Ω outcome/sample space $\quad 2^{\Omega} = \{\phi, \{H\}, \{T\}, \{H,T\}\}$

null event

$= \{H, T\}$

$\{1, 2, 3, 4, 5, 6\}$ $\quad 2^{\Omega} = \{\phi, \{R\}, \{B\}, \{G\}, \{R,B\},$

$\{R, B, G\}$ $\quad \{R,G\}, \{B,G\}, \{R,G,B\}\}$

Cardinality $= 2^{x}$ $\quad x =$ # of elements

$P: 2^{\Omega} \to \mathbb{R}$ $\quad A, B \in 2^{\Omega}$ $\quad A \cup B = \{\omega \in \Omega :$ either $\omega \in A$ OR $\omega \in B$, OR $\omega \in A$ AND $\omega \in B\}$

$P(\phi) = 0 \quad P(\Omega) = 1$

$0 \leq P(A) \leq 1$ FOR ALL $A \in 2^{\Omega}$ $\quad A \cap B = \{\omega \in \Omega : \omega \in A$ AND $\omega \in B\}$

disjoint

$P(A \cup B) = P(A) + P(B)$ IF $A \cap B = \phi$ $\quad A^{c} = \{\omega \in \Omega : \omega \notin A\}$

KOLMOGOROV'S AXIOMS $\quad A \cup A^{c} = \{\omega : \Omega\} = \Omega$

$\Omega = \{x_1, x_2, \ldots, x_n\}$

$P(\{x_1\}) = P(\{x_2\}) = \ldots = P(\{x_n\}) = \frac{1}{N}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) > 0$

$P(\{1\}) = 1/6$

$P(\{1,3,5\}) = 1/2 = P(\{1\} \cup \{3\} \cup \{5\}) = P(\{1\}) + P(\{3\}) + P(\{5\})$

$= 1/6 + 1/6 + 1/6$

$$P(\{1\} | \{1,3,5\}) = \frac{P(\{1\} \cap \{1,3,5\})}{P(\{1,3,5\})} = \frac{P(\{1\})}{P(\{1,3,5\})} = \frac{\frac{1}{6}}{\frac{1}{2}} = 1/3$$

$$P(\{1\} | \{2,4,6\}) = \frac{P(\{1\} \cap \{2,4,6\})}{P(\{2,4,6\})} = \frac{P(\emptyset)}{P(\{2,4,6\})} = \frac{0}{\frac{1}{2}} = 0$$

$P(A) \leftarrow$ prior probability

$P(A|B) \leftarrow$ posterior probability

A, B ARE INDEPENDENT EVENTS

IF $P(A \cap B) = P(A)\,P(B)$

A, B ARE INDEPENDENT

IF $P(A|B) = P(A)$

AND

$P(B|A) = P(B)$

$\Omega = A_1 \cup \ldots \cup A_k \quad P(A_i) > 0$

$A_i \cap A_j = \emptyset$

IF $i \neq j$

$$B = (B \cap A_1) \cup (B \cap A_2) \cup \ldots \cup (B \cap A_k)$$

$$P(B) = P(B \cap A_1) + P(B \cap A_2) + \ldots + P(B \cap A_k)$$

$$= P(B|A_1)\,P(A_1) + P(B|A_2)\,P(A_2) + \ldots + P(B|A_k)\,P(A_k)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B)\,P(B)$$

$$P(A_i|B) = \frac{P(B|A_i)\,P(A_i)}{P(B|A_1)\,P(A_1) + P(B|A_2)\,P(A_2) + \ldots + P(B|A_i)\,P(A_i)}$$

$$= \frac{P(A_i \cap B)}{P(B)} = \frac{P(B \cap A_i)}{P(B)} = \frac{P(B|A_i)\,P(A_i)}{P(B)} = \frac{P(B|A_i)\,P(A_i)}{P(B|A_1)\,P(A_1) + \ldots + P(B|A_i)\,P(A_i)}$$

BAYES' THEOREM

$P(B) = .0005 \quad P(B^c) = 1 - .0005 = .9995 \qquad P(B|A) + P(B^c|A) = 1$

IF A PERSON HAS CANCER, THEN POSITIVE TEST 98% $\quad P(A|B) = .98$ $\quad + \quad 0 \leq x \leq 2$

IF A PERSON DOESN'T HAVE CANCER, POSITIVE 2% $\quad P(A|B^c) = .02$

WHAT IS THE PROBABILITY A PERSON HAS CANCER IF TEST RESULT IS POSITIVE

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)\,P(B) + P(A|B^c)\,P(B^c)}$$

$$= \frac{(.98)(.0005)}{(.98)(.0005) + (.02)(.9995)}$$

$$= .024$$

$52 \cdot 51 \cdot 50 \cdot \ldots \cdot 2 \cdot 1 = 52!$ $\quad$ 86,400 SEC/DAY

$52 \cdot 51 \cdot 50$

$75 \cdot 10 \cdot 5 \cdot 10 \cdot 5 \cdot 10 > 4^{10} \times 10^{10}$

$> 175 \times 10^3$

366 DAYS IN YEAR 31,622,400

400 YEARS 1.264896×10^{10} sec

10 BILLION

1.264896×10^{20} SHUFFLES

$X : \Omega \to \mathbb{R}$

$\Omega = \{H, T\}$ $P(\{H\}) = \frac{1}{2}$

$P(\{T\}) = \frac{1}{2}$

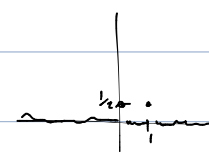
$X(\{H\}) = 1$

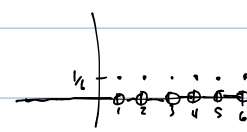
$X(\{T\}) = 0$

PROBABILITY MASS FUNCTION FOR X

$P_X : \mathbb{R} \to \mathbb{R}$

$P_X(z) = P\{X = z\}$





THE (CUMULATIVE) DISTRIBUTION FUNCTION OF X

$F_X : \mathbb{R} \to \mathbb{R}$

$F_X(z) = P\{X \leq z\}$

$X(\{H\}) = 1$ $P(\{H\}) = P(\{T\}) = \frac{1}{2}$

$X(\{T\}) = 0$

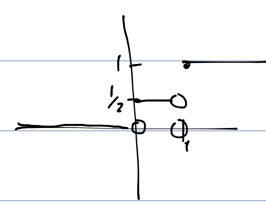
$z < 0 : F_X(z) = P\{X \leq z\} = 0$

$z = 0 : F_X(0) = P\{X \leq z\} = \frac{1}{2}$

$0 < z < 1 : F_X(z) = P\{X \leq z\} = \frac{1}{2}$

$z = 1 : F_X(1) = P\{X \leq 1\} = 1$

$z > 1 : F_X(z) = P\{X \leq z\} = 1$



$\Omega = \{A, B\}$ $0 < p < 1$

$X(\{A\}) = 1$ $P(\{A\}) = p$

$X(\{B\}) = 0$ $P(\{B\}) = 1 - p$

BERNOULLI

X = NUMBER OF DEFECTIVE CHIPS (OUT OF SAMPLE OF N CHIPS)

BINOMIAL RAND. VARIABLE WITH

$P\{X = z\}$

$P\{X = k\}$

$k = 0, \ldots, N$

$= \binom{N}{k} p^k (1-p)^{N-k}$

PARAMETERS N AND P