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T4 ________	**B**	F4 ________

2016

Mathematical Contest in Modeling (MCM/ICM) Summary Sheet

The Modeling of Space Debris Cleaning

Summary

Until now, there are over 500,000 pieces of space debris orbiting around the Earth, which pose a threat to satellites in earth orbit. A series of realistic, sensible and useful mathematical models about the evolution of debris and cost with different cleaning methods are developed to offer best solution to clean space debris.

First, we set up a method to obtain parameters (especially cleaning cost) of our model. We choose 4 basic cleaning methods in our model :**Space-based laser**(SBL) ,**Ground-based laser**(GBL), **ElectroDynamic Debris Eliminator** (EDDE) and **GEO thruster**. Based on NASA DAS simulation and analysis of physical process, we are able to estimate the cost and efficiency of each cleaning methods and cleaning cost of debris as standard input of our model.

Then, we consider a two-phase cleaning operation: first reduce potential collision rate of the space debris to an acceptable number, then keep potential collision rate under this level. We develop a **Multi-Objective Optimization Model** (MOO model) to estimate the optimal profit for required cleaning plan. This model is applied to 3 strategies (they contain mixture of different cleaning methods) and we make a profit comparison among them.

To restate the clean process more precisely, we develop a **Stochastic Evolution Simulating Model** (SES model) to directly simulate the evolution of debris over time. Taking factors such as the production of debris via collisions into consideration, we are able to simulate debris evolution process in our model. We simulate the Free-Evolution Process and Evolution with Cleaning Process for every strategy. By the result of simulation, we find the best time for each strategy to launch and compare the efficiency and economic effect of these strategies. After comparison, we propose our strategy for space debris issue : If we only need to clean debris in LEO, the best solution (strategy B) enables us to achieve this goal with an initial investment of 6 billion dollars and get investment back in 20 years. Nevertheless, investment on GEO in 50 or even 80 years might not be a lucrative strategy. So in our model, cleaning LEO orbit can be adopted as a commercial opportunity for commercial firms. We also consider the risk and give some suggestions of collision avoidance.

In out sensitivity analysis, we independently consider the influence of several factors, such as the coefficient rate of annual collision, and a sudden increase of debris number. These change will vary the total profit of the industry and several results might meaningful while making decisions.

Contents

1 Introduction

1.1 Problem Background

The problem of space debris is becoming more and more serious. According to NASA, it is estimated that there are about half a million pieces of space debris orbiting around the Earth. Due to their various size/mass and high orbiting velocity, space debris are seen as potential hazards to satellites and spacecrafts which is hard to clean.

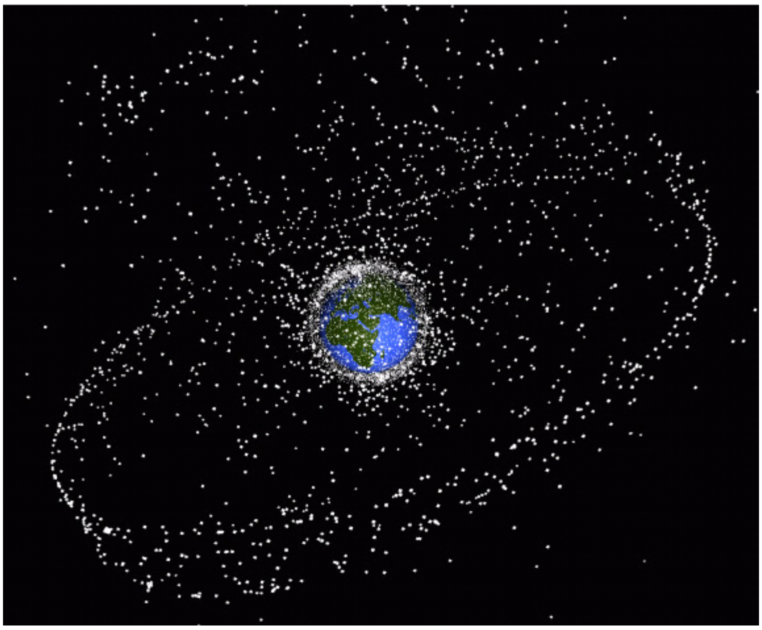

Figure 1: Computer generated image of the distribution of the catalogued space debris around the Earth(source : [1])

1.2 Previous Research

Scientists had realised the hazards of space debris since 1960s, when *United States Space Surveillance Network* (SSN) began to track the trajectories of these debris. The radial distribution of space shows in Fig.2. This distribution shows that most of space debris are located between *Low Earth orbit*(LEO) and *Geostationary Orbit*(GEO).

Space debris can be divided to three parts by their size : Small , Medium and Large. Table 1 shows the range of size of each type :

Table 1: Different kinds of debris and their sizes

Type	Range of Size
Midium	$RAS < 0.01$, about 1 10cm
Medium	$0.01 < RAS < 1$, about 10cm 2m
Large	$RAS > 1$, larger than 2m

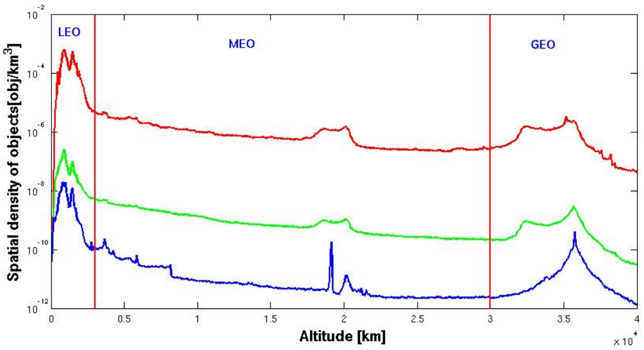

Figure 2: Spatial density of objects as a function of altitude for three different size thresholds : objects with diameter larger than 1 mm (red line), 1 cm (green line) and 10 cm (blue line).(source : [1])

Laixing Lin[2] reviewed the situation of space debris on LEO, MEO and GEO, getting the result that debris in MEO has little hazards comparing to the debris on LEO and GEO.

If the number of space debris continues to increase, the possibility of collision between satellite and debris will keep increasing, and collisions will largely increase the amount of debris, which will in turn increase the collision probability. Finally the density of objects on LEO so high that we will be unable to ensure the safety of satellites on LEO, which is called *Kessler Syndrome*.[3] The problem of space debris is becoming more and more serious, and we humans must find some ways to fix it.

2 General Assumptions

1. Debris in our model are distributed in 3 regions : LEO, GEO and MEO. According to [2], we ignore debris in MEO.
2. There are three types of debris in our model : Large, Medium and Small(based on Fig.2).
3. We will only consider following 4 cleaning methods and their combinations:
    - Space-Based Laser(SBL)
    - Ground-Based Laser(GBL)
    - ElectroDynamic Debris Eliminator (EDDE)
    - GEO thruster

    We will explain their mechanism and estimate corresponding parameters in section 4.
4. The deflation and inflation are ignored.
5. The number of satellites launched to space every year can be seen as a normal distribution.

3 Notation and Symbols Description

3.1 Notations

- **Initial Period** : The action of cleaning can be separated into two parts. In this initial period we aim to reduce the amount of space debris to a safe level.
- **Maintaining Period** : After initial period, we enter this maintaining period aiming to keep the amount of debris in a low level.
- **Space Collision Rate** : The space collision rate is the parameter of the corresponding poisson process of collision process.
- **M$** : In our model, the unit of cost is 1 million, and we'll denote 'million' as 'M' in some part of our paper.

3.2 Symbol Description

Table 2: Symbols Description List

Symbol	Description
x_{sat}	The value of a satellite
x_{ri}	Average cost for cleaning debris(depending on region, size and strategy)
T_{expect}	The lifetime of a satellite
T_{age}	The working time of a satellite
m_r	Amount of satellites in region(depending on region)
$M_{r,lau}$	Newly launched satellite in a year(depending on region)
$M_{r,ret}$	Newly retired satellite in a year(depending on region)
$M_{r,lea}$	Newly retired and leaved satellite in a year(depending on region)
n_{ri}^*	Amount of debris of each size in each region(depending on region and size)
n_{ri}	Target amount of debris of each size in each region(depending on region and size)
v_r^*	Space collision factor for single satellite(depending on region)
v_r	Target space collision factor for single satellite(depending on region)
α_{ri}	Space collision factor caused by single debris(depending on region and size)
X_r^*	Number of space collision in a year(depending on region)
X_r	Target number of space collision in a year(depending on region)
μ_r	Average number of retired satellite per year(depending on region)
σ_r	The standard deviation of number of retired satellite per year(depending on region)
λ_i	Average debris per collision(depending on region and size)
σ_i	The standard deviation of debris per collision(depending on region and size)
C_{Lr}	Maintaining cost per year during maintaining period
C_{Pr}	Positive economic effect for maintaining per year during maintaining period
y	Time length of maintaining period
T_{start}	The starting year for cleaning
$z_{R\&D}$	The R&D cost
w_j	Maintaining cost for each facility(depending on strategy)
Q_{zr}	Potential loss of free evolution(depending on year)
z_r	Expenditures of cleaning (depending on year)
u_r	Financial restrict per year for maintaining (depending on region)
k_r	Tolerance of space collision rate(depending on region)
η_r	Average number of new satellite launched per year(depending on region)
Y_r	Average number of new satellite launched per year(depending on region)

4 Mathematical Models

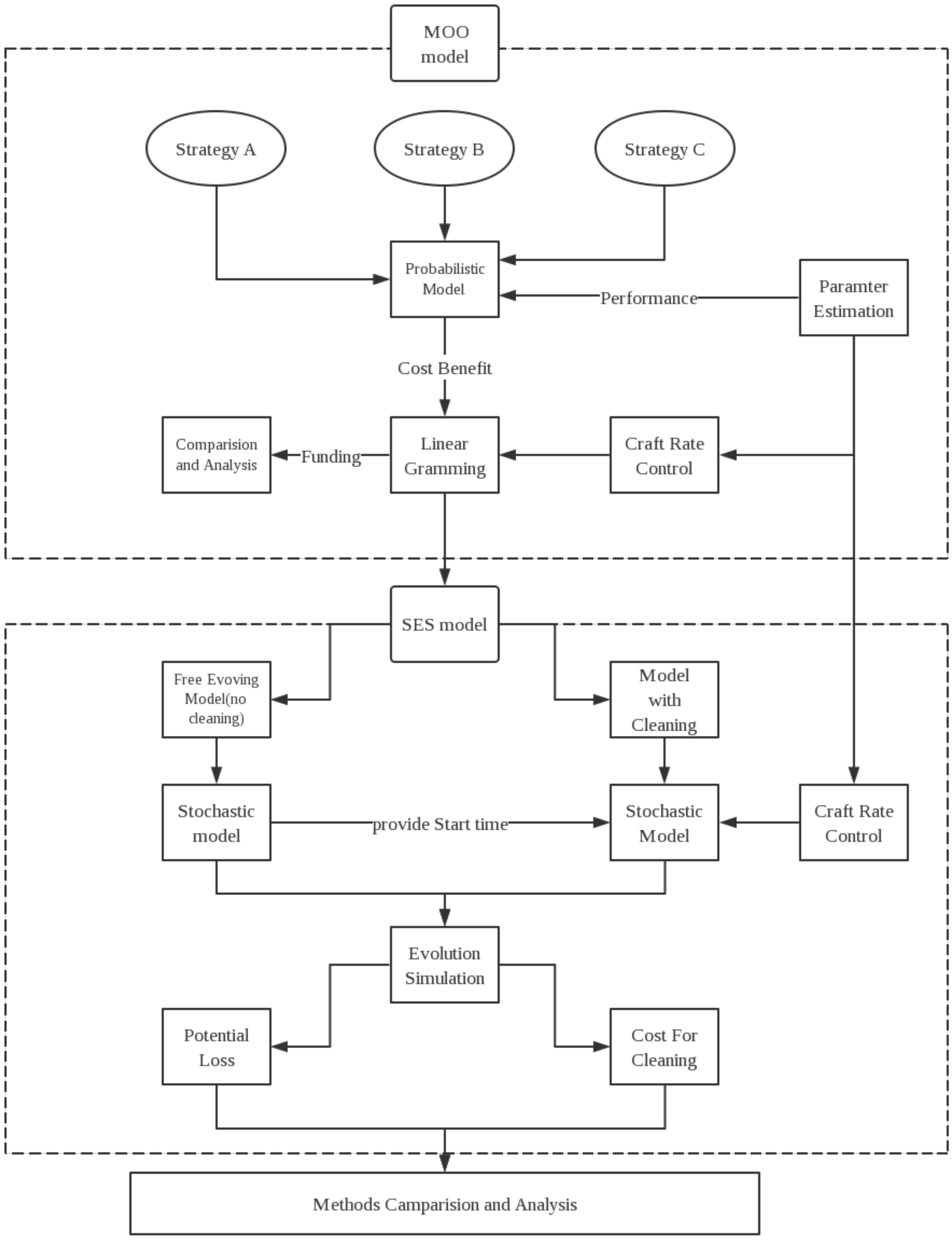

Figure 3: General Structure of Our Models

4.1 Problem Analysis & Model Objective

The problem requires us to compare various cleaning methods with estimating the efficiency, costs and risks of them. To solve this problem, we develop models to analyze solutions for cleaning strategies of the debris in orbit and judge whether the solutions attractive for commercial opportunities. The aim of the model can be separated into five components as shown below :

1. Mathematically describe the danger of space collision caused by debris in orbit.
2. Reveal the connection between economic loss and spatial distribution of orbital debris.
3. Simulate the commercial benefits and costs of different solutions and find the optimal solution among them.
4. Analyse the result and give answer to the practical question : If a viable commercial opportunity exists considering those methods?
5. Consider risk of different solutions and provide alternative method for avoiding collisions if there is no commercial opportunity.

4.2 Parameters Estimation

4.2.1 General Considerations

Statistical and physical parameters play a vital rule in our problem. Only with this parameter could we find the optimal solution. However, the space junk clean is far from a mature industry, which means that there are no standard values of these parameters. For example, no one know exactly the cost of building a space-based laser broom to clean small debris, because no such laser broom had been built before.

In order to get the most possible precise parameters, we need to do some estimation. We hope to get each method's cost and efficiency when dealing with different types of debris, so as to do optimization and simulation.

We know average value of satellite in space is about 100 M $. For simplicity, we use this average value in our model. In addition, the launch to LEO will be carried by SpaceX Falcon 9 rocket(costs 60 M $)[4], while all the launch to GEO will be carried by SpaceX Falcon Heavy rocket(costs 90 M $)[5]. Considering this two rockets has very strong power, we can send two satellites to space with one launch.

Most of methods in our model are not technologically mature now, so we need to set a R&D cost. Comparing to normal R&D cost in a space project, we estimate this cost to be 5 times of the manufacturing cost.

In the following, we will do detailed parameter analysis and cost estimation of every cleaning methods based on corresponding data and physical process.

4.2.2 Satellites and Debris Statistics

We use RSS data on*space-track.org*[16]. Based on their SATCAT , we can count the number of tracked debris and satellites on orbit and calculate factors.

We determine initial state and some parameters about random variables as following tables:

Satellite Number in space now:

- LEO : 2486
- GEO : 1306

Table 3: Debris Number(n_{ij})

	Small(approximation)	Medium	Large	Unknown(Not Calculated)
LEO	353280	11039	3717	8579
GEO	4762	120	98	195

Table 4: Satellite Launch Number(LEO)

year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Launch Number	92	98	94	104	97	103	96	141	111	99

We can calculate the average number η_1 and the standard deviation σ_1 of data in table 4:

$$\eta_1 = 103.5 \qquad \sigma_1 = 42.8$$

Table 5: Satellite Launch Number(GEO)

year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Launch Number	52	47	62	51	55	58	63	35	66	70

We can calculate the average number η_2 and the standard deviation σ_2 of data in table 5:

$$\eta_2 = 55.9 \qquad \sigma_2 = 30.8042$$

According to the general assumption, the number of satellite to retire every year $M_{r,ret}$ satisfies the normal distribution :

$$M_{r,ret} \approx 0.4 M_{r,lau}(\mu_r, \sigma_r)$$

Table 6 shows the values of μ_r and σ_r:

Table 6: Approximation of debris satellite number

Orbit	μ_r	σ_r
LEO(r=1)	41.4	6.85
GEO(r=2)	22.4	4.93

The mean and variance of distribution of debris after a collision will be approximated as the following table 7:

Table 7: Approximation of number of debris after a collision incident

Size of Debris	μ_i	σ_i
Small(i=1)	15000	2415
Medium(i=2)	856	22.5
Large(i=3)	0.5	0.1

4.2.3 Laser Impulse(SBL & GBL)

High-energy pulsed laser is a good way to mitigate the debris in the size range from 1-10cm. We irradiate the debris of this size by the laser, which will ablated part of debris and deflects the orbit of the debris to make it reentry the atmosphere. For now, there are two versions of this method: ground-based laser and space-based.

Space-Based Laser(SBL) Space-based laser is a kind of satellite that can locate space debris and launch high-energy laser impulse to de-orbit the debris.

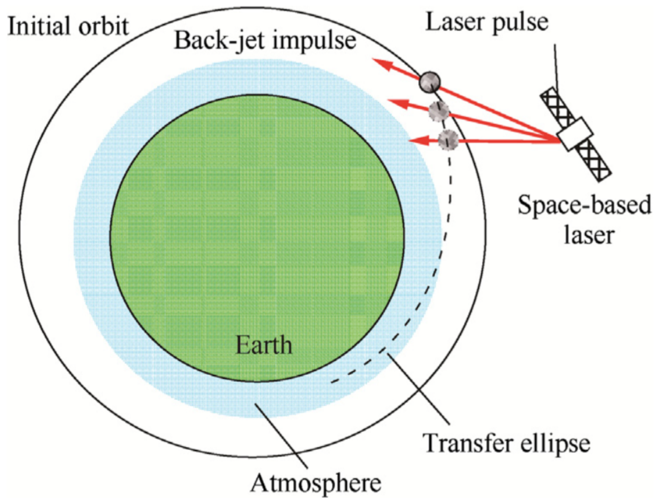

Figure 4: Removal process of a debris particle(source :[6])

From the result of performance model for this SBL system[7],we have following analysis result :

- Average time between two firings will be between 30 min and 40 min.
- Average required energy $\approx$ 3kWh for each fire.

In addition to these results, we do following estimations :

- Average time between two firing is 30 min for a SBL on LEO, and 40min for a SBL on GEO.(Because the debris locating part works faster when SBL is near the ground)
- Manufacturing cost of such a laser satellite is 100 M \$.
- Lifetime of a SBL satellite is 10 years.(Estimated based on the normal lifetime of a telescopic satellite)

- We can launch two laser satellite by one rocket, which will half the launch cost.
- Cost of maintenance is 1 M $ / year.
- Because the power of SBL is too small for Midium debris cleaning, SBL can only clean small debris.
- R&D cost = 100$\times$ 5 = 500 M $
- The time needed to build one SBL is about 1 year.

Table 8 lists all the actions above and their corresponding cost :

Table 8: Money and time cost of different Actions of SBL

Action	Cost
Manufacture	100M$
Launch to LEO	30M$
Launch to GEO	45M$
Lifetime	10 year
R&D	500M$
Maintain per system	1M$ / year
Time to Build	1 year

Then we'll calculate the cost of this method :

1. Manufacture of the laser satellite : 100 M $
2. Launch of the each laser satellite :
   - LEO : 30 M $
   - GEO : 45 M $
3. Lifetime : 10 years

So we get the number of debris SBL can clean in it's lifetime :

- LEO : 10year $\div$ (1 shot / 30min) = 175320
- GEO : 10year $\div$ (1 shot / 40min) = 131490

So we have the cost of cleaning 1 small debris on LEO using SBL :

$$c_{SBL\_LEO\_Small} = (100 + 30 + 10M \$ \div 175320 = 0.0008 \text{ M } \$$$

The cost of cleaning 1 small debris on GEO using SBL :

$$c_{SBL\_GEO\_Small} = (100 + 45 + 10) \text{ M } \$ \div 131490 = 0.00118 \text{ M } \$$$

Figure 5: Artistic representation of GBL(source:[10])

Ground-based laser(GBL) As we mentioned in introduction, NASA proposed a project *Orion* in 1990s to build a GBL[8]. This project was estimated to cost 500 M$ in the 1990s.

We estimate each process in the following way :

Manufacture of the laser system : 20 years have passed from start of *Orion Project* to now. In this 20 years, the currency inflation will increase this cost, while the technology advance will decrease this cost. Taken these two effect together into consideration, we can probably say that the cost of this whole project now is still about 500 M. In this project, the cost of a near-term system costs 60-80 M, and a far-term system costs 150-200 M(according to the data of *Orion Project* in [9]). So we average manufacturing cost of one system will be about 100 M.

Cost of maintaining the system : According to a normal maintaining fee of a satellite, we estimated that that it's about 1 M $ per year.

The system's lifetime : Due to the development of laser technology in this 20 years, the system now will have a better lifetime than the system in *Orion* project, which is about 10 years.

The efficiency of this system : Due to the convenience of changing target orientation in the ground and higher power, GBL works 4 times faster than the space-based laser system. So this system have the efficiency of cleaning 1 debris per 6 minutes.

Then in it's lifetime, the number of small debris it can shoot is :

$$10 \text{ year} \times 1 \text{ debris} / 6\text{min} = 876600$$

We ignore the cost of shooting, for it has been considered in the maintaining process, so finally we have:

- System Manufacture : 100 M $
- System Maintaining : 1 M $ / year
- Cleaning Efficiency : 1 debris / 6min
- R&D cost = 100M $\times$ 5 = 500 M $

The manufacturing time of GBL is estimated to be the same as SBL, which is 1 year.

Table 9: Money and time cost of different Actions of GBL

Action	Cost
Manufacture	100M$
Maintain per system	1M$/year
R&D	500M$
Lifetime	10 year
Time to Build	1 year

We list all these actions in table 9.

The cost of cleaning 1 small debri on LEO using GBL :

$$c_{GBL\_LEO\_Small} = 250 + 10\text{M \$} \div 876600 = 0.0003 \text{ M \$}$$

Because the GBL has sufficient power supply to shoot stronger laser impulse, we could us it to clean Midium debris. Considering the size of Midium debris is larger than small debris, the efficiency ratio of cleaning Midium and small debris is about 1 : 10.

So we can get the cost of cleaning 1 Midium debris on LEO using GBL :

$$c_{GBL\_LEO\_Midium} = \text{cGBL\_LEO\_Small} \times 10 = 0.003 \text{ M \$}$$

This power is still too small for large debris(about 1000kg) cleaning,so we couldn't clean large debris in this way.

4.2.4 Mechanical Capture(EDDE & GEO thruster)

Laser impulse can't handle the Large debris such as discarded satellite (usually over 100 kg), because it's still too weak. To clean those large debris, mechanical capture is a natural and effective idea. We need to estimate the cost of this in two orbits : LEO and GEO.

Capture Debris on LEO (Using EDDE) In the sense of LEO, we fortunately find that there is already such a project called *ElectroDynamic Debris Eliminator*(EDDE)[11]. It's a solar-powered space vehicle that uses electric current in a long conductor to change it's position, and capture large debris by using a net which is made of aluminium.

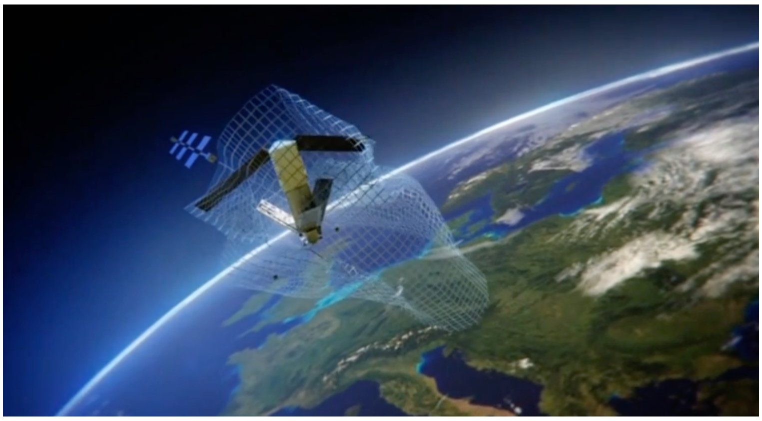

Figure 6: Mechanism of EDDE Capture(source : [12])

J.Pearson et .al analysed this vehicle and got following list of it's parameters[13]:

- 12 EDDE could remove all large debris form LEO in 7 years.
- EDDE will be launched on one ESPA ring(two per slot)
- 12 EDDE costs 90M$ per year to keep them operating.
- 1 EDDE has 5 year's lifetime.

With these data and mechanism of EDDE in [13], we can do following estimations:

- The launch cost of on ESPA ring is the same to Falcon 9 ,which is about 60M to LEO.
- Manufacturing cost of one EDDE is 50M$.
- Different EDDEs have exactly same performance.
- EDDE is 4 times more efficient when dealing with Midium debris. Because the size of capture net is too big for small debris, we can't capture small debris in this way.
- R&D cost = 50M $\times$ 5 = 250M$.
- The building time of EDDE is 1 year.

We list cost of EDDE actions in table 10.

Table 10: Money and time cost of different Actions of EDDE

Action	Cost
Manufacture	50M$
Launch to LEO	30M$
Lifetime	5 year
R&D	250M$
Maintain per system	7.5M$ / year
Time to Build	1 year

We calculate these parameters :

1.The efficiency of cleaning

The number of large debris one EDDE could clean per year : n_1 = 23748 $\div$ 12 $\div$ 7 = 282.7

So in it's lifetime ,one EDDE could clean up to 282.7$\times$5 = 1413.5 large debris.

In this case , the main process of capturing a debris is to put it into the net, so there is actually no big difference between capturing large and Midium debris. So the ratio of efficiency of cleaning large/Midium debris is 1 : 2.

So one EDDE could clean up to 1413.5$\times$2 = 2827 Midium debris in it's lifetime.

2.The total cost of one EDDE :

Total cost = manufacture + launch + maintenance $\times$ lifetime

Manufacture = 50 M $

Launch = 30 M $

Maintenance = (90M / 12) / year = 7.5 M $/ year

so we have total cost = 50M + 30M + 7.5M $\times$ 5 = 117.5 M $

the cost of cleaning one Large debris on LEO :

$$c_{EDDE\_LEO\_Large} = 117.5M \div 1413.5 = 0.083 \text{ M } \$$$

the cost of cleaning one Midium debris on LEO :

$$c_{EDDE\_LEO\_Midium} = 117.5M / 1413.5 / 2 = 0.0415 \text{ M } \$$$

the cost of cleaning one small debris on LEO :

$$c_{EDDE\_LEO\_Small} = \infty$$

Capture debris on GEO From the result of EDDE above, we can see that EDDE is a very cost-saving way of capturing debris on LEO. However, EDDE can't be used to clean up the debris on GEO. In order to clean debris on GEO, we have to launch a thruster(we call it GEO thruster)[14] to the GEO orbit and thrust the debris to leave from earth.

Figure 7: Concept of GEO thruster(source:[15])

We have following process and cost:

- Capture Satellite manufacture : 50M$.
- Launch to GEO : using Falcon Heavy,which is about 90M$. Considering the huge capacity of Falcon Hevay, we could launch 2 satellite at the same time, so the launch cost per satellite is 45M$.
- We could clean 5 large debris, 10 Midium debris in one mission. Because the size of capture net is too big for small debris, we can't capture small debris in this way.
- In this case, we don't need to consider the maintianing cost.
- R&D cost = 50M$ $\times$ 5 = 250M$

- The building time of 1 GEO thruster is 1 year.

We list these paramters in table 11.

Table 11: Money and time cost of different Actions of GEO thruster

Action	Cost
Manufacture	50M$
Launch to LEO	30M$
Launch to GEO	45M$
R&D	250M$
Time to Build	1 year

So we have the total cost = 50M + 45M = 95M$

the cost of cleaning one Large debris on GEO :

$$c_{gtr\_GEO\_Large} = 95M\$ / 5 = 19\ M\$$$

the cost of cleaning one Midium debris on GEO :

c2 = 95M / 10 = 9.5 M$

the cost of cleaning one small debris on GEO :

c3 = infinity

We can make a summary of all unit cost and Speed of different methods in following table 12 and 13:

Table 12: Cleaning Unit Cost of Different Methods

Unit Cost / year	SBL	GBL	EDDE	GEO thruster
LEO Large	∞	∞	0.083	19
LEO Midium	∞	0.003	0.0415	9.5
LEO Small	0.0008	0.0003	∞	∞
GEO Large	∞	∞	∞	19
GEO Midium	∞	∞	∞	9.5
GEO Small	0.00118	∞	∞	∞

Table 13: Cleaning Speed of Different Methods

Cleaned debris / year	SBL	GBL	EDDE	GEO thruster
LEO Large	0	0	282.7	10
LEO Midium	0	8766	565.4	20
LEO Small	17532	87660	0	0
GEO Large	0	0	0	10
GEO Midium	0	0	0	20
GEO Small	13149	0	0	0

4.2.5 Estimation of Poisson parameter α

The parameter α plays an important rule in our model. From the properties of Poisson distribution, we know that $P(X = 1) \sim m_r v_r$, when the value of $m_r v_r$ is very small. Recall that in our model, $v_r = \alpha_r n_r^* + \frac{1}{2} y\tau$, we could get $v_r \propto \alpha_r n_r$.

From the debris data, we have the ratio of debris : small : Midium : large $\approx 100 : 3 : 1$. Since $P(X = 1) \propto v_r$, we just need to get the probability ratio of number of collision between small / Midium / large debris.

We use NASA's Debris Assessment Software(DAS)[17] to get this ratio. DAS can simulate the collision events of object on LEO. We set the start time T = 2010 yr and make this system evolve for 50 years, getting the number of impacts vs altitude(Fig.8) :

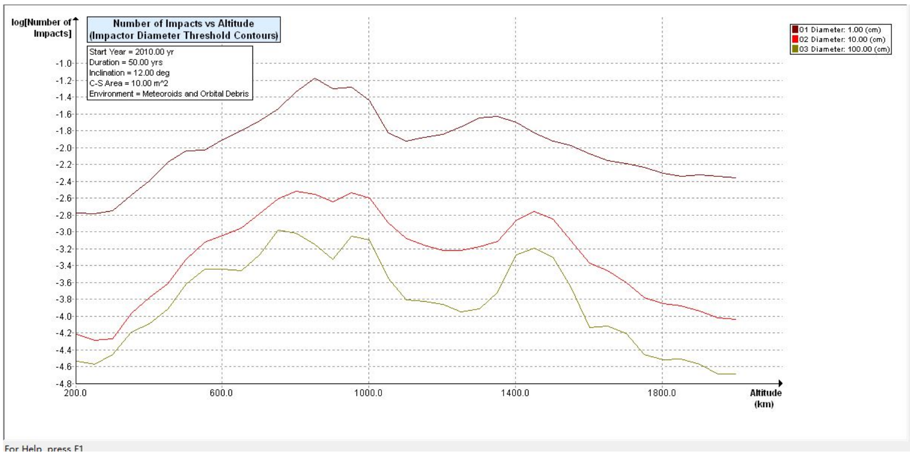

Figure 8: Number of Collisions vs Altitude.(DAS simulation)

from Fig.8, we can roughly get the ratio of number of collision between small/Midium/large is about 100 : 10 : 1. So we have $v_1 : v_2 : v_3 = 100 : 10 : 1$. Recalling that $\alpha_i \propto v_i$, we have:

$$\alpha_1 : \alpha_2 : \alpha_2 = \frac{v_1}{n_1} : \frac{v_2}{n_2} : \frac{v_3}{n_3} = 1 : \frac{1}{3} : 1$$

m,a,n satisfy following equation : $m_r \sum \alpha_r n_r = P_0$. P_0 is the annual collision rate between debris and satellites. According to data in [19], we have $P_0 = 0.06$.

In the last we find the alpha is about 10^{-8}.

4.3 Optimization Model : Finding the Minimal Cost with Required Safety Level

4.3.1 Model Construction

As mentioned in 2.1, action of cleaning debris should be considered separately in two periods. In this standard model, we consider the costs of initial period, the costs and benefits of maintaining period which will last y years. In order to maximize the total income(benefits-costs), we develop the stochastic and linear programming model. This model will be improved and consist more rational elements in SES model.Then in SES model it will become a time-dependent model.

During maintaining period, the process of space collision is Poisson process. The parameter $m_r v_r$ of this Poisson process depends on the amount of three kind of debris and the amount of satellites in each space region :

$$v_r = \alpha_{r1} n_{r1} + \alpha_{r2} n_{r2} + \alpha_{r3} n_{r3}$$

where n_{ri} represent the amounts of each sizes of debris in each areas.

In each year, the number of space collisions in each space region X_r obeys Poisson distribution.

$$P(X_r = k) = e^{-m_r v_r} \frac{m_r v^k}{k!}$$

For every lost satellite caused by collision, the age of it T_{age} obeys the uniform distribution on $(0, T_{expect})$.In order to maintain the space condition, the satellites which retire in this year and new debris should be cleaned. So the costs in this space region for maintaining each year C_{Lr} should satisfy this equation :

$$C_{Lr} = \sum_{j=1}^{X_r} \frac{T_{expect} - T_{age}}{T} x_{sat} + X_r \lambda_1 x_{r1} + X_r \lambda_2 x_{r2} + X_r \lambda_3 x_{r3} + \mu_r x_{r1}$$

Moreover, the action to clean also brings about positive benefits. It prevents some potential collisions. If we don't clean it, in y years the mean parameter $m_r v_r^*$ of Poisson distributions of space collision each year could be predicted as :

$$m_r v_r^* = (\alpha_{r1} n_{r1}^* + \alpha_{r2} n_{r2}^* + \alpha_{r3} n_{r3}^* + \frac{1}{2} \alpha_{r1} \mu_r y) m_r$$

The potential benefits could be calculated below :

$$C_{\mathrm{Pr}} = \sum_{j=1}^{X_r^* - X_r} \frac{T_{expect} - T_{age}}{T_{expect}} x_{sat} + (X_r^* - X_r)\lambda_1 x_{r1} + (X_r^* - X_r)\lambda_2 x_{r2} + (X_r^* - X_r)\lambda_3 x_{r3}$$

Obviously, the mathematical expectation can be calculated, the values of them are :

$$E(C_{Lr}) = \frac{1}{2} m_r v_r x_{sat} + m_r v_r (S_1 x_{r1} + S_2 x_{r2} + S_3 x_{r3}) + \mu_r x_{r1}$$

$$E(C_{\mathrm{Pr}}) = \frac{1}{2} m_r (v_r^* - v_r) x_{sat} + m_r (v_r^* - v_r)(S_1 x_{r1} + S_2 x_{r2} + S_3 x_{r3})$$

Apart from costs and benefits during maintaining period, the costs of initial cleaning should also be taken into consideration. In order to cut down the amounts of debris to the level mentioned above, amount of each size of debris in each region should be cleaned by $n_{ri}^* - n_{ri}$,and for different plans, the R&D cost is$\sum w_j$.

So the cost for initial period is :

$$\sum_{i=1}^{3} \sum_{r=1}^{2} x_{ri}(n_{ri}^* - n_{ri}) + \sum w_j$$

In sum, the total loss of cleaning including initial costs and y years maintaining should be calculated in this way :

$$z = \sum_{i=1}^{3} \sum_{r=1}^{2} x_{ri}(n_{ri}^{*} - n_{ri}) + \sum w_j + y(E(C_{Lr}) - E(C_{\mathrm{Pr}}))$$

In addition, we should also consider the tolerance of funding(not including benefits)each year, because it may be too heavy to support and may get beyond the tolerance of society years later. So a maximum budget for maintaining each should be set. A set of constraint conditions could be set :

$$m_r v_r(\lambda_1 x_{r1} + \lambda_2 x_{r2} + \lambda_3 x_{r3}) + \mu_r x_{r1} \leq u_r$$

Apart from financial restricts, the tolerance of dangers of space collision should also be considered. So the tolerance of $m_r v_r$ should be set :

$$m_r v_r \leq k_r$$

Combining all conditions altogether, we will receive the first MOO model as below:

Object function :

$$z = \sum_{i=1}^{3} \sum_{r=1}^{2} x_{ri}(n_{ri}^{*} - n_{ri}) + \sum w_j + y(E(C_{Lr}) - E(C_{\mathrm{Pr}}))$$

$$\begin{cases} m_r v_r \leq k_r \\ m_r v_r(\lambda_1 x_{r1} + \lambda_2 x_{r2} + \lambda_3 x_{r3}) + \mu_r x_{r1} \leq u_r \end{cases}$$

Implementing stochastic process and linear programming, we will consequently receive a set of optimal solutions, in which we will get the value of n_{ri} when reaching maximal profit. We will base on parameters provided in 4.2 to compute the model and compare several strategies.

4.3.2 Model Results and Comparison

In this part, we are going to use MOO model to optimize for several proposed strategies of cleaning space debris and calculate their minimal loss (or maximum profit) under different collision rate tolerance.

Here propose the strategies waiting for choose :

- Strategy A : EDDE + SBL just for LEO
- Strategy B : EDDE + GBL + SBL + GEO thruster
- Strategy C : EDDE + SBL + GEO thruster

Here we compare the advantages and shortage of each plan in table 14:

Table 14: Advantages and Shortages of Different Strategies

Strategy	Advantage(s)	Shortage(s)
A (EDDE+SBL) for LEO	Least R&D Cost; Efficient for LEO	Can't Solve GEO Problem Completely
B (EDDE+GBL +SBL+GEO thruster)	Can Clean both GEO and LEO efficiently;	Most R&D Cost
C (EDDE+SBL+GEO thruster)	Get a Balance Between R&D Cost and Cleaning Efficiency	Get a Balance Between R&D Cost and Cleaning Efficiency

Result of Strategy A For the different values of different collision rate tolerance., implement MOO model to optimize the best $(n_{ri})_{2\times3}$ for economic results. Strategy A is only capable for cleaning the debris on LEO. For this reason, the solution of strategy A only including LEO. In SES model, we will concentrate on the results of strategy A both on LEO and GEO (Fig.9) :

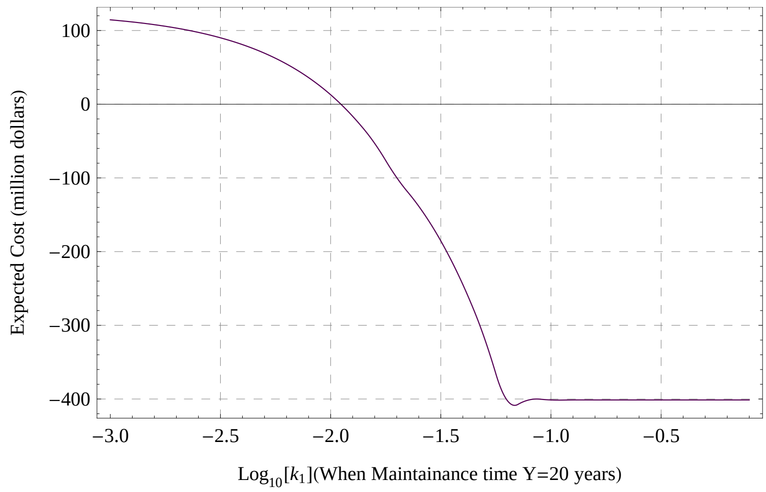

Figure 9: Economic results with different rate tolerance on Strategy A(only LEO) (y=20)

Result of Strategy B Using strategy B, we can clean debris on both LEO and GEO. Optimize the best $(n_{ri})_{2\times3}$ for economic results by MOO model and we can get the results of minimal costs :

For LEO and GEO, the result is Fig.10.

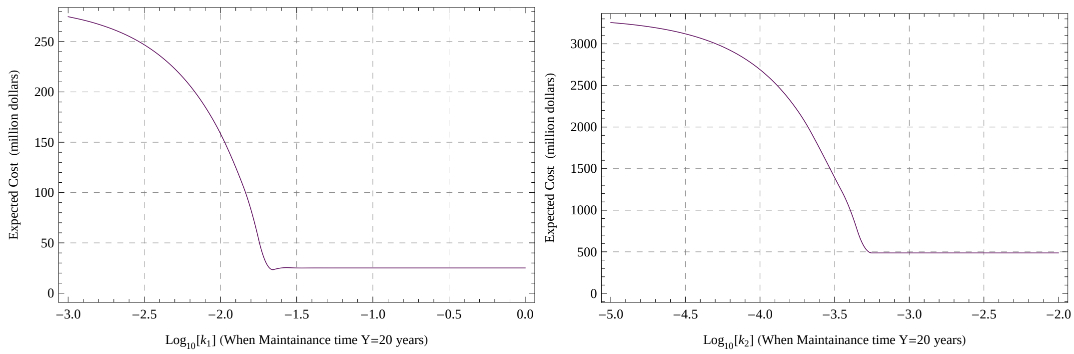

Figure 10: Economic results with different rate tolerance on Strategy B on LEO(left) and GEO(right)(y=20)

Result of Strategy C Using strategy C, we can also clean debris on both LEO and GEO. Optimize the best $(n_{ri})_{2\times3}$ for economic results by MOO model to get the results :

For LEO and GEO, the result is shown in Fig.11:

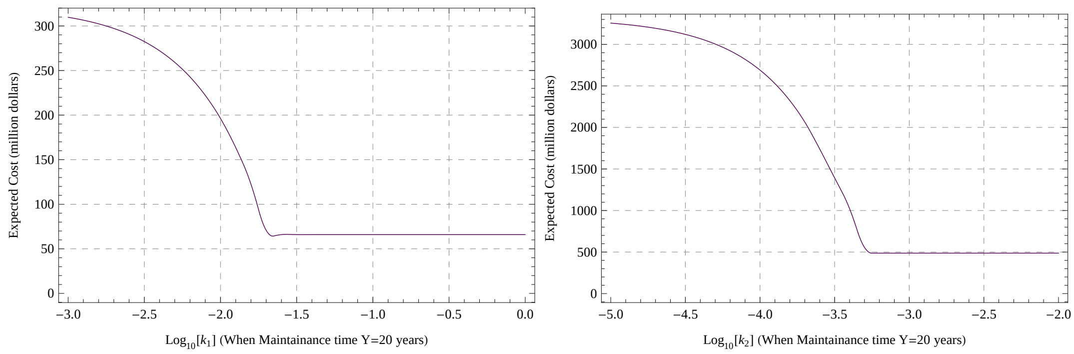

Figure 11: Economic results with different rate tolerance on Strategy C on LEO(left) and GEO(right) (y=20)

Total Cost Now we can put the three set of figures into one set and the contrast becomes visible. From Fig.12 we can figure out the total economic effect of two regions :

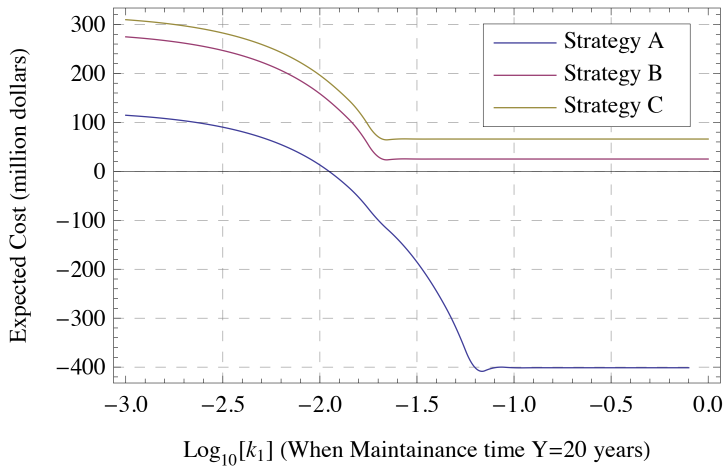

Figure 12: Compare economic results with different rate tolerance on all strategies (y=20)

From this figure we can notice that every strategy has a point of inflexion. When the rate is controlled smaller than this point, the repay goes quickly. But beyond this point, the repay stand still. So this point is just the most appropriate target rate for controlling, considering the safety and economic effect. For each strategy at its own target rate, the target amount of debris of each size in each region could be calculated from the optimization model.

Table 15 and 16 show the LEO and GEO part of

Table 15: LEO part Optimium Endurance Rate Control(T=Y=20)

Strategy Number	Space Collision Rate of LEO	Debris Endurance		
		Large	Medium	Small
A	0.045	3649.74	11039	0
B	0.025	3568.12	0	0
C	0.025	3568.12	0	0

Table 16: GEO part of Optimium Endurance Rate Control(T=Y=20)

Strategy Number	Space Collision Rate of GEO	Debris Endurance		
		Large	Medium	Small
A None	None	None	None	
B	0.0004	78.5292	120	0
C	0.0004	78.5292	120	0

For different length of maintaining consideration, the result is shown in Fig.13:

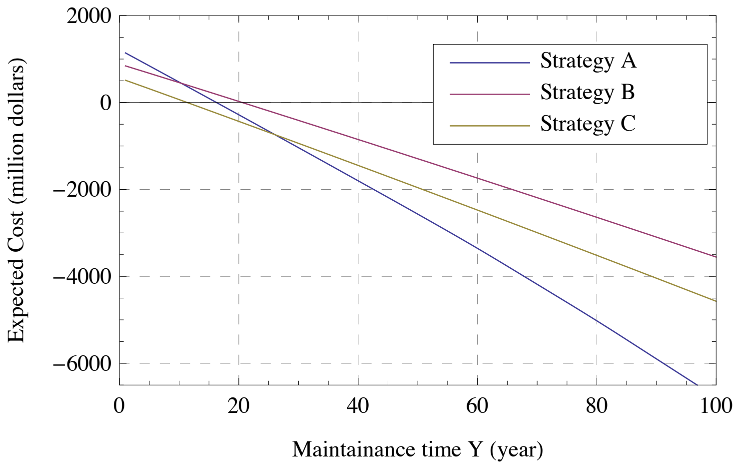

Figure 13: Effects of Duration of Maintenance on LEO Economic results

4.3.3 MOO Model Discussion

When we set maintaining period to 80 years, from Fig.13, its easy to figure out that the most economic attractive strategy is Strategy A. Only in strategy A, the retribution would cover the costs. Strategy B and Strategy C are similar, and the cost of Strategy B is less than that of Strategy C. However, all strategies are acceptable from financial aspect.

In Fig.13 , the optimized economic effects of all strategies are almost linear. The longer the maintaining duration we consider, the more economical attractive strategies are.

The optimization model gives us reasonable collision rate and the target amount of debris, which are pretty useful in our later research. Different control rate results in different economic effect and the standard of each strategies become visible from our model. However, the model still needs to be improved, such as the long maintaining duration in consideration might result in large deviation. Moreover, the time dependent model needs to be created for deeper analysis of initial period and maintaining period. So we developed the another model to analyse more accurately.

Stochastic model will be utilized in the following models.

4.4 Stochastic Evolution Simulating model(SES model)

4.4.1 Free Evolution Model

In this part, we will develop a model to simulate the condition of debris in space when no method applied for cleaning them. It is obvious that the debris would increase in future years. In each year, the amount of new launched satellites $M_{r,lau}$ obey normal distribution $N(\mu_r, \sigma_r^2)$. So for every year, the satellite change :

$$m_{r,next} = m_{r,this} + M_{r,lau} - M_{r,ret}$$

Y_r names the number of collision of scrapped satellites (not satellites in use).It obeys the Poisson distribution and the parameter of it is $n^*_{r1,this}(\alpha_{r1}n^*_{r1,this}+\alpha_{r2}n^*_{r2,this}+\alpha_{r3}n^*_{r3,this})$. And amount of each size of debris caused by the collision S_{ij} also obey each normal distribution. So we can describe the change of each size of debris :

$$n^*_{r1,next} = n^*_{r1,this} + M_{r,lea} + S_1X_r - Y_r + \sum_{j=1}^{Y_r} S_{1j}$$

$$n^*_{r2,next} = n^*_{r2,this} + S_2X_r + \sum_{j=1}^{Y_r} S_{2j}$$

$$n^*_{r3,next} = 1.03n^*_{r3,this} + S_3X_r + \sum_{j=1}^{Y_r} S_{3j}$$

And we should calculate the potential loss until this year :

$$Q_{zr} = x_{r1}(n^*_{r1} - n_{r1}) + x_{r2}(n^*_{r2} - n_{r2}) + x_{r3}(n^*_{r3} - n_{r3})$$

Simulate the process each year and print vector every year :

$$(m_r, n^*_{r1}, n^*_{r2}, n^*_{r3}, Q_{zr}, m_r(\alpha_{r1}n^*_{r1} + \alpha_{r2}n^*_{r2} + \alpha_{r3}n^*_{r3}), X_r, Y_r)$$

4.4.2 Model With Cleaning Evolution

In MOO model, we estimate the total economic effects of each strategy under each financial restricts. The solutions $(n_{ri})_{2\times3}$ calculated in MOO model should be used as standards in the process of simulation in time dependent model. Comparing with MOO model, SES model considers enough about the time dependence of initial period and maintaining period. Including stochastic elements in simulation, SES model predicts the rate of space collision and economic effects in future y years. In addition, simulating the tendency of space debris without any cleaning methods and using the statistics from the prediction in simulation, we can also predicts the economic effects when each strategy starts from different year.

Lets start with the vector of the condition in starting year :

$$(m_r, n^*_{r1}, n^*_{r2}, n^*_{r3}, z_{R\&D})$$

Consider the vector for each year under each strategy. For feasibility, in initial period, the debris on LEO should take the first and the speed of cleaning should be as fast as possible :

Step1 : In each year, the amount of new launched satellites $M_{1,lau}$ obey normal distribution $N(\mu_1, \sigma_1^2)$. So does the amount of new retired satellites $M_{1,set}$ in this year.

$$m_{1,next} = m_{1,this} + M_{1,lan} - M_{1,ret}$$

$M_{1,lea}$ means the amount of new produced scrapped satellites which leave in orbit. $M_{1,lea}$ obey the normal distribution $N(\mu_1^0, \sigma_1^0)$. Y_1 names the number of collision of scrapped satellites (not satellites in use).It obeys the Poisson distribution and the parameter of it is $n^*_{11,this}(\alpha_{11}n^*_{11,this} + \alpha_{12}n^*_{12,this} + \alpha_{13}n^*_{13,this})$. And amount of each size of debris caused by the collision S_{ij} also obey each normal distribution. So we can describe the change of each size of debris :

$$n^*_{11,next} = n^*_{11,this} + M_{1,lea} + S_1X_1 - n^*_{11,clean} - Y_1 + \sum_{j=1}^{Y_1} S_{ij}$$

$$n^*_{12,next} = n^*_{12,this} + S_2X_1 - n^*_{12,clean} + \sum_{j=1}^{Y_1} S_{2j}$$

$$n^*_{13,next} = 1.03n^*_{13,this} + S_3X_1 - n^*_{13,clean} + \sum_{j=1}^{Y_1} S_{3j}$$

Calculate the cost for this year :

$$z_1 = x_{11}n^*_{11,clean} + x_{12}n^*_{12,clean} + x_{13}n^*_{13,clean}$$

NOTE : In each strategy, the facilities need to be replaced every ten years. So when the year number ,some fee need to be added:

$$z_1 = x_{11}n^*_{11,clean} + x_{12}n^*_{12,clean} + x_{13}n^*_{13,clean} + fee$$

Simulate the process each year and print vector :

$$(m_1, n^*_{11}, n^*_{12}, n^*_{13}, z_1, m_1(\alpha_{11}n^*_{11} + \alpha_{12}n^*_{12} + \alpha_{13}n^*_{13}), X_1, Y_1)$$

every year. In addition, the cost should be added into total funding every year. And calculate the time. When $n^*_{11} \leq n_{11}, n^*_{12} \leq n_{12}, n^*_{13} \leq n_{13}$, we stop the step1 and then begin the next step. During the first step, the debris on GEO also changes in a free condition. So we also ,need to simulate this process.

$$n^*_{21,next} = n^*_{21,this} + M_{2,lea} + S_1X_2 - Y_2 + \sum_{j=1}^{Y_2} S_{1j}$$

$$n^*_{22,next} = n^*_{22,this} + S_2X_2 + \sum_{j=1}^{Y_2} S_{2j}$$

$$n^*_{22,next} = n^*_{22,this} + S_2X_2 + \sum_{j=1}^{Y_2} S_{2j}$$

Then print the vector :

$$(m_2, n^*_{21}, n^*_{22}, n^*_{23}, z_2, m_2(\alpha_{21}n^*_{21} + \alpha_{21}n^*_{21} + \alpha_{21}n^*_{21}), X_2, Y_2)$$

Step2 : In step2, we also need to consider LEO and GEO separately. For LEO, the cleaning work turns into maintaining period and what we need to do is to control the collision rate. For GEO, the initial period starts.

1.maintain the LEO : The changes of satellites and each size of debris each year are below :

$$m_{1,next} = m_{1,this} + M_{1,lau} - M_{1,ret}$$

$$n^*_{11,next} = n^*_{11,this} + M_{1,lea} + S_1X_1 - Y_1 + \sum_{j=1}^{Y_1} S_{1j}$$

$$n^*_{12,next} = n^*_{12,this} + S_2X_1 + \sum_{j=1}^{Y_1} S_{2j}$$

$$n^*_{13,next} = 1.03n^*_{13,this} + S_3X_1 + \sum_{j=1}^{Y_1} S_{3j}$$

The debris would increase, so the maintaining work needs to control the collision rate. So if

$$m_1(\alpha_{11}n^*_{11} + \alpha_{12}n^*_{12} + \alpha_{13}n^*_{13}) > k_1$$

Then measures should be launched to cut down the debris to keep each sizes of debris under the level :

$$n^*_{11}, n^*_{12}, n^*_{13}$$

The cost for maintaining(if maintaining work occurs in this year) is

$$z_1 = x_{11}n^*_{11,clean} + x_{12}n^*_{12,clean} + x_{13}n^*_{13,clean} + fee$$

The cost should be added into total funding. And for each year we print the vector :

$$(m_1, n^*_{11}, n^*_{12}, n^*_{13}, z_1, m_1(\alpha_{11}n^*_{11} + \alpha_{12}n^*_{12} + \alpha_{13}n^*_{13}), X_1, Y_1)$$

2.GEOs initial period Very similar to the initial period of LEOs initial period in step1, the change of satellites and debris on GEO are :

$$m_{2,next} = m_{2,this} + M_{2,lau} - M_{2,ret}$$

$$n^*_{21,next} = n^*_{21,this} + M_{2,lea} + S_1X_2 - n^*_{21,clean} - Y_2 + \sum_{j=1}^{Y_2} S_{1j}$$

$$n^*_{22,next} = n^*_{22,this} + S_2X_2 - n^*_{22,clean} + \sum_{j=1}^{Y_2} S_{2j}$$

$$n^*_{23,next} = 1.03n^*_{23,this} + S_3X_2 + \sum_{j=1}^{Y_2} S_{3j}$$

Calculate the cost for this year :

$$z_2 = x_{21}n^*_{21,clean} + x_{22}n^*_{22,clean} + x_{23}n^*_{23,clean} + fee$$

Then print the vector :

$$(m_2, n^*_{21}, n^*_{22}, n^*_{23}, z_2, m_2(\alpha_{21}n^*_{21} + \alpha_{22}n^*_{22} + \alpha_{23}n^*_{23}), X_2, Y_2)$$

When $n^*_{21} \leq n_{21}, n^*_{22} \leq n_{22}, n^*_{23} \leq n_{23}$, we stop the step2 and then begin the final step.

Step3 : For both regions, the work turn into maintaining period. Similar to step2, the changes in each year are :

$$m_{r,next} = m_{r,this} + M_{r,lau} - M_{r,ret}$$

$$n^*_{r1,next} = n^*_{r1,this} + M_{r,lea} + S_1 X_r - Y_r + \sum_{j=1}^{Y_r} S_{1j}$$

$$n^*_{r2,next} = n^*_{r2,this} + S_2 X_r + \sum_{j=1}^{Y_r} S_{2j}$$

$$n^*_{r3,next} = 1.03 n^*_{r3,this} + S_3 X_r + \sum_{j=1}^{Y_r} S_{3j}$$

To control the debris below the collision rate. So if

$$m_r(\alpha_{r1} n^*_{r1} + \alpha_{r2} n^*_{r2} + \alpha_{r3} n^*_{r3}) > k_r$$

We should cut down the debris to keep each sizes of debris under the level :

$$n^*_{r1}, n^*_{r2}, n^*_{r3}$$

The cost for maintaining(if maintaining work occurs in this year) is

$$z_r = x_{r1} n^*_{r1,clean} + x_{r2} n^*_{r2,clean} + x_{r3} n^*_{r3,clean} + fee$$

For each year we print the vector of both regions :

$$(m_r, n^*_{r1}, n^*_{r2}, n^*_{r3}, z_r, m_r(\alpha_{r1} n^*_{r1} + \alpha_{r2} n^*_{r2} + \alpha_{r3} n^*_{r3}), X_r, Y_r)$$

4.4.3 Result of Free Evolution Model

In simulating, we can predict the complete information about the debris conditions on GEO and LEO by the model in 4.4.1.

The relational graph of collision rate and year in GEO and LEO(Fig.14) :

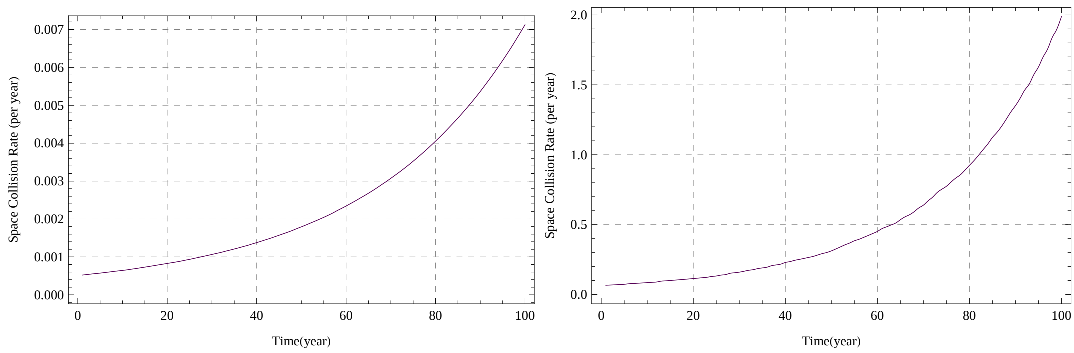

Figure 14: Free Evolution result of collision rate on GEO(left) and LEO(right)

The collision rate increase quickly. So from this figure we can infer that if no influential method taken to clean the debris, the accidents on orbit will be much more frequent in future.

The relational graph of economic effect and year in GEO and LEO(Fig.15):

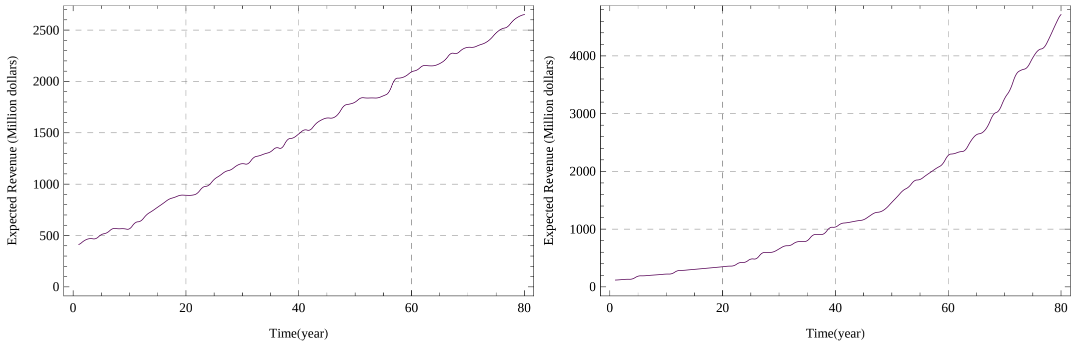

Figure 15: Free Evolution result of Potential Loss on GEO(left) and LEO(right)

The potential loss caused by the increase of debris grows sharply after 20 years. So we shouldnt wait too much time to start a strategy cleaning the debris.

The space collision each year(Fig.16):

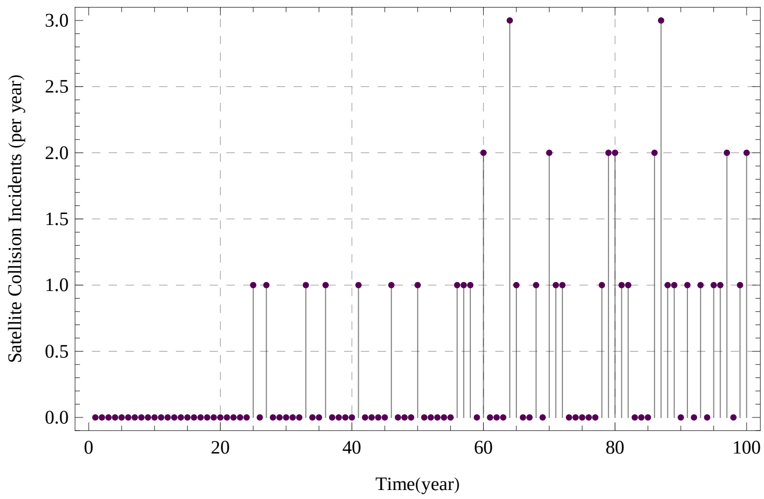

Figure 16: Free Evolution result of collision incidents on LEO

The space collision will happen gradually frequent in future. The safety problem of the orbit will become much more urgent in future if no method implemented.

4.4.4 Result of Model with Cleaning

In evolving simulating, we will predict every year conditions and compare the economic effects and collision rate. For different starting year, the result of simulating differs. And for different tolerant rate of space collision, the result also differs :

Strategy A Start the strategy immediately, after 10 years or after 20 years. The return of the method would show up along with the cleaning work.

The relational graph of collision rate and year(Fig.17) :

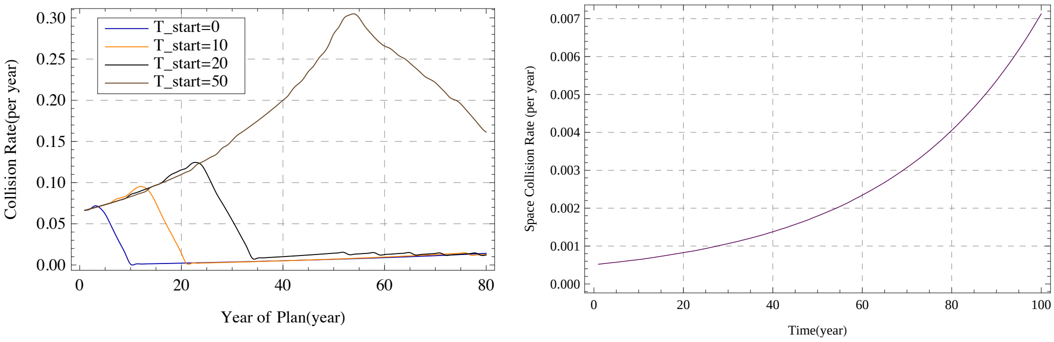

Figure 17: Strategy A on LEO(left) and GEO(right) : Change of Collision rate with different project launch time

The relational graph of economic effect and year(Fig.18) :

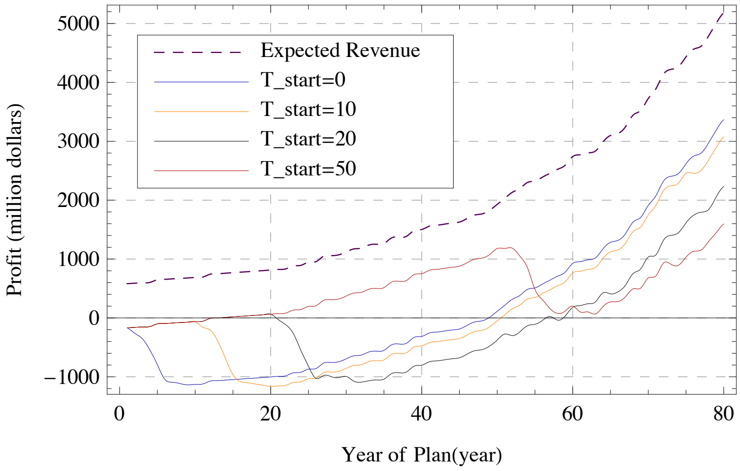

Figure 18: Strategy A on LEO : Total profit at certain year with different satellite launch time (Initial Status based on data of year 2015)

Table 17: Strategy A LEO

T_{start}	Time to Recoup Investment	Total Profit/M$ (for 80 Year Plan)	Total Profit/M$ (for 50 Year Plan)
0	49	3365	113
10	51	3074	-49.3
20	57	2233	-376
50	75	501	None

Strategy B Start the strategy immediately, after 10 years or after 20 years. The return of the method would show up along with the cleaning work. The relational graph of collision rate and year is Fig.19.

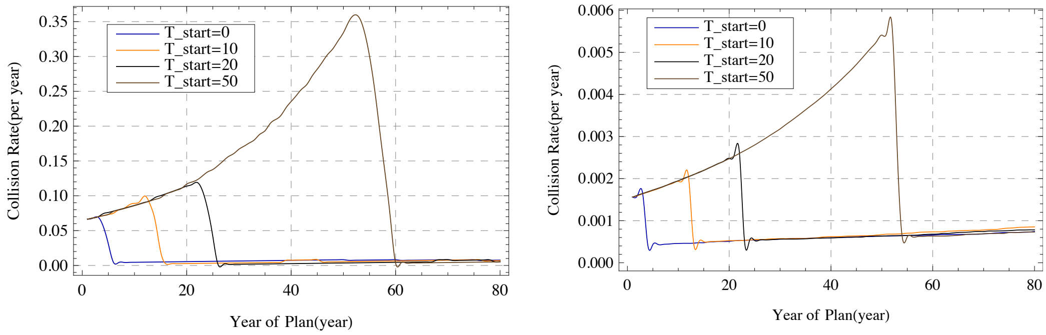

Figure 19: Strategy B on LEO(left) and GEO(right) : Change of Collision rate with different project launch time

The relational graph of economic effect and year is Fig.20.

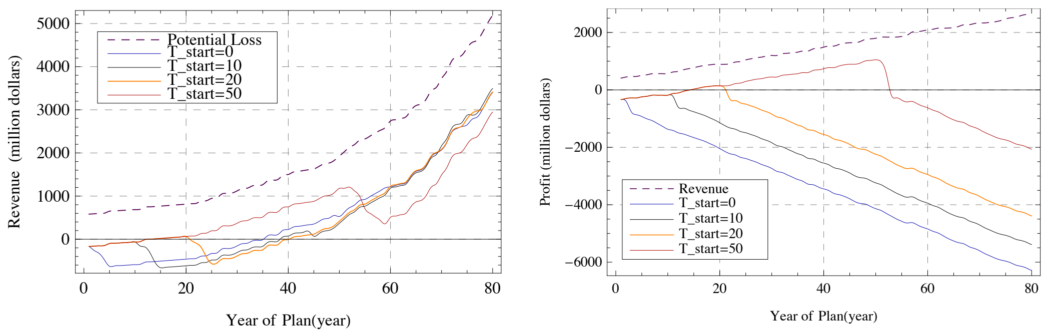

Figure 20: Strategy B on LEO(left) and GEO(right) : Total Profit at certain year with different project launch time (Initial Status is based on data of year 2015)

Table 18: Strategy B LEO

T_{start}	Time to Recoup Investment	Total Profit/M$ (for 80 Year Plan)	Total Profit/M$ (for 50 Year Plan)
0	36	3400	527
10	39	3411	3494
20	40	3411	419
50	Instantly	2946	None

Table 19: Strategy B GEO

T_{start}	Time to Recoup Investment	Total Profit/M$ (for 80 Year Plan)	Total Profit/M$ (for 50 Year Plan)
0	Never	-3755	-4008
10	Never	-2857	-3110
20	Never	-1860	-2112
50	70	464	None

4.4.5 SES model Result Discussion and Model Analysis

In first three solutions, for both Strategy A&B, the collision rate will be cut down sharply in initial period. However, if starting time is 50 years later, the collision rate will achieve a high level and become extremely hard to cut down. From financial aspect, the costs will be high during initial period. Then the positive effects will show up and recover the cost years later.

Results of Strategy A The GEO condition is the same with free evolving condition. So we should mainly consider the LEO in Strategy A. From the left part of Fig.17, we can find it is unacceptable to wait for 50 years to start the figure because of the high collision rate. Combined with Fig.18, we can find that the best solution is to wait 10 years to start the project. Here are the reasons :

- Quickly recover the cost;
- Most considerable economic effects
- Very low collision rate

However, the collision rate on GEO would increase quickly. So the Strategy A should only work for no more than 80 years. After 80 years, new project needs to be launched to replace this strategy.

Result of Strategy B The right part of Fig.19 and right part of Fig.20 shows that the costs for cleaning debris on GEO are heavy, but all solutions can keep the collision rate on GEO in a very low rate.

From the left part of Fig.19, we can find the collision rates of first three solutions keep low. Combined with the left part of Fig.20, we can find the solution to wait 20 years to start the project is the best. Here are the reasons :

- Quickly recover the cost;
- Low cost on GEO
- Very low collision rate

Compare Strategy A&B For both A and B, we choose the best solutions. Now we should compare these two strategies.

- Strategy A avoid heavy cost on GEO; Strategy B spend much on it
- Strategy A and B have similar future Profit on LEO
- Strategy A and B have similar collision rate in future
- Strategy A discuss the GEO, and so increase the potential risk of this region. For this reason, Strategy A couldnt work more than 80 years and new heavy problem may arise in future.

To sum up, Strategy A is more efficient to solve urgent problem of LEO debris. However, Strategy B can prevent the more serious problem in future (the costs for solving GEOs problem are much heavier than LEOs).

SES model simulate the process of cleaning debris under the guidance of each strategy. The prediction and stochastic process are significant components for developing this time dependent model. The results are valuable for making decision to choose strategy and starting time.

The results of SES model are reasonable. The initial funding will be high because we should quickly clean the debris which have accumulated for so many years. On the other hand,, the advantage of such action is that we can control the collision rate in a low level for future, so that we can prevent potential loss caused by accumulation of amount of debris number.

4.4.6 Analysis of Risk

There are following potential risks of debris cleaning operation.

For laser cleaning :

1. The laser cleaning method strongly depends on accurate positioning, which will be affected by weather or some mechanical breakdown. The mis-targeting of the laser may makes it hit some 'good' objects, which will cause economical loss.

2. High-power laser jet can also be used as a anti-satellite weapon. This may cause some political concern and make it hard to allowed by governments. This effect is very hard to estimate, thus we only consider it qualitatively.

For mechanical capture:

1. The EDDE will throw space debris into atmosphere, and those big debris may hit the ground and cause economical loss. We could show this effect in our model by increase the cost of EDDE capture.

2. EDDE needs to get very close to the debris to be able to capture it, which makes it more likely for a collision between EDDE and the debris to happen.We could add a sudden increase of debris number in our model to reflect this effect.

4.5 Exploration and Answer of What if

4.5.1 What if ISS explode?

International Space Station(ISS) is a habitable satellite on LEO. It's has a length of 72.8m , a width of 108.5m and a height of 20m. Let's suppose there is a anti-satellite missile hit it and blows up ISS(it's just a crazy imagine). In addition to save astronauts in ISS, we need to clean a huge amount of space debris causing by this explosion.

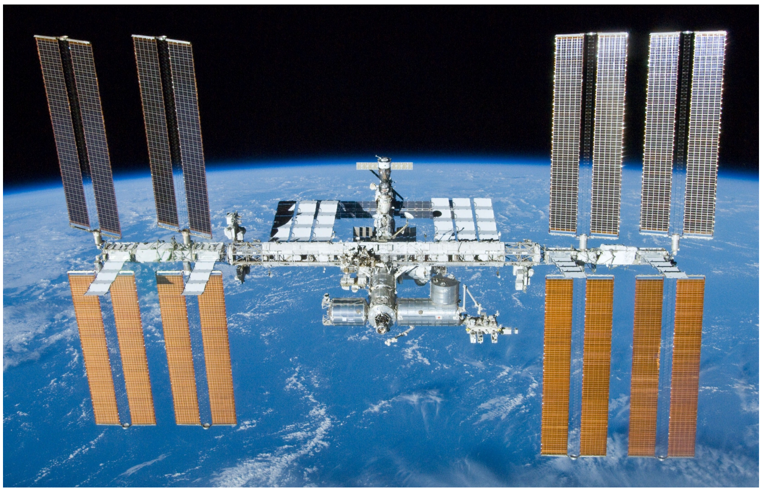

Figure 21: The International Space Station on 23 May 2010 (source : [18])

In our model, this means a big sudden increase of number of space debris on LEO. We estimate the exact numbers by following way:

1. Considering the shape of the ISS, we suppose that there is about 1/20 of it's length height * width, so we have the volume of ISS :

$$V_{ISS} = 72.8 \times 108.5 \times 20 \ \div 20 = 7812m^3$$

2. According to the size of different debris above, we set debris sizes as : small 5cm, medium 50cm, large 2.5m. Then we have the volume of debris :

$$V_{small} = 0.000125m^3, \qquad V_{medium} = 0.125m^3, \qquad V_{large} = 15.625m^3$$

. 3.We suppose the number ratio of debris produced by explosion of ISS is about the same to size ratio of debris, which is small : medium : large = 50 : 5 : 1

By solving following equations :

$$\begin{cases} \sum_i n_i V_i = 7812m^3 \\ n_{small} : n_{medium} : n_{large} = 50 : 5 : 1 \end{cases}$$

we get $n_{small} = 24027.7, n_{medium} = 2402.8, n_{large} = 480.6$.

Now we can use SES model to simulate the process and then answer the question. Considering this cleaning operation is just another "initial" process in SES model, we just need to change the number of debris in the year of collision. For simplicity, we suppose the collision happen in 30th year of the start of SES model.(The initial process of SES model lasts 20 years.)

Simulate the process and compare it with normal process of Strategy B (Fig.22):

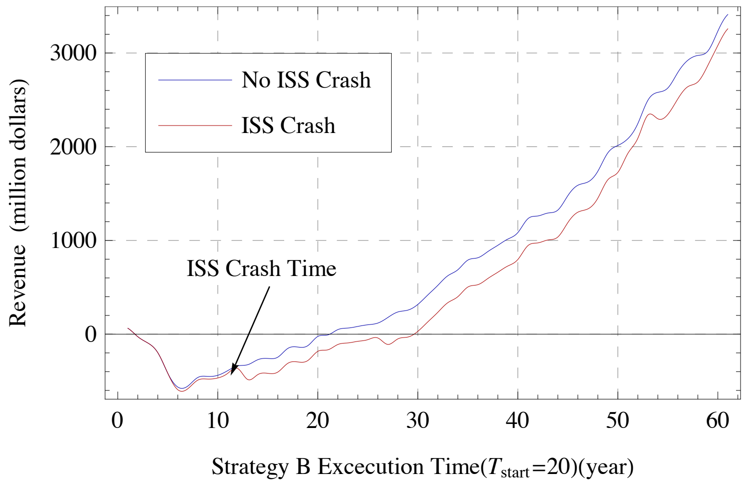

Figure 22: Effects on profit after a severe crash scenario

From the figure we can find that the crash would cause some loss immediately. However, the loss will be recovered years later and the later process would be very similar to the normal one.

4.5.2 Avoid Collision

Though our strategy can clean most debris on LEO, it's still better to have avoiding collision methods as backup.

We considered following methods :

1. We can track the location of debris and make collision warning in advance.
2. Satellites should install protective device around it's core part to protect it from damaged by colliding with small debris.
3. Satellites can add a EDDE-like Electrodynamic moving part in order to change their orbit convenient.

5 Sensitivity Analysis

The optmization model and SES model depend on some significant parameters. So the fluctuation of these parameters may lead to variety of the model results.

5.1 Space Collision Factor α_{ri}

The parameter α_{ri} (Space collision factor caused by single debris) decides the collision rate. Collision rate also influence the costs. So we use Strategy B (start after 20 years) to consider the parameter α_{ri} (Space collision factor caused by single debris) first.

5.1.1 Fluctuation of α_{1i}

The estimation of α_{1i} is $(10^{-8}, 10^{-9}, 10^{-10})$. Now we consider $0.1 \times (10^{-8}, 10^{-9}, 10^{-10}), 0.5 \times (10^{-8}, 10^{-9}, 10^{-10}), 1 \times (10^{-8}, 10^{-9}, 10^{-10}), 5 \times (10^{-8}, 10^{-9}, 10^{-10}), 10 \times (10^{-8}, 10^{-9}, 10^{-10})$ in SES model and compare the collision rates figure of them(Fig.23).

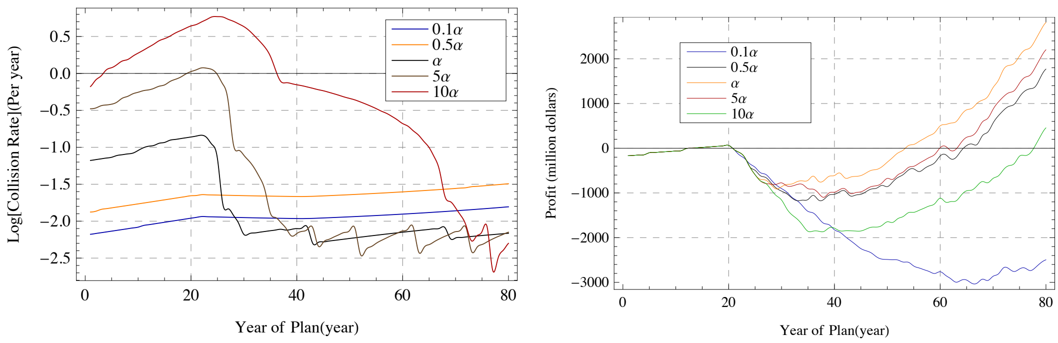

Figure 23: Collision rate(left) and profit(right) change with different $\vec{\alpha}_1$ (For Strategy B on LEO with a 20-year delay)

From Fig.23 we can find that in initial period, the fluctuation of collision rate caused by fluctuation of α_{1i} will be very sharp. Especially when α_{1i} is $10 \times (10^{-8}, 10^{-9}, 10^{-10})$, on LEO, the collision rate would be 3 times per year which is extremely unacceptable.

Except for $0.1 \times (10^{-8}, 10^{-9}, 10^{-10})$, in other situations the economic effects would be similar.

5.1.2 Fluctuation of α_{2i}

On GEO, let us consider the fluctuation of α_{2i}. The same as before, we also consider following value of α_{2i} : $0.1 \times (10^{-8}, 10^{-9}, 10^{-10}), 0.5 \times (10^{-8}, 10^{-9}, 10^{-10}), 1 \times (10^{-8}, 10^{-9}, 10^{-10}), 5 \times (10^{-8}, 10^{-9}, 10^{-10}), 10 \times (10^{-8}, 10^{-9}, 10^{-10})$.

From SES model we get following collision rates comparison figure.(Fig.24)

The initial collision rates would fluctuate sharply in initial period, very likely to LEO, collision rate on GEO depend a lot to the accuracy of α_{2i} .

The result of this change are shown in following Fig.24 .

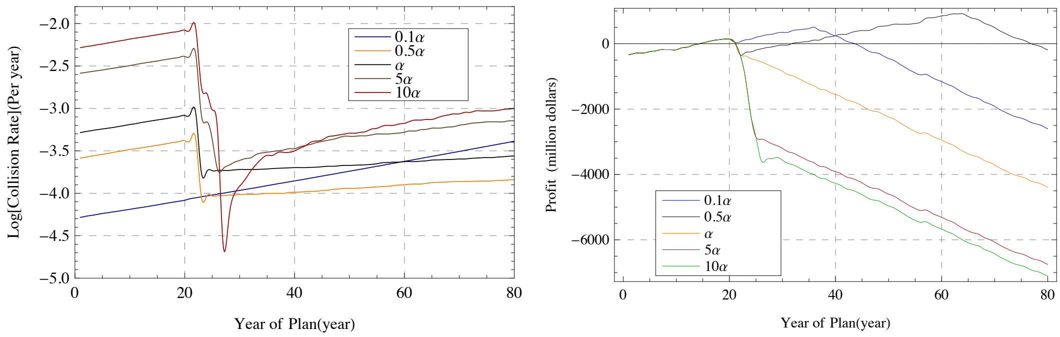

Figure 24: Collision rate(left) and profit(right) change with different $\vec{\alpha}_2$ (For Strategy B on GEO with a 20-year delay)

The economic effects would fluctuate sharply in maintaining period, so on GEO the economic effect relied much on the accuracy of α_{2i}.

5.2 Growth Rate of New Satellite η_r

η_r decides the amount of satellites on orbit every year. So we use Strategy B (start after 20 years) to consider the parameter η_r.

5.2.1 Fluctuation of η_1

The esatimation of η_1 is 103.5 . Now we consider η_1 = 83.5, 103.5, 123.5 in SES model and compare the collision rates figure of them. The results are shown in Fig.25 .

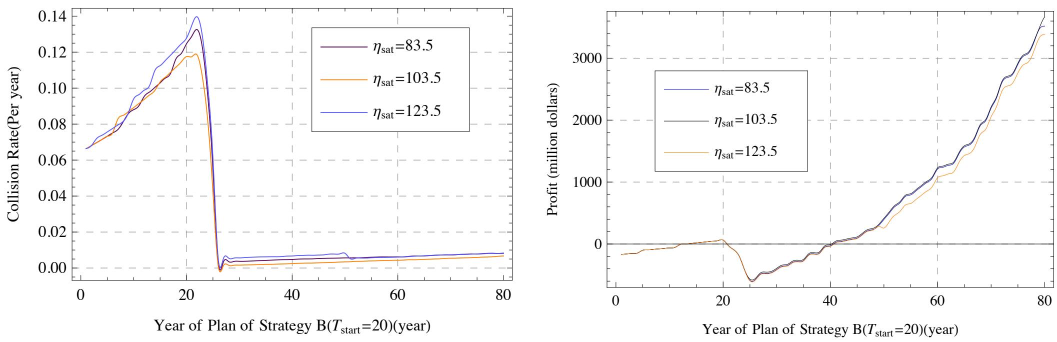

Figure 25: Collision rate change(left) and profit change(right) with different η_1 (For Strategy B on LEO with a 20-year delay)

Both figures show that for all values of η_1 the collision rates every year are very similar. Different η_1 result in almost the same economic effect. So simulation for LEO relies very little on η_1.

5.2.2 Fluctuation of η_2

On GEO, let us consider the fluctuation of η_2 . We set η_2 = 45.9, 55.9, 65.9 seperately.

Following Fig.26 shows the result of this.

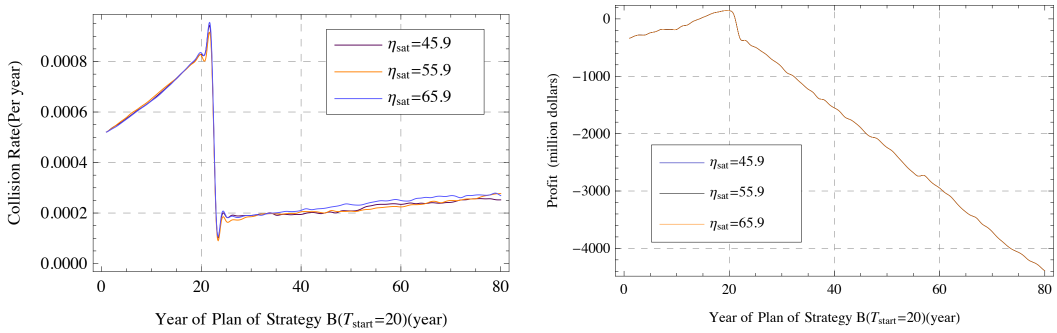

Figure 26: Collision rate change(left) and profit change(right) with different η_2 (For Strategy B on LEO with a 20-year delay)

Similar to LEO, the collision rate and economic effect rely very little on η_1.

6 Conclusion

6.1 Future Debris Trajectories

We have developed an MOO model and SES model. From our models we get the cost of each strategy and simulate the collision rate. For every strategy, restricted by tolerance rate of collision, the initial cost would be high and the maintaining cost would be cheaper. Compared with Strategy B, Strategy C costs more and has no advantages. So we won't choose Strategy C. Strategy A only concentrate on solving the problem on LEO and sharply cut down the collision rate. The prices of technologies in Strategy A are cheaper, so from simulating results we can find that Strategy A is efficient to solve collision problems with higher return. On the contrast, though Strategy B also perform well in solving LEO problems, it spend much on GEO to control the collision rate. For this reason, the funding of Strategy B is higher.

Time is also decisive in choosing the strategies. From our model we can find Strategy A needs to wait 10 years to launch and Strategy B needs 20 years. In addition, Strategy A need 30 years to recover the cost while Strategy B need 40 years. However, it only takes 20 years for Strategy B to recover the cost on LEO while the cost on GEO would not be recovered in 50 years.. However, for the reason that the costs for cleaning debris on GEO are much higher, so Strategy A would result in much more serious problem on GEO in future 70 years. If GEOs debris accumulate, it would cost too much for our descendants to clean. Strategy B will succeed in control the increase of debris on GEO, though spend high in initial period.

6.2 Recommened Strategy

We have compared the strategies from the aspects of cost, collision rate, time and future. To sum up, though efficient at the beginning and quick to recover the cost, Strategy A would lead to a much more serious problem in future which would be too hard to solve. For this reason, we recommend to take Strategy B for cleaning the debris. However, we should wait for 20 years to start this project for maximum the return and avoid unnecessary cost. After launching the project, it will take nearly 40 years to recover the cost and top deficit will be about 4 billion dollars. It seems hard for a company to endure deficit for 40 years. Nevertheless, if we only consider LEO project, Strategy B will have a better performance: it will get payback in 20 years and maximal deficit will only be about 1 billion dollars. So we finally recommend several related companied to cooperate for this project, especially on LEO orbit project, so the pressure would be not so heavy.

To sum up, the project is pretty attractive for commercial firms. The contract would repay more if the length of time is longer.

6.3 Strengths & Weaknesses

6.3.1 Strengths

1. Consider both safety of orbit and commercial value of the methods
2. The model balance the integrity and accuracy. Simulate the process of orbit conditions without heavy computation complexity.

3. Our model has comprehensive physics background. All parameters are estimated based on authoritative theses and official statistics. The result is promising and valuable.

4. Every set of results will be optimized and the directive opinions could be provided. The potential elements and uncertain elements are considered.

6.3.2 Weaknesses

1. In SES model, we take the speed of producing the facilities during initial period as a constant value. However, the speed of producing may vary in a range actually;

2. The distribution of the amount of new satellites every year is constant. However, the requirement for satellites may increase in future.

6.4 Future Work

1. We can use stochastic dynamic programming for solving the first weakness named in last paragraph, so that we can consider more detailed about the producing process.

2. The requirement for satellites could be estimated from the level of economy, science and technology of each country on the earth. The statistics about the space industry of each country could help us to develop a model for solving this problem.

3. We can divide the space orbits into more regions and consider them in a more detailed situation.

4. Consider the decrease of the prices of facilities in future, and give more *What If* about the change of prices.

7 Executive Summary to high-level policy makers and news media analysts

There is a growing concern on space debris issue. Both government and commercial satellite industries are seeking for a solution to create a safe space zone for satellites. This report provides an analysis and evaluation on the prospective availability and profitability.

According to NASA's comment, there are approximately 20,000 trackable large space debris and 500,000 small objects with diameter larger than 1cm. Based on the evolutionary simulation of collision rate and debris amount on orbit, there will be over 2 million space debris after 50 years. Moreover, incident rate will raise sharply, as 3 times after 50 years, which indicates a severe collision impact for every five years. There are still over thousands of tracked debris after *2009 Satellite Collision Incident*. If no manners of mitigation of the problem executed in next 50 years, governments cannot afford the risk and potential loss of valuable satellites.

Although the cleaning method is not realized, this report draws a promising conclusion after considering about duration of paybacks and consecutive plans by several stages. Based on present relative researches, four programs are likely to be applied in this problem on LEO(Low Earth Orbit) and GEO(Geostationary Earth Orbit) in next following decades :

- EDDE: For large debris on LEO
- Ground-Based Laser (GBL): For small and medium debris on LEO
- GEO thruster: For large debris on GEO
- Sky-Based Laser (SBL) : For small debris on both GEO and LEO

The report mainly focuses on three strategies:

- A: EDDE + SBL only on GEO
- B: EDDE + GBL for LEO; GEO thruster + SBL on GEO
- C: EDDE + SBL for LEO; GEO thruster + SBL on GEO

Thus, based on simulation and valid prediction, the best solution might be the combination of all four cleaning methods i.e. Strategy B). There are three phases to execute the plan:

- Wait 20 years before launching the project
- launch the program and quickly control the collision risk to a settled point
- maintain the collision rate constantly

This can be applied on LEO and function well. However, report reveals that none of the solution would create lucrative opportunity on GEO, and strategy B&C might waste some expenditure if applied on GEO part in 80 years.

Comparison of three strategies are mainly covered in the report. Since we cannot solve out the problem on LEO or GEO completely without combination of raised methods, the combination should be considered. Therefore the report proposed three mixed solution. Moreover, although this illustrated plan will require relatively larger amount of researching funding, it will have a lower cost for clean-up of small scatters compared with Strategy A and Strategy C (thanks to SBL project), which, in long term, could save tons of expenditure. Meanwhile, the profit comes from the meet of decrease of space debris on orbit, and the initial time for clean-up plan can seriously influence the total interest for a fixed strategy. Based on comparison, a 20-year delay on LEO is reasonable as well as lucrative, and the blank time can be used to research, test and implement the technique for advent of commercial opportunities.

If LEO part of Strategy B is taken after 20 years, according to estimation in the report, the company will face approximately one billion dollars instant loss at first six years after the clean-up program launched. The company will receive payback at 20th years after Strategy B is launched.

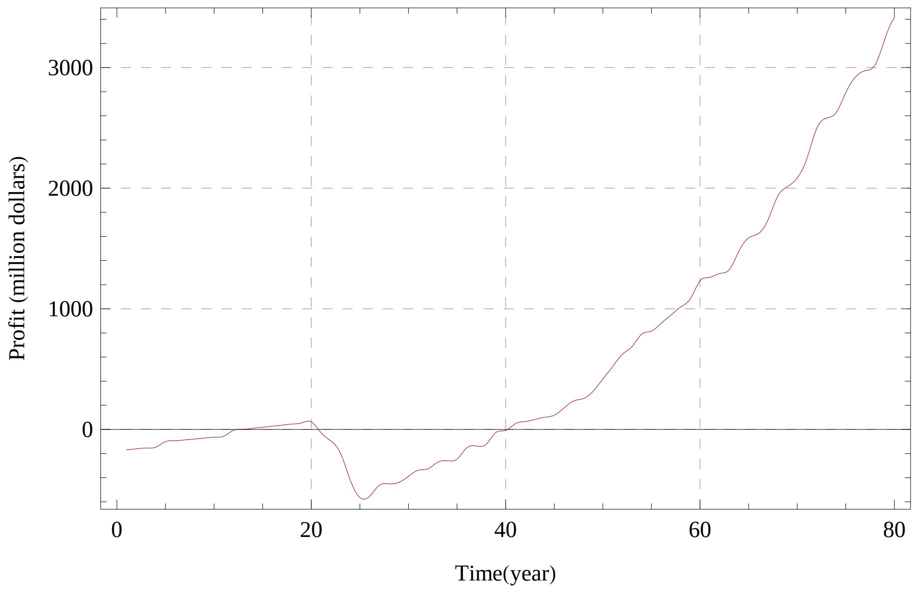

Figure 27: Net Profit Estimation about Strategy B on LEO(launched after 20 years)

In addition, the report indicates the estimation about collision rate severely impacts the commercial plan, and some artificial or man-made collision will increase expenditure of maintenance and decrease the expected profit. Therefore, it is important for decision makers to consider these latent, unstable factors.

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