DAY 5 LINEAR ALGEBRA (PART 2)
90-DAY AGENTIC AI CHALLENGE

"From theory to intelligent systems mastering matrices today builds AI tomorrow."

Today's Focus

The following topics are critical for understanding the foundational role of linear algebra in AI development.

Topic	Purpose in AI
Matrices	Representing data, models, and relationships
Matrix $\times$ Vector	Transforming inputs in neural layers
Dot Product	Measuring similarity (used in NLP, vision, search engines)
Vector Length (Norm)	Normalization, distance, and similarity comparisons

Table 1: Key linear algebra concepts and their AI applications.

1 What is a Matrix?

A matrix is defined as a rectangular array of numbers organized into rows and columns, serving as a fundamental structure in linear algebra.

1.1 Prevalence of Matrices in AI

Matrices are ubiquitous in artificial intelligence, underpinning various data representations:

- Images are encoded as matrices of pixel values.
- Text is transformed into numerical vectors or embeddings using models such as BERT or Word2Vec.
- Tabular data utilizes matrices where rows represent samples and columns denote features.

1.2 Shape of a Matrix

The dimensions of a matrix are described by its shape. For instance, a matrix with 3 rows and 4 columns is characterized by a shape of 3×4.

Linear Algebra Concepts

July 03, 2025

Introduction

This document explores fundamental linear algebra operations, including matrix-vector multiplication and the dot product, with applications in artificial intelligence and machine learning.

1 Matrix-Vector Multiplication

Matrix-vector multiplication is a core operation used to transform vectors using matrices, widely applied in machine learning and deep learning.

Example

Matrix:

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

Vector:

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Result:

$$\begin{bmatrix} 8 \\ 14 \end{bmatrix}$$

Calculation:

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} (2 \cdot 1 + 3 \cdot 2) \\ (4 \cdot 1 + 5 \cdot 2) \end{bmatrix} = \begin{bmatrix} 8 \\ 14 \end{bmatrix}$$

1.1 Application in AI

This operation forms the basis of forward propagation in neural networks, where the weight matrix is multiplied by the input vector to produce the output.

2 Dot Product (Inner Product)

The dot product combines two vectors into a single number, indicating their similarity in direction.

Example

Vectors:

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Dot Product:

$$\mathbf{a} \cdot \mathbf{b} = (1 \cdot 4) + (2 \cdot 5) + (3 \cdot 6) = 4 + 10 + 18 = 32$$

Interpretation: A larger value indicates greater directional alignment.

2.1 Geometric Interpretation

The dot product is also related to the cosine of the angle between two vectors: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between them.

➡️ KEY USES:

- Measures alignment between vectors
- Forms the basis of cosine similarity
- Helps in understanding relationships between items in high-dimensional space

➡️ IN AI:

- 🔍 **Search Engines:** Rank documents by similarity to your query
- 💬 **Chatbots:** Match input text to the closest intent
- 🧠 **Transformers (like ChatGPT):** Use dot products to calculate attention scores between tokens

➡️ 4. VECTOR LENGTH (NORM)

The length (or norm) of a vector tells you how long or large the vector is — like the distance from the origin to that point in space.

➡️ WHY IT MATTERS:

- 📏 **Normalize vectors (scale them to unit length)**
- ✏️ **Calculate distances between data points**
- 🚫 **Prevent numerical instability in deep learning models (like exploding gradients)**

➡️ CODE PLAYGROUND: NUMPY IMPLEMENTATION

import numpy as np

Matrix & Vector

KEY USES:

Measures **alignment** between vectors

Forms the basis of **cosine similarity**

Helps in understanding relationships between items in high-dimensional space

IN AI:

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🔄 Prevent numerical instability in deep learning models (like exploding gradients)

💻 CODE PLAYGROUND: NUMPY IMPLEMENTATION

```
import numpy as np

# Matrix & Vector
```

```
A = np.array([[2, 3], [4, 5]])
x = np.array([1, 2])

# Matrix × Vector
result = A @ x
print("Matrix-Vector Product:", result)

# Dot Product
a = np.array([1, 2, 3])
b = np.array([4, 5, 6])
dot = np.dot(a, b)
print("Dot Product:", dot)

# Vector Length
length = np.linalg.norm(a)
print("Vector Length:", length)
```

🧠 REAL AI/INDUSTRY USE CASES

⚙️ Application	💬 Explanation
🎥 Netflix/Youtube Recs	Dot product to find which video/movie best matches your profile vector
📚 ChatGPT / BERT	Uses attention scores (dot product between tokens) to understand language
🧠 Neural Nets (Deep NN)	Weights × input (matrix × vector) → transforms into hidden features
📷 Image Recognition	Image pixels → matrix → CNN filters via dot products for object detection
🚗 Autonomous Cars	Sensor fusion matrices (LiDAR, cameras) + vector motion modeling

🎓 STUDY RESOURCE

📘 Book: Gilbert Strang – Introduction to Linear Algebra

📺 Video: [MIT OCW Linear Algebra (Lectures 2–3)]

📄 **Chapters to Focus**: Matrix-vector multiplication, inner product, vector spaces

💥 POWER LINE

> “Neural networks don’t think in language — they think in vectors, and vectors live in linear algebra.”