Homework 5: Math 15200

1. Let f be an increasing function on $[a, b]$ and let f^{-1} be the inverse of f. Show that

$$\int_a^b f(x)\,dx + \int_{f(a)}^{f(b)} f^{-1}(x)\,dx = bf(b) - af(a).$$

You may assume that the graph of $f(x)$ lies in the first quadrant. That is, you may assume that $0 \leq a < b$ and $f(x) > 0$ for all x in $[a, b]$.

Hint: Draw a picture.

2. Show that

(a) $\sec(\operatorname{arccot}(x)) = \dfrac{\sqrt{1+x^2}}{x}$

(b) $\sec(\operatorname{arccsc}(x)) = \dfrac{x}{\sqrt{x^2-1}}$

(c) $\tan(\operatorname{arcsec}(x)) = \sqrt{x^2-1}$