

MATH 15100 HOMEWORK

DUE WEDNESDAY, NOVEMBER 14, 2018

1. PROBLEM 1.A

Prove the following theorem.

Theorem 1.1. Suppose f and g are functions, differentiable on (a, b) , such that $(f \circ g)(x) = x$. Then

$$g'(x) = \frac{1}{f'(g(x))}$$

for all x in (a, b) .

We say that g is the *inverse function* of f and write $g = f^{-1}$.

2. PROBLEM 1.B

Give examples of two functions f and g , defined on $(0, 1)$ such that $(f \circ g)(x) = x$. Do not use $f(x) = g(x) = x$.

3. PROBLEM 2

Suppose f is a differentiable function such that

$$f(2x) = (f(x))^2.$$

Show that if $f'(0) \neq 0$, then

$$f(0) = 1.$$

Hint: Use the Chain Rule.