🚀 DAY 4: LINEAR COMBINATIONS, SPAN & BASIS – INTUITION + CODE

1. WHAT IS A LINEAR COMBINATION?

A linear combination of vectors means:

- Taking vectors, scaling (multiplying) them by numbers (scalars), and adding the results.

Mathematically:

If `v₁ = [a, b]` and `v₂ = [c, d]`,

then any vector of the form:

`v = α·v₁ + β·v₂`

is a linear combination of `v₁` and `v₂`.

✅ This helps us understand how vectors can "build" other vectors.

2. WHAT IS THE SPAN?

The span of vectors is:

- The set of all possible vectors you can create by linear combinations of given vectors.

Example:

- 1 vector ⇒ Line (in 2D).
- 2 non-parallel vectors ⇒ A plane (in 2D, it can cover all of 2D).
- In 3D: 3 non-coplanar vectors ⇒ Space.

In Python (Visualized):

```
import numpy as np
import matplotlib.pyplot as plt
```

```
plt.style.use('seaborn-v0_8-whitegrid')

v1 = np.array([2, 1])
v2 = np.array([1, 2])

# Create the parallelogram (span area)
span_points = np.array([
    [0, 0],
    v1,
    v1 + v2,
    v2
])

plt.figure(figsize=(6,6))
plt.plot(*span_points.T, 'ro-')
plt.fill(*span_points.T, alpha=0.3, color='skyblue')
plt.axhline(0, color='black')
plt.axvline(0, color='black')
plt.quiver(0, 0, *v1, angles='xy', scale_units='xy',
scale=1, color='r', label='v1')
plt.quiver(0, 0, *v2, angles='xy', scale_units='xy',
scale=1, color='g', label='v2')
plt.grid()
plt.legend()
plt.title("Span of v1 and v2 - Forms a Parallelogram")
plt.show()
```

3. WHAT IS A BASIS?

Basis is a minimum set of independent vectors that can span a space.

- For 2D space: 2 non-parallel vectors form a basis.

They must be:

- Linearly Independent (not multiples of each other)
- Span the Space

Think of it like:

- The X and Y axes are basis vectors for 2D.

- You can reach any point using combinations of them.

4. HOW TO CHECK LINEAR INDEPENDENCE?

Vectors `v1` and `v2` are independent if:

- No scalar `λ` exists such that: `v1 = λ·v2`

In NumPy:

```
# Are v1 and v2 linearly independent?
from numpy.linalg import matrix_rank

M = np.array([v1, v2])
rank = matrix_rank(M)
print("Rank:", rank)
# Rank == 2 ⇒ linearly independent (basis for 2D)
```

5. SUMMARY TABLE

Concept	Meaning
Linear Combination	$\alpha \cdot v_1 + \beta \cdot v_2$ (scaling + adding vectors)
Span	All vectors we can create using linear combinations
Basis	Smallest set of linearly independent vectors that span the space
Independence	Vectors aren't scalar multiples of each other