MATH 15100 HOMEWORK

DUE MONDAY, OCTOBER 8, 2018

1. Problem 1

Determine where the function

$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{for } x < 0 \\ \frac{x-2}{x^2-2}, & \text{for } x \geq 0 \end{cases}$$

is continuous. Show your work

2. Problem 2

Compute the following limits.

(1) $\displaystyle\lim_{x\to 0} \frac{\sin(7x)}{3x}$

(2) $\displaystyle\lim_{x\to 0} \frac{1-\cos(x)+\sin(x)}{x}$

(3) $\displaystyle\lim_{x\to 0} \frac{1-\cos(x^2)}{x^4}$

Problem 1| We break this up into two parts

1) When $x \geq 0$, $f(x) = \frac{x-2}{x^2-2}$. Since rational functions are continuous everywhere they are defined, $\frac{x-2}{x^2-2}$ is continuous whenever $x^2-2 \neq 0$, i.e. $x \neq \pm\sqrt{2}$.

Since $x=\sqrt{2}$ is the only one of these points such that $x \geqslant 0$, we have ~~that~~ f is continuous for all $x > 0$, $x \neq \sqrt{2}$. (We do not yet know about $x=0$, since this is an endpoint for the interval given by $x \geqslant 0$, so we only know the right-handed limit exists.)

2) When $x<0$, $f(x) = \frac{x^2-1}{x-1}$. The rational function $\frac{x^2-1}{x-1}$ is continuous everywhere except at $x=1$. Since 1 does not satisfy $1<0$, f is continuous for all $x<0$.

Therefore, f is continuous <u>at least</u> on $(-\infty, 0) \cup (0, \sqrt{2}) \cup (\sqrt{2}, \infty)$. This is everywhere except 0 and $\sqrt{2}$.

It is discontinuous at $x=\sqrt{2}$ since $f(\sqrt{2})$ is not defined.

To see if it is continuous at $x=0$, we calculate.

$$\left.\begin{aligned} \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+} \frac{x-2}{x^2-2} = \frac{-2}{-2} = 1 \\ \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-} \frac{x^2-1}{x-1} = \frac{-1}{-1} = 1 \end{aligned}\right\} \Longrightarrow \lim_{x\to 0} f(x) \text{ exists and } = 1.$$

Since $f(1) = \frac{0-2}{0^2-2} = \frac{-2}{-2} = 1$, we see that $\lim_{x\to 0} f(x) = f(0)$ and so f is continuous at $x=0$.

Therefore, f is continuous on $(-\infty, \sqrt{2}) \cup (\sqrt{2}, \infty)$.

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Problem 2

1) $\lim_{x\to 0} \frac{\sin(7x)}{3x} = \lim_{x\to 0} \frac{\sin(7x)}{7x} \cdot \frac{7}{3}$

$= 1 \cdot \frac{7}{3}$

$= \boxed{\frac{7}{3}}$

2) $\lim_{x\to 0} \frac{1-\cos(x)+\sin(x)}{x} = \lim_{x\to 0} \frac{1-\cos(x)}{x} + \lim_{x\to 0} \frac{\sin(x)}{x}$

$= 0 + 1$

$= \boxed{1}$

3) $\lim_{x\to 0} \frac{1-\cos(x^2)}{x^4} = \lim_{x\to 0} \frac{1-\cos(x^2)}{(x^2)^2}$

$= \boxed{\frac{1}{2}}$