VECTORS

"A vector is a quantity having both magnitude as well as direction such as displacement, velocity, force & acceleration."

Scalars:

"A scalar is a quantity having magnitude but no direction e.g. mass, distance, temperature etc.

Scalar field:

"If to each point (x, y, z) of a region 'R' in space there corresponds a number or scalar then is called a scalar function of position or scalar point function."

for example:

- Temperature at any point within or on the earth's surface at a certain time defines a scalar field.

Vector field:

"If to each point (x, y, z) of a region R space there corresponds a vector $V(x, y, z)$, then V is called vector function of position or vector point function & we say that a vector field V has been defined in R."

For Example:
- If the velocity at any point (x, y, z) within a moving field fluid is known at a certain time, then a vector field is defined.

Q. $\vec{r_1} = 3i - 2j + k$, $\vec{r_2} = 2i - 4j - 3k$, $\vec{r_3} = -i + 2j + 2k$

Find the magnitudes of.

(a) $|\vec{r_3}|$, (b) $|\vec{r_1} + \vec{r_2} + \vec{r_3}|$, (c) $|2\vec{r_1} - 3\vec{r_2} - 5\vec{r_3}|$

Sol:
$\because \vec{r_1} = 3i - 2j + k$

$|\vec{r_1}| = \sqrt{(3)^2 + (-2)^2 + (1)^2}$

$|\vec{r_1}| = \sqrt{9 + 4 + 1}$

$|\vec{r_1}| = \sqrt{14}$

$\because \vec{r_2} = 2i - 4j - 3k$

$|\vec{r_2}| = \sqrt{4 + 16 + 9}$

$|\vec{r_2}| = \sqrt{29}$

$\because \vec{r_3} = -i + 2j + 2k$

$|\vec{r_3}| = \sqrt{1 + 4 + 4}$

$|\vec{r_3}| = \sqrt{9}$

$\boxed{|\vec{r_3}| = 3} \rightarrow$ (a) (b). $|\vec{r}_1 + \vec{r}_2 + \vec{r}_3| = (3i - 2j + k) + (2i - 4j - 3k) +$

$+ (-i + 2j + 2k)$

$= 4i - 4j + 0k$

$= \sqrt{16 + 16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Ans

(c). $2r_1 - 3r_2 - 5r_3 = 2(3i - 2j + k) - 3(2i - 4j - 3k) -$

$+ 5(-i + 2j + 2k)$

$= (6i - 4j + 2k) + (-6i + 12j + 9k) +$

$+ (5i - 10j - 10k)$

$= 5i - 2j + k$

$= \sqrt{25 + 4 + 1}$

$= \sqrt{30}$

Ans

Vector Differentiation:

There are two types of differentiation

- Ordinary derivative of a vector (vector depends on one variable)
- Partial derivatives of a vector (vector depends on more than one variable)

Differential formulae:

1. $\frac{d}{du}(A+B) = \frac{dA}{du} + \frac{dB}{du}$

2. $\frac{d}{du}(A\cdot B) = A\cdot\frac{dB}{du} + \frac{dA}{du}\cdot B$

3. $\frac{d}{du}(A\times B) = A\times\frac{dB}{du} + \frac{dA}{du}\times B$

4. $\frac{d}{du}(\phi A) = \phi\frac{dA}{du} + \frac{d\phi}{du}A$

5. $\frac{d}{du}(A\cdot B\times C) = A\cdot B\times\frac{dC}{du} + A\cdot\frac{dB}{du}\times C + \frac{dA}{du}\cdot B\times C$

6. $\frac{d}{du}\{A\times(B\times C)\} = A\times\left(B\times\frac{dC}{du}\right) + A\times\left(\frac{dB}{du}\times C\right) + \frac{dA}{du}\times(B\times C)$

Q. Given. $R = \sin t\, i + \cos t\, j + t k$

find. (a) $\frac{dR}{dt}$, (b). $\frac{d^2R}{dt^2}$, (c) $\left|\frac{dR}{dt}\right|$, (d) $\left|\frac{d^2R}{dt^2}\right|$

Sol:

$R = \sin t\, i + \cos t\, j + t k$

(a). $\frac{dR}{dt} = \frac{d}{dt}(\sin t) i + \frac{d}{dt}(\cos t) j + \frac{d}{dt}(t) k$

$\frac{dR}{dt} = \cos t\, i - \sin t\, j + k$

Ans

(b). $\frac{d^2R}{dt^2} = -\sin t\, i - \cos t\, j + 0$

$\frac{d^2R}{dt^2} = -\sin t\, i - \cos t\, j$

Ans

(c). $\left|\frac{dR}{dt}\right| = \sqrt{(\cos t)^2 + (-\sin t)^2) + (1)^2}$

$= \sqrt{\cos^2 t + \sin^2 t + 1}$

$= \sqrt{1+1}$

$\left|\frac{dR}{dt}\right| = \sqrt{2}$

Ans

(d) $\left|\frac{d^2R}{dt^2}\right| = \sqrt{(-\sin t)^2 + (-\cos t)^2}$

$= \sqrt{\sin^2 t + \cos^2 t}$

$\left|\frac{d^2R}{dt^2}\right| = 1$

Ans

d) $\left|\frac{d^2R}{dt^2}\right|$

Q: A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$ & $z = 2\sin 3t$ where t is the time.

(a). Determine its acceleration & velocity at any time.

(b). Find the magnitudes of the acceleration & velocity at $t=0$.

Sol:

(a). The position vector, r of the particle is,

$r = x\hat{i} + y\hat{j} + z\hat{k}$

$r = e^{-t}\hat{i} + 2\cos 3t\,\hat{j} + 2\sin 3t\,\hat{k}$

$V = \frac{dr}{dt} = -e^{-t}\hat{i} - 6\sin 3t\,\hat{j} + 6\cos 3t\,\hat{k}$

$a = \frac{d^2r}{dt^2} = e^{-t}\hat{i} - 18\cos 3t\,\hat{j} - 18\sin 3t\,\hat{k}$

(b). At $t=0$

$V = \frac{dr}{dt} = -e^{-0}\hat{i} - 6\sin 3(0)\hat{j} + 6\cos 3(0)\hat{k}$

$= -\hat{i} + 6\hat{k}$

$|\bar{V}| = \sqrt{1+36} = \sqrt{37}$

$a = \frac{d^2r}{dt^2} = e^{-0}\hat{i} - 18\cos 3(0)\hat{j} - 18\sin 3(0)\hat{k}$

$a = \frac{d^2r}{dt^2} = \hat{i} - 18\hat{j}$

$|\bar{a}| = \sqrt{1+324} = \sqrt{325}$

Ordinary Derivative Of a Vectors

Let $R(u)$ be a vector depending on a single scalar variable u.

$$\frac{\Delta R}{\Delta u} = \frac{R(u+\Delta u) - R(u)}{\Delta u}$$

The ordinary derivative of $R(u)$ with respect to the scalar u is given by if the limit exists.

$$\frac{dR}{du} = \lim_{\Delta u \to 0} \frac{\Delta R}{\Delta u} = \lim_{\Delta u \to 0} \frac{R(u+\Delta u) - R(u)}{\Delta u}$$

Partial Differentiation of a Vector:

If A is a vector depending on more than one scalar variable, say x, y & z for example, then we write $A = A(x,y,z)$. The partial derivative of A with respect to A is defined as,

$$\frac{\partial A}{\partial x} = \lim_{\Delta x \to 0} \frac{A(x+\Delta x, y, z) - A(x,y,z)}{\Delta x}$$

IIy,

$$\frac{\partial A}{\partial y} = \lim_{\Delta y \to 0} \frac{A(x, y+\Delta y, z) - A(x,y,z)}{\Delta y}$$

$$\frac{\partial A}{\partial z} = \lim_{\Delta z \to 0} \frac{A(x, y, z+\Delta z) - A(x,y,z)}{\Delta z}$$

Q. If $A = (2x^2y - x$ …

find $\frac{\partial A}{\partial x}, \frac{\partial A}{\partial y}$ …

Sol:

$A = (2x^2y - 2$ …

$\frac{\partial A}{\partial x} = (4xy - 4$ …

$\frac{\partial A}{\partial y} = (2x^2)\hat{i}$ …

$\frac{\partial^2 A}{\partial x^2} = (4y -$ …

$\frac{\partial^2 A}{\partial y^2} = x^2 e$ …

$\frac{\partial^2 A}{\partial x \partial y} = \frac{\partial}{\partial x}[$ …

$\frac{\partial^2 A}{\partial x \partial y} = 4$ …

$\frac{\partial^2 A}{\partial y \partial x} = \frac{\partial}{\partial y}$ …

$\frac{\partial^2 A}{\partial y \partial x} = 4$ …

Q. If $A = (2x^2y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + (x^2\cos y)\hat{k}$,

find $\frac{\partial A}{\partial x}$, $\frac{\partial A}{\partial y}$, $\frac{\partial^2 A}{\partial x^2}$, $\frac{\partial^2 A}{\partial y^2}$, $\frac{\partial^2 A}{\partial x \partial y}$, $\frac{\partial^2 A}{\partial y \partial x}$

Sol:

$$A = (2x^2y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + (x^2\cos y)\hat{k}$$

$$\frac{\partial A}{\partial x} = (4xy - 4x^3)\hat{i} + (ye^{xy} - y\cos x)\hat{j} + (2x\cos y)\hat{k}$$

$$\frac{\partial A}{\partial y} = (2x^2)\hat{i} + (xe^{xy} - \sin x)\hat{j} - (x^2\sin y)\hat{k}$$

$$\frac{\partial^2 A}{\partial x^2} = (4y - 12x^2)\hat{i} + (y^2e^{xy} + y\sin x)\hat{j} + (2\cos y)\hat{k}$$

$$\frac{\partial^2 A}{\partial y^2} = x^2e^{xy}\hat{j} - x^2\cos y\,\hat{k}$$

$$\frac{\partial^2 A}{\partial x \partial y} = \frac{\partial}{\partial x}\left[2x^2\hat{i} + (xe^{xy} - \sin x)\hat{j} - (x^2\sin y)\hat{k}\right]$$

$$\frac{\partial^2 A}{\partial x \partial y} = 4x\hat{i} + (\underset{xye^{xy}+e^{xy}}{\cancel{\quad\quad}} - \cos x)\hat{j} - (2x\sin y)\hat{k}$$

$$\frac{\partial^2 A}{\partial y \partial x} = \frac{\partial}{\partial y}\left[(4xy - 4x^3)\hat{i} + (ye^{xy} - y\cos x)\hat{j} + (2x\cos y)\hat{k}\right]$$

$$\frac{\partial^2 A}{\partial y \partial x} = 4x\hat{i} + (xye^{xy} + e^{xy} - \cos x)\hat{j} - (2x\sin y)\hat{k}$$

Vector Differential Operator (∇):

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

The Gradient

Let $\phi(x,y,z)$ be defined & differentiable at each point (x,y,z) in a certain region of space (i.e. ϕ defines a differentiable scalar field). Then the gradient of ϕ written $\nabla\phi$ or grad, is defined by ϕ

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi$$

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k\right)$$

- $\nabla\phi$ defines a vector field.
- The component $\nabla\phi$ in the direction of a nit vector given by $\nabla\phi \cdot a$ & is called directional derivative of $\nabla\phi$ in the direction of

Q. If $\phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\phi$ (or grad ϕ) at the point $(1,-2,-1)$.

Sol:

$$\nabla\phi = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(3x^2y - y^3z^2)$$

$$\nabla\phi = 6xy\,\hat{i} + (3x^2 - 3y^2z^2)\hat{j} - 2zy^3\,\hat{k}$$

$$\nabla\phi = 6(1)(-2)\hat{i} + \{3(1)^2 - 3(-2)^2(-1)^2\}\hat{j} - 2(-1)(-2)^3\hat{k}$$

$$= -12\hat{i} + \{3 - 3(4)(1)\}\hat{j} - 16k$$

$$= -12\hat{i} - 9\hat{j} - 16\hat{k}$$

Q. Find a unit normal to surface $x^2y + 2xz = 4$ at point $(2,-2,3)$.

Sol:

$$\nabla\phi = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(x^2y + 2xz - 4)$$

$$= (2xy + 2z)i + x^2 j + 2xk$$

$$= \{2(2)(-2) + 2(3)\}i + (2)^2 j + 2(2)k$$

$$\nabla\phi = -2i + 4j + 4k$$

$$\hat{\nabla\phi} = \frac{-2i + 4j + 4k}{\sqrt{4+16+16}}$$

$$= \frac{-\cancel{2}}{\cancel{6}\,3} i + \frac{4}{6} j + \frac{4}{6} k$$

$$\hat{\nabla\phi} = -\frac{1}{3} i + \frac{2}{3} j + \frac{2}{3} k$$

opposite in direction $\left[\frac{1}{3} i - \frac{2}{3} j - \frac{2}{3} k\right]$

The Divergence:

Let $V(x,y,z) = V_1 i + V_2 j + V_3 k$ be defined & differentiable at each point (x,y,z) in a certain region of space (i.e. V defines a differentiable vector field)

Then the divergence of V written $\nabla \cdot V$ or div. V, is defined by.

$$\nabla \cdot V = \left(\frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k\right)(V_1 i + V_2 j + V_3 k)$$

$$\nabla \cdot V = \frac{dV_1}{dx} + \frac{dV_2}{dy} + \frac{dV_3}{dz}$$

- Note that analogy $A \cdot B = A_1B_1 + A_2B_2 + A_3B_3$.

Also. $\nabla \cdot V \neq V \cdot \nabla$

Q. $A = x^2 z i - 2y^3 z^2 j + xy^2 z k$ find $\nabla \cdot A$ at point $(1,-1,1)$

Sol:

$$\nabla \cdot A = \left(\frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k\right)(x^2 z i - 2y^3 z^2 j + xy^2 z k)$$

$$= (x^2 z)\frac{d}{dx} + \frac{d}{dy}(-2y^3 z^2) + \frac{d}{dz}(xy^2 z)$$

$$= 2xz - 6y^2 z^2 + xy^2$$

$$= 2(1)(1) - 6(-1)^2(1)^2 + (1)(-1)^2$$

$$= 2 - 6 + 1$$

$$= -3$$

The curl:

If $V(x,y,z)$

the curl

- curl V o

$\nabla \times V$

$\nabla \times V$

Q. If $A =$

Sol: curl

$\Rightarrow \nabla$

The curl:

If $V(x,y,z)$ is a differentiable vector field then the curl or rotation of V, written as $\nabla \times V$, curl V or rot.V, is defined by

$$\nabla \times V = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \times (V_1 i + V_2 j + V_3 k)$$

$$\nabla \times V = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_1 & V_2 & V_3 \end{vmatrix}$$

Q. If $A = x^2y\, i - 2xz\, j + 2yz\, k$, find curl curl A.

Sol: curl curl $A = \nabla \times (\nabla \times A)$

$$\Rightarrow \nabla \times (\nabla \times A) = \nabla \times \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & -2xz & 2yz \end{vmatrix}$$

$$= \nabla \times \left[i\left\{\frac{\partial}{\partial y}(2yz) + \frac{\partial}{\partial z}(2xz)\right\} - j\left\{\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial z}(x^2y)\right\} + k\left\{-\frac{\partial}{\partial x}(2xz) - \frac{\partial}{\partial y}(x^2y)\right\} \right]$$

$$= \nabla \times \left[(2z + 2x)\, i + (-2z - x^2)\, k \right]$$

$$\Rightarrow \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (2x+2z) & 0 & -(x^2+2z) \end{vmatrix}$$

$\Rightarrow (0-0)i - j(-2x-2) + k(0-0)$

$\Rightarrow (2x+2)j$ Ans

Q. $A = xz^3 i - 2x^2yz j + 2yz^4 k$, find $\nabla \times A$ at point $(1,-1,1)$

Sol:

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xz^3 & -2x^2yz & 2yz^4 \end{vmatrix}$$

$= i(2z^4 + 2x^2y) - j(0 - 3xz^2) + k(-4xyz - 0)$

$= \{2(1)^4 + 2(1)^2(-1)\} i + 3(1)(1)^2 j - 4(1)(-1)(1)k$

$= 3j + 4k$

Ans

Gradient

Gradient is

It gives di

normal vecto

used to fi

defined by

Divergence

by $\vec{\nabla} \cdot \vec{A}$

$\vec{\nabla} \cdot \vec{A} <$

field is

Diverg

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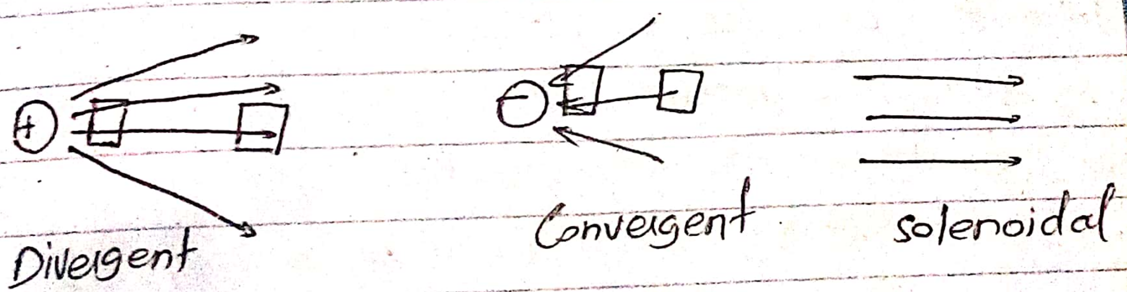
field

Gradient, Divergence & Curl

- Gradient is always founds for scalar quantities. It gives direction to scalar by defining a normal vector associated with scalar quantity. used to find directional derivative. It is defined by $\vec{\nabla}\phi$.

- Divergence is always found for vector, defined by $\vec{\nabla}\cdot\vec{A}$. If $\vec{\nabla}\cdot\vec{A} = 0$ field is solenoidal, $\vec{\nabla}\cdot\vec{A} < 0$ field is convergent, $\vec{\nabla}\cdot\vec{A} > 0$ field is divergent.



- Curl is also found for vectors. It is defined by $\vec{\nabla}\times\vec{A}$. It is usually used to find if the field is conservative or non-conservative. As all conservative fields have straight lines of force that mean that the curl of such fields will be zero. If $\vec{\nabla}\times\vec{A} = 0$; field is conservative, $\vec{\nabla}\times\vec{A} \neq 0$; field is non-conservative.

Vector Differentiation

Q. If $\phi = 2xz^4 - x^2y$, find $\nabla\phi$ & $|\nabla\phi|$ at point $(2, -2, -1)$.

$$\nabla\phi = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(2xz^4 - x^2y)$$

$$= (2z^4 - 2xy)i + (-x^2)j + (8xz^3)k$$

$$\nabla\phi = (2z^4 - 2xy)i - x^2 j + 8xz^3 k$$

at $(2, -2, -1)$

$$\Rightarrow \nabla\phi = \{2(-1)^4 - 2(2)(-2)\}i - (2)^2 j + 8(2)(-1)^3 k$$

$$= \{2 + 8\}i - 4j - 16k$$

$$\boxed{\nabla\phi = 10i - 4j - 16k}$$ Ans

$$|\nabla\phi| = \sqrt{100 + 16 + 256}$$

$$\boxed{|\nabla\phi| = 2\sqrt{93}}$$ Ans

Q. If $A = 2x^2 i - 3yz j + xz^2 k$ & $\phi = 2z - x^3 y$, find $A \cdot \nabla\phi$ & $A \times \nabla\phi$ at point $(1, -1, 1)$

$\because \phi = 2z - x^3 y$

$$\nabla\phi = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(2z - x^3 y)$$

$$= (-3x^2 y) i + (-x^3) j + (2) k$$

$$\nabla\phi = -3x^2 y i - x^3 j + 2k$$

at $(1, -1, 1)$

$$\nabla\phi = -3(1)^2(-1) i - (1)^3 j + 2k$$

$$\boxed{\nabla\phi = 3i - j + 2k}$$

Now.

$\because A = 2x^2 i - 3yz j + xz^2 k$

at $(1, -1, 1)$

$$\Rightarrow A = 2(1)^2 i - 3(-1)(1) j + (1)(1)^2 k$$

$$\boxed{A = 2i + 3j + k}$$

$$A \cdot \nabla\phi = (2i + 3j + k) \cdot (3i - j + 2k)$$

$$= (2)(3) + (3)(-1) + (1)(2)$$

$$= 6 - 3 + 2$$

$$= 8 - 3$$

$$\boxed{A \cdot \nabla\phi = 5}$$

Now.

$$A \times \nabla\phi = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= i(7) - j(1) + k(-11)$$

$$\boxed{A \times \nabla\phi = 7i - j - 11k}$$

Q. If $f = x^2z + e^{y/x}$ & $G = 2x^2y - xy^2$, find (a) $\nabla(F+G)$ & (b) $\nabla(FG)$ at point $(1,0,-2)$

$$F+G = (x^2z + e^{y/x}) + (2x^2y - xy^2)$$

$$F+G = 2x^2y + x^2z - xy^2 + e^{y/x}$$

$$\nabla(F+G) = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(2x^2y + x^2z - xy^2 + e^{y/x})$$

$$\nabla(F+G) = \left\{4xy + 2xz - y^2 - \frac{ye^{y/x}}{x^2}\right\} i + \left\{2x^2 - 2xy + \frac{e^{y/x}}{x}\right\} j + \left\{x^2\right\} k$$

at $(1, 0, -2)$

$$\nabla(F+G) = \left\{4(1)(0) + 2(1)(-2) - (0)^2 - \frac{(0)e^{0/1}}{(1)^2}\right\} i + \left\{2(1)^2 - 2(1)(0) + \frac{e^{0/1}}{(1)}\right\} j + (1)^2 k$$

$$= (-4)i + (2+1)j + k$$

$$\boxed{\nabla(F+G) = -4i + 3j + k}$$

$FG = (x^2z + e^{y/x}) \ldots$

$FG = 2x^4yz - \ldots$

$\nabla(FG) = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} \ldots\right.$

$= \{8x^3yz - \ldots$

$+ -y^2(\ldots$

$+ 2x^2 \ldots$

$+ \{2 \ldots$

at $(1, 0, \ldots$

$\nabla(FG) = \{ \ldots$

$+ 2 \ldots$

$\nabla(FG) = \ldots$

$\nabla(FG) = \ldots$

$$FG = (x^2 z + e^{y/x})(2x^3 y - xy^2)$$

$$FG = 2x^4 y z - x^3 y^2 z + 2x^2 y e^{y/x} - xy^2 e^{y/x}$$

$$\nabla(FG) = \left(\frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k\right)(2x^4 y z - x^3 y^2 z + 2x^2 y e^{y/x} - xy^2 e^{y/x})$$

$$= \Big\{ 8x^3 y z - 3x^2 y^2 z + 2y\left(2x e^{y/x} - \frac{y e^{y/x}}{x^2} x^2\right) - y^2\left(e^{y/x} + \frac{x y e^{y/x}}{x^2}\right)\Big\} i + \Big\{ 2x^4 z - 2x^3 y z + 2x^2\left(e^{y/x} + \frac{y e^{y/x}}{x}\right) - x\left(2y e^{y/x} + \frac{y^2 e^{y/x}}{x}\right)\Big\} j + \Big\{ 2x^4 y - x^3 y^2 \Big\} k$$

at $(1, 0, -2)$

$$\nabla(FG) = \{0 - 0 + 0 - 0\} i + \{2(1)^4(-2) - 0 + 2(1)^2(e^{0/1} + 0) - x(0 + 0)\} j + (0 - 0) k$$

$$\nabla(FG) = 0i + (-4 + 2) j + 0k$$

$$\boxed{\nabla(FG) = -2j}$$

~~find curl~~

find $A \times (\nabla \times B)$ & $(A \times \nabla) \times B$ at the point $(1,-1,2)$, if $A = xz^2 i + 2yj - 3xzk$ & $B = 3xzi +$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3xz & 2yz & -z^2 \end{vmatrix}$$

$$= i(0-2y) - j(0-3x) + k(0-0)$$

$(\nabla \times B) = -2yi + 3xj$

at $(1,-1,2)$

$(\nabla \times B) = -2(-1)i + 3(1)j$

$\boxed{(\nabla \times B) = 2i + 3j}$

$A = xz^2 i + 2yj - 3xzk$

at $(1,-1,2)$

$A = (1)(2)^2 i + 2(-1)j - 3(1)(2)k$

$\boxed{A = 4i - 2j - 6k}$

Now

$$A \times (\nabla \times B) = \begin{vmatrix} i & j & k \\ 4 & -2 & -6 \\ 2 & 3 & 0 \end{vmatrix}$$

oint

$B = 3xzi + 2yz$

$$A \times (\nabla \times B) = i(0+18) - j(0+12) + k(12+4)$$

$$\boxed{A \times (\nabla \times B) = 18i - 12j + 16k}$$

$$(A \times \nabla) = \begin{vmatrix} i & j & k \\ xz^2 & 2y & -3xz \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix}$$

$$= i(0+0) - j(2xz + 3z) + k(0-0)$$

$$(A \times \nabla) = -j(2xz + 3z)$$

at $(1, -1, 2)$

$$(A \times \nabla) = -j\{2(1)(2) + 3(2)\}$$

$$= -j(4+6)$$

$$= -j(10)$$

$$\boxed{(A \times \nabla) = -10j}$$

$$B = 3xzi + 2yzj - z^2k$$

at $(1, -1, 2)$

$$B = 3(1)(2)i + 2(-1)(2)j - (2)^2k$$

$$\boxed{B = 6i - 4j - 4k}$$

$$(A \times \nabla) \times B = \begin{vmatrix} i & j & k \\ 0 & -10 & 0 \\ 6 & -4 & -4 \end{vmatrix}$$

$= \cancel{i(\ )}$

$$\boxed{(A \times \nabla) \times B = 40i + 60k}$$

wrong ans?

Q $\nabla|r|^3$

Let $\vec{r} = xi + yj + zk$

$|r| = \sqrt{x^2+y^2+z^2}$

$$\nabla|r|^3 = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(x^2+y^2+z^2)^{3/2}$$

$$= \left\{ \frac{3}{\cancel{2}} (x^2+y^2+z^2)^{1/2} (\cancel{2}x) \right\} i + \left\{ \frac{3}{\cancel{2}} (x^2+y^2+z^2)^{1/2} (\cancel{2}y) \right\} j + \left\{ \frac{3}{\cancel{2}} (x^2+y^2+z^2)^{1/2} (\cancel{2}z) \right\} k$$

$$= 3|r|xi + 3|r|yj + 3|r|zk$$

$$= 3|r|(xi + yj + zk)$$

$$\boxed{\nabla|r|^3 = 3|r|\vec{r}}$$

Ans

Q. $\nabla\left(3r^2 - 4\sqrt{r} + \frac{6}{\sqrt[3]{r}}\right)$

let $\vec{r} = xi + yj + zk$

$|\vec{r}| = \sqrt{x^2+y^2+z^2}$

Now.

$$\nabla\left\{3(x^2+y^2+z^2) - 4(x^2+y^2+z^2)^{1/4} + \frac{6}{(x^2+y^2+z^2)}\right.$$

$$\left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)\left\{3(x^2+y^2+z^2) - 4(x^2+y^2+z^2)^{1/4} + 6(x^2+y^2+z^2)\right.$$

$$\Rightarrow \left\{6x - \frac{4}{4}(x^2+y^2+z^2)^{-3/4}(2x) - \frac{6}{6}(x^2+y^2+z^2)^{-7/6}(2x)\right\}$$

$$+ \left\{6y - \frac{4}{4}(x^2+y^2+z^2)^{-3/4}(2y) - \frac{6}{6}(x^2+y^2+z^2)^{-7/6}(2y)\right\}$$

$$+ \left\{6z - \frac{4}{4}(x^2+y^2+z^2)^{-3/4}(2z) - \frac{6}{6}(x^2+y^2+z^2)^{-7/6}(2z)\right\}$$

$$\Rightarrow \left\{6x - 2x r^{-3/4} - r^{-7/6}(2x)\right\} i +$$

$$+ \left\{6y - 2y r^{-3/4} - 2y r^{-7/6}\right\} j +$$

$$+ \left\{6z - 2z r^{-3/4} - 2z r^{-7/6}\right\} k$$

$$\Rightarrow x\left(6 - 2r^{-3/4} - 2r^{-7/6}\right) i +$$

$$+ y\left(6 - 2r^{-3/4} - 2r^{-7/6}\right) j +$$

$$+ z\left(6 - 2r^{-3/4} - 2r^{-7/6}\right) k$$

$\Rightarrow \left(6 - 2r^{-3/4} - 2r^{-7/6}\right)(xi + yj + zk)$

$\Rightarrow \boxed{\left(6 - 2r^{-3/4} - 2r^{-7/6}\right) r}$ Ans

Q. ~~Find~~ Prove :

(a). $\nabla \times (A + B) = \nabla \times A + \nabla \times B$

LHS

let $A = ai + bj + ck$

$B = xi + yj + zk$

$$\nabla \times (A \times B) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (a+x) & (b+y) & (c+z) \end{vmatrix}$$

$= i$

Prove (a). $\nabla \times (A+B) = \nabla \times A + \nabla \times B$

let

$A = ai + bj + ck$

$B = xi + yj + zk$

LHS.

$$\nabla \times (A+B) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (a+x) & (b+y) & (c+z) \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(c+z) - \frac{\partial}{\partial z}(b+y)\right] - j\left[\frac{\partial}{\partial x}(c+z) - \frac{\partial}{\partial z}(a+x)\right]$$

$$+ k\left[\frac{\partial}{\partial x}(b+y) - \frac{\partial}{\partial y}(a+x)\right] \rightarrow (1)$$

RHS

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ a & b & c \end{vmatrix}$$

$$\nabla \times A = i\left[\frac{\partial}{\partial y}(c) - \frac{\partial}{\partial z}(b)\right] - j\left[\frac{\partial}{\partial x}(c) - \frac{\partial}{\partial z}(a)\right] + k\left[\frac{\partial (b)}{\partial x}\right.$$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(z) - \frac{\partial}{\partial z}(y)\right] - j\left[\frac{\partial}{\partial x}(z) - \frac{\partial}{\partial z}(x)\right] + k\left[\frac{\partial (y)}{\partial x}\right.$$

$(\nabla \times A) + (\nabla \times B) = i\left[\frac{\partial}{\partial y}\right.$

$+ k\left[\right.$

from (1) & (2)

$\nabla \times (A+B) =$

(b). $\nabla \times (\phi A)$

$A =$

$\phi =$

$$(\nabla\times A)+(\nabla\times B) = i\left[\frac{\partial}{\partial y}([illegible]) - \frac{\partial}{\partial z}([illegible])\right] - j\left[\frac{\partial}{\partial x}([illegible]) - \frac{\partial}{\partial z}([illegible])\right]$$

$$+\, k\left[\frac{\partial}{\partial x}([illegible]) - \frac{\partial}{\partial y}([illegible])\right] \quad (2)$$

from (1) & (2)

$$\nabla\times(A+B) = (\nabla\times A)+(\nabla\times B)$$

Proved

(b) $\nabla\times(\phi A) = (\nabla\phi)\times A + \phi(\nabla\times A)$

$$A = x^2y\,\hat{i} - 2xz\,\hat{j} + 2xz\,\hat{k}$$

$$\phi = 3xy^2z$$

Prove $\nabla f(r) = \frac{f'(r)\, r}{r}$

let $\vec{r} = xi + yj + zk$

$|r| = \sqrt{x^2+y^2+z^2}$

Now.

$\nabla f(r) = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) f(r)$

$= \frac{\partial}{\partial x} f(r)\, i + \frac{\partial}{\partial y} f(r)\, j + \frac{\partial}{\partial z} f(r)\, k$

$= f'(r)\left[i \frac{\partial(r)}{\partial x} + j \frac{\partial(r)}{\partial y} + k \frac{\partial(r)}{\partial z} \right]$

$= f'(r)\left[i \frac{\partial(x^2+y^2+z^2)^{1/2}}{\partial x} + j \frac{\partial(x^2+y^2+z^2)^{1/2}}{\partial y} + k \frac{\partial(x^2+y^2+z^2)^{1/2}}{\partial z} \right]$

$= f'(r)\Big[i \times \frac{1}{\cancel{2}} (x^2+y^2+z^2)^{-1/2} \times \cancel{2}x +$

$+ j \times \frac{1}{\cancel{2}} (x^2+y^2+z^2)^{-1/2} \times \cancel{2}y +$

$+ k \times \frac{1}{\cancel{2}} (x^2+y^2+z^2)^{-1/2} \times \cancel{2}z \Big]$

$= f'(r)(x^2+y^2+z^2)^{-1/2}(xi+yj+zk)$

$\boxed{\nabla f(r) = \frac{f'(r)\, \vec{r}}{r}}$ Proved.