INTERMEDIATE MICROECONOMICS

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Session 10: Second Best Mechanisms

Fall 2018

Problem 1: Public Good Provision

Two flatmates are considering whether to buy a coffee machine that costs $c = \$50$. They each have private valuation v_i, and report $\hat{v}_i$, $i = 1, 2$. To decide whether they will buy the coffee machine, the flatmates use an incentive compatible mechanism M that outputs the decision $x^M(\hat{v}_1, \hat{v}_2) \in 0, 1$ and implements some transfers $t^M(\hat{v}) = (t_1^M(\hat{v}), t_2^M(\hat{v}))$, with no deficits.

Problem 1: Public Good Provision

1. What would be the conditions for the mechanism to be:
   - Incentive compatible?
   - Without deficit?
   - Budget balanced?
   - Efficient?

   In the following, we just need the decision mechanism to run no deficits and be incentive compatible

Problem 1: Public Good Provision

Incentive Compatible

Incentive compatibility means that we want agents to reveal their true valuations. That is for every $i = 1, 2$, and every report $\hat{v}_{-i}$, it is better to report v_i than any other $\hat{v}_i$:

$$v_i x^M(v_i, \hat{v}_{-i}) - t_i^M(v_i, \hat{v}_{-i}) \geq v_i x^M(\hat{v}_i, \hat{v}_{-i}) - t_i^M(\hat{v}_i, \hat{v}_{-i})$$

Problem 1: Public Good Provision

Without Deficit

The no deficit constraint is:

$$t_1^M(\hat{v}) + t_2^M(\hat{v}) \geq cx^M(\hat{v})$$

Problem 1: Public Good Provision

Budget balanced

The no budget balanced constraint is:

$$t_1^M(\hat{v}) + t_2^M(\hat{v}) = cx^M(\hat{v})$$

Problem 1: Public Good Provision

Efficient

A mechanism is efficient if it maximizes total utility, that is if the coffee machine is bought when the flatmates' sum of valuations is more than \$50. The condition for efficiency is therefore:

$$v_1 + v_2 \geq c \;\Rightarrow\; x^M(\hat{v}_1, \hat{v}_2) \;=\; 1$$

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Problem 1: Public Good Provision

2. Assume flatmate 1 has private valuation $v_1 = 25$ and flatmate 2 has $v_2 = 30$. If the flatmates were to use the VCG mechanism to decide whether to buy the coffee machine, what would be the outcome? Would it be efficient? Would it run a deficit?

Problem 1: Public Good Provision

- It is a dominant strategy to report truthfully with VCG mechanism, therefore the flatmates report $\hat{v}_1 = 25$ and $\hat{v}_2 = 30$. Based on these truthful reports, the VCG mechanism implements the efficient choice $x^M(\hat{v}_1, \hat{v}_2) = 1$, since total utility is 55 with the coffee machine and 0 without the coffee machine.

Problem 1: Public Good Provision

- Flatmates' transfers are:
  - $F_1 \;=\; 30 - 0 = 30$
  - $F_2 \;=\; 25 - 0 = 25$

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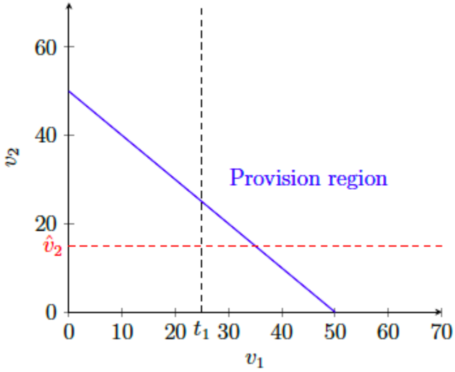
- Flatmates' transfers are:
  - $F_1 \;=\; 30 - 0 = 30$
  - $F_2 \;=\; 25 - 0 = 25$
- The VCG mechanism runs a deficit because flatmates do not put money on the table to buy their coffee machine

Problem 1: Public Good Provision

3. Consider the following mechanism: the coffee machine is bought as soon as $\hat{v}_1 + \hat{v}_2 = c$, and the flatmates make a transfer $t_1 = t_2 = \$25$ each if $x^M(\hat{v}_1, \hat{v}_2) = 1$. What does the decision rule look like on a graph? Will the flatmates report truthfully? Does this mechanism run a deficit?

Problem 1: Public Good Provision

The provision region is a line:



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  - If $v_1 \geq 25$ then flatmate 1 is better off reporting $\hat{v}_1 = \$35$, paying \$25 and getting positive payoff, whereas if he reports anything different from \$35 the good will not be provided

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  - If $v_1 \geq 25$ then flatmate 1 is better off reporting $\hat{v}_1 = \$35$, paying \$25 and getting positive payoff, whereas if he reports anything different from \$35 the good will not be provided
  - If $v < 25$ then flatmate 1 truthfully reports and get utility 0 (the good is not provisioned), instead of negative utility if the good is provisioned.

Problem 1: Public Good Provision

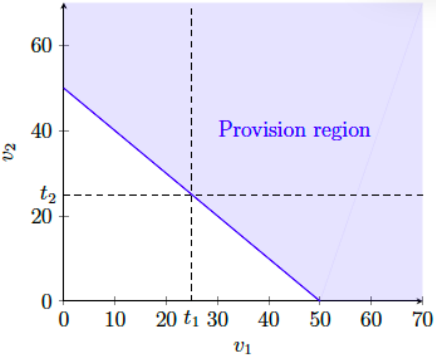
- The mechanism is budget balanced as flatmates pay each \$25 as soon as $x^M(\hat{v}_1, \hat{v}_2) \; = \; 1$

Problem 1: Public Good Provision

4. Consider another mechanism: the coffee machine is bought as soon as $\hat{v}_1 + \hat{v}_2 \geq c$, and the flatmates make a transfer $t_1 = t_2 = \$25$ each if $x^M(\hat{v}_1, \hat{v}_2) = 1$. What does the decision rule look like on a graph? Will the flatmates report truthfully? Does this mechanism run a deficit?

Problem 1: Public Good Provision

The provision region is now a large area:



Problem 1: Public Good Provision

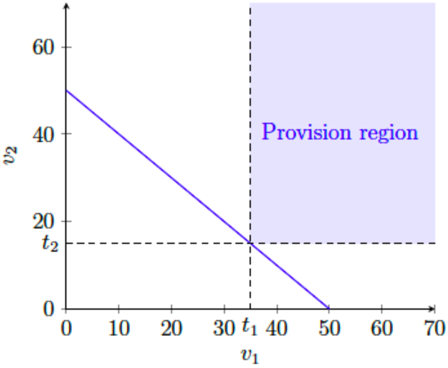
- As the previous decision rule, it is not incentive compatible but it is budget balanced
- It is efficient in the sense that $x^M = 1$ as soon as both flatmates' valuations are above \$50.

Problem 1: Public Good Provision

5. Now consider the mechanism in which flatmate 1 makes payment $t_1 = \$35$ and flatmate 2 makes payment $t_2 = \$15$ each if $x^M(\hat{v}_1, \hat{v}_2) = 1$ if $\hat{v}_1 > t_1$ and $\hat{v}_2 > t_2$. What does the decision rule look like on a graph? Will the flatmates report truthfully? Does this mechanism run a deficit?

Problem 1: Public Good Provision

The provision region is now a rectangle:



Problem 1: Public Good Provision

- This mechanism is incentive compatible: from flatmate 1's point of view, either
  - If $v_1 \geq 35$, then the flatmate 1 is better off reporting his true value than reporting $\hat{v}_1 < 35$: if flatmate 2 reports below 15, the good is not provisioned no matter what flatmate 1 does, and if flatmate 2 reports above 15, flatmate 1 prefers reporting v_1 to get the good provisioned

Problem 1: Public Good Provision

- This mechanism is incentive compatible: from flatmate 1's point of view, either
  - If $v_1 \geq 35$, then the flatmate 1 is better off reporting his true value than reporting $\hat{v}_1 < 35$: if flatmate 2 reports below 15, the good is not provisioned no matter what flatmate 1 does, and if flatmate 2 reports above 15, flatmate 1 prefers reporting v_1 to get the good provisioned
  - If $v_1 < 35$, then flatmate 1 is better off reporting v_1 so that the good is not provisioned, in which case his payoff is 0, rather than $v_1 - 35 < 0$

Problem 1: Public Good Provision

- The mechanism does not run a deficit since the flatmates pay $35 + 15 = 50$ if $x^M = 1$

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- The mechanism does not run a deficit since the flatmates pay $35 + 15 = 50$ if $x^M = 1$
- It can only be second-best efficient since the good is not provisioned every time $v_1 + v_2 \geq 50$

Problem 2: Trade in large markets

Assume there are 4 buyers and 4 sellers for a single type of good. Each seller has one unit of this good to sell. Buyers' and sellers' good's valuations are the following: $v_B^1 = 10$, $v_B^2 = 10$, $v_B^3 = 8$, $v_B^4 = 5$ and $v_S^1 = 12$, $v_S^2 = 10$, $v_S^3 = 9$, $v_S^4 = 4$.

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Problem 2: Trade in large markets

1. How many trades take place if the market designer arbitrarily sets the price to (a) 12, (b) 10, (c) 8 or (d) 4?

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a. No buyer is willing to pay 12, so no trade takes place.
b. 2 buyers are ready to buy the good for 10, and 3 sellers are ready to sell it for 10, so 2 exchanges take place
c. 3 buyers are ready to buy the good for 8, and 1 sellers are ready to sell it for 8, so 1 exchange takes place

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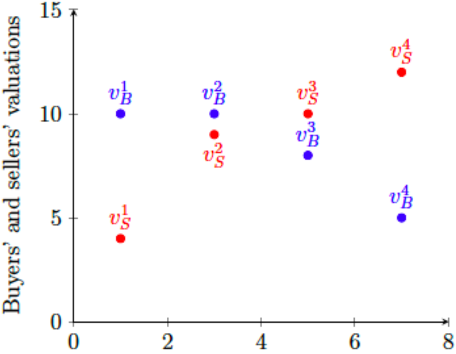
a. No buyer is willing to pay 12, so no trade takes place.
b. 2 buyers are ready to buy the good for 10, and 3 sellers are ready to sell it for 10, so 2 exchanges take place
c. 3 buyers are ready to buy the good for 8, and 1 sellers are ready to sell it for 8, so 1 exchange takes place
d. All buyers are ready to buy the good for 4, and 1 seller is ready to sell it for 4, so 1 exchange takes place

Problem 2: Trade in large markets

2. Place sellers and buyers values on a graph. What prices result from the adapted second price auction described in the slides for large markets? How many trades are realized?

Problem 2: Trade in large markets



Problem 2: Trade in large markets

- Only one trade is realized:
  - Buyer 1 pays 10 and seller 1 pays 9, such that buyer 1 makes a transfer of 1 to buying seller

Problem 2: Trade in large markets

3. Illustrate why this mechanism is incentive compatible by looking at buyers and sellers' possible deviations from revealing their true valuations.

Problem 2: Trade in large markets

Let us take the buyers' side

- Buyer 1:
  - If buyer 1 reveals $\hat{v}_1 > v_1$, he still gets to trade at the same price, hence he is indifferent
  - If he reveals $\hat{v}_1 < v_1$, then either $\hat{v}_1 > \hat{v}_2$ and he still gets to trade at the same price, or $\hat{v}_1 < \hat{v}_2$ and he misses the trade

Problem 2: Trade in large markets

- Buyer 2:
  - If buyer 2 reveals $\hat{v}_2 > v_2$, then either $\hat{v}_1 > \hat{v}_2$ and he does not get to trade, or $\hat{v}_1 < \hat{v}_2$ and he gets to trade at price $\hat{v}_1 = v_1 = 10$ but still gets zero payoff.
  - If buyer 2 reveals $\hat{v}_2 < v_2$ he does not get to trade

Problem 2: Trade in large markets

- Buyer 2:
  - If buyer 2 reveals $\hat{v}_2 > v_2$, then either $\hat{v}_1 > \hat{v}_2$ and he does not get to trade, or $\hat{v}_1 < \hat{v}_2$ and he gets to trade at price $\hat{v}_1 = v_1 = 10$ but still gets zero payoff.
  - If buyer 2 reveals $\hat{v}_2 < v_2$ he does not get to trade
- Buyers 3 and 4 behave identically to buyer 2