=b=5.$$

Data 1: 7 tail, 3 heads.

$$p(D_1 \mid P_{tail}) = \text{Binomial}(10, 7, P_{tail}).$$

$$= C_{10}^{7} P_{tail}^{7} (1-P_{tail})^{3}$$

$$p(P_{tail} \mid D_1) \propto p(P_{tail}) \cdot p(D_1 \mid P_{tail}).$$

$$\propto P_{tail}^{a-1} (1-P_{tail})^{b-1} P_{tail}^{7} (1-P_{tail})^{3}$$

$$\propto P_{tail}^{7+a-1} (1-P_{tail})^{3+b-1}$$

$$p(p_{tail} \mid D_1) \propto p_{tail}^{7+a-1} (1-p_{tail})^{3+b-1}$$

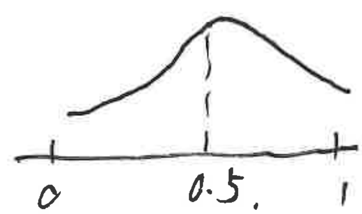
$$Beta(a, b) \propto p_{tail}^{a-1} (1-p_{tail})^{b-1}$$

posterior: $p_{tail} \mid D_1 \sim Beta(a+7,\ b+3)$.

where $a = b = 5$.

prior

posterior:

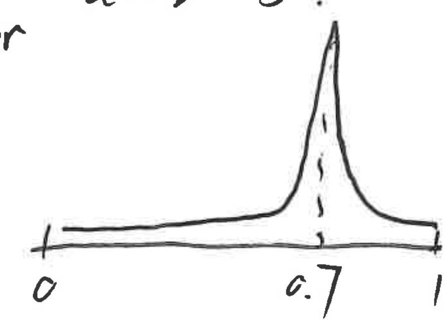


D_2 : 700 tails, 300 heads.

$p_{tail} \mid D_2 \sim Beta(a+700,\ b+300)$.

$a = b = 5$.

posterior



Intuition.

	CA ✓	NY.	NE.
population	40M	20M	2M.
voting rate.	50%	50%	100%.
Trump	30%	40%	100%.
→	6M	4M.	2M.

Bayes method: $p(H|E) = \dfrac{p(H)\,p(E|H)}{p(E)}$

$$\propto p(H)\,p(E|H).$$

priors: $p(CA)$ $\quad p(NY)$ $\quad p(NE)$.

$$= \frac{40M}{300M} \qquad = \frac{20M}{300M} \qquad = \frac{2M}{300M}.$$

$p(E|H)$: $p(\text{Trump}|CA)$ $\quad p(\text{Trump}|NY)$ $\quad p(\text{Trump}|NE)$

$$= 50\% \times 30\% \qquad = 50\% \times 40\% \qquad = 100\% \times 100\%$$

$p(H|E)$ $\quad p(CA|\text{Trump})$ $\quad p(NY|\text{Trump})$ $\quad p(NE|\text{Trump})$

$$\propto 6/300 \qquad \propto 4/300 \qquad \propto 2/300$$

$$p(D) = 3\%.$$

$$p(+ \mid D) = 98\%$$

$$p(+ \mid ND) = 4\%$$

E: $++$.

$$p(D \mid ++) = \frac{p(D) \cdot p(++ \mid D)}{p(++)}.$$

$$= \frac{p(D)\ p(++ \mid D)}{p(++ \mid D)\, p(D) + p(++ \mid ND)\, p(ND)}$$

$$\left(\begin{array}{l} p(++, D) + p(++, ND) \\ = p(++). \end{array} \right).$$

$$p(++ \mid D) = p(+ \mid D) \cdot p(+ \mid D).$$
$$= (98\%)^2$$

$$p(++ \mid ND) = p(+ \mid ND) \cdot p(+ \mid ND)$$
$$= (4\%)^2$$

$p(D) = 3\%$ $\quad$ $p(ND) = 1 - 3\% = 97\%$

$$\Rightarrow p(D \mid ++) = 94.9\%$$

what if one test is taken and the result is "+" ?

$$p(D \mid +) = 43.1\%.$$