Midterm 2 Review

13 November 2018

1. Differentiate the following functions at the given points using the limit definition of derivative (difference quotient).

   (a) $f(x) = x^3 - 4x$ at $x = 1$

   (b) $f(x) = \dfrac{1}{x+4}$ at $x = 4$

   (c) $f(x) = \sqrt{x-4}$ at $x = 3$

   (d) $\dfrac{x-1}{x+2}$ at $x = 0$

2. Differentiate the following functions.

(a) $f(x) = \dfrac{x^5}{7} - \dfrac{x^2}{3} - \dfrac{1}{x^2}$

(b) $f(x) = \dfrac{x^2 - 2x - 3}{x^3 + 2x - 1}$

(c) $f(x) = (x^4 - 2x^3 + 5x^2 - 2x + 1)(x^5 - x^4 + 4x^3 - 3x^2 + 1)$

(d) $f(x) = \dfrac{1 + \cos^2(x)}{1 + \sin^2(x)}$

(e) $f(x) = \cot(x)(1 - \sec(x))$

(f) $f(x) = \csc(x) + \tan(x)\sin(x)$

(g) $f(x) = (x^3 - 1)^{17}$

(h) $f(x) = \big(\cos(x) - \sin(x)\big)^{12}$

(i) $f(x) = \cos^7(1 - x^2)$

3. Find the equation of the tangent line to the graph of the following function at the given point.

   (a) $f(x) = x^2 - 3x + 4$ at $(0, 4)$

   (b) $f(x) = \cos(x) + 2x$ at $\left(\frac{\pi}{4}, \frac{\sqrt{2}+\pi}{2}\right)$

   (c) $\frac{1}{x-2}$ at $(3, 1)$

4. Calculate

(a) $\dfrac{d^2}{dx^2}\Big[(2x+3)\cos(x)\Big]$

(b) $\dfrac{d^3}{dx^3}\Big[x^3-2x^2+x+5\Big]$

(c) $\dfrac{d^2}{dx^2}\Big[\csc(x)\Big]$

(d) $\dfrac{d^4}{dx^4}\Big[\cos(x)\Big]$

5. A bucket which is in the shape of an inverted cone. The height of the bucket is 10 m and the radius of its base is 5 m. The bucket is being filled with paint at a rate of 0.25 m^3/s. Find the rate at which the height of the water is rising when there is 1 m^3 of water in the bucket.

   Given a cone of height h with base radius r, its volume is given by $V = \frac{1}{3}\pi r^2 h$.

   Hint: You will need to use similar triangles to find a relationship between the base radius and height of the water in the bucket ($r = h/2$).

6. A rubber band is stretched along the equator of a spherical balloon which is being filled at a rate of 3 cm^3/s. How fast is the rubber band stretching when the balloon contains 50 cm^3 of air?

   The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$ *and the circumference of a circle of radius r is* $C = 2\pi r$.

7. Find the **equation of the tangent line** to the curve given by the following equation at the given point.

   (a) $2x^3 + y^2 - 3xy = 0$ at $(1, 2)$

   (b) $\cos(xy) = x^2 - 1$ at $\left(1, \frac{\pi}{2}\right)$.