Mach-Zehnder Interferometry and Erasure of 'Which-path' Information

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1 Abstract

The famous Einstein-Bohr debate at the Fifth Solvay Conference occurred in 1927. It ended with Niels Bohr having the last word. He claimed that there would be no interference pattern if we obtain the which-way information regarding the path the photon is taking through a standard double slit experiment. This is an either-or distinction. That is, either you obtain the which way-information regarding photon in which case you lose out on interference pattern, or either you obtain the interference pattern in which case you lose out on which-way information. Although Bohr's statement was backed by experimental results of those times, in our experiment we are going to explore how things are not as black-and-white. Subsequent explorations into the subject matter reveals a far more nuanced relationship between which-way information and interference pattern than Bohr envisioned which continues to excite and befuddle experimentalists and theorists alike.

2 Theoretical Background

2.1 Copenhagen Interpretation revisited

The Copenhagen interpretation splits the classical double-slit experiment into two scenarios.

1. The "wave" scenario. The general wavefunction is that of a two-level system with the wave-function being in a superposition state $\psi = \frac{1}{\sqrt{2}}\left[\psi_1(x) + \psi_2(x)\right]$. If we refrain from measuring through which slit each particle has passed, the particle density observed at the level of the detecting screen corresponds to an interference pattern given by $|\psi|^2 = \frac{1}{2}|\psi_1(x) + \psi_2(x)|^2$

2. The "measurement" scenario. If we place a detector at one of the slits to find out whether the particle has passed through the slit, the interference pattern disappears. In that case, by invoking the collapse postulate of Quantum Mechanics, we say that the wavefunction has collapsed, say, for example to $\psi_1(x)$ such that we have $\psi(x) \to \psi_1(x)$. Now the density pattern on the distant screen is simply equal to a classical addition of the pattern created by all particles that have traversed one of the slit and the pattern created by all particles that have passed through the other slit: $p(x) = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2$

2.2 Density Matrix formulation

In this exposition, we are going to build towards a wonderful, exciting development in the foundation of Quantum Mechanics. For a double slit-experiment, Wojciech Hubert Zurek and William Wootters established that we can actually retain parts of the interference pattern by gathering only some which-path information, e.g, by performing an imprecise measurement of which slit the particle has traversed. To discuss the features which allow us to do this, the description in terms of wave-function collapse will no longer be suitable. By postulate, the collapse of the wavefunction is a discontinuous, irreversible process and therefore cannot account for smooth, reversible changes in the amount of which-path information and the degree of interference. Instead, we shall pursue an entirely new formalism in terms of density matrices.

Let us denote the quantum states of the particle corresponding to passage through slit 1 and 2 by $|\psi_1\rangle$ and $|\psi_2\rangle$ respectively. By von neumann measurement scheme, we place a detector at each of the two slits, with both detectors initially in the "ready" state. We can prepare our particle, in say, the state $|\psi_1\rangle$ by covering slit two, and placing the particle source directly behind slit one such that the particle will be guaranteed to pass through this slit. Consequently, the detector associated with slit one will trigger, while the detector at slit two will remain in the untriggered "ready" state.

We denote the joint "ready" state of the (composite) detector by $|ready\rangle$ and the quantum state of this detector system after preparation of the state $|\psi_1\rangle$ by $|1\rangle$, indicating the passage of the particle through slit one. Thus, in this case, the evolution of the composite particle-detector system will be of the form:

$$|\psi_1\rangle\,|ready\rangle \rightarrow |\psi_1\rangle\,|1\rangle\,. \tag{1}$$

Repeating the above argument with the role of the slits reversed yields the evolution:

$$|\psi_2\rangle\,|ready\rangle \rightarrow |\psi_2\rangle\,|2\rangle\,. \tag{2}$$

On the other hand, if both slits are open, the particle must be described by a superposition of states $|\psi_1\rangle$ and $|\psi_2\rangle$ of the form $|\psi\rangle = \frac{1}{\sqrt{2}}\big(\,|\psi_1\rangle + |\psi_2\rangle\big)$. Therefore, we obtain a dynamical evolution of the von Neumann type, leading to an entangled composite particle-detector state

$$\frac{1}{\sqrt{2}}\big(\,|\psi_1\rangle + |\psi_2\rangle\big)\,|ready\rangle \rightarrow \frac{1}{\sqrt{2}}\big(\,|\psi_1\rangle\,|1\rangle + |\psi_2\rangle\,|2\rangle\big). \tag{3}$$

Expressing (3) in terms of density matrix:

$$\begin{aligned}
\hat{\rho} &= |\psi\rangle\,\langle\psi|\,,\\
&= \frac{1}{2}\Big[\big(\,|\psi_1\rangle\,|1\rangle + |\psi_2\rangle\,|2\rangle\big)\big(\,\langle\psi_1|\,\langle 1| + \langle\psi_2|\,\langle 2|\big)\Big],\\
&= \frac{1}{2}\Big[\,|\psi_1\rangle\,\langle\psi_1|\,|1\rangle\,\langle 1| + |\psi_1\rangle\,\langle\psi_2|\,|1\rangle\,\langle 2| + |\psi_2\rangle\,\langle\psi_1|\,|2\rangle\,\langle 1| + |\psi_2\rangle\,\langle\psi_2|\,|2\rangle\,\langle 2|\Big],\\
&= \frac{1}{2}\Big[\,|\psi_1\rangle\,\langle\psi_1| + |\psi_1\rangle\,\langle\psi_2|\,|1\rangle\,\langle 2| + |\psi_2\rangle\,\langle\psi_1|\,|2\rangle\,\langle 1| + |\psi_2\rangle\,\langle\psi_2|\Big].
\end{aligned}$$

Where we utilized $|1\rangle\,\langle 1| = |2\rangle\,\langle 2| = 1$ to simplify the above expression. Applying Born's Rule and the symmetry of inner product, the density matrix above gives the following probability density for a two-state quantum system which is entangled with the detector:

$$|p(x)|^2 = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2 + \frac{1}{2}\,\langle\psi_1|\psi_2\rangle\,\langle 1|2\rangle\,. \tag{4}$$

The last term in equation (4) describes the interference pattern, and we see that the visibility of the interference pattern is quantified by the overlap $\langle 1|2\rangle$. In particular, the limiting case of distinguishability of the detector states $|1\rangle$ and $|2\rangle$, the overlap $\langle 1|2\rangle = 0$ corresponds to the "particle" regime defined by

$$|p(x)|^2 = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2. \tag{5}$$

Conversely, if $|1\rangle$ and $|2\rangle$ are unable to resolve the path of the particle entirely, we have the overlap $\langle 2|1\rangle = 1$. In this limiting case, the wave-scenario of full interference of full pattern applies:

$$|p(x)| = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2 + \langle\psi_1|\psi_2]\rangle\,. \tag{6}$$

2.3 Looking beyond Copenhagen Interpretation

Using equation (4), we are in a position to formulate a physical model of double slit experiment that is much more causal and continuous than the Copenhagen interpretation which introduces the controversial collapse postulate. In particular, note that the situation in which the detector obtains some but not full which-path information is formally represented by an overlap of the two detector states $|1\rangle$ and $|2\rangle$ that is nonzero but less than one. From equation (4), we see that we will be able to observe an interference pattern, but the pattern will decay progressively as the overlap $\langle 2|1\rangle$ decreases and thus the amount of which path information obtained by the detector increases.

Thus we see that it is indeed possible to simultaneously observe an interference pattern and obtain some information about the path of the particle through the slits in this representation, provided this information remains incomplete [1]. As soon as the information acquired by the detector is sufficient to enable us to infer with certainty which path the particle has taken, the interference pattern disappears.

In summary, the degree to which an interference pattern can be observed is simply determined by the available which-path information encoded in some system entangled with the object of interest, and this amount can be changed without necessarily influencing the spatial wave function of the object itself. Thus complementarity can be regarded as a consequence of quantum entanglement. To sum up, the overlap integral $\langle 2|1\rangle$ highlights how the original scheme of either observing interference pattern or obtaining which-way information is incorrect. Instead, there seems to be a trade-off between these quantities. Again, the introduction of overlap integral $\langle 2|1\rangle$ does not make measurement a reversible process. However, what it shows is that the collapse is not instantaneous like conventionally taught in freshmen quantum mechanics course.

2.4 Polarization Along Arbitary Angle

Since we would be utilizing lasers polarized along various angle throughout the experiment, let us formulate how polarization along any arbitrary angle may be expressed mathematically. The derivation can be taken from any elementary textbook on Jones calculus and therefore we would not go into details. Instead we would provide a brief summary of the derivation: Firstly, take the original electric field E_{org} as a plane wave with E_x component oriented along $\hat{x}$ axes and E_y oriented along $\hat{y}$ axes. Denote a transmission axis of the polarizer by $\hat{e_1}$ and the absorption axis of the polarizer by $\hat{e_2}$ (see figure below)

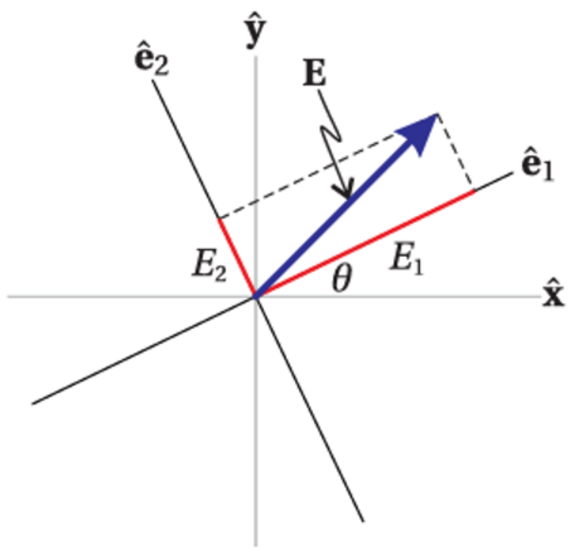

Figure 1: Electric field through a Polarizer

Suppose now that the electric field passes through the polarizer. How will E_{org} change? To answer that, note that the component of the electric field E after it passes through the transmission axes is E_1 and is along e_1 and the component of the electric field that passes through the absorption axes is E_2 and is along e_2.Therefore, we must express the components of E_{org} in terms of the new basis specified by $\hat{e_1}$ and $\hat{e_2}$. To do this, we utilize geometry of the figure above to invert the equations of $\hat{x}$ and $\hat{y}$ in terms of $\hat{e_1}$ and $\hat{e_2}$ and make them equations of $\hat{e_1}$ and $\hat{e_2}$ expressed in terms of $\hat{x}$ and $\hat{y}$. Finally, we introduce the effect of polarizer. E_1 is transmitted unaffected while E_2 is extinguished. To account for the effect of the device, we multiply E_2 by a parameter ξ (for polarizers $\xi = 0$ but for half-wave plates it may have some values other than 0 or 1) and then express our result in the following matrix form:

$$\hat{P}(\theta) = \begin{pmatrix} cos^2(\theta) & sin(\theta)cos(\theta) \\ sin(\theta)cos(\theta) & sin^2(\theta) \end{pmatrix}. \tag{7}$$

3 Experimental Procedure

3.1 Background and Interferometer Setup

During the course, we had the opportunity to work with both the Mach-Zehnder interferometer and the Michelson Interferometer. As such, a brief explanation of what distinguishes the two setup is in order. In Michelson interferometer, we use a single beam splitter. When a beam passes through the beam splitter, it splits the beam, causing the beam to hit two mirrors at right angle to the splitter. After hitting the mirrors, the beam comes back to the beam splitter which then recombines them and send them to the detector. On the other hand, in the case of Mach-Zehnder interferometer, we utilize two beam splitter. One beam splitter splits the beam into two and the second beam splitter combines the split beams again and then this combined beam hit the detector again. Keeping this in mind, a pictorial representation of the setup is given below in Figure 2:

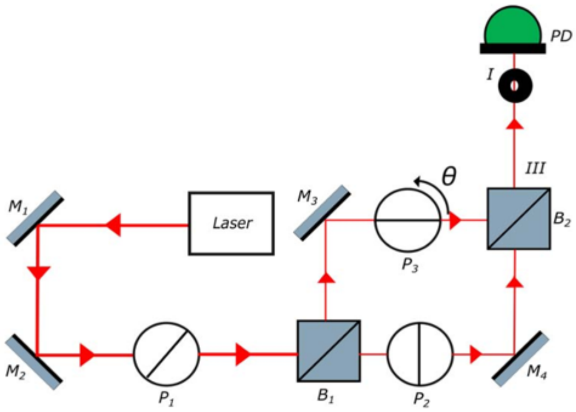

Figure 2: Pictorial Representation of the Experiment

Alignment is perhaps the most challenging part of this experiment and requires considerable care and attention. Majority of our time in lab was spent in adjusting the mirrors with respect to the laser beam such that an interference pattern at the screen can be observed. Some rules of thumb that finally helped us attain an interference pattern after a long and tiring vigil was the following: firstly, make the most of the optical mounts, ensuring that the light from the incident beam is parallel to a particular row of mounts and does not deviate horizontally from the chosen row. Ensure that this is the case when light is deflected perpendicularly through the mirror also such that light continues to follow a chosen row of mount. This means that all the mirrors and beam splitters should ideally be placed in a manner such that their center bisects a particular row of optical mount. Secondly, ensure that the height of all the optical equipment is the same and that the laser maintains a constant height as it travels through the setup. One should be pedantic about both of these criterion being met and ideally a meter rule should be employed to root out even the slightest of deflection in either horizontal or vertical direction. Perhaps the strongest indicator of obtaining an interference pattern after the lens is the following: both the split lasers beam perfectly align on the screen placed right after PD (see figure above), and also on the screen that is placed to the right of B_2 splitter. Most of the time, although the beam is aligning on the screen placed after PD, it does not align on the screen placed to the right of B_2 . This indicates that perfect alignment has not been attained and further work needs to done in setting up the apparatus properly.

After the setup has taken place, an unpolarized light is passed through a 45 degrees polarizer P_1. When it passes through the polarizer P_1 which is oriented at 45 degrees, it picks up the following polarization:

$$E_{in} = \frac{1}{\sqrt{2}}\big[\,|V\rangle + |H\rangle\big], \tag{8}$$

When the beam passes through a perfect beam splitter, a fraction r of it is reflected and fraction t is transmitted. For a 50:50 non-polarizing beam splitter, t = r = $\frac{1}{2}$. In our setup, the output field is given by:

$$E_{out} = r^2 P_3 E_{in} + t^2 P_2 E_{in}, \tag{9}$$

If the light remains on the same path (from B_1 to M_4, it meets a polarizer which is vertical to the setup. That is, it is at 90 degrees. Keeping P_3 at any arbitrary direction, we have using equation (9) and (7):

$$E_{out} = \frac{1}{4}\begin{pmatrix} cos^2(\theta) & sin(\theta)cos(\theta) \\ sin(\theta)cos(\theta) & sin^2(\theta) \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$E_{out} = \begin{pmatrix} cos^2(\theta) + sin(\theta)cos(\theta) \\ sin^2(\theta) + sin(\theta)cos(\theta) + 1 \end{pmatrix}, \tag{10}$$

Since Intensity is $I_{out} = E^*_{out}E_{out}$, we obtain:

$$I_{out} = 2 + 2sin(\theta) + 2sin^2(\theta), \tag{11}$$

3.2 Experimental Result with P_3 at various angles

On rotating the Polarizer P_3, we find that a distinct pattern is observed when the Polarizer has the same orientation as P_2 which then subquently wanes at various angles, being extinguished entirely at 90 degrees. Subsequently, the intereference pattern reappears at 180 degrees (when the Polarizer P_3 is once again parallel to P_2:

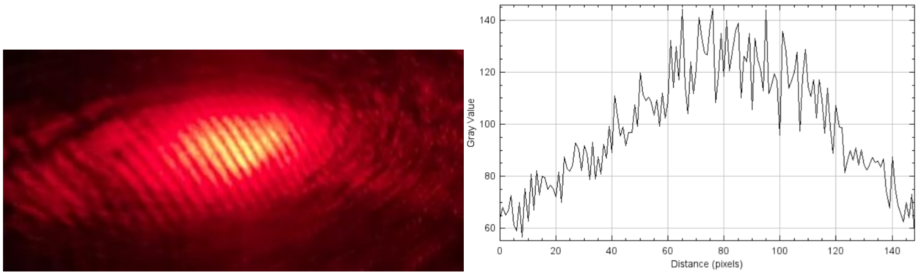

Figure 3: Orientation of Polarizer P_3 at 0 ϕ / rad

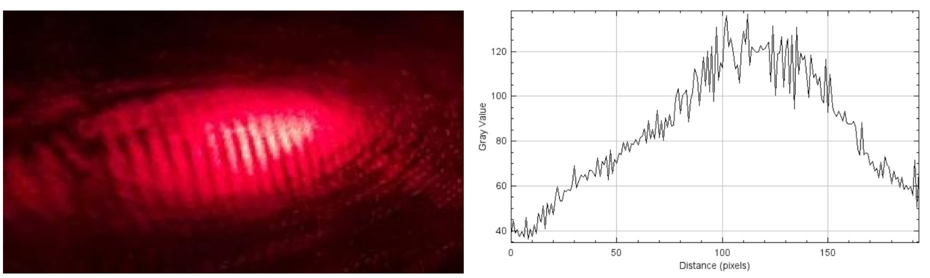

Figure 4: Orientation of Polarizer P_3 at $\frac{\pi}{12}$ ϕ / rad

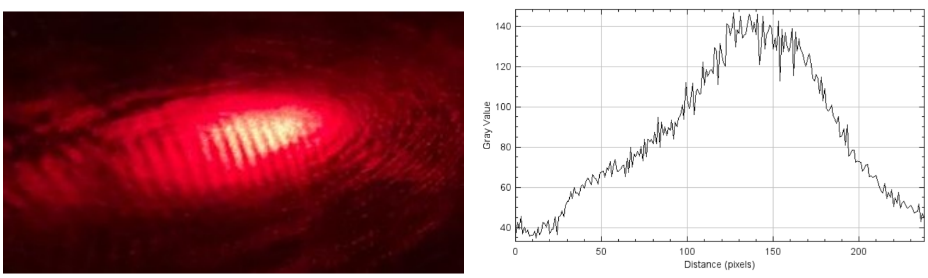

Figure 5: Orientation of Polarizer P_3 at $\frac{\pi}{6}\phi$ / rad

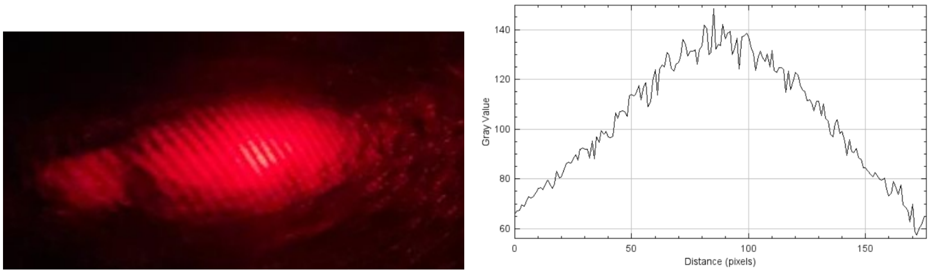

Figure 6: Orientation of Polarizer P_3 at $\frac{\pi}{6}\phi$ / rad

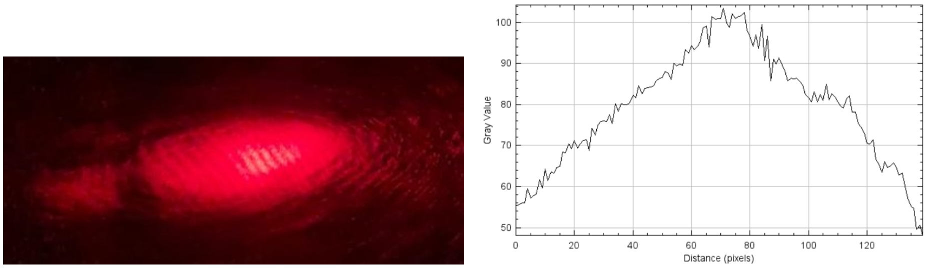

Figure 7: Orientation of Polarizer P_3 at $\frac{\pi}{3}$ ϕ / rad

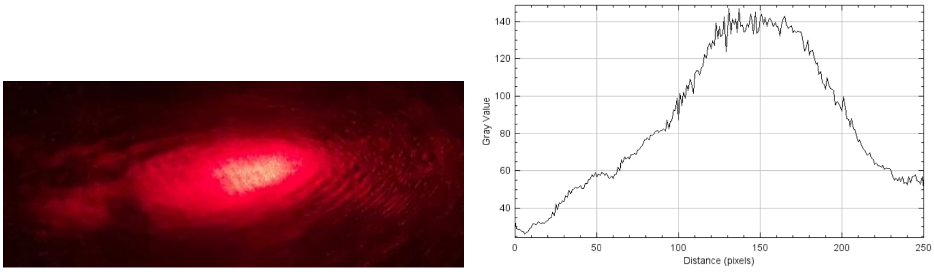

Figure 8: Orientation of Polarizer P_3 at $\frac{5\pi}{12}$ ϕ / rad

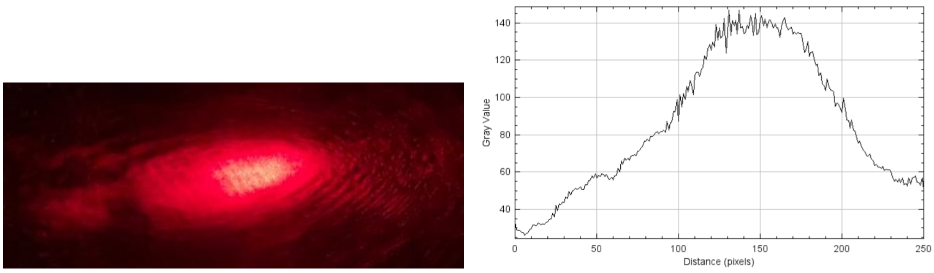

Figure 9: Orientation of Polarizer P_3 at $\frac{\pi}{2}$ ϕ / rad

Finally, we replace the lens and instead place a photo-detector in its position. We then find the values of the output voltage using IV convertor for varying values of polarization of P_3. The graph we obtained is plotted below:

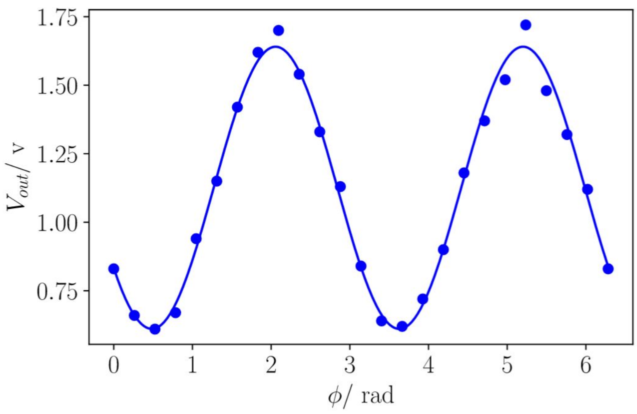

Figure 10: Intensity as P_3 is rotated

3.3 Explanation of Interference

We can explain the reason why the interference pattern appears the most pronounced at 0 ° degrees and then gradually diminishes until it disappears entirely at 90 ° degrees by considering the effect of relative polarization between two waves on the eventual interference pattern. In particular, it can be accounted by the fact that parallel polarization interferes while perpendicular polarization does not.

When the Electric field is perpendicularly polarized, we can represent the first wave as:

$$E_1 = E_0 cos(\omega t)\hat{x},$$

On the other hand, the second wave has the orientation:

$$E_2 = E_0 cos(\omega t + \alpha)\hat{y},$$

where α represents a phase difference. The combined wave that occurs after they meet at the beam splitter is:

$$E = \begin{pmatrix} E_0 cos^2(\omega t) \\ E_0 cos^2(\omega t + \alpha) \end{pmatrix}. \tag{12}$$

Taking $I = E^* E$, we find $I = E^2$ which is independent on the phase difference between waves (and therefore does not cause interference). On the other hand, if both the electric field components have a parallel polarization, then the combined electric field becomes:

$$E = \begin{pmatrix} E_0 cos^2(\omega t) + E_0 cos^2(\omega t + \alpha) \\ 0 \end{pmatrix}. \tag{13}$$

Taking $I = E^* E$,

$$I = E_0^2 (cos^2(\omega t) + 2cos(\omega t) + cos(\omega t + \alpha) + cos^2(\omega t + \alpha)),$$

which depends on the phase shift and therefore the observed interference between the waves when P_3 and P_2 are parallel. We can use the density matrix formulation to generalize the result when it would be carried out in the Quantum Mechanical limit. When both the waves have the same polarization, the overlap integral $\langle 2|1 \rangle$ is maximum and therefore pronounced interference is seen. This overlap integral then decreases continuously and reaches zero when the polarizations are perpendicular to one another

3.4 Introduction of P_4 polarizer

We now introduce modification to our experiment. We cross P_2 and P_3 such that the overlap integral $\langle 2|1\rangle$ introduced in equation 4 becomes 0. This then gives us the particle regime in which we should have classical addition of probability

$$p(x) = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2. \tag{14}$$

However, as we rotate P_4 , we find that pronounced interference pattern reappear at 120 degrees and 300 degrees. Somehow the quantum regime of probabilities has been restored by P_4 and therefore the which-way information for the trajectory the photon has been "erased".

2

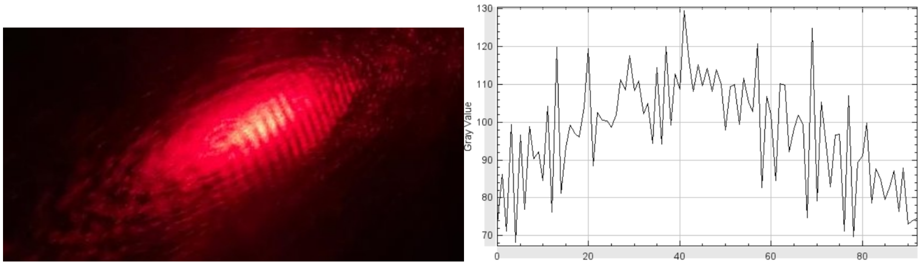

Figure 11: Orientation of Polarizer P_4 at $\frac{120\pi}{180}$ ϕ / rad

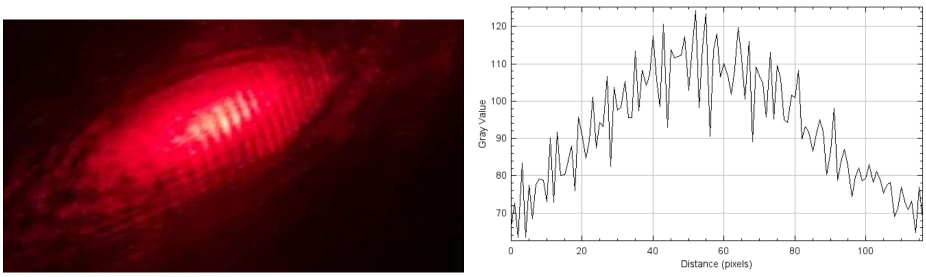

Figure 12: Orientation of Polarizer P_4 at $\frac{135\pi}{180}$ ϕ / rad

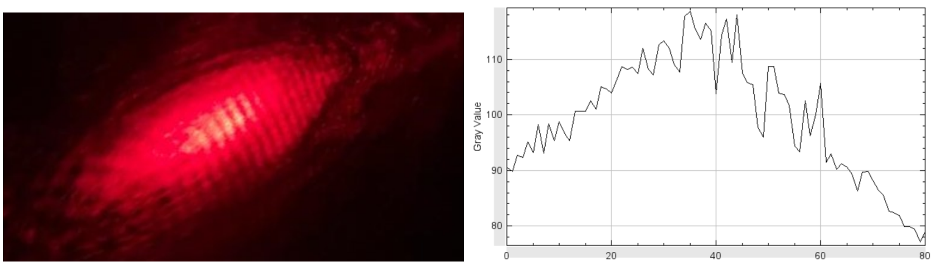

Figure 13: Orientation of Polarizer P_4 at $\frac{150\pi}{180}$ ϕ / rad

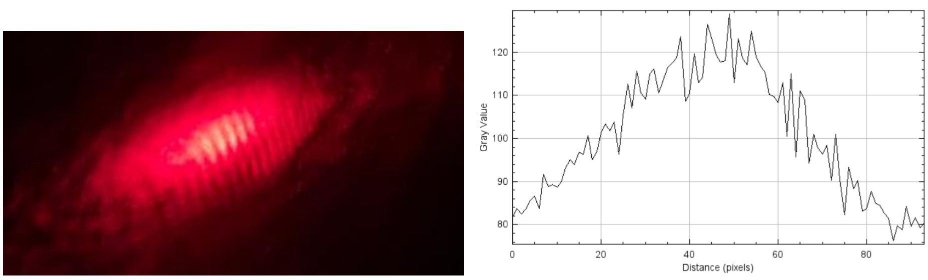

Figure 14: Orientation of Polarizer P_4 at $\frac{165\pi}{180}$ ϕ / rad

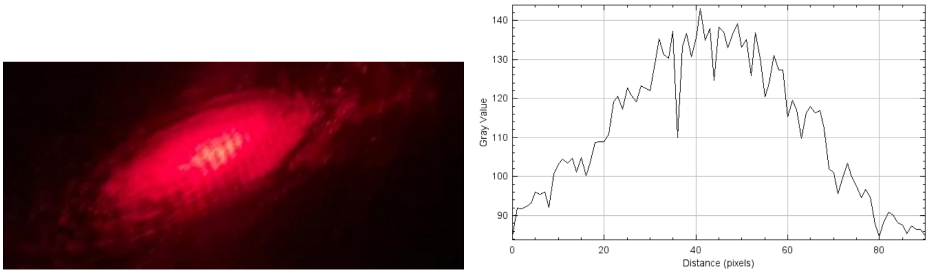

Figure 15: Orientation of Polarizer P_4 at π ϕ / rad

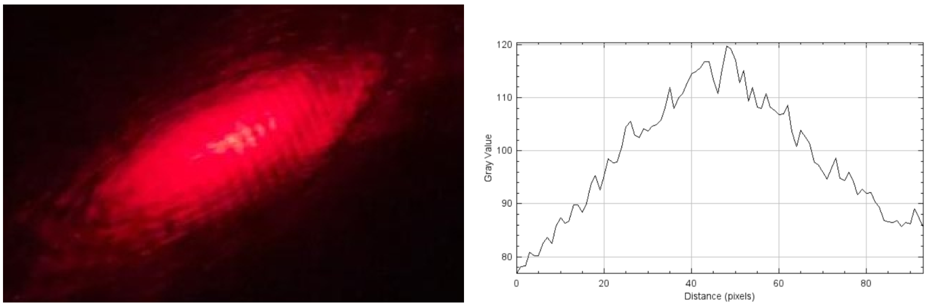

Figure 16: Orientation of Polarizer P_4 at $\frac{195\pi}{180}$ ϕ / rad

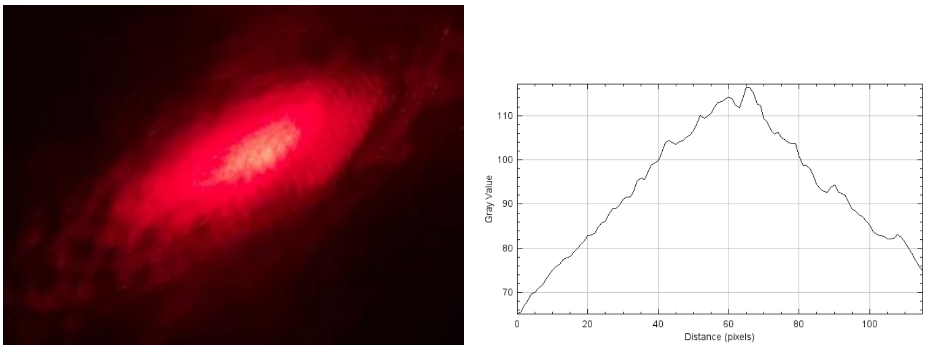

Figure 17: Orientation of Polarizer P_4 at $\frac{210\pi}{180}$ ϕ / rad

Once again, we replace the lens and instead place a photo-detector in its position. We then find the values of the output voltage using IV convertor for varying values of polarization of P_4. The graph we obtained is plotted below:

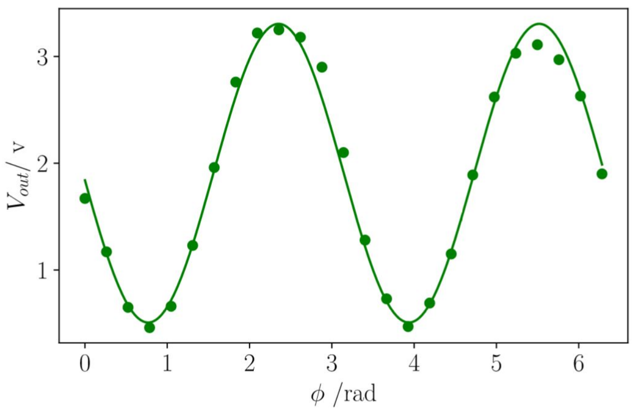

Figure 18: Intensity as P_4 is rotated

3.5 Quantum Erasure Demystified

Since we are working with classical light, first, lets explain the effect using a description that does not invoke Quantum mechanics. In particular, consider that when P_1 and P_3 are crossed, the electric field assumes the form

$$E = \begin{pmatrix} E_0 cos^2(\omega t) \\ E_0 cos^2(\omega t + \alpha) \end{pmatrix}, \tag{15}$$

where the phase difference α is introduced due to a small asymmetry in one of the beam splitters or in one of the two mirrors. When a polarizer whose action is given in equation (7) would act on this electric field, the x-components and y-components would mix such that the intensity would become a function of α, thereby causing it to depend on the phase shift between the combined waves and hence the interference. Note that this explanation is far from the actual quantum erasure and therefore this experiment can best be described as being analogical to the quantum erasure experiment rather than actually being one

On that note, we end this section by running with the analogy and giving a quantum mechanical explanation for interference, imagining that a single photon passes through the entire apparatus To understand the path that this photon would take, we fall back to the pointer states introduced when we explored Density matrices in subsection 2. From equation 1 and 2, pointer states $|1\rangle$ and $|2\rangle$ are related with the wavefunction of the photon as following:

$$|V\rangle |1\rangle , \tag{16}$$

$$|H\rangle |2\rangle , \tag{17}$$

where $|V\rangle$ denotes the wavefunction of the photon travelling from B_1 to M_4 and is vertically polarized while $|H\rangle$ denotes the wavefunction of the photon travelling from B_1 to M_3 and is horizontally polarized.

The state $|1\rangle$ and $|2\rangle$ are sometimes called "recording qubits" since they store the information about $|V\rangle$ and $|H\rangle$ respectively. We can now understand the situation that arises when P_2 and P_3 are crossed as following: Assume that we are in the Eigenbasis of $|1\rangle$ and $|2\rangle$ and we measure $|1\rangle$. This implies that the photon travelled from B_1 to M_4. On the other hand, if we measure $|2\rangle$ in its Eigenbasis, then we know that the photon travelled from B_1 to M_3. However, it is not necessary that we measure $|1\rangle$ and $|2\rangle$ along its eigenbasis. We can choose an arbitrary axes we denote by $|\alpha\rangle$ and $|\beta\rangle$. For example, if we are considering the classic stern-Gerlach experiment and replace $|1\rangle$, $|2\rangle$ states with $|+z\rangle$, $|-z\rangle$ states respectively, then their decomposition along horizontal axes may read:

$$|1\rangle = \frac{1}{\sqrt{2}}\big[\, |\alpha\rangle + |\beta\rangle\big], \tag{18}$$

$$|2\rangle = \frac{1}{\sqrt{2}}\big[\, |\alpha\rangle - |\beta\rangle\big], \tag{19}$$

Hence, in some basis, the states $|1\rangle$ and $|2\rangle$ are again in a superposition state such that we can no longer discern what the original path of the photon was. In this basis spanned by $|\alpha\rangle$ and $|\beta\rangle$, we can therefore have interference pattern again. The real central paradox of Quantum erasure is the following: if we didn't measure $|1\rangle$ and $|2\rangle$ along the axis in which they were eigenbasis, there was no interference. However, choosing to measure these pointers states in some arbitrary basis reintroduced interference. But since the photon must choose a particular path through the apparatus, our decision to not measure $|1\rangle$ and $|2\rangle$ is essentially a signal to the photon that travels back in time to remain in interference. This counterintuitive interpretation has been constructively refuted by reputed physicists like Sabine Hossenfelder and Sean Carroll. In particular, the interested reader is referred to Sean Caroll's wonderfully written blog, "The Notorious Delayed Choice Quantum Erasure".

4 Acknowledgements

This experiment was carried alongside my lab partner Daniel Shafi Batla. His contributions in the lab and the rich theoretical discussions I held with him were imperative in helping me write this. I would like to extend my gratitude to him from the bottom of my heart.

References

[1] W. K. Wootters and W. H. Zurek, "Complementarity in the double-slit experiment: Quantum nonseparability and a quantitative statement of bohr's principle," *Physical Review D*, vol. 19, no. 2, p. 473, 1979.