Gravitational microlensing and its effects on polarization signals received from stars

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Summary

We study gravitational lensing and microlensing. Formulas have been developed to calculate the shape deformation of the source image and the light curves of a microlensing event as a result of gravitational lensing and microlensing. Also, we looked at polarization signals produced by scattering in stellar atmospheres and how they are viewed in a faraway star regardless of whether they have been distorted by lensing in the first place. The Python codes for making the simulations can be found in the appendices.

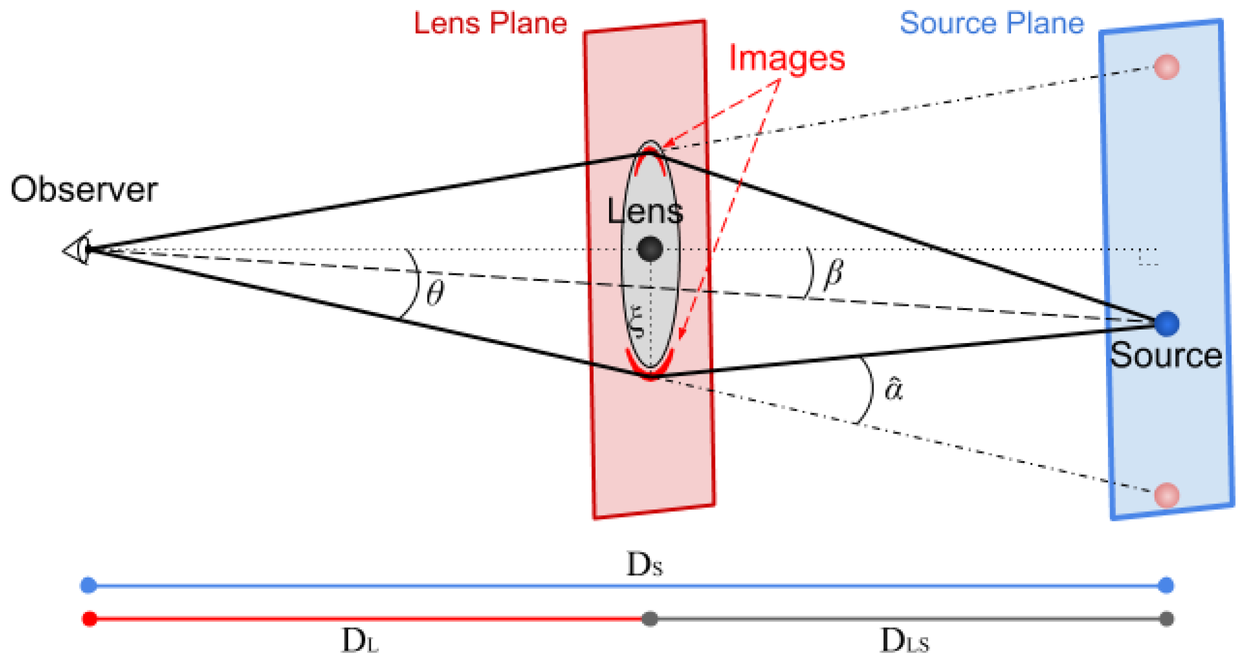

FIG. 1. Schematic setup of a microlensing system.

I. GRAVITATIONAL MICROLENSING

Gravitational microlensing is defined as the increase in the brightness of a background star due to passing through the gravitational field of a foreground object. Einstein showed that when light passes by a massive object it will bend towards that mass and the inclination angle is

$$\alpha = \frac{4GM}{c^2 b}. \tag{1}$$

Here M is the mass of the lens, G is the universal gravitational constant and c is the speed of light. the impact parameter b is assumed to be much larger than the Schwarzschild radius which holds for almost all cases. This approximation is called the weak field approximation.

Formula shows that the inclination angle is independent from the relativistic mass and therefore frequency of photon. So one of the main characteristic features of microlensing events is it's color independence.

A. Lens equation

We assume source and lens have the distances D_S and D_L to the observer respectively. And $D_{LS} = D_S - D_L$ is the distance between them. We wish to find an equation to calculate radial location of produced images (θ) with respect to radial location of source (β). The setup is shown in Fig.(1). In galactic distances, angles are very small so we can write $sin(\alpha) = \alpha$. We have

$$\theta D_S = \beta D_S + \alpha D_{LS} \,. \tag{2}$$

In Fig.(1), impact parameter $\xi = \theta D_L$. by placing this and Eq.(1) into Eq.(2) we get

$$\theta^2 - \beta\theta = \theta_E^2 \,, \tag{3}$$

and

$$\theta_E = \sqrt{\frac{4GMD_{LS}}{c^2 D_L D_S}} \tag{4}$$

is called angular Einstein radius.

B. Magnification coefficient

In Eq.(3) if $\beta = 0$, meaning that observer, lens and source are all on a line, star's image appears as a ring with radius of θ_E called Einstein ring. but in other cases, Eq.(3) has two solutions

$$\theta = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2} \,. \tag{5}$$

This gives two images with opposite parities, one inside the Einstein ring and the other outside. as in Fig.(2).

In most cases, images can not be optically resolved and we only see their combined brightness. Irradiance received is directly proportional to area of images. So we can calculate the brightness of images relative to source

$$\frac{I_{image}}{I_{star}} = \frac{D_S^2\, d\theta\, sin(\theta)\, d\phi}{D_S^2\, d\beta\, sin(\beta)\, d\phi} = \frac{\theta\, d\theta}{\beta\, d\beta} \,. \tag{6}$$

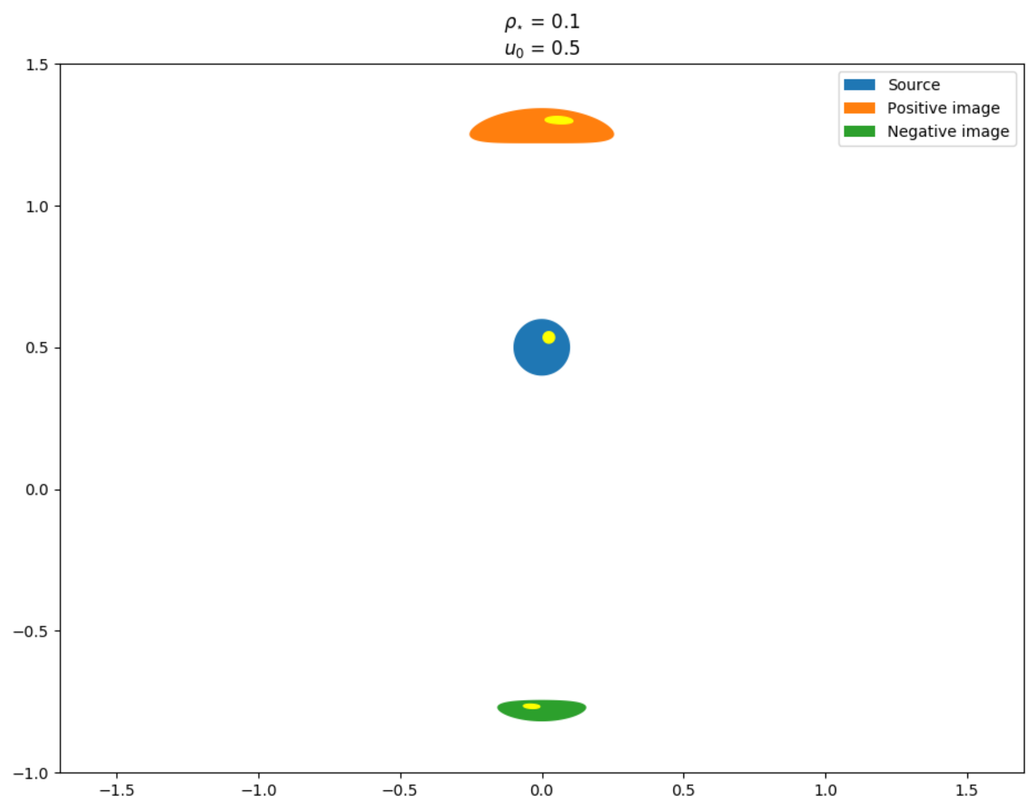

FIG. 2. Source star and it's images due to microlensing. Here lens is positioned at the origin of coordinate system. $\rho_\star$ is the normalized radius of source and u_0 is the normalized impact parameter. Also a stain on the surface of star is placed to show parities of images. Code to produce this figure can be found in appendix A.

Adding this value for both images, gives the magnification

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}\,. \tag{7}$$

Here $u = \theta/\theta_E$ is the normalized impact parameter.

Studying plots of magnification versus time, the so-called lightcurve of the microlensing event, is the main job of astrophysicists who work on gravitational microlensing.

Here it's necessary to note that Eq.(7) only describes the magnification coefficient of a

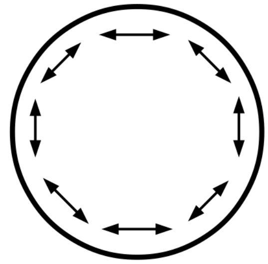

FIG. 3. A schematic representation of polarization map on the surface of a star.

point-like source. Sometimes it's a good approximation but usually the situation is more complex. For example in the case of source star being very large or when we want to study properties of received light that vary on the surface of source star, such as polarization or spectral properties. In those situations we need to integrate the magnification coefficient over the surface of source star.

II. STOKES PARAMETERS

Observations on the sun and nearby stars show that light emitted from each point on the star is at least partially polarized. It is believed that this is due to scattering in the stellar atmosphere. The direction of polarization as shown if Fig.(3) is perpendicular to the radius of the picture of star.

The brightness and the degree of linear and circular polarization of the source can best be described by the Stokes parameters, denoted by I, Q, U, V, where I is the total intensity, $Q = I_y - I_x$ is the difference in intensity measured in two perpendicular directions, x and y, U is the difference in intensity measured in two perpendicular directions at 45° to the x- and y-axes, and V is the net circular polarization.

First we need to note that there is usually no net circular polarization so we only need to calculate Q and U. In the case of distant stars without microlensing, star's image has a circular symmetry and linear polarization signals cancel out. As we have shown in previous section, gravitational lensing, breaks this symmetry and a net polarization can be seen. Chandrasekhar obtained these formulas for a simplified Thompson scattering. Here we also didn't consider limb-darkening effects.

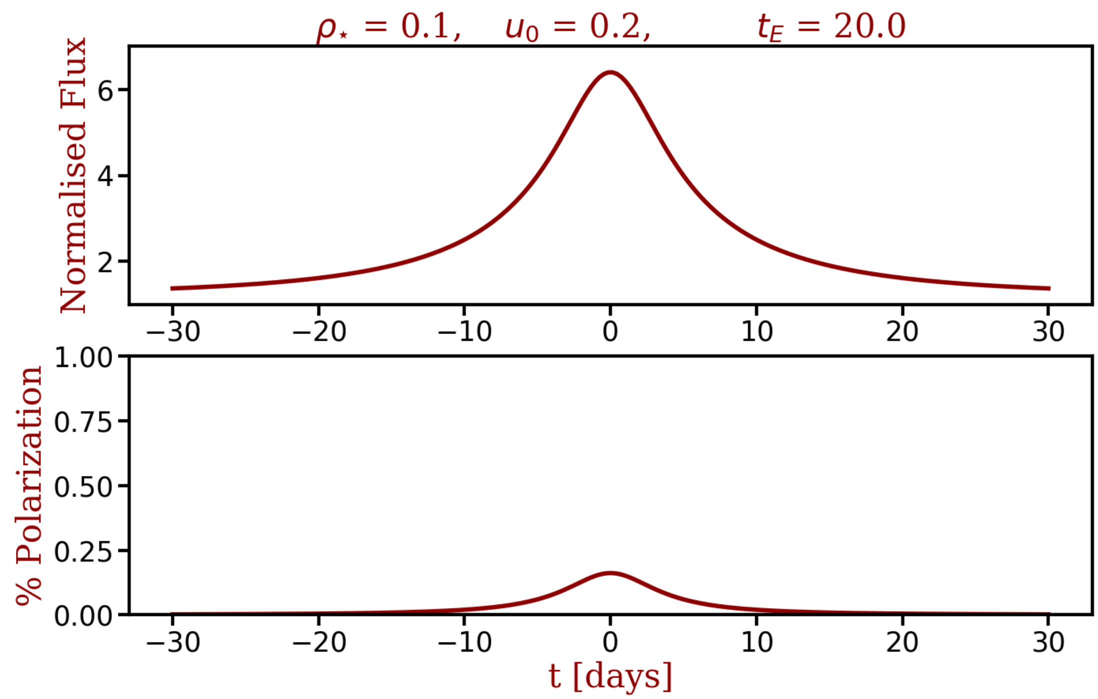

FIG. 4. Effect of microlensing on polarization signals received from a source. $\rho_\star$ is the normalized radius of source, u_0 is the normalized closest impact parameter and t_E called Einstein time, is the time it takes the source to pass the Einstein ring of the lens. Parameters used here, are similar to that of a typical star in the center of the galaxy. Code to produce this figure can be found in appendix B.

$$I = I_0 \iint_{surface} A(u)\, r\, dr\, d\phi, \tag{8}$$

$$U = I_0 \iint_{surface} A(u)\, cos(2\phi)\, r\, dr\, d\phi, \tag{9}$$

$$Q = I_0 \iint_{surface} A(u)\, sin(2\phi)\, r\, dr\, d\phi. \tag{10}$$

If $\vec{u}_0$ is the closest impact parameter of the lens and center of source, $u = |\vec{r} \cdot \vec{u}_0|$. I_0 is the total intensity of unlensed source. The degree of polarization is given by

$$p = \frac{\sqrt{U^2 + Q^2}}{I} \tag{11}$$

We preformed a simple simulation to confirm that in a microlensing event a polarization signal can be seen. The code comes in Appendix B and the results are shown in Fig.(4).

III. CONCLUSION

Microlensing of stellar sources, produce variable polarization. Depending on the geometry of the lensing system, the amount of polarization differs but it is usually below 1 percent. The measurement of the variable polarization gives much more information about the lensing system compared to simple photometry. In principle it can yield the Einstein radius of the lens and it's velocity direction on the sky.

The low levels of polarization produced by lensing means the stars have to be sufficiently large for a given size of telescope. During lensing, the rise in polarization generally takes place significantly later than the rise in flux. this would allow astronomers to first detect a microlensing event and implement an early warning system to study the polarization variability with a larger telescope in detail.

In this semester project, we have concentrated on the simplest stellar atmosphere model. Papers studying more sophisticated models are available for decades now and it is still an open field for study.

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[1] Frank L. Pedrotti, Leno S. Pedrotti, Introduction to Optics, Second edition, *Prentice Hall*, 1993

[2] Chandrasekhar S., Radiative Transfer, *Dover Press, New York*, 1960

[3] Parisa Sangtarash, A review of microlensing and it's applications, *Master's thesis*, 2019

[4] Einstein A., Lens-like action of a star by the deviation of light in the gravitational field, *Science, 84, L506*, 1936

[5] Simmons J. F. L., Newsam A. M., Willis J. P., Variable polarization and luminosity for microlensing of extended stellar sources, *MNRAS, 276, 182*, 1995b

Appendix A: Code to produce Fig.(2)

```
import numpy as np
import matplotlib.pyplot as plt

#Given values
loc_star = [0, 0.5]          #Star's center
loc_stain = [0.025, 0.535]     #Stain's center
rho_star = 0.1               #Normalized radius of star
rho_stain = 0.02             #Normalized radius of stain

#This function builds the location vectors of images
def lens(loc, rho):
    n = 1000
    alpha = np.linspace(0, 2*np.pi, n).reshape((n, 1))                #Azimuthal angle
    location = np.ones((n, 2), float) * loc
    u = rho * np.hstack((np.cos(alpha), np.sin(alpha))) + location    #Source
    U = np.linalg.norm(u, axis=1)

    theta_p = (U + np.sqrt(U**2 + 4))/2
    pi = u * (theta_p/U)[:, None]          #Positive image

    theta_n = (U - np.sqrt(U**2 + 4))/2
    ni = u * (theta_n/U)[:, None]          #Negative image

    return u, pi, ni

#Plotting
star, p_star, n_star = lens(loc_star, rho_star)
stain, p_stain, n_stain = lens(loc_stain, rho_stain)

plt.title("$\\rho_{\star}$ = %.1f\n$u_{0}$ = %.1f" % (rho_star, loc_star[1]))
```

```
plt.fill(star[:,0], star[:,1], label="Source")
plt.fill(p_star[:,0], p_star[:,1], label="Positive image")
plt.fill(n_star[:,0], n_star[:,1], label="Negative image")
plt.legend()

plt.fill(stain[:,0], stain[:,1], color='yellow')
plt.fill(p_stain[:,0], p_stain[:,1], color='yellow')
plt.fill(n_stain[:,0], n_stain[:,1], color='yellow')

plt.gca().set_aspect('equal')
plt.xlim(-1.7, 1.7)
plt.ylim(-1, 1.5)
plt.show()
```

Appendix B: Code to produce lightcurves

```
import numpy as np
import matplotlib.pyplot as plt

#Given values
rho_star = 0.1  #Normalized radius of star
u0 = 0.2        #Normalized impact parameter
t0 = 0
tE = 20         #days
I0 = 1200

#Main function
def lens(t, rho):
    n = 6800
    phi = np.linspace(0, 2*np.pi, n)    #Azimuthal angle
    phi_r = phi.reshape((n, 1))
    location = np.ones((n, 2), float) * [(t-t0)/tE, u0]
```

```
    r = rho * np.hstack((np.cos(phi_r), np.sin(phi_r))) + location
    u = np.linalg.norm(r, axis=1)

    A = (u**2 + 2)/(u * np.sqrt(u**2 + 4))  #Magnification
    I = (I0/n) * np.sum(A)                   #First Stokes parameter
    U = (I0/n) * np.sum(A * np.cos(2 * phi))    #Second Stokes parameter
    Q = (I0/n) * np.sum(A * np.sin(2 * phi))    #Third Stokes parameter

    return I, Q, U

#Initialization
Nt = 200
Nr = 31
t = np.linspace(-30, 30, Nt)
rho = np.linspace(0, rho_star, Nr)
I = np.zeros(Nt)
Q = np.zeros(Nt)
U = np.zeros(Nt)
p = np.zeros(Nt)

#Calculation loop
for j in range(Nt):
    for k in range(1, Nr):
        S = np.pi * (rho[k]**2 - rho[k-1]**2)   #Area
        Ij, Qj, Uj = lens(t[j], rho[k])
        I[j] += (Ij * S)
        Q[j] += (Qj * S)
        U[j] += (Uj * S)

    p[j] = np.sqrt(Q[j]**2 + U[j]**2)/I[j]      #The degree of polarization
```

```
#Plotting
font = {'family': 'serif',
        'color':  'darkred',
        'weight': 'normal',
        'size': 30,
        }

plt.subplot(211)
plt.plot(t, I/Nr, 'darkred', linewidth=4)
plt.tick_params(labelsize=25, width=3, length=10)
plt.gca().spines['top'].set_linewidth(3)
plt.gca().spines['right'].set_linewidth(3)
plt.gca().spines['bottom'].set_linewidth(3)
plt.gca().spines['left'].set_linewidth(3)
plt.ylabel('Normalised Flux', fontdict=font)
plt.ylim([1, 7])

plt.subplot(212)
plt.plot(t, p*100/Nr, 'darkred', linewidth=4)
plt.tick_params(labelsize=25, width=3, length=10)
plt.gca().spines['top'].set_linewidth(3)
plt.gca().spines['right'].set_linewidth(3)
plt.gca().spines['bottom'].set_linewidth(3)
plt.gca().spines['left'].set_linewidth(3)
plt.ylabel('% Polarization', fontdict=font)
plt.ylim([0, 1])
plt.xlabel('t [days]', fontdict=font)
plt.show()
```