Work Energy Theorem, Law of Conservation of Mechanical Energy

Lecture 6

PH-122

Work Energy Theorem

- Work is done when force changes the state of motion of an object. In other words work should be equal to change in kinetic energy of the object.
- In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.

$$W = \Delta K.E$$

- Above equation is known as work-energy theorem and is valid for constant force, variable force, conservative and non-conservative force.

Work Energy Theorem; proof for constant force

A constant force moves an object.

$W = \vec{F}.\vec{d}$ if force and displacement are parallel then $W = Fd$

According to Newton's second law of motion $F = ma$. Therefore,

$W = (ma)d$ (1)

But, $2ad = V_f^2 - V_i^2$ or $ad = \frac{V_f^2 - V_i^2}{2}$

Therefore, equation (1) becomes:

$$W = m\left(\frac{V_f^2 - V_i^2}{2}\right)$$

$$W = \Delta K.E$$

Work Energy Theorem; proof for variable force

A variable force moves an object.

$$W = \int_i^f F dr$$

According to Newton's second law of motion $F = m(\frac{dV}{dt})$. Therefore,

$$W = \int_i^f m\left(\frac{dV}{dt}\right) dr \qquad (1)$$

After mathematical manipulation equation (1) becomes

$$W = \int_i^f m\left(\frac{dr}{dt}\right) dV$$

Work Energy Theorem; proof for variable force

$$W = \int_{i}^{f} m\,(V)dV$$

$$W = \frac{1}{2}m\left(V_f^2 - V_i^2\right)$$

$$W = \Delta K.E$$

Problem

- During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = -3m\hat{\imath}$ while a steady wind pushes against the crate with a force $\vec{F} = 2N\hat{\imath} + (-6N\hat{\jmath})$.

(a) How much work does this force do on the crate during the displacement?

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement , what is its kinetic energy at the end of $\vec{d}$?

Solution

(a)

$$W = \vec{F}.\vec{d}$$

$$W = [2N\hat{\imath} + (-6N\hat{\jmath})].[-3m\hat{\imath}]$$

$$W = -6J$$

(b)

$$W = \Delta K.E$$

$$K.E_f = K.E_i + W$$

$$K.E_f = 10J + (-6J)$$=4J

Problem

- A block of mass 3.63 kg slides on a frictionless horizontal table with a speed of 1.22m/s. It is brought to rest in compressing a spring in its path. By how much is the spring compressed if its spring constant is 135N/m.

Solution

$$\Delta K.E = K.E_f - K.E_i$$

$$\Delta K.E = -\frac{1}{2}\mathrm{mV}^2$$

$$W = -\frac{1}{2}kx^2$$

According to work-energy theorem:

$$W = \Delta K.E$$

$$x = V\left(\frac{m}{k}\right)^{\left(\frac{1}{2}\right)}$$

Law of Conservation of Mechanical Energy

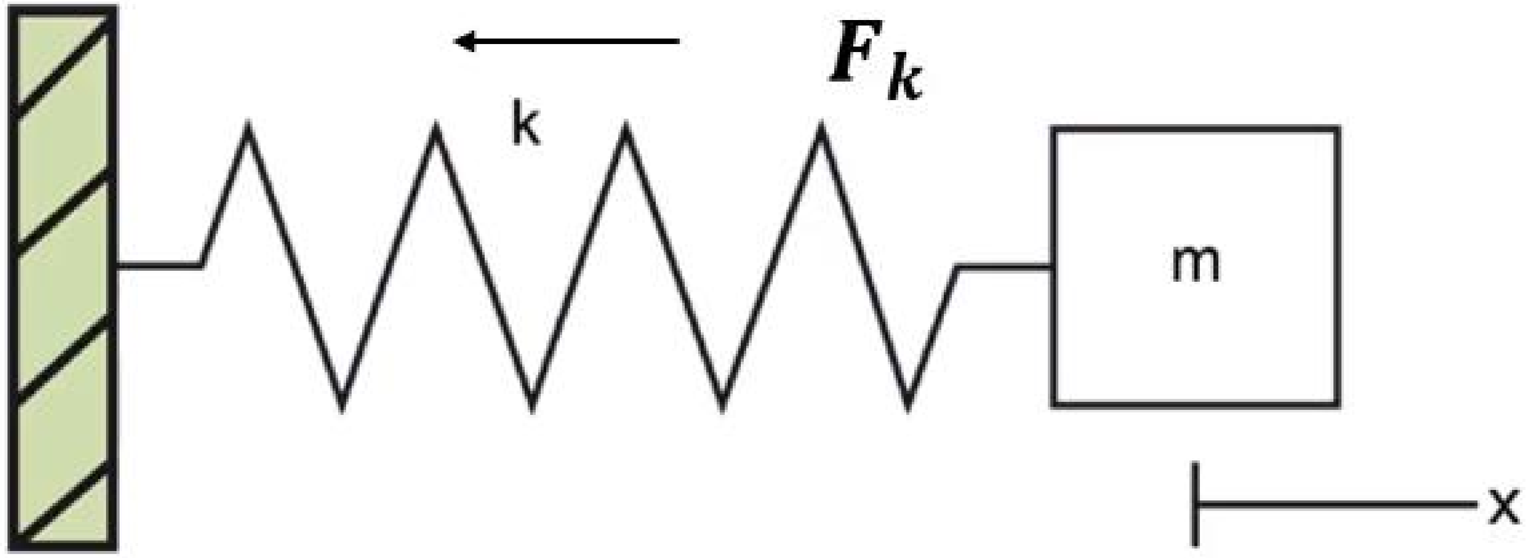
- A special class of force that is able to store energy (known as potential energy) in objects due to relative position of object or orientation of parts of object is called conservative force.
- For such forces work done around a closed is zero. Moreover work done depends only on initial and final conditions and does not depend upon the path followed between the two states.
- With in a conservative field, work is done at the cost of potential energy stored in the object.

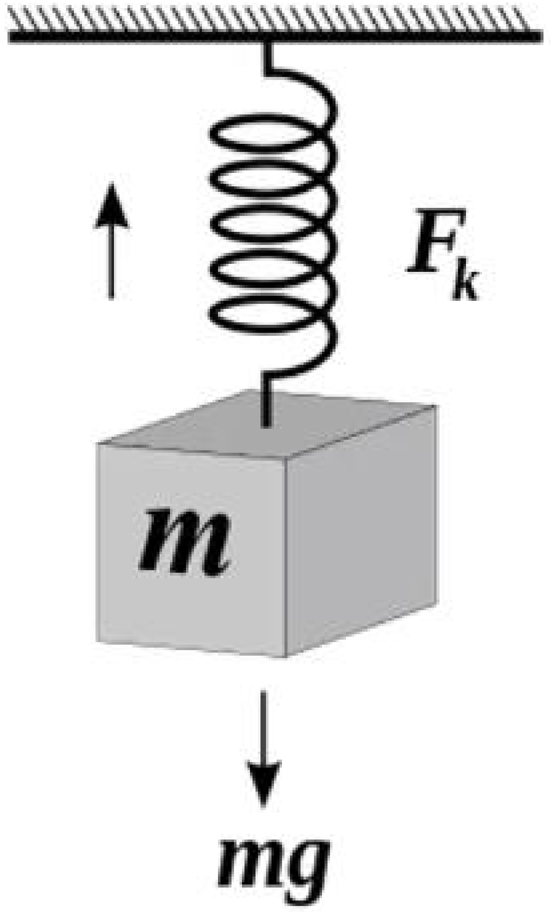
$$W = -\Delta P.E$$

Law of Conservation of Mechanical Energy

- From work energy theorem, we know that $W = \Delta K.E$.
- By combining the two equations $\Delta K.E = -\Delta P.E$
- $\Delta K.E + \Delta P.E = 0$
- $\Delta(K.E + P.E) = 0$
- $\Delta\, mechanical\ energy = 0$.

Law of Conservation of Mechanical Energy





There is one conservative force acting on mass so it will acquire only one kinetic energy and one potential energy i.e

$$E = K.E + P.E$$

There are two conservative forces acting on mass so it will acquire only one kinetic energy and two potential energies i.e

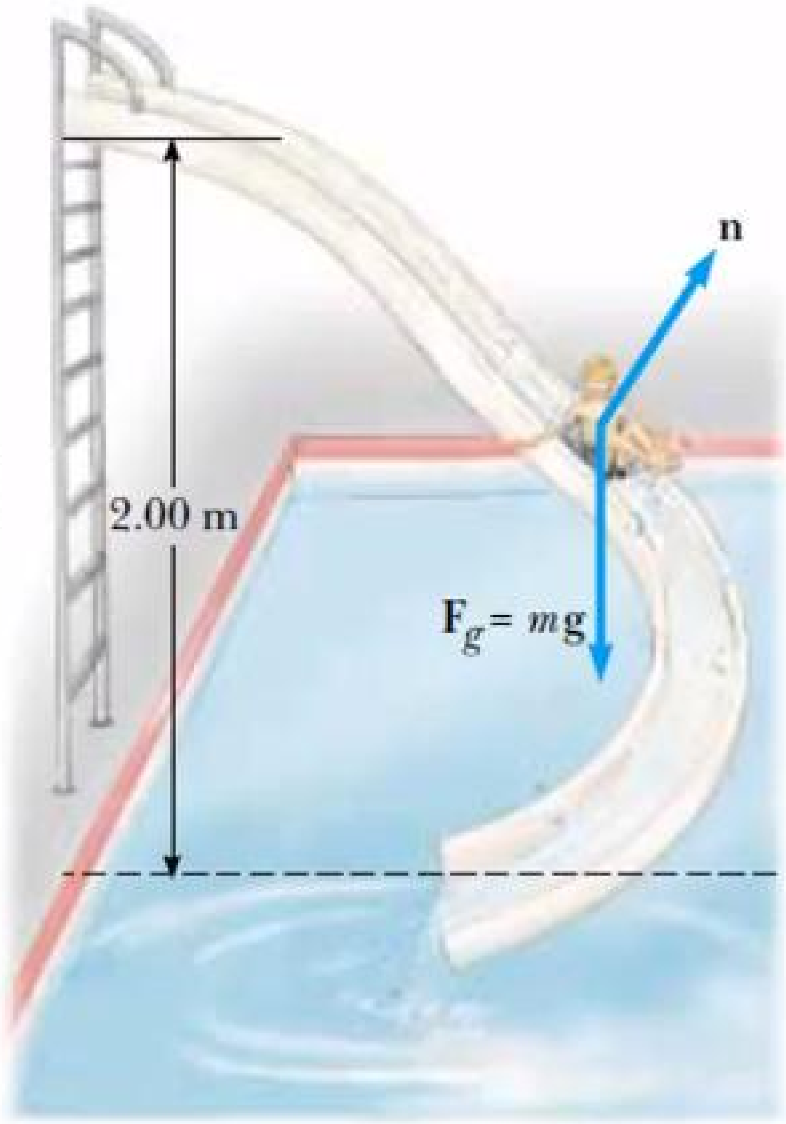
$$E = K.E + P.E_g + P.E_k$$

Problem

- A child of mass 'm' rides on an irregularly curved Slide of height h=2m. The child starts from rest at the top.

(a) Determine his speed at the bottom, assuming nc present.

(b) If force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume $V_f = 3m/s$ and $m = 20kg$.



Solution

$$(a) E_f = E_i \qquad K.E_f + P.E_f = K.E_i + P.E_i$$

$$\frac{1}{2} m V_f^2 + 0 = 0 + mgh$$

$$V_f = (2gh)^{\frac{1}{2}}$$

(b) $\Delta E = \left(K.E_f + P.E_f\right) - (K.E_i + P.E_i)$

$$\Delta E = \left(\frac{1}{2} m V_f^2 + 0\right) - (0 + mgh)$$

$$\Delta E = \frac{1}{2} m V_f^2 - mgh$$