MEC205, Problem Session Wed/Thurs

Basics of probability

$E[a\bar{X}+bY] = aE[\bar{X}] + bE[Y]$

$VAR[a\bar{X}+b] = a^2 VAR[\bar{X}]$

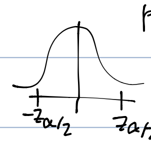
parameter def 3:20 $\quad \hat{\Theta} \quad \Theta$

$\bar{X} = \dfrac{X_1 + \cdots + X_N}{N}$ $\quad BIAS[\hat{\Theta}] = E[\hat{\Theta}] - \Theta$

$E[\hat{\Theta}] = \Theta$

$Y = \dfrac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$

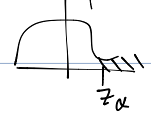
Z



$P[-z_{\alpha/2} \le Z \le z_{\alpha/2}] = 1-\alpha$

$\dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{N}}} = Z$

$P[Z \ge z_\alpha] = \alpha$



$\left[\bar{X} - z_\alpha \frac{\sigma}{\sqrt{N}},\ \bar{X} + z_\alpha \frac{\sigma}{\sqrt{N}}\right]$ $\quad \sigma \to s$

$H_0: \Theta = \Theta_0$ $\quad H_0: \mu = \mu_0$ $\quad 0 < \alpha < 1$ $\quad \beta(\mu')$ POWER

$H_1: \Theta \neq \Theta_0$ $\quad H_1: \mu \neq \mu_0$ $\quad -z_{\alpha/2} \quad z_{\alpha/2}$

$\Theta > \Theta_0$

$\Theta < \Theta_0$

$\dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{N}}}$

$H_0: p = p_0$ $\quad Np_0 \ge 10$ $\quad H_0: \sigma^2 = \sigma_0^2$

$N(1-p_0) \ge 10$ $\quad \dfrac{(N-1)s^2}{\sigma_0^2}$ $\quad \chi^2$

$\dfrac{\hat{p} - p_0}{\quad}$

$$\sqrt{\frac{(1-p_0)p_0}{N}}$$

$H_0: \mu_1 - \mu_2 = \Delta_0$

$$\frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}} \rightarrow S_1^2 \quad S_2^2$$

ASSUME $\sigma_1^2 = \sigma_2^2$

$$\frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{S_p^2}{N_1} + \frac{S_p^2}{N_2}}}$$

$\sigma_1^2 \neq \sigma_2^2$

Ugly DF

PAIRED t-TEST

$\{\bar{X}_{11}, \bar{X}_{12}, \ldots \bar{X}_{1N}\}$ $\{\bar{X}_{21}, \bar{X}_{22}, \ldots \bar{X}_{2N}\}$

$d_i = \bar{X}_{1i} - \bar{X}_{2i}$ $\quad \bar{D} = \frac{\sum_{i=1}^{N} d_i}{N}$

$$\frac{\bar{D} - \Delta_0}{\frac{S_D}{\sqrt{N}}}$$ DF $N-1$

$H_0: p_1 - p_2 = 0$

$N_1 \hat{p}_1 \geq 10 \quad N_2 \hat{p}_2 \geq 10$

$N_1(1-\hat{p}_1) \geq 10 \quad N_2(1-\hat{p}_2) \geq 10$

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\tilde{p}(1-\tilde{p})}{N_1} + \frac{\tilde{p}(1-\tilde{p})}{N_2}}}$$

$$\tilde{p} = \frac{X_1 + X_2}{N_1 + N_2}$$

$H_0: \sigma_1^2 = \sigma_2^2$ $\quad 1 > 2$ UPPER

$\frac{S_1^2}{S_2^2}$ F $N_1 - 1$ NUM, $N_2 - 1$ DEN

$$F_{1-\alpha, N_1-1, N_2-1} = \frac{1}{F_{\alpha, N_1-1, N_2-1}}$$

No Q's on INDEPENDENCE OR HOMOGENEITY

K CATEGORIES

$H_0: p_1 = (p_1)_0, p_2 = (p_2)_0, \ldots, p_k = (p_k)_0$

$E_i = N(p_i)_0$

$$\sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \quad \chi^2 \; k-1 \text{ DF}$$

$E_i \geq 5$

SINGLE-FACTOR ANOVA

$H_0: \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_I$

NORMAL & VARIANCES SAME

$\boxed{\text{ALL SIZES SAME}}$ $SST = \sum_{i=1}^{I} \sum_{j=1}^{J} (X_{ij} - \bar{X}_{..})^2$

$SSE = \sum_{i=1}^{I} \sum_{j=1}^{J} (X_{ij} - \bar{X}_{i.})^2$

$SSTR = \sum_{i=1}^{I} \sum_{j=1}^{J} (\bar{X}_{i.} - \bar{X}_{..})^2$

SOURCE	DF	SS	MS	f
TREATMENT	$I-1$	SSTR	$\frac{SSTR}{I-1}$	$\frac{MSTR}{MSE}$
ERROR	$I(J-1)$	SSE	$\frac{SSE}{I(J-1)}$	
TOTAL	$IJ-1$	SST		

NO TUKEY

$F_{I-1, I(J-1)}$

X Y

$X = x$

$Y = \beta_0 + \beta_1 x + \epsilon$ NOT A FUNCTIONAL RELATIONSHIP

$E[Y|_{X=x}] = \beta_0 + \beta_1 x$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 x$

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x$

$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = SSE$ $\hat{\sigma}^2 = \frac{SSE}{n-2}$

$r^2 = 1 - \frac{SSE}{SST}$ $H_0: \beta_1 = 0$

$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$

$\frac{\hat{\beta}_1}{s} = \frac{\hat{\beta}_1}{s_{\hat{\beta}}}$ t $n-2$ DF

$\bar{S}_{XX}$ $\quad \supset \beta_1$

$$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{x})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}}$$

$H_0: \rho = 0$

$$\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

t $n-2$ DF

$H_0: \mu = 0 \longrightarrow \mu = \mu_0$ NO LARGE SAMPLE

SYMMETRIC, μ = MEDIAN

$$P_0[S_+ \geq c_1] = P_0\left[S_+ \leq \frac{n(n+1)}{2} - c_1\right]$$

$m < n$ ONLY UPPER-SIDED

$H_0: \mu_x - \mu_y = \cancel{0}\ \Delta_0 \quad W =$

NO THROW 0's SUB ONLY FROM X

6-7 Q, 1 has 5-7 parts, some quick, HW & midterm

CALC