MEC302: Embedded Computer Systems

Theme I: Modeling Dynamic Behaviors

Lecture 3 – Discrete Dynamics: Discrete and Hybrid Systems

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Lecture outline

- Discrete dynamics:
  - Discrete systems;
  - State machines;
  - Finite-state machines;
  - State machine properties;
  - Extended state machines;
- Hybrid system modelling:
  - Hybrid systems;
  - Timed automata;
  - Supervisory control.

Discrete dynamics

A **discrete system** operates in a sequence of discrete steps and is said to have **discrete dynamics**.

Discrete systems operate with **discrete signals** of the form $e\colon \mathbb{R} \to \{absent\} \cup X$:

- It is absent most of the time and we can count, in order, the times at which it is not *absent*;
- If $X = \{present\}$, the signal carries no value and it is called a **pure signal**;
- Otherwise, the signal is not pure and X can be any set of values ($X \subseteq \mathbb{R}$).

在离散系统中，纯信号是指只包含有限个离散时间点上的数值，而且在这些时间点上的数值都是确定的、单一的、不受干扰和噪声的信号。

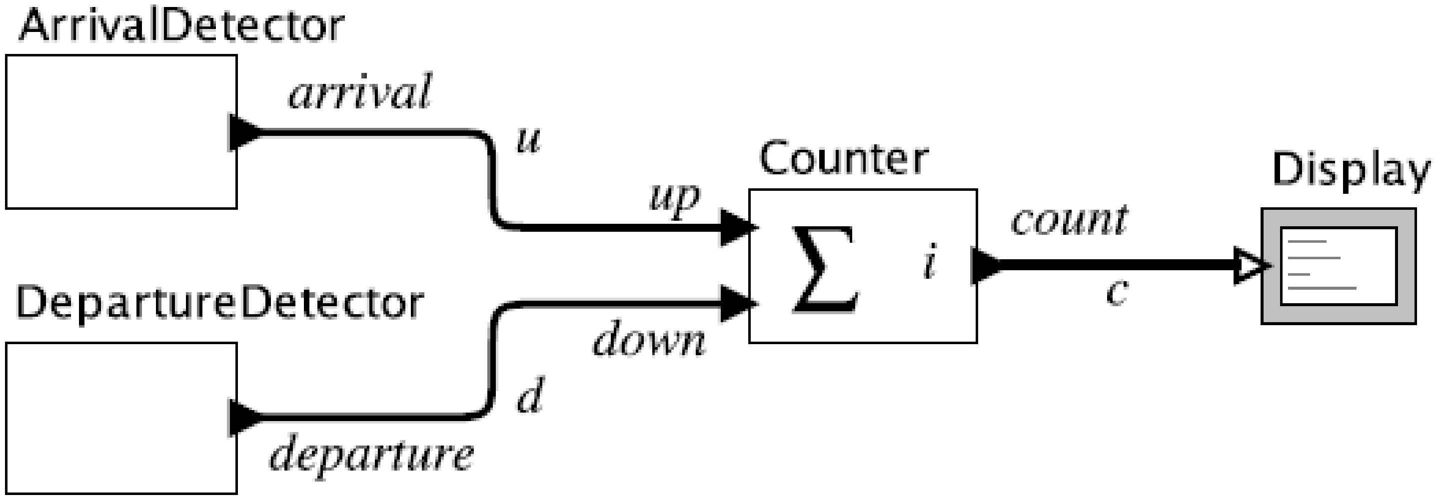
Examples of discrete systems:

- Calculators/computers/controllers;
- Traffic lights;
- Board games;
- Etc.

Discrete systems

Consider a counter system (e.g., at a parking garage):

- **Inputs**:
  - $u: \mathbb{R} \rightarrow \{absent, present\}$;
  - $d: \mathbb{R} \rightarrow \{absent, present\}$;
- **States***:
  - $s: \mathbb{R} \rightarrow \mathbb{Z}$;
- **Output** (two options):
  - $c: \mathbb{R} \rightarrow \{absent\} \cup \mathbb{Z}$ – **Mealy machine** (produces output during transition);
  - $c: \mathbb{R} \rightarrow \mathbb{Z}$ – **Moore machine** (produces output when in a state).



* – Discrete models with finite state space are called finite-state machines.
George H. Mealy and Edward F. Moore are Bell Labs engineers in 1950s.

State machines

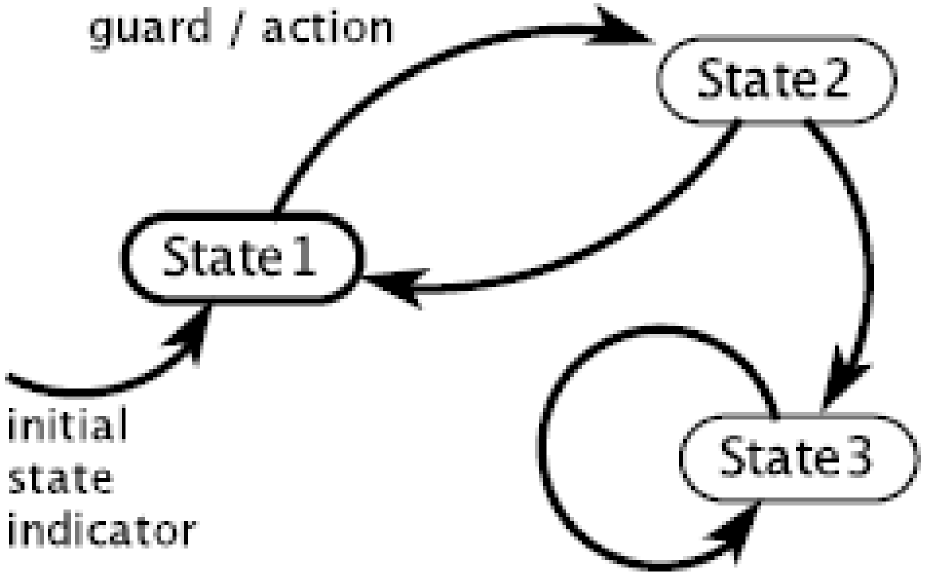
A **State Machine (SM)** is a model of a system with **discrete dynamics** that at each reaction maps valuations of the inputs to valuations of the outputs, where the map may depend on its current state.

It can be represented **graphically** with:

- *Inputs* – specified with names and types/sets;
- *Outputs* – specified with names and types/sets;
- *System states* – distinguished with balloons;
- *Transitions* – represented with arrows;
- *Predicate* (logical expression) – guards;
- *Reaction* (update) – action on a guard.

inputs: name : type/set

outputs: name : type/set



Or, it can be explicitly formulated with a Mathematical model of a **Finite-SM (FSM)**:

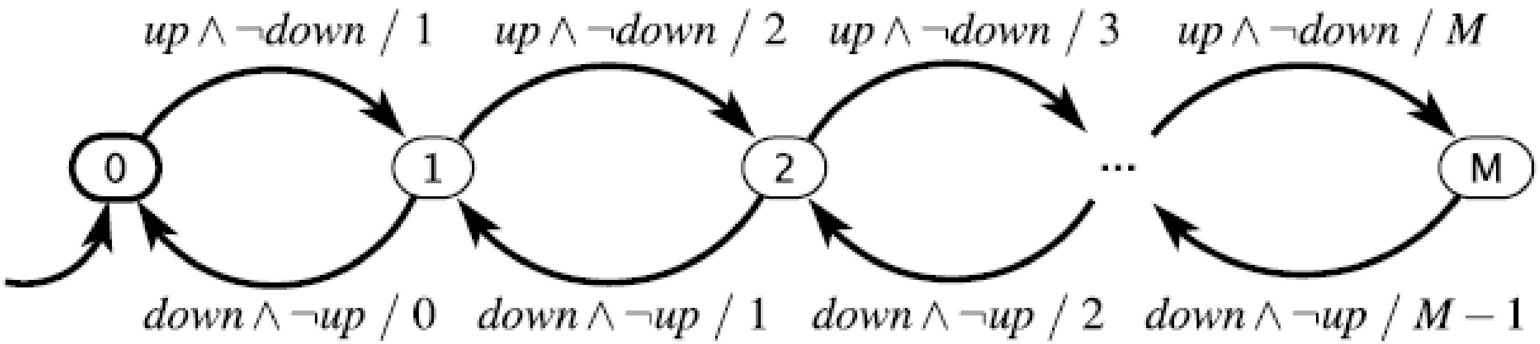
$$FSM(States;\ Inputs;\ Outputs;\ update;\ initialState)$$

Finite-state machines

A **Finite-State Machine (FSM)** is an **SM** where the set *States* of possible states is finite.

FSM model for the garage counter (graphical representation):

inputs: $up, down$: pure
output: $count : \{0, \cdots, M\}$



Predicate standard code (key):

true	Transition is always enabled.	$p_1 \wedge p_2$	Transition is enabled if both p_1 and p_2 are *present*.
p_1	Transition is enabled if p_1 is *present*.	$p_1 \vee p_2$	Transition is enabled if either p_1 or p_2 is *present*.
$\neg p_1$	Transition is enabled if p_1 is *absent*.	$p_1 \wedge \neg p_2$	Transition is enabled if p_1 is *present* and p_2 is *absent*.

Finite-state machines

Mathematical model of an **FSM** is formulated with a five-tuple:

$$FSM(States; Inputs; Outputs; update; initialState) = \Bigg($$

$$States = \{0,1,\ldots,M\}$$

$$Inputs = (\{up, down\} \to \{present, absent\})$$

$$Outputs = (\{count\} \to \{0,1,\ldots,M, absent\})$$

$$initialState = 0$$

$$update(s,i) = \begin{cases} (s+1, s+1) \text{ if } s < M \wedge i(up) = present \wedge i(down) = absent \\ (s-1, s-1) \text{ if } s > 0 \wedge i(up) = absent \wedge i(down) = present \\ (s, abs) \qquad \text{otherwise} \end{cases}$$

$$\Bigg)$$

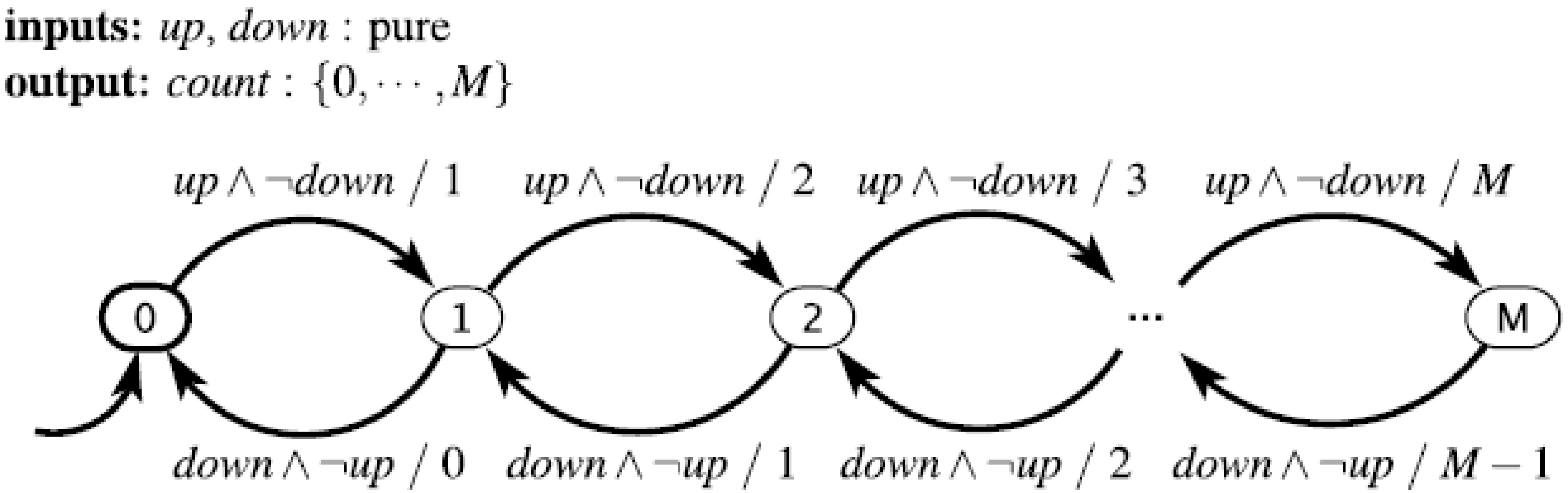
State machine properties

Important properties of **State Machines (SM)**:

- **Determinacy** – An SM is said to be deterministic if, for each state, there is at most one transition enabled by each input value.
- **Receptiveness** – An SM is said to be receptive if, for each state, there is at least one transition possible on each input symbol.

As a result, if an SM is both deterministic and receptive, there is exactly one transition possible on each input value.

Is the FSM model of the garage counter deterministic, receptive, neither or both?

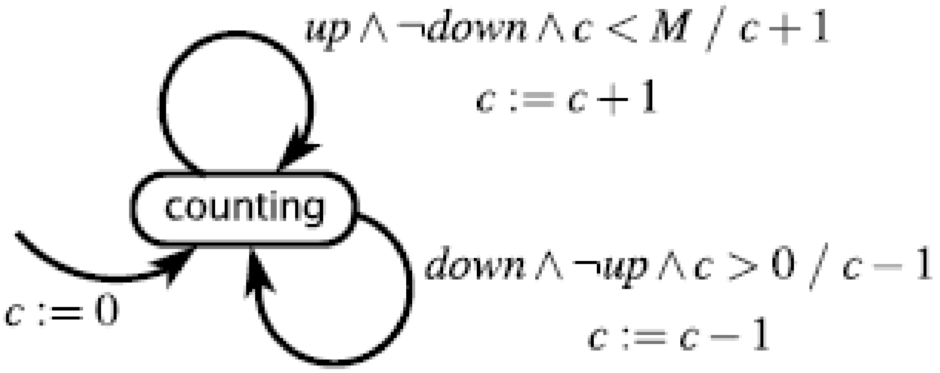


Extended state machines

An **FSM** with large number of states can be generalized with an **Extended State Machine (ESM)**.

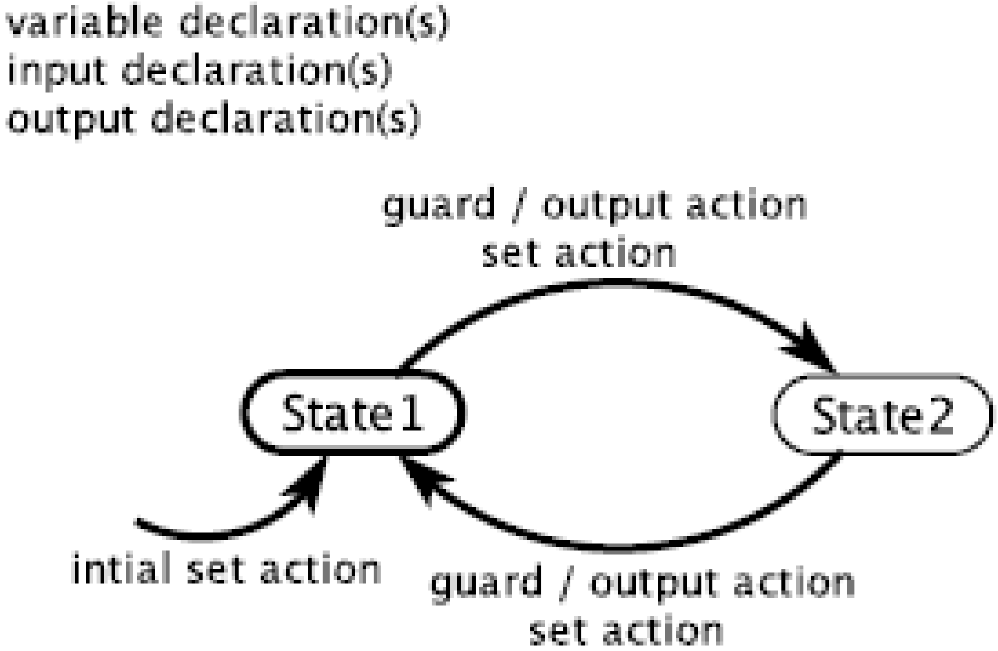
- Consider ESM model for garage counter:

variable: c: $\{0, \cdots, M\}$
inputs: *up*, *down*: pure
output: *count*: $\{0, \cdots, M\}$



Extended state machines

The general notation for an **ESM** is as follows:



There are three main differences:

- Variable declarations;
- Variables initialization – initial set action;
- Assignments to variables – set actions.

The state of an **ESM** is defined by both the discrete state (a “bubble”) and states of variables (ie, their values).

Further reading on SM in the textbook (sections: **3.5 Nondeterminism; 3.6 Behaviors and Traces**):
Lee, E.A. and Seshia, S.A., 2016. Introduction to embedded systems: A cyber-physical systems approach. Mit Press.

Hybrid system modelling

Modelling techniques considered previously were:

- **Continuous dynamics** uses differential/integral equations to model continuous and “smooth” processes;
- **Discrete dynamics** uses state machines to model **discrete systems**.

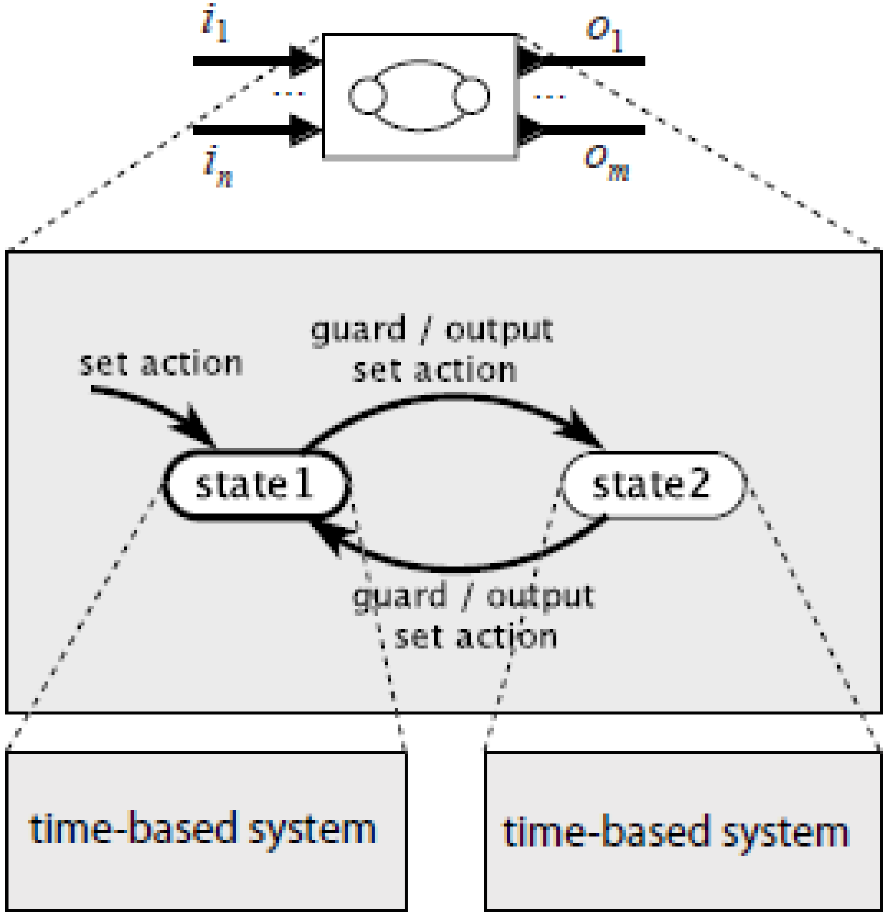
Hybrid system modelling is a formalism that incorporates both.

Hybrid systems can be very powerful to model cyber-physical systems by integrating their continuous physical dynamics with discrete computational systems.

Hybrid systems

Discrete **state machines** can be extended to **hybrid systems** by allowing any of the following:

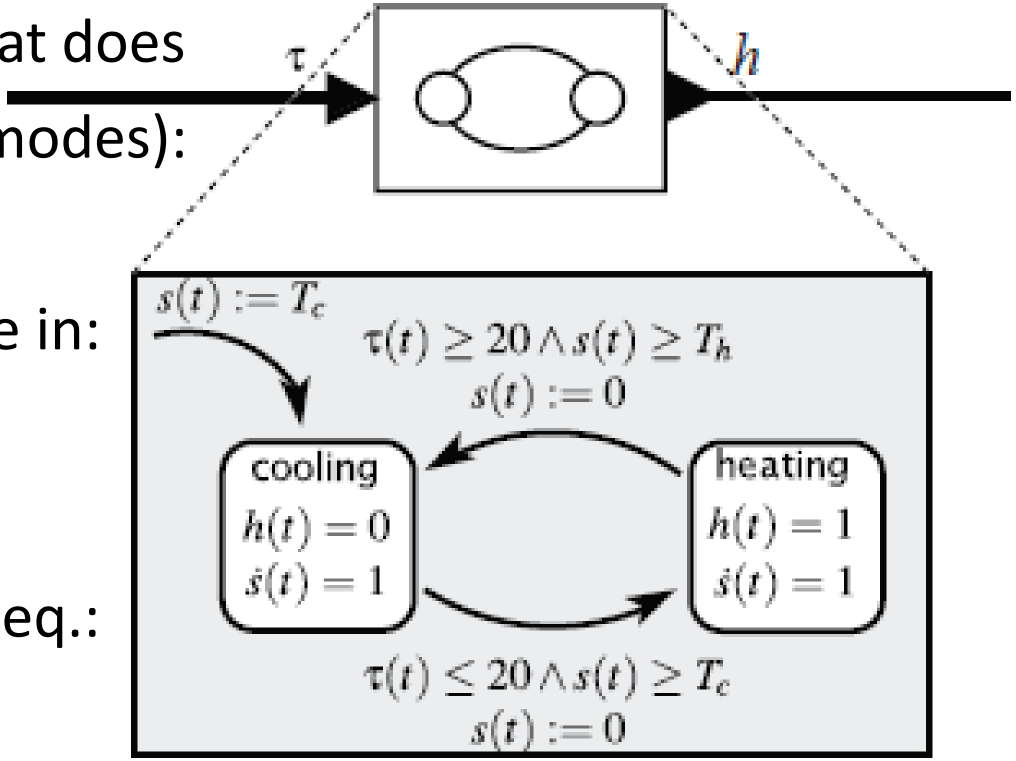
- Continuous inputs (i.e., continuous-time signals);
- Continuous dynamics of the output(s);
- State refinement (i.e., continuous state variables).



Timed automata

Timed Automata (TA) is a modal model with time-based refinement of states.

For example, let us consider a thermostat model that does not allow chattering (frequent transition between modes):



The main differences from **discrete machines** reside in:

- “Bubbles” (States) become **modes**;
- In each **mode**, state is defined by it variables;
- The model tracks the **clock** using a first-order diff.eq.:

$$\dot{s}(t) = 1.$$

Timed automata

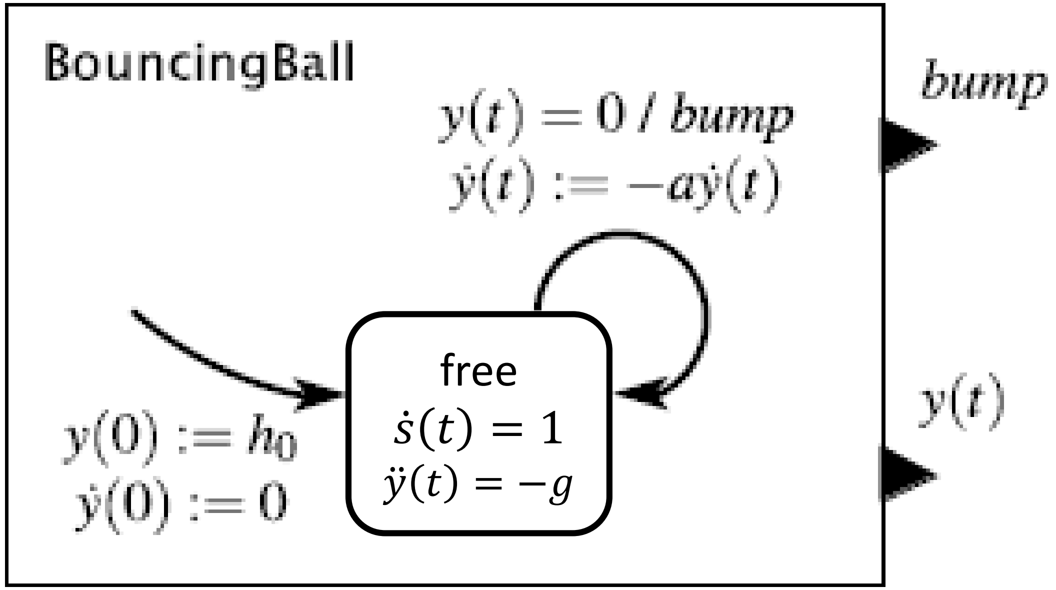
Let us consider another example of an **TA**, which models both continuous and discrete dynamics of a bouncing ball:

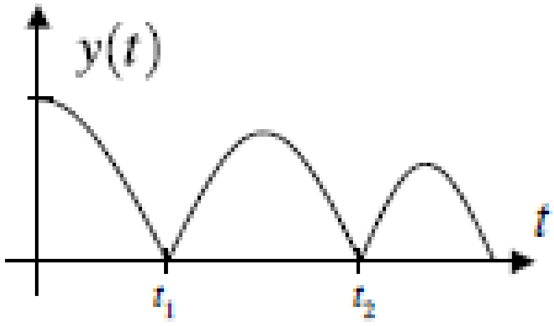
Problem description:

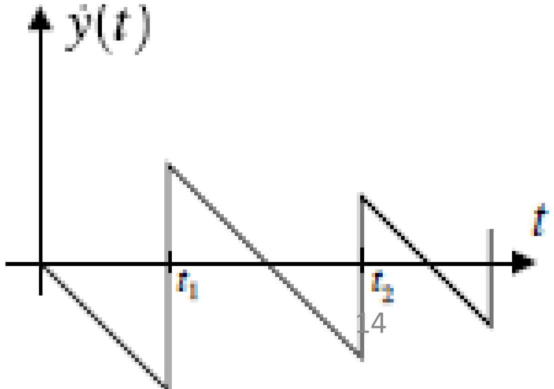
- Initial parameters at $t = 0$:
  - Ball is released at high h_0 ($y(0) := h_0$);
  - Initial speed is zero ($\dot{y}(0) := 0$);
- State refinement:
  - Passage of time ($\dot{s}(t) = 1$);
  - Ball position in space ($\ddot{y}(t) = -g$);
- Predicate (guard):
  - Ball hits the ground ($y(t) = 0$)
- Set action (on the predicate):
  - Change ball speed ($\dot{y}(t) = -a\dot{y}(t)$), where $a \in R_{(0,1)}$ – inelastic collision.
  - Pure signal *bump* is produced.

TA characteristics:

- Two outputs (and no inputs);
- Only one mode “free” (instantaneous collision).







Supervisory control

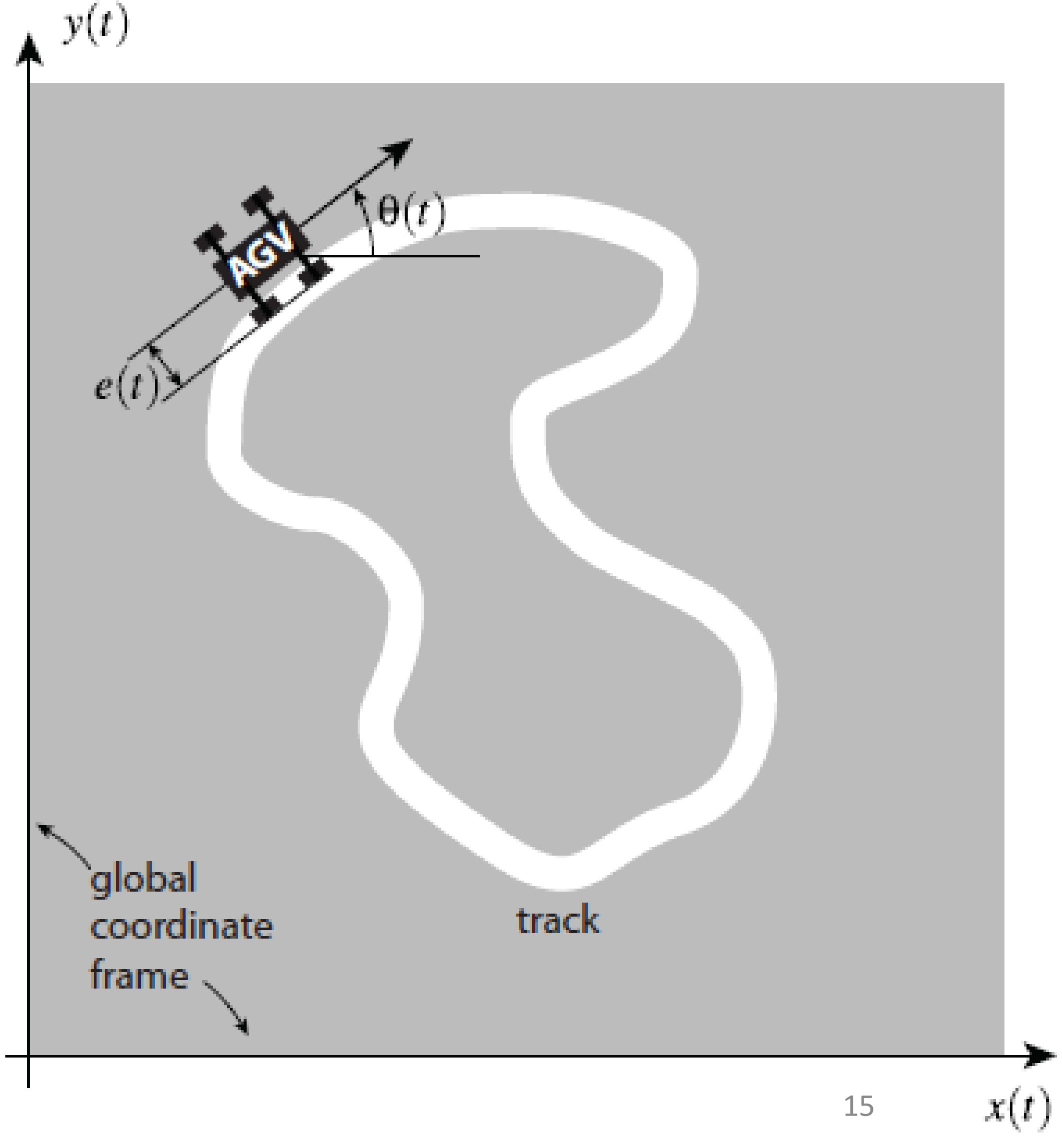
A control system includes four main components:

- A system to be controlled (ie, **plant**);
- **Environment** (where system operates);
- **Sensors** (measure some variables of the plant and environment);
- **Controller** (provide control signals to the plant).

The **controller** has two levels:

- **Low-level control** – determines time-based inputs to the plant;
- **Supervisory control** – determines control strategy to follow.

#Let us consider an Automated Guided Vehicle (AGV) that moves along a closed track painted on a warehouse floor.

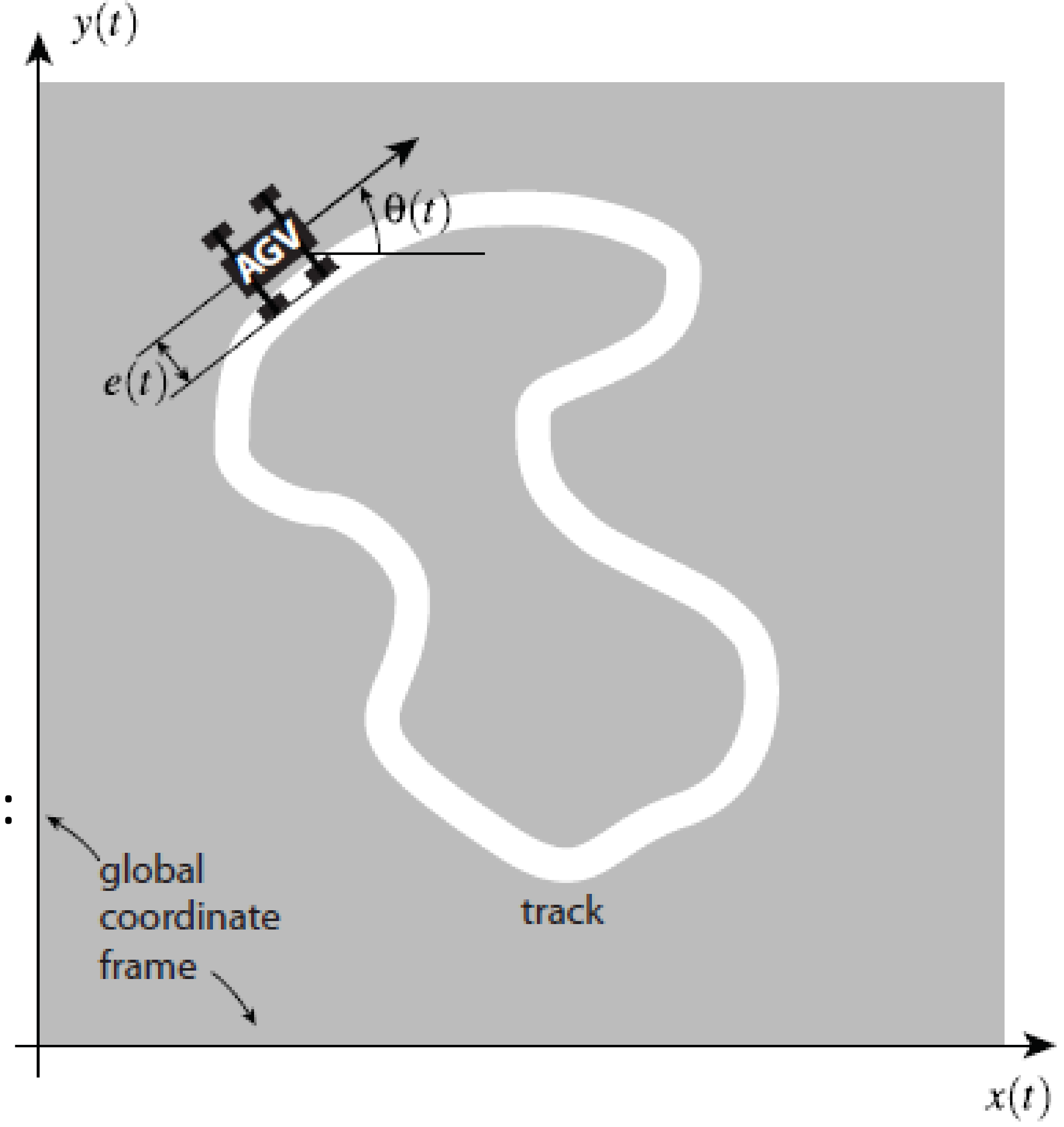


Supervisory control

#Let us consider an Automated Guided Vehicle (AGV) that moves along a closed track painted on a warehouse floor.

Problem description:

- Two control variables (of AGV):
  - Translational speed $u(t) \in R_{[0,10]}$ [m/s];
  - Rotational speed $\omega(t) \in R_{[-\pi,\pi]}$ [rad/s];



AGV position can be defined with:

- Coordinates $\big(x(t), y(t)\big) \in R^2$;
- Orientation $\theta(t) \in R_{(-\pi,\pi]}$.

AGV position can be modelled by the **ODE** system:

$$\begin{cases} \dot{x}(t) = u(t)\cos\theta(t)\,, \\ \dot{y}(t) = u(t)\sin\theta(t)\,, \\ \dot{\theta}(t) = \omega(t). \end{cases}$$

Supervisory control

Suppose AGV has a sensor that measures distance from the track:

$$e(t) = f\big(x(t), y(t)\big),$$

where

$e(t) < 0$ - to the right of the track;

$e(t) > 0$ - to the left of the track.

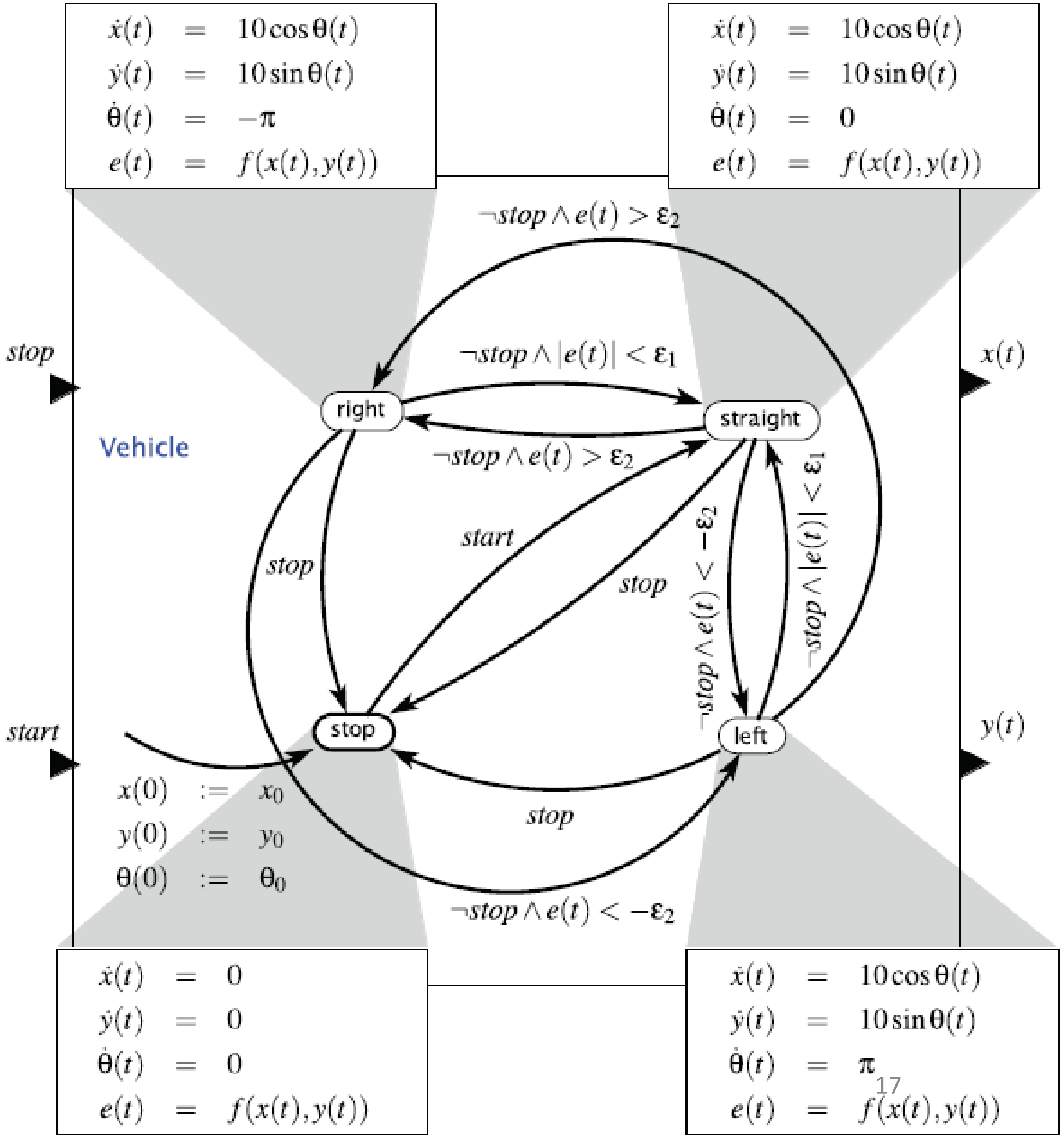
Specify two thresholds $0 < \varepsilon_1 < \varepsilon_2$:

- ε_1 - AGV is close enough;
- ε_2 - AGV far away.

Let's assume the simplest low-level control:

- $u(t) \in \{0,10\}$ m/s;
- $\omega(t) \in \{-\pi, 0, \pi\}$ rad/s.

AGV **Supervisory control** model:



To sum up

- A **discrete system** operates in a sequence of discrete steps and is said to have **discrete dynamics**;
- **Discrete systems** can be modelled with **state machines**:
    - Graphical notation useful for FSM and ESM;
    - Mathematical notation to ensure precise model formulation;
- Two main properties of **state machines** are **determinacy** and **receptiveness**, determining if the system can be effectively controlled;
- **Hybrid systems** provide a bridge between continuous and discrete dynamics:
    - **State machines** are used to reflect discrete dynamics (i.e., mode transition);
    - **State refinement** is used to model continuous dynamics within a mode;
- **Timed automata** is the particular example of a hybrid system, where **time-based refinement of states** is modelled;
- **Supervisory control** model has been considered to illustrate the use of **hybrid systems** when designing multi-level control.

The End

See you next time (March 13)