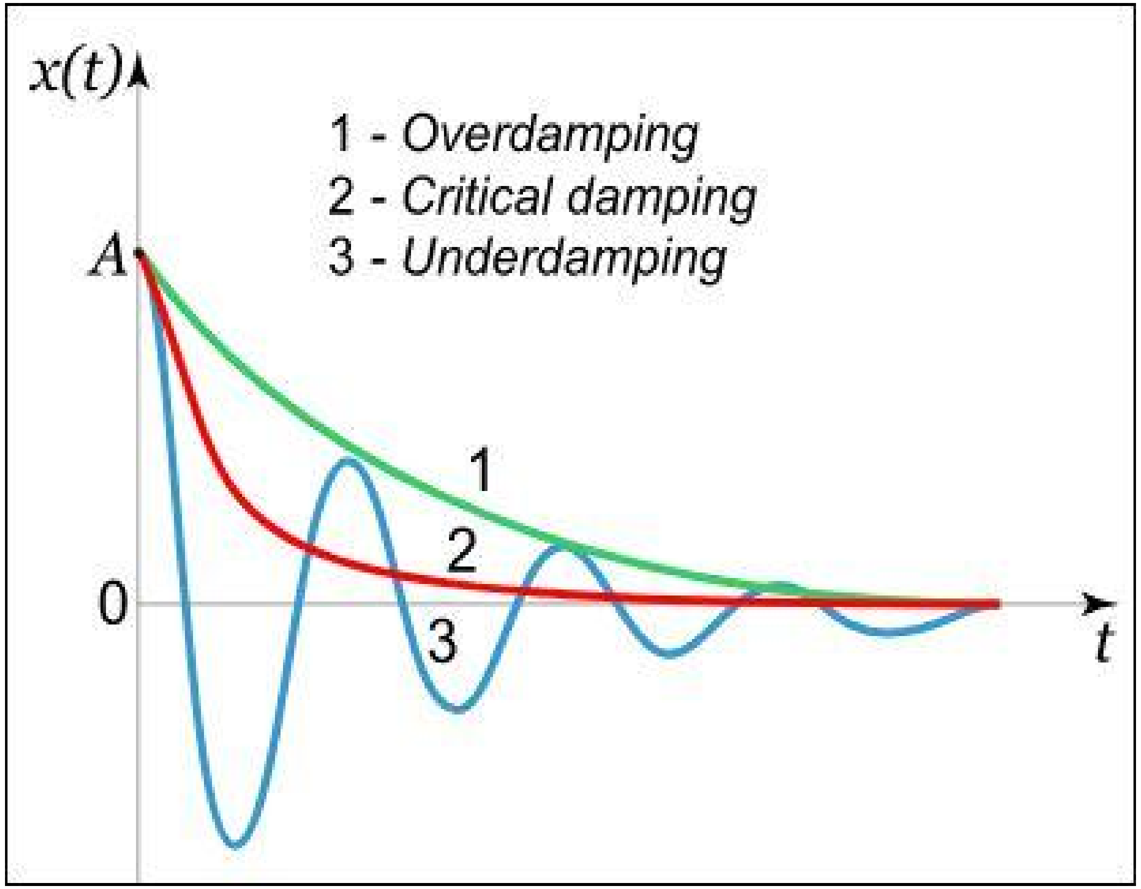
Underdamped motion: b/2m$<\ \omega_o$
Overdamped b/2m $>\omega_o$
Critically damped b/2m= ω_o

There is no periodic motion in over
Damped and critically damped oscillations



- **Driven oscillator** :
- If the external time dependent force is present the harmonic oscillator is described as a driven oscillator.

- Example: First a mass hanging on a spring, going up and down. When the object is moving upwards, the driving force is also pointed upwards, and the other way around. The object is making a sinusoidal movement and therefore the driving force is mostly a sinusoidal force.

- Suppose in addition to spring and damping forces the mass m is acted on by a periodic force.
- $F(t) = Fo \cos(\omega_d t)$
- Where Fo is a constant and ω_d is the angular velocity of the driving force.
- The differential equation for this motion is
- $m\, d^2x/dt^2 + b\, dx/dt + kx = Fo \cos(\omega_d t)$
- After a sufficiently long time this driving force is performing work on the mass at the same rate at which energy is dissipated by friction. In this case we reach what is called the steady state condition and x(t) settles down to a simple harmonic oscillation defined by
- $x(t) = A \cos(\omega_d t - \phi_o)$

Equation for Forced (Driven) Oscillations

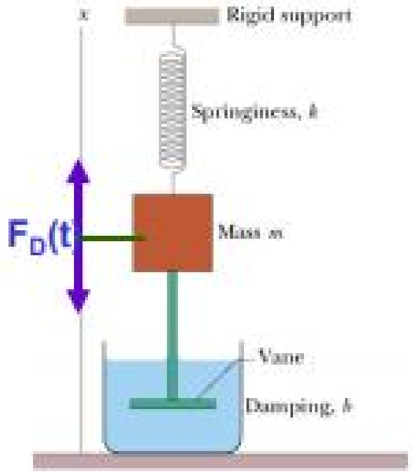
ω_0 = natural frequency $\quad \omega_0 = \sqrt{\dfrac{k}{m}}$

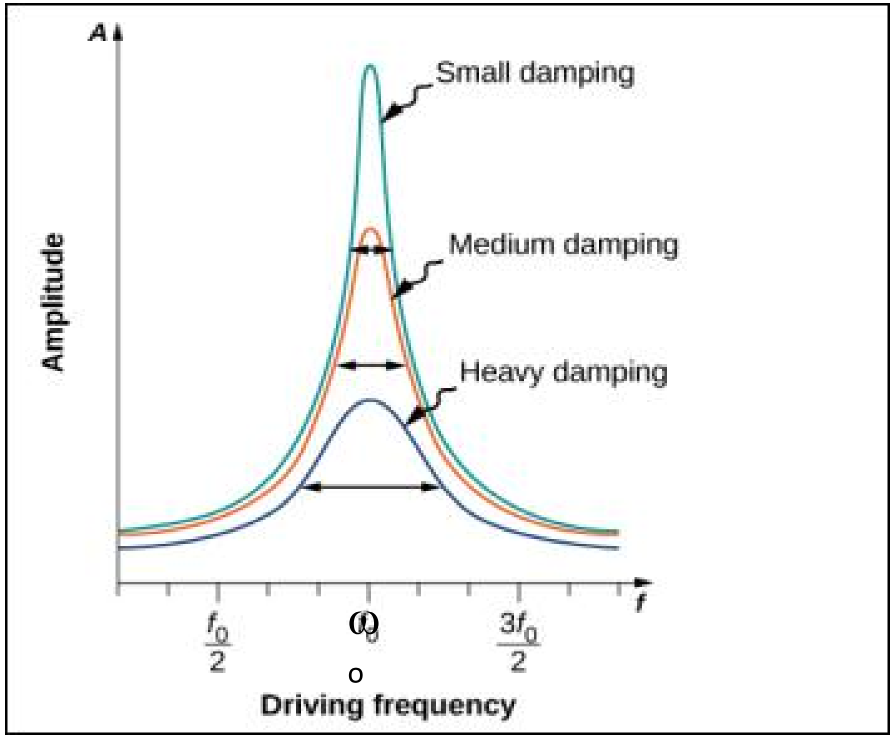
ω_D = driving frequency of external force

External driving force function:

$$F_D(t) = F_0 \cos(\omega_D t + \phi')$$

$$F_{net} = F_D(t) \ -b\,\frac{dx(t)}{dt} \ - k\,x(t) \ = \ m\frac{d^2x(t)}{dt^2}$$





Steady state amplitude of a driven harmonic oscillator as a function of driving Frequency ω_d

- Important points from graph:
- Amplitude versus driving frequency
- Steady state oscillation amplitude depends on ω_d
- For larger b …damping is large and peak is broad and low
- For small value of b… damping is small and peak is high and centered closer to ω_d
- If b=0, no damping, amplitude would grow without bound as ω_d approaches to ω_o
- ω_o is natural frequency and dramatic increase in amplitude near this frequency is called resonance.
- As damping becomes weaker …. resonance sharpens & amplitude at resonance increases.

- **Resonance:** Resonance occurs when an oscillating system is driven (made to oscillate from an outside source) at a frequency which is the same as its own natural frequency.
- Amplitude is greater at resonant frequency.
- Examples: (i) machines like automobile,
- ii) marching of soldiers on a bridge
- iii) swing
- iv) guitar
- v) pendulum
- vi)breaking of glass

- **Mechanical resonance** is the tendency of a mechanical system to respond at greater amplitude when the frequency of its oscillations matches the system's natural frequency of vibration than it does at other frequencies.
- Avoiding resonance disasters is a major concern in every building and bridge’s construction.

- **Optical resonance:** LASER produced by optical resonator
- **Acoustic Resonance**
- Acoustic resonance is a phenomenon in which an acoustic system amplifies sound waves whose frequency matches one of its own natural frequencies of vibration. Acoustic resonance is an important consideration for instrument builders as most acoustic instruments such as the length of tube in a flute, the strings and body of a violin and the shape of a drum membrane use resonators. Acoustic resonance is also important for hearing.

- **Electrical Resonance**
- In a circuit when the inductive reactance and the capacitive reactance are equal in magnitude electrical resonance occurs. The resonant frequency in an LC circuit is given by the formula

- $\omega = 1/\sqrt{LC}$

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