2a)

$x, x' \in R^d$

$$K(x, x') = \sum_{i=1}^{d} \cos(x_i - x_i')$$

$$\cos(x_i - x_i') = \cos x_i \cos x_i' + \sin x_i \sin x_i'$$

written as the dot product of

$$\phi(x_i) = \begin{bmatrix} \cos(x_i) \\ \sin(x_i) \end{bmatrix} \qquad \phi(x_i') = \begin{bmatrix} \cos(x_i') \\ \sin(x_i') \end{bmatrix}$$

Since it can be written as the dot product of $\phi(x_i)$ and $\phi(x_i')$, $K(x, x')$ is positive definite symmetric.

2b)

$$K(x, x') = (c + x^T x')^p$$

$\underline{\text{let } x' = y}$

$$K(x, y) = (c + x^T y)^p$$

$\underline{d = 2}$ $\quad x = x_1, x_2 \quad y = y_1, y_2$

$\underline{p = 2}$

$$= (c + x_1 y_1 + x_2 y_2)^2$$

$$= c^2 + 2c x_1 y_1 + 2c x_2 y_2 + x_1^2 y_1^2 + x_2^2 y_2^2 + 2x_1 y_1 x_2 y_2$$

$$\phi(x) = \begin{bmatrix} c \\ \sqrt{2c}\, x_1 \\ \sqrt{2c}\, x_2 \\ x_1^2 \\ x_2^2 \\ \sqrt{2} x_1 x_2 \end{bmatrix} \qquad \phi(y) = \begin{bmatrix} c \\ \sqrt{2c}\, y_1 \\ \sqrt{2c}\, y_2 \\ y_1^2 \\ y_2^2 \\ \sqrt{2} y_1 y_2 \end{bmatrix}$$

$K(x, x')$ is positive definite symmetric b/c it is an inner product of $\phi(x)$ and $\phi(x')$

2 b)

$x, x' \in R^d$

$K(x, x') = (c + x^T x')^p$

let $p = 2$

$K(x, x') = (c + x^T x')^2$

$= \sum_i^n \sum_j^n x_i x_j (x')_i (x')_j + \sum_i^n (\sqrt{2c}\, x_i)(\sqrt{2c}\, x_i) + c^2$

let $d = 2$

associated mapping $\phi(x) = \begin{bmatrix} x_1 x_1 \\ x_1 x_2 \\ x_2 x_1 \\ x_2 x_2 \\ \sqrt{2c}\, x_1 \\ \sqrt{2c}\, x_2 \end{bmatrix}$ $\to d + p$ dimension, $\to p$ dimension

$\boxed{\text{feature map } \phi(x) = \binom{d+p}{p} \text{ dimension}}$