$$y_{ij} = \underbrace{\alpha + \beta x_{ij} + \gamma_i} + \varepsilon_{ij}$$

$$\gamma_i \sim N(0, s^2).$$

$$p(y_{ij} \mid \alpha, \beta, s^2, \sigma^2)$$

$$= \int p(y_{ij}, \gamma_i \mid \alpha, \beta, s^2, \sigma^2)\, d\gamma_i$$

$$= \int p(y_{ij} \mid \gamma_i, \alpha, \beta, \sigma^2)\, p(\gamma_i \mid s^2)\, d\gamma_i. \quad (*)$$

$$\gamma_i^{(n)} \sim N(0, s^2)$$

Reparameterization trick.

$$\Rightarrow \gamma_i^{(n)} = s \cdot z^{(n)}, \quad z^{(n)} \overset{iid}{\sim} N(0, 1^2).$$

$$(*) = \frac{1}{N} \sum_{n=1}^{N} p(y_{ij} \mid \alpha, \beta, \sigma^2, \gamma_i = s \cdot z^{(n)}).$$

$$= \frac{1}{N} \sum_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2\sigma^2}(y_{ij} - \alpha + \beta x_{ij} + s \cdot z^{(n)})^2}$$

what if $\gamma_i \sim Exp(s)$.

$$\int p(y_{ij} \mid \gamma_i, \alpha, \beta, \sigma^2)\, p(\gamma_i \mid s)\, d\gamma_i. \quad (*).$$

CDF: $F(\gamma_i) = 1 - e^{-s\gamma_i} \sim Unif(0,1)$.

We draw $u \sim Unif(0,1)$ and

let $1 - e^{-s\gamma_i} = u$.

$$\gamma_i^{(n)} = -\frac{1}{s}\log(1-u^{(n)})$$

$$(*) = \frac{1}{N}\sum_{n=1}^{N} p(y_{ij} \mid \gamma_{ij}^{(n)} = -\frac{1}{s}\log(1-u^{(n)}), \alpha, \beta, \sigma^2)$$

However, the trick is not always available.

Eg, $\gamma_i \sim Gamma(shape = a, rate = b)$.

$a$, $b$ are unknown. The trick isn't available.