# Network Flows

- Start of network = source & end = sink.
- Capacity of arc = no. neat go it.
- Saturated arcs are at full capacity.

- Maximum-flow minimum-cut theorem:
  - States that the flow through any network cannot exceed the value of any cut
  - maximum flow = value of minimum cut

- Add super sinks and super sources to questions containing multiple sinks and sources. These should have a sensible capacity and removed after finding a maximum flow.

- A node may have a restricted capacity : add in an extra arc to replace the node.

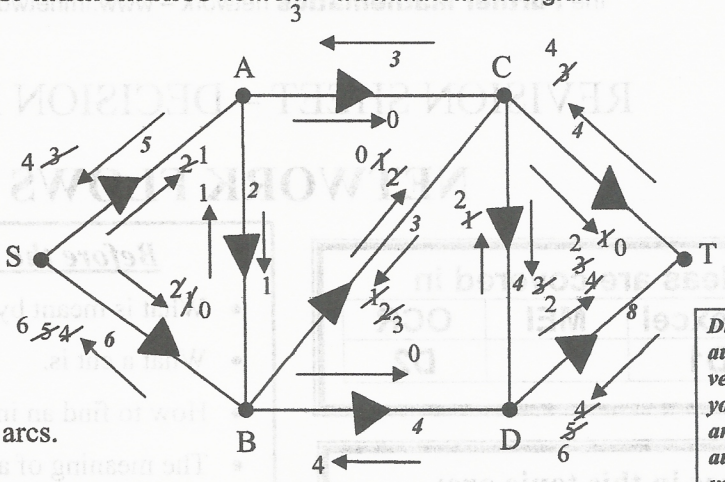
- For networks with minimum flows, the value of any cut is now the sum of the upper capacities from S to T minus the sum of the lower capacities from T to S.

c) Flow augmenting paths

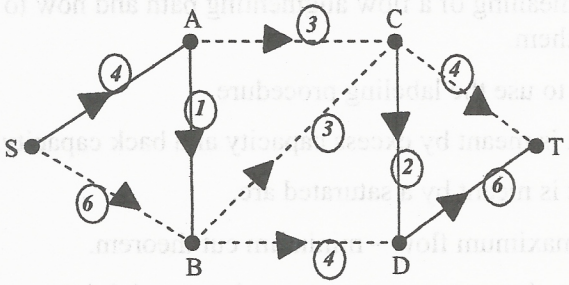
SBCT with capacity 1

SBCDT with capacity 1

SABCDT with capacity 1




This gives a maximum total flow of 10. The flow is shown on this diagram, along with the saturated arcs.



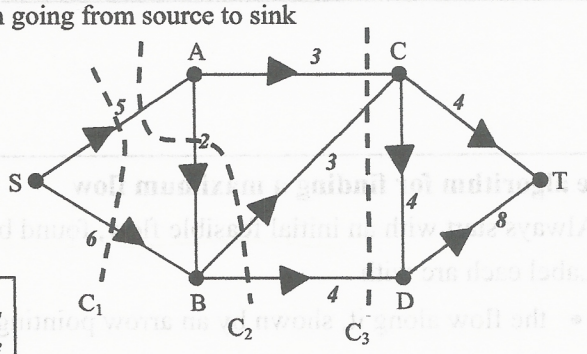

***Diagrams showing the flow augmenting paths can be very messy. Try to keep yours as tidy as possible and always list the flow augmenting paths you have used***

## *Cuts*

A cut partitions the vertices into two sets, one containing the source and one containing the sink.

The capacity of a cut is the total of all the cut edges with direction going from source to sink

***Find the capacity of the cuts shown on the directed network:***




Note that only three cuts have been shown here, but there are many more cuts in this network.

$C_1$ is the cut {S}, {A, B, C, D, T}

It has capacity 5 + 6 = 11

$C_2$ is the cut {S, B}, {A, C, D, T}

It has capacity 5 + 0 + 3 + 4 = 12

$C_3$ is the cut {S, A, B}, {C, D, T}

It has capacity 3 + 3 + 4 = 10

***Note that we do not add the capacity of arc AB as it is directed from the sink side of the cut to the source side***

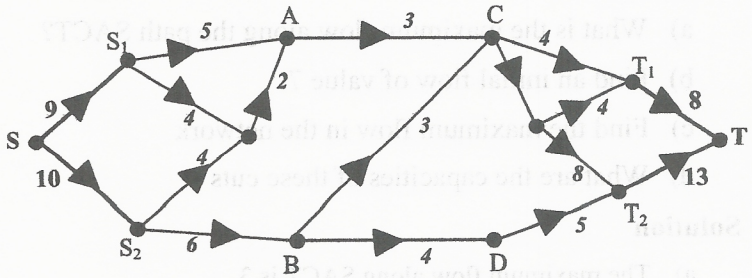
## *Maximum flow- minimum cut theorem.*

The theorem states that the maximum flow in a directed network is equal to the capacity of the minimum cut.

In the example above the cut $C_2$ is the minimum cut and it has a value 10. This confirms that the flow of 10 found in (c) above is the maximum flow.

## *Networks with many sources and sinks*

If there is more than one source ($S_1$ and $S_2$ on the diagram) or sink ($T_1$ and $T_2$ on the diagram) you must introduce ***supersource (S)*** and/or ***supersink (T).***




$SS_1$ must have a capacity 5 + 4 = 9

$SS_2$ must have capacity 4 + 6 = 10

$T_1T$ must have capacity 4 + 4 = 8

$T_2T$ must have capacity 8 + 5 = 13