Describing the Orbit of (911) Agamemnon Relative to the Jovian L4 Lagrange Point

Rei R. Johnson
Department of Mathematics, Statistics, and Data Science, Montgomery College Rockville Campus

Abstract

This study will describe the behavior of the Greek Camp Trojan 911 Agamemnon. 911 Agamemnon is the largest regular-shaped asteroid in the Greek Camp. Above it is 624 Hektor, the largest Trojan, and 617 Patroclus, the two-asteroid system, which have incongruent shapes with what would be considered regular. For the purposes of this study, we will describe the orbits of 911 Agamemnon around Jupiter's L4 Lagrange point as proof of a Lagrange-point-relative orbit determination.

Keywords: (911), Agamemnon, Lagrange Point, Greek Camp, Trojan, orbit.

Heliocentric Orbit Parameters

1.0 *Parameter Tables*

Determining Agamemnon's orbit based on its movement around the sun will give us a basis for where it will be at any given time. We use this data along with Jupiter's orbital parameters to determine where Agamemnon is relative to the L4 Lagrange point.

The orbital parameters of Agamemnon are already determined in a heliocentric reference frame. Using these parameters, as well as the parameters of the center of the L4 point, we can determine the relative distance between the asteroid at any point in time and the center of the Lagrange point at any point in time. We should begin by determining the orbital parameters of the asteroid. Agamemnon was first discovered in 1919 (Reinmuth, 1926), and its orbit has been continually refined over 105 years and 299 days as of 2025/01/12 (NASA Jet Propulsion Laboratory, 2025). Agamemnon's orbital parameters are the following:

Element	Value	Reference	Note
Eccentricity (e)	0.0671889	(International Astronomical Union, 1937)	Uncertainty of 4.9582E-9
Semi-major axis (a)	5.2859443au	(International Astronomical Union, 1937)	Uncertainty of 3.0303E-8au
Semi-minor axis (b)	5.2739995au	Hand calculation	Calculated as $b = a\sqrt{(1 - e^2)}$, certainty based upon IAU measurements for all hand calculations.
Semi-parameter (p)	5.2620817au	Hand calculation	Calculated as $p = a(1 - e^2)$, certainty based upon IAU measurements for all hand

			calculations.
Apoapsis (r_a)	5.6411010au	Hand calculation	Calculated as $r_a = p \div (1 - e)$, certainty based upon IAU measurements for all hand calculations.
Periapsis (r_P)	4.9307875au	(NASA Jet Propulsion Laboratory, 2025)	Calculated as $r_p = p \div (1 + e)$ Uncertainty of 5.357E-8au
Inclination (i)	21.755921°	(NASA Jet Propulsion Laboratory, 2025)	Uncertainty of 7.4775E-8°
Period	12.153240y	(NASA Jet Propulsion Laboratory, 2025)	Uncertainty of 1.0451E-7y, period in days is ~4438d ± 3.8172E-5d
Longitude of the Ascending Node (Ω)	338.02013°	(NASA Jet Propulsion Laboratory, 2025)	Uncertainty of 2.8377E-7°
Argument of Periapsis (ω)	82.358383°	(NASA Jet Propulsion Laboratory, 2025)	Uncertainty of 3.3206E-6°
Mean Anomaly (M)	86.089087°	(NASA Jet Propulsion Laboratory, 2025)	Uncertainty of 2.4149E-6°
Time of Perihelion Passage (tp)	2022-Jun-08	(NASA Jet Propulsion Laboratory, 2025)	Uncertainty of 3.2724E-5s
Mean Motion (n)	0.0810998°/d	(NASA Jet Propulsion Laboratory, 2025)	Uncertainty of 6.9739E-10°/d

Agamemnon's orbit parameters are now defined. This gives us the ability to place it at a near-exact position at a certain time. These parameters will be used later in the process of determining the Lagrange-relative orbit. To use this data later, we require the orbital parameters of Jupiter. Luckily, Jupiter has a very well-described orbit:

Element	**Value**	**Reference**	**Note**
Eccentricity (e)	0.0489	(NASA, 2024)	Certain
Semi-major axis (a)	5.2038au	(NASA, 2024)	Certain
Semi-minor axis (b)	5.2739995au	(NASA, 2024)	Certain

Semi-parameter (p)	5.1913566au	Hand Calculation	Calculated as $p = a(1 - e^2)$ Uses certain values
Apoapsis (r_a)	5.4570au	(NASA, 2024)	Certain
Periapis (r_P)	4.9506au	(NASA, 2024)	Certain
Inclination (i)	1.304°	(NASA, 2024)	Certain
Period	11.856523y	(Willman Jr., 2021)	Certain
Longitude of the Ascending Node (Ω)	100.464°	(Simon et al., 1994, 678-679)	Certain
Argument of Periapsis (ω)	273.867°	(Simon et al., 1994, 678-679)	Certain
Mean Anomaly (M)	20.020°	(Simon et al., 1994, 678-679)	Certain
Time of Perihelion Passage (tp)	2023-Jan-21	(NASA Jet Propulsion Laboratory, 2025)	Certain
Mean Motion (n)	0.0831294°/d	(Willman Jr., 2021)	Certain

Now, with Jupiter's parameters, we can describe those of its fourth Lagrange point. These will be mostly similar to the parameters of Jupiter's orbit; however, certain parameters will be offset by 60°. We will redescribe these parameters in a new table:

Element	**Value**	**Reference**	**Note**
Eccentricity (e)	0.0489	(NASA, 2024)	Certain
Semi-major axis (a)	5.2038au	(NASA, 2024)	Certain
Semi-minor axis (b)	5.2739995au	(NASA, 2024)	Certain
Semi-parameter (p)	5.1913566au	Hand Calculation	Calculated as $p = a(1 - e^2)$
Apoapsis (r_a)	5.4570au	(NASA, 2024)	Certain
Periapis (r_P)	4.9506au	(NASA, 2024)	Certain
Inclination (i)	1.304°	(NASA, 2024)	Certain
Period	11.856523y	(Willman Jr., 2021;	Certain

		Seidelmann, 1992)	
Longitude of the Ascending Node (Ω)	160.464°	Hand Adjusted	Calculated as Ω for Jupiter +60°
Argument of Periapsis (ω)	213.867°	Hand Adjusted	Calculated as ω for Jupiter -60°
Mean Anomaly (M)	20.020°	(Simon et al., 1994, 678-679)	Certain
Time of Perihelion Passage (tp)	2025-Jan-23	Hand Calculated	Calculated as tp for Jupiter +16.6% of Jupiter's orbital period in days.
Mean Motion (n)	0.0831294°/d	(Willman Jr., 2021)	Certain

We finally have all of the required parameters to describe our system. With these parameters, we can tell exactly where Agamemnon, Jupiter, and its L4 point are at any given time.

Keywords: orbital parameters, Jupiter, L4, offset.

Methods

2.0 *Coordinate Determination*

To solve Agamemnon or any other object's position relative to the Lagrange point, we need to solve its position coordinates. Luckily, these position coordinates can be calculated easily. For demonstration, we will use this as the setup for our calculations:

T_0 = 01/01/2026, an arbitrary future point in which both objects will likely exist

T_{Next} is a variable used as the sample rate

Using these parameters, we can determine the angle of each object from the sun. This angle will also allow us to find the distance from the sun at a given point in time. The distance is calculated as:

$$r_{current} = a(1 - e^2) \div (1 + e \cos \theta)$$

Theta is the current angle between the object's periapsis and its current location. r_{current} is the current distance from the sun. Once we have determined this for both Agamemnon and Jupiter's L4 point at a given time, we can use this to determine the first two coordinates of our 3D system. The origin for these coordinates needs to be a constant, so we will use the Sun. The sun will always be located at a focus of the ellipse, which makes it easier to calculate trigonometrically. The calculations for each coordinate are:

$$x = a \cos \theta$$
$$y = b \sin \theta$$

Theta remains the same between the equations. We now have a coordinate with the same unit as our distance and our orbital parameters. For this paper, this unit is au. Now that the coordinates in a flat plane are identified, we can determine the relative elevation to the sun. This will use both the angle and the inclination. For a body like Agamemnon, this means the relative up-down will just be congruent to a point on a sine wave similar to its angle on orbit and its x, and y coordinates. The elevation equation is:

$$c = ea$$
$$z_{Ascending} = r_a \sin \theta - c$$
$$z_{Descending} = r_p \sin \theta + c$$

Elevation is measured in au as well, so with this, we can determine the x, y, and z coordinates for each point or body in space relative to the elliptical center point. All of these can then be adjusted by their required values to reach the heliocentric coordinates. These equations can be described as one of the following dependent on the coordinate:

$$x_{Adjusted} = x_{Original}$$
$$y_{Adjusted} = y_{Original} + c$$
$$z_{Adjusted} = z_{Original} + (c \cos \theta)$$

With the adjusted coordinates, a simple matrix division operation can give us the offset of the object relative to the Lagrange point. This matrix subtraction operation can be represented as:

$$\left[x_{L4}, \, y_{L4}, \, z_{L4}\right] - \left[x_{Agamemnon}, \, y_{Agamemnon}, \, z_{Agamemnon}\right] = \left[x_{Relative}, \, y_{Relative}, \, z_{Relative}\right]$$

These coordinates can then be input to GNU Octave to be represented as a point in space.

Keywords: matrix, ellipse, focus, system, coordinate, inclination, elevation, relative

2.1 *GNU Octave Programming*

Using the previously established equations, we can use GNU Octave, an open-source alternative to MatLab, to describe the orbits visually. The reason for using GNU Octave is that it is an accessible alternative with the same functionality as MatLab. This paper will cover the programming required as well as being published alongside the necessary .m files in the interest of reproducing the experiments. To begin, we copy the aforementioned algorithm into GNU Octave using the MatLab standard language.

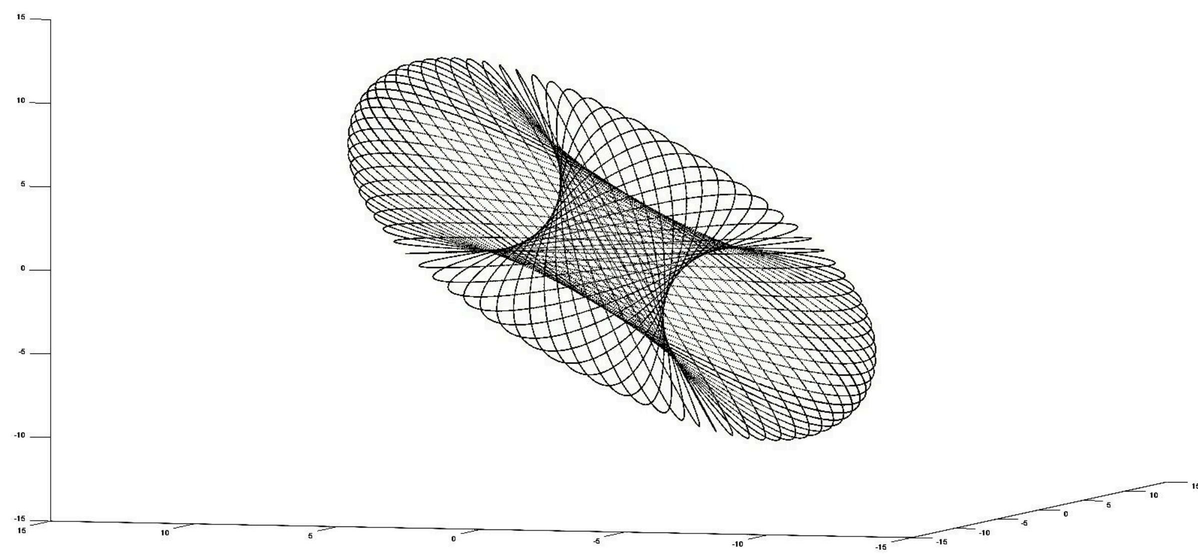
The code representing the algorithm intakes the required variables T_0, T_{Step}, $r_{current}$, a, b, e, r_a, r_b, i, as well as the mean motion for both objects and the time of passage for each of the objects. It will need these variables for each of the points, so we represent this as two matrices with the a, b, e, r_a, r_b variables in that order. Each of these matrices is defined in any manner, however, the example code defines the L4 matrix as 'lagrange' and the 911 Agamemnon matrix as 'agamemnon' to avoid syntax confusion. The matrices are hence defined at the top of the file as:

lagrange = [5.2038, 5.2739995, 0.0489, 5.4570, 4.9506, 1.304];
agamemnon = [5.2859443, 5.2739995, 0.0671889, 5.6411010, 4.9307875, 21.755921];

The data is accessed by calling these matrices like so:

lagrange[1,1]; (Accesses the first row, first column, which corresponds to 5.2038)

With these established in our file, we can then translate all of the equations into MatLab's programming language. In Octave it is recommended to use a function to translate the algorithm, so we call the function as determineOrbit in the example file. These methods return us a plot of the object's orbit. When simulating the orbit over 10 years with points every day, the orbit is returned as this shape:



The graph above shows a 100x scaled version of the orbit, meaning all measures are 100x their normals. This places the r_a and r_p at 0.11 Au instead of the shown 11 Au.

This shape shows something interesting about the orbit of Agamemnon relative to the L4 Lagrange point. Not only was this 100 years of simulation, but the simulation eventually overlaps. If you follow any one line, you realize that it connects to itself. This means that there is theoretically an orbital period within the orbit that is between zero and 100 years long. Not only is this information extremely interesting to see, but it provides a really interesting fit to the orbit of an object around a point in space that simply has nothing there.

Results

According to these calculations, we can derive a very specific shape. This shape is a spindle torus, whose inner spindle is very large by comparison to other spindle toruses. The orbit shape is highly self-intersecting as stated before, which causes a complicated to follow path. This path based on my calculations is overlapping, meaning the above shape is the fully repeating shape that Agamemnon should always fall upon at any given time. So, the orbit is derived and we can create a table with all of the extractable parameters thus far. These are all of the parameters I have extracted although using the program and the data provided anyone should be able to recreate this orbit.

Helio-relative Inclination	~39.4 °
r_a and r_p	~0.11 Au
Orbit Shape	Spindle Torus
Orbital Period	4-40 Y
Average distance	~0.11 Au

This isn't much information in terms of a standard orbit determination as the process is not yet fully worked out, i.e. there are ways to use the data points to determine the period, average distance, etc. more accurately than the rough figures providing you store them and give the simulation more time to run.

Keywords: spindle torus, algorithm, function, helio-relative,

References

Chebotarev, G. A., Beliaev, N. A., & Eremenko, R. P. (1974). Orbits of Trojan Asteroids. *International Astronomical Union Symposium*, (62), 63-69. SAO/NASA Astrophysics Data System. Retrieved 03 12, 2025, from https://ui.adsabs.harvard.edu/abs/1974IAUS...62...63C/abstract

International Astronomical Union. (1937). *(911) Agamemnon*. International Astronomical Union Minor Planet Center. Retrieved 03 12, 2025, from https://www.minorplanetcenter.net/db_search/show_object?object_id=911

NASA. (2024, October 2). *Jupiter Fact Sheet*. the NSSDCA. Retrieved March 12, 2025, from https://nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html

NASA Jet Propulsion Laboratory. (2025, January 30). *HORIZONS Planet-center Batch call for January 2023 Perihelion*. NASA Jet Propulsion Laboratory. Retrieved March 12, 2025, from https://ssd.jpl.nasa.gov/horizons_batch.cgi?batch=1&COMMAND=%27599%27&START_TIME=%272023-01-01%27&STOP_TIME=%272023-01-31%27&STEP_SIZE=%273%20hours%27&QUANTITIES=%2719%27

NASA Jet Propulsion Laboratory. (2025, March 08). *911 Agamemnon*. Small Body Database. Retrieved March 12, 2025, from https://ssd.jpl.nasa.gov/tools/sbdb_lookup.html#/?sstr=911

Reinmuth, K. W. (1926). Die Herschel-Nebel nach Aufnahmen der Koenigstuhl-Sternwarte. *Veroeffentlichungen der Badischen Landes-Sternwarte zu Heidelberg*, *9*, 103. SAO/NASA Astrophysics Data System. Retrieved 03 12, 2025, from https://ui.adsabs.harvard.edu/abs/1926ViHei...9....1R/abstract

Seidelmann, K. P. (Ed.). (1992). *Explanatory Supplement to the Astronomical Almanac*. University Science Books. https://ui.adsabs.harvard.edu/abs/1992esaa.book.....S/abstract

Seidelmann, K. P. (2007, July). Report of the IAU/IAG Working Group on cartographic coordinates and rotational elements: 2006. *Celestial Mechanics and Dynamical Astronomy*, *98*(3), 155-180. SAO/NASA Astrophysics Data System. 10.1007/s10569-007-9072-y

Seidelmann, K. P. (2018, February 23). Report of the IAU/IAG Working Group on cartographic coordinates and rotational elements: 2015. *Celestial Mechanics and Dynamical Astronomy*, *130*(22). 10.1007/s10569-017-9805-5

Simon, J. L., Brentagon, P., Chapront, J., Chapront-Touze, M., Francou, G., & Laskar, J. (1994, February). Numerical expressions for precession formulae and mean elements for the Moon and the planets. *Astronomy and Astrophysics*, *282*, 663-689. SAO/NASA Astrophysical Data System. https://ui.adsabs.harvard.edu/abs/1994A%26A...282..663S/abstract

Willman Jr., A. J. (2021, July 12). *Planet Jupiter Data*. Alexander J. Willman, Jr. Retrieved March 12, 2025, from https://www.princeton.edu/~willman/planetary_systems/Sol/Jupiter/

Programming files have been published in parallel with this paper. Access is available on GitHub here: