

# Quantum Decoherence of a Central Spin Systems

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# Quantum Decoherence

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# Quantum Decoherence

Formally speaking, decoherence is a dynamical development of quantum correlations between the central system and its environment.

# Quantum Decoherence

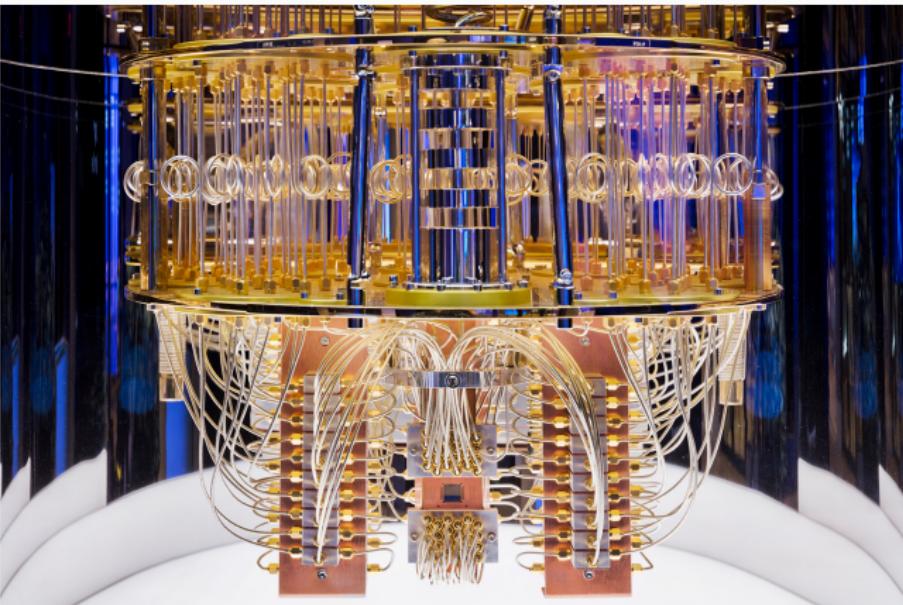


Figure 1: IBM's Quantum Computer

# Quantum Decoherence

System initial state is

$$|\Psi(t=0)\rangle = |\psi_0\rangle \otimes |\chi_0\rangle \quad (1)$$

Density matrix

$$\rho_S(t) = \text{Tr}_B |\Psi(t)\rangle\langle\Psi(t)| \quad (2)$$

# Quantum Decoherence

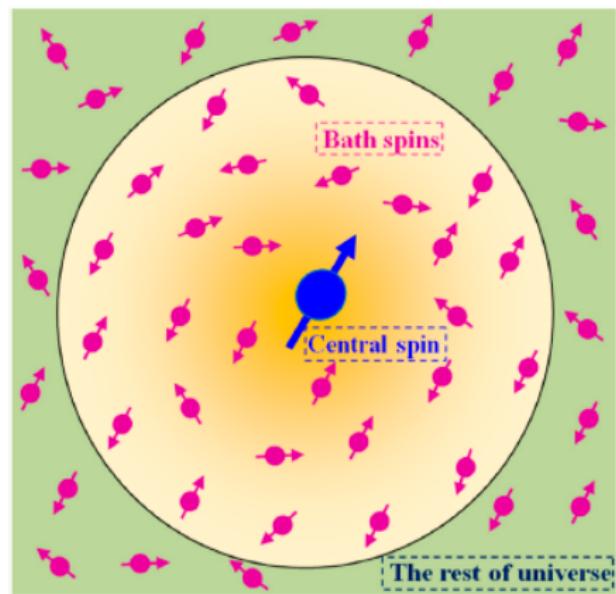


Figure 2: Central spin System[3]

# Model

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# Model

We consider the following class of models[1]. There is a central system made of  $M$  coupled spins  $S_m$

$$S_m = 1/2, \quad m = 1, \dots, M \quad (3)$$

The spins  $S_m$  interact with a bath consisting of  $N$  environmental spins  $I_n$

$$I_n = 1/2, \quad n = 1, \dots, N \quad (4)$$

The Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{V} = \mathcal{H}_S + \mathcal{H}_B + \mathcal{V} \quad (5)$$

# Model

$$\begin{aligned}\mathcal{H}_S &= \sum_{\langle m, m' \rangle} \sum_{\alpha=x,y,z} J_{mm'}^\alpha S_m^\alpha S_{m'}^\alpha, + \sum_m \sum_{\alpha=x,y,z} H_m^\alpha S_m^\alpha, \\ \mathcal{H}_B &= \sum_{\langle n, n' \rangle} \sum_{\alpha=x,y,z} \Gamma_{nn'}^\alpha I_n^\alpha I_{n'}^\alpha + \sum_n \sum_{\alpha=x,y,z} H_n^\alpha I_n^\alpha, \\ \mathcal{V} &= \sum_{\langle m, n \rangle} \sum_{\alpha=x,y,z} A_{mn}^\alpha S_m^\alpha I_n^\alpha.\end{aligned}\tag{6}$$

# Model

Schrödinger equation

$$i\partial\Psi(t)/\partial t = \mathcal{H}\Psi(t) \quad (7)$$

For a time-independent Hamiltonian

$$\Psi(t) = \exp(-it\mathcal{H})\Psi_0 = U(t)\Psi_0, \quad (8)$$

# Chebyshev Polynomials

$$T_n(x) = \cos(n \cos^{-1} x), \quad n \in [-1, 1] \quad (9)$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n = 1, 2, \dots \quad (10)$$

# Chebyshev Polynomials

$$f(x) = \sum_{n=0}^{\infty} c_n T_n(x), \quad x \in [-1, 1] \quad (11)$$

where

$$c_0 = \frac{1}{\pi} \int_{-1}^1 \frac{f(x)T_0(x)}{\sqrt{1-x^2}} dx, \quad (12)$$

and

$$c_n = \frac{2}{\pi} \int_{-1}^1 \frac{f(x)T_n(x)}{\sqrt{1-x^2}} dx, \quad n = 1, 2, 3, \dots \quad (13)$$

# Model

$$\begin{aligned} E_c &= \frac{1}{2} (E_{\max} + E_{\min}), & E_0 &= E_{\max} - E_{\min} \\ E_{\min} &= \min \langle \mathcal{H} \rangle = \min_{\langle \Phi | \Phi \rangle = 1} \langle \Phi | \mathcal{H} | \Phi \rangle \\ E_{\max} &= \max \langle \mathcal{H} \rangle = \max_{\langle \Phi | \Phi \rangle = 1} \langle \Phi | \mathcal{H} | \Phi \rangle \end{aligned} \tag{14}$$

The rescaled operator

$$\mathcal{G} = 2(\mathcal{H} - E_c)/E_0 \tag{15}$$

# Model

The evolution operator

$$U(t) = \exp(-i\tau\mathcal{G}) = \sum_{k=0}^{\infty} c_k T_k(\mathcal{G}), \quad (16)$$

where  $\tau = E_0 t / 2$ ,

$$c_k = \frac{a_k}{\pi} \int_{-1}^1 \frac{T_k(x) \exp(-ix\tau)}{\sqrt{1-x^2}} dx = a_k (-i)^k J_k(\tau), \quad (17)$$

$$a_k = \begin{cases} 1, & \text{for } k = 0 \\ 2, & \text{for } k \geq 1 \end{cases}$$

and

$$T_{k+1}(\mathcal{G}) = 2\mathcal{G}T_k(\mathcal{G}) - T_{k-1}(\mathcal{G}), \quad T_0(\mathcal{G}) = 1, T_1(\mathcal{G}) = \mathcal{G} \quad (18)$$

## Simulation Results

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# Simulation Results

Hamiltonian

$$\mathcal{H}_S = JS_1 \cdot S_2, \quad \mathcal{H}_B = 0, \quad \mathcal{V} = \sum A_n (S_1 + S_2) \cdot I_n \quad (19)$$

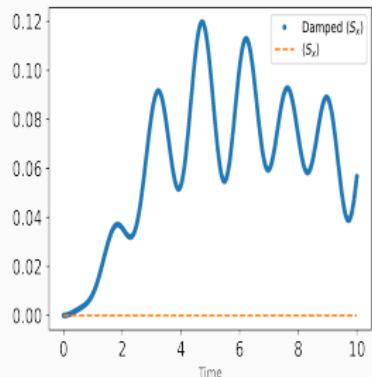
where

$$J = 1, \quad A_n = [-1, 0]. \quad (20)$$

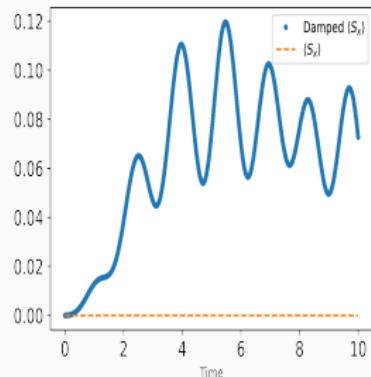
System's initial state

$$|\Psi(t)\rangle = |\psi_0\rangle \otimes |\chi_0\rangle \quad (21)$$

# Simulation Results



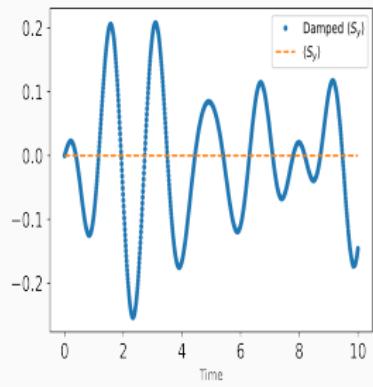
(a)  $S_1^X(t)$



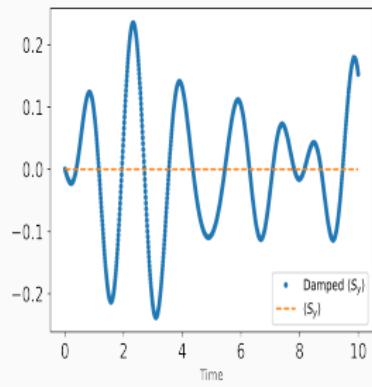
(b)  $S_2^X(t)$

Figure 3

# Simulation Results



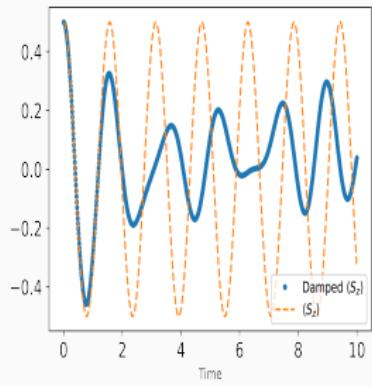
(a)  $S_1^x(t)$



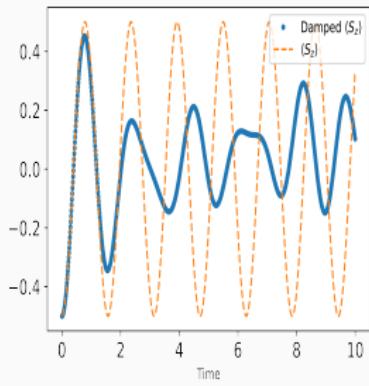
(b)  $S_2^x(t)$

Figure 4

# Simulation Results



(a)  $S_1^x(t)$



(b)  $S_2^x(t)$

Figure 5

Qutip

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# Qutip

QuTiP, short for the Quantum Toolbox in Python, is an open-source computational physics software library for simulating quantum systems, particularly open quantum systems.

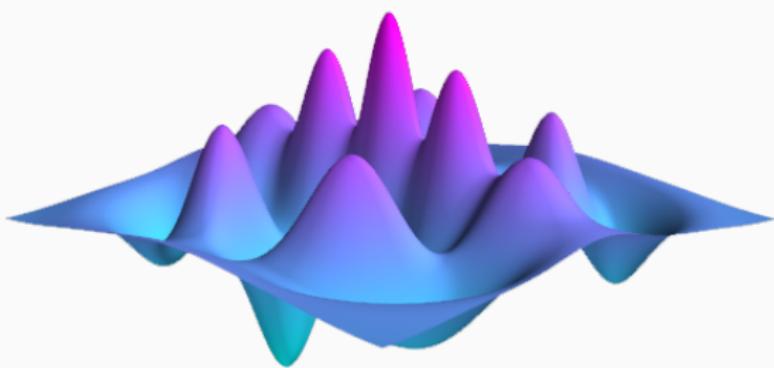


Figure 6: Qutip Logo

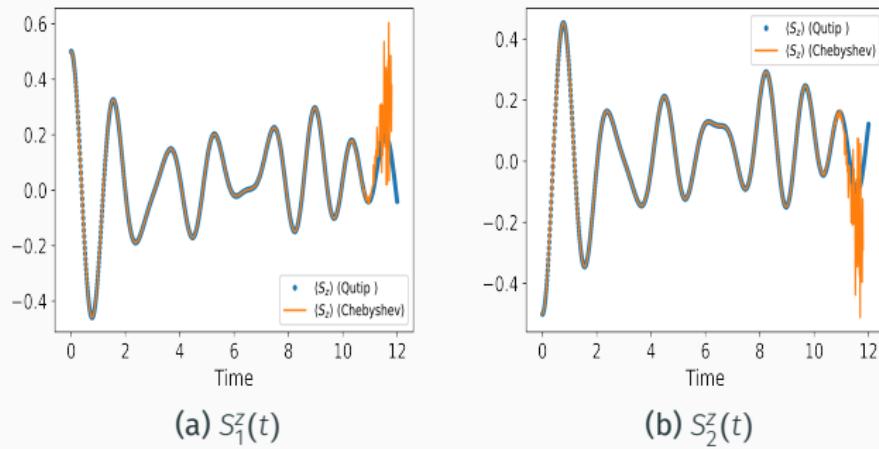


Figure 7

# Error

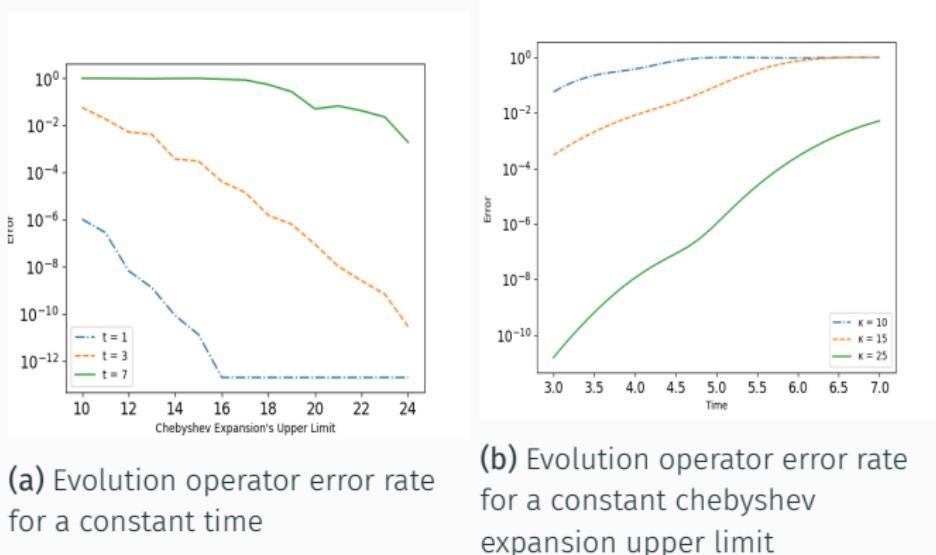


Figure 8

Thanks For Your Attention :)

## References i

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