November 14 — § 4.2 Increasing and Decreasing Functions.

Definition A function $f(x)$ is said to be increasing on S if for all $x_1, x_2 \in I$, $x_1 < x_2 \Longrightarrow f(x_1) < f(x_2)$

decreasing on S if for all $x_1,x_2 \in I$, $x_1<x_2 \Longrightarrow f(x_1) > f(x_2)$

nondecreasing on S if for all $x_1,x_2\in I$, $x_1<x_2 \Longrightarrow f(x_1) \le f(x_2)$

nonincreasing on S if for all $x_1,x_2\in I$, $x_1<x_2 \Longrightarrow f(x_1) \geq f(x_2)$.

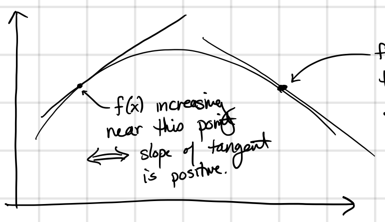
Examples 1) $f(x) = \frac{1}{x}$ is decreasing on $(0,\infty)$

2) $f(x) = -x$ is decreasing on $(-\infty,\infty)$

3) $f(x) = x^2$ is increasing on $[0,\infty)$
decreasing on $(-\infty, 0]$

A linear function $f(x) = mx+b$ is increasing if $m>0$ and decreasing if $m<0$.

The derivative offers a test for determining whether a function is increasing or decreasing.



Theorem Let $f(x)$ be a function and I an interval. Then f is

1) $f'(x) > 0$ for all x in $I \Longrightarrow$ increasing on I

2) $f'(x) < 0$ for all x in $I \Longrightarrow$ decreasing on I

Example 1) $f(x) = \frac{1}{x} \longrightarrow f'(x) = -\frac{1}{x^2} < 0$ on $(0,\infty)$

Therefore, f is decreasing on $(0,\infty)$.

2) $f(x) = x^3 - 12x^2 + 36x - 1$

$f'(x) = 3x^2 - 24x + 36$

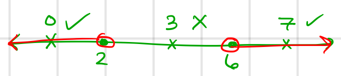
On what intervals is f increasing?

$f'(x) > 0$

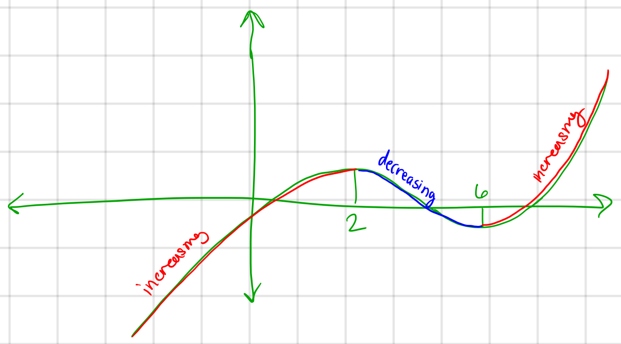
$3x^2 - 24x + 36 > 0$

$x^2 - 8x + 12 > 0$

$(x-2)(x-6) > 0$

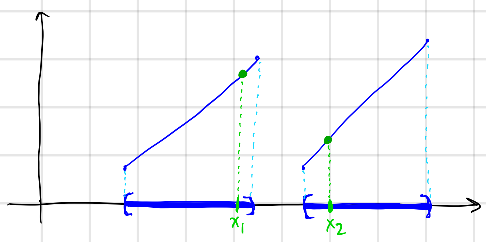


f is increasing on $(-\infty, 2)$ and on $(6,\infty)$



f is not increasing on $(-\infty,2) \cup (6,\infty)$ since $1.5 < 6.5$ but $f(1.5) > f(6.5)$
$f(1.5) = 29.375$, $f(6.5) = 0.625$

It may happen that a function increases on two intervals but is not increasing on their union.



The graph of some function is shown on the right.
The function increases on each of the blue intervals but not on their union because we may pick x_1 and x_2 with $x_1 < x_2$ but $f(x_1) > f(x_2)$

Example $f(x) = \sin(x) + \cos(x)$ is increasing on $(0, \pi/4)$.

$f'(x) = \cos(x) - \sin(x) > 0$

$\cos(x) > \sin(x)$ is true for x in $(0, \frac{\pi}{4})$.

Example On what intervals is $f(x) = x^3 - 3x + 5$ decreasing?

$f'(x) = 3x^2 - 3 < 0$

$x^2 - 1 < 0$

$x^2 < 1$

$-1 < x < 1$

Therefore, the function f is decreasing on $(-1, 1)$.

Theorem If f is continuous on $[a,b]$ and increasing (decreasing) on (a,b),
then f is increasing (decreasing) on $[a,b]$.

Example $f(x) = x^3 - 3x + 5$ decreases on $(-1,1)$ and is continuous on $[-1,1]$.
Therefore, by this theorem, f decreases on $[-1,1]$.

Example Let $f(x) = x^3 - 12x^2 + 36x - 1$

We found that f is increasing on $(-\infty, 2)$ and $(6,\infty)$

By the theorem, we may include the endpoints of these intervals
f is increasing on $(-\infty, 2]$ and
f is increasing on $[6,\infty)$.