SF - Exponential Distribution

Introduction

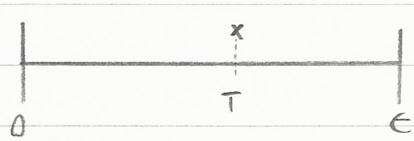
- Let λ = mean no. of occurences in unit time
  $\Rightarrow \lambda t$ = mean no. in time t
- For Poisson, we know $P(x) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$.
- We can use an exponential dist^{n} to describe the waiting times between Poisson events.

Let $X \sim Po(\lambda)$ so X_t can be the time in which no events occur.

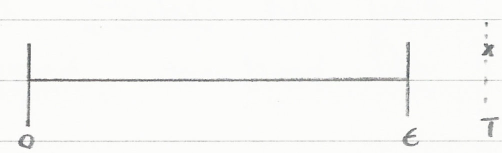
$\Rightarrow P(X_t = 0) = e^{-\lambda t}$

& X_T can be the time until the first occurence (i.e., $T > t$)

We have two outcomes:



or



$P(T \leq t) = 1 - e^{-\lambda t}$ $\qquad$ $P(T \geq t) = e^{-\lambda t}$

Since this is the probability, this is a cumulative function

$\Rightarrow \text{pdf} = \dfrac{d}{dt}\left(1 - e^{-\lambda t}\right) = \lambda e^{-\lambda t}$

Mean & Variance

$$E(X) = \int_0^{\infty} x f(x)\, dx = \lambda \int_0^{\infty} x e^{-\lambda x}\, dx$$

$$= \lambda \left\{ \left[x \frac{e^{-\lambda x}}{-\lambda} \right]_0^{\infty} - \int_0^{\infty} \frac{e^{-\lambda x}}{-\lambda}\, dx \right\}$$

$$= \lambda \left\{ 0 - \left[\frac{e^{-\lambda x}}{\lambda^2} \right]_0^{\infty} \right\} = \lambda \left(\frac{1}{\lambda^2} \right)$$

$$= \frac{1}{\lambda}$$

$$E(X^2) = \int_0^{\infty} x^2 f(x)\,dx = \lambda \int_0^{\infty} x^2 e^{-\lambda x}\,dx$$

$$= \lambda \left\{ \left[x^2 \frac{e^{-\lambda x}}{-\lambda} \right]_0^{\infty} + \int_0^{\infty} \frac{e^{-\lambda x}}{\lambda} 2x\,dx \right\}$$

$$= \lambda \left\{ 0 - 2\left(\left[x \frac{e^{-\lambda x}}{\lambda^2} \right]_0^{\infty} - \int_0^{\infty} \frac{e^{-\lambda x}}{\lambda^2}\,dx \right) \right\}$$

$$= \lambda \left\{ 0 - 2\left(0 + \left[\frac{e^{-\lambda x}}{\lambda^3} \right]_0^{\infty} \right) \right\} = \lambda \frac{2}{\lambda^3} = \frac{2}{\lambda^2}$$

$$\therefore \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Memory-less Property Imagine starting to wait for a call @ $t = 0$. No matter how long we wait, the dist^n 'forgets' it as though it had not happened. Starting afresh each time.