MATH 15100 HOMEWORK

DUE MONDAY, OCTOBER 8, 2018

1. PROBLEM 1

For each integer $n \geq 1$, define
$$T_n = 1 + 2 + 3 + \cdots + n.$$
Therefore,
$$\begin{aligned} T_1 &= 1, \\ T_2 &= 1 + 2 = 3, \\ T_3 &= 1 + 2 + 3 = 6, \\ &\vdots \end{aligned}$$

These numbers are called *triangular numbers* since, for any n, T_n dots can be arranged into a regular triangular patter (in the same way square numbers of dots can form a square pattern.

Prove, using mathematical induction, that
$$T_n = \frac{n(n+1)}{2}.$$

Hint: You may find it necessary to use the relation
$$T_n = T_{n-1} + n.$$

2. PROBLEM 2

You have three pegs and a collection of disks of different sizes. Initially all of the disks are stacked on top on each other according to size on the first peg – the largest disk being on the bottom, and the smallest on the top. A move in this game consists of moving a disk from one peg to another, subject to the condition that a larger disk may never rest on a smaller one. The objective of the game is to find a number of permissible moves that will transfer all of the disks from the first peg to the third peg, making sure that the disks are assembled on the third peg according to size. The second peg is used as an intermediate peg.

Show that you can move n disks from the first peg to the third peg in only $2^n - 1$ moves.