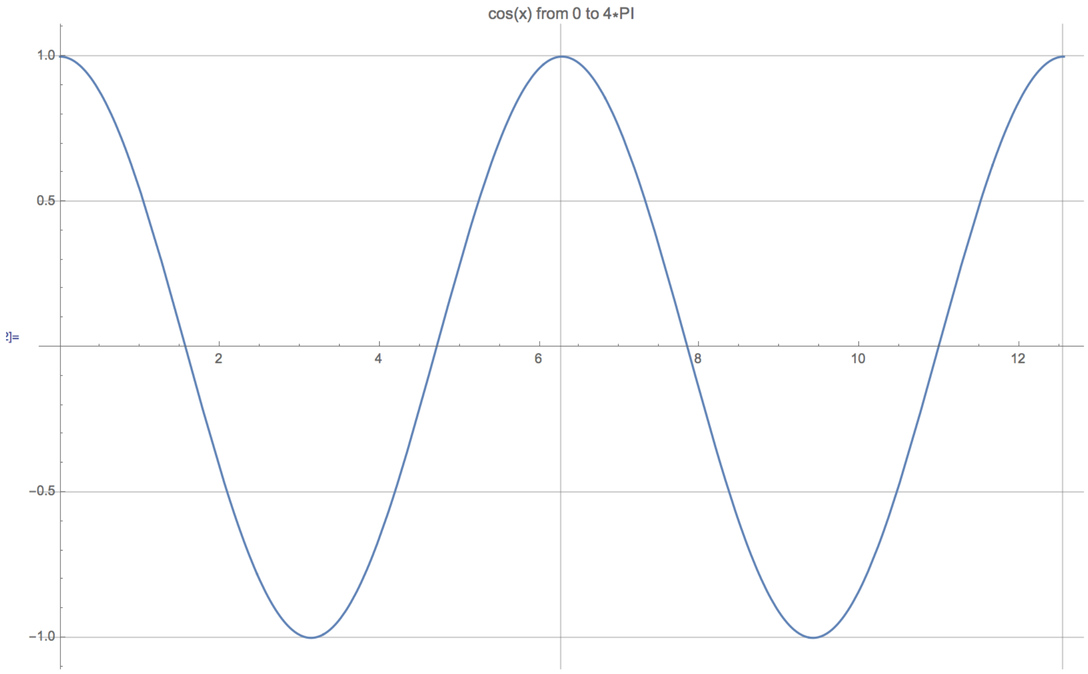
Chem/Stat3240: Homework 1b
Mathematica

August 31, 2016

4 Write the code that takes real numbers L and R and determines the maximum value of $f(x) = \cos(x)$ on the interval $[L, R]$. Assume $L < R$. Recall that the cosine achieves a maximum value of 1 at integral multiples of 2π. The intention of the problem is to use a conditional to determine if the interval contain's a multiple of 2π. If so, cosine is equal to 1 somewhere on the interval. Otherwise, one of the interval endpoints is the maximum. Useful commands for this problem are `Floor, Ceiling`, and `Max`. See the plot below for insight into how to construct the test.

Submit your code as a function in the file `intervalMax.nb` and the tests in the .zip to the collab site.



5 For large n,

$$
\begin{aligned}
R_n &= 1 - \frac{1}{3} + \cdots + \frac{(-1)^{n+1}}{2n-1} = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{2k-1} \approx \frac{\pi}{4} \\
T_n &= 1 + \frac{1}{2^2} + \cdots + \frac{1}{n^2} = \sum_{k=1}^{n} \frac{1}{k^2} \approx \frac{\pi^2}{6} \\
U_n &= 1 + \frac{1}{2^4} + \cdots + \frac{1}{n^4} = \sum_{k=1}^{n} \frac{1}{k^4} \approx \frac{\pi^4}{90}
\end{aligned}
$$

giving two different ways to estimate π:

$$
\begin{aligned}
\rho_n &= 4R_n \\
\tau_n &= \sqrt{6T_n} \\
\mu_n &= (90U_n)^{1/4}
\end{aligned}
$$

For a given value of n, write the code to compute the estimates ρ_n, τ_n, and μ_n in the body of the function template `piEst1` and run the test suite until it passes the tests.

Now write a script `hwk2_1.nb` that uses the code for your estimates (without the function wrapper) and displays the value of $|\pi - \rho_n|$, $|\pi - \tau_n|$, and $|\pi - \mu_n|$ for $n = 100, 200, 300, \ldots, 1000$. Remember to initialize variables to store partials sums. You should also be able to use one `Do` loop and avoid recomputing previous partial sums, instead of separate loops for each partial sum. Useful commands are `Do`,`Abs`, `Mod`, `Sqrt`, and `Power`.

Format the output as a 3-column table with the first column being values of n, and appropriate headers for each column as shown below.

Commands you will find useful in formatting the output are `PaddedForm` for the first column and `ScientificForm` for the subsequent columns. It is also useful to speicify the degree of precision with the `N` command.

```
<< hwk2_1.m
```

n	R_error	T_error	U_error
100	9.99975×10^{-3}	9.51612×10^{-3}	2.38282×10^{-7}
200	4.99997×10^{-3}	4.76635×10^{-3}	3.00098×10^{-8}
300	3.33332×10^{-3}	3.17941×10^{-3}	8.91407×10^{-9}
400	2.50000×10^{-3}	2.38525×10^{-3}	3.76533×10^{-9}
500	2.00000×10^{-3}	1.90853×10^{-3}	1.92929×10^{-9}
600	1.66667×10^{-3}	1.59063×10^{-3}	1.11705×10^{-9}
700	1.42857×10^{-3}	1.36351×10^{-3}	7.03699×10^{-10}
800	1.25000×10^{-3}	1.19314×10^{-3}	4.71550×10^{-10}
900	1.11111×10^{-3}	1.06062×10^{-3}	3.31253×10^{-10}
1000	1.00000×10^{-3}	9.54597×10^{-4}	2.41524×10^{-10}

Submit `piEst1.nb`, the script `hwk2_1.nb`, and the test suite in the .zip file to the collab site.

Remember to comment your code as shown in the example below

```
In[11]:= sphereArea[r_] :=
         (* sphereArea(r):Calculates the surface area of a sphere given the radius
          INPUT:
            r:sphere radius
          OUTPUT:
            A=surface area *)

          (* Compute the area of a sphere of radius r *)
          A = 4. * Pi * r ^ 2;
```