Proof By Induction

Example for Series

Prove $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

When $n=1$, LHS $= \sum_{r=1}^{1} r = 1$ & RHS $= \frac{1}{2}(1)(2) = 1$

$\therefore$ true when $n=1$.

Assume true when $n=k \Rightarrow \sum_{r=1}^{k} r = \frac{1}{2}k(k+1)$

Show true when $n=k+1 \Rightarrow \sum_{r=1}^{k+1} r = \sum_{r=1}^{k} + (k+1)$

$= \frac{1}{2}k(k+1) + (k+1)$

$= \frac{1}{2}(k+1)\left[k+2\right] = \frac{1}{2}(k+1)(k+2)$

$= \frac{1}{2}\left[k+1\right]\left[(k+1)+1\right]$

$\therefore$ true when $n=k+1$ & since true for $n=1$, true for $n=2$, $3, 4, \ldots$ for all numbers $n \in \mathbb{N}$

Example for Divisibility

Show that $3^{2n}-1$ is divisible by 8 for all positive integers.

Let $f(n) = 3^{2n}-1$. When $f(1) = 3^{2}-1 = 8 \Rightarrow$ true for $n=1$.

Assume true for $n=k \Rightarrow f(k)$ is divisible by 8.

When $n=k+1 \Rightarrow f(k+1) - f(k) = 3^{2(k+1)} - 1 - (3^{2k}-1)$

$= 3^{2k+2} - 1 - 3^{2k} + 1$

$= 3^{2k+2} - 3^{2k}$

$= 3^{2k} \times 3^{2} - 3^{2k}$

$= 9(3^{2k}) - 3^{2k}$

$= 8(3^{2k})$

$\therefore f(k+1) = 8(3^{2k}) + f(k)$ $\therefore$ both terms are divisible by 8

Example for Multiplication (Matrix)

$A = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$, prove $A^n = \begin{pmatrix} 2n+1 & -n \\ 4n & 1-2n \end{pmatrix}$ where $n \in \mathbb{N}$

When $n=1$, $A^1 = \begin{pmatrix} 2(1) & -(1) \\ 4(1) & 1-2(1) \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$

$\therefore$ true when $n=1$

Assume true when $n=k \Rightarrow A^k = \begin{pmatrix} 2k+1 & -k \\ 4k & 1-2k \end{pmatrix}$

$\therefore \quad A^{k+1} = A^k A = \begin{pmatrix} 2k+1 & -k \\ 4k & 1-2k \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$

$= \begin{pmatrix} 2k+3 & -k-1 \\ 4k+4 & -2k-1 \end{pmatrix} = \begin{pmatrix} 2(k+1)+1 & -(k+1) \\ 4(k+1) & 1-2(k+1) \end{pmatrix}$

$\therefore$ true for $n=1$ & for $n=k+1$, true for $n=2,3,4,\ldots$ for all numbers $n \in \mathbb{N}$