Continuous Charge Distribution

Lecture 9

PH-122

Continuous Charge Distribution

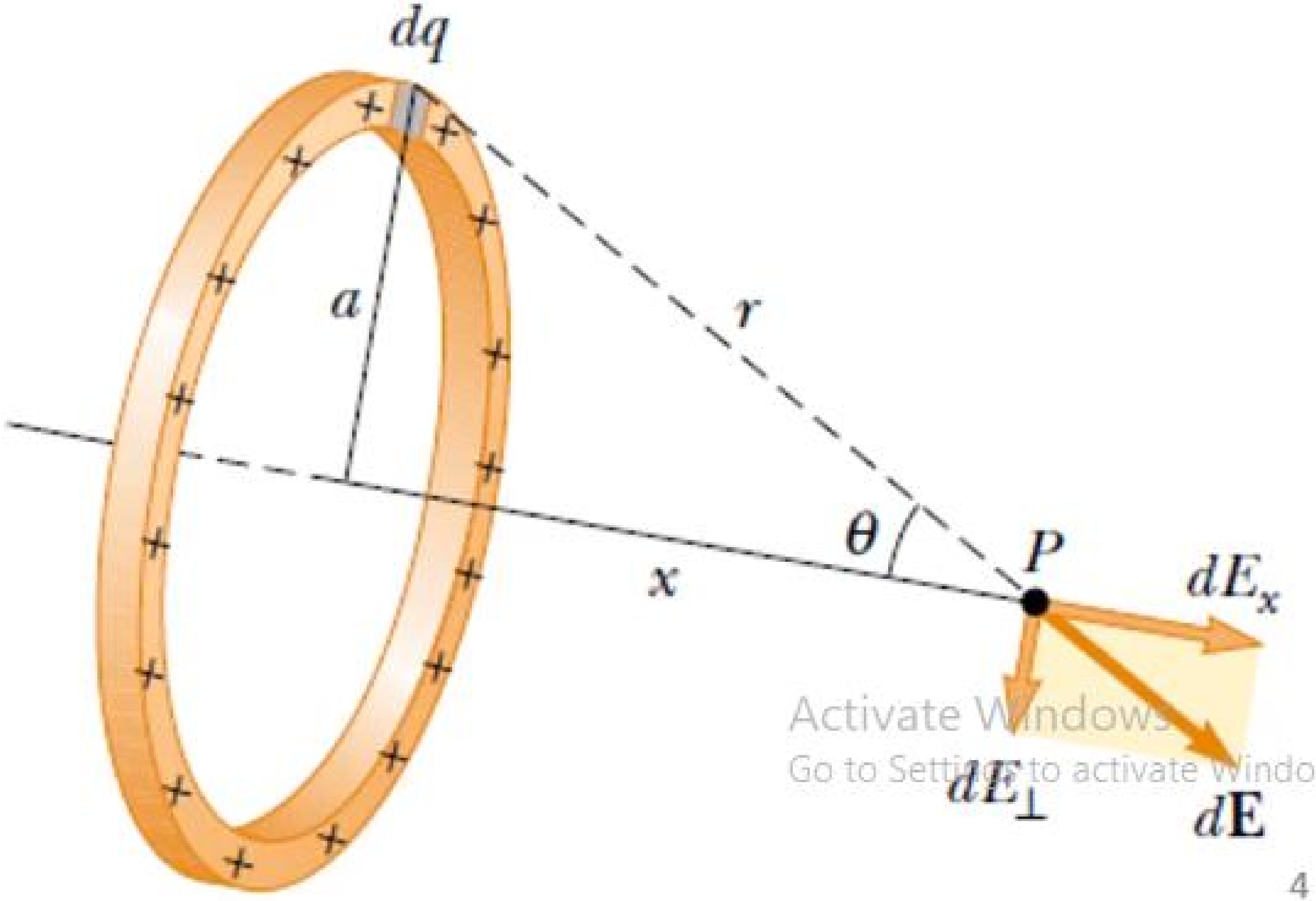
- Charges are quantized; then what is continuous charge distribution?
- An assembly of charges that has a high density i.e the charges are so closely packed that it is not possible to identify each charge in isolation or individually.
- Charge density can be of three different types: linear charge density (λ), surface charge density (σ) and volume charge density (ρ).

Linear Charge Distribution

- Linear charge density is defined as $\lambda = \frac{q}{l}$
- $\frac{q}{l} = \frac{\mathrm{dq}}{\mathrm{dl}}$ (Uniform charge distribution).
- $\frac{q}{l} \neq \frac{\mathrm{dq}}{\mathrm{dl}}$ (non-uniform charge distribution).
- Electric field due to an element of distribution $dE = k(\frac{dq}{r^2})$
- Electric field due to entire distribution $E = \int k(\frac{\lambda dl}{r^2})$

Example

- A ring of radius *a* carries a uniformly distributed positive total charge *Q*. Calculate the electric field due to the ring at a point *P* lying a distance *x* from its center along the central axis perpendicular to the plane of the ring (see Fig.)



Example

$$dE = k_e \frac{dq}{r^2}$$

But according to figure only x- components add up to give the electric field at point P.

$$dE_x = dE \cos\theta$$

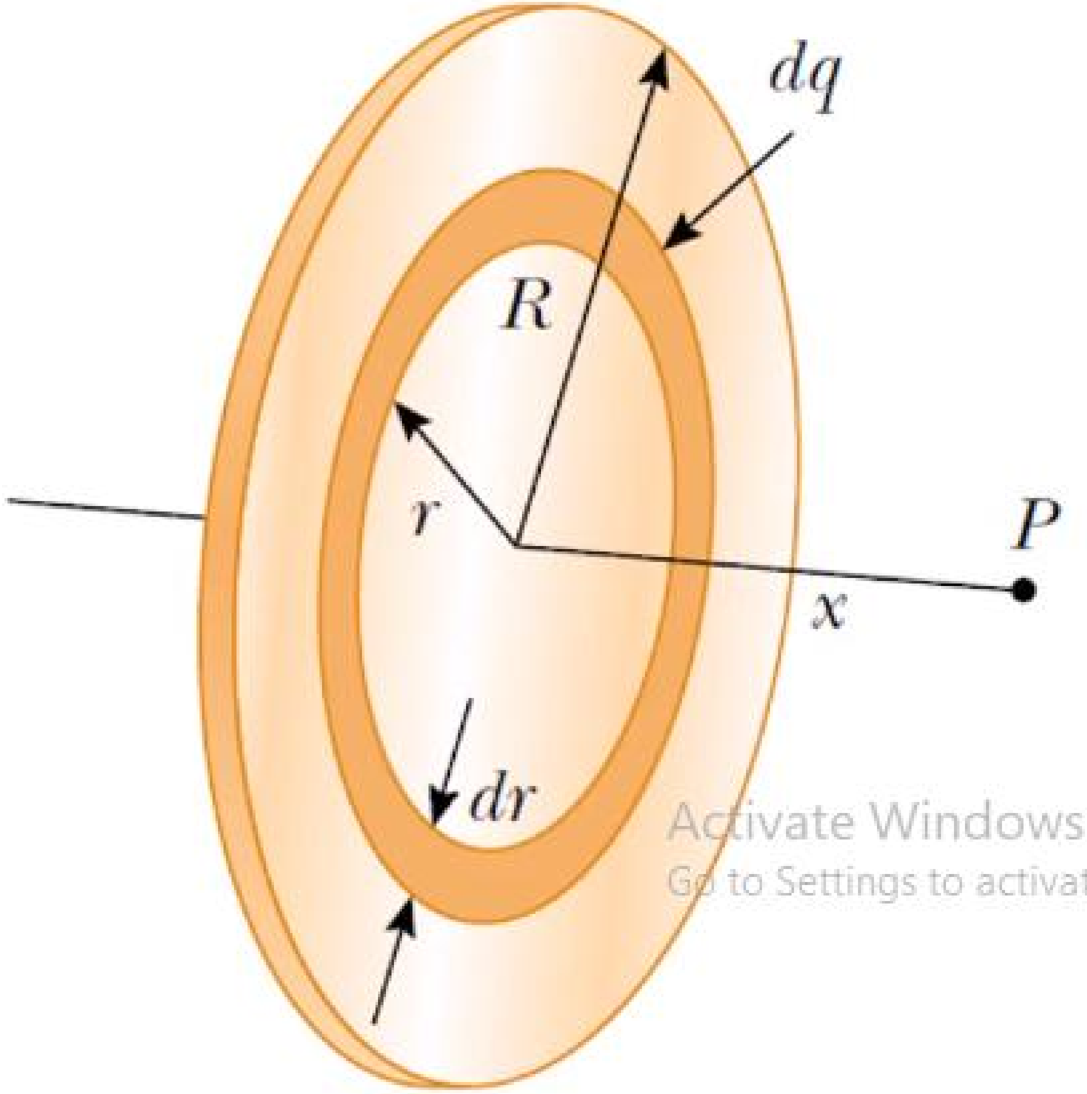
From figure $r = (x^2 + a^2)^{1/2}$ and $\cos\theta = x/r$

$$dE_x = dE\cos\theta = \left(k_e \frac{dq}{r^2}\right)\frac{x}{r} = \boxed{\frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq}$$

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq = \frac{k_e x}{(x^2 + a^2)^{3/2}} \int dq = \frac{k_e x}{(x^2 + a^2)^{3/2}}\, Q$$

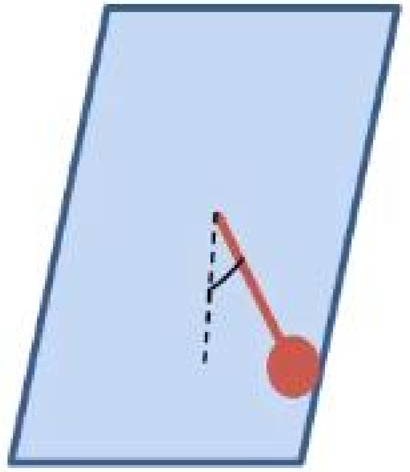
Practice Problem

- A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk (see Fig.).



Problem

- A small sphere of mass 1.12kg carries a charge of 19.7nC, hangs in Earth's gravitational field force by a silk thread that makes an angle of 27.4 degrees with a large uniformly charged non-conducting sheet (see Fig.). Calculate the uniform charge density.



Solution

$\sum F_x = T\sin\theta - F_e = 0$ $\sum F_y = T\cos\theta - mg = 0$

By definition: $F_e = q_0 E$

If the sheet is large it can be visualized as an Infinite sheet of charge so that the electric field due to it is given by: $E = \sigma/2\epsilon_0$.

Therefore, $tan\theta = \dfrac{q_0\sigma}{2\epsilon_0 mg}$

We can use this and find the value of surface charge density $\sigma = 5.11nC/m^2$