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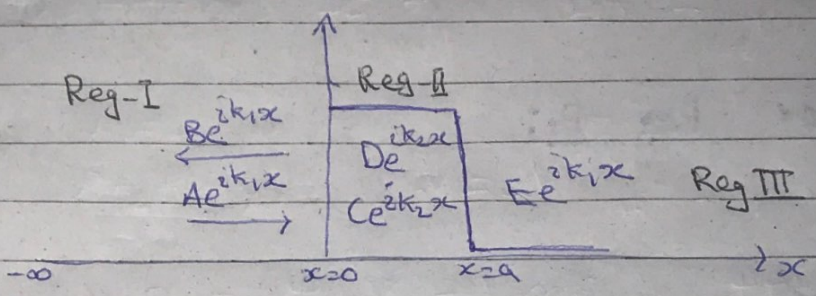
Potential Barrier:-



Case 1 $E > V_0$:-

Beam of Particles of mass 'm'

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 < x \le a \\ 0 & x > a \end{cases}$$

For Reg I:-

From Schrodinger wave eq:

$V(x) = 0 \qquad x < 0$

$$\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi_1 = 0$$

$$|V=0|$$

$$\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2}(E-0)\psi_1 = 0$$

where,

$$\frac{2mE}{\hbar^2} = K_1^2$$

$$\frac{d^2\psi_1}{dx^2} + K_1^2\psi = 0 \longrightarrow (1)$$

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By solving this we get:

$$\psi_1 = Ae^{2K_1x} + Be^{-2K_1x} \longrightarrow (A)$$

For Reg 2:

$$V_x = V_0 \qquad 0 < x \leq a$$

$$\frac{d^2\psi_2}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi_2 = 0$$

where;

$$\frac{2m}{\hbar^2}(E - V_0) = K_2^2$$

where;

$$\frac{2m}{\hbar^2}(E - V_0) = K_2^2$$

$$\frac{d^2\psi_2}{dx^2} + K_2^2\psi_2 = 0 \longrightarrow (ii)$$

By solving this we have:

$$\psi_2 = Ce^{2K_2x} + De^{-2K_2x} \longrightarrow (B)$$

For Reg 3:

$$V(x) = 0 \qquad x > a$$

$$\frac{d^2\psi_3}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi_3 = 0$$

$$\frac{d^2\psi_3}{dx^2} + \frac{2m}{\hbar^2}(E - 0)\psi_3 = 0$$

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$$\frac{d^2\psi_3}{dx^2} + \frac{2m}{\hbar^2} E \psi_3 = 0$$

where;

$$\frac{2mE}{\hbar^2} = K_1^2$$

So,

$$\frac{d^2\psi_3}{dx^2} + K_1^2 \psi_3 = 0 \longrightarrow (iii)$$

By salving this we have:

$$\psi_3 = Ee^{ik_1x} + Fe^{-ik_1x} \longrightarrow (C)$$

* where in eq (A) 'A' gave the information of incident wave in Reg-I, and "B" gave the information of reflected wave in Reg-I.

* In eq (B) "C" gave the information of transmitted wave in Reg-II. and "D" gave information of reflected wave in Reg-II.

* In eq (C) "E" gave information of transmitted wave in Reg-III and "F" gave information of reflected wave in Reg-III. In Reg III Here is no reflected wave So, $Fe^{-ik_1x} = 0$

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Find the values of Constant:

To find A, B, C, D, E we will apply Boundary conditions:

at $x = 0$

$\psi_1 = \psi_2$

$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx}$

at $x = a$

$\psi_2 = \psi_3$

$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$

For eq A, B at $x = 0$

when:

$\psi_1 = \psi_2$

By Comparing:

$$Ae^{ik_1x} + Be^{-ik_1x} = Ce^{ik_2x} + De^{-ik_2x}$$

$$Ae^{ik_1(0)} + Be^{-ik_1(0)} = Ce^{ik_2(0)} + De^{-ik_2(0)}$$

$$A + B = C + D \longrightarrow (D)$$

when: $\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx}$

Taking Differentiate of eq A & B

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$$

$$\frac{d\psi_1}{dx} = ik_1Ae^{ik_1x} - ik_1Be^{-ik_1x} \longrightarrow (E)$$

$$\psi_2 = Ce^{ik_2x} + De^{-ik_2x}$$

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$$\frac{d\psi_2}{dx} = ik_2 Ce^{ik_2x} - ik_2 De^{-ik_2x} \longrightarrow (F)$$

Comparing eq (E) & (F)

$$ik_1 Ae^{ik_1x} - ik_1 Be^{-ik_1x} = ik_2 Ce^{ik_2x} - ik_2 De^{-ik_2x}$$

$$ik_1 Ae^{ik_1(0)} - ik_1 Be^{-ik_1(0)} = ik_2 Ce^{ik_2(0)} - ik_2 De^{-ik_2(0)}$$

$$ik_1 A - ik_1 B = ik_2 C - ik_2 D \longrightarrow (G)$$

Now For eq 'B' and 'C' at

$x = a$

when:

$$\psi_2 = \psi_3$$

By Comparing:

$$Ce^{ik_2x} + De^{-ik_2x} = Ee^{ik_1x}$$

$$Ce^{ik_2(a)} + De^{-ik_2(a)} = Ee^{ik_1(a)} \longrightarrow (H)$$

when: $\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$

Taking differentiation of eq 'B' & 'C'

$$\psi_2 = Ce^{ik_2x} + De^{-ik_2x}$$

$$\frac{d\psi_2}{dx} = ik_2 Ce^{ik_2x} - ik_2 De^{-ik_2x} \longrightarrow (I)$$

$$\psi_3 = Ee^{ik_1x}$$

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$$\frac{d\psi_3}{dx} = iK_1 E e^{iK_1 x}$$

$$\frac{d\psi_3}{dx} = iK_1 E e^{iK_1 (a)} \longrightarrow \text{(J)}$$

Compare I & J

$$iK_2 C e^{iK_2 a} - iK_2 D e^{-iK_2 a} = iK_1 E e^{iK_1 a} \longrightarrow \text{(K)}$$

$$K_2 \left(iCe^{iK_2 a} - iDe^{-iK_2 a} \right) = K_1 iEe^{iK_1 a}$$

$$iCe^{iK_2 a} - iDe^{-iK_2 a} = \frac{K_1}{K_2} iEe^{iK_1 a}$$

$$i\left(Ce^{iK_2 a} - De^{-iK_2 a}\right) = i\frac{K_1}{K_2} Ee^{iK_1 a}$$

$$Ce^{iK_2 a} - De^{-iK_2 a} = \frac{K_1}{K_2} Ee^{iK_1 a} \longrightarrow \text{(L)}$$

Now:

$$Ce^{iK_2 a} + De^{-iK_2 a} = Ee^{iK_1 a} \longrightarrow \text{(H)}$$

$$Ce^{iK_2 a} - De^{-iK_2 a} = \frac{K_1}{K_2} Ee^{iK_1 a} \longrightarrow \text{(L)}$$

For transmission coefficient:

$$T = \frac{K_1}{K_1} \frac{|E|^2}{|A|^2}$$

$$T = \left[1 + \frac{V_0^2}{4E(E-V_0)} \sin^2\left(a\sqrt{\frac{2mV_0}{\hbar^2}} \sqrt{E/V_0 - 1} \right) \right]^{-1}$$

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$$\lambda = a\sqrt{\frac{2mV_0}{\hbar^2}} \qquad \frac{E}{V_0} = \varepsilon$$

$$T = \left[1 + \frac{1}{4\varepsilon(\varepsilon-1)} \sin^2\lambda\sqrt{\varepsilon-1}\right]^{-1}$$

$$T = \left[\frac{1 + \sin^2\lambda\sqrt{\varepsilon-1}}{4\varepsilon(\varepsilon-1)}\right]^{-1}$$

$$R + T = 1$$

$$R = 1 - T$$

$$R = \left[1 + \frac{4\varepsilon(\varepsilon-1)}{\sin^2\lambda\sqrt{\varepsilon-1}}\right]^{-1}$$

Case-I

$E > V_0$

$E >> V_0 \qquad \varepsilon >> 1$

$T = 1 \qquad R = 0$

Case-II

$T = 1$ when:

$$\sin^2\lambda\sqrt{\varepsilon-1} = 0$$

$$\lambda = \sqrt{\varepsilon-1} = n\pi$$

$$\sqrt{\varepsilon-1} = \frac{n\pi}{\lambda}$$

$$\varepsilon = \frac{n^2\pi^2}{\lambda^2} - 1$$

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$$E = \frac{n^2\pi^2 V_0}{\frac{a^2 2mV_0}{\hbar^2}} + V_0$$

$$E_n = \frac{n^2\hbar^2\pi^2}{2a^2m} + V_0$$

$$n = 1, 2, 3 \ldots\ldots$$

Total Transmission is possible when energy of the incident particles in integral multiple of $\left(\frac{\hbar^2\pi^2}{2a^2m} + V_0\right)$