Biot-savart law and Ampere’s law

Applied Physics

PH-122

Biot-savart Law

- The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law. This law asserts that the contribution to the field produced by a current-length element at a point P located a distance r from the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{\mathrm{r}}}{r^2} \qquad \text{(Biot–Savart law)}$$

$\mu_0 =$ permiability of free space
$= 4\pi \times 10e\text{-}7$ h/m

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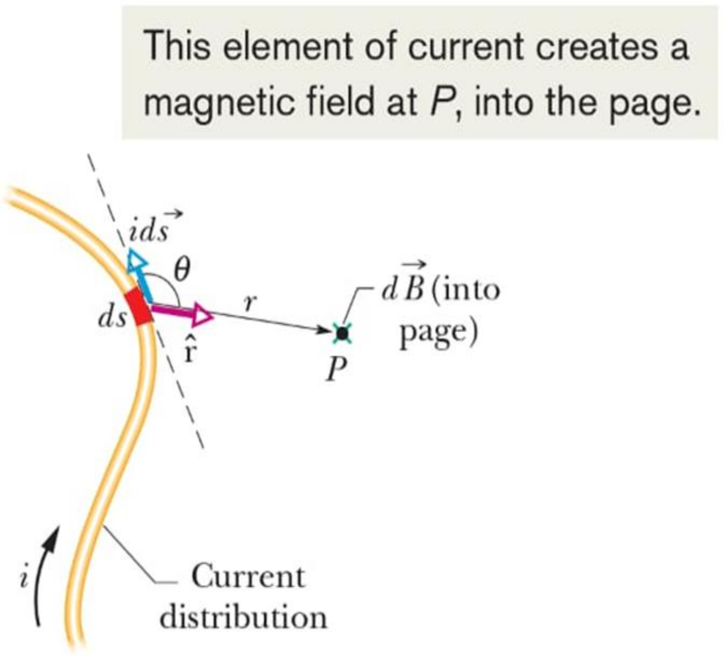
- Here r is a unit vector that points from the element toward P.

Calculating the Magnetic Field Due to a Current

- Figure shows a wire of arbitrary shape carrying a current *i*.
- The magnetic field at a nearby point *P*.
- First divide the wire into differential elements *ds* and then define for each element a length *ds*.
- We can then define a differential *current-length element* to be *ids*.
- Now we calculate the field *dB* produced at *P* by a typical current-length element



Continued

- The magnitude of the field produced at point P at distance r by a current-length element i turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

μ_0 = permiability of free space

ϴ is angle between ds and vector r

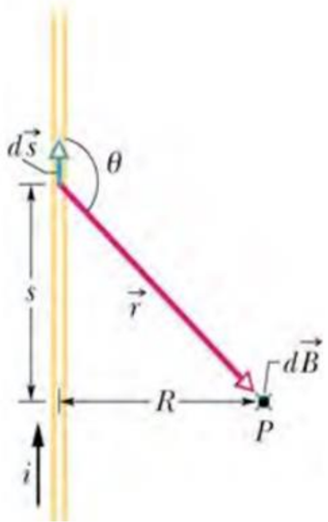
- we can calculate the net field at *P* by summing, via integration, the
- contributions from all the current-length elements.
- The direction shown as being into the page in Figure, is that of the cross product .We can the[illegible] as

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} \quad \text{(Biot–Savart law)}$$

Magnetic Field Due to a Current in a Long Straight Wire

- For a long straight wire carrying a current i, the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance R from the wire

$$B = \frac{\mu_0 i}{2\pi R}$$



- An infinitely long straight wire carries a current i.
- The magnetic field generated at a point located at a perpendicular distance R from the wire.

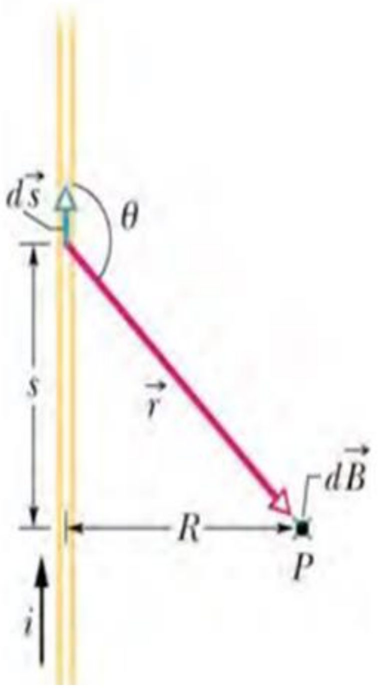
Continued

- Choose an element ds as shown
- Biot-Savart Law: dB points INTO the page
- Integrate over all such elements

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi}\frac{ids(r\sin\theta)}{r^3}$$

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{ds(r\sin\theta)}{r^3}$$



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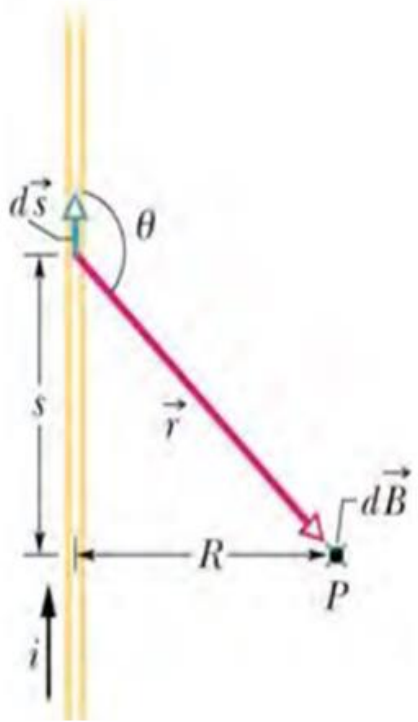
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3} \qquad dB = \frac{\mu_0}{4\pi}\frac{i ds(r\sin\theta)}{r^3}$$

$$\sin\theta = R/r \qquad r = (s^2 + R^2)^{1/2}$$

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{ds(r\sin\theta)}{r^3} = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{R ds}{\left(s^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi}\int_{0}^{\infty}\frac{R ds}{\left(s^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 i R}{2\pi}\left[\frac{s}{R^2\left(s^2 + R^2\right)^{1/2}}\right]_0^{\infty} \quad \boxed{= \frac{\mu_0 i}{2\pi R}}$$



Magnetic Field Due to a Current in a Circular Arc of Wire

- The magnitude of the magnetic field at the center of a circular arc, of radius R and central angle φ (in radians), carrying current i

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

- A circular arc of wire of radius R carries a current i.
- Magnetic field at the center of the loop is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\overrightarrow{ds}\times\vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi}\frac{idsR}{R^3} = \frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2}$$

$$B = \frac{\mu_0}{4\pi}\int\frac{id\phi}{R} = \frac{\mu_0 i\Phi}{4\pi R}$$

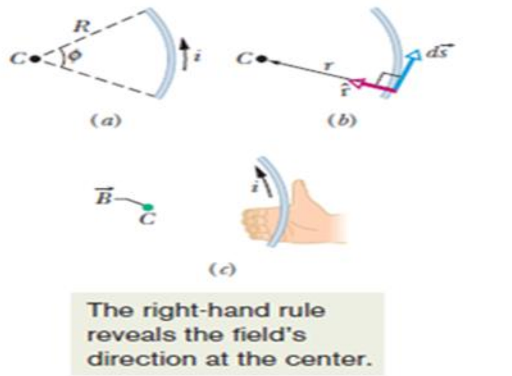


Figure 29-7 (*a*) A wire in the shape of a circular arc with center *C* carries current *i*. (*b*) For any element of wire along the arc, the angle between the directions of $d\vec{s}$ and $\hat{r}$ is 90°. (*c*) Determining the direction of the magnetic field at the center *C* due to the current in the wire; the field is out of the page, in the direction of the fingertips, as

Problem

- The wire in Fig. carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field (magnitude and direction) does the current produce at C?

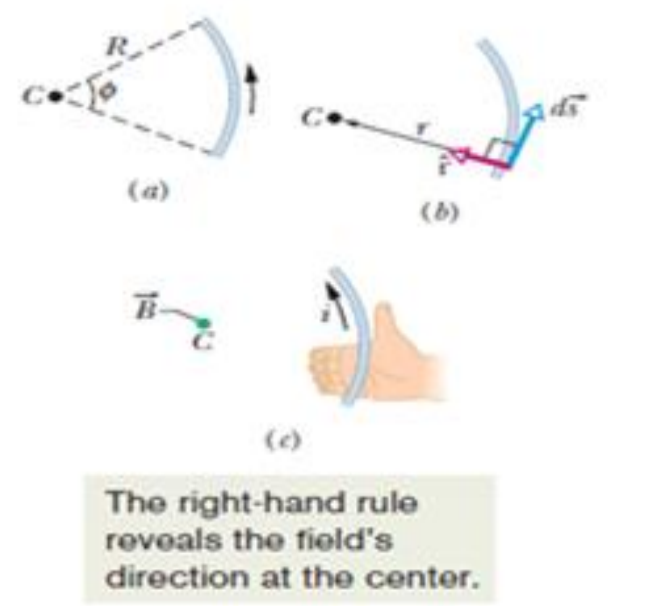


Figure (*a*) A wire in the shape of a circular arc with center *C* carries current *i*. (*b*) For any element of wire along the arc, the angle between the directions of $d\vec{s}$ and $\hat{r}$ is 90°. (*c*) Determining the direction of the magnetic field at the center *C* due to the current in the wire; the field is out of the page, in the direction of the fingertips, as

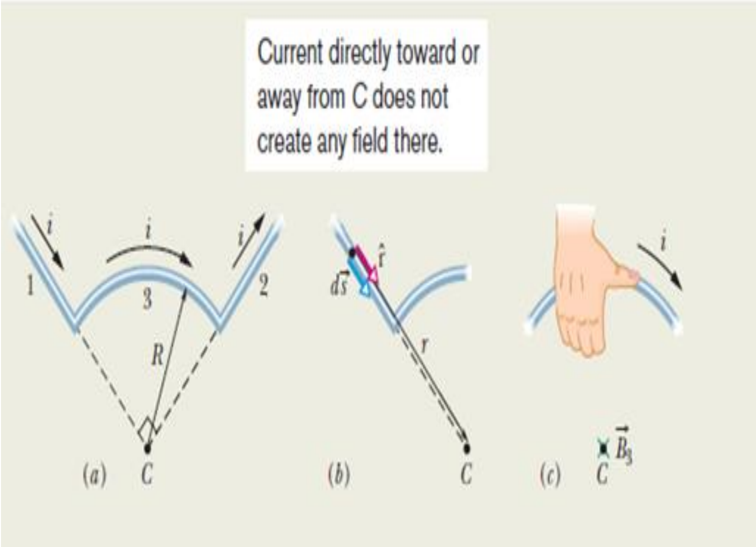
Continued

- Section 1: angle between ds and r is zero

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at *C*:

$$B_1 = 0.$$



- Section 2:The same situation prevails in straight section 2, where
- the angle between ds and r for any current-length element
- is 180°.Thus,

$$B_2 = 0.$$

Continued

- Section 3:Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc. Here the central angle f of the arc is $\pi/2$ rad. Thus

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

- Net field:

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \qquad \text{(Answer)}$$

Ampere’s law

- Ampere's Law states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's law).}$$

- The line integral in this equation is evaluated around a closed loop called an Amperian loop. The current i on the right side is the net current encircled by the loop

Continued

- The figure shows cross sections of three long straight wires that carry currents i_1, i_2, and i_3 either directly into or directly out of the page.
- An arbitrary Amperian loop lying in the plane of the page encircles two of the currents but not the third.
- To apply Ampere's law, we mentally divide the loop into differential vector element dS that are everywhere directed along the tangent to the loop in the direction of integration.

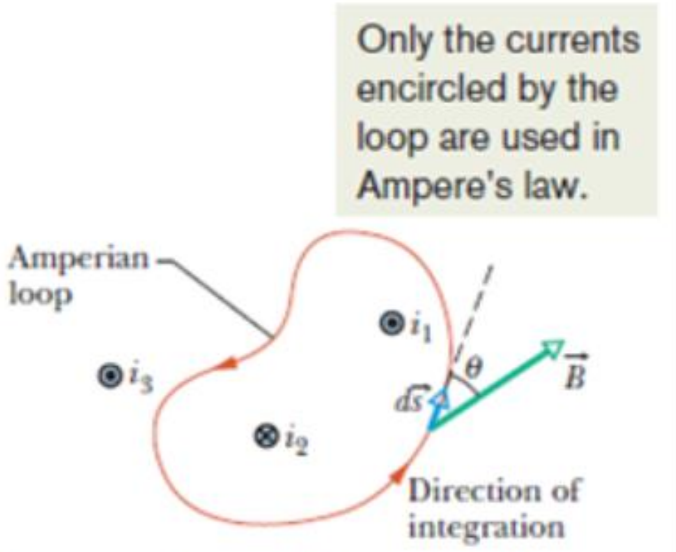


Figure Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

Continued

- Assume that at the location of the element shown in Fig. the net magnetic field due to the three currents is B because the wires are perpendicular to the page.
- In fig B is arbitrarily drawn at an angle ϴ in the direction of dS.
- The scalar product of B.dS on the left side is equal to BcosϴdS. So amperes law states that

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = \mu_0 i_{enc},$$

- we use the following curled–straight right-hand rule to assign a plus sign or a minus sign to each of the currents that make up the net encircled current i_{enc}:

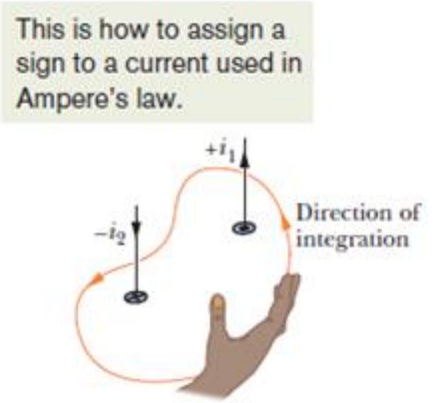


Figure: A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29-12.

Continued

- we apply the curled–straight right-hand rule for Ampere’s law to the situation of Fig. With the indicated counterclockwise direction of integration, the net current encircled by the loop is

$$i_{net} = i_2 - i_1$$

- Current i_3 is not encircled by the loop.so we can then rewrite Eq.

$$\oint B \cos \theta \, ds = \mu_0(i_1 - i_2).$$

Magnetic Field Outside a Long Straight Wire with Current

- Figure shows a long straight wire that carries current i directly out of the page. We know that the magnetic field produced by the current has the same magnitude at all points that are the same distance r from the wire; that is, the field has cylindrical symmetry about the wire.

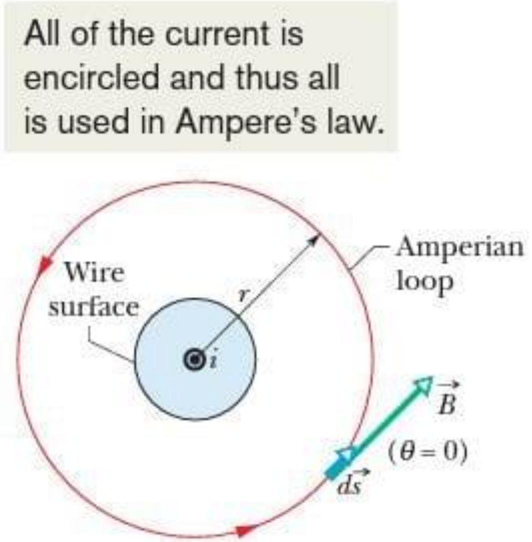


Figure Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

Continued

- Now we simplify equation

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = \mu_0 i_{enc}.$$

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r).$$

- Our right-hand rule gives us a plus sign for the current of Fig..The equation is now

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straight wire).}$$

Magnetic Field Inside a Long Straight Wire with Current

- Figure shows the cross section of a long straight wire of radius R that carries a uniformly distributed current i directly out of the page. Because the current is uniformly distributed over a cross section of the wire, the magnetic field produced by the current must be cylindrically symmet

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r).$$

- i_{enc} encircled by the loopis proportional to the area encircled by the loop;

$$i_{\text{enc}} = i \frac{\pi r^2}{\pi R^2}.$$

Only the current encircled by the loop is used in Ampere's law.

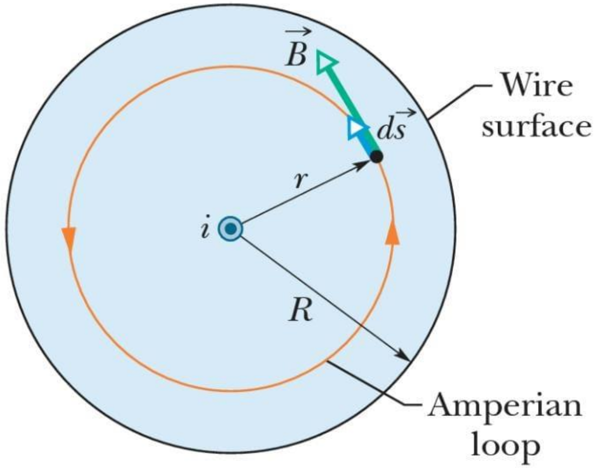


Figure Using Ampere's law to find the magnetic field that a current i produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Continued

- Our right-hand rule tells us that i_{enc} gets a plus sign. Then Ampere's law gives us

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \qquad \text{(inside straight wire).}$$

If you have any questions regarding this lecture, please ask in the live session

Thank you