



Subdivision Surfaces

COS 426, Spring 2021
Felix Heide
Princeton University



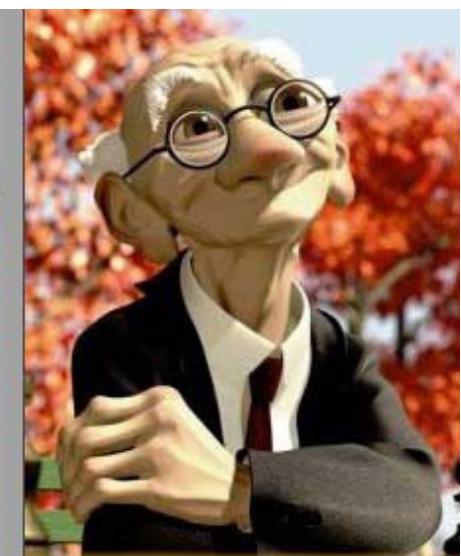
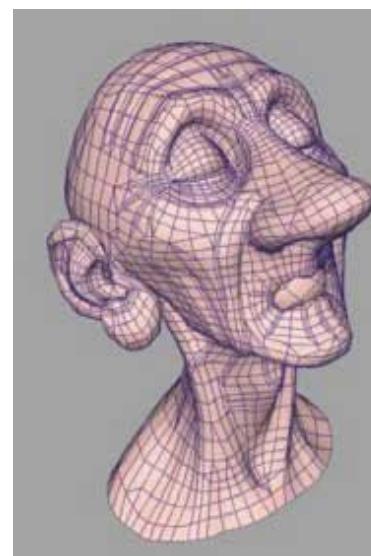
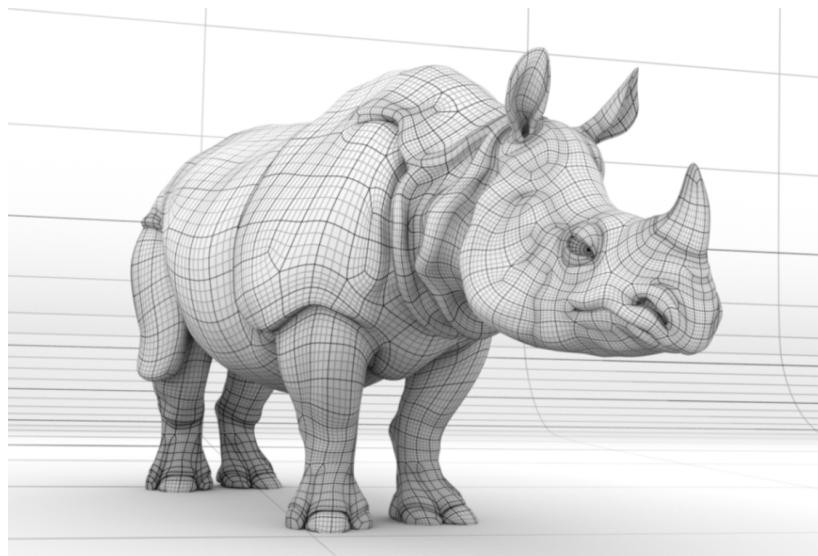
3D Object Representations

- Raw data
 - Range image
 - Point cloud
- Surfaces
 - Polygonal mesh
 - Parametric
 - Subdivision
 - Implicit
- Solids
 - Voxels
 - BSP tree
 - CSG
 - Sweep
- High-level structures
 - Scene graph
 - Application specific



Subdivision Surfaces

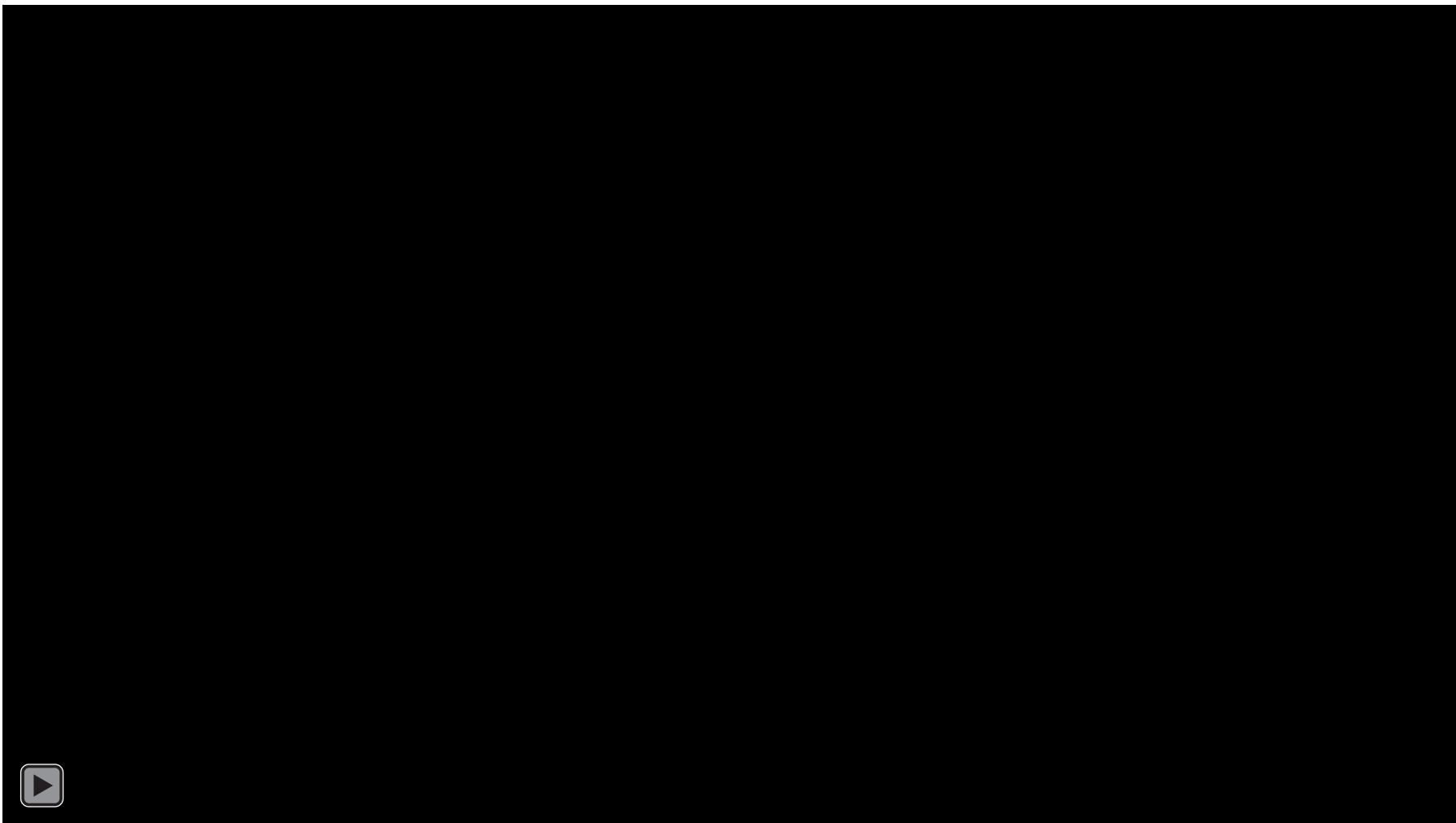
- Used in movie and game industries
- Supported by most 3D modeling software



Geri's Game © Pixar Animation Studios



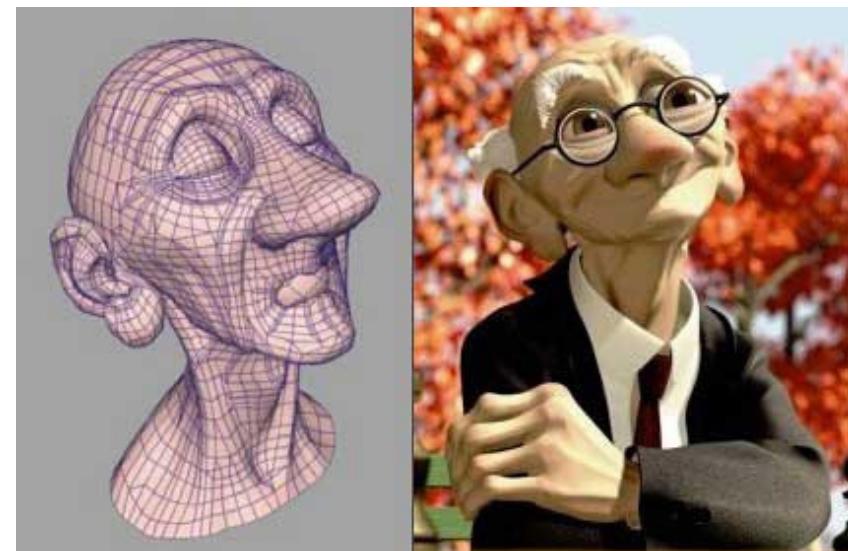
Geri's Game





Geri's Game

- “served as a demonstration of a new animation tool called subdivision surfaces” (Wikipedia)
- Subdivision used for head, hands & clothing
- Academy Award winner



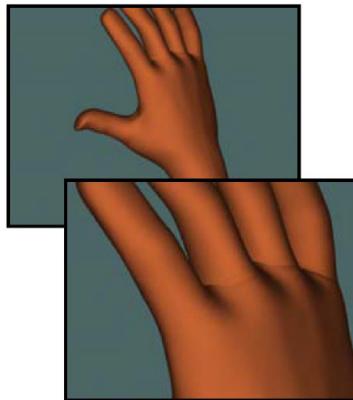
Geri's Game © Pixar Animation Studios



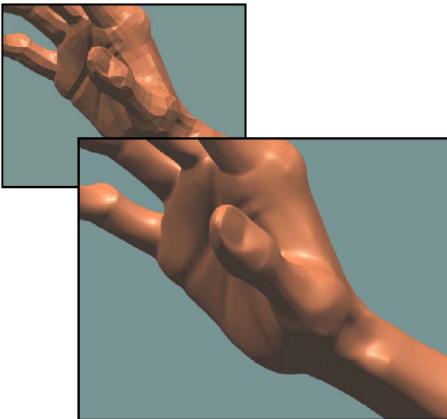
Subdivision Surfaces

- Alternative to parametric surfaces, overcoming:
 - Many patches
 - Difficult to mark sharp features
 - Irregularities after deformation

Woody's hand (NURBS)



Geri's hand (subdivision)

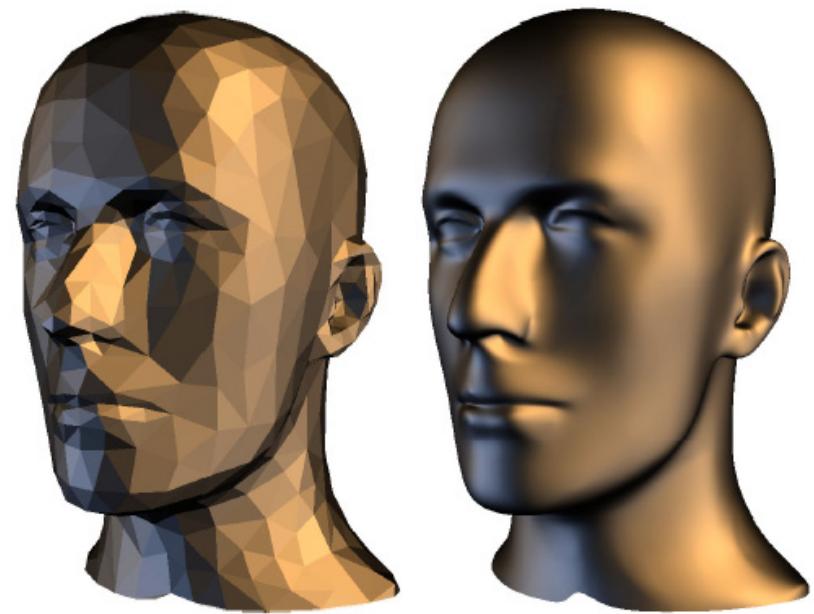


Stanford Graphics course notes



Subdivision Surfaces

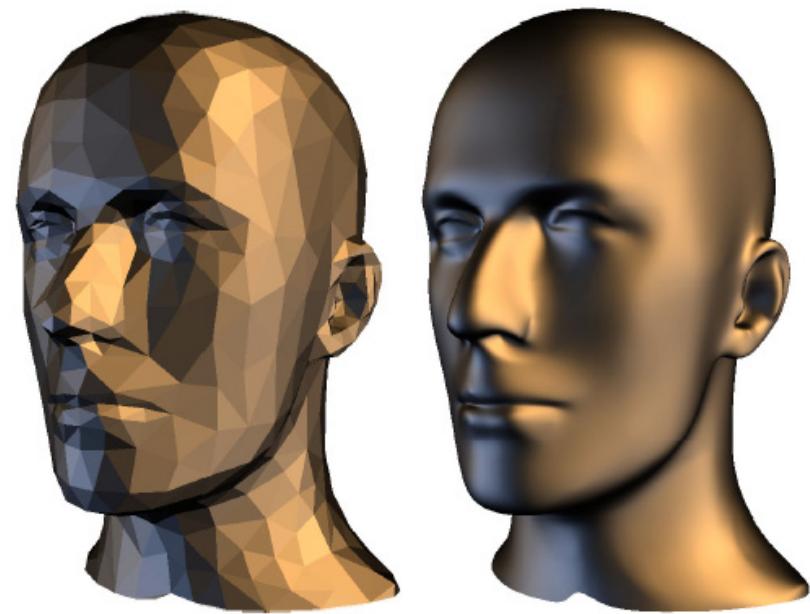
- What makes a good surface representation?
 - Accurate
 - Concise
 - Intuitive specification
 - Local support
 - Affine invariant
 - Arbitrary topology
 - Guaranteed continuity
 - Natural parameterization
 - Efficient display
 - Efficient intersections





Subdivision Surfaces

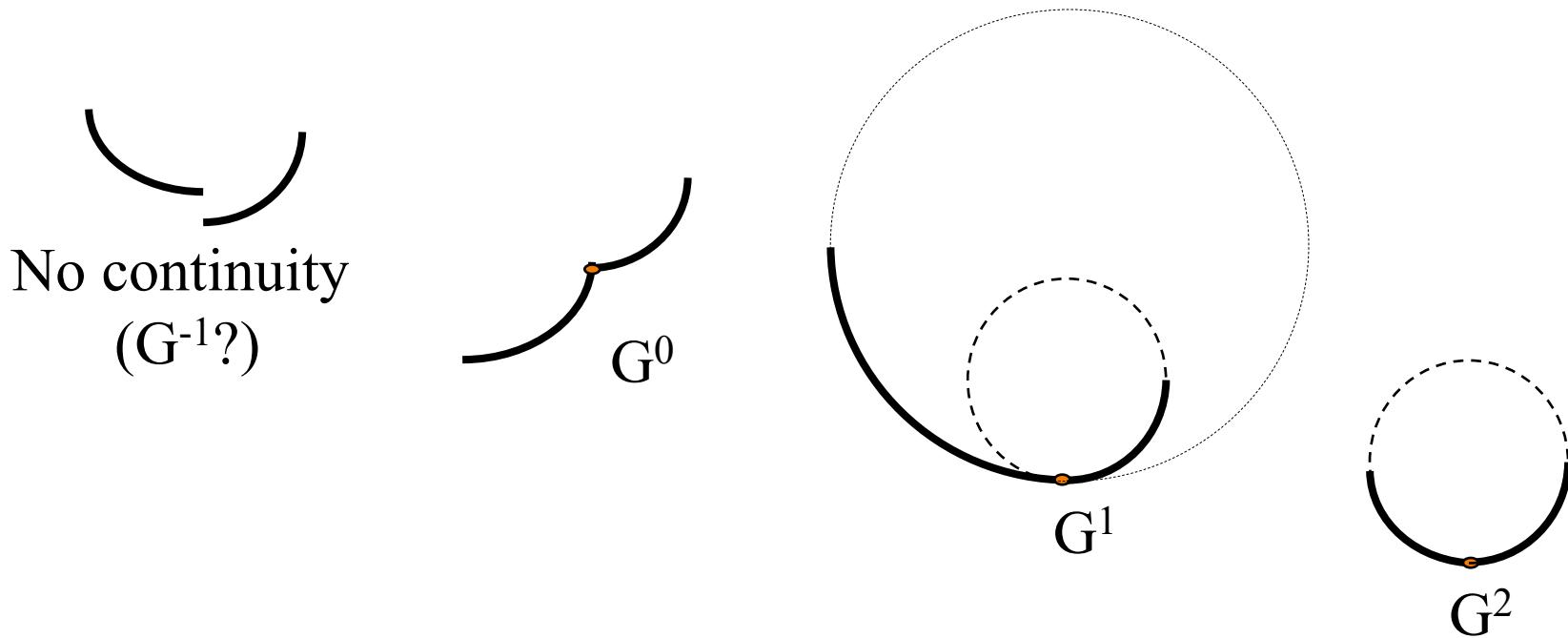
- What makes a good surface representation?
 - Accurate
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 - Affine invariant
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 - **Guaranteed continuity**
 - Natural parameterization
 - Efficient display
 - Efficient intersections





Note on Continuity

A curve / surface with G^k continuity has a continuous k -th derivative, geometrically.

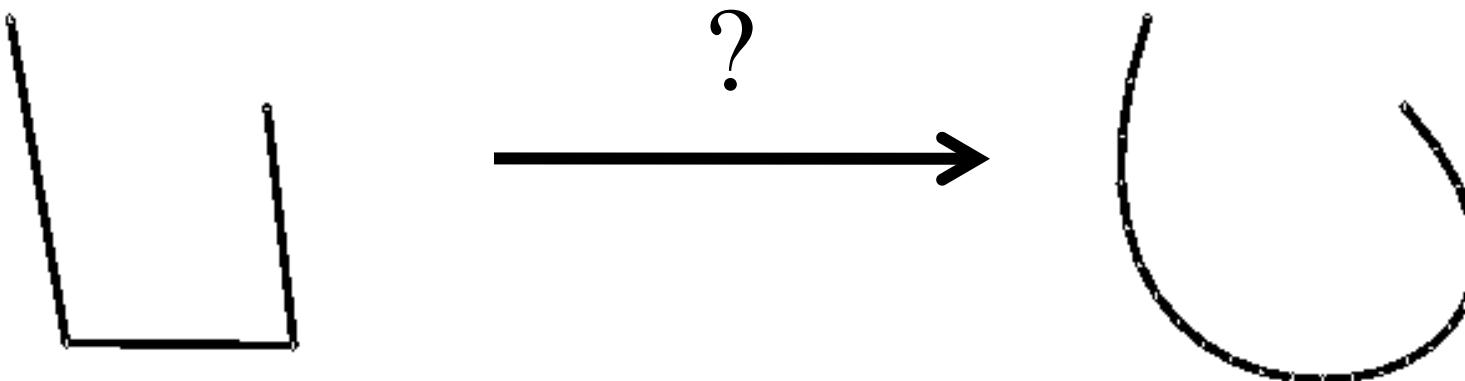


Similar to (but not the same as) C^k continuity, which refers to continuity with respect to parameter
e.g.: $f_x(u) = r_x \cos(2\pi u)$ *(but we're going to say C^k from now on...)*



Subdivision

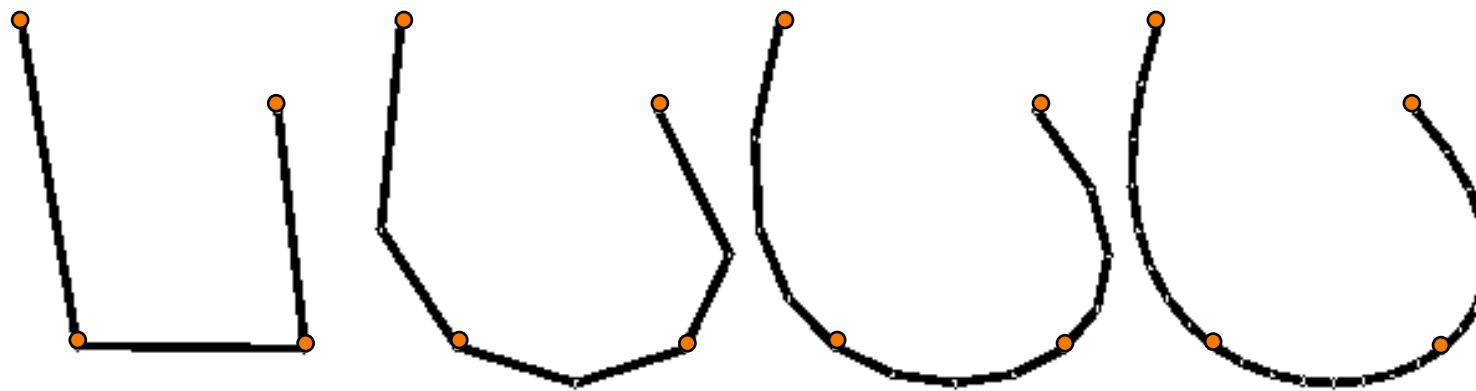
- How do you make a curve with guaranteed continuity?





Subdivision

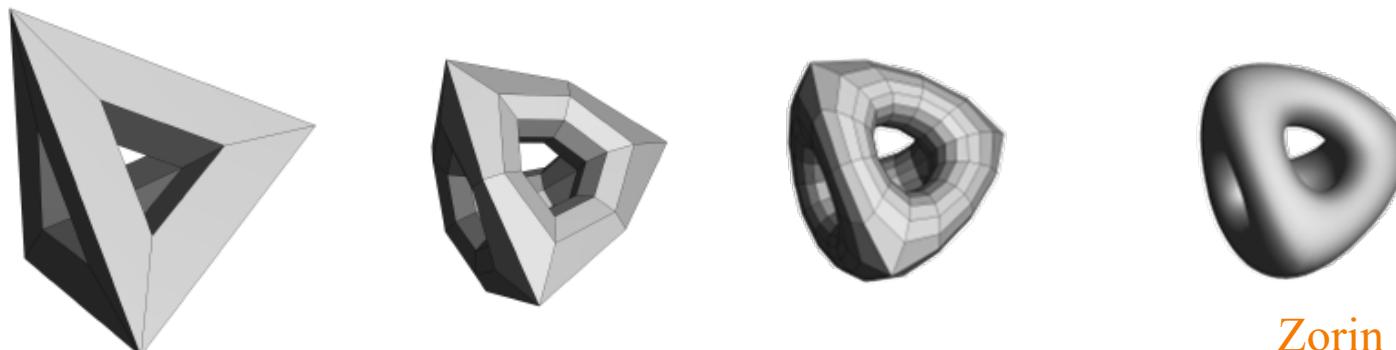
- How do you make a curve with guaranteed continuity? ...





Subdivision

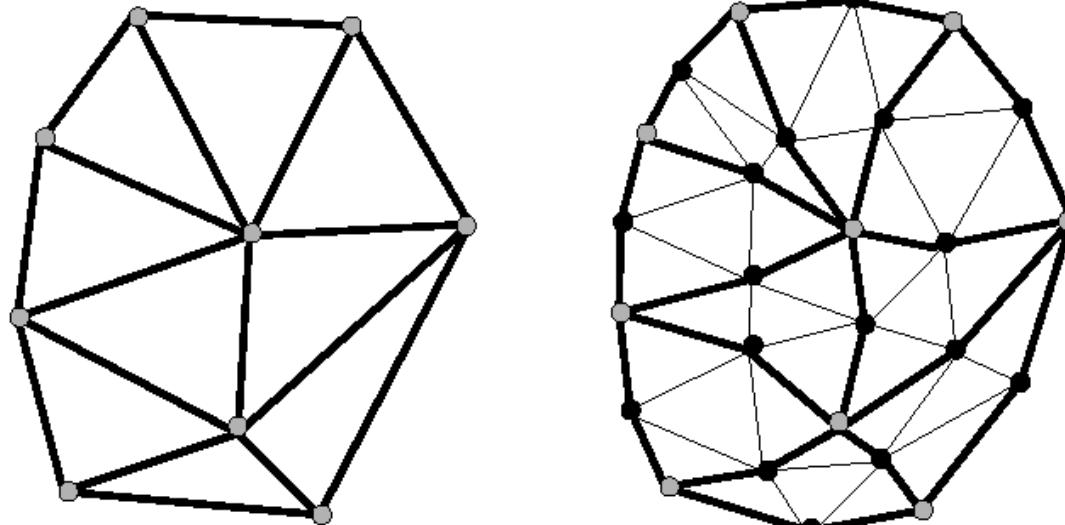
- How do you make a surface with guaranteed continuity?





Subdivision Surfaces

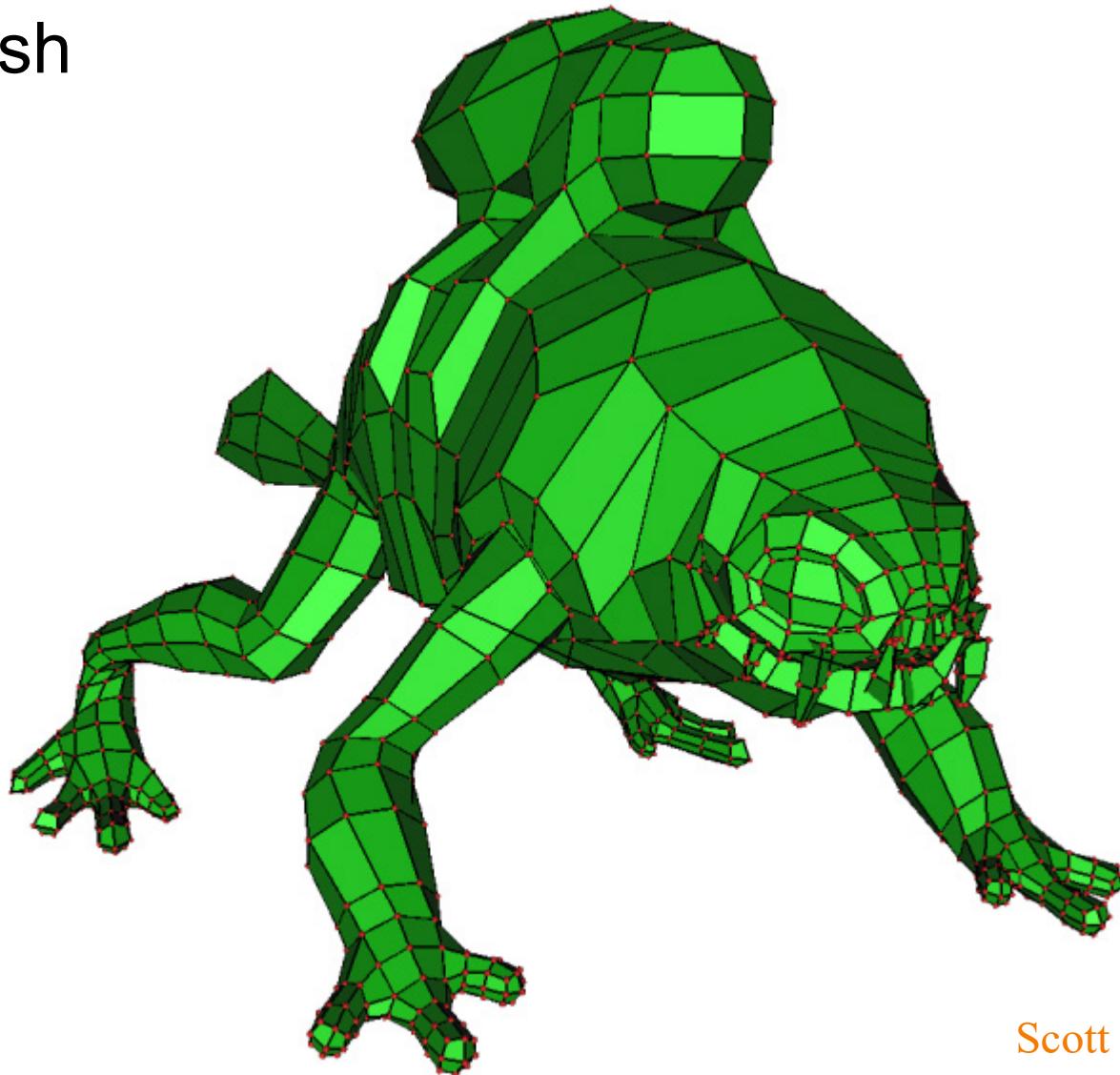
- Repeated application of
 1. Topology refinement (splitting faces)
 2. Geometry refinement (weighted averaging)





Subdivision Surfaces – Examples

- Base mesh

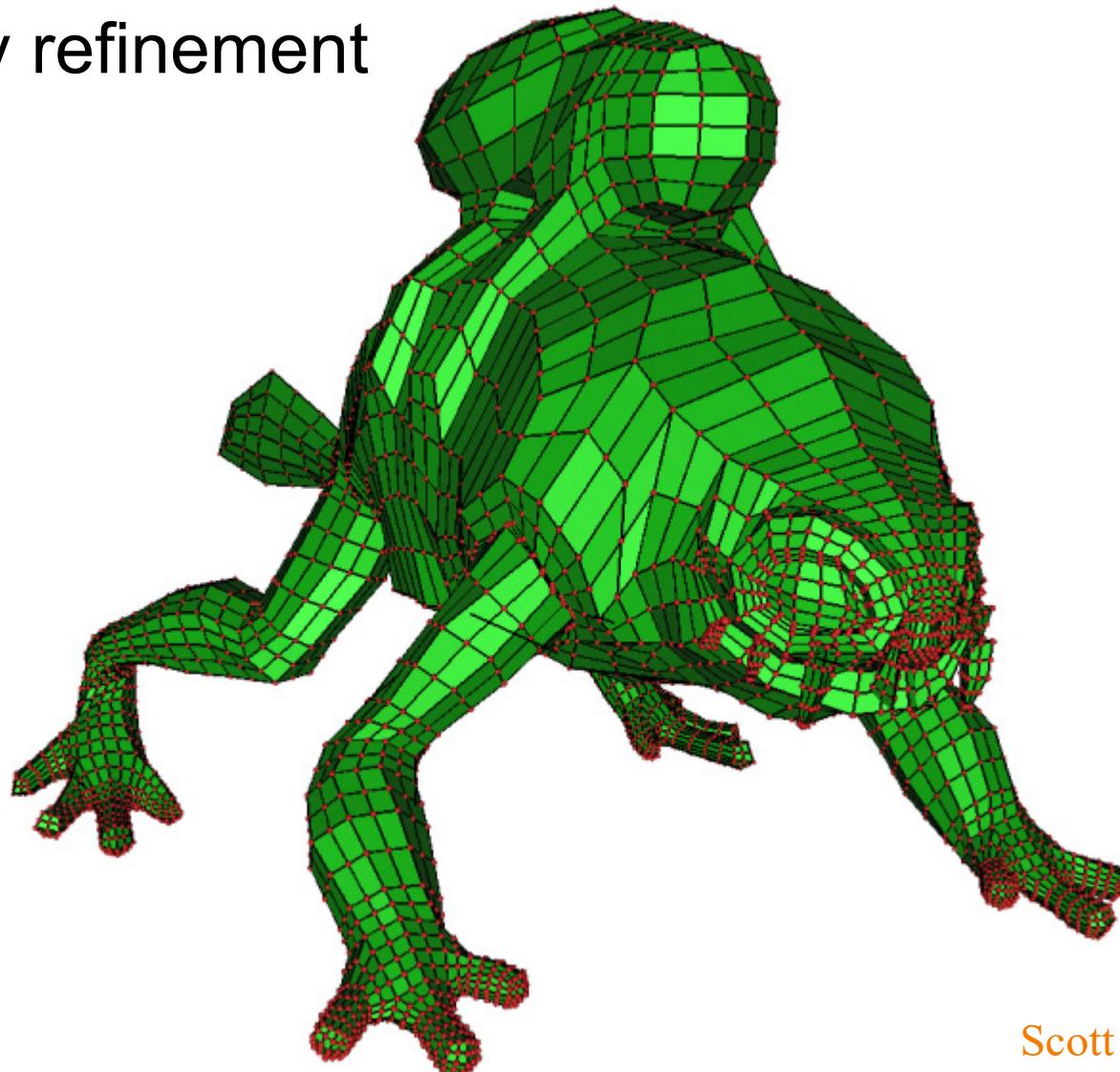


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Subdivision Surfaces – Examples

- Topology refinement

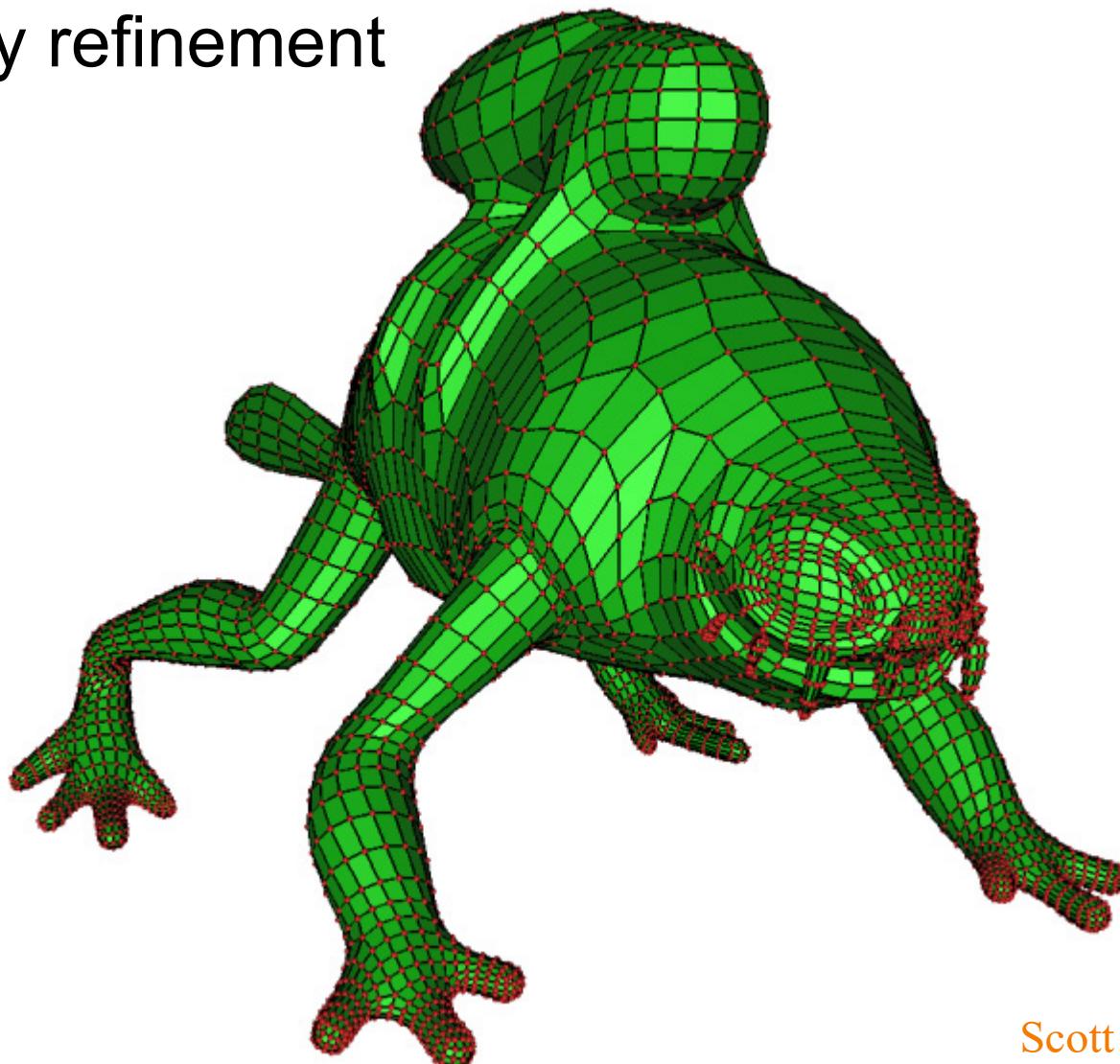


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Subdivision Surfaces – Examples

- Geometry refinement

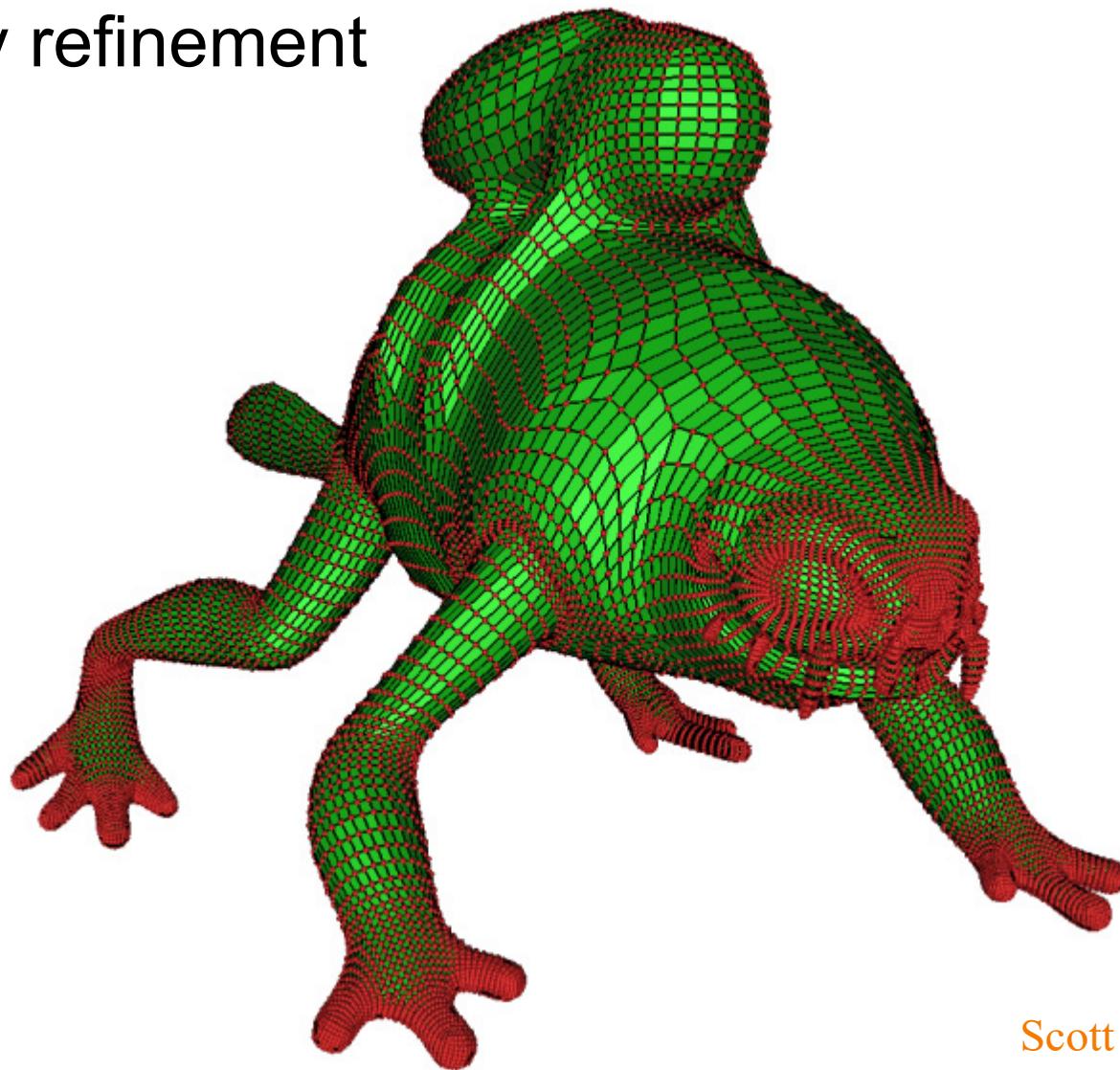


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Subdivision Surfaces – Examples

- Topology refinement

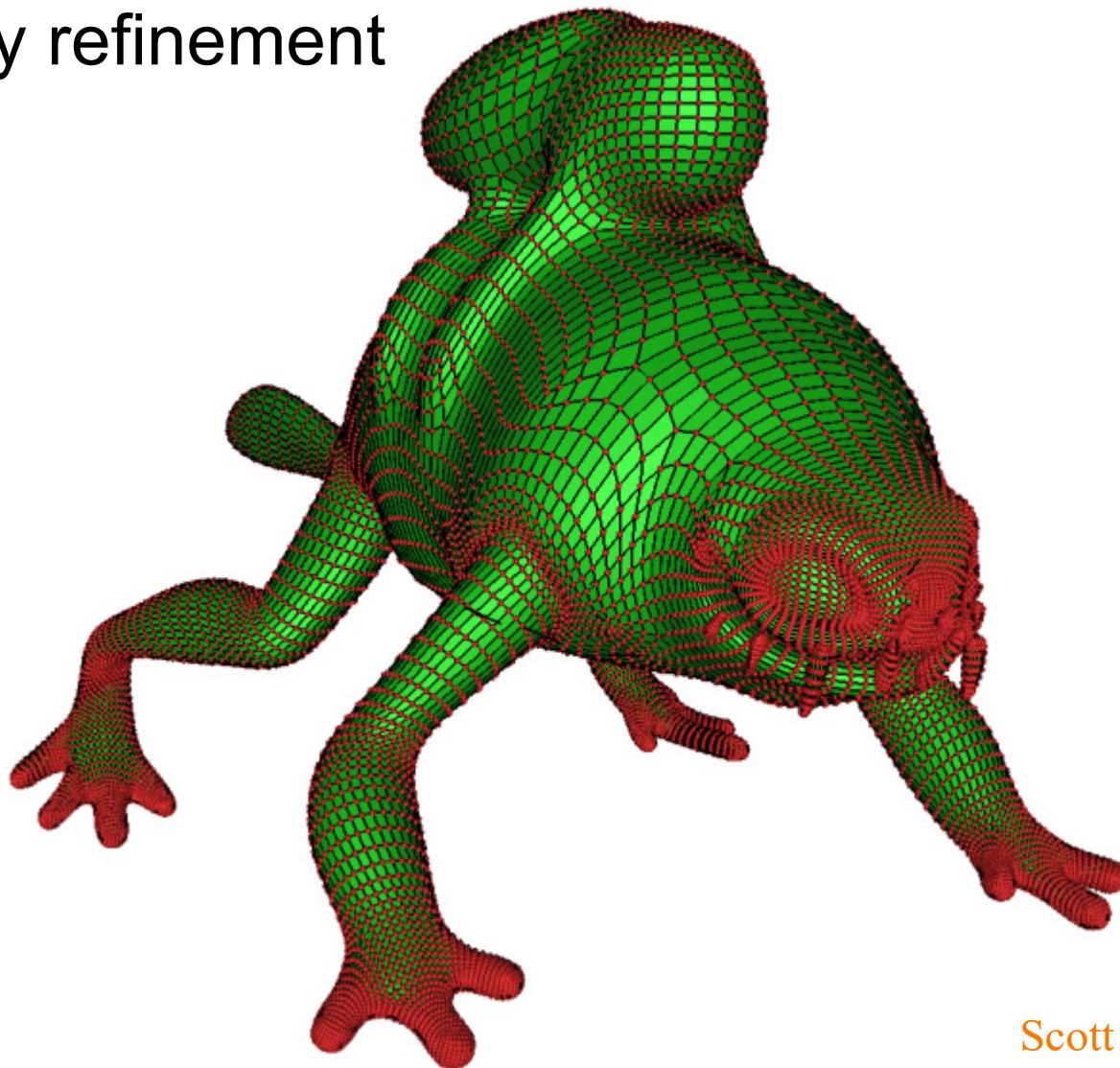


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Subdivision Surfaces – Examples

- Geometry refinement

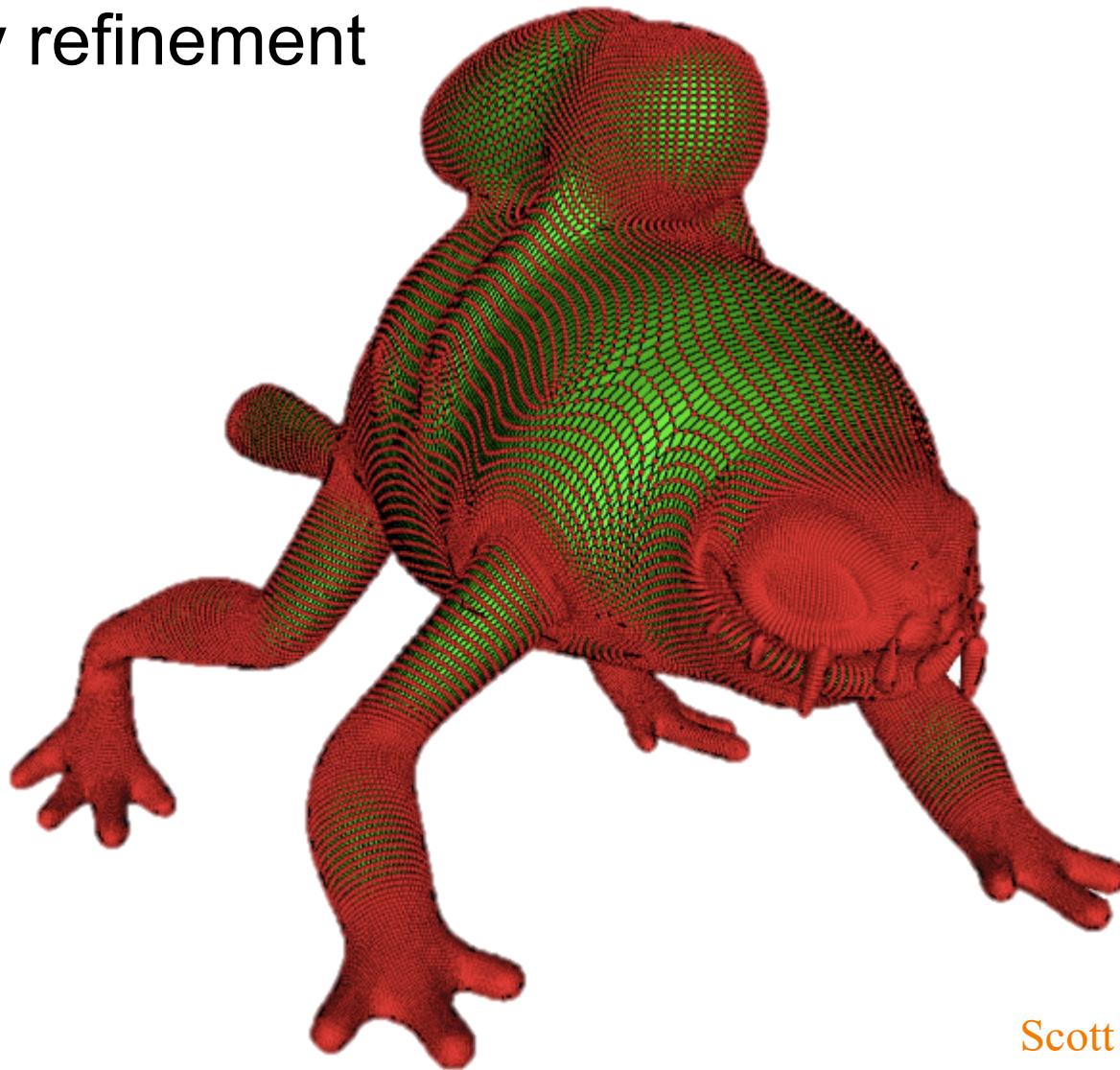


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Subdivision Surfaces – Examples

- Topology refinement

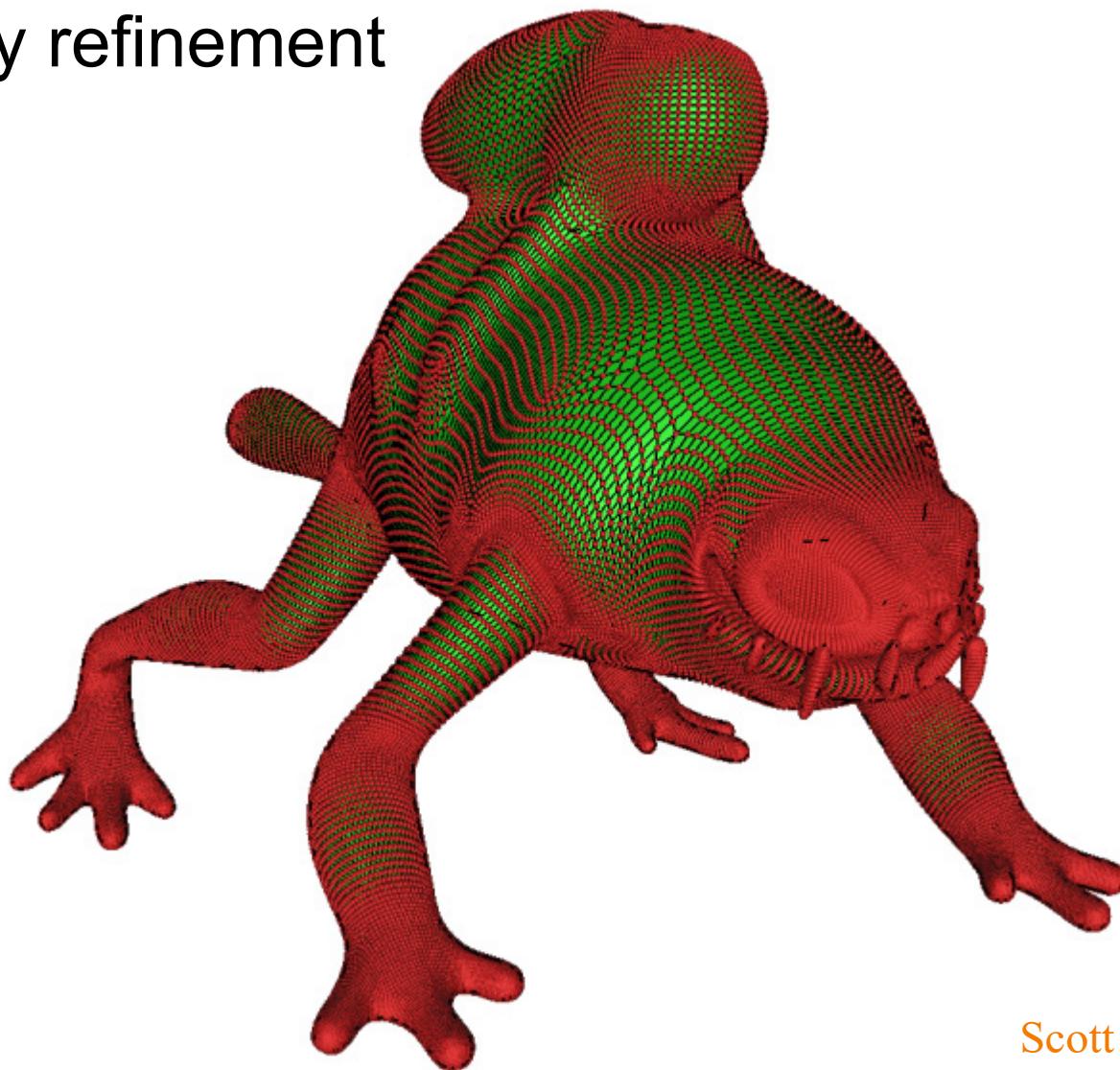


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Subdivision Surfaces – Examples

- Geometry refinement

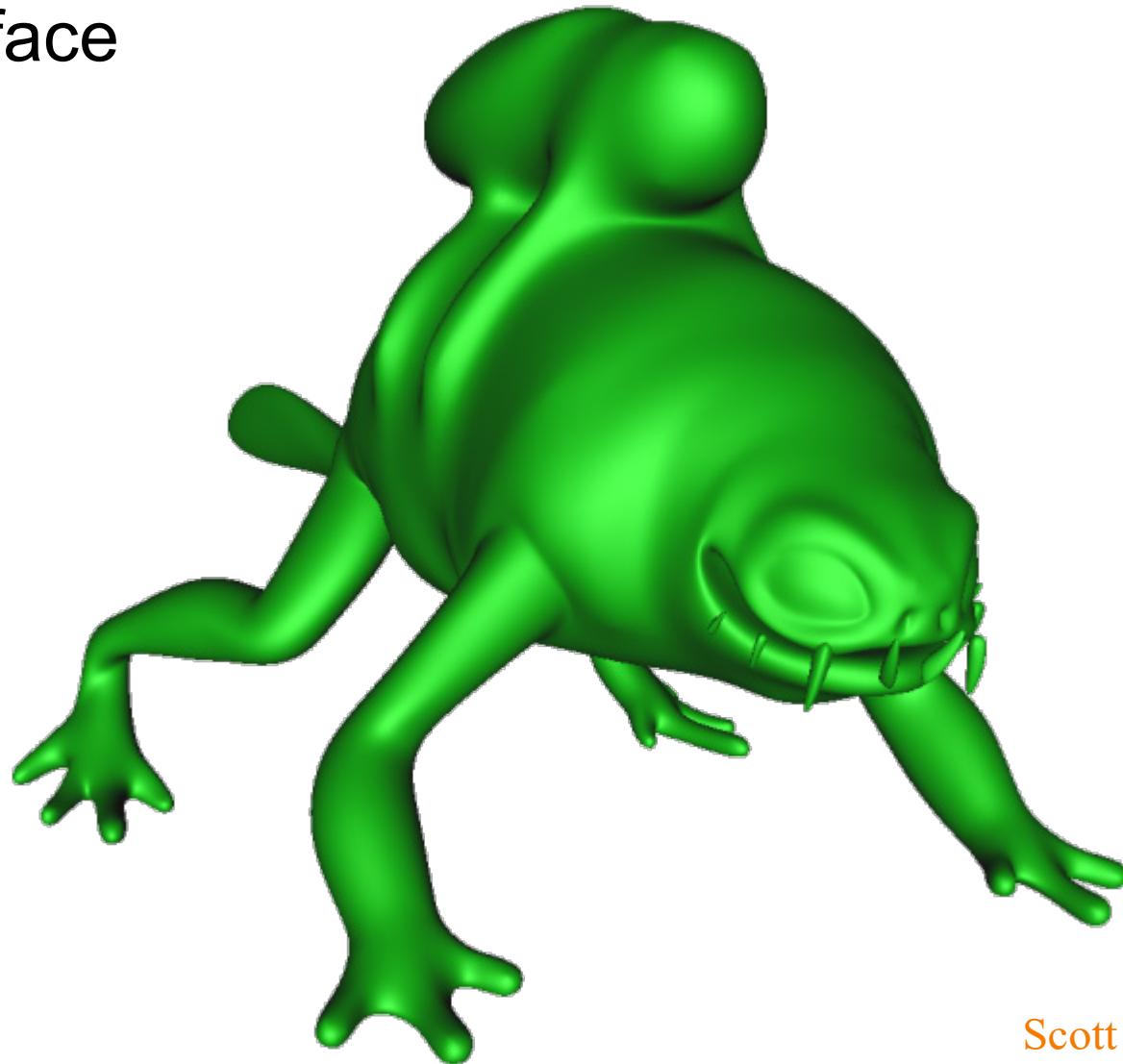


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Subdivision Surfaces – Examples

- Limit surface

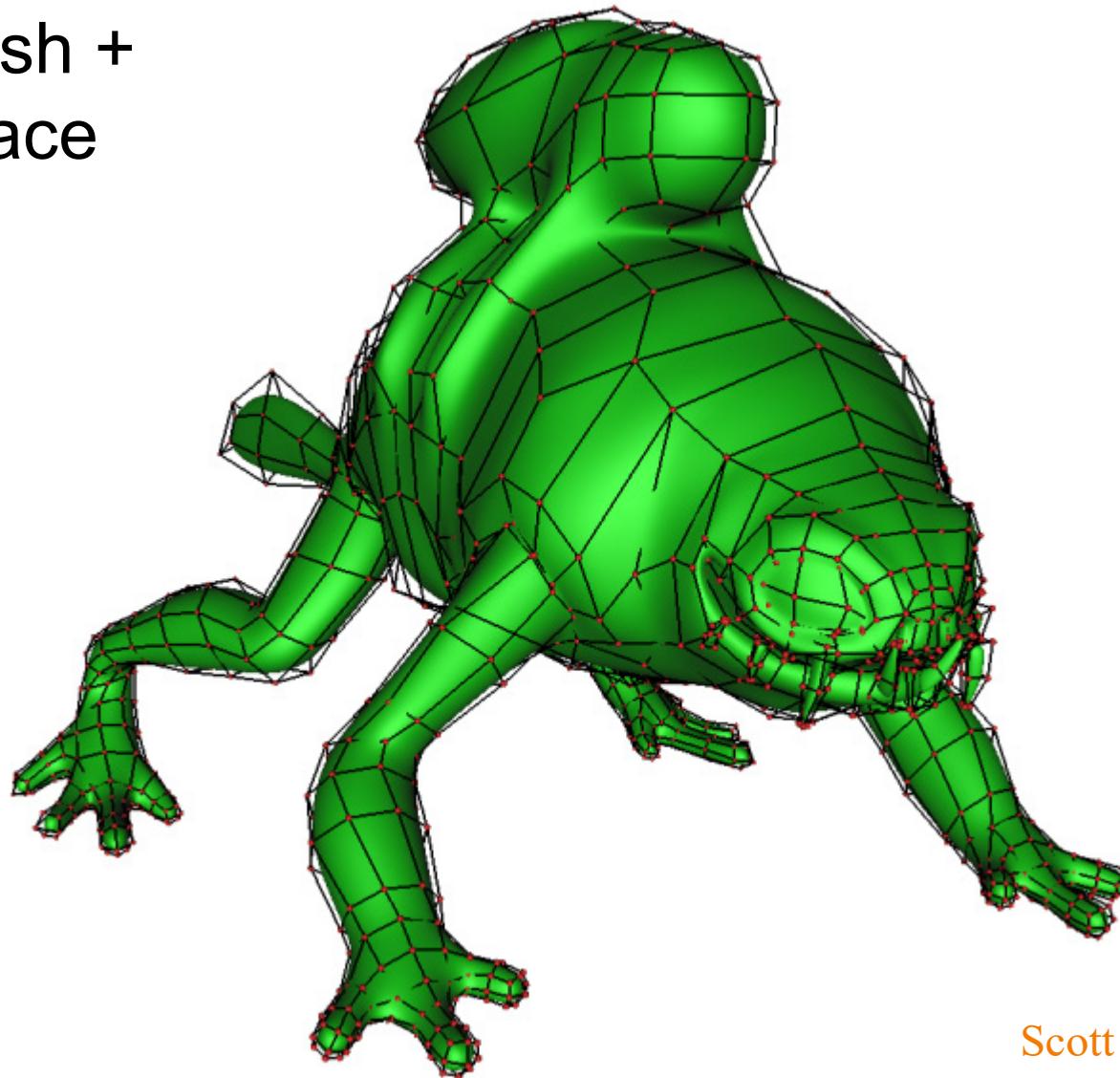


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Subdivision Surfaces – Examples

- Base mesh + limit surface



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Design of Subdivision Rules

- What types of input?
 - Quad meshes, triangle meshes, etc.
- How to refine topology?
 - Simple implementations
- How to refine geometry?
 - Smoothness guarantees in limit surface
 - » Continuity ($C^0, C^1, C^2, \dots?$)
 - Provable relationships between limit surface and original control mesh
 - » Interpolation of vertices?
 - » Surface within their convex hull?





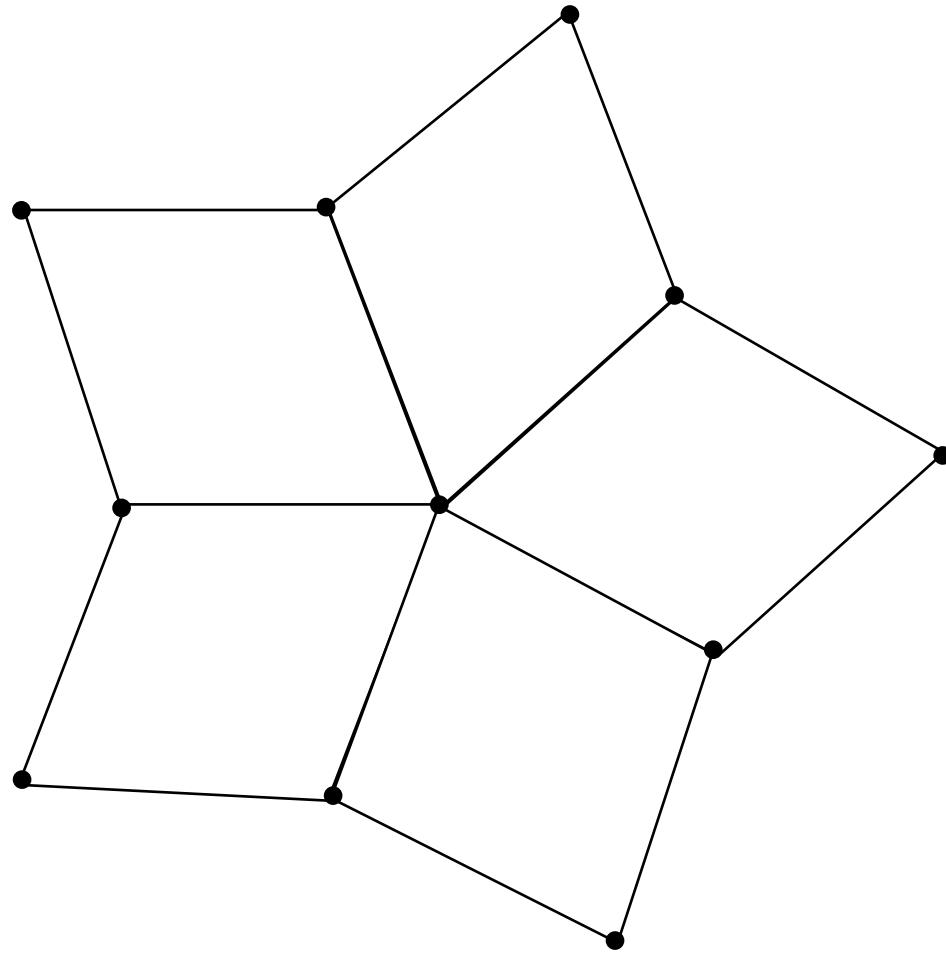
Linear Subdivision

- Type of input
 - Quad mesh -- four-sided polygons (*quads*)
- Topology refinement rule
 - Split every quad into four at midpoints
- Geometry refinement rule
 - Average vertex positions

Note: simple example to demonstrate how such schemes work, but not the best scheme...



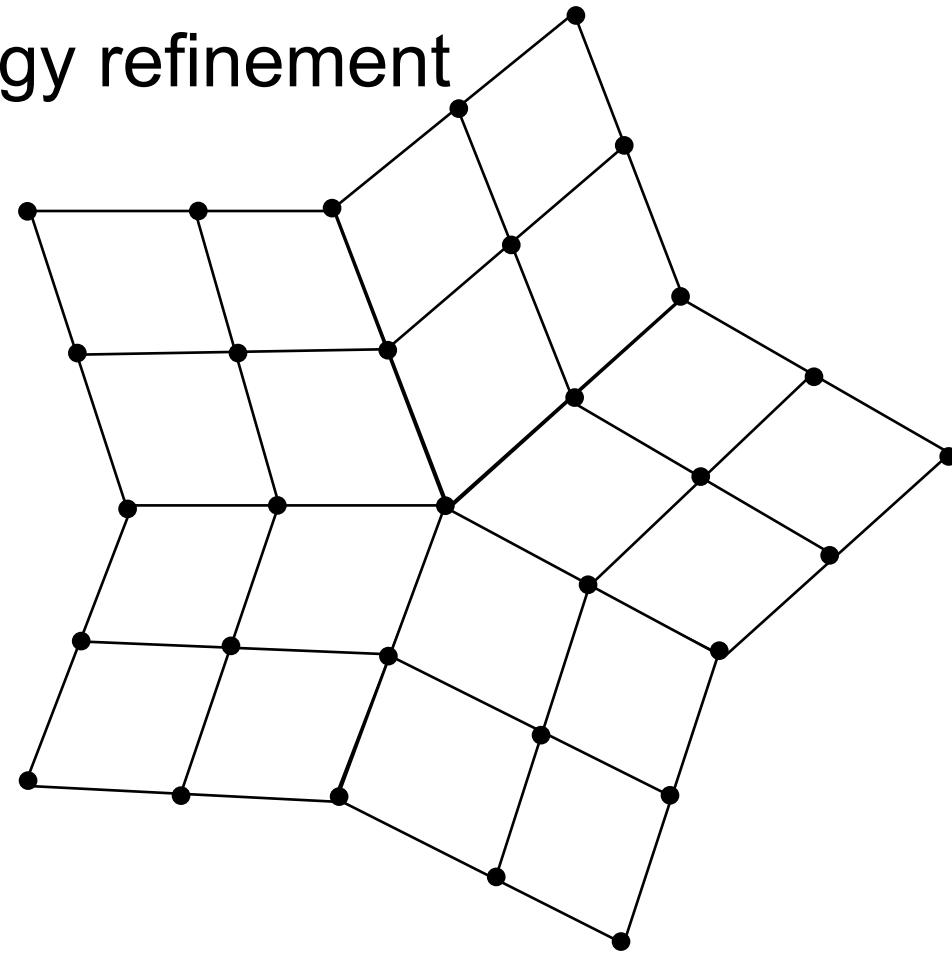
Linear Subdivision





Linear Subdivision

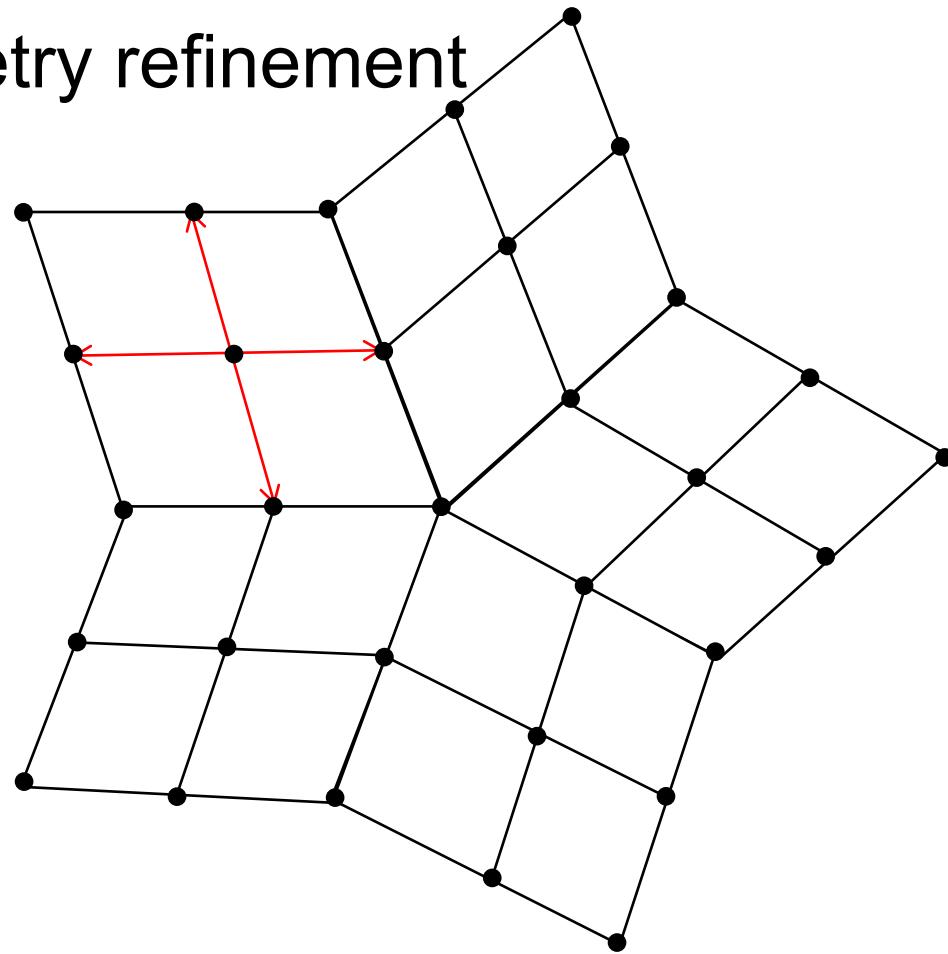
- Topology refinement





Linear Subdivision

- Geometry refinement





Linear Subdivision

LinearSubivision (F_0, V_0, k)

for i = 1 ...k levels

$(F_i, V_i) = \text{RefineTopology}(F_{i-1}, V_{i-1})$

$\text{RefineGeometry}(F_i, V_i)$

return (F_k, V_k)



Linear Subdivision

RefineTopology (F, V)

$newV = V$

$newF = \{\}$

for each face F_i

 Insert new vertex c at centroid of F_i into $newV$

 for $j = 1$ to 4

 Insert in $newV$ new vertex e_j at
 centroid of each edge ($F_{i,j}, F_{i,j+1}$)

 for $j = 1$ to 4

 Insert new face ($F_{i,j}, e_j, c, e_{j-1}$) into $newF$

return ($newF, newV$)



Linear Subdivision

RefineGeometry(F, V)

$newV = V$

$newF = F$

for each vertex V_i in $newV$

$weight = 0;$

$newV[i] = (0,0,0)$

for each face F_j connected to V_i

$newV[i] +=$ centroid of F_j

$weight += 1.0;$

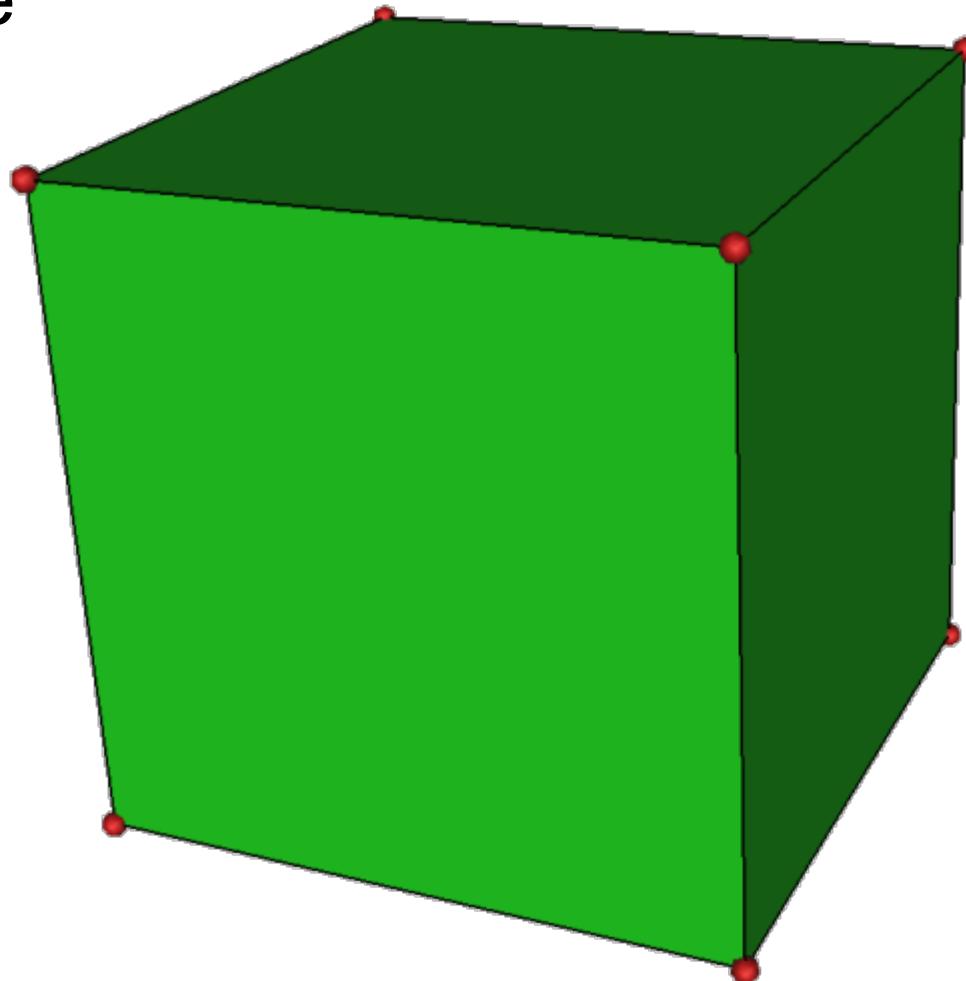
$newV[i] /= weight$

return ($newF, newV$)



Linear Subdivision

- Example



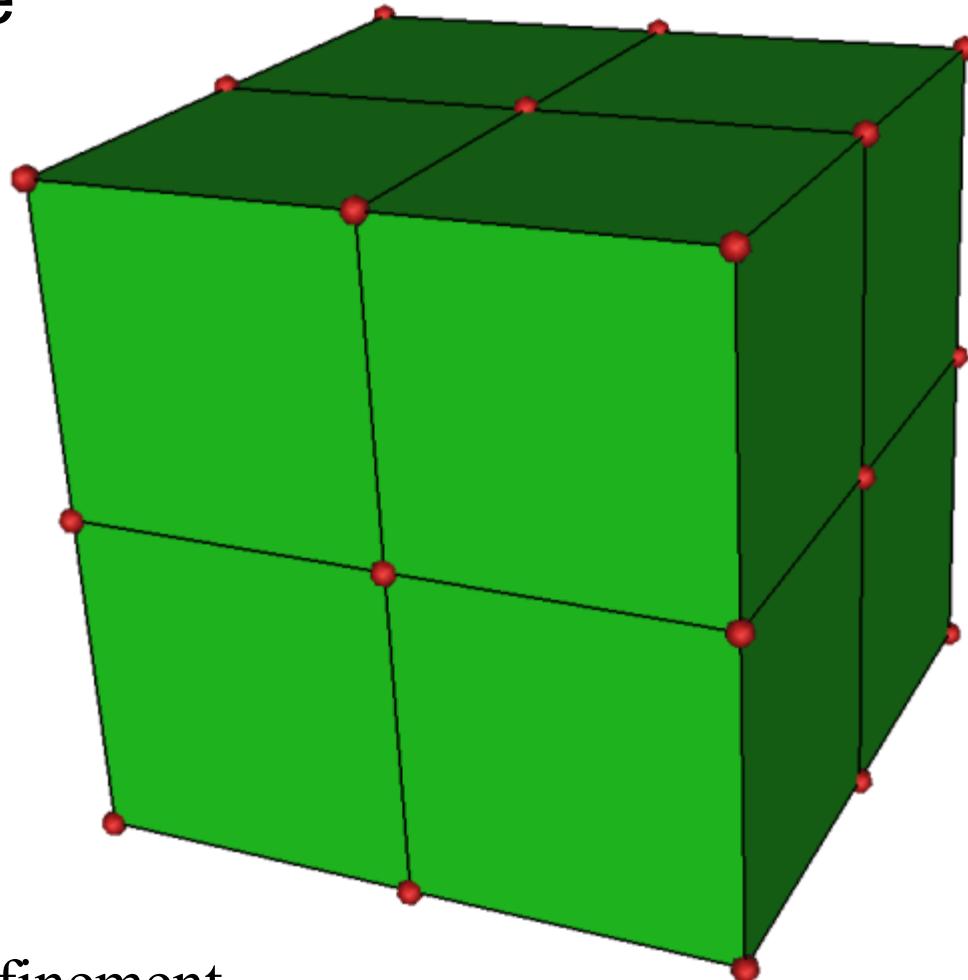
Input mesh

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Linear Subdivision

- Example



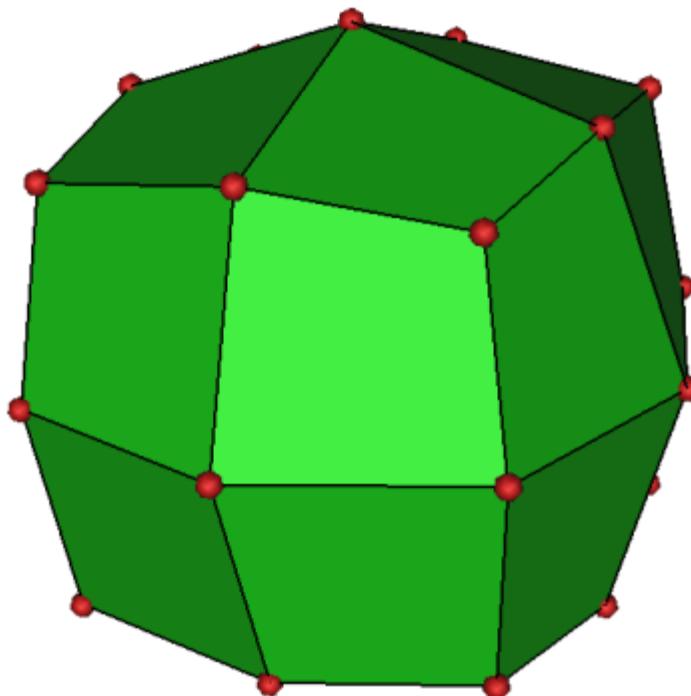
Topology refinement

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Linear Subdivision

- Example



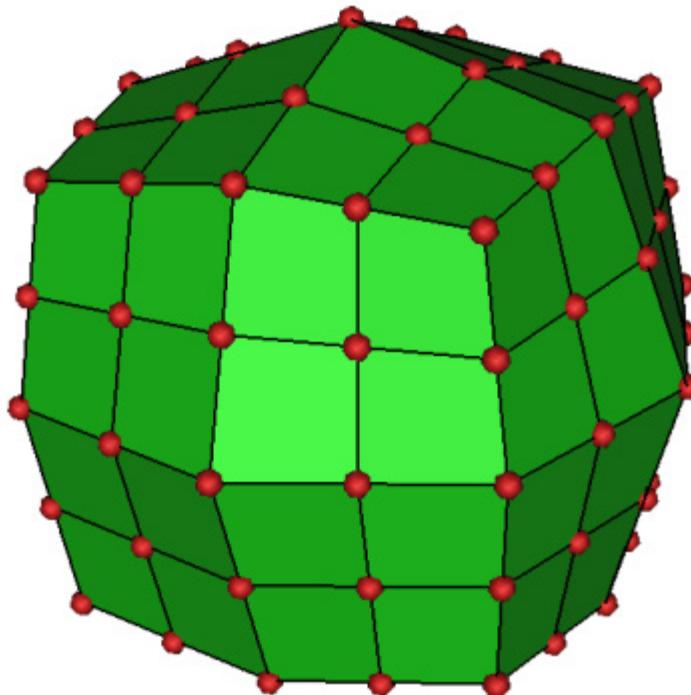
Geometry refinement

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Linear Subdivision

- Example



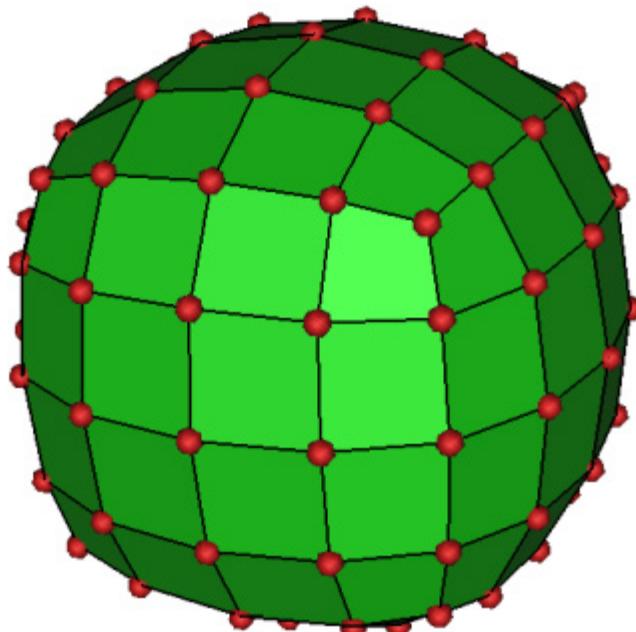
Topology refinement

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Linear Subdivision

- Example



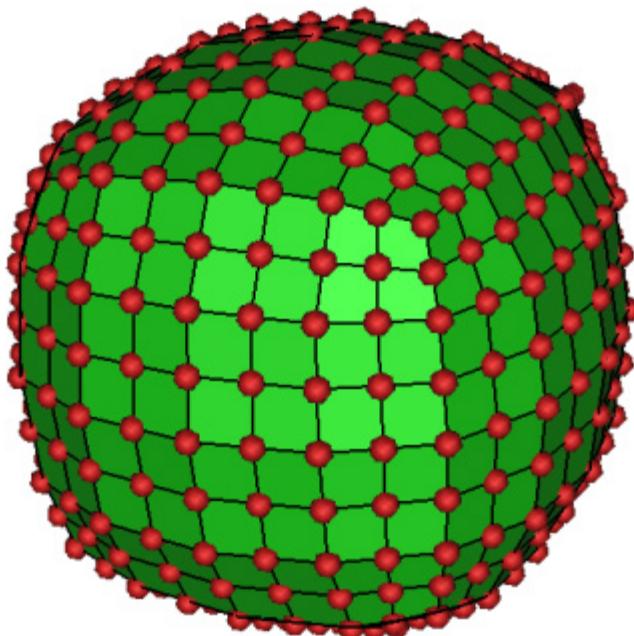
Geometry refinement

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Linear Subdivision

- Example



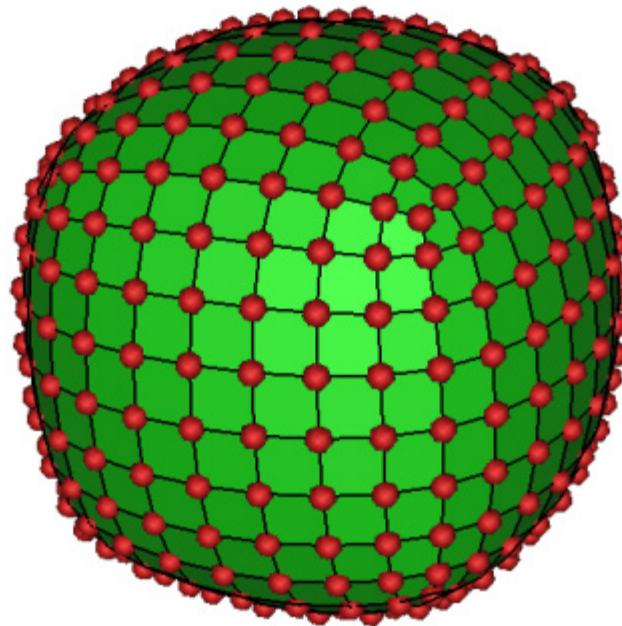
Topology refinement

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Linear Subdivision

- Example



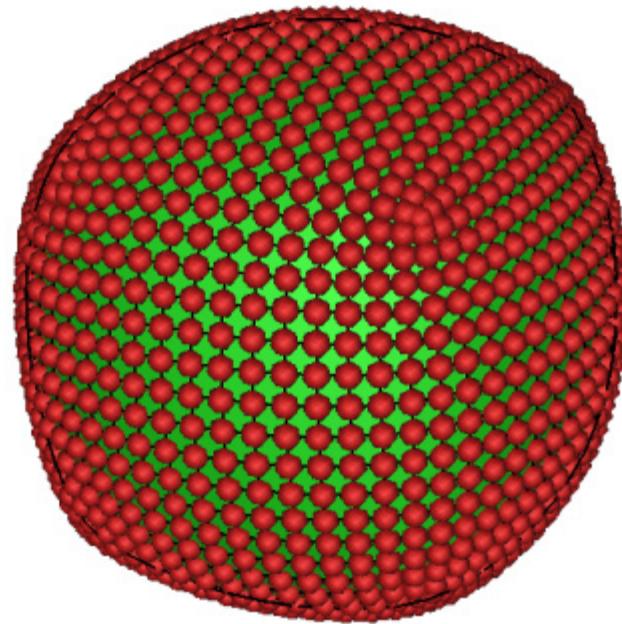
Geometry refinement

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Linear Subdivision

- Example



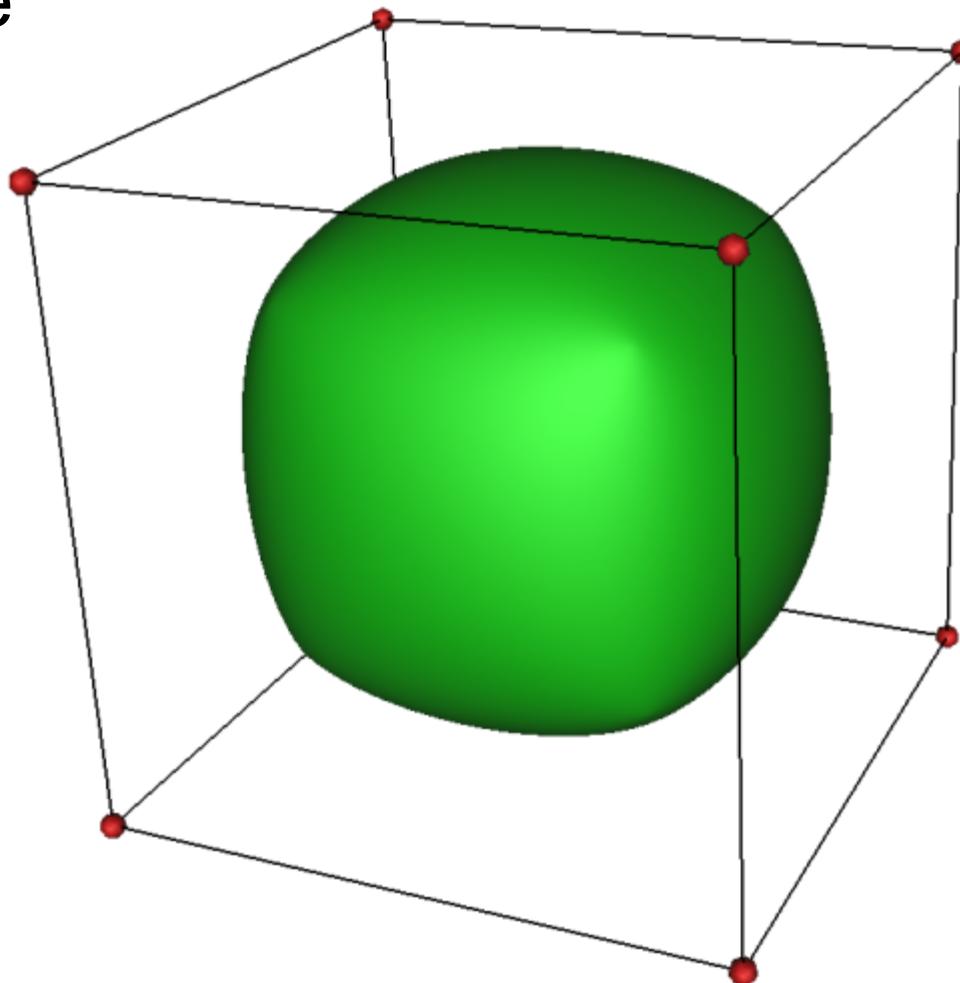
Topology refinement

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Linear Subdivision

- Example



Final result

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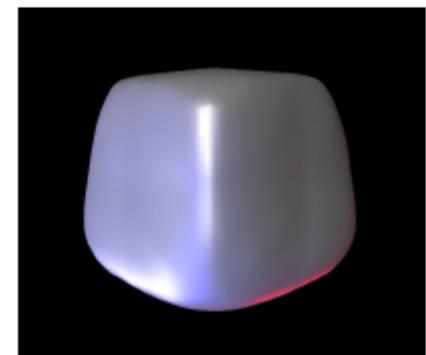
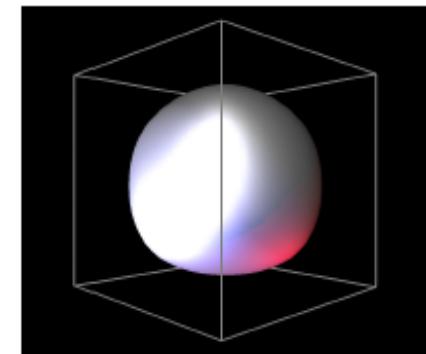
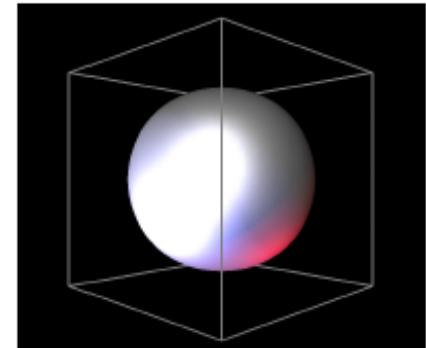
Subdivision Demo

https://threejs.org/examples/webgl_modifier_subdivision.html



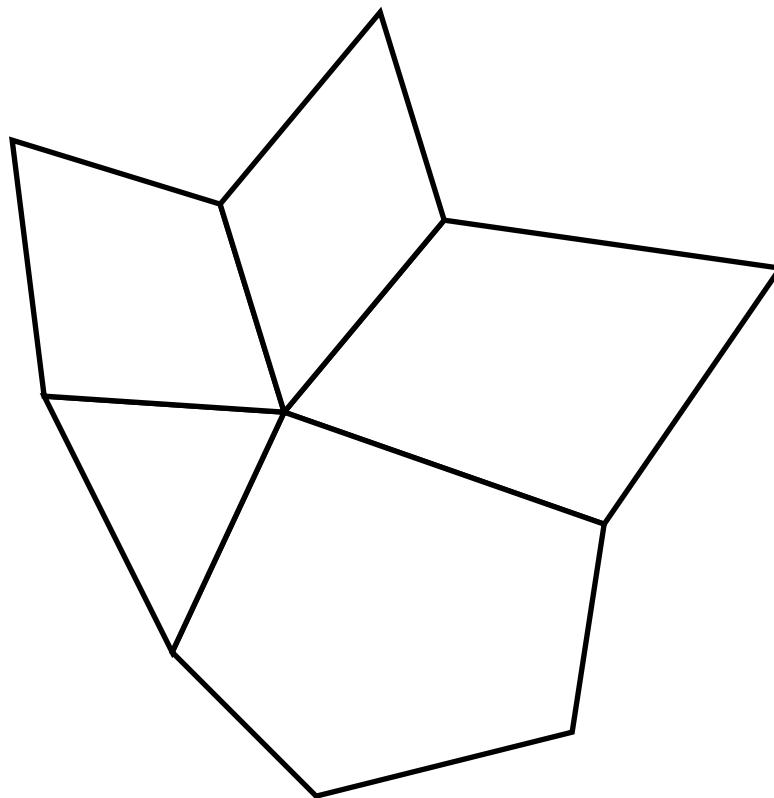
Subdivision Schemes

- Common subdivision schemes
 - Catmull-Clark
 - Loop
 - Many others
 - Differ in ...
 - Input topology
 - How refine topology
 - How refine geometry
- ... which makes differences in ...
- Provable properties





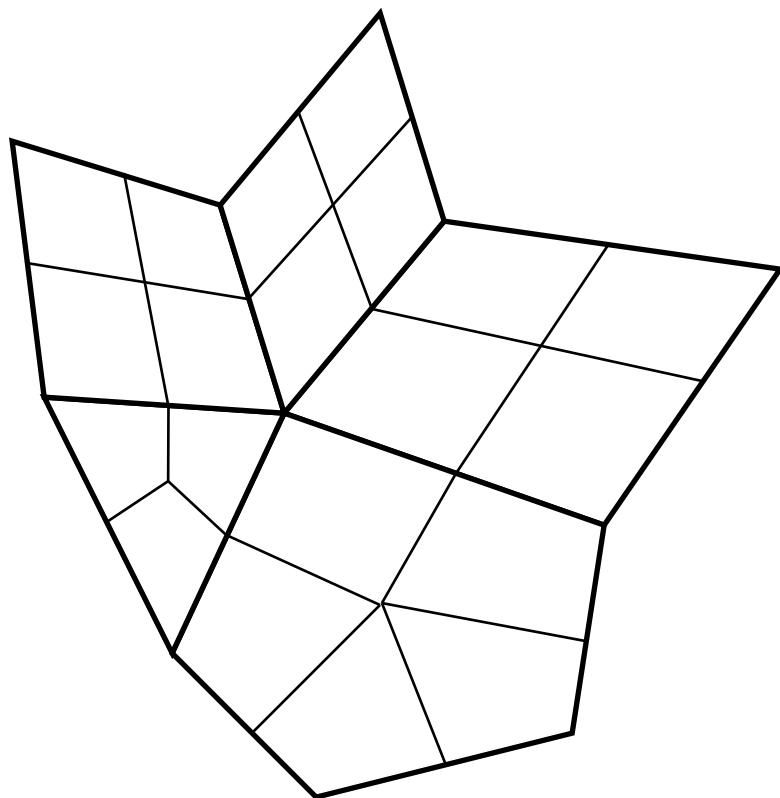
Catmull-Clark Subdivision



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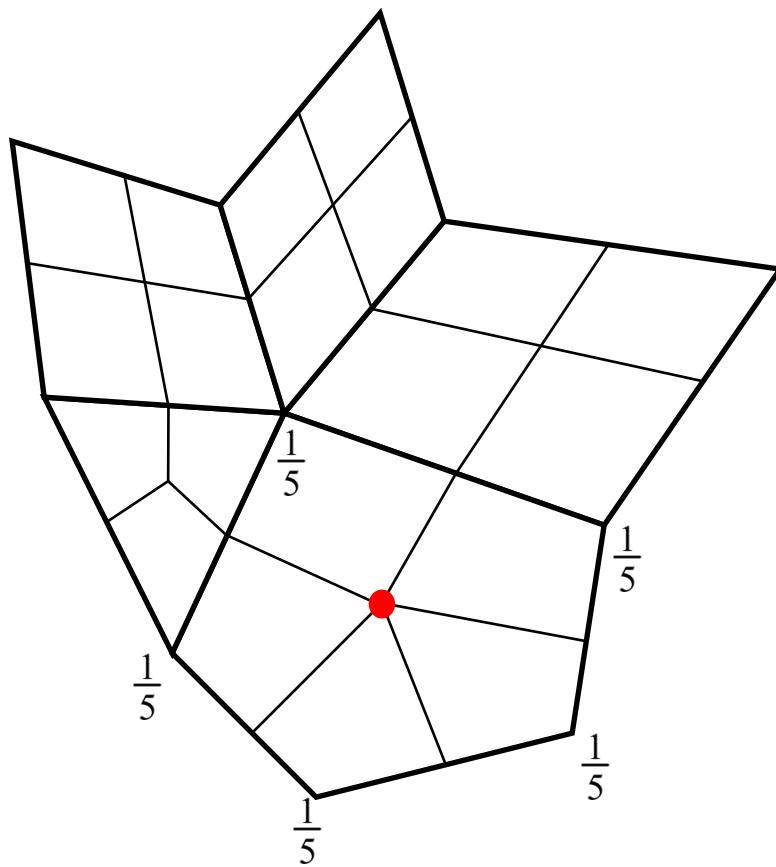
Catmull-Clark Subdivision



Scott Schaefer



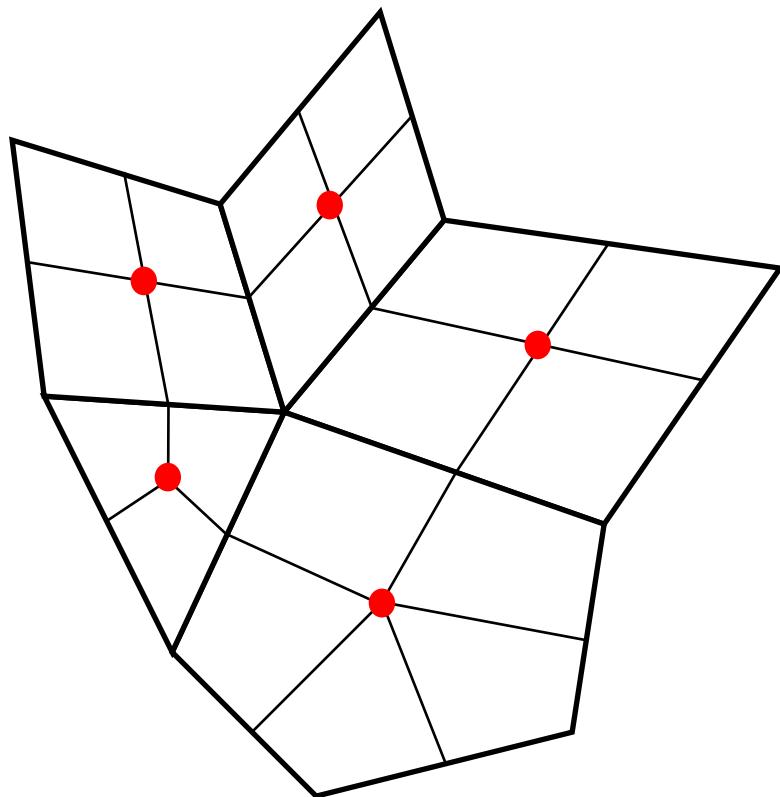
Catmull-Clark Subdivision



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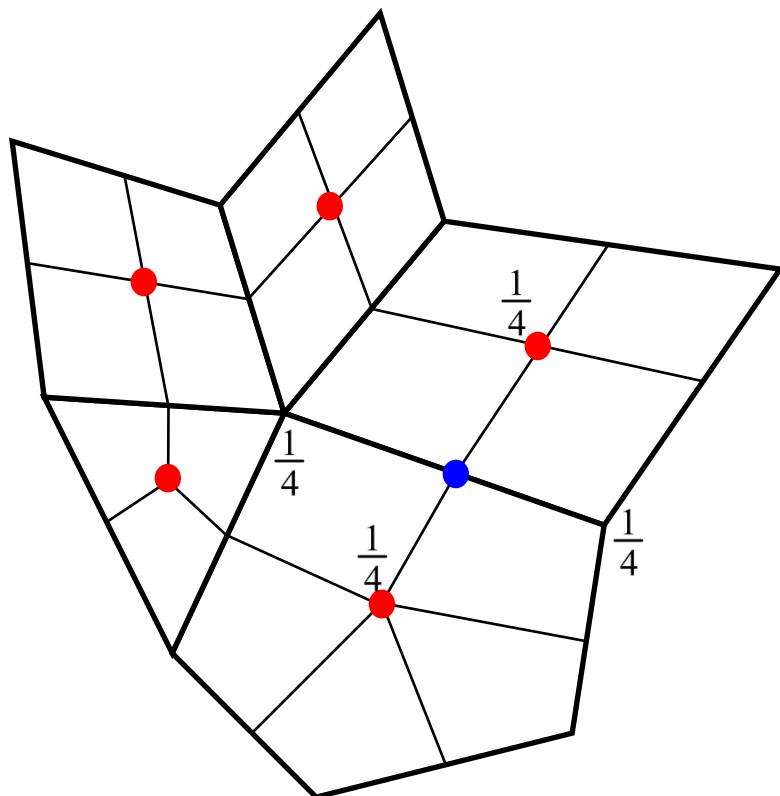
Catmull-Clark Subdivision



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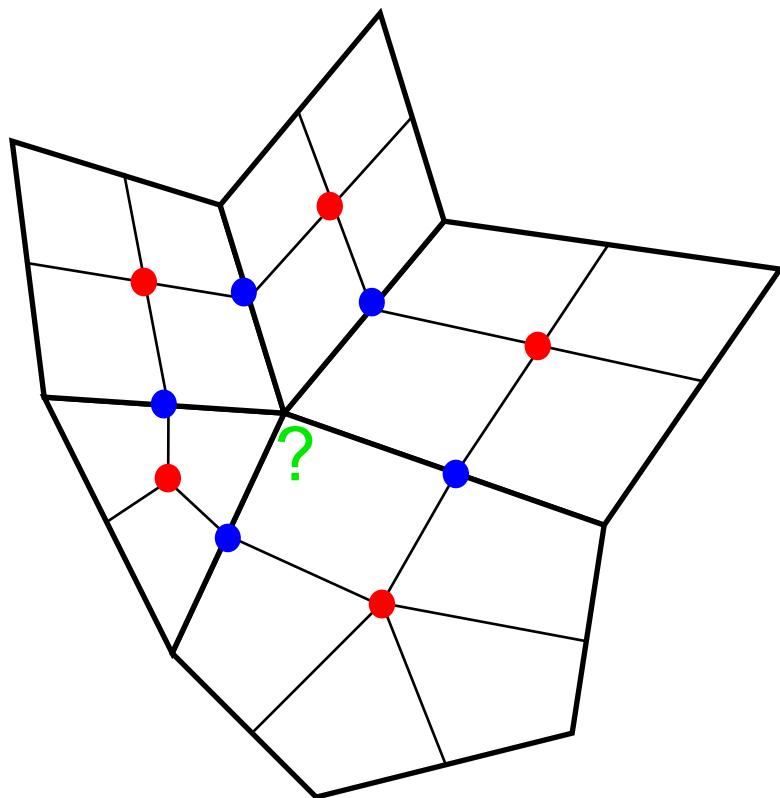
Catmull-Clark Subdivision



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Catmull-Clark Subdivision

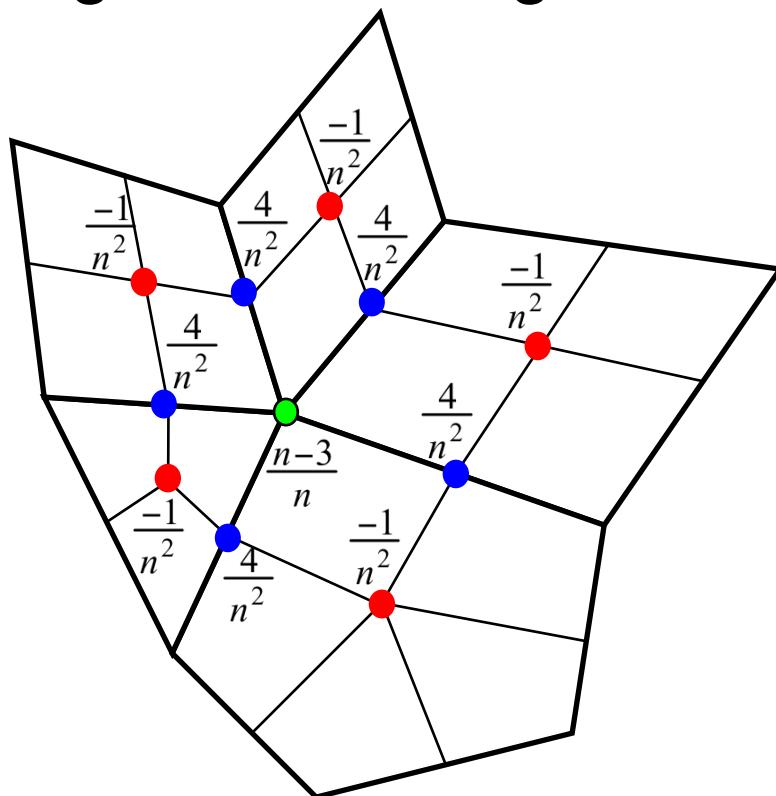


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Catmull-Clark Subdivision

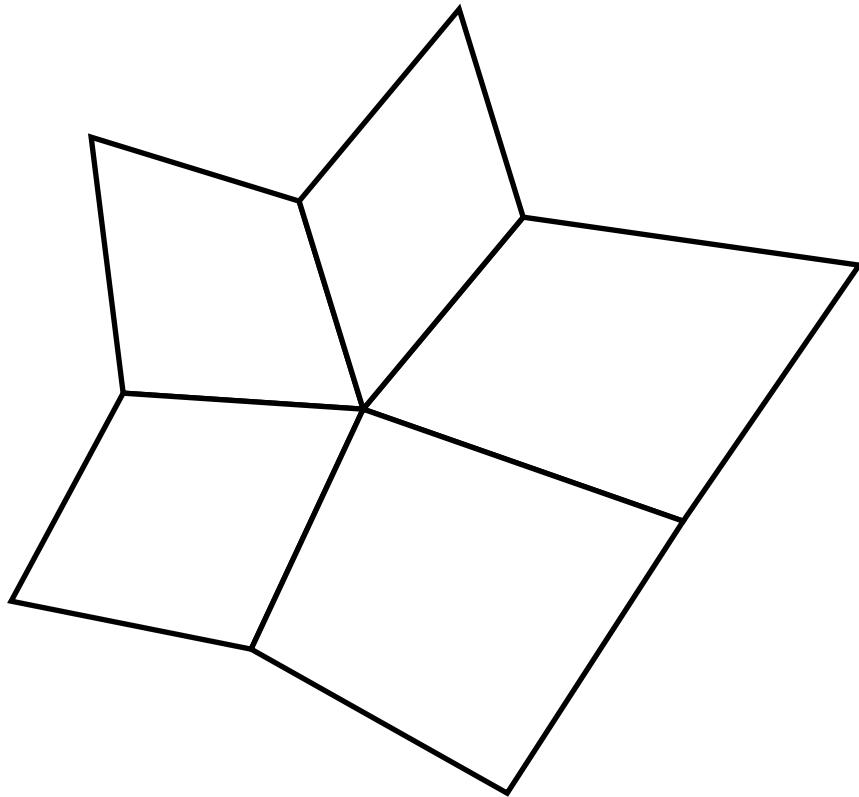
$$\text{New green dot} = \left(4 * \text{avg of blue dots} - 1 * \text{avg of red dots} + (n-3) * \text{current green dot} \right) / n$$



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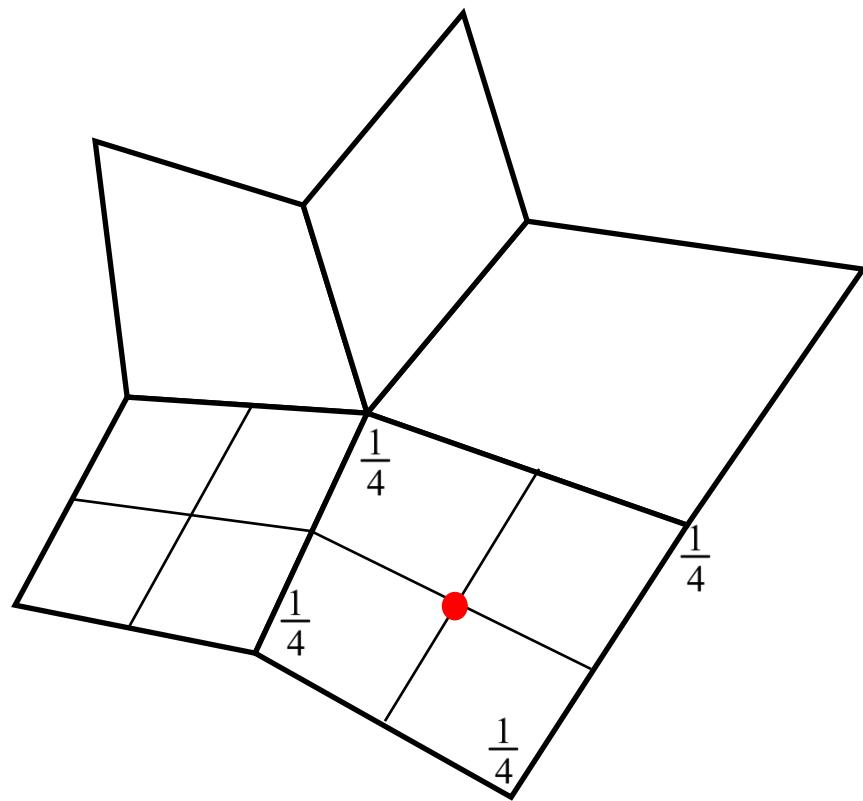
Catmull-Clark Subdivision



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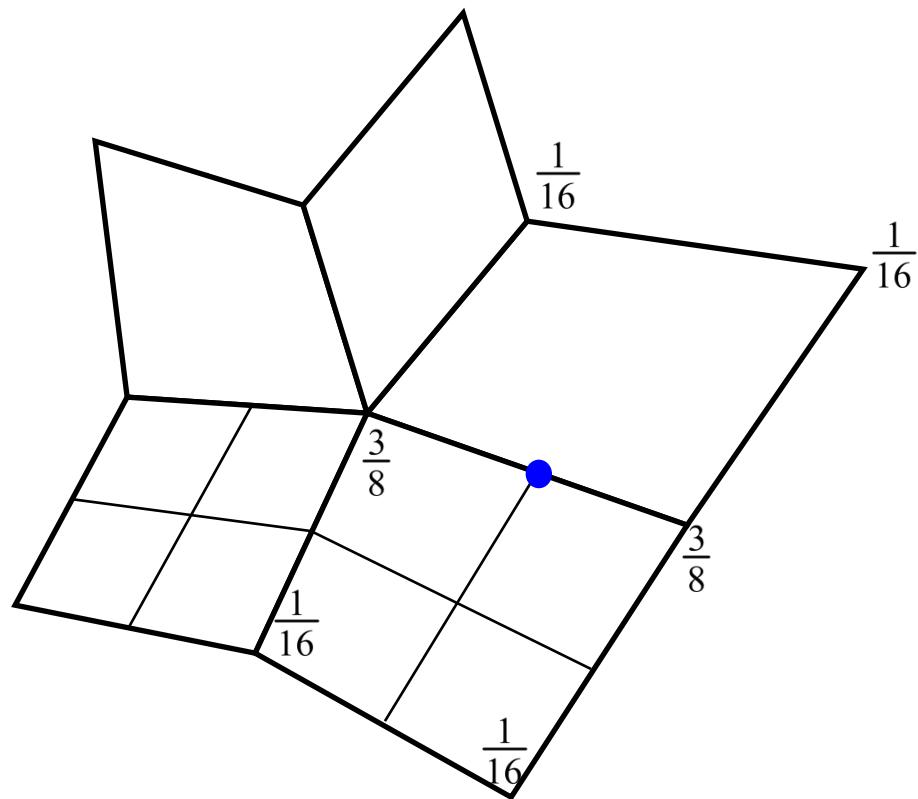
Catmull-Clark Subdivision



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Catmull-Clark Subdivision



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Catmull-Clark Subdivision



Linear
Subdivision

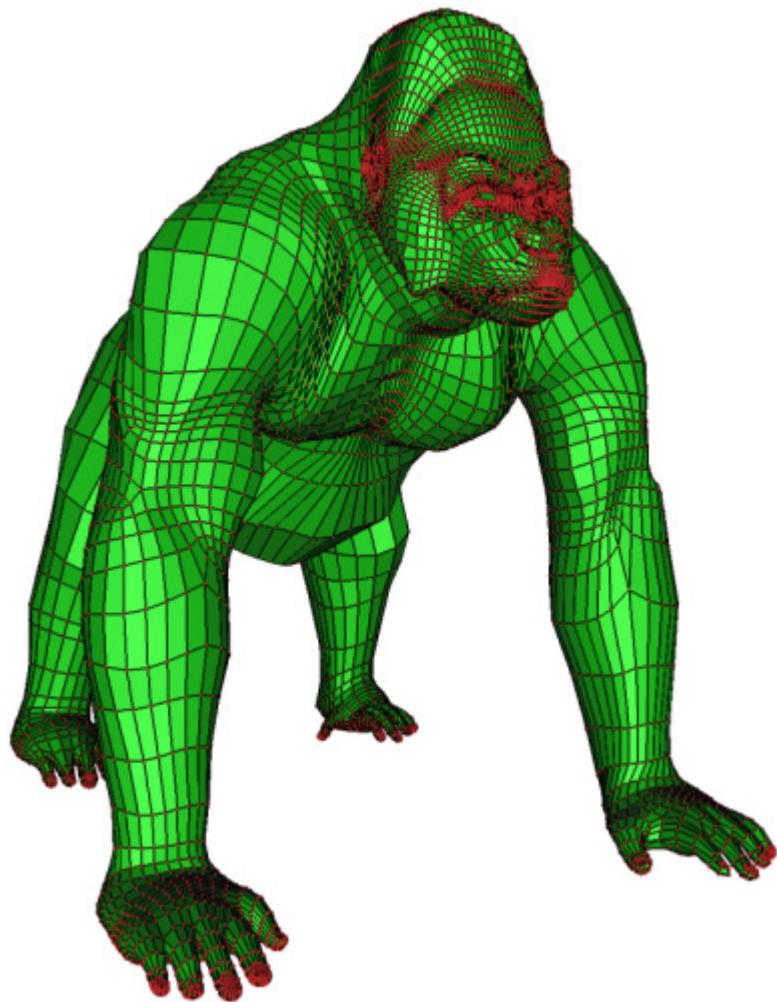


Catmull-Clark
Subdivision

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Catmull-Clark Subdivision



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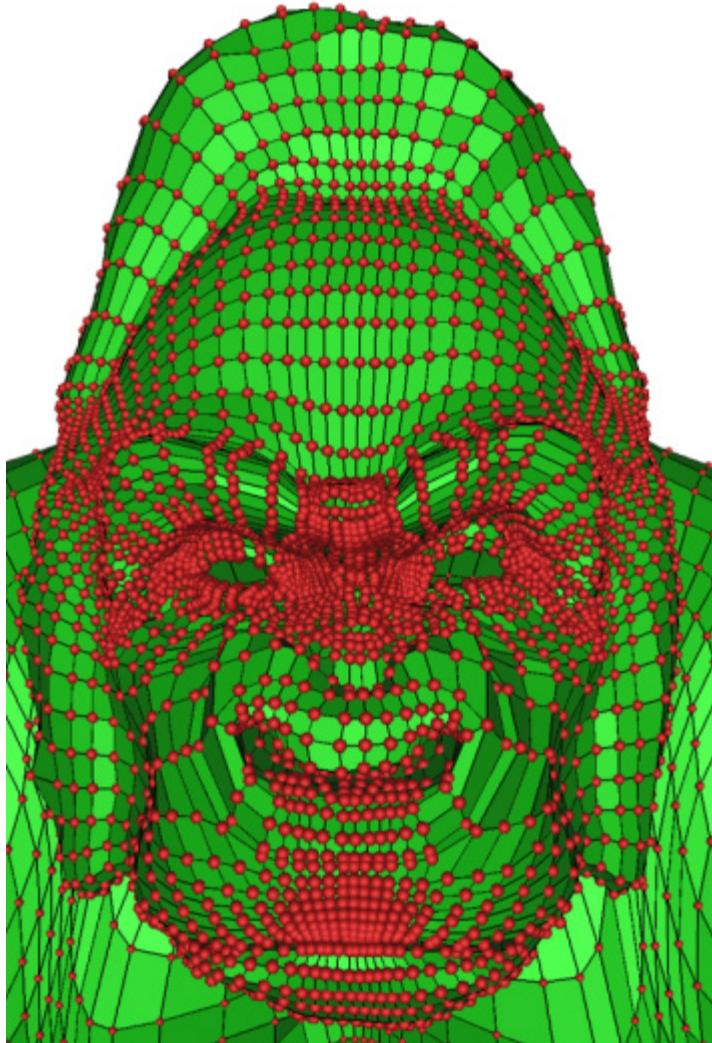
Catmull-Clark Subdivision



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Catmull-Clark Subdivision



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Catmull-Clark Subdivision



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Catmull-Clark Subdivision

- One round of subdivision produces all quads
- Smoothness of limit surface
 - C^2 almost everywhere
 - C^1 at vertices with valence $\neq 4$
- Relationship to control mesh
 - Does not interpolate input vertices
 - Within convex hull
- Most commonly used subdivision scheme in the movies...

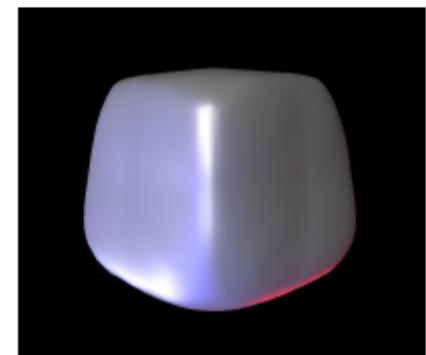
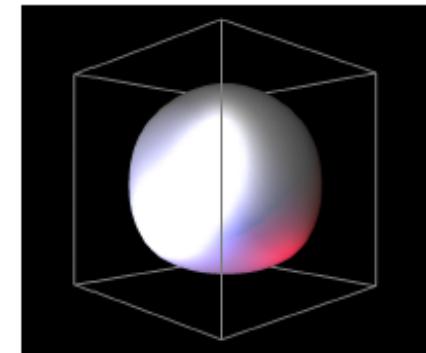
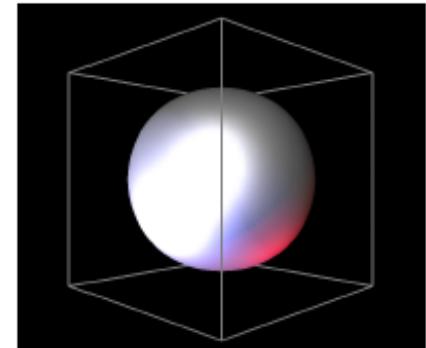


Pixar



Subdivision Schemes

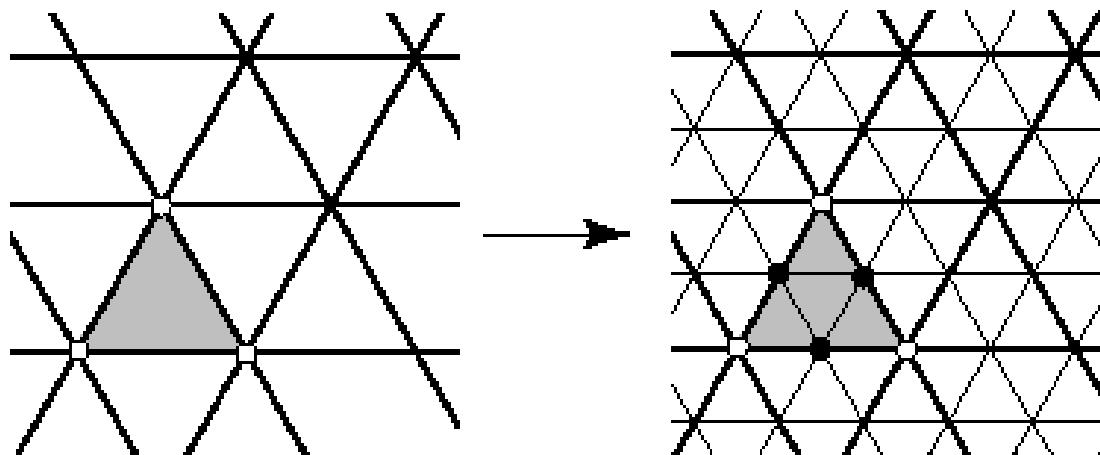
- Common subdivision schemes
 - Catmull-Clark
 - Loop
 - Many others
 - Differ in ...
 - Input topology
 - How refine topology
 - How refine geometry
- ... which makes differences in ...
- Provable properties





Loop Subdivision

- Operates on pure triangle meshes
- Subdivision rules
 - Linear subdivision
 - Averaging rules for “even / odd” (white / black) vertices

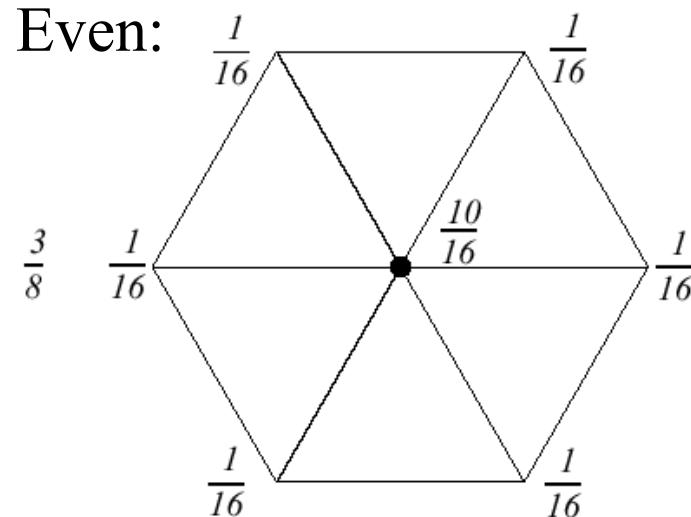
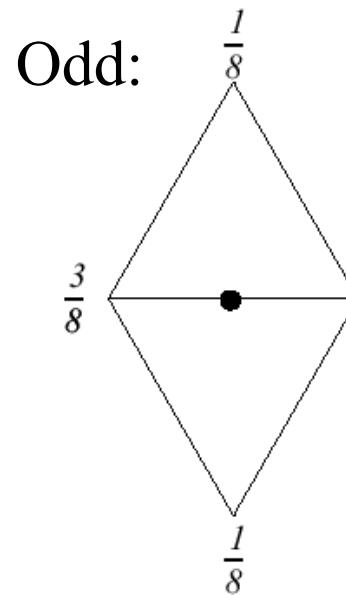


Zorin & Schroeder
SIGGRAPH 99
Course Notes



Loop Subdivision

- Averaging rules
 - Weights for “odd” and “even” vertices

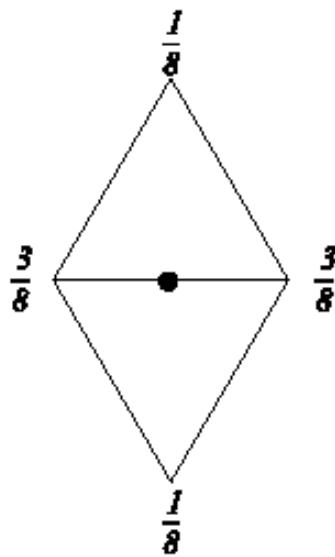


... but what about vertices with valence $\neq 6$?

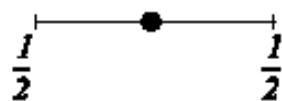
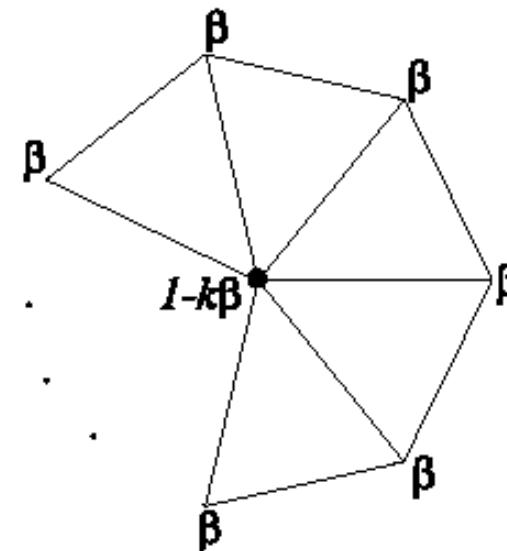


Loop Subdivision

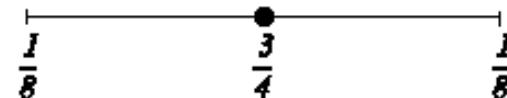
- Rules for extraordinary vertices and boundaries:



Interior



Crease and boundary



a. Masks for odd vertices

b. Masks for even vertices



Loop Subdivision

- How to choose β ?
 - Analyze properties of limit surface
 - Interested in continuity of surface and smoothness

» Original Loop

$$\beta = \frac{1}{n} \left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right)^2 \right)$$

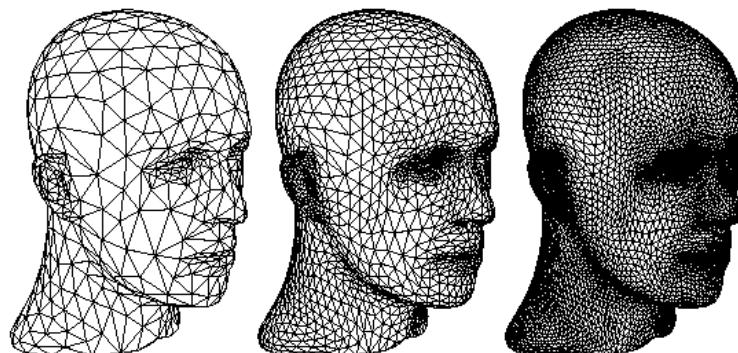
» Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$



Loop Subdivision

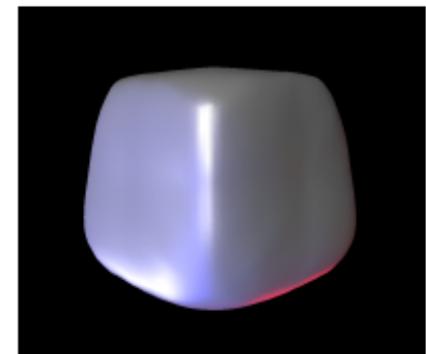
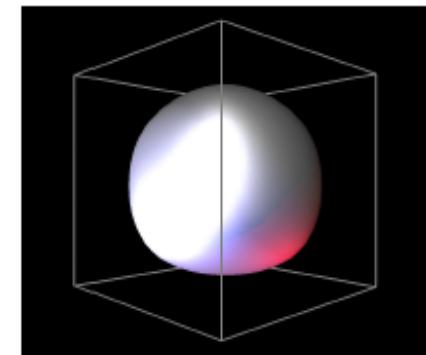
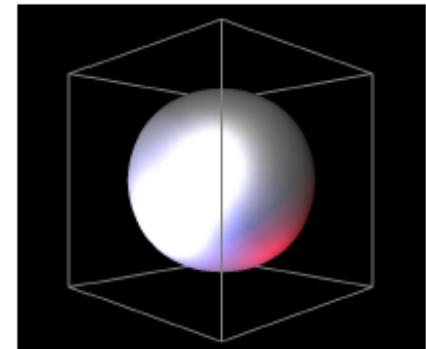
- Operates only on triangle meshes
- Smoothness of limit surface
 - C^2 almost everywhere
 - C^1 at vertices with valence $\neq 6$
- Relationship to control mesh
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 - Within convex hull





Subdivision Schemes

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Subdivision Schemes

- Other subdivision schemes

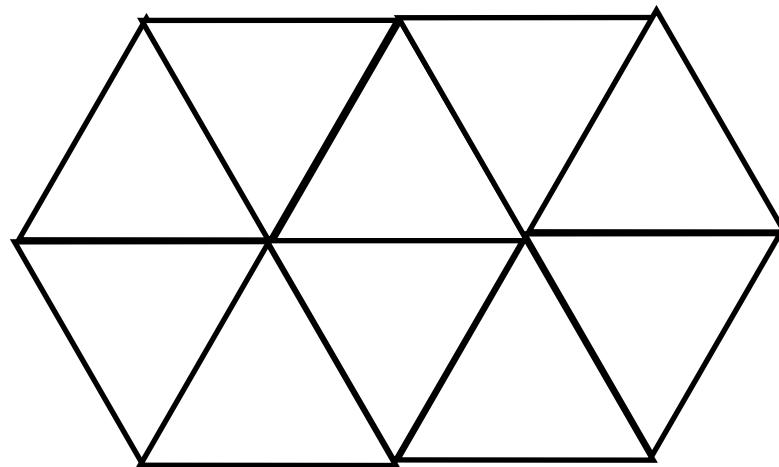
Face split		
	<i>Triangular meshes</i>	<i>Quad. meshes</i>
<i>Approximating</i>	Loop (C^2)	Catmull-Clark (C^2)
<i>Interpolating</i>	Mod. Butterfly (C^1)	Kobbelt (C^1)

Vertex split
Doo-Sabin, Midedge (C^1)
Biquartic (C^2)



Other Subdivision Schemes

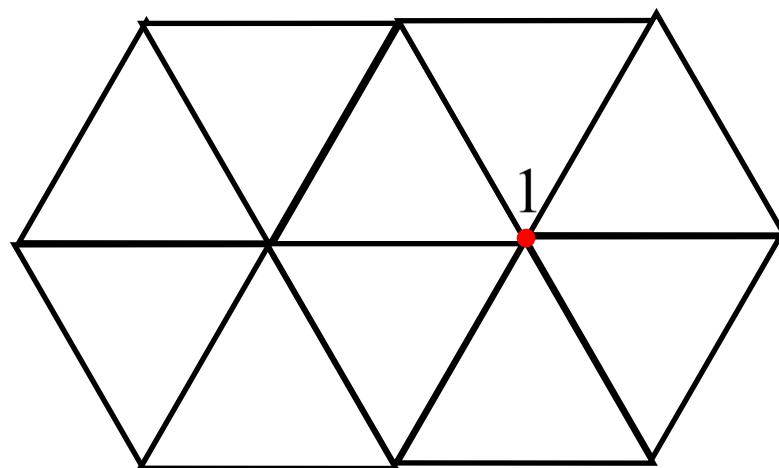
- Butterfly subdivision





Other Subdivision Schemes

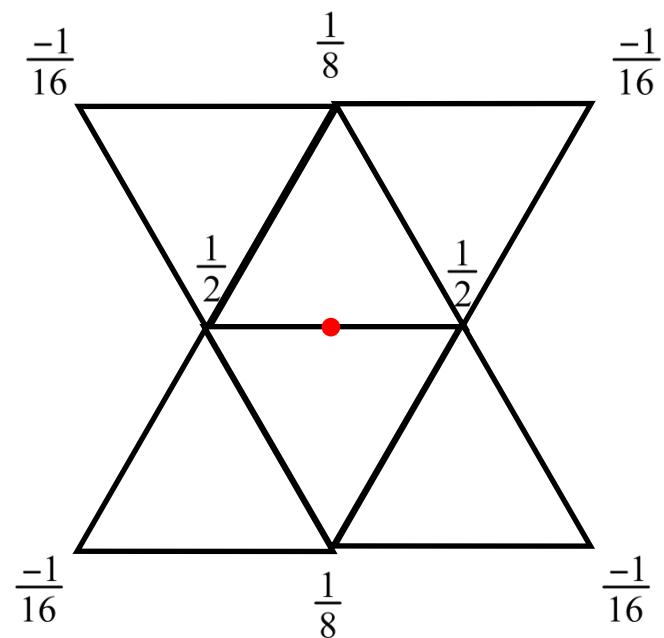
- Butterfly subdivision





Other Subdivision Schemes

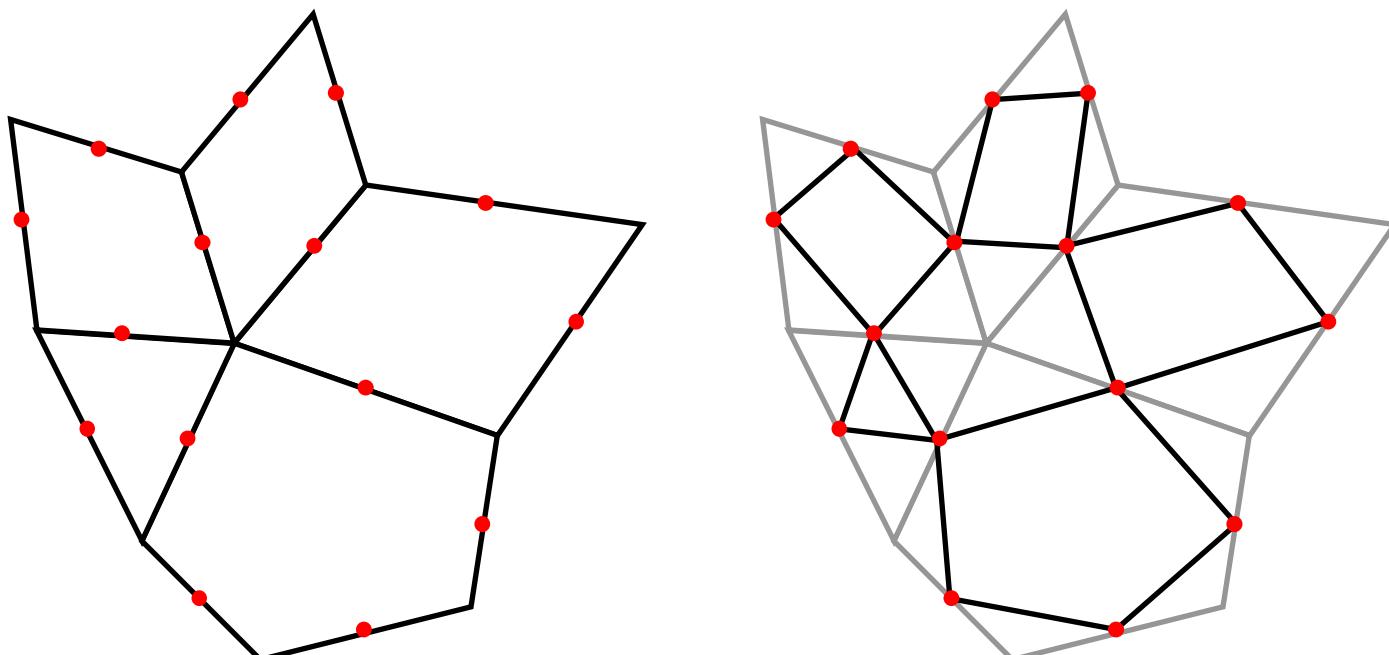
- Butterfly subdivision





Other Subdivision Schemes

- Vertex-split subdivision
(Doo-Sabin, Midedge, Biquartic)

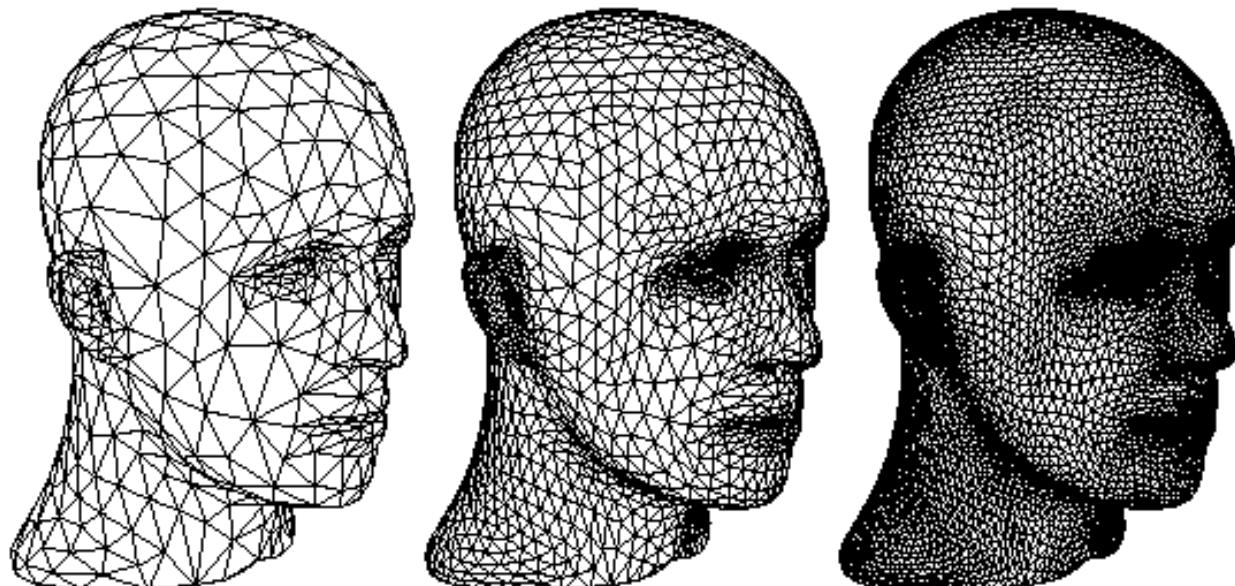


One step of Midedge subdivision



Drawing Subdivision Surfaces

- Goal:
 - Draw best approximation of smooth limit surface
 - With limited triangle budget



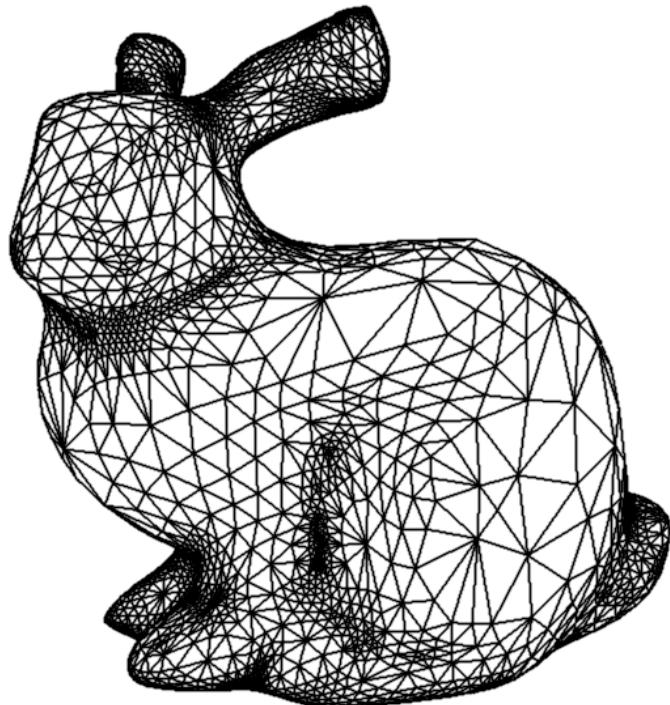


Drawing Subdivision Surfaces

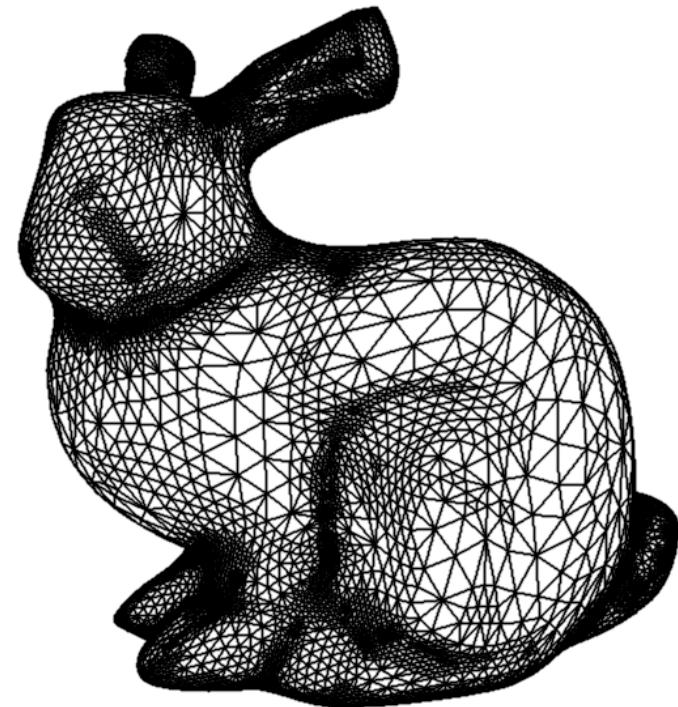
- Goal:
 - Draw best approximation of smooth limit surface
 - With limited triangle budget
- Solution:
 - Stop subdivision at different levels across the surface
 - Stop-criterion depending on quality measure
- Quality of approximation can be defined by
 - Projected (screen) area of final triangles
 - Local surface curvature



Adaptive Subdivision



10072 Triangles



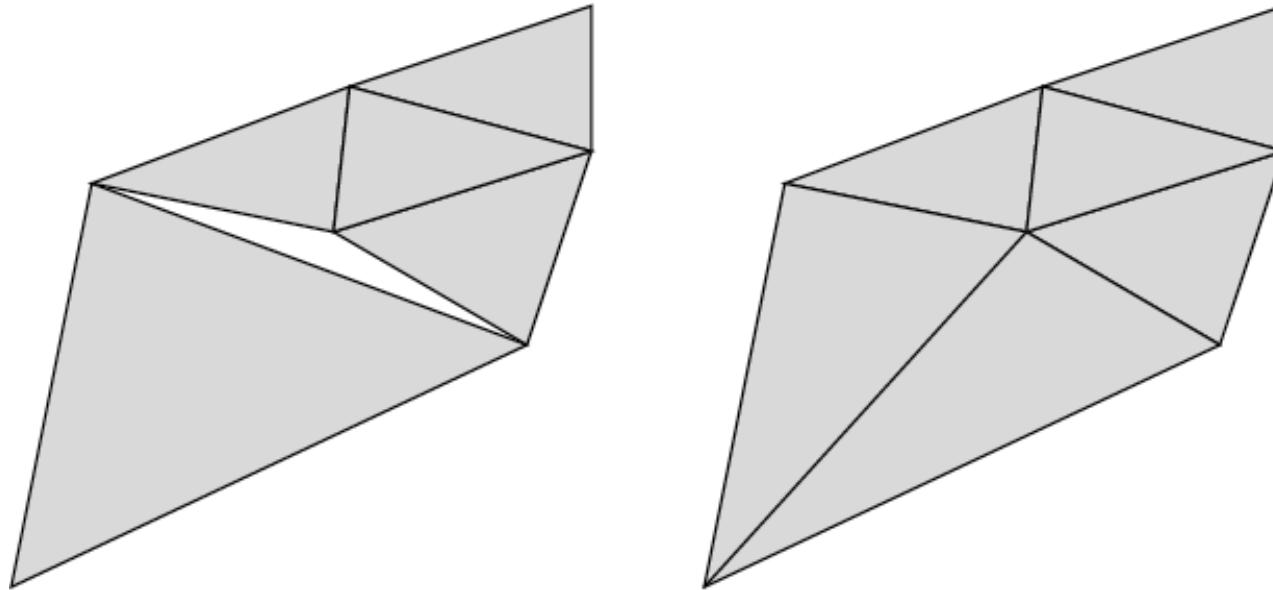
228654 Triangles

[Kobbelt 2000]



Adaptive Subdivision

- Problem:
 - Different levels of subdivision may lead to gaps in the surface

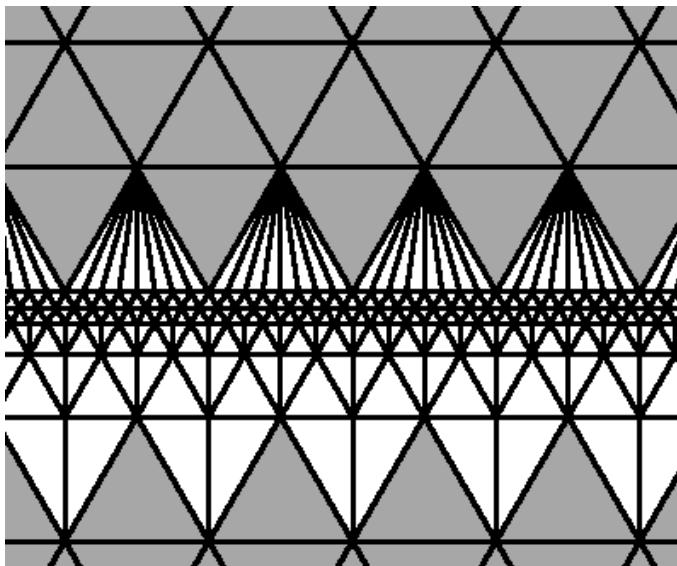


[Kobbelt 2000]

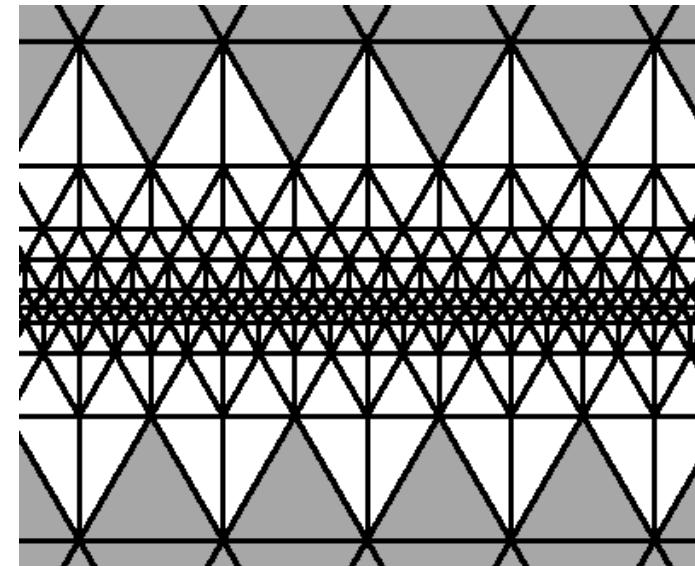


Adaptive Subdivision

- Solution:
 - Replacing incompatible coarse triangles by *triangle fan*
 - Balanced subdivision: neighboring subdivision levels must not differ by more than one



Unbalanced



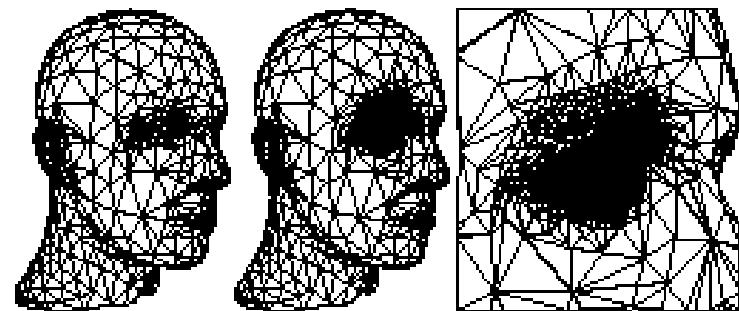
Balanced

[Kobbelt 2000]



Subdivision Surface Summary

- Advantages:
 - Simple method for describing complex surfaces
 - Relatively easy to implement
 - Arbitrary topology
 - Intuitive specification
 - Local support
 - Guaranteed continuity
 - Multiresolution
- Difficulties:
 - Parameterization
 - Intersections





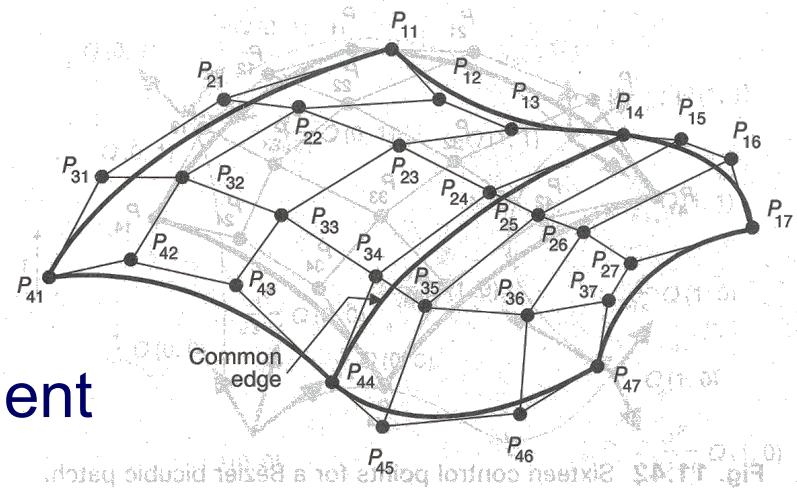
Comparison

Parametric surfaces

- Provide parameterization
- More restriction on topology of control mesh
- Some require careful placement of control mesh vertices to guarantee continuity (e.g., Bezier)

Subdivision surfaces

- No parameterization
- Subdivision rules can be defined for arbitrary topologies
- Provable continuity for all placements of control mesh vertices





Comparison

Feature	Polygonal Mesh	Parametric Surface	Subdivision Surface
Accurate	No	Yes	Yes
Concise	No	Yes	Yes
Intuitive specification	No	Yes	Yes
Local support	Yes	Yes	Yes
Affine invariant	Yes	Yes	Yes
Arbitrary topology	Yes	No	Yes
Guaranteed continuity	No	Yes	Yes
Natural parameterization	No	Yes	No
Efficient display	Yes	Yes	Yes
Efficient intersections	No	No	No