Ordinary Integrals Of A Vector

Let $R(u) = R_1(u)i + R_2(u)j + R_3(u)k$ be a vector depending on a single scalar variable u, where $R_1(u), R_2(u), R_3(u)$ are supposed continuous in specified interval. Then

$$\int R(u)du = i\int R_1(u)du + j\int R_2(u)du + k\int R_3(\ldots$$

This is called an indefinite integral of $R(u)$, where c is an arbitrary constant vector independent of u.

Integrals can also be defined in a limit of a sum in a manner analogous to that of elementary integral calculus.

$$\int_a^b R(u)du = \int_a^b \frac{d}{du}(s(u))du = s(u)+c\Big|_a^b =$$

$S(b)-$

Q. If $R(u) = (u-u^2)i + 2u^3 j - 3k$

find (a) $\int R(u)\,du$

(b) $\int_1^2 R(u)\,du$

Sol:

(a). $\int R(u)\,du = i\int (u-u^2)\,du + j\int 2u^3\,du - 3k\int du$

$$\int R(u)\,du = i\left[\frac{u^2}{2} - \frac{u^3}{3}\right] + j\left[\frac{u^4}{2}\right] - 3k\left[u\right] + c$$

$$\int R(u)\,du = \left(\frac{u^2}{2} - \frac{u^3}{3}\right)i + \frac{u^4}{2}j - 3uk + c$$

Ans

(b). $\int_1^2 R(u)\,du = i\int_1^2 (u-u^2)\,du + j\int_1^2 2u^3\,du - 3k\int_1^2 du$

$$\int_1^2 R(u)\,du = \frac{u^2}{2} - \frac{u^3}{3}\Big|_1^2 i + \frac{u^4}{2}\Big|_1^2 j - 3u\Big|_1^2 k$$

$$= \left[\frac{u^2}{2}\Big|_1^2 - \frac{u^3}{3}\Big|_1^2\right]i + \frac{u^4}{2}\Big|_1^2 j - 3u\Big|_1^2 k$$

$$= \left[2 - \frac{1}{2} - \frac{8}{3} + \frac{1}{3}\right]i + \left[8 - \frac{1}{2}\right]j - \left[6 - 3\right]k$$

$$= \frac{-5}{6}i + \frac{15}{2}j - 3k$$

Ans

Line Integral

• Let $r(u) = x(u)i + y(u)j + z(u)k$, where $r(u)$ is the position vector of (x,y,z) define a curve C joining points P1 & P2, where $u = u_1$ & $u = u_2$ respectively.

• We assume that C is composed of a finite number of curves for each of $r(u)$ has a continuous derivative. Let $A(x,y,z) = A_1 i + A_2 j + A_3 k$ be a vector function of position defined & continuous along C.

$$\int_{P_1}^{P_2} A\cdot dr = \int_C A\cdot dr = \int_C A_1 dx + A_2 dy + A_3 dz$$

• Then the integral of the tangential component of A along C from P1 to P2, written as,

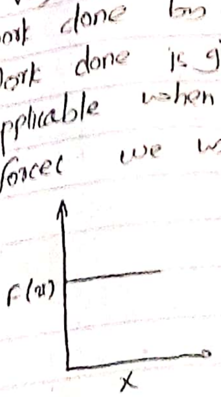
$$\oint A\cdot dr = \oint A_1 dx + A_2 dy + A_3 dz$$

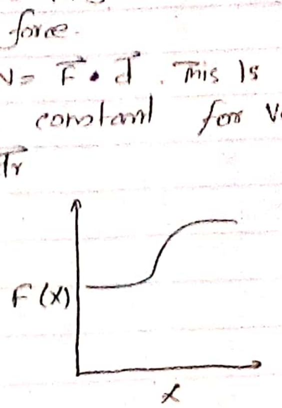
$\frac{dx}{dt} = 2t$ $\qquad$ $\frac{dy}{dt} = 4t$

$\frac{dz}{dt} = 3t^2$

• A very common use of line integral is to find work done by variable force.

• Work done is given by $W = \vec{F}\cdot\vec{d}$. This is applicable when force is constant for variable forces we use $\int \vec{F}\cdot d\vec{r}$





Q. Find the total work done in moving a particle in a force field given by $F = 3xy\,i - 5z\,j + 10x\,k$ along the curve $x = t^2+1$, $y = 2t^2$, $z = t^3$ from $t=1$ to $t=2$.

Sol^n

$$\int_C \vec{F}\cdot d\vec{r} = \int_C (3xy\,i - 5z\,j + 10x\,k)(dx\,i + dy\,j + dz\,k)$$

$$= \int_C (3xy\,dx - 5z\,dy + 10x\,dz)$$

$$= \int_{t=1}^{2} 3(t^2+1)(2t^2)(2t\,dt) - \int_1^2 5(t^3)(4t$$

$$+ \int_1^2 10(t^2+1)(3t^2\,dt)$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_1^2 (3t^2+3)(4t^3)\,dt - \int_1^2 20t^4\,dt + \int_1^2 (10$$

$\frac{dx}{dt} = 2t$ $\frac{dy}{dt} = 4t$

$\frac{dz}{dt} = 3t^2$

- A very common use of line integral is to find work done by variable force.
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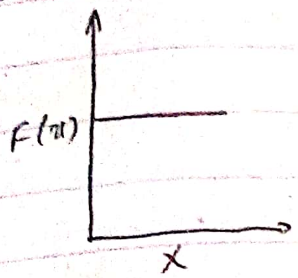
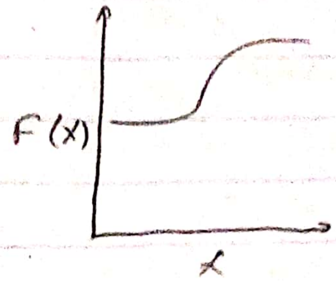




Q. Find the total work done in moving a particle in a force field given by $F = 3xyi - 5zj + 10xk$ along the curve $x = t^2+1$, $y = 2t^2$, $z = t^3$ from $t=1$ to $t=2$.

Sol:

$$\int_C \bar{F} \cdot dr = \int_C (3xyi - 5zj + 10xk)(dxi + dyj + dzk)$$

$$= \int_C (3xy\,dx - 5z\,dy + 10x\,dz)$$

$$= \int_{t=1}^{2} 3(t^2+1)(2t^2)(2t\,dt) - \int_1^2 5(t^3)(4t\,dt) + \int_1^2 10(t^2+1)(3t^2\,dt)$$

$$\int_C \bar{F} \cdot \bar{dr} = \int_1^2 (3t^2+3)(4t^3)\,dt - \int_1^2 20t^4\,dt + \int_1^2 (10t^2+10)\,3t^2\,dt$$

$$\int_C \bar{F}\cdot d\bar{r} = \int_1^2 \left(12t^5 + 12t^3\right) dt - \int_1^2 (20t^4)\, dt + \int_1^2 (30t^4 + 30t^2 \ldots$$

Evaluate $\oint_C A\cdot$ ... fig.

$$= 12\int_1^2 t^5\, dt + 12\int_1^2 t^3\, dt - 20\int_1^2 t^4\, dt + 30\int_1^2 t^4\, dt +$$

$$+ 30\int_1^2 t^2\, dt$$

$$= 12\left[\frac{t^6}{6}\right]_1^2 + 12\left[\frac{t^4}{4}\right]_1^2 - 20\left[\frac{t^5}{5}\right]_1^2 + 30\left[\frac{t^5}{5}\right]_1^2 +$$

$$+ 30\left[\frac{t^3}{3}\right]_1^2$$

$$= 2(64-1) + 3(16-1) - 4(32-1) + 6(32-1) + 10(8$$

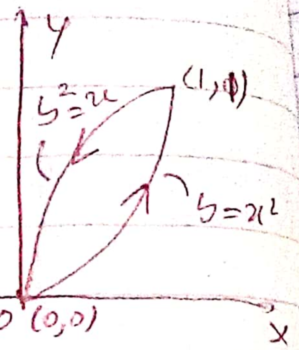
$$= 126 + 45 - 124 + 186 + 70$$

$$= 303$$

Ans

$\frac{dy}{dx} = 2x$

Q. Evaluate $\oint_c A \cdot dr$ around the closed curve c of fig. if $A = (x-y)i + (x+y)j$.



The plane curve $y = x^2$ & $y^2 = x$ intersect at $(0,0)$ & $(1,1)$.

Along $y = x^2$ the line integral is,

$$\int_{x=0}^{1} \bar{A} \cdot d\bar{r}$$

$$\int_0^1 \{(x-y)i + (x+y)j\}\{dx\,i + dy\,j\}$$

$$\int_0^1 (x-y)dx + (x+y)dy$$

$$\int_0^1 (x-x^2)dx + (x+x^2)(2x\,dx)$$

$$\int_0^1 x\,dx - \int_0^1 x^2 dx + 2\int_0^1 x^2 dx + 2\int_0^1 x^3 dx$$

$$\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 + 2\left[\frac{x^3}{3}\right]_0^1 + 2\left[\frac{x^4}{4}\right]_0^1$$

$$\frac{1}{2} - \frac{1}{3} + \frac{2}{3} + \frac{1}{2}$$

$$= \frac{4}{3}$$

Along $y^2 = x$ the

$$\int_{y=1}^{0} (x-y)d\ldots$$

$$\int_1^0 (y^2-y)\ldots$$

$$2\int_1^0 y^3 dy - \ldots$$

$$\Rightarrow 2\left[\frac{y^4}{4}\right]_1^0 - \ldots$$

$$\Rightarrow \frac{1}{2}(-1)\ldots$$

$$\Rightarrow \ldots \frac{2}{3}$$

$$\Rightarrow -\frac{1}{2}$$

$$\Rightarrow -\frac{2}{3}$$

Now the

$$\Rightarrow \frac{4}{3}$$

$$\Rightarrow$$

curve

)j.

(1,1)

y=x^2

x

$\frac{dx}{dy} = 2y$

Along $y^2 = x$ the line integral is,

$\int_{y=1}^{0} (x-y)\,dx + (x+y)\,dy$

$\int_{1}^{0} (y^2-y)\,2y\,dy + (y^2+y)\,dy$

$2\int_1^0 y^3\,dy - 2\int_1^0 y^2\,dy + \int_1^0 y^2\,dy + \int_1^0 y\,dy$

$\Rightarrow 2\left[\frac{y^4}{4}\right]_1^0 - 2\left[\frac{y^3}{3}\right]_1^0 + \left[\frac{y^3}{3}\right]_1^0 + \left[\frac{y^2}{2}\right]_1^0$

$\Rightarrow \frac{1}{2}(-1) - \frac{2}{3}(-1) + \frac{1}{3}(-1) + \frac{1}{2}(-1)$

$\Rightarrow -\frac{1}{2} + \frac{2}{3} - \frac{1}{3} - \frac{1}{2}$

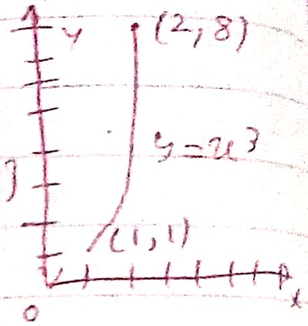
$\Rightarrow -\frac{2}{3}$

Now the required line integral will be,

$\Rightarrow \frac{4}{3} + \left(-\frac{2}{3}\right)$

$\Rightarrow \frac{2}{3}$ Ans

If $F = (5xy-6x^2)i + (2y-4x)j$, evaluate $\int_c F\cdot dr$ along the curve C in the xy-plane, $y=x$ from the point $(1,1)$ to $(2,8)$.



$\int_c F\cdot dr$

$\Rightarrow \int_c \{(5xy-6x^2)i + (2y-4x)j\}(dxi+dyj)$

$\Rightarrow \int_c (5xy-6x^2)dx + (2y-4x)dy$

$\because y = x^3$, $\frac{dy}{dx} = 3x^2 \Rightarrow dy = 3x^2 dx$

$\Rightarrow \int_c \{5x(x^3)-6x^2\}dx + (2x^3-4x)(3x^2 dx)$

$\Rightarrow \int_{x=1}^{2} (5x^4-6x^2)dx + (6x^5-12x^3)dx$

$\Rightarrow 5\int_{x=1}^{2} x^4 dx - 6\int_1^2 x^2 dx + 6\int_1^2 x^5 dx - 12\int_1^{2} x^3 dx$

$\Rightarrow \frac{5}{5}[x^5]_1^2 - \frac{6}{3}[x^3]_1^2 + \frac{6}{6}[x^6]_1^2 - \frac{12}{4}[x^4]_1^2$

$\Rightarrow (32-1) - 2(8-1) + (64-1) - 3(16-1)$

$\Rightarrow 35$ Ans

Q $\nabla|r|^3$

let $\vec{r} = xi+$

$|r| = \sqrt{}$

$\nabla|r|^3 = ($

$\boxed{\nabla|r|^3}$

Q $\nabla|r|^3$

let $\vec{r} = xi + yj + zk$

$|r| = \sqrt{x^2+y^2+z^2}$

$$\nabla|r|^3 = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(x^2+y^2+z^2)^{3/2}$$

$$= \left\{\cancel{2x}\,\frac{3}{\cancel{2}}(x^2+y^2+z^2)^{1/2}(\cancel{2}x)\right\} i + \left\{\frac{3}{\cancel{2}}(x^2+y^2+z^2)^{1/2}(\cancel{2}y)\right\} j + \left\{\frac{3}{\cancel{2}}(x^2+y^2+z^2)^{1/2}(\cancel{2}z)\right\} k$$

$$= 3|r|xi + 3|r|yj + 3|r|zk$$

$$= 3|r|(xi+yj+zk)$$

$$\boxed{\nabla|r|^3 = 3|r|\vec{r}}$$

Ans

The acceleration a of a particle at any time $t \geq 0$ is given by $a = e^{-t} i - 6(t+1) j + 3 \sin t k$. If the velocity v & displacement r are zero at $t=0$, find v & r at any time.

$\because a = e^{-t}\hat{i} - 6(t+1)\hat{j} + 3\sin t \hat{k}$

$\int a\,dt = \int e^{-t} i - 6(t+1) j + 3 \sin t k$

$v = i\int e^{-t}\,dt - 6j\int t\,dt - 6j\int dt + 3k\int \sin t\,dt$

$v = \Big| i\Big[\frac{e^{-t}}{\;}\Big] - 6i\Big[\frac{t^2}{2} + t\Big] + 3k\big[\cos t\big] + c$

$$V = i\left[\frac{e^{1-t} - e^{t}e^{t}}{(1-t)}\right] - (3t^2+6t)j + 3k - 3\cos t\, k$$

$$V = i\left[\frac{e^{2-t} - e^{t}}{(1-t)}\right].$$

$$V = i\left[\frac{e^{-t}}{(-1)}\right] - 6j\left[\frac{t^2}{2} + t\right] + 3k\left[-\cos t\right] + c$$

$$= i\left[-e^{-t}\right] - \frac{\cancel{6}^{3}j}{\cancel{2}}\left[t^2 + 2t\right] - 3k\cos t + c$$

$$V = i\left[-e^{-t}\right] - (3t^2+6t)j - 3k\cos t + c \longrightarrow ①$$

$\because$ at $t=0$, V is also 0.

$$① \Rightarrow 0 = i(-e^{0}) - 0j - 3k\cos(0) + c$$

$$0 = -i - 3k + c$$

$$\boxed{c = i + 3k}$$

$$① \Rightarrow V = i(-e^{-t}) - (3t^2+6t)j - 3k\cos t + (i + 3k)$$

$$V = -e^{-t}i + i - (3t^2+6t)j - 3k\cos t + 3k$$

$$\boxed{V = (1-e^{-t})i - (3t^2+6t)j + (3-3\cos t)k}$$

$\int v\,dt = \int (1-e^{-t})i - (3t^2+6t)j + (3-3\cos t)k$

$r = i\int (1-e^{-t})dt - j\left[3\int t^2 dt + 6\int t\,dt\right] + k\left[3\int\right.$

$\left. - 3\int \cos t\,dt\right]$

$r = i\left[\{t\} - \left\{\frac{e^{-t}}{(-1)}\right\}\right] - j\left[\frac{3}{3}(t^3) + \frac{6}{2}(t^2)\right] +$

$+ k\left[3(t) - 3(\sin t)\right] + c$

$r = i\left[t + e^{-t}\right] - j\left[t^3 + 3t^2\right] + k\left[3t - 3\sin\right.$

$\because$ at $t=0$, $r=0$

$\Rightarrow 0 = i(0+1) - j(0+0) + k(0-0) + c$

$0 = i + c$

$\boxed{c = -i}$

(2) $\Rightarrow r = i(t+e^{-t}) - i - j(t^3+3t^2) + k(3t-3\sin$

$r = (t-1+e^{-t})i - j(t^3+3t^2) + k(3t-3\sin$

Q. If $F = (2x$
where c
consisting
$(0,0)$ to

$\int_c \vec{F}\cdot d\vec{r} =$

$\int_{c_1} \vec{F}\cdot d\vec{r} =$

$\int_{c_1} \vec{F}\cdot d\vec{r}$

$\Rightarrow x$

$\Rightarrow$

$\Rightarrow \int_{x=0}^{2}($

$\Rightarrow 2\int$

$\Rightarrow \frac{2}{2}$

$\Rightarrow$

Now

$\int_{c_2}$

Q. If $F=(2x+y)i+(3y-x)j$, evaluate $\int_c F\cdot dr$ where c is the curve in xy-plane consisting of the straight lines from $(0,0)$ to $(2,0)$ & then to $(3,2)$.

$$\int_c \bar{F}\cdot d\bar{r} = \int_{c_1} \bar{F}\cdot d\bar{r} + \int_{c_2} \bar{F}\cdot d\bar{r} \longrightarrow (1)$$

$$\int_{c_1} \bar{F}\cdot d\bar{r} = \int_{c_1} (2x+y)i+(3y-x)j\,(dx\,i+dy\,j)$$

$$\int_{c_1} \bar{F}\cdot d\bar{r} = \int_{c_1} (2x+y)dx+(3y-x)dy$$

as x varies from $0 \to 2$ $\quad (0,0)$ to $(2,0)$

$\Rightarrow \quad y=0 \quad \& \quad dy=0$

$$\Rightarrow \int_{x=0}^{2} (2x+0)dx + (0-x)(0)$$

$$\Rightarrow 2\int_0^2 x\,dx$$

$$\Rightarrow \frac{2}{2}\left[x^2\right]_0^2$$

$$\Rightarrow \boxed{\int_{c_1} \bar{F}\cdot d\bar{r} = 4}$$

Now.

$$\int_{c_2} \bar{F}\cdot d\bar{r} = \int_{c_2} (2x+y)dx + (3y-x)dy$$

in coordinates $(2,0)$ to $(3,2)$
using two point formula.

$$(y-y_1) = \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1)$$

$$y-0 = \frac{2}{1}(x-2)$$

$$\boxed{y = 2x-4} \qquad dy = 2dx$$

Now.

$$\int_{C_2} \bar{F}\cdot\bar{dr} = \int_{x=2}^{3} (2x + \overset{2x-4}{2x-12})dx + \int_2^3 \{(6x-12)-2x\}$$

$$= \int_2^3 (4x-4)dx + \int_2^3 (5x-12)dx$$

$$= 4\int_2^3 x\,dx - 4\int_2^3 dx + 5\int_2^3 x\,dx - 12\int_2^3 dx$$

$$= \frac{4}{2}\left[x^2\right]_2^3 - 4\left[x\right]_2^3 + \frac{5}{2}\left[x^2\right]_2^3 - 12\left[x\right]_2^3$$

$$= 2(9-4) - 4(3-2) + \frac{5}{2}(9-4) - 12(3-2)$$

$$= 2(5) - 4(1) + \frac{5}{2}(5) -$$

$$\int_{C_2} \bar{F}\cdot\bar{dr} = \int_2^3 (4x-4)dx + \int_2^3 (10x-24)dx$$

$\Rightarrow 4\int_2^3 x\,dx - 4\int_2^3 dx + 10\int_2^3 x\,dx - 24\int_2^3 dx$

$\Rightarrow \frac{4}{2}\left[x^2\right]_2^3 - 4\left[x\right]_2^3 + \frac{10}{2}\left[x^2\right]_2^3 - 24\left[x\right]_2^3$

$\Rightarrow 2(5) - 4(1) + 5(5) - 24(1)$

$\Rightarrow 10 - 4 + 25 - 24$

$\Rightarrow 7$

$\boxed{\int_{c_2} F \cdot dr = 7}$

(1) $\Rightarrow \boxed{\int_c \bar{F} \cdot \bar{dr} = 4 + 7 = 1}$ Ans

~~Electric Potential~~ & Potential Diff.

Q. find the work done in moving a particle once around a circle in the xy-plane if the circle has center at the origin & radius 3 & if the force field is given by. $F = (2x-y+z)i+(x+y-z^2)j+(3x-2y+4z)k$

$\because$ we are in xy-plane

$\Rightarrow z=0$

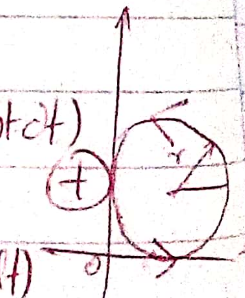
$$\Rightarrow \int_c \bar{F}\cdot d\bar{r} = \int_c \{(2x-y)i+(x+y)j+(3x-2y)k\}\cdot(dx\,i+dy\,j+dz\,k)$$

$$\int_c \bar{F}\cdot d\bar{r} = \int_c (2x-y)dx+(x+y)dy$$

using parametric eq. of circle

$x = r\cos t$ & $y = r\sin t$ to $0 \to 2\pi$

$dx = -r\sin t\,dt$ & $dy = r\cos t\,dt$



$$\int_c \bar{F}\cdot d\bar{r} = \int_{t=0}^{2\pi} \{2(3\cos t)-3\sin t\}(-3\sin t\,dt) + \{(3\cos t+3\sin t)(3\cos t\,dt)\}$$

$$= \int_0^{2\pi} (6\cos t-3\sin t)(-3\sin t\,dt)+(3\cos t+3\sin t)(3\cos t\,dt)$$

$$= \int_0^{2\pi}\left(-18\cos t\sin t+9\sin^2 t\right)dt+\left(9\cos^2 t+9\cos t\sin t\right)$$

$= \int_0^{2\pi} \{7 \cos t \sin t - 18 \cos t \sin t + 9(\cos^2 t + \sin^2 t)\}\,dt$

$= \int_0^{2\pi} (9 - 9 \cos t \sin t)\,dt$

$= 9\int_0^{2\pi} dt - 9 \int_0^{2\pi} \cos t \sin t\,dt$

$9t\Big|_0^{2\pi} \quad 9\left[\frac{\cos^2 t}{2}\right]_0^{2\pi}$

let $u = \cos t$

$du = -\sin t\,dt$

$9(2\pi) + \frac{9}{2} \quad 9t - \ldots$

18π Ans

Pg (102) Schaum's

Q37. If $A = (2y+3)i + xzj + (yz-x)k$, evaluate $\int_c A \cdot dr$ along the following paths C.

(a). $x = 2t^2$, $y = t$ & $z = t^3$ from $t=0$ to $t=1$.

$dx = 4t\,dt$ $dy = dt$ & $dz = 3t^2\,dt$

$$\int A \cdot dr = \int_c (2y+3)i + xzj + (yz-x)k\ (dx\,i + dy\,j + dz\,k)$$

$$= \int_0^1 (2t+3)dx + 2t^5 dy + (t^4 - 2t^2)dz$$

$$= \int_0^1 (2t+3)4t\,dt + 2t^5\,dt + (t^4 - 2t^2)\,3t^2\,dt$$

$$= \int_0^1 (8t^2 + 12t)dt + 2t^5\,dt + (3t^6 - 6t^4)dt$$

$$= 8\int_0^1 t^2\,dt + 12\int_0^1 t\,dt + 2\int_0^1 t^5\,dt + 3\int_0^1 t^6\,dt - 6\int_0^1 t^4\,dt$$

$$= \frac{8}{3}\left[t^3\right]_0^1 + \frac{12}{2}\left[t^2\right]_0^1 + \frac{2}{6}\left[t^6\right]_0^1 + \frac{3}{7}\left[t^7\right]_0^1 - \frac{6}{5}\left[t^5\right]_0^1$$

$$= \frac{8}{3} + 6 + \frac{1}{3} + \frac{3}{7} - \frac{6}{5}$$

$$\boxed{\int A \cdot dr = \frac{288}{35}}$$

(b) the straigh

(0,0,0) –

$\Rightarrow z$

$x =$

$\Rightarrow \int_0^1 (yz -$

$\Rightarrow 0$

(0,

$\Rightarrow \int_0^8$

$\Rightarrow 0$

$\Rightarrow \int_{x=0}^2$

$\Rightarrow 5$

$\Rightarrow$

$\Rightarrow$

(b) the straight lines $(0,0,0)$ to $(0,0,1)$ then to $(0,1,1)$ then to $(2,1,1)$

$(0,0,0) \longrightarrow (0,0,1)$

as z varies $0 \longrightarrow 1$

$x=0$, $y=0$ $dx=0$, $dy=0$

$$\Rightarrow \int_{z=0}^{1} (yz - x)\,dz$$

$$\Rightarrow 0$$

$(0,0,1)$ to $(0,1,1)$

as y varies from $0 \longrightarrow 1$

$x=0$, $dx=0$, $z=1$ & $dz=0$

$$\Rightarrow \int_{y=0}^{1} \cancel{(2x+3)}\ \cancel{(yz-x)dx}$$

$$\Rightarrow 0$$

$(0,1,1) \longrightarrow (2,1,1)$

as $x \Rightarrow 0 \longrightarrow 2$

$y=1$ & $z=1$ & dy & $dz=0$

$$\Rightarrow \int_{x=0}^{2} (2+3)\,dx$$

$$\Rightarrow 5\,[x]_0^2$$

$$\Rightarrow 5(2)$$

$$\Rightarrow 10$$

$$\Rightarrow 0+0+10 = 10$$ Ans

(c). $\overset{x_1, y_1, z_1}{(0,0,0)} \longrightarrow \overset{a\ b\ c}{(2,1,1)}$

$$\int_c A \cdot dr = \int_c (2y+3)dx + xz\,dy + (yz-x)dz \quad \text{--- (1)}$$

Now the eq. of straight line $(0,0,0) \rightarrow (2,1,1)$ is,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\frac{(x-0)}{2} = \frac{y-0}{1} = \frac{z-0}{1}$$

$$\frac{x}{2} = y = z$$

$\Rightarrow z = y$ & $x = 2y$

$dz = dy$ $\quad dx = 2dy$

(1) $\Rightarrow \int_0^1 (2y+3)2dy + 2y^2 dy + (y^2-2y)dy$

$\Rightarrow \int_0^1 (4y+6)dy + 2y^2 dy + (y^2-2y)dy$

$\Rightarrow 4\int_0^1 y\,dy + 6\int_0^1 dy + 2\int_0^1 y^2 dy + \int_0^1 y^2 dy - 2\int_0^1 y\,dy$

$\Rightarrow \frac{4}{2}\left[y^2\right]_0^1 + 6\left[y\right]_0^1 + \frac{2}{3}\left[y^3\right]_0^1 + \frac{1}{3}\left[y^3\right]_0^1 - \frac{2}{2}[$

$\Rightarrow 2 + 6 + \frac{2}{3} + \frac{1}{3} - 1$

$\Rightarrow 8$

Q. The net flux
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