

2a)

$$x, x' \in \mathbb{R}^d$$

$$K(x, x') = \sum_{i=1}^d \cos(x_i - x'_i)$$

$$\cos(x_i - x'_i) = \cos x_i \cos x'_i + \sin x_i \sin x'_i$$

written as the dot product of

$$\phi(x_i) = \begin{bmatrix} \cos(x_i) \\ \sin(x_i) \end{bmatrix} \quad \phi(x'_i) = \begin{bmatrix} \cos(x'_i) \\ \sin(x'_i) \end{bmatrix}$$

Since it can be written as  
the dot product of  $\phi(x_i)$   
and  $\phi(x'_i)$ ,  $K(x, x')$  is positive  
definite symmetric.

2b)

$$K(x, x') = (c + x^T x')^p$$

$$\text{let } x' = y$$

$$K(x, y) = (c + x^T y)^p$$

$$\underline{d=2} \quad x = x_1, x_2 \quad y = y_1, y_2$$

$$\underline{p=2}$$

$$= (c + x_1 y_1 + x_2 y_2)^2$$

$$= c^2 + 2c x_1 y_1 + 2c x_2 y_2 + x_1^2 y_1^2 + x_2^2 y_2^2 + 2x_1 y_1 x_2 y_2$$

$$\Phi(x) = \begin{bmatrix} c \\ \sqrt{2c} x_1 \\ \sqrt{2c} x_2 \\ x_1^2 \\ x_2^2 \\ \sqrt{2} x_1 x_2 \end{bmatrix}$$

$$\Phi(y) = \begin{bmatrix} c \\ \sqrt{2c} y_1 \\ \sqrt{2c} y_2 \\ y_1^2 \\ y_2^2 \\ \sqrt{2} y_1 y_2 \end{bmatrix}$$

$K(x, x')$  is positive definite  
symmetric b/c it is an  
inner product of  $\Phi(x)$  and  $\Phi(x')$

$$x, x' \in \mathbb{R}^d$$

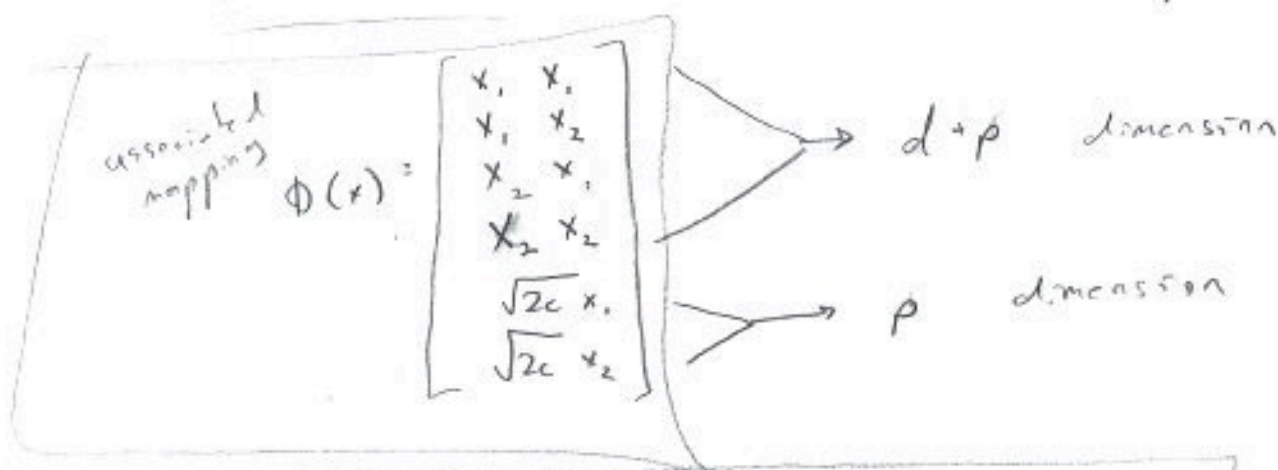
$$K(x, x') = (c + x^T x')^p$$

let  $p=2$

$$K(x, x') = (c + x^T x')^2$$

$$= \sum_i \sum_j x_i x_j (x'_i x'_j) + \sum_i (\sqrt{2c} x_i) (\sqrt{2c} x'_i) + c^2$$

let  $d=2$



feature map

$$\Phi(x) = \begin{pmatrix} d+p \\ p \end{pmatrix} \text{ dimension}$$