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Part a) What happens if we do not neglect pressure from the perturbation equation?

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_0\delta - \frac{c_s^2}{a}\nabla^2\delta = 0$$

First, I am going to solve this equation neglecting pressure so I can compare it later.

Fourier Modes:

$$\rho_1 = \rho_s e^{i(kx-\omega t)}$$

$$\vec{v}_1 = \vec{v}_s e^{i(kx-\omega t)}$$

Starting w/ $\ddot{\delta}$ term by plugging in the scalar Fourier mode (assuming perturbations are scalars):

$$\ddot{\delta} = \frac{\partial^2}{\partial t^2}\left[\delta_s e^{i(kx-\omega t)}\right]$$

$$= \frac{\partial}{\partial t}\left[-i\omega\,\delta_s e^{i(kx-\omega t)}\right]$$

$$= i^2\omega^2\delta_s e^{i(kx-\omega t)}$$

$$\ddot{\delta} = -\omega^2\delta_s e^{i(kx-\omega t)}$$

Now moving onto $\dot{\delta}$

$$\dot{\delta} = \frac{\partial}{\partial t}\left[\delta_s e^{i(kx-\omega t)}\right]$$

$$= -i\omega \delta_s e^{i(kx-\omega t)}$$

Now plugging the derivatives into the original equations:

$$\left(-\omega^2 \delta_s e^{i(kx-\omega t)}\right) + 2H\left(-i\omega\delta_s e^{i(kx-\omega t)}\right) -$$

$$4\pi G\rho_0\left(\delta_s e^{i(kx-\omega t)}\right) - 0 = 0$$

setting pressure term to 0

$$-\delta_s e^{i(kx-\omega t)}\omega^2 - 2Hi\delta_s e^{i(kx-\omega t)}\omega - 4\pi G\rho_0 \delta_s e^{i(kx-\omega t)} = 0$$

$$-\cancel{\delta_s e^{i(kx-\omega t)}}\left[\omega^2 + 2Hi\omega + 4\pi G\rho_0\right] = 0$$

$$\omega^2 + 2Hi\omega + 4\pi G\rho_0 = 0$$

$$\omega = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(2Hi) \pm \sqrt{(2Hi)^2 - 4(1)(4\pi G\rho_0)}}{2(1)}$$

$$= \frac{-2Hi \pm \sqrt{-4H^2 - 16\pi G\rho_0}}{2}$$

$$= \frac{-2Hi \pm 2i\sqrt{H^2 + 4\pi G\rho_0}}{2} = \boxed{-iH \pm i\sqrt{H^2 + 4\pi G\rho_0}}$$

Starting with the positive value:

$$\delta_1 = \delta_s e^{i(kx - \omega t)}$$

$$= \delta_s e^{i(kx - (-iH + i\sqrt{H^2 + 4\pi G\rho_0})t)}$$

$$= \delta_s e^{i(kx + iHt - it\sqrt{H^2 + 4\pi G\rho_0})}$$

$$= \delta_s e^{ikx - Ht + t\sqrt{H^2 + 4\pi G\rho_0}}$$

$$\boxed{\delta_1 = \frac{\delta_s e^{ikx} e^{t\sqrt{H^2+4\pi G\rho_0}}}{e^{Ht}}}$$

Now plugging in the negative term

$$\delta_2 = \delta_s e^{i(kx-\omega t)}$$

$$= \delta_s e^{i(kx-(-iHt-i\sqrt{H^2+4\pi G\rho_0})t)}$$

$$= \delta_s e^{i(kx+iHt+it\sqrt{H^2+4\pi G\rho_0})}$$

$$= \delta_s e^{ikx-Ht-t\sqrt{H^2+4\pi G\rho_0}}$$

$$\boxed{\delta_2 = \frac{\delta_s e^{ikx}}{e^{Ht} e^{t\sqrt{H^2+4\pi G\rho_0}}}}$$

The δ_1 is the growth term where the δ_2 is the decay term. Now, I will include pressure in the differential equation:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_0 \delta - \frac{c_s^2}{a}\nabla^2\delta = 0$$

Fourier Modes:

$$\rho_1 = \rho_s e^{i(kx-\omega t)}$$

$$\vec{v}_1 = \vec{v}_s e^{i(kx-\omega t)}$$

I will use the same derivatives as last time, but I will now derive the $\frac{c_s^2}{a}\nabla^2\delta$ term

$$\nabla^2\delta = \frac{\partial^2}{\partial x^2}\left(\delta_s e^{i(kx-\omega t)}\right)$$

$$= \frac{\partial}{\partial x}\left(\delta_s ik e^{i(kx-\omega t)}\right)$$

$$= -\delta_s k^2 e^{i(kx-\omega t)}$$

Now plugging in all the derivatives into the differential equations:

$$\left(-\omega^2 \delta_s e^{i(kx-\omega t)}\right) + 2H\left(-i\omega \delta_s e^{i(kx-\omega t)}\right) -$$

$$4\pi G\rho_0\left(\delta_s e^{i(kx-\omega t)}\right) + \frac{c_s^2}{a}\left(\delta_s k^2 e^{i(kx-\omega t)}\right) = 0$$

$$\cancel{\delta_s e^{i(kx-\omega t)}}\left[-\omega^2 - 2iH\omega - 4\pi G\rho_0 + \frac{k^2 c_s^2}{a}\right] = 0$$

$$-\omega^2 - 2iH\omega - 4\pi G\rho_0 + \frac{k^2 c_s^2}{a} = 0$$

$$\omega = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\omega = \frac{-(-2iH) \pm \sqrt{(-2iH)^2 - 4(-1)\left(4\pi G\rho_0 + \frac{k^2 c_s^2}{a}\right)}}{2(-1)}$$

$$= \frac{2iH \pm \sqrt{-4H^2 + 4\left(4\pi G\rho_0 + \frac{k^2 c_s^2}{a}\right)}}{-2}$$

$$= \frac{2iH \pm 2i\sqrt{H^2 - \left(4\pi G\rho_0 + \frac{k^2 c_s^2}{a}\right)}}{-2}$$

$$= -\left(iH \pm i\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}\right)$$

$$\omega = -\left(iH \pm i\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}\right)$$

Starting to sub in the positive term first into the Fourier mode.

$$\delta_1 = \delta_s e^{i\left(kx - \left(-iH - i\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}\right)t\right)}$$

$$= \delta_s e^{i\left(kx + iHt + it\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}\right)}$$

$$= \delta_s e^{ikx - Ht - t\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}}$$

$$\boxed{\delta_1 = \frac{\delta_s e^{ikx}}{e^{Ht}\, e^{t\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}}}}$$

Now plugging in the negative term into the Fourier Mode:

$$\delta_2 = \delta_S \, e^{i\left(kx - \left(-iH + i\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}\right)t\right)}$$

$$= \delta_S \, e^{i\left(kx + iHt - it\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}\right)}$$

$$= \delta_S \, e^{ikx - Ht + t\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}}$$

$$\boxed{\delta_2 = \frac{\delta_S \, e^{ikx} \, e^{t\sqrt{H^2 - 4\pi G\rho_0 - \frac{k^2 c_s^2}{a}}}}{e^{Ht}}}$$

In this case, δ_1 is the decaying term while δ_2 is the growing term.

B) This instability illustrates a competition between which two forces?

This instability illustrates the competition between radiative pressure (the k space) and the gravitational force (the omega space)

C) Do you think a similar equation may also explain one of the gas clumping processes at work in galaxies? If yes, which one?

Yes, there is a similar equation that may explain gas clumping processes in galaxies. It is the Jean's instability. The densest parts of the cold patches in a galaxy will collapse due to gravity since there is a lack of pressure.