Electric Potential & Electrostatic Potential Energy

Lecture 10

PH-122

Electric Potential & Potential Difference

- Every point in an electric field has acquired certain level of energy known as it's 'Electric Potential.'
- Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.
- The work done to move a unit positive charge from one point to another in an electric field is known as potential difference.

$$\Delta V = work/charge$$

$$\Delta V = \Delta U/q_0$$

Electric Potential & Potential Difference

$$\mathbf{F}\cdot d\mathbf{s} = q_0\mathbf{E}\cdot d\mathbf{s}.$$

$$\Delta U = -q_0\int_A^B \mathbf{E}\cdot d\mathbf{s}$$

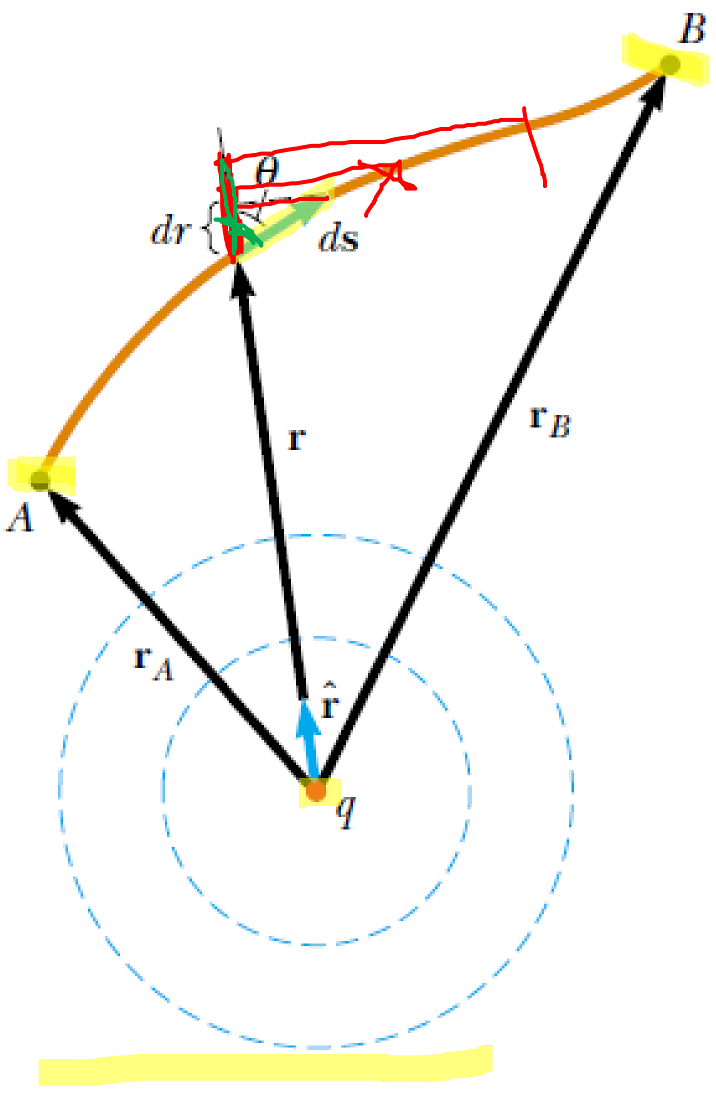
$$\boxed{\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E}\cdot d\mathbf{s}}$$

$$\mathbf{E}\cdot d\mathbf{s} = k_e\frac{q}{r^2}\,\hat{\mathbf{r}}\cdot d\mathbf{s} \qquad \hat{\mathbf{r}}\cdot d\mathbf{s} = ds\cos\theta$$

Any displacement *ds* along the path from point *A* to point *B* produces a change *dr* in the magnitude of r, the position vector of the point relative to the charge creating the field. This means

$dsCos\theta = dr.$

$$\mathbf{E}\cdot d\mathbf{s} = (k_eq/r^2)\,dr$$



Electric Potential & Potential Energy

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

$$\int \frac{1}{r^2} = -\frac{1}{r}$$

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

$$\mathbf{E} \cdot d\mathbf{s} = (k_e q / r^2)\, dr$$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V = k_e \frac{q}{r}$$

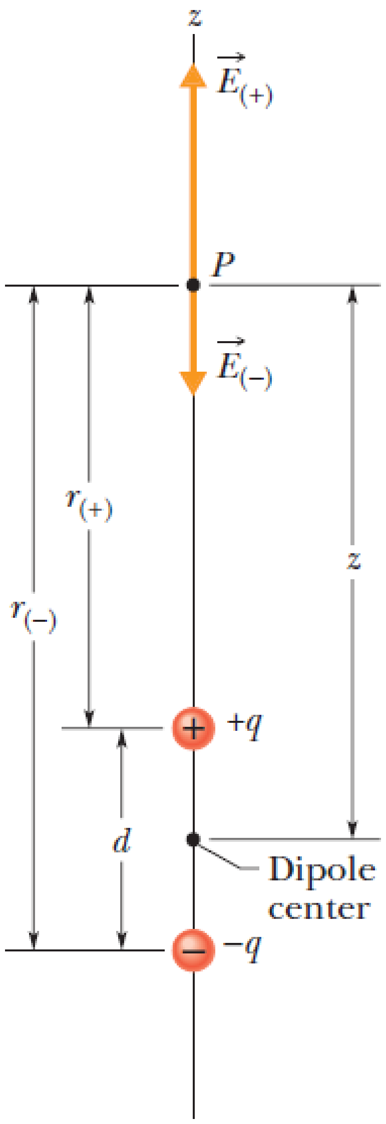
$$V = k_e \sum_i \frac{q_i}{r_i}$$

Problem

- Find electric potential due to an electric dipole at some point ‘P.’

Using the same setup as we did earlier for electric field due to dipole.

Hint: Remember electric potential is scalar quantity and hence sign of Charge must be included in calculation.

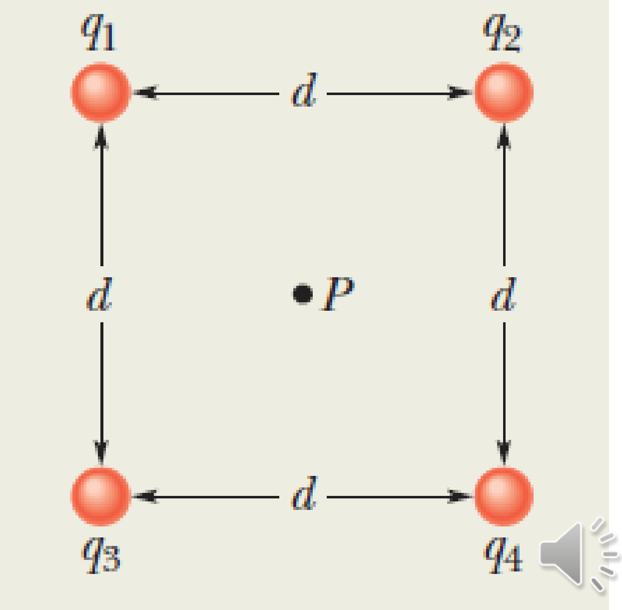


Problem

What is the electric potential at point P, located at the center of the square of charged particles shown in Fig. 24-11a? The distance d is 1.3 m, and the charges are

$$q_1 = +12\text{ nC}, \qquad q_3 = +31\text{ nC},$$

$$q_2 = -24\text{ nC}, \qquad q_4 = +17\text{ nC}.$$

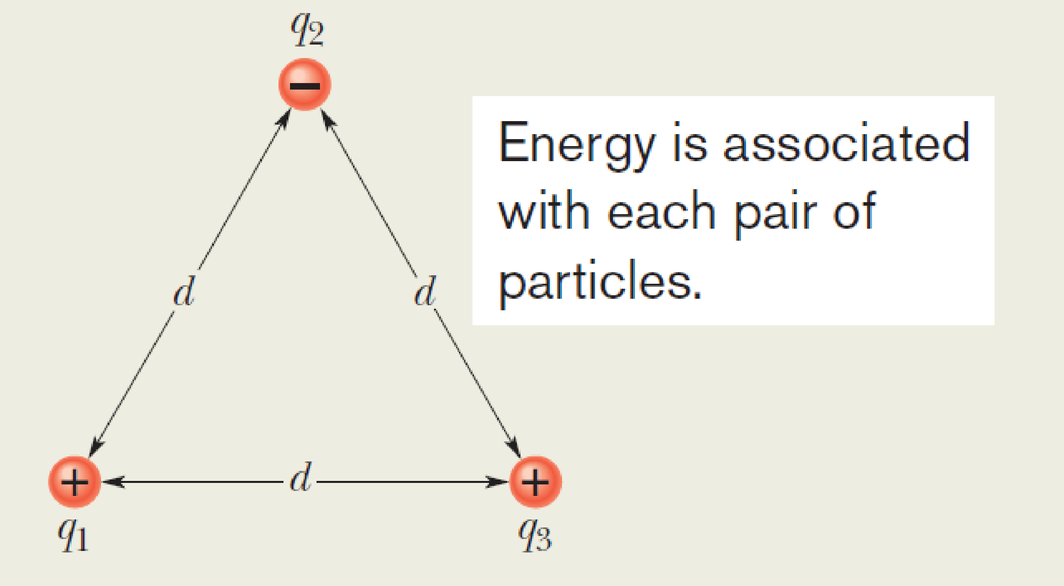


Problem

Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.



Solution

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right)$$

$$= -\frac{10q^2}{4\pi\varepsilon_0 d}$$

$$= -\frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9} \text{ C})^2}{0.12 \text{ m}}$$

$$= -1.7 \times 10^{-2} \text{ J} = -17 \text{ mJ}. \qquad \text{(Answer)}$$

Relationship between Electric Potential & Electric Field

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

$$dV = -\mathbf{E} \cdot d\mathbf{s}$$

If the electric field has only one component E_x, then $\mathbf{E} \cdot d\mathbf{s} = E_x\, dx$.

$$dV = -E_x\, dx,$$

$$E_x = -\frac{dV}{dx}$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

$$E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Combining these we get

$$\vec{E} = -\vec{\nabla} V$$

Problem

•35 The electric potential at points in an xy plane is given by $V = (2.0\ \text{V/m}^2)x^2 - (3.0\ \text{V/m}^2)y^2$. In unit-vector notation, what is the electric field at the point (3.0 m, 2.0 m)?

••37 SSM What is the magnitude of the electric field at the point $(3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})$ m if the electric potential in the region is given by $V = 2.00xyz^2$, where V is in volts and coordinates x, y, and z are in meters?