Oscillation and Simple Harmonic Motion

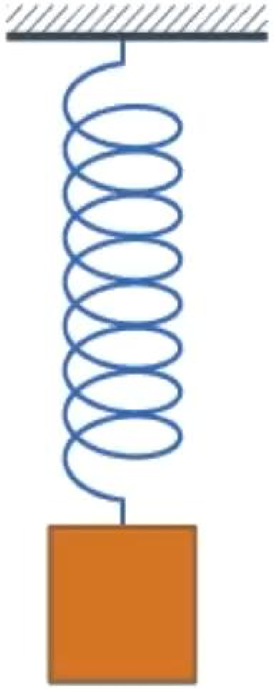
PH-122

Applied physics

Periodic motion

Repeats its value after a regular interval of time.

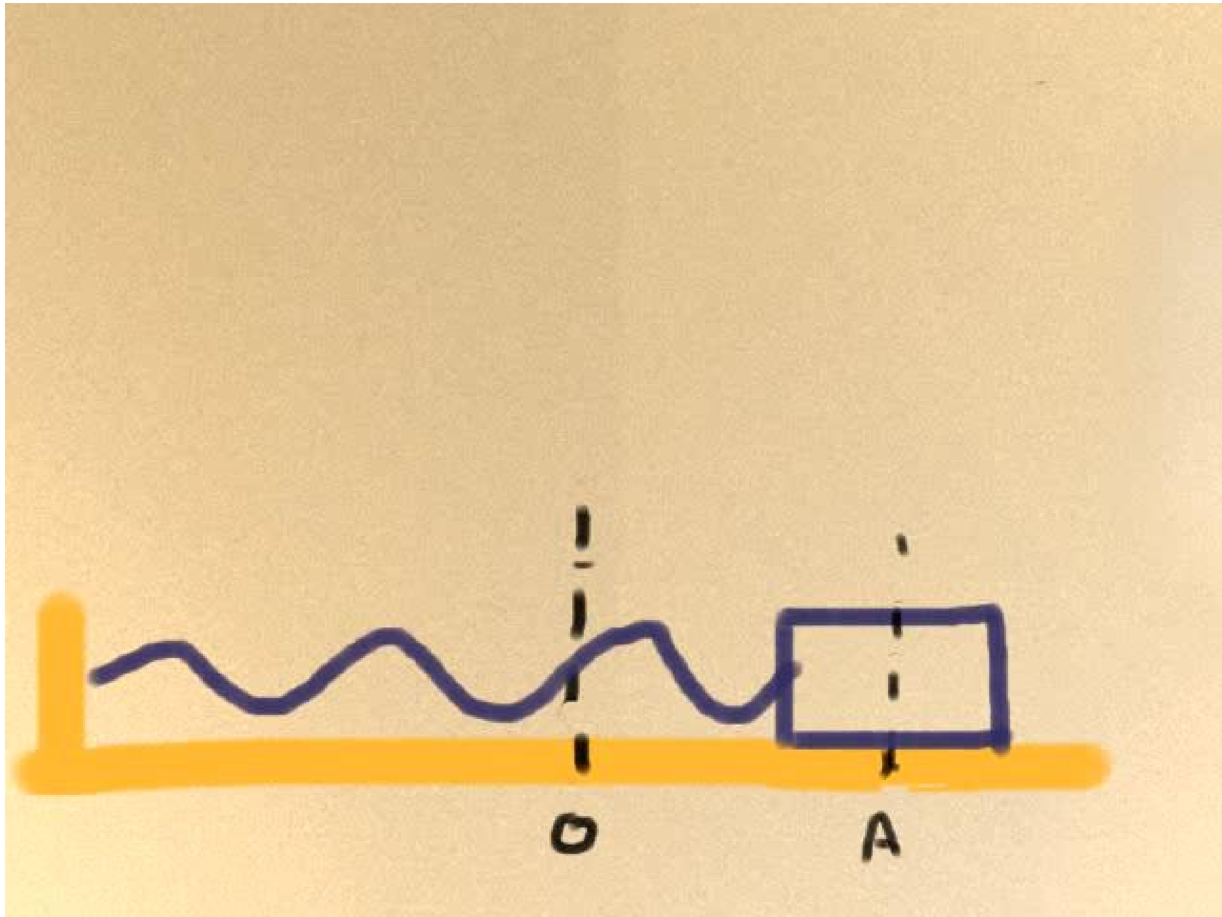
- Examples of periodic motions are
  1. motion of a pendulum,
  2. the motion of a spring,
  3. the vibration of a guitar string,
  4. the rotation of the Earth over its axis



Oscillatory Motion

- The periodic motion in which there is existence of a restoring force and the body moves along the same path to and fro about a definite point called equilibrium position(or mean position), is called oscillatory motion.

- **Types of oscillation**: free oscillation, damped oscillation, forced oscillation.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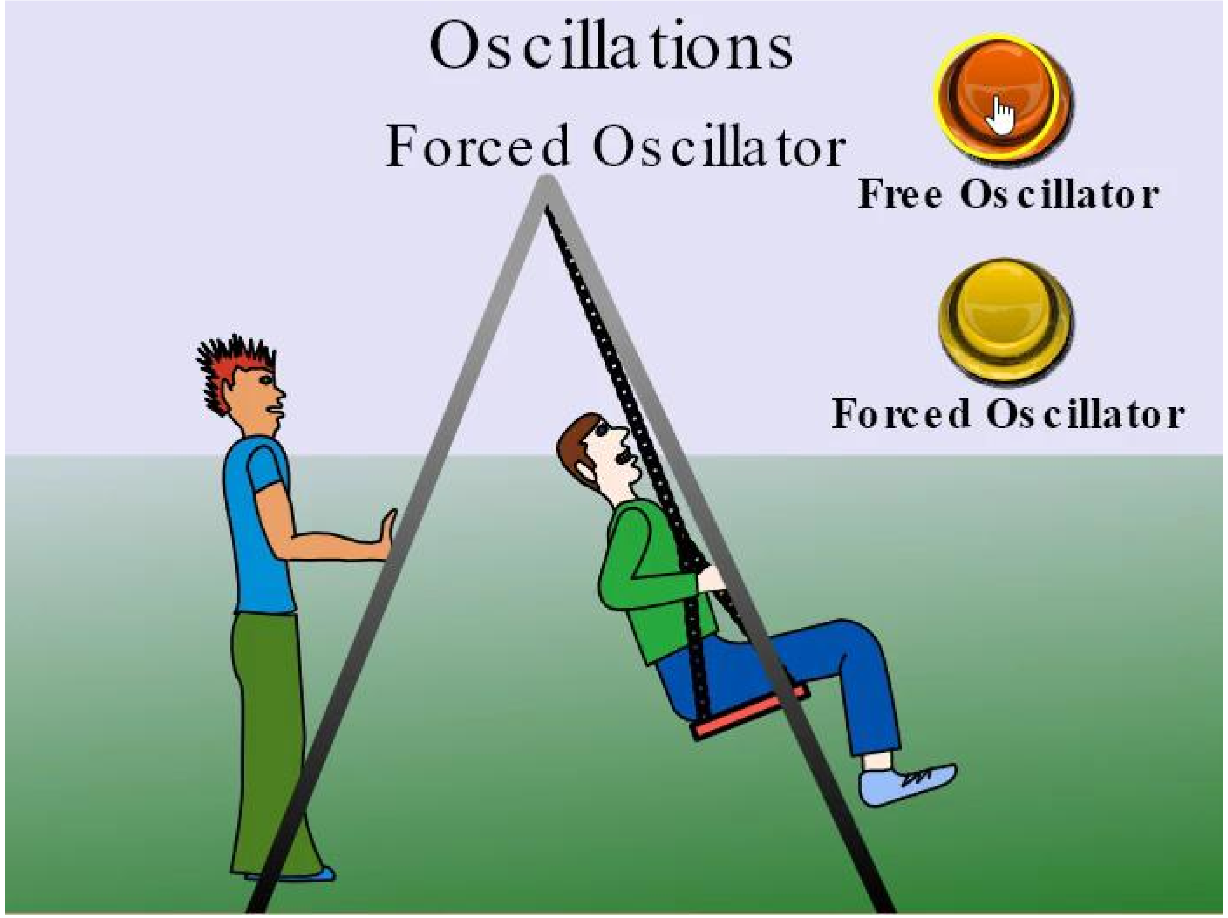
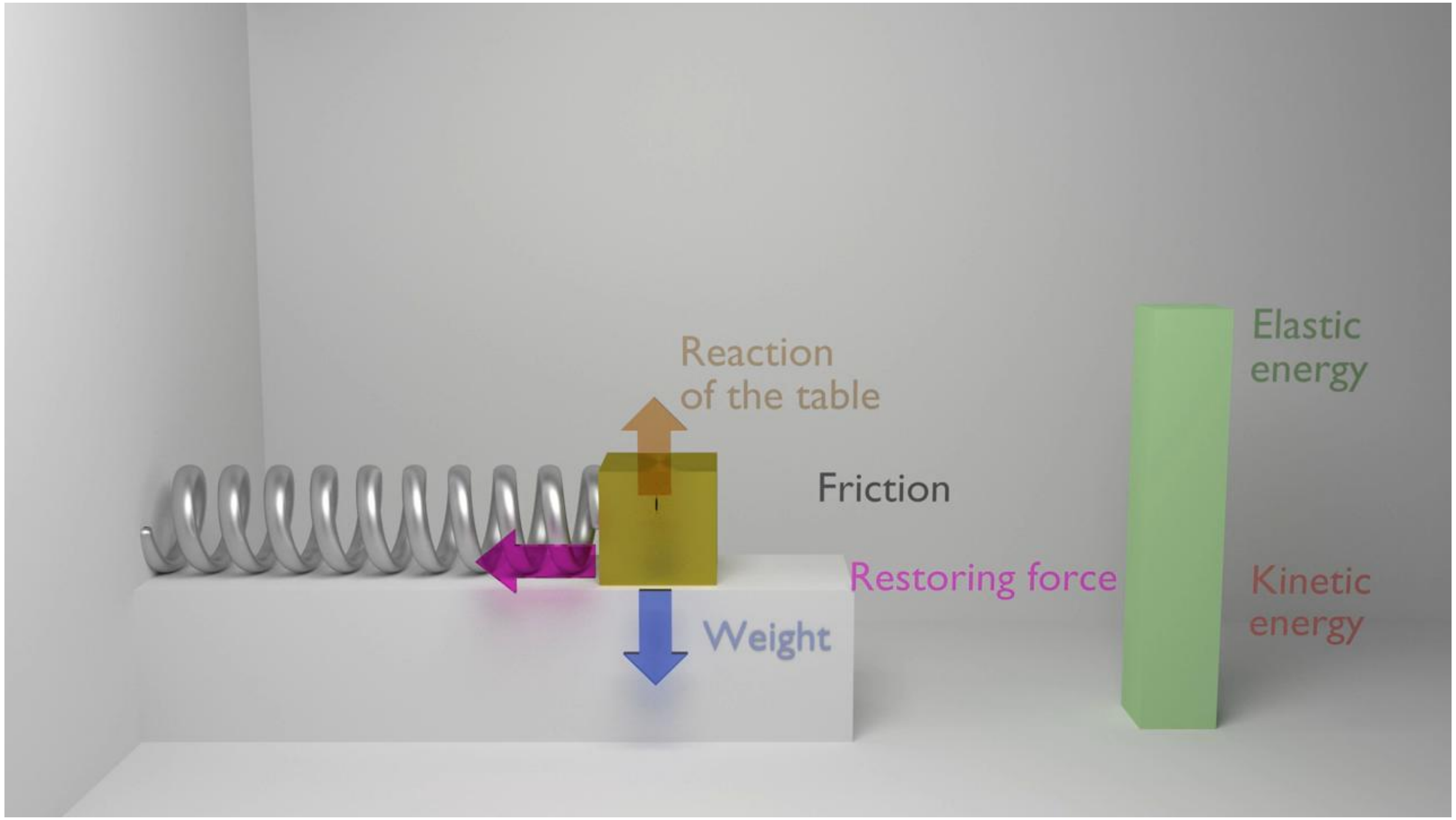
Types of Oscillations

- **Free oscillations:** No resistive force like friction , air, fluid resistance
  - Its amplitude remains constant with time and no energy is lost to the surroundings.
- **Damped oscillations**: Oscillations in the presence of frictional forces are called damped oscillations.
- **Forced Oscillations:** Oscillations in the presence of an external forces


Oscillations
Forced Oscillator
Free Oscillator
Forced Oscillator


Reaction
of the table
Friction
Restoring force
Weight
Elastic
energy
Kinetic
energy

Simple Harmonic Motion

It is the simplest type of oscillatory motion.

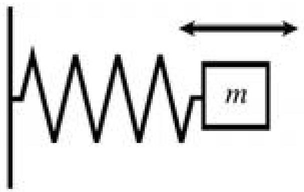
Acceleration α -x

Conditions:

- If F_s is the restoring force on the oscillator when its displacement from the equilibrium position is x,

  then $F_s \propto -x$,
- -ve sign shows that the direction of restoring force is opposite to the displacement of object.

  $F_s = -kx$ (Hooke's law)........(1)
- F_s is restoring force and always directed toward equilibrium position,
- k is spring constant and x is displacement.
- i) If x is positive then Fs is directed to the left, ii) if x=0 then Fs is zero (neither compressed nor stretched)
- iii) If x is negative then Fs is directed to the right.



Spring mass system

Spring mass system

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- If F_s is the only force acting on the system, the system is called a simple harmonic oscillator and it undergoes simple harmonic motion: sine or cosine waves about the equilibrium point, with a constant amplitude and a constant frequency .
- $X(t) = A\cos(\omega t + \phi)$……(2) or $X(t) = A\sin(\omega t + \phi)$
- A is the amplitude of oscillation,
- ϕ is the phase constant
- ω= angular frequency, It is related to the period of oscillation T by the formula: $\omega = 2\pi/T$

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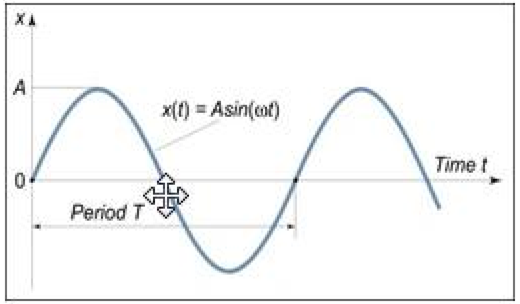
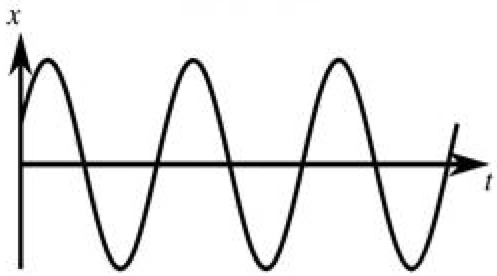


Sine wave

x(t) = A cos (ωt +ϕ)



Cosine wave

Position: X (t) = A cos ($\omega t + \phi$)……(2)

Velocity and Acceleration:

Differentiate eq. 2 dx(t) /dt = $-A\omega \sin(\omega t + \phi)$ = v(t)……….(3)

Differentiate eq. 3 dv(t)/dt = $-A\omega^2 \cos(\omega t + \phi)$ = a(t)………(4)

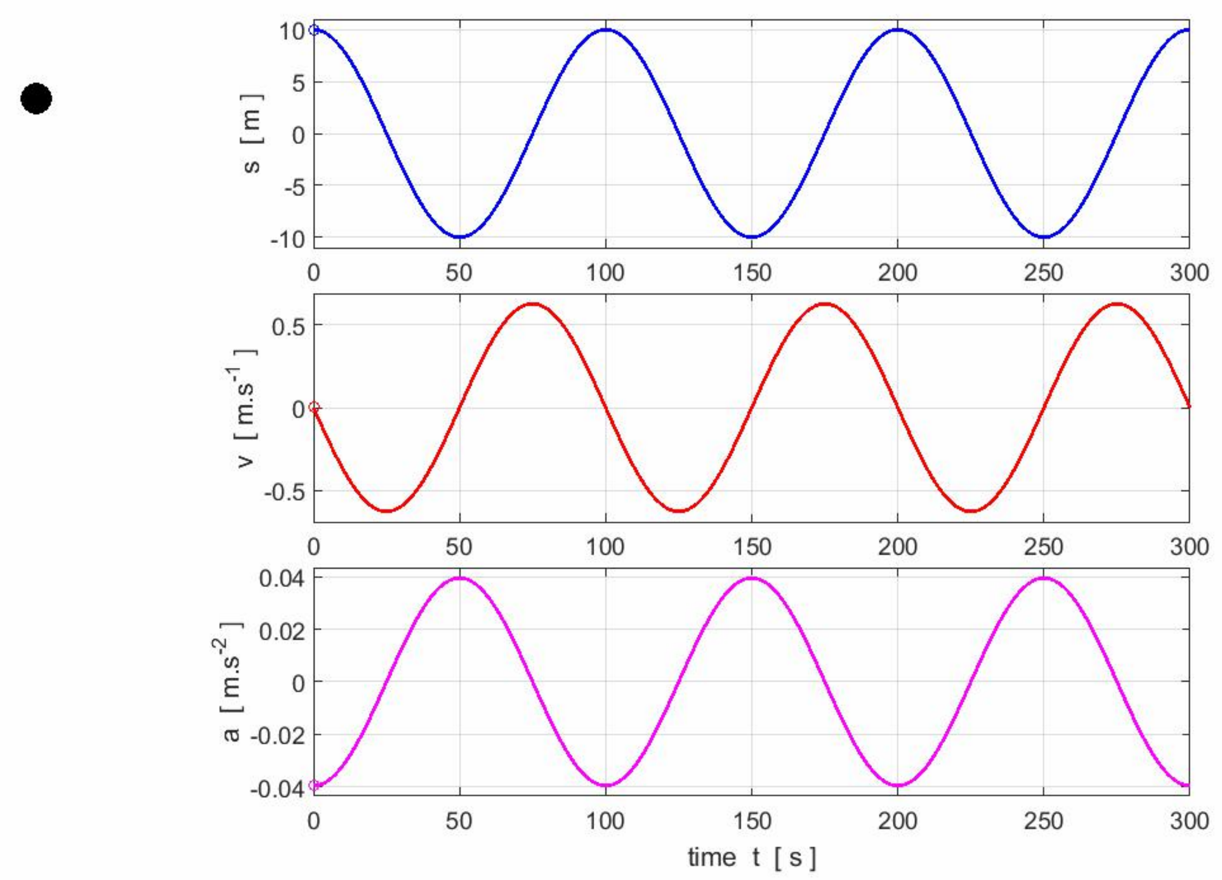
Vmax. = $A\omega$

a max. = $A\omega^2$

$X(t) = A\cos(\omega t + \phi)$

$V(t) = -A\omega \sin(\omega t + \phi)$

$a(t) = -A\omega^2 \cos(\omega t + \phi)$



Relationship for T, f, ω

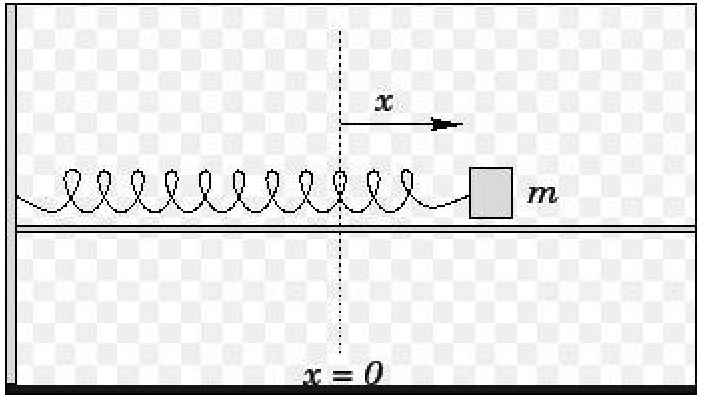
- cosine repeats after every multiple of 2π
- $\Delta(\omega t + \phi) = 2\pi$
- Initial time t, final time (t +T)
- $[\omega(t+T) + \phi] - (\omega t + \phi) = 2\pi$
- $\omega t + \omega T + \phi - \omega t - \phi = 2\pi$
- $\omega T = 2\pi$ or $\omega = 2\pi / T$
- $\omega = 2\pi f$

Dynamics of simple Harmonic motion

- Example:
- **(i) Mass attached to a spring:**
- Consider frictionless spring mass system , when mass m is given a displacement x from equilibrium it experiences a restoring force
- $F_s = -kx$ ....(5)
- Newton's second law : $\Sigma F = ma$......(6)

- $Kx = ma$

- $a = -(k/m)\,x$............(7)
- K and m are constant
- Eq.7 tells us that the acceleration of the mass m is proportional to but in the opposite direction from the displacement x. Therefore, The condition for SHM is satisfied.
- Both a and x vary in time, rewrite eq. 7
- $d^2x(t)/dt + (k/m)\,x(t) = 0$..........(8)
- Eq.8 is second order differential equation.
- As we know that $X(t) = A\cos(\omega t + \phi)$
- And $dx(t)/dt = -A\omega\sin(\omega t + \phi) = v(t)$
- $dv(t)/dt = -A\omega^2\cos(\omega t + \phi) = a(t)$



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Substitute these values in eq.(8)

- $-A\,\omega^2 \cos(\omega t + \phi) + (k/m)\,A \cos(\omega t + \phi) = 0$
- Term $A \cos(\omega t + \phi)$ is common to each side so cancelling it and solving for ω,
- $\omega = \sqrt{\frac{k}{m}}$
- $\omega^2 = k/m$ for spring mass system
- Solving for T and f:
- $\omega = 2\pi / T$ then $T = 2\pi / \omega = 2\pi / \sqrt{k/m}$
- $f = \omega / 2\pi = (1/2\pi)\sqrt{k/m}$………(9)

Conclusion:

- Two springs with the same size of masses but different spring constants k have different frequencies of vibration.
- Eq. (8) in terms of angular frequency
- $d^2x(t)/dt + \omega^2 x(t) = 0$

Problem

A spring of 200 N/m constant is fixed at one end and a 2.0 kg mass is attached to other end. The mass is pulled 10 cm from equilibrium and released. As the mass first passes through x=0 position a stopwatch is started. (a) what are the angular frequency and time period of the mass's motion? (b) what is the equation of motion x(t) for the mass when $\phi = \pi/2$ rad. ? (c) what is the mass's velocity and acceleration at t=1.50 s?

Solution:

(a) $\omega^2 = k/m$, $\omega = \sqrt{200\ \text{N.m}/2\ \text{kg}} = 10$ radian/sec.

$T = 2\pi/\omega = 2\pi/10$ rad./s = 0.528 sec.

(b) $X(t) = A\cos(\omega t + \phi)$

$X(t) = (10\ \text{cm})\cos[(10\ \text{rad./s})t + \pi/2\ )$

(c) $V(t) = dx(t)/dt = -d/dt[(10\ \text{cm})\cos\{(10\ \text{rad./s})t + \pi/2\ \}]$

At t=1.50sec

$V(t) = -(10)(10)\sin\{(10\ \text{rad./s})(1.50\text{s}) + \pi/2$

$V(t) = 76$ cm/s

$a(t) = d^2x/dt^2 = d/dt\ [-(100)\sin\{(10\ \text{rad./s})(1.50\text{s}) + \pi/2\ \}$

$a(t) = 650\ \text{cm/sec}^2$

Energy of the Simple Harmonic Oscillator

- Kinetic energy of a simple harmonic oscillator

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

spring $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$

- The total mechanical energy of the simple harmonic oscillator

$$E = K + U = \frac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

From the identity $\sin^2\theta + \cos^2\theta = 1$,

Total energy of a simple harmonic oscillator $E = \frac{1}{2}kA^2$

- That is, the total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude.
- **Not that U is zero at x zero and K is zero at x max where v=0**
- We can use the principle of conservation of energy to obtain the velocity for an arbitrary position by expressing the total energy at some arbitrary position x as

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{A^2 - x^2}$$

$mv^2+kx^2=Ax^2$

$mv^2=kA^2-kx^2$

$V^2=(kA^2-kx^2)/m$