CSC 420 - Fall 2021 - Assignment 4

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In solving the questions in this assignment, I worked together with my classmate [Haining Tan, 1004153364]. I confirm that I have written the solutions/code/report in my own words.

Question 1.

1.1

$P = P_0 + td$ is the line and we project it,

$$
\begin{aligned}
p = \begin{pmatrix} wx \\ wy \\ w \end{pmatrix} &= KP \\
&= \begin{pmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{pmatrix} \\
&= \begin{pmatrix} fX_0 + ftd_x + p_x Z_0 + p_x td_z \\ fY_0 + p_y td_y + p_y Z_0 + p_y td_z \\ Z_0 + td_z \end{pmatrix}
\end{aligned}
$$

To get the coordinates of the vanishing point, we can assume t goes to infinity.

$$
\begin{aligned}
x = \lim_{t\to\infty} \frac{wx}{w} &= \lim_{t\to\infty} \frac{fX_0 + ftd_x + p_x Z_0 + p_x td_z}{Z_0 + td_z} \\
&= \lim_{t\to\infty} \frac{ftd_x + p_x td_z}{td_z} = \frac{fd_x + p_x d_z}{d_z} \\
y = \lim_{t\to\infty} \frac{wy}{w} &= \frac{fd_y + p_y d_z}{d_z}
\end{aligned}
$$

The coordinate of the vanishing points are $(\frac{fd_x+p_xd_z}{d_z}, \frac{fd_y+p_yd_z}{d_z})^T$

1.2

Since all lines on the plane are perpendicular to the normal vector n,

$$n_x d_x + n_y d_y + n_z d_z = 0$$
$$n_x \frac{d_x}{d_z} + n_y \frac{d_y}{d_z} + n_z = 0$$
$$\frac{d_y}{d_z} = -\frac{n_z}{n_y} - \frac{n_x}{n_y}\frac{d_x}{d_z}$$

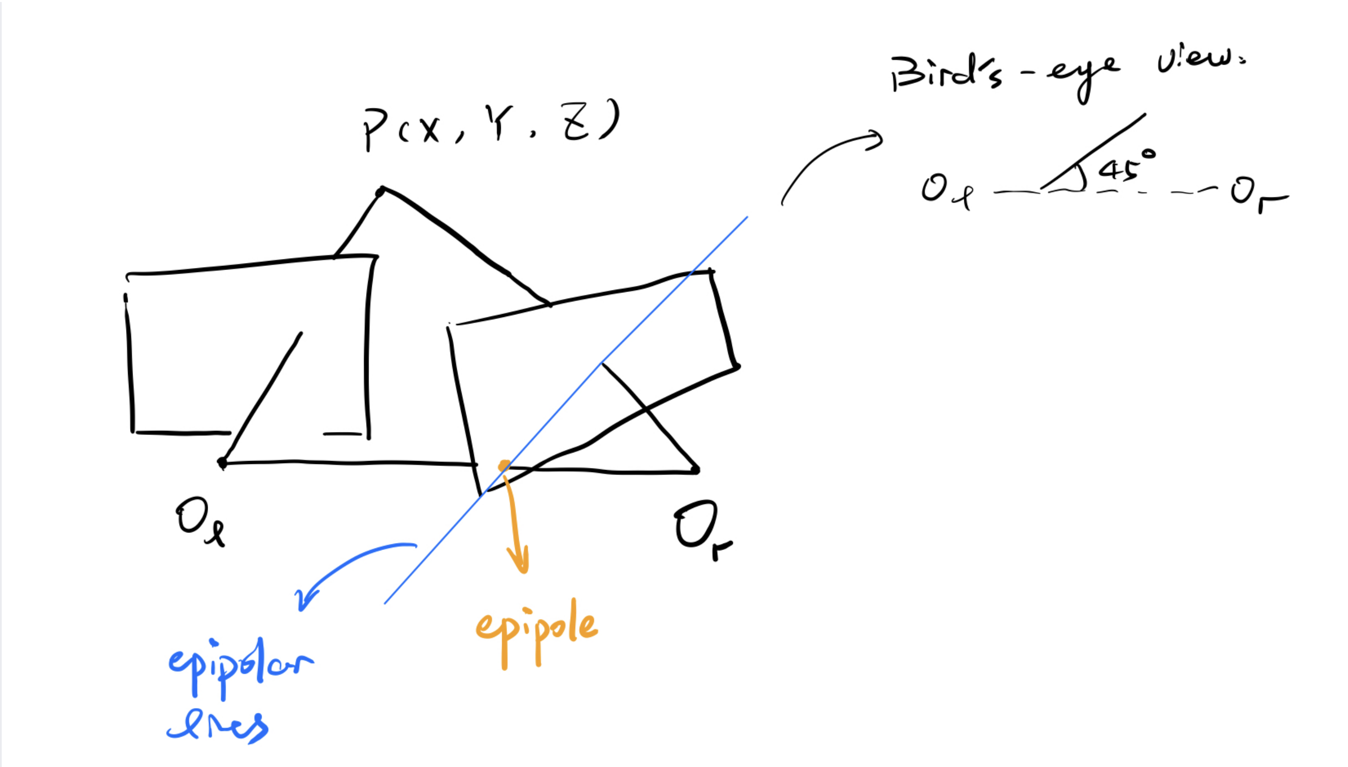
By 1.1, we know the vanishing points are of the form $(\frac{fd_x+p_xd_z}{d_z}, \frac{fd_y+p_yd_z}{d_z})$, which is equal to

$$\begin{aligned}(p_x, p_y) + (f\frac{d_x}{d_z}, f\frac{d_y}{d_z}) &= (p_x, p_y) + f(\frac{d_x}{d_z}, -\frac{n_z}{n_y} - \frac{n_x}{n_y}\frac{d_x}{d_z}) \\ &= (p_x, p_y) - (0, f\frac{n_z}{n_y}) + f(\frac{d_x}{d_z}, -\frac{n_x}{n_y}\frac{d_x}{d_z}) \\ &= (p_x, p_y - f\frac{n_z}{n_y}) + f\frac{d_x}{d_z}(1, -\frac{n_x}{n_y})\end{aligned}$$

Because the vanishing point only depends on d, for arbitrary d, $\frac{d_x}{d_z}$ is also arbitrary real number.
Treat $\frac{d_x}{d_z}$ as the unknown variable, by the equation above, all the vanishing points must be in the straight line passing $(p_x, p_y - f\frac{n_z}{n_y})$ with slope $(-\frac{n_x}{n_y})$.
Since the point coordinate and the slope does not depend on d, the vanishing points of all lines on the plane must be colinear.

Question 2.

The left image plane is parallel to the line crossing optical centers thus there is no left epipole.
The right epipolar line is the line goes through the epipole and the projection point.



Question 3.

3.1

Let $l : a_1x + b_1y + c_1 = 0$ and $l' : a_2x + b_2y + c_2 = 0$ be arbitrary 2D lines that intersect.
Solve the system given by the above two lines,

$$\begin{aligned} a_2(a_1x + b_1y + c_1) - a_1(a_2x + b_2y + c_2) = 0 \\ (a_2b_1 - a_1b_2)y = a_1c_2 - a_2c_1 \\ y = \frac{a_1c_2 - a_2c_1}{a_2b_1 - a_1b_2} \end{aligned}$$

Similarly,

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

Note the denominator is non-zero because l, l' are not parallel.
In homogeneous coordinate, (x, y) can be written as (xw, yw, w), take $w = a_1b_2 - a_2b_1$ here,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xw \\ yw \\ w \end{pmatrix} = \begin{pmatrix} b_1c_2 - b_2c_1 \\ a_2c_1 - a_1c_2 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

Notice

$$
\begin{aligned}
l \times l' &= (a_1, b_1, c_1) \times (a_2, b_2, c_2) \\
&= \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{pmatrix} b_1c_2 - b_2c_1 \\ a_2c_1 - a_1c_2 \\ a_1b_2 - a_2b_1 \end{pmatrix} \\
&= \begin{pmatrix} x \\ y \end{pmatrix}
\end{aligned}
$$

The intersection point $p = l \times l'$ is showed as required.

3.2

Let $p = (x_1, y_1)^T$ and $p' = (x_2, y_2)^T$ be arbitrary.
The line, denoted as $ax + by + c = 0$ goes through p, p' must satisfy

$$
\begin{cases} ax_1 + by_1 + c = 0 \\ ax_2 + by_2 + c = 0 \end{cases}
$$

Solve the system get

$$
\begin{cases} a = \frac{c(y_1 - y_2)}{x_1y_2 - x_2y_1} \\ b = \frac{c(x_1 - x_2)}{x_2y_1 - x_1y_2} \end{cases}
$$

Using the homogeneous coordinate, thus the line vector is

$$
\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{c(y_1 - y_2)}{x_1y_2 - x_2y_1} \\ \frac{c(x_1 - x_2)}{x_2y_1 - x_1y_2} \\ c \end{pmatrix} = \begin{pmatrix} y_1 - y_2 \\ x_2 - x_1 \\ x_1y_2 - x_2y_1 \end{pmatrix}
$$

Notice

$$
\begin{aligned}
p \times p' = (x_1, y_1)^T \times (x_2, y_2)^T &= \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} \times \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \\
&= \begin{vmatrix} i & j & k \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} \\
&= \begin{pmatrix} y_1 - y_2 \\ x_2 - x_1 \\ x_1y_2 - x_2y_1 \end{pmatrix}
\end{aligned}
$$

Therefore the line $l = p \times p'$