$$t = \frac{\bar{X} - \bar{Y}}{S_p\sqrt{\frac{1}{m} + \frac{1}{n}}} = \bar{X}$$

$$S_p = \sqrt{\frac{(m-1)S_x^2 + (n-1)S_y^2}{m+n-2}}$$

$$\bar{X} = \frac{\sum_{i=1}^{m} X_i}{m} = \frac{T_1}{m} \qquad \bar{Y} = \frac{\sum_{j=1}^{n} Y_j}{n}$$

$$S_x^2 = \frac{\left(\sum_{i=1}^{m} X_i - \bar{X}\right)^2}{m} \qquad S_y^2 = \frac{\left(\sum_{j=1}^{n} Y_j - \bar{Y}\right)^2}{n}$$

$$S_x^2 = \frac{\left(T_1 - \frac{T_1}{m}\right)^2}{m} = \frac{(n-1)^2 T_1^2}{m} = \frac{\left(m + \frac{1}{m} - 2\right)T_1^2}{m}$$

$$S_y^2 = \frac{(n-1)^2(T - T_1)^2}{n} = \frac{(n-1)^2(T^* - T_1^2)}{n}$$

$$S_p = \sqrt{\frac{(m-1)^3 T_1^2}{m}}$$

$$S_x^2 = (m-1)^2$$

$$S_p = \sqrt{\frac{\frac{(m-1)^3 T_1^2}{m} + \frac{(n-1)^3(T^* - T_1^2)}{n}}{m+n-2}}$$

$$Y_i = T - T_1$$

$$Y_i^2 = (T^* - T_1^2) = (T - T_1)^2$$

$$T^* = T^2 - 2T_1(T - T_1)$$

$$= T^2 - 2TT_1 - 2T_1^2$$

$$= \sqrt{T_1^2\left(\frac{\frac{(m-1)^3}{m} + \frac{(n-1)^3 T^*}{n}}{m+n-2}\right)}$$

$$T_1\sqrt{\qquad}$$

$$\frac{T_1}{m} - \frac{T - T_1}{n} = \frac{T_1}{m} - \frac{T - T_1}{n} = T_1\left(\frac{1}{m} + \frac{1}{n}\right) - \frac{T}{n}$$

$$= \frac{1}{m} + \frac{1}{n} - \frac{T}{nT_1}$$

$$\frac{1}{m} + \frac{1}{n} - \frac{1}{n}\left(\frac{X_i + Y_i}{X_i}\right) \rightarrow \left(1 + \frac{Y_i}{X_i}\right)$$

$$\left(1 + \frac{T - T_1}{T_1}\right)$$

$$\frac{(m-1)^3 T_1^2}{m} + \frac{(n-1)^3(T^* - T_1^2)}{n}$$

$$\frac{(m-1)^3}{m} T_1^2 + \frac{(n-1)^3}{n}(T^* - T_1^2) = T_1^2\left(\frac{(m-1)^3}{m} + \frac{(n-1)^3}{n}\right) - \frac{(n-1)^3}{n} T^*$$

$$T^2 - 2TT - 2T_1^2$$

$$T_1^2\left(\frac{(m-1)^3}{m}+\frac{(n-1)^3}{n}\right)-\frac{(n-1)^3}{n}\left(T^2-2TT_1-2T_1^2\right)$$

$$T_1^2(M+N)-N\left(T^2-2TT_1-2T_1^2\right)$$

$$T_1^2(M+3N)-NT(T-2T_1)$$

$$MT_1^2+NT_1^2-NT^2+2NTT_1+2NT_1^2$$

$$MT_1^2+3NT_1^2-NT^2+2NTT_1$$

$$MT_1^2+N\left(3T_1^2-T^2+2TT_1\right)$$

$$\frac{T_1^2\left(\frac{(m-1)^3}{m}+\frac{(n-1)^3}{n}\right)-\frac{(n-1)^3}{n}\left(T^2-2TT_1-2T_1^2\right)\left(\frac{1}{m}+\frac{1}{n}\right)}{m+n-2}$$

$$\frac{(m+n)\left(mS_x^2+nS_y^2-S_x^2-S_y^2\right)}{mn(m+n-2)}=(m+n)\left((m-1)^2T_1^2+(n-1)^2(T-T_1)^2-T_1^2\right.$$

$$t=\frac{T_1}{m}-\frac{T-T_1}{n}=\frac{T_1}{m}+\frac{T_1-T}{n}=\frac{T_1(m+n)}{mn}-\frac{T}{n}$$

$$\frac{(mT_1-m\bar{T}+n\bar{T}_1)\frac{1}{mn}}{S_p\sqrt{\frac{m+n}{mn}}}$$

$$S_p=\sqrt{\frac{S_x^2((m-1)n+(m-1)m)+S_y^2\left(m(n-1)+n(n-1)\right)}{n(mn+(m-2)n)}}$$