Gauss' Law & Applications

Lecture 11

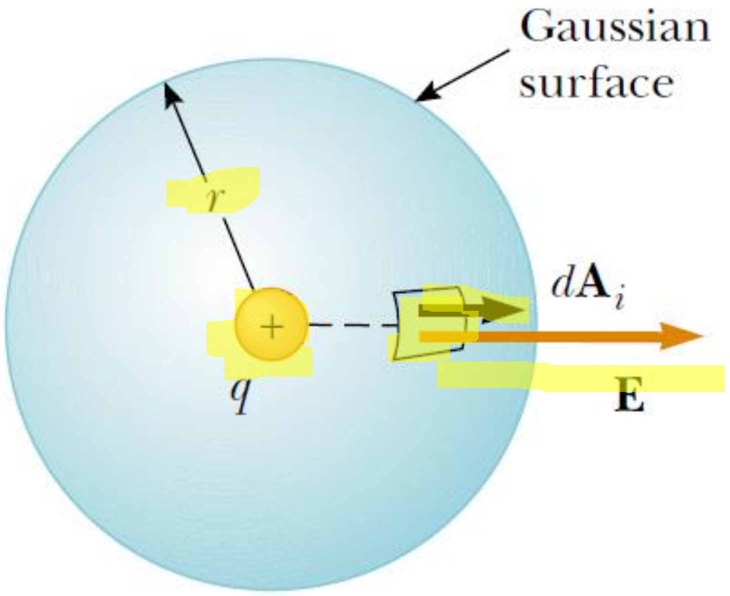
PH-122

Gauss' Law

- We know that flux through a surface is given by:

$$\mathbf{E}\cdot\Delta\mathbf{A}_i = E\,\Delta A_i$$

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \oint E\,dA = E\oint dA$$



But,

$$\oint dA = A = 4\pi r^2$$

$$\Phi_E = \frac{k_e q}{r^2}(4\pi r^2) = 4\pi k_e q$$

$$\Phi_E = \frac{q}{\epsilon_0}$$

Gauss' Law

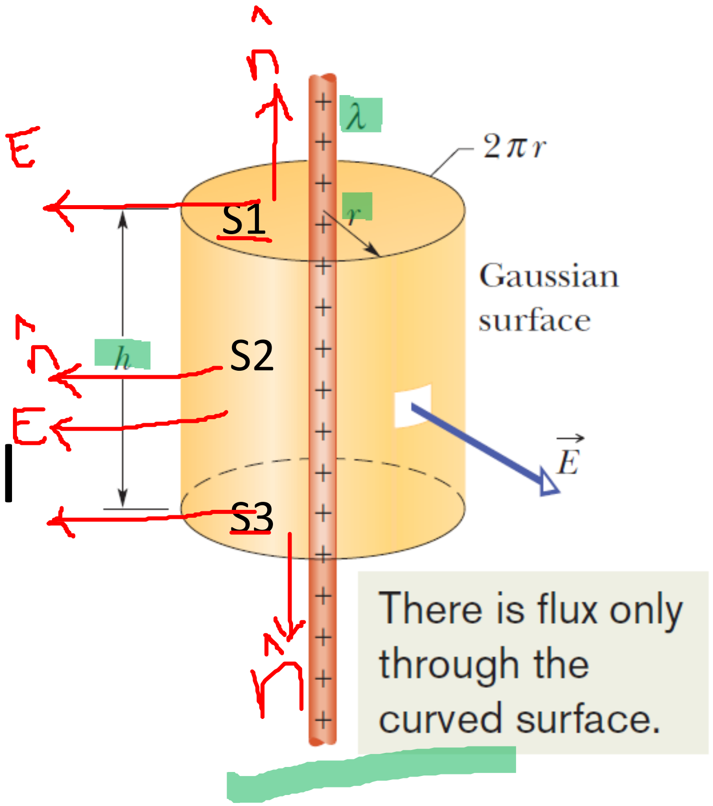
- The net flux through any closed surface surrounding a point charge q is given by $\frac{q}{\epsilon_0}$ and is independent of the shape of that surface.
- Gauss's law, which is a generalization of what we have just described, states that the net flux through *any* closed surface is

$$\Phi_E = \boxed{\oint \mathbf{E} \cdot d\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

Application of Gauss's law

- Find electric field due to a line of charge by using Gauss's law.

We first consider a Gaussian Surface that surrounds the line of charge which has a linear charge density λ.
The Gaussian surface considered is cylindrical in shape and hence is made up of three surface S1, S2 and S3.



Application of Gauss's law

- Applying Gauss's law

$\int_{S2} E\,dA = EA$

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

- The integral will be broken down into three parts over S1, S2 and S3. But dot product is $EA \cos\theta$ non-zero only over S2. Therefore;

$$\Phi = EA \cos\theta = E(2\pi rh)\cos 0 = E(2\pi rh)$$

$$\varepsilon_0 \Phi = q_{\text{enc}},$$

$$\varepsilon_0 E(2\pi rh) = \lambda h$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

Problem

- The net flux through each face of a dice has magnitude in units of $10^3 Nm^2/C$ equal to number 'N' of dots on the face (1-6). The flux is inward for 'N' odd and outward for 'N' even. What is the net charge inside the dice.

Solution

- Flux directed inward is taken as negative

$$\varphi_{odd} = -\varphi_1 - \varphi_3 - \varphi_5$$
$$\varphi_{odd} = -1X10^3 - 3X10^3 - 5X10^3 = -9X10^3 \mathrm{Nm}^2/\mathrm{C}$$

- Flux directed outward is taken as positive.

$$\varphi_{even} = \varphi_2 + \varphi_4 + \varphi_6 = 12X10^3 \mathrm{Nm}^2/\mathrm{C}$$

- Total flux;

$$\varphi = 3X10^3 Nm^2/C$$
$$q = \varphi\epsilon_0 = 2.66X10^{-8}\mathrm{C}$$