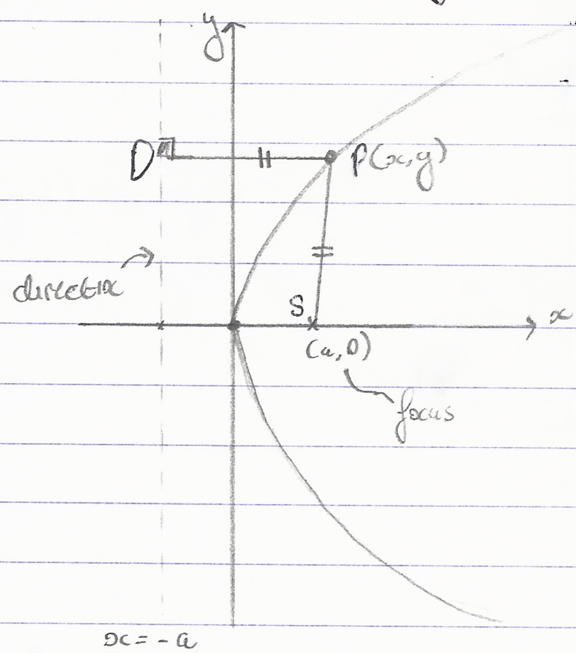
Parabolas, Ellipses, & Hyperbolae

Parabola



Defined as a point P equidistant from the focus & directrix.

The directrix & focus are the same distance from the y-axis.

$PS = PD \Rightarrow (PS)^2 = (PD)^2$

Don't need to know derivation

By pythagoras $(PS)^2 = y^2 + (x-a)^2 =$

$\because PD = x + a$

$(PS)^2 = x + a \Rightarrow$

$$y^2 + (x-a)^2 = (x+a)^2$$

$$y^2 + \cancel{x^2} - 2ax + \cancel{a^2} = (x+a)^2$$

$$= \cancel{x^2} + 2ax + \cancel{a^2}$$

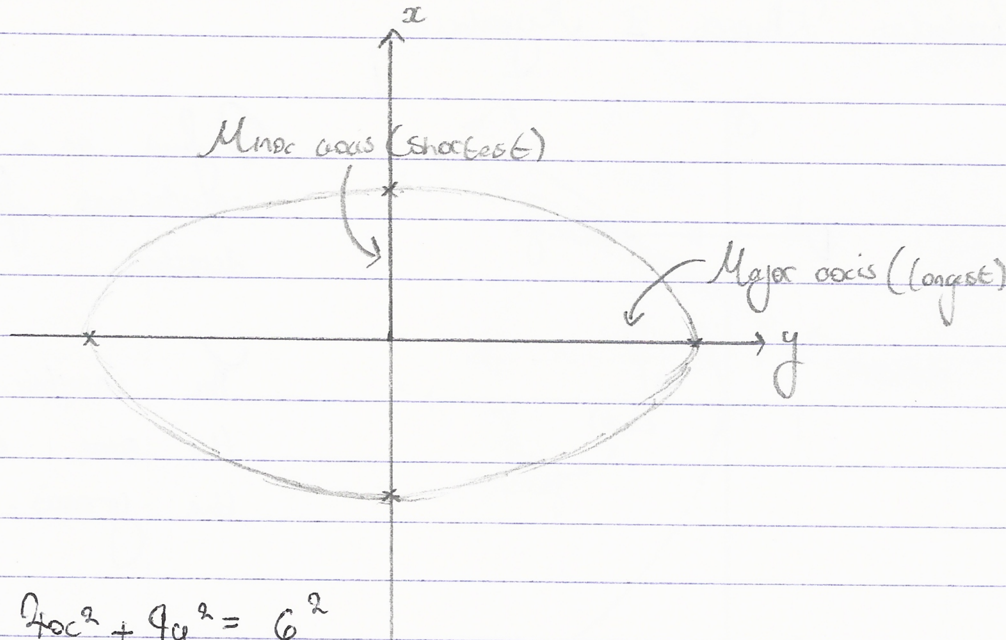
$$y^2 - 2ax = 2ax$$

$$y^2 = 4ax$$

E.g., $y^2 = 12x \Rightarrow 12 = 4a \Rightarrow a = 3$

focus $= (3,0)$ & direction $\Rightarrow x = -3$

Ellipse

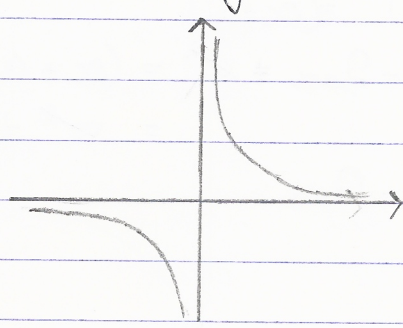


E.g., $4x^2 + 9y^2 = 6^2$

$$\frac{4x^2}{6^2} + \frac{9y^2}{6^2} = 1 \Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$

$\Rightarrow$ intercepts x-axis @ $(3,0)$ & $(-3,0)$
y-axis @ $(2,0)$ & $(-2,0)$

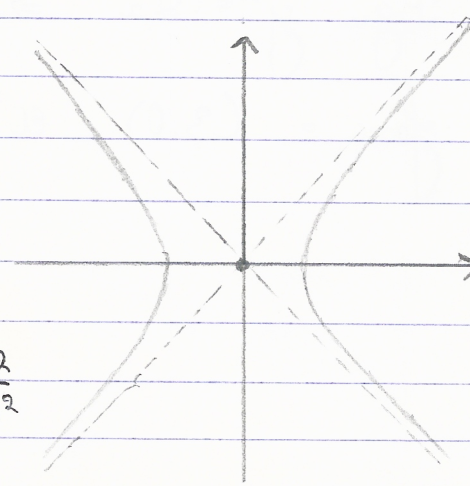
Rectangular Hyperbolae



Form: $xy = c^2$
with the x-axis & y-axis as asymptotes

Hyperbolae

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Asymptotes: $\frac{x^2}{a^2} = 1 + \frac{y^2}{b^2}$

as x & $y \longrightarrow \infty$, $\frac{x^2}{a^2} \approx \frac{y^2}{b^2}$

$\Rightarrow y^2 = \frac{b^2}{a^2} x \Rightarrow y = \pm \frac{b}{a} x$