Homework 4

Due January 31, 2017

1. Consider the following functions

   - $f(x) = 2x^2 - 6x + 7 \qquad \text{on } (-\infty, \infty)$
   - $f(x) = 3x^5 - x^3 \qquad \text{on } (-\infty, \infty).$
   - $f(x) = \dfrac{x^2 - 2x + 4}{x - 2} \qquad \text{on } (-\infty, \infty)$
   - $f(x) = \sin(x) + x \qquad \text{on } [0, 2\pi]$

   For each of the above functions, answer the following (worth 2 points each, for a total of 32 points).

   (a) Determine where the functions are monotone (increasing and decreasing).

   (b) Identify the local extrema of the function on the given interval and classify them as local maxima or local minima.

   (c) Determine the concavity of the above functions (where are they concave up and concave down?).

   (d) Sketch a quick graph of each function.

2. Let $f(x)$ be a twice differentiable function (that is, both $f'(x)$ and $f''(x)$ exist at all points) on $(-\infty, \infty)$ which is increasing. Let $g(x)$ be any twice differentiable function on $(-\infty, \infty)$.

   (a) (1 point) What are the possible types (endpoint, singular, stationary) of critical points that $g(x)$ could have?

   (b) (3 points) Show that if c is a stationary point of $g(x)$ then c is a critical point of $f \circ g(x)$.

   (c) (3 points) Show that if $g(x)$ attains a maximum at c then $f \circ g(x)$ attains a maximum at c.

3. (3 points) If $f(x)$ and $g(x)$ are differentiable functions increasing on $(-\infty, \infty)$, then $f \circ g(x)$ is also an increasing function.

4. For parts (c) and (d) below, you will need to use Questions 2, 3, 4(a), and 4(b).

   (a) (1 point) Show that $f(x) = x^3 + x$ is increasing.

   (b) (1 point) Show that $g(x) = \sin(x) + 2x$ is increasing.

   (c) (1 point) Show that $f \circ g(x) = (\sin(x) + 2x)^3 + (\sin(x) + 2x)$ is increasing.

(d) (3 points) Find the minimum value of the function

$$F(x) = \left(\sin(x^2 - \pi) + 2(x^2 - \pi)\right)^3 + \left(\sin(x^2 - \pi) + 2(x^2 - \pi)\right).$$