Geometric Shapes

Module 3

Lecture 1

Geometric Shapes

- Geometry has no color and texture
- Points – 0 degree of freedom shape
- Curves – 1 degree of freedom shape
- Surfaces – 2 degree of freedom shape
- Solid objects – 3 degree of freedom shape

- 2 and 3 dimensional spaces
- Time is yet another dimension

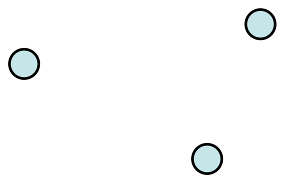
- Displayed as pixels, voxels, polylines, and shaded polygons

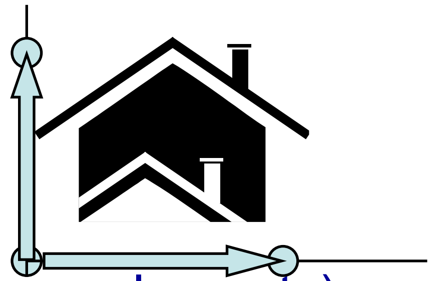
Learning objectives

- To understand how points and curves can be used in solving data visualization problems
- To understand curves as objects with 1 degree of freedom
- To understand what mathematical representation is the most efficient for defining and displaying curves
- To understand how different coordinate systems can be used together for deriving mathematical representations of curves

Points

- Individual points
- Reference points
- Point rendering
- Splat (e.g. little oriented disks) rendering
- 2D pixels (picture elements) and 3D voxels (volume elements)
- Defined by Cartesian coordinates (x, y, z), polar (r, α), spherical (r, α, β) or cylindrical (h, r, α) coordinates





Curves

- 2D and 3D
- Polylines – interpolation by connected straight line segments
- Explicit (only 2D)

  $y=f(x)$ *or* $x=f(y)$
- Implicit (only 2D)

  $f(x,y)=0$
- Parametric (2D and 3D)

  $x=x(t)$

  $y=y(t)$

  $t=[t_1, t_2]$

  $x=x(t)$

  $y=y(t)$

  $z=z(t)$

  $t=[t_1, t_2]$

Study by Example:

- Straight Line
- Circle
- Ellipse

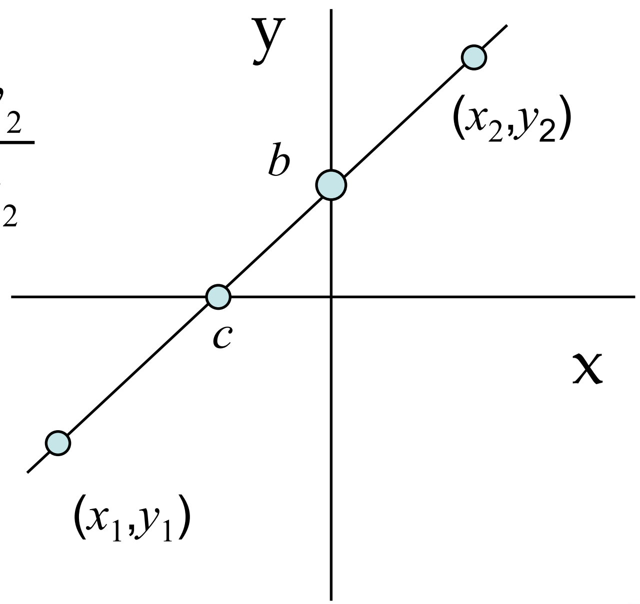
2D Straight Line
Explicit Representation

- Straight line

$$y = ax + b$$

$$x = dy + c$$

$$a = \frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$



- Straight line segment ?
- Straight line ray ?

- No the same formula for each straight line (axes dependent)

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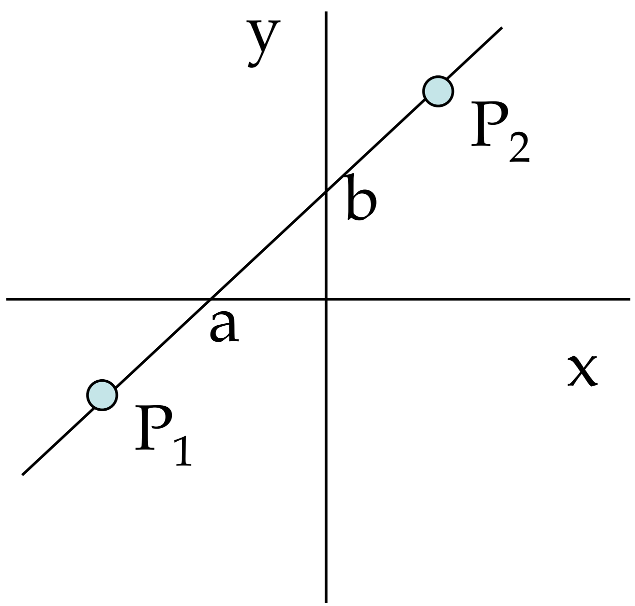
2D Straight Line
Implicit Representation

- Straight line

$$Ax + By + C = 0$$

$$\frac{y - y_1}{x - x_1} - \frac{y - y_2}{x - x_2} = 0$$



- Equation in intercepts

$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$$

- Straight line segment ?
- Straight line ray ?

2D Straight Line Parametric Representation

- Parametric definition of a straight line segment

$$x = x_1 + \tau(x_2 - x_1) = x_1(1-\tau) + \tau x_2$$

$$y = y_1 + \tau(y_2 - y_1) = y_1(1-\tau) + \tau y_2$$

$\tau = [0,1]$ One parameter !

- Straight line

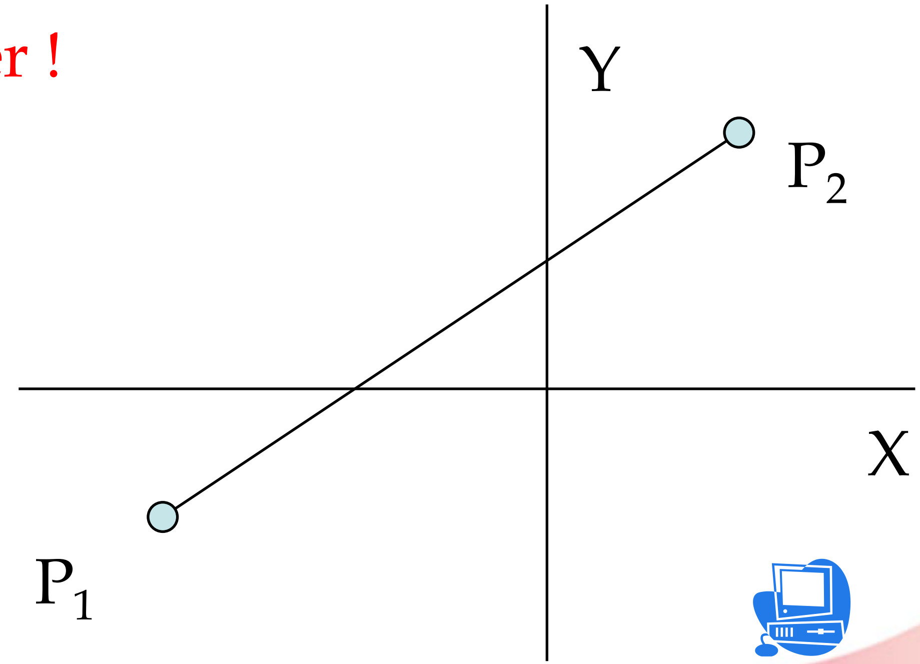
$$\tau = (-\infty, \infty)$$

- Straight line ray

$$\tau = [0, \infty)$$

$$\tau = (-\infty, 1]$$

- Axes independent



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Geometric Shapes: curves

Module 3

Lecture 2

We have learnt that

- A curve is defined by a point moving with one degree of freedom (forward and backward)
- Polylines – interpolation by connected straight line segments
- Analytically curves are defined by:

  - **Explicit functions**
    $y=f(x)$ *or* $x=f(y)$ *– axes dependent, no arcs and segments*
  - **Implicit functions**
    $f(x,y)=0$ *– no arcs and segments. Slow for rendering*
  - **Parametric functions**
    One parameter only. *Any curve or part. Fast rendering with different resolution.*
    $x=x(t), \quad y=y(t) \qquad t=[t_1, t_2]$

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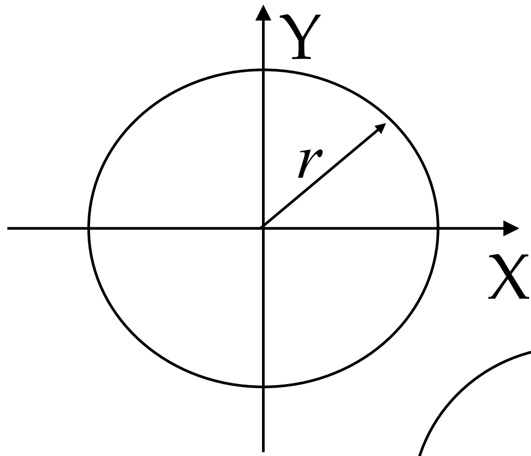
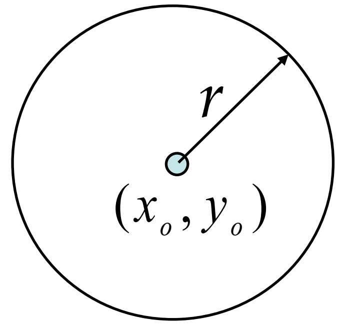
2D Circle
Implicit Representation

$$r^2 - x^2 - y^2 = 0$$

$$r^2 - (x - x_o)^2 - (y - y_o)^2 = 0$$



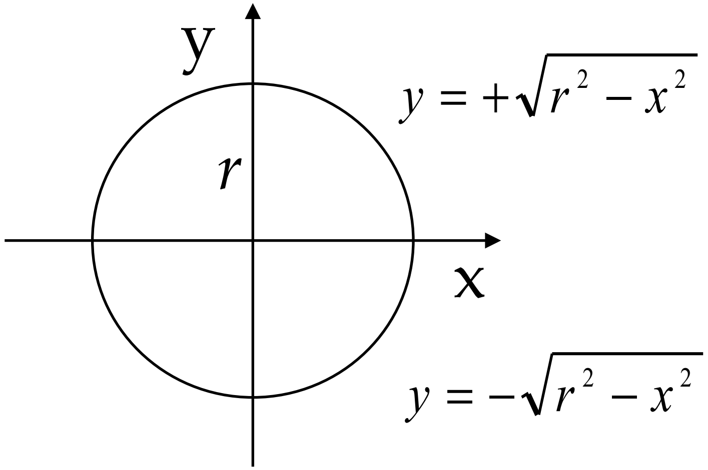


- Arc?

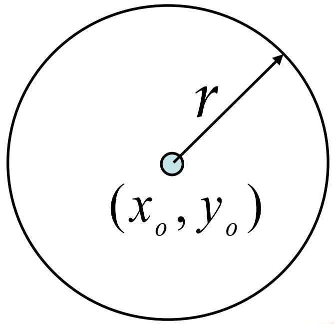
2D Circle
Explicit Representation

$$y = \pm\sqrt{r^2 - x^2}$$

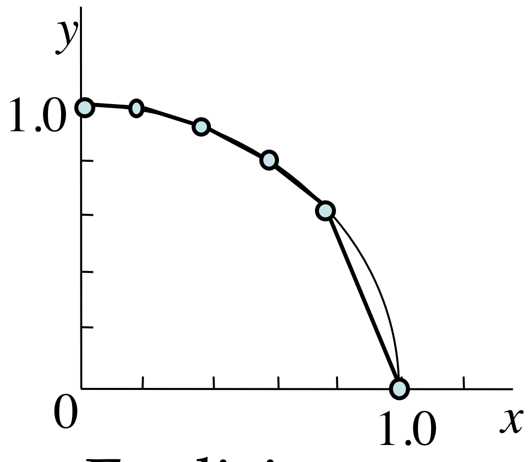
$$y = \pm\sqrt{r^2 - (x - x_o)^2} + y_o$$



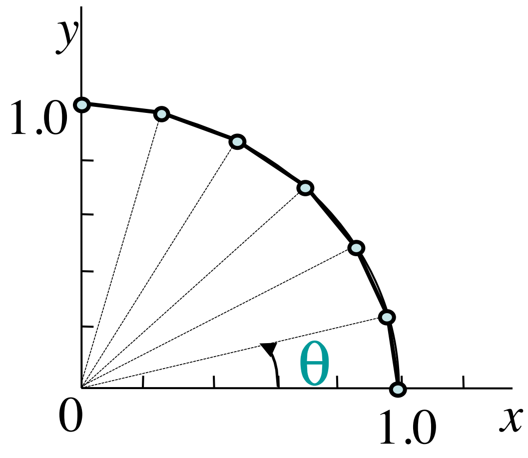
- 2 formulas for the upper and lower semicircles
- Arc?



2D Circle
Parametric Representation



Explicit



Parametric

$$x = r\cos(\theta) + x_o \qquad 0 \le \theta \le 2\pi \qquad \text{for a circle}$$

$$y = r\sin(\theta) + y_o \qquad \theta_1 \le \theta \le \theta_2 \qquad \text{for an arc}$$

One parameter!

CZ2003

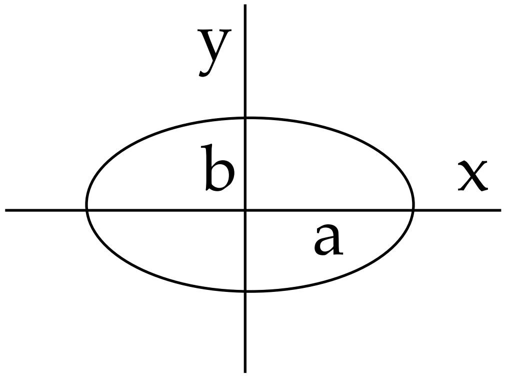
Circle in Polar Coordinates

- Origin-centred: r=*radius.*
- Other locations - problematic

Ellipse
Implicit Representation

$$1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2=0 \qquad 1-\left(\frac{x-x_0}{a}\right)^2-\left(\frac{y-y_0}{b}\right)^2=0$$

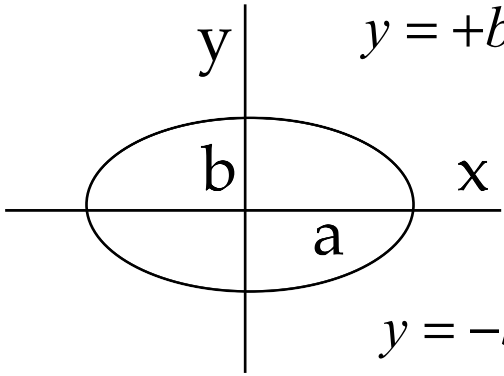
- Elliptic Arc?



2D Ellipse
Explicit Representation

$$y = \pm b\sqrt{1-\left(\frac{x}{a}\right)^2} \qquad\qquad y = \pm b\sqrt{1-\left(\frac{x-x_o}{a}\right)^2} + y_o$$

- 2 formulas for the upper and lower semiellipses
- Elliptic Arc?



$$y = +b\sqrt{1-\left(\frac{x}{a}\right)^2}$$

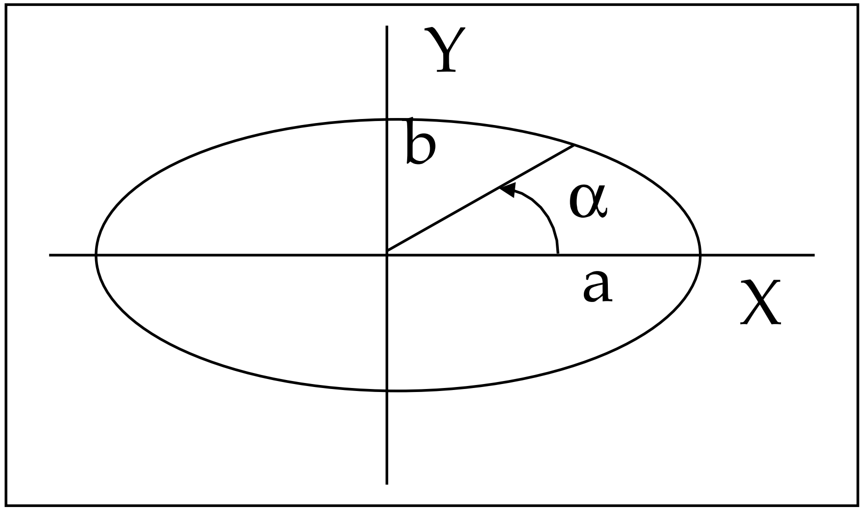
$$y = -b\sqrt{1-\left(\frac{x}{a}\right)^2}$$

2D Ellipse Parametric Representation

$$x = a\cos\theta + x_0$$

$$y = b\sin\theta + y_0$$

Scaling of a unit circle along X and Y by *a* and *b*



$$0 \le \theta \le 2\pi \qquad \text{for an ellipse}$$

$$\theta_1 \le \theta \le \theta_2 \qquad \text{for an arc}$$

Angle $\alpha \neq$ parameter θ !

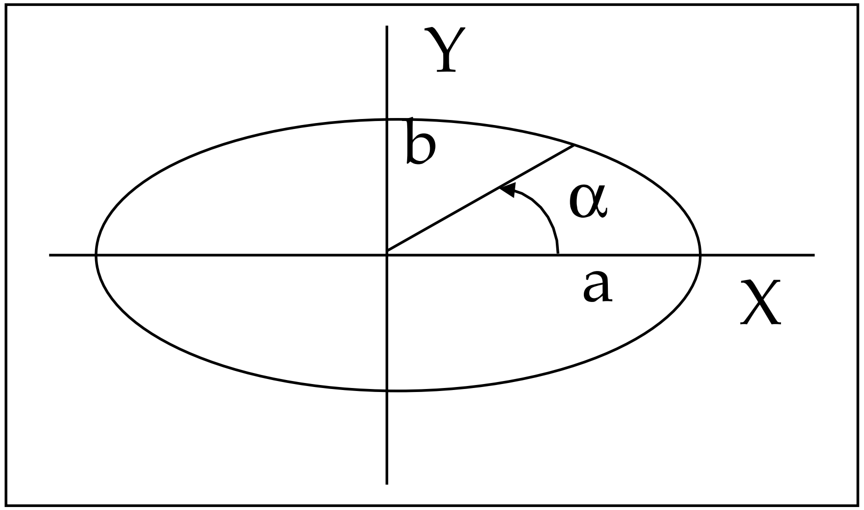
2D Origin-centred Ellipse in Polar Coordinates

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\left(\frac{r\cos\alpha}{a}\right)^2+\left(\frac{r\sin\alpha}{b}\right)^2$$

$$r^2\left(\left(\frac{\cos\alpha}{a}\right)^2+\left(\frac{\sin\alpha}{b}\right)^2\right)=1$$

$$r=\frac{1}{\sqrt{\left(\frac{\cos\alpha}{a}\right)^2+\left(\frac{\sin\alpha}{b}\right)^2}}$$

$$0\le\alpha<2\pi$$



3D Curves

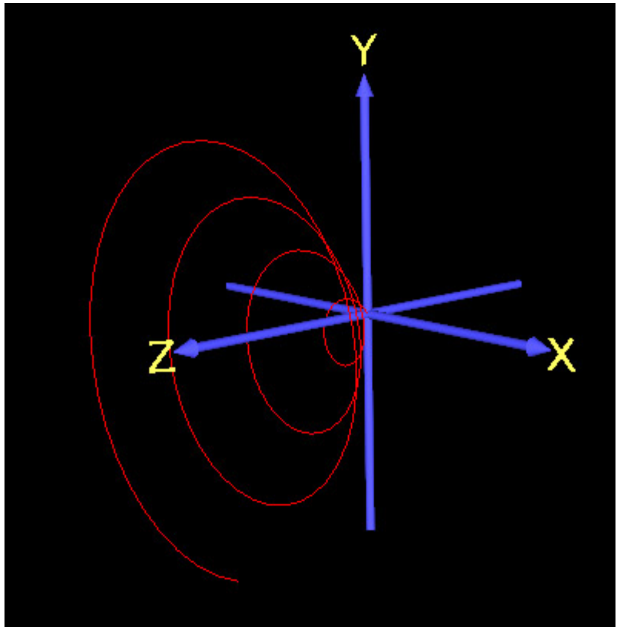
Can be only defined parametrically!

$$x = f_x(\tau)$$

$$y = f_y(\tau)$$

$$z = f_z(\tau)$$

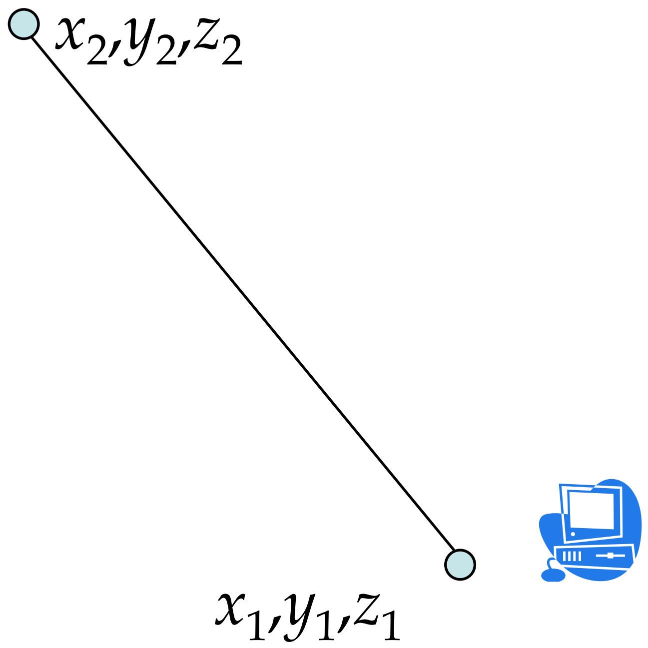
$$\tau = [\tau_1, \tau_2]$$



Still one parameter !

3D Straight Line

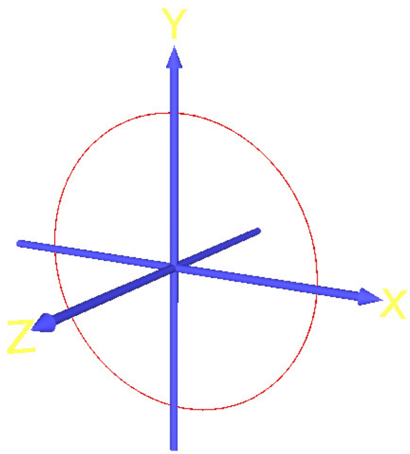
$$x = x_1 + \tau(x_2 - x_1)$$
$$y = y_1 + \tau(y_2 - y_1)$$
$$z = z_1 + \tau(z_2 - z_1)$$
$$\tau = [0,1] \quad \text{Segment}$$
$$\tau = [0,\infty) \quad \text{Ray}$$
$$\tau = (-\infty,\infty) \quad \text{Straight line}$$



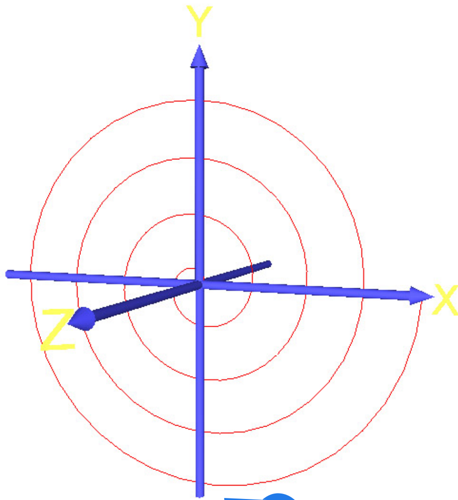
No explicit and implicit representation !

Experimenting with Parametric Curves

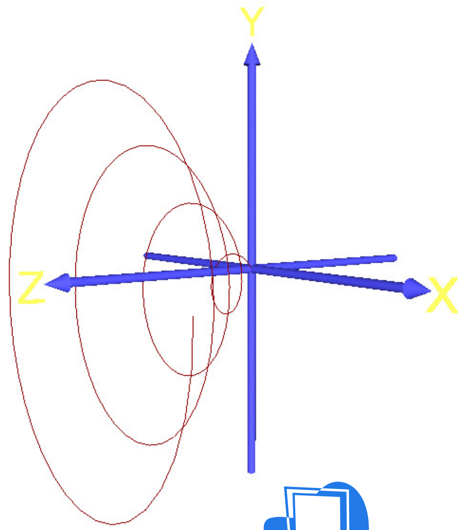
$x=6\cdot cos(u\cdot\pi)/8$
$y=6\cdot sin(u\cdot\pi)/8$
$z=0$
$0\leq u\leq 8$



$x=\mathbf{u}\cdot cos(u\cdot\pi)/8$
$y=\mathbf{u}\cdot sin(u\cdot\pi)/8$
$z=0$
$0\leq u\leq 8$



$x=u\cdot cos(u\cdot\pi)/8$
$y=u\cdot sin(u\cdot\pi)/8$
$z=\mathbf{u/8}$
$0\leq u\leq 8$



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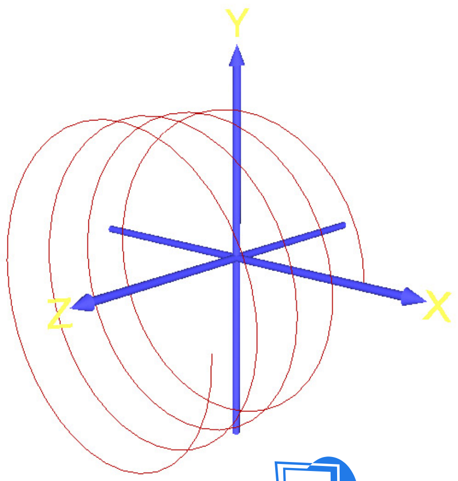
Experimenting with Parametric Curves

$x = \mathbf{6} \cdot \cos(u \cdot \pi)/8$

$y = \mathbf{6} \cdot \sin(u \cdot \pi)/8$

$z = \boldsymbol{u/8}$

$0 \le u \le 8$

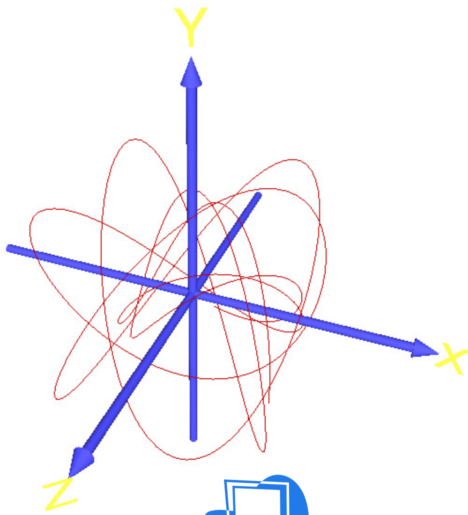


$x = \cos(u \cdot \pi + u)/2$

$y = \sin(u \cdot \pi + 3 \cdot u)/2$

$z = \cos(u \cdot \pi + 2 \cdot u)/2$

$0 \le u \le 8$



Curves. Summary

- 2D and 3D. A point moving with one degree of freedom (forward and backward)
- Polylines – interpolation by connected straight line segments
- 2D:
  - **Explicit**
    y=f(x) or x=f(y) – axes dependent, no arcs and segments
  - **Implicit**
    f(x,y)=0 – no arcs and segments
  - **Parametric**
    One parameter only. Any curve, even with self-intersections. The concept of a moving point as a function of the parameter. Curve interpolation.
    $x=x(t), \quad y=y(t) \qquad t=[t_1, t_2]$
- 3D:
  - **Parametric**
    One parameter only. Any curve, even with self-intersections. The concept of a moving point as a function of the parameter. Curve interpolation.
    $x=x(t), \quad y=y(t), \quad z=z(t) \qquad t=[t_1, t_2]$

Labs. Experiment 2

- *Download file* ***curve.wrl*** *from the course-site (Fig. 3). Use it as a template for the following exercises.*
- *Define parametrically in different files*
  - *straight line segment,*
  - *circle and its arc,*
  - *ellipse and its arcs,*
  - *2D spiral,*
  - *3D helix.*
- *Convert the explicitly defined curve y=sin(x) to parametric representation x(u), y(u) and define it in FVRML file.*
- *Explore what happens when you change the curves resolutions to as little as 2 and see how the shape of the curves changes.*
- *Change the curves parameter domain to see how they elongate or shorten.*
- *Create a folder with name Lab2 and copy there all the FVRML files you have experimented with.*
- *Write a brief report explaining what each file defines and also copy it to Lab2 folder.*

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Geometric Shapes

Module 3

Lecture 3

We have learnt that

- 2D and 3D. A point moving with one degree of freedom (forward and backward)
- Polylines – interpolation by connected straight line segments
- 2D:
  - **Explicit**

    $y=f(x)$ or $x=f(y)$ – axes dependent, no arcs and segments
  - **Implicit**

    $f(x,y)=0$ – no arcs and segments
  - **Parametric**

    One parameter only. Any curve, even with self-intersections. The concept of a moving point as a function of the parameter. Curve interpolation.

    $x=x(t), \quad y=y(t) \qquad t=[t_1, t_2]$
- 3D:
  - **Parametric**

    One parameter only. Any curve, even with self-intersections. The concept of a moving point as a function of the parameter. Curve interpolation.

    $x=x(t), \quad y=y(t), \quad z=z(t) \qquad t=[t_1, t_2]$

Geometric Shapes

- Geometry has no color and texture
- Points
- Curves
- **Surfaces**
- Solid objects

Learning objectives

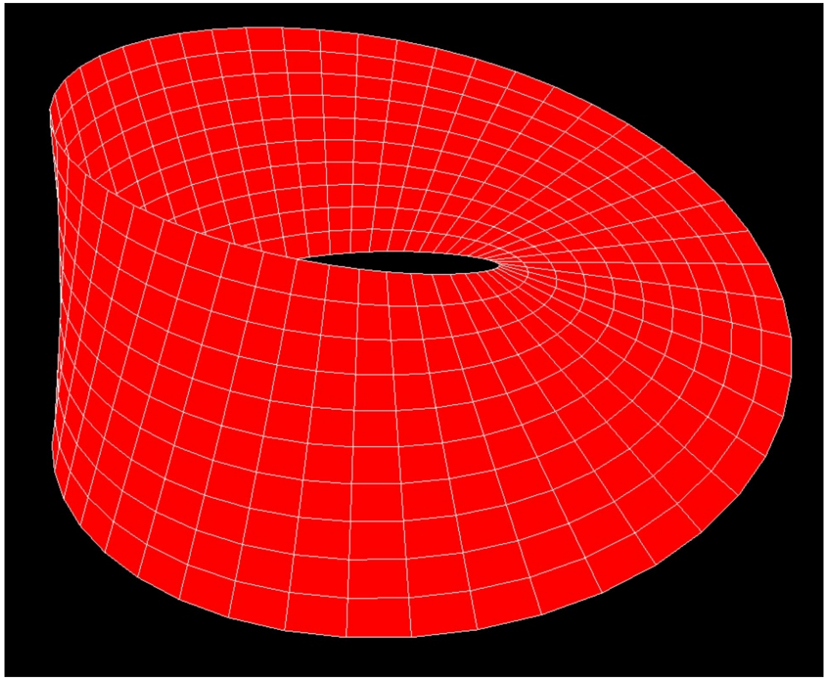
- To understand how surfaces can be used in solving data visualization problems
- To understand surfaces as objects with 2 degree of freedom
- To understand what mathematical representations are the most efficient for defining and displaying surfaces
- To understand how different coordinate systems can be used together for deriving mathematical representations of surfaces
- To understand surfaces as objects created by moving curves

Surfaces

- Polygonal representation - polygon meshes
- Analytic representations
  – Explicit representation
  – Implicit representation
  – Parametric representation

Polygonal Representation

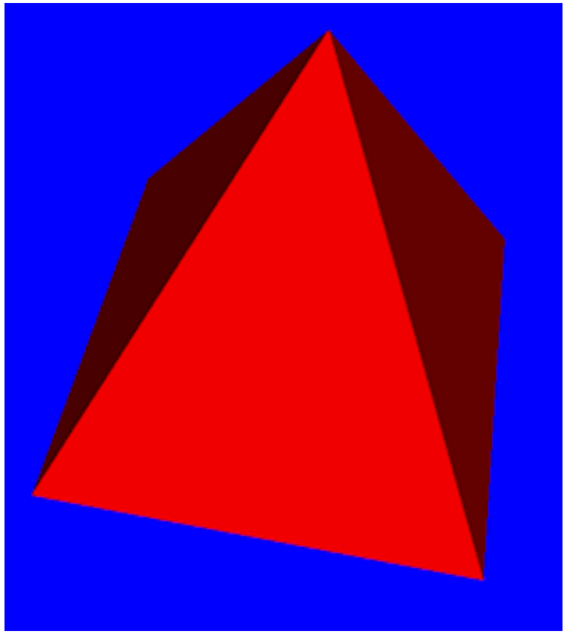
- List of vertices
- List of polygons formed by the vertices
- List of normals built at the vertices



- Order of vertices is important
- Usually only one visible side
  where the normal is pointing out from

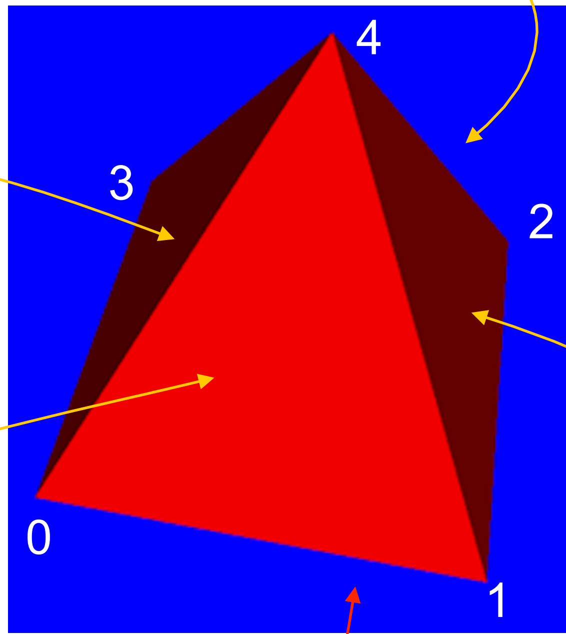
VRML Polygon Mesh

```
coord Coordinate { point [
 0,000000  0.000000   0.000000,
-0.005000 -0.505000 -1.619200,
-0.105000 -0.485000 -1.619200,
. . . . . . . . . . . . . . . . . . . . . . . . . . ]   }
normal Normal { vector [
 1.000000  0.000000  0.000000
 0.176000 -0.601000 -0.780000
-0.306000 -0.340000 -0.889000
. . . . . . . . . . . . . . . . . . . . . . . . . . ]   }
coordIndex [
70 65 62 -1
23 63 6 -1
18 19 17 -1
. . . . . . . . . . . . . . . . . . . . . . . . . . ] }
```



VRML Polygon Mesh

```
geometry IndexedFaceSet {
coord Coordinate { point [
-1.0  -1.0   1.0,     #vertex 0
 1.0   -1.0   1.0,    #vertex 1
 1.0   -1.0  -1.0,    #vertex 2
-1.0  -1.0  -1.0,     #vertex 3
 0.0    1.0   0.0     #vertex 4
 ]  }
coordIndex [
0,  3,  2,  1,  -1,
0,  1,  4, -1,
1,  2,  4, -1,
2,  3,  4, -1,
3,  0,  4, -1,
                    ]
    }
```



Geometric Shapes: surfaces

Module 3

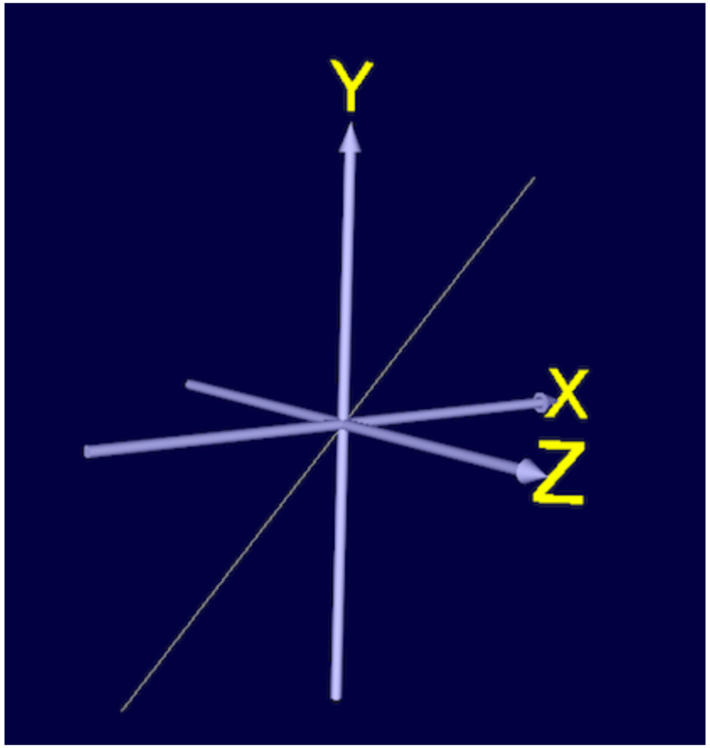
Lecture 4

We have learnt that

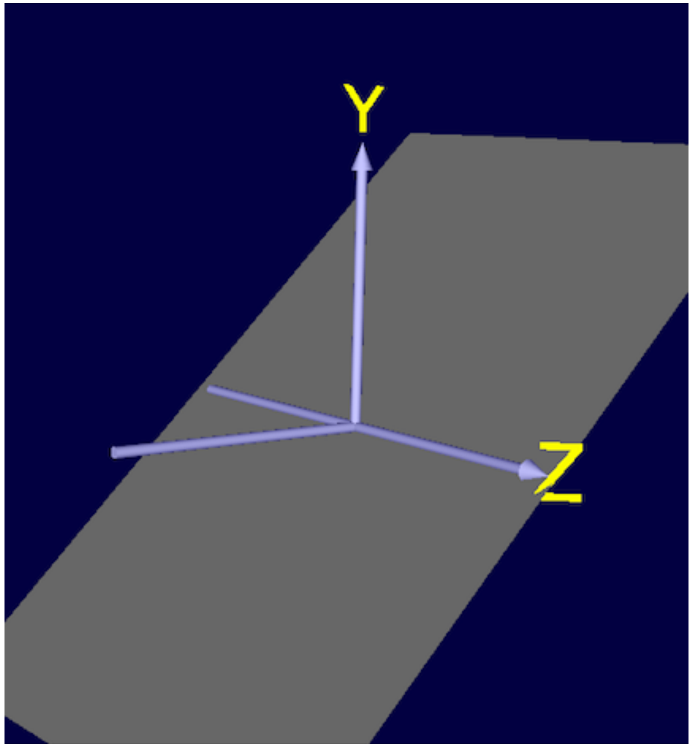
- Surfaces can be defined (interpolated) by polygon meshes
- Polygon meshes are defined by set of vertices, set of normals at the vertices, and polygons formed by the vertices
- Order how the vertices are listed to form a polygon define the direction of the normal to the polygon. Right-hand rule is used to define the normal direction
- Examples of common polygon mesh data formats:
  .OBJ (Wavefront), .STL (STereo Lithography), Indexed FaceSet in VRML and X3D
- Explicit, implicit and parametric functions can be used for defining surfaces analytically for any precision and compactness of the model

Let's Increase the Dimension

2D: $y - x = 0$

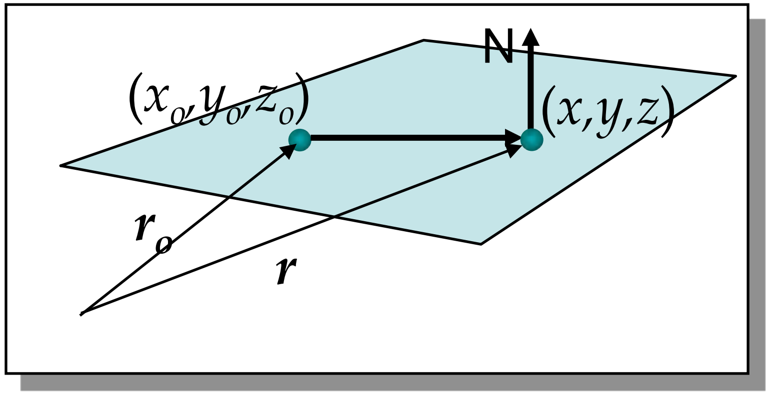


3D: $y - x = 0$



Plane Surface

$$Ax + By + Cz + D = 0$$



- Implicit function
- Explicit functions can be derived from the implicit but are seldom used
- The values of the coefficients A,B,C, and D can be obtained by solving a set of three plane equations using the coordinates for three non-collinear points in the plane. $\frac{A}{D}x + \frac{B}{D}y + \frac{C}{D}z + 1 = 0$
- N=[A B C] – normal vector to the plane: cross product of two vectors
- For any point $\boldsymbol{r}_o = (x_o, y_o, z_o)$: $\mathbf{N} \bullet (\boldsymbol{r} - \boldsymbol{r}_o) = 0$ i.e., 90° angle

$$A(x - x_o) + B(y - y_0) + C(z - z_o) = 0$$

See the detailed video exercise in Lecture Supplement

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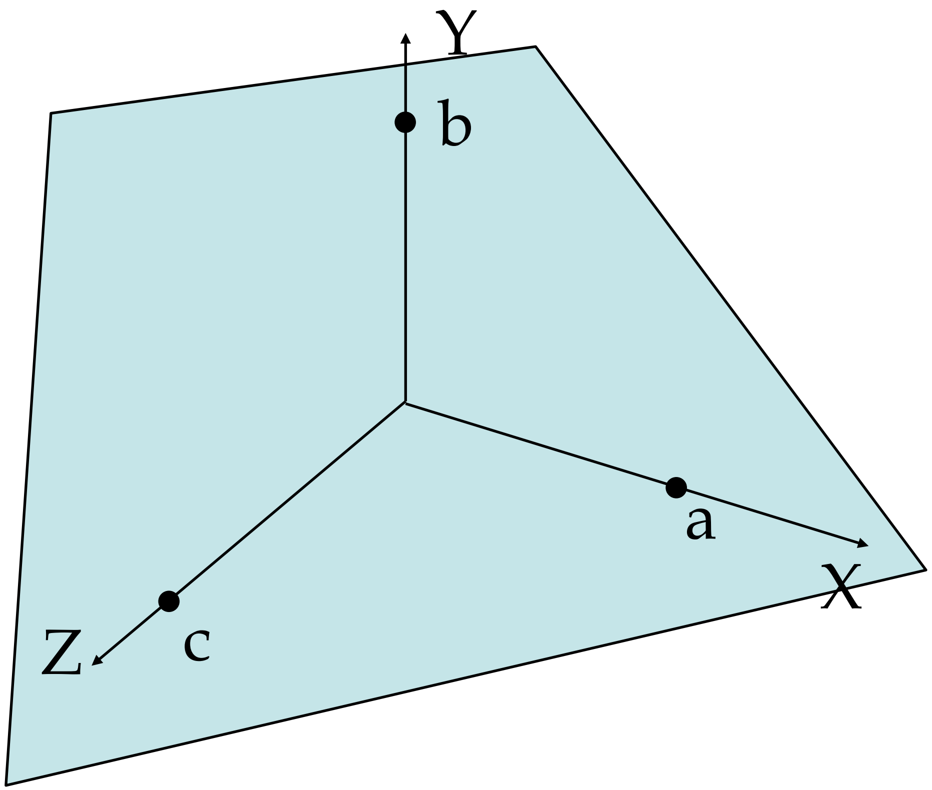
Plane Surface

- Implicit equation in intercepts



$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1=0$$

$$Ax+By+Cz+D=0$$

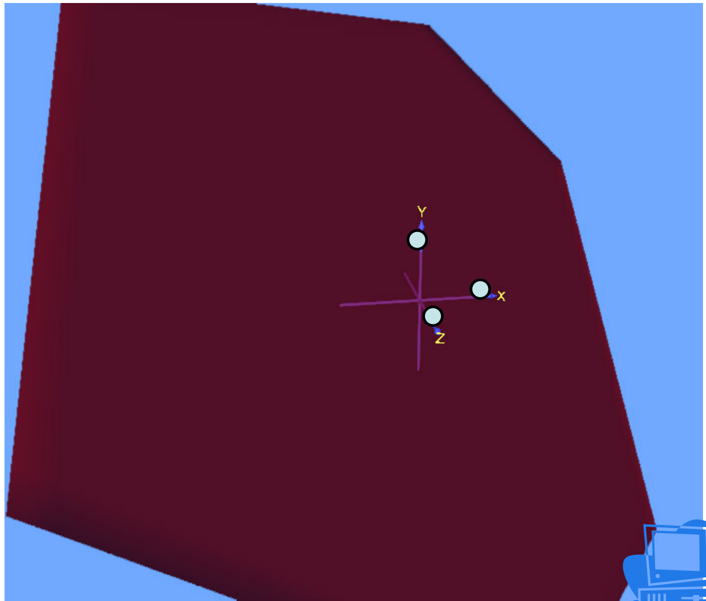
$$\frac{A}{D}x+\frac{B}{D}y+\frac{C}{D}z+1=0$$

$$\frac{A}{-D}x+\frac{B}{-D}y+\frac{C}{-D}z=1$$

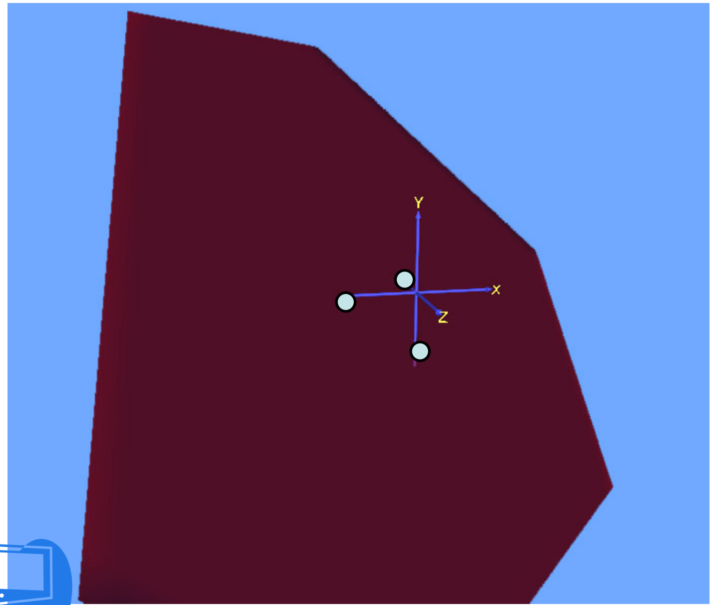
$$a=-\frac{D}{A},\quad b=-\frac{D}{B},\quad c=-\frac{D}{C}$$

Plane Surface

$x/1.2+y/1+z/1-1 = 0$



$x/1.2+y/1+z/1+1 = 0$



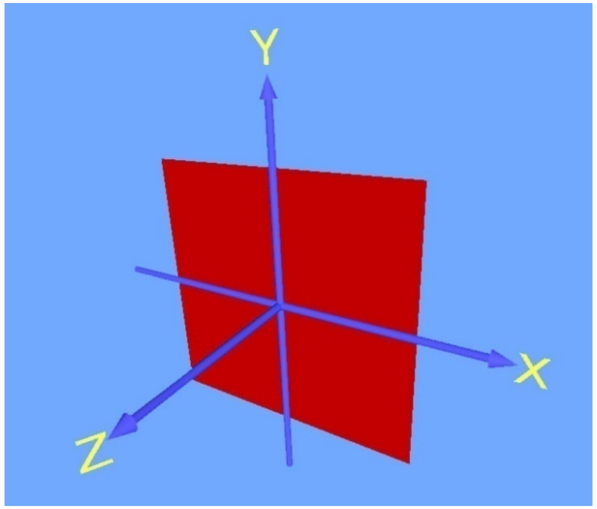
Plane Parametrically

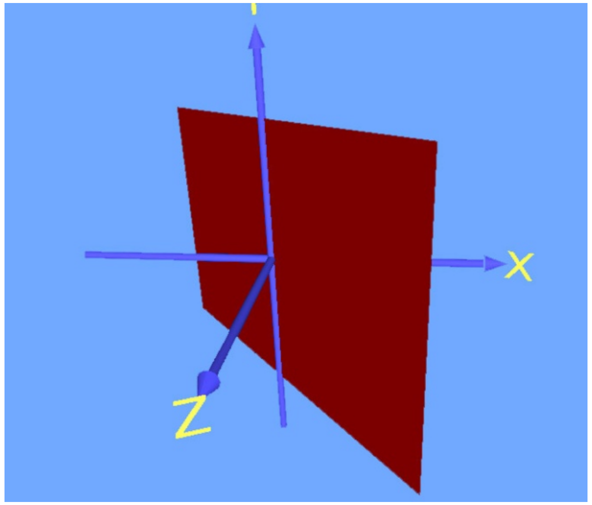
x=u   y=v   z=0

u,v=[−∞, ∞] - plane

x=u   y=v   z=u

u=[-1, 1] v=[-1, 1] – rectangle





Two parameters !

Parametrically defined plane

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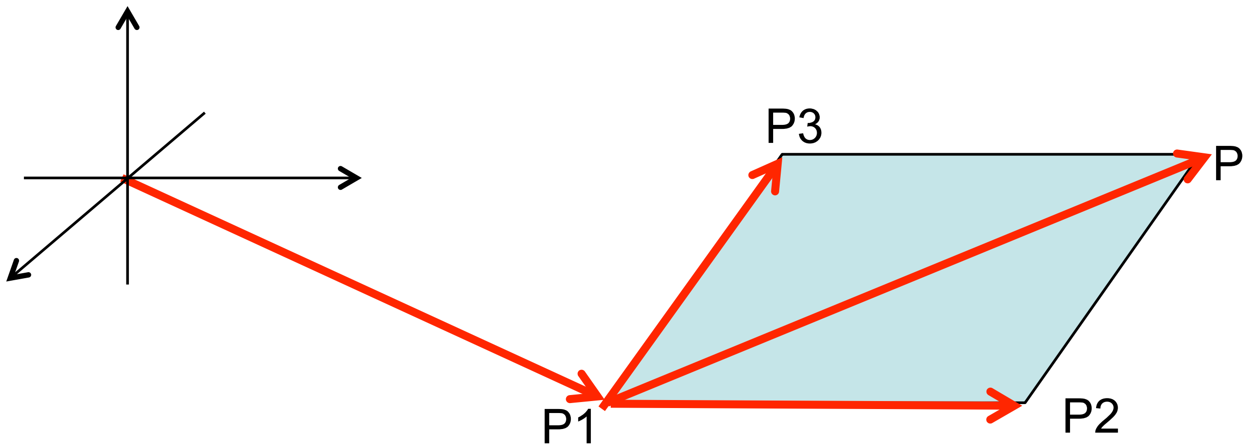
Plane Parametricaly

By the cum of two vectors:

P=P1+u*(P2-P1)+v*(P3-P1), $u,v=[-\infty,\infty]$

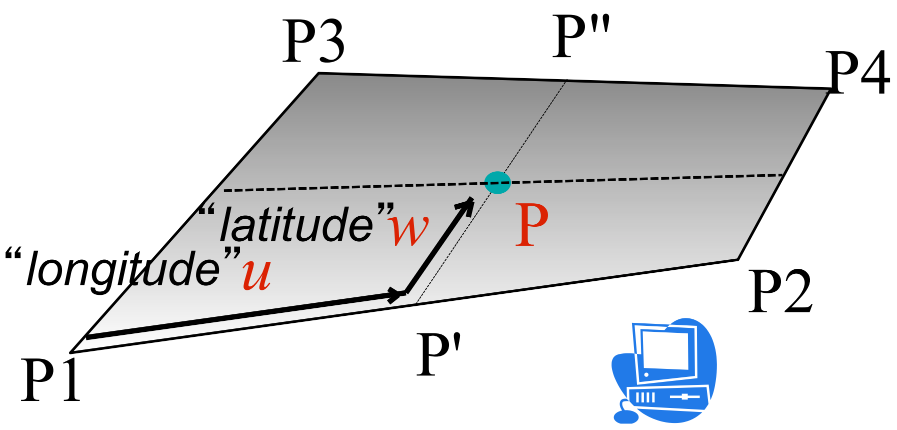


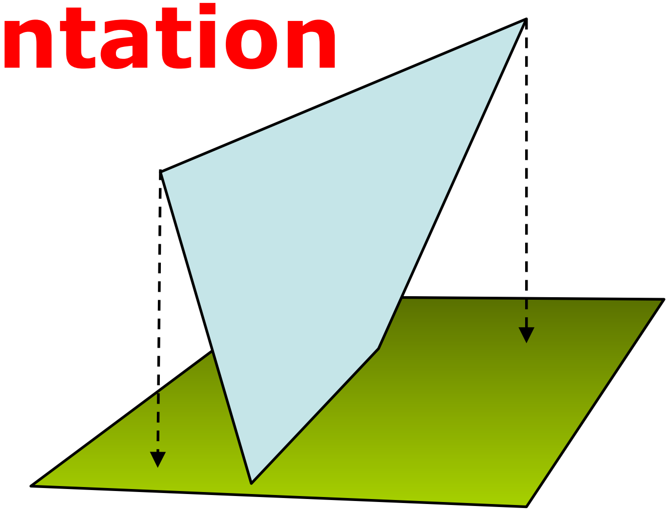
Bilinear Surface Parametric Representation

- Line equation parametrically: $P' = P_1 + u(P_2 - P_1)$
- One parametric coordinate: u
- Surface has 2 parametric coordinates (e.g. u, v)
- Let's use crossing lines to define points on the surface

Bilinear Surface
Parametric Representation

Two parameters !





$$P' = P_1 + u(P_2 - P_1)$$
$$P'' = P_3 + u(P_4 - P_3)$$
$$P = P' + w(P'' - P')$$

$$P = P_1 + u(P_2 - P_1) + w(P_3 - P_1 + u(P_4 - P_3 - (P_2 - P_1)))$$

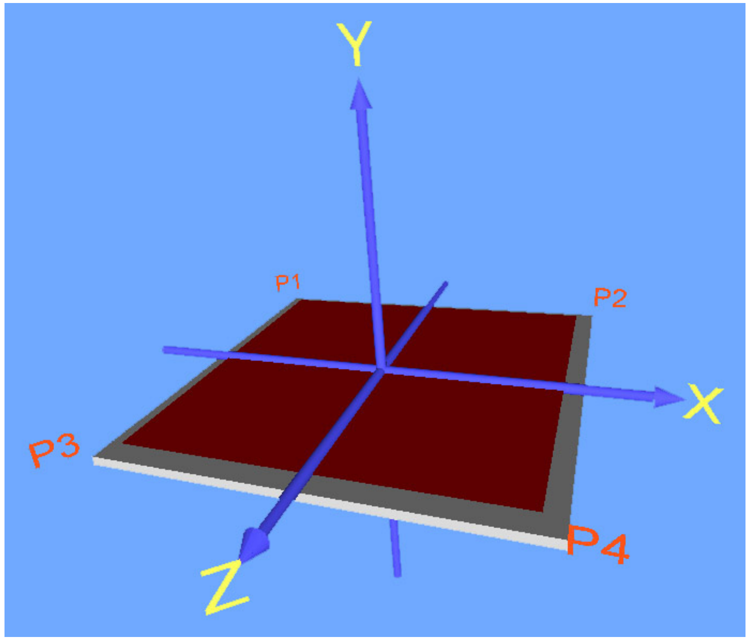
Point P_1, P_2, P_3 and P_4 may be any points in a 3D space so that even "twisted" surfaces may result

Bilinear Surface Parametric Representation

$x_1=-1\ x_2=1\ x_3=-1\ x_4=1$ $x(u,v)=x_1+u\cdot(x_2-x_1)+v\cdot(x_3-x_1+u\cdot(x_4-x_3-x_2+x_1))$

$y_1=0\ y_2=0\ y_3=0\ y_4=0$ $y(u,v)=y_1+u\cdot(y_2-y_1)+v\cdot(y_3-y_1+u\cdot(y_4-y_3-y_2+y_1))$

$z_1=-1\ z_2=-1\ z_3=1\ z_4=1$ $z(u,v)=z_1+u\cdot(z_2-z_1)+v\cdot(z_3-z_1+u\cdot(z_4-z_3-z_2+z_1))$

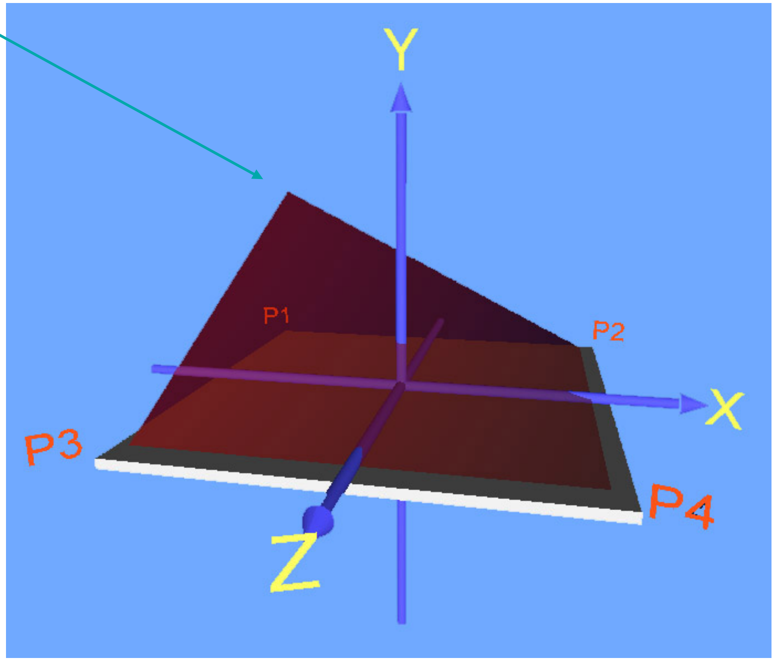


Bilinear Surface Parametric Representation

x_1=-1 x_2=1 x_3=-1 x_4=1 $\quad x(u,v)=x_1+u\cdot(x_2-x_1)+v\cdot(x_3-x_1+u\cdot(x_4-x_3-x_2+x_1))$

y_1=**1** y_2=0 y_3=0 y_4=0 $\quad y(u,v)=y_1+u\cdot(y_2-y_1)+v\cdot(y_3-y_1+u\cdot(y_4-y_3-y_2+y_1))$

z_1=-1 z_2=-1 z_3=1 z_4=1 $\quad z(u,v)=z_1+u\cdot(z_2-z_1)+v\cdot(z_3-z_1+u\cdot(z_4-z_3-z_2+z_1))$

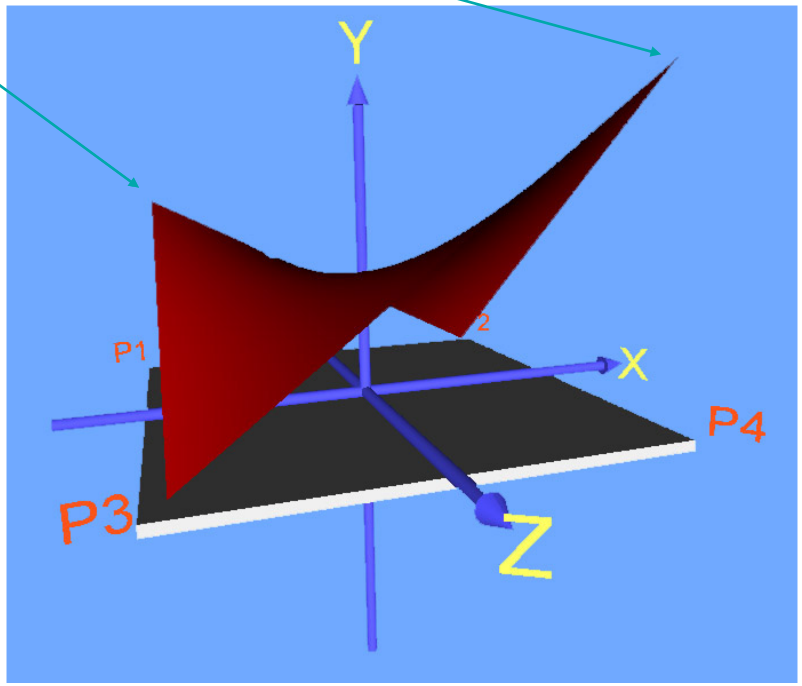


Bilinear Surface
Parametric Representation

$x_1=-1$ $x_2=1$ $x_3=-1$ $x_4=1$ $\quad x(u,v)=x_1+u\cdot(x_2-x_1)+v\cdot(x_3-x_1+u\cdot(x_4-x_3-x_2+x_1))$

$y_1=\mathbf{1}$ $y_2=0$ $y_3=0$ $y_4=\mathbf{1.5}$ $\quad y(u,v)=y_1+u\cdot(y_2-y_1)+v\cdot(y_3-y_1+u\cdot(y_4-y_3-y_2+y_1))$

$z_1=-1$ $z_2=-1$ $z_3=1$ $z_4=1$ $\quad z(u,v)=z_1+u\cdot(z_2-z_1)+v\cdot(z_3-z_1+u\cdot(z_4-z_3-z_2+z_1))$

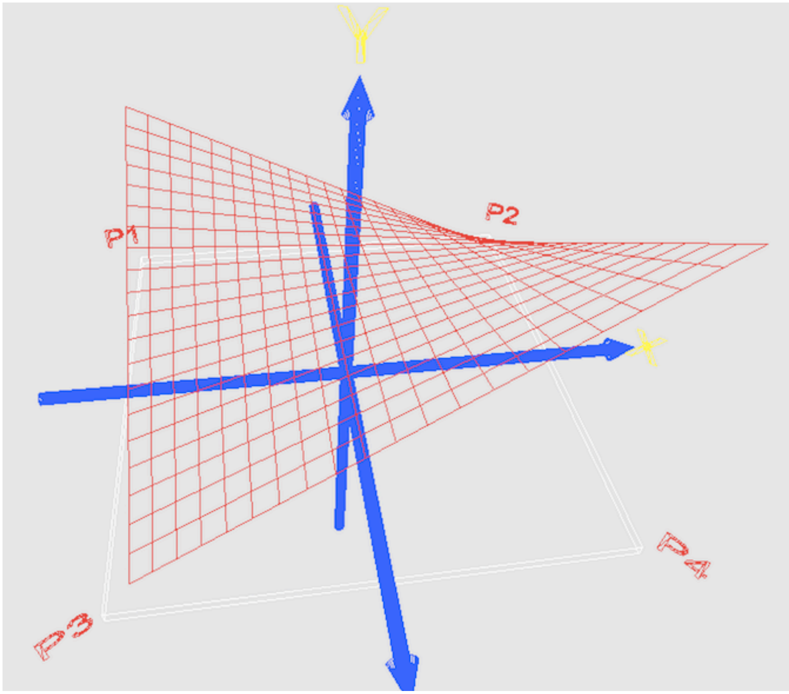


Bilinear Surface
Parametric Representation

$x_1=-1\ x_2=1\ x_3=-1\ x_4=1 \qquad x(u,v)=x_1+u\cdot(x_2-x_1)+v\cdot(x_3-x_1+u\cdot(x_4-x_3-x_2+x_1))$

$y_1=\mathbf{1}\ \ y_2=0\ y_3=0\ y_4=\mathbf{1.5} \qquad y(u,v)=y_1+u\cdot(y_2-y_1)+v\cdot(y_3-y_1+u\cdot(y_4-y_3-y_2+y_1))$

$z_1=-1\ z_2=-1\ z_3=1\ z_4=1 \qquad z(u,v)=z_1+u\cdot(z_2-z_1)+v\cdot(z_3-z_1+u\cdot(z_4-z_3-z_2+z_1))$

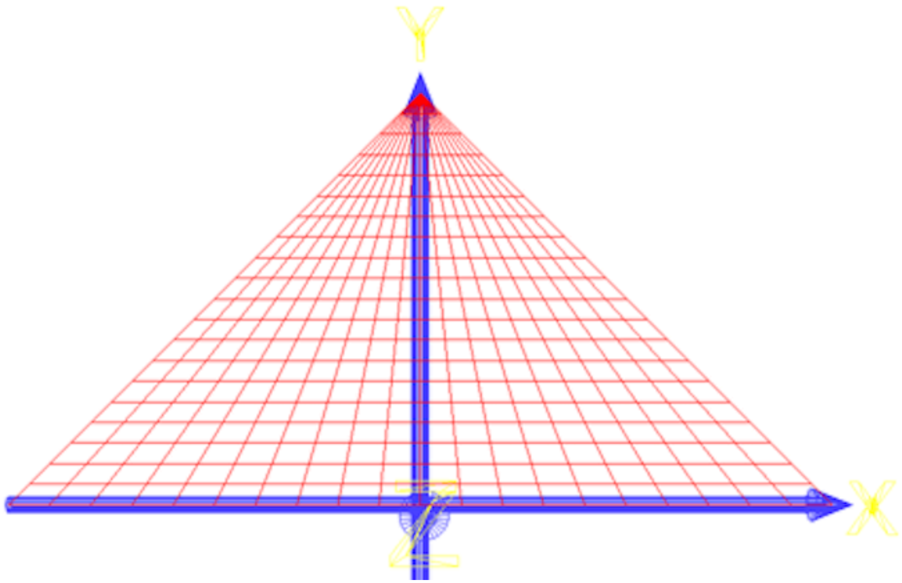


Bilinear Surface Parametric Representation

x_1=-1 x_2=1 x_3=0 x_4=0 $\quad x(u,v)=x_1+u\cdot(x_2-x_1)+v\cdot(x_3-x_1+u\cdot(x_4-x_3-x_2+x_1))$

Y_1=0 y_2=0 y_3=1 y_4=1 $\quad y(u,v)=y_1+u\cdot(y_2-y_1)+v\cdot(y_3-y_1+u\cdot(y_4-y_3-y_2+y_1))$

z_1=0 z_2=0 z_3=0 z_4=0 $\quad z(u,v)=z_1+u\cdot(z_2-z_1)+v\cdot(z_3-z_1+u\cdot(z_4-z_3-z_2+z_1))$



Geometric Shapes: surfaces

Module 3

Lecture 5

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We have learnt that

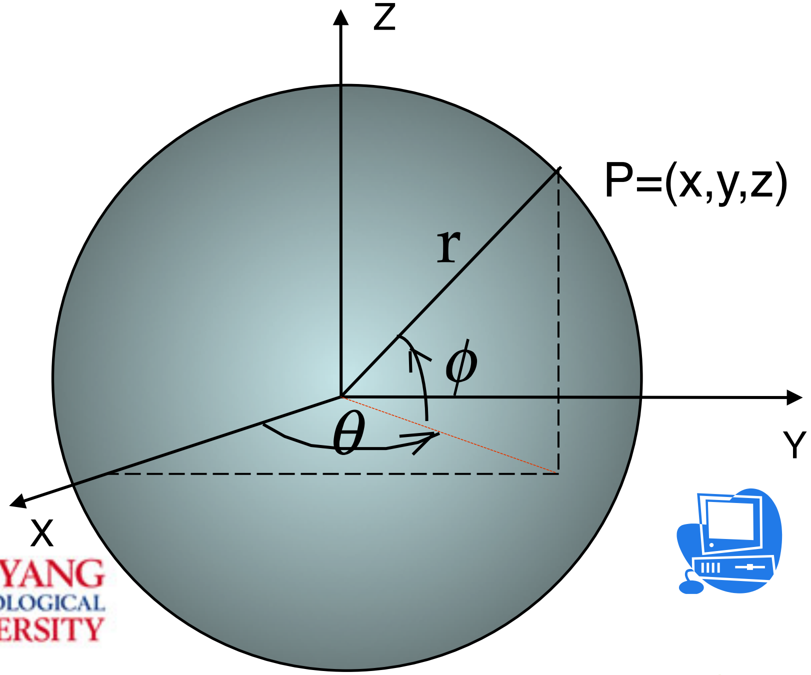
- Plane surfaces can be defined by explicit, implicit and parametric functions
- In implicit (linear) equation Ax+By+Cz+D=0, **N**=[A B C] while D defines displacement from the origin
- To get the plane equation, derive **N** first (e.g., by cross product of two vectors), then substitute any *x,y,z* on the plane to derive D.
- Implicit equation in intercepts can be easily written x/a+y/b+z/c=1
- Parametric definition of plane is based on linear equations of two parameters
- Bilinear interpolation is an extension of linear interpolation for interpolating functions of two variables (e.g., *u, v*). It performs linear interpolation first in one direction, and then again in the other direction. Although each step is linear, the interpolation as a whole is not linear but rather quadratic in the sample location
- Bilinear surface defines a 4-sided polygon, including non-planar surfaces.
- Bilinear surface can be used for writing parametric functions of a triangle as well: two of the vertices are simply merged together which will also simplify the defining formulas.

Quadric Surfaces

- $ax^2+by^2+cz^2+dxy+eyz+fxz+gx+hy+kz+m=0$
- Ellipsoids (including spheres), Elliptic Paraboloid, Hyperbolic Paraboloid, Hyperboloid of One Sheet, Hyperboloid of Two Sheets, Cone, Elliptic Cylinder (including circular cylinders), Hyperbolic Cylinder, Parabolic Cylinder
- Only highlighted surfaces are examinable

Sphere

- Implicit $\quad r^2 - x^2 - y^2 - z^2 = 0$
- Explicit $\quad z = \pm\sqrt{r^2 - x^2 - y^2}$
- Parametric



$$x = r\cos\phi\cos\theta$$

$$y = r\cos\phi\sin\theta$$

$$z = r\sin\phi$$

$$-\pi/2 \leq \phi \leq \pi/2$$

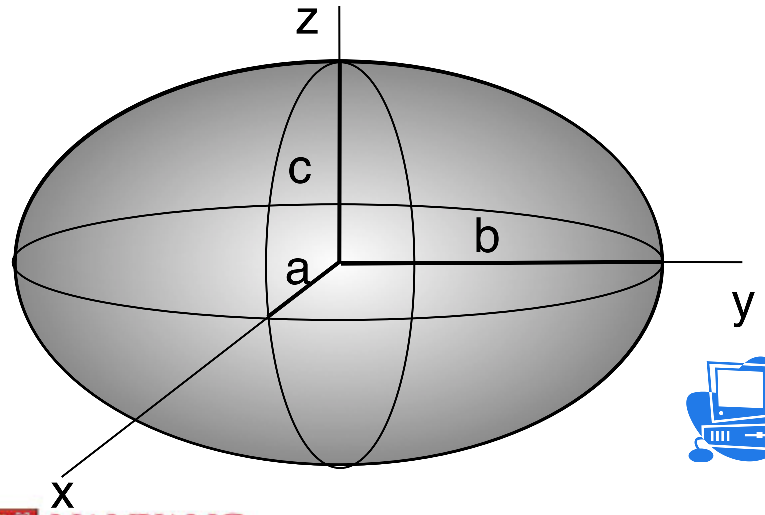
$$-\pi \leq \theta \leq \pi$$

Two parameters !

Ellipsoid

- Implicit $$1-(x/a)^2-(y/b)^2-(z/c)^2=0$$
- Explicit $$z=\pm c\sqrt{1-(x/a)^2-(y/b)^2}$$
- Parametric

By scaling of a unit sphere by a, b, c



$$x=a\cos\phi\cos\theta$$

$$y=b\cos\phi\sin\theta$$

$$z=c\sin\phi$$

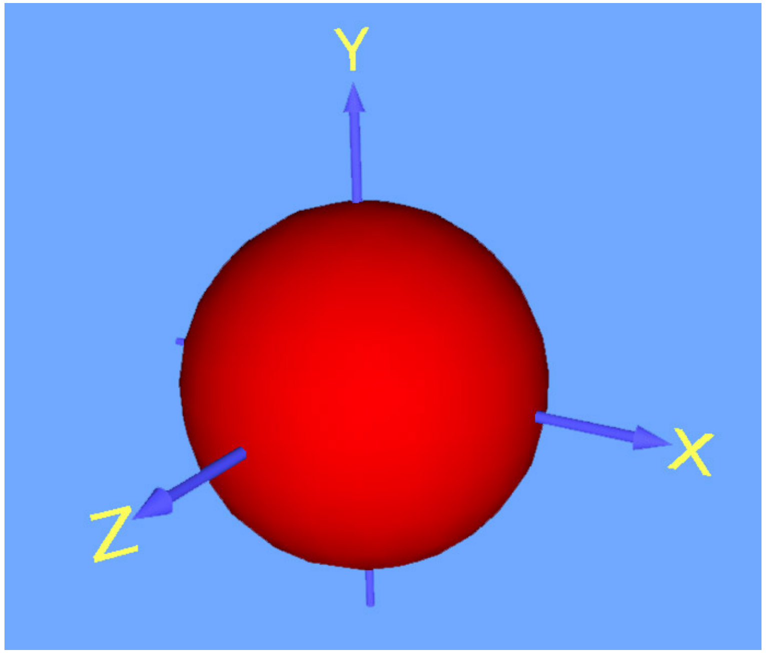
$$-\pi/2\le\phi\le\pi/2$$

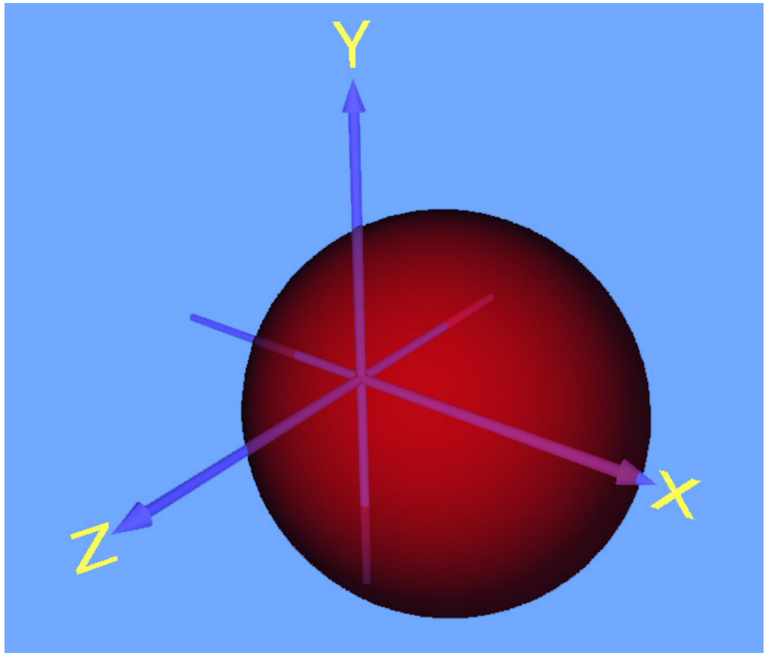
$$-\pi\le\theta\le\pi$$

Two parameters !

Experimenting with Quadrics

$2^2-x^2-y^2-z^2 = 0$ $\qquad$ $2^2-(x-1)^2-y^2-z^2 = 0$

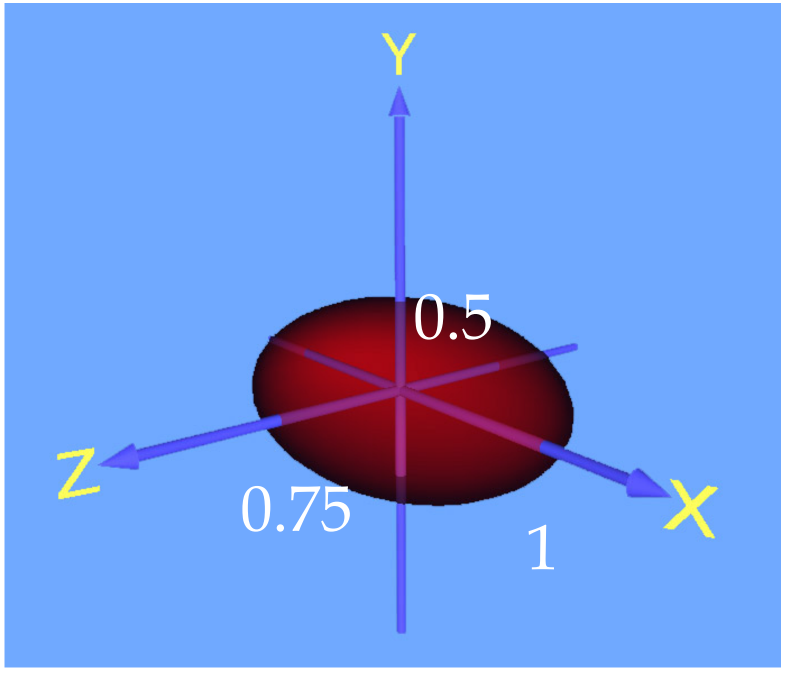




sphere

Experimenting with Quadrics

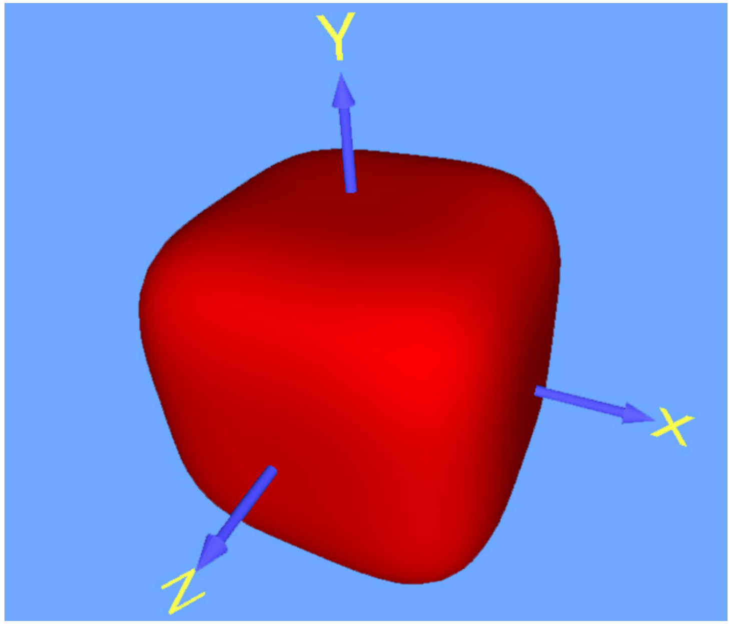
$$1-(x/1)^2-(y/0.5)^2-(z/0.75)^2 = 0$$

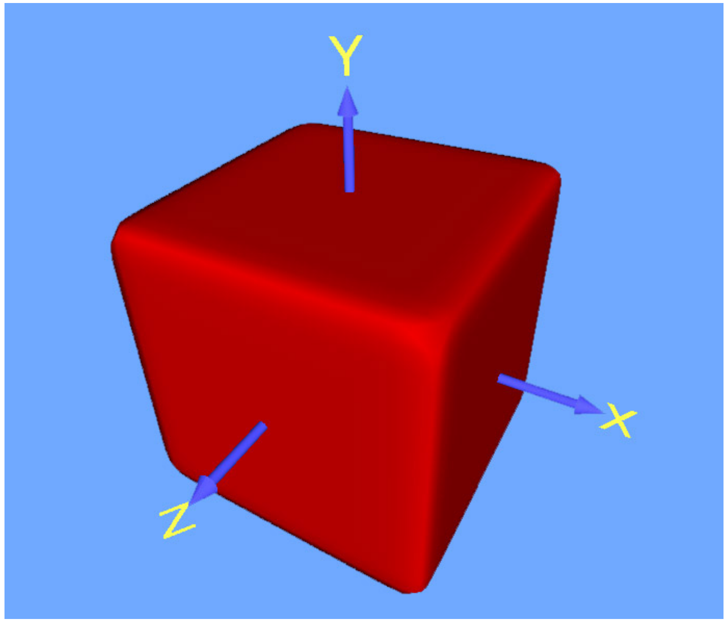


ellipsoid

Experimenting with Quadrics

$1^2 - x^4 - y^4 - z^4 = 0$ $\qquad$ $1^2 - x^{16} - y^{16} - z^{16} = 0$



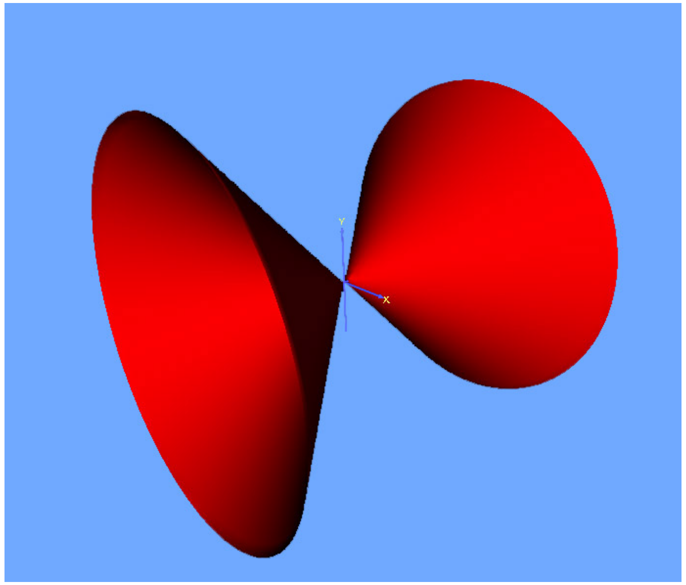


Super-ellipsoid

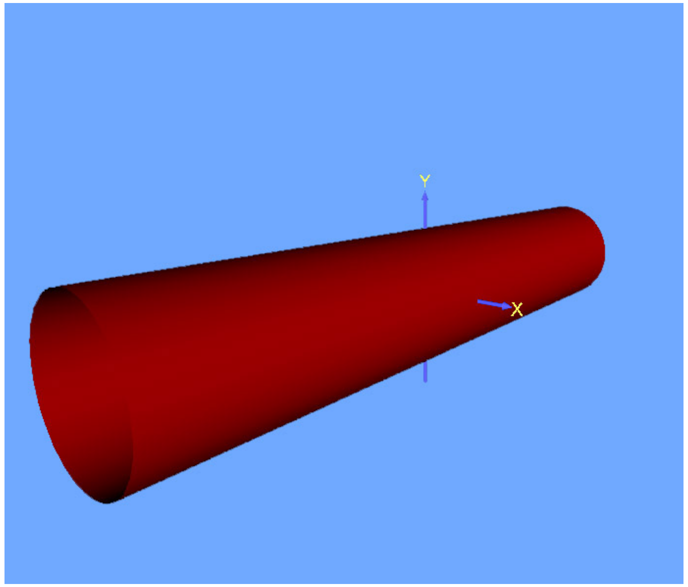
Experimenting with Quadrics

$(z/a)^2-(x/b)^2-(y/c)^2=0$

$1-(x/b)^2-(y/c)^2=0$



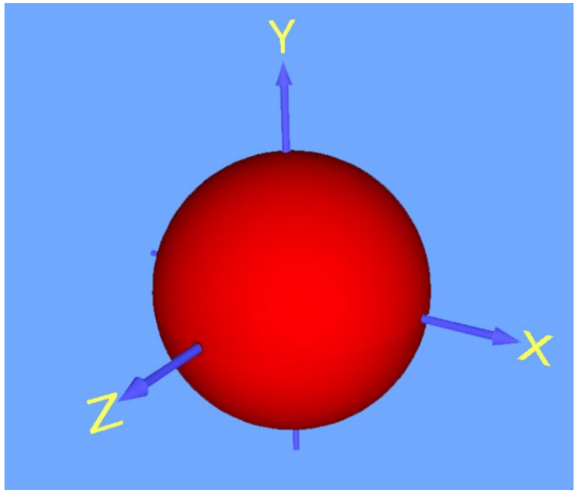
Cone



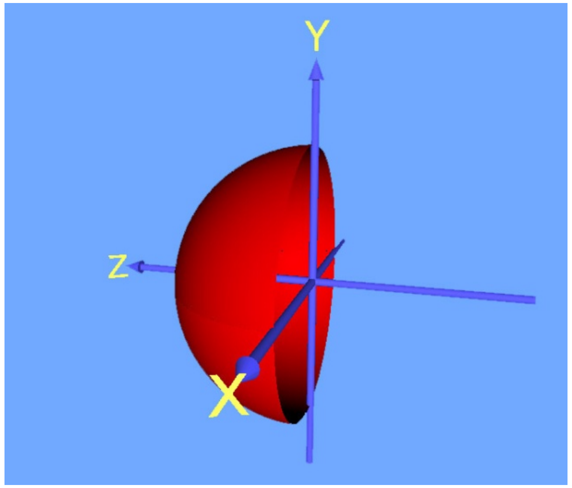
Elliptic Cylinder

Experimenting with Parametric Sphere

$X=0.7*\cos(u)\cdot\cos(v)$
$y=0.7*\cos(u)\cdot\sin(v)$
$z=0.7*\sin(u)$

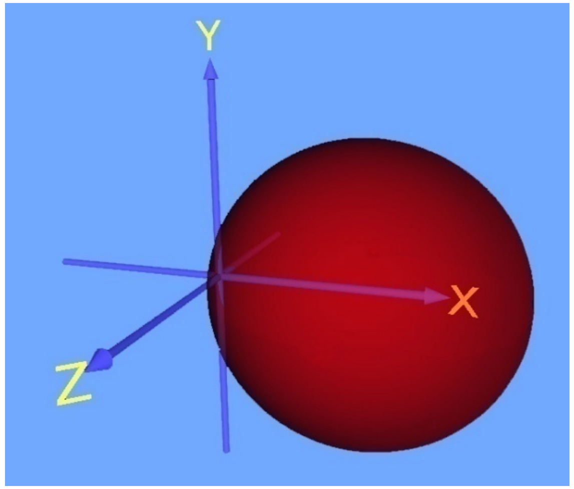


u=[0,2π] v=[0, π]



u=[0, π] v=[0, π]

$x=0.7*\cos(u)\cdot\cos(v)+1$
$y=0.7*\cos(u)\cdot\sin(v)$
$z=0.7*\sin(u)$

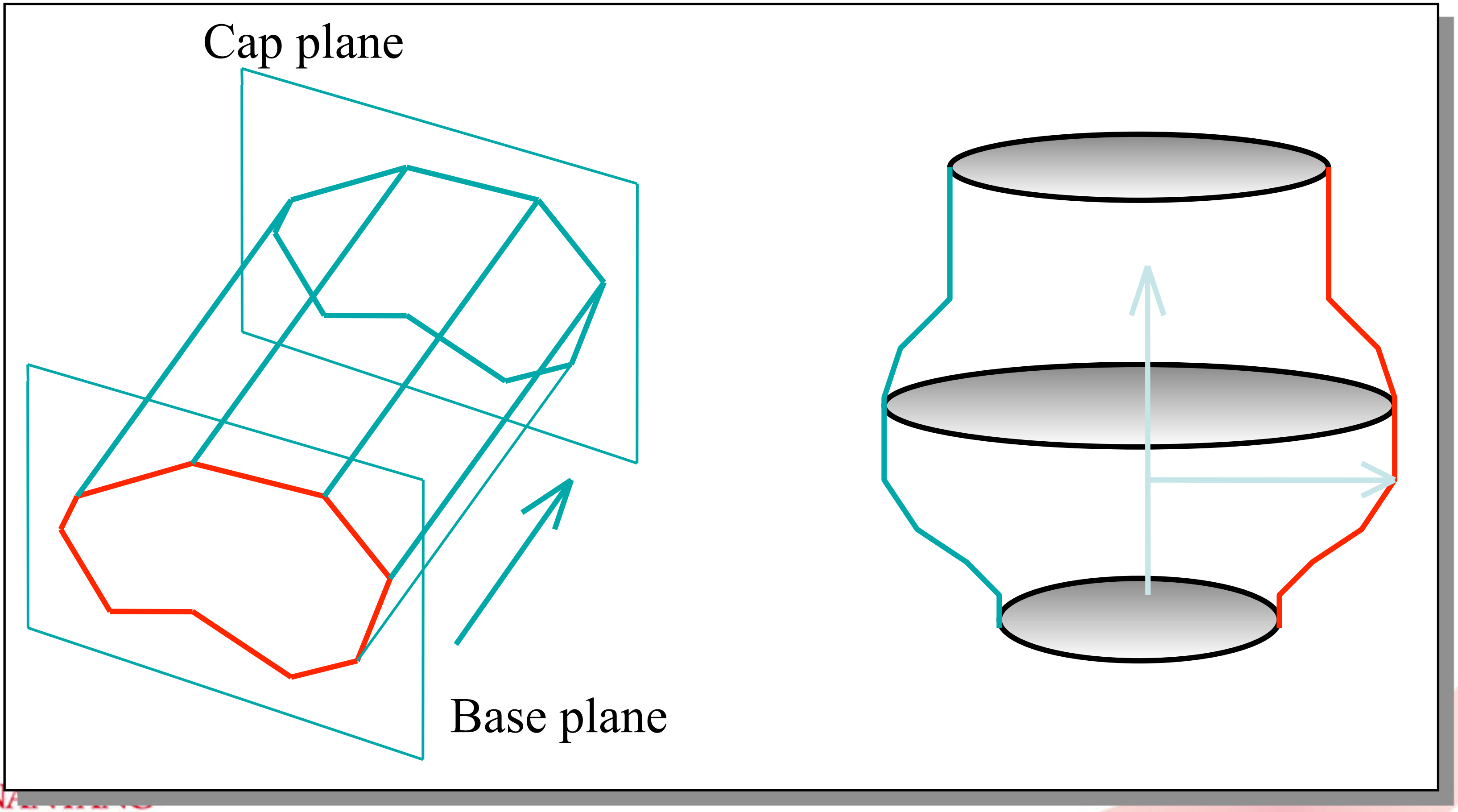


u=[0,2π] v=[0, π]

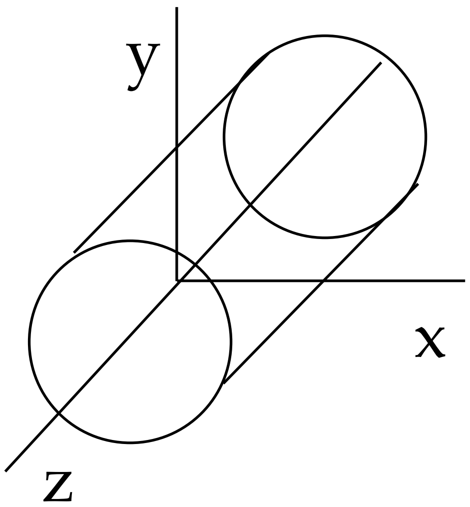
Sweeping

- Shapes are created by curve moving along some path
- Two particular cases of sweeping--*translational* and *rotational sweeping*--can be easily defined parametrically

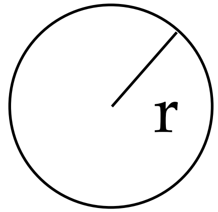
Translational and Rotational Sweeping



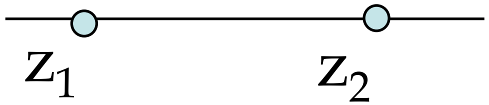
Parametric Representation of Translational Sweeping



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$z = 0 \qquad 0 \le \varphi \le 2\pi$$



$$z = z_1(1-\tau) + z_2\tau$$
$$0 \le \tau \le 1$$



$$x = r\cos\varphi$$
$$y = r\sin\varphi \qquad 0 \le \varphi \le 2\pi$$
$$z = z_1(1-\tau) + z_2\tau \qquad 0 \le \tau \le 1$$

Cylinder by Translational Sweeping a Circle

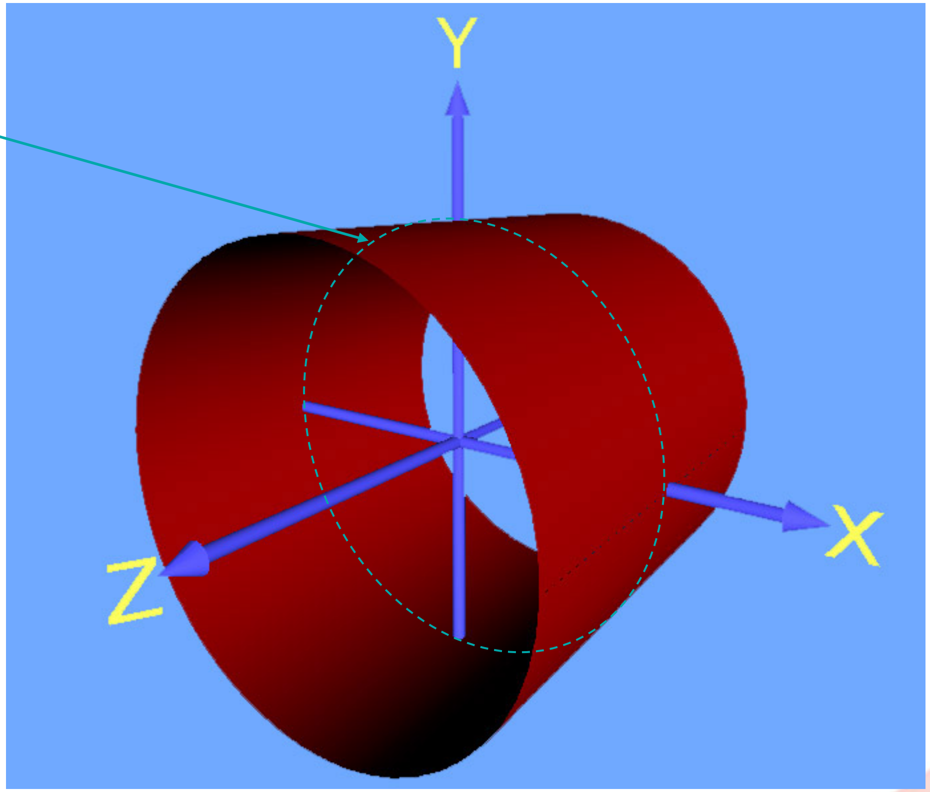
$x=0.5*cos(u)$

$y=0.5*sin(u)$

$z=v$

$u=[0, 2\pi]$

$v=[-1, 1]$



Translational sweeping

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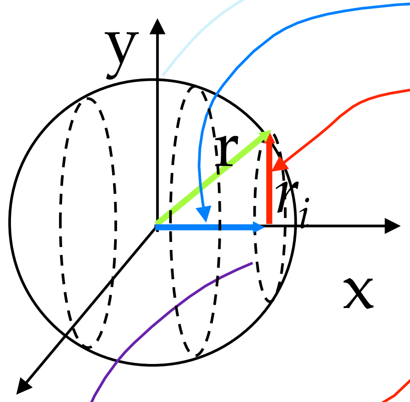
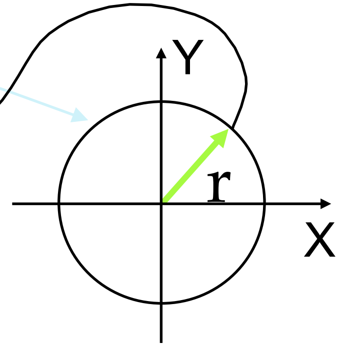
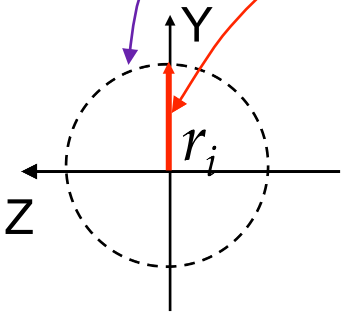
Parametric Representation of Rotational Sweeping. Making a Sphere



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$0 \le \varphi \le 2\pi$$





$$y = r_i\cos\theta$$
$$z = r_i\sin\theta$$
$$0 \le \theta \le \pi$$

$$x = r\cos\varphi$$
$$y = r\sin\varphi\cos\theta$$
$$z = r\sin\varphi\sin\theta$$
$$0 \le \varphi \le 2\pi$$
$$0 \le \theta \le \pi$$

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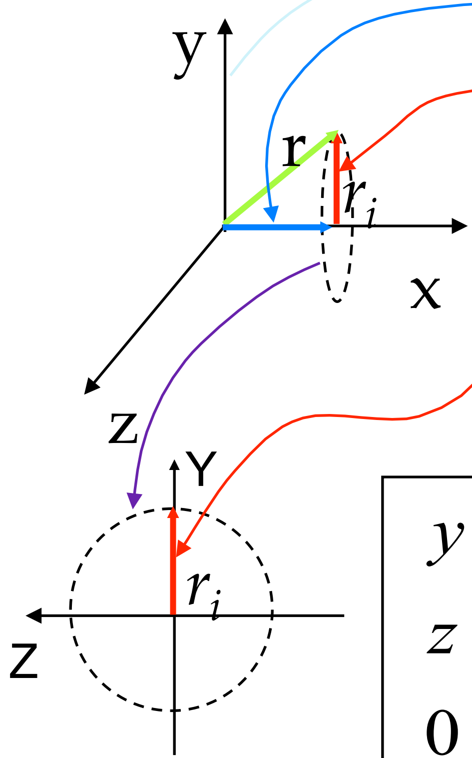
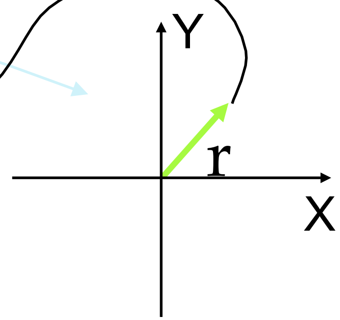
Parametric Representation of Rotational Sweeping. Making a Cone



$$x = u$$
$$y = u$$
$$0 \le u \le r/\sqrt{2}$$



$$y = r_i \cos\theta$$
$$z = r_i \sin\theta$$
$$0 \le \theta \le \pi$$

$$x = u$$
$$y = u\cos\theta$$
$$z = u\sin\theta$$
$$0 \le u \le r/\sqrt{2}$$
$$0 \le \theta \le \pi$$

Cone by Rotational Sweeping of a Straight Line Segment

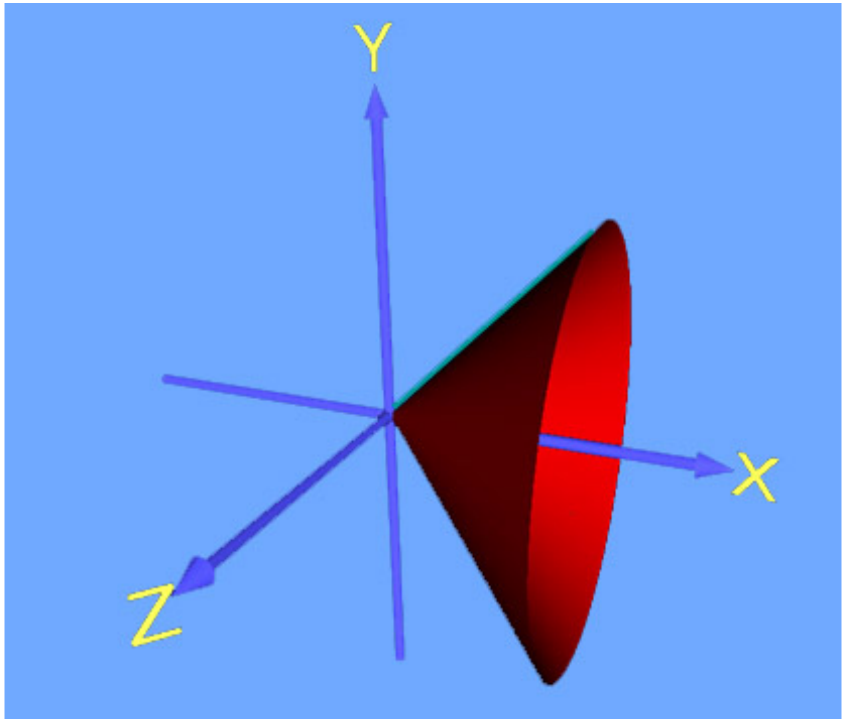
$$x = u$$

$$y = u\cos v$$

$$z = u\sin v$$

$$0 \le u \le 1$$

$$0 \le v \le 2\pi$$



Rotational sweeping

Surfaces. Summary

- Polygonal representation - polygon meshes
- Analytic representations
  Define how a point can move along the surface with 2 degrees of freedom (forward, backward, left, right).
  - Explicit representation.
    Seldom used because of axes dependency. Can be derived from implicit functions if needed.
  - Implicit representation
  - Parametric representation.
    Two parameters must be used to define a surface. Often, the parameters can be considered as a kind of latitude and longitude coordinates for locating a point on the surface.

Geometric Shapes: solids

Module 3

Lecture 6

Geometric Shapes

- Geometry has no color and texture
- Points
- Curves
- Surfaces
- Solid objects

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Learning objectives

- To understand how solids can be used in solving data visualization problems
- To understand solids as objects with *3 degree of freedom*
- To understand what mathematical representations are the most efficient for defining and displaying solids
- To understand how different coordinate systems can be used together for deriving mathematical representations of solids
- To understand solids as objects created *by moving surfaces*
- To understand how complex solids can be created from *combinations of other solids*

Geometric Shapes

- Geometry has no color and texture
- Points – 0 degree of freedom shape
- Curves – 1 degree of freedom shape
- Surfaces – 2 degree of freedom shape
- Solid objects – 3 degree of freedom shape

- 2 and 3 dimensional spaces
- Time is yet another dimension

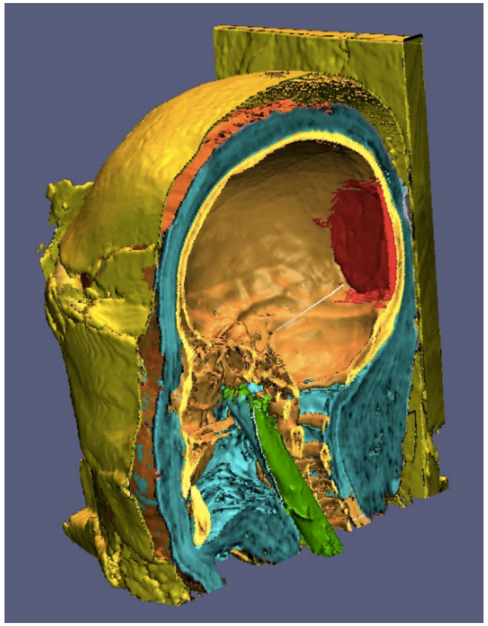
- Displayed as pixels, voxels, polylines, and shaded polygons

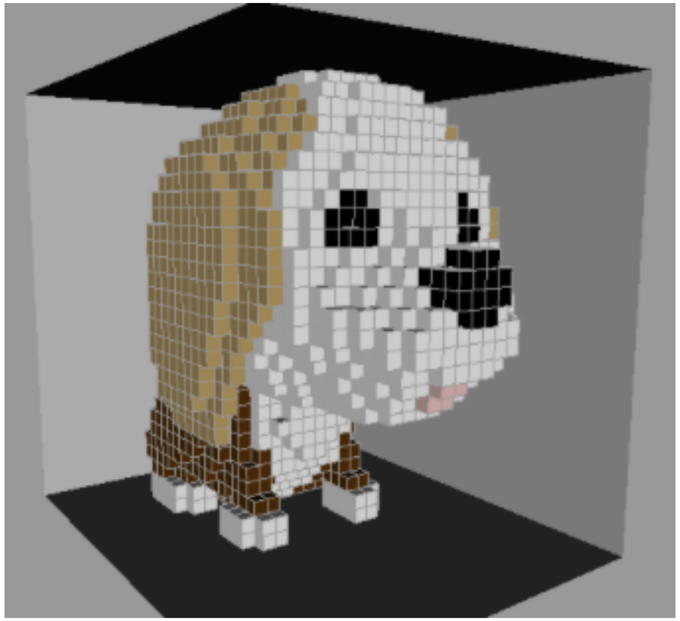
Solid Objects

- Voxels (volume elements)
- Parametric representation
- Explicit (variant of implicit) representation

Voxels

- Voxel (volumetric pixel or Volumetric Picture Element) is a volume element, representing a value on a regular grid in three dimensional space.





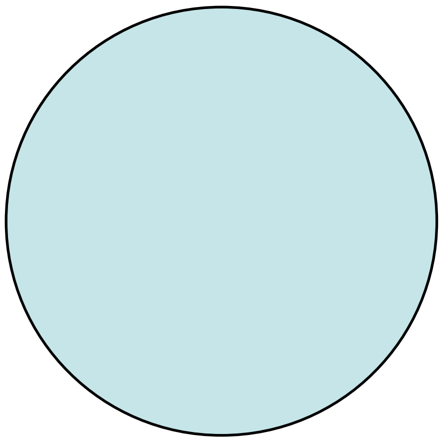
Solid Objects

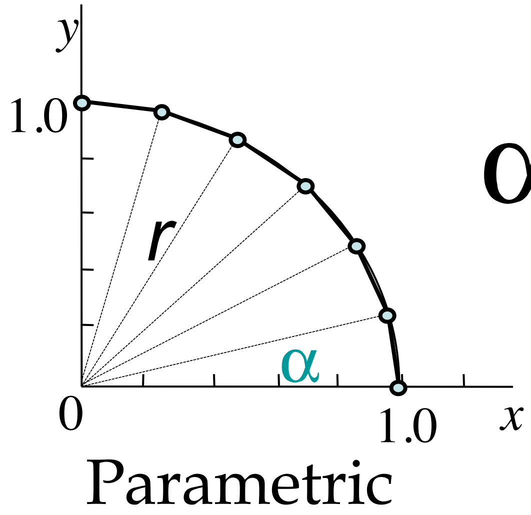
- Voxels (volume elements)
- Parametric representation
- Explicit (variant of implicit) representation

Using parametric functions for defining solid objects

- Let's add one more degree of freedom which is an additional parameter

Circular disk
Parametric Representation





$\alpha = \theta$

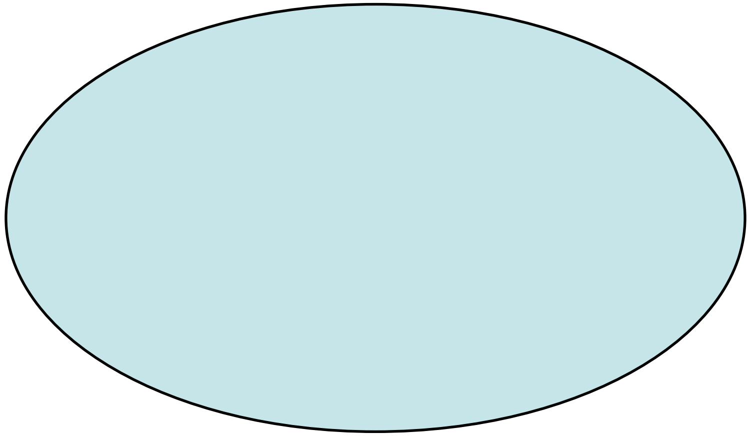
$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

r =[0, R], θ =[θ_1, θ_2]

Two parameters to define a 2D disk or its part !

Elliptical disk
Parametric Representation



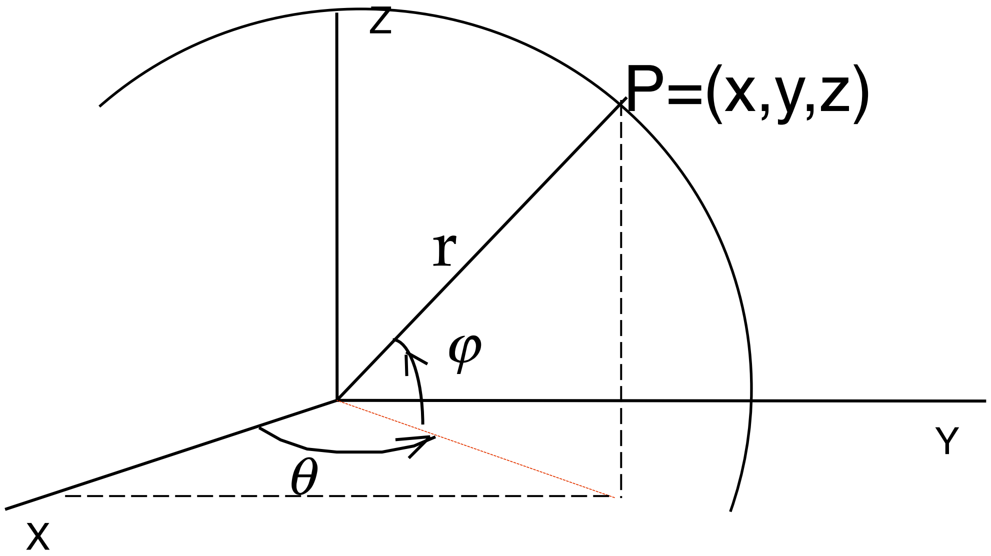
$$x = a * k * \cos(\theta)$$
$$y = b * k * \sin(\theta)$$

$$k = [0, 1],\ \theta = [\theta_1, \theta_2]$$

Two parameters to define an elliptical disk or its sector !

Solid Sphere

- Parametric



$$x = r\cos\varphi\cos\theta$$

$$y = r\cos\varphi\sin\theta$$

$$z = r\sin\varphi$$

$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$

$$-\pi \le \theta \le \pi$$

$$0 \le r \le R$$

Three parameters !

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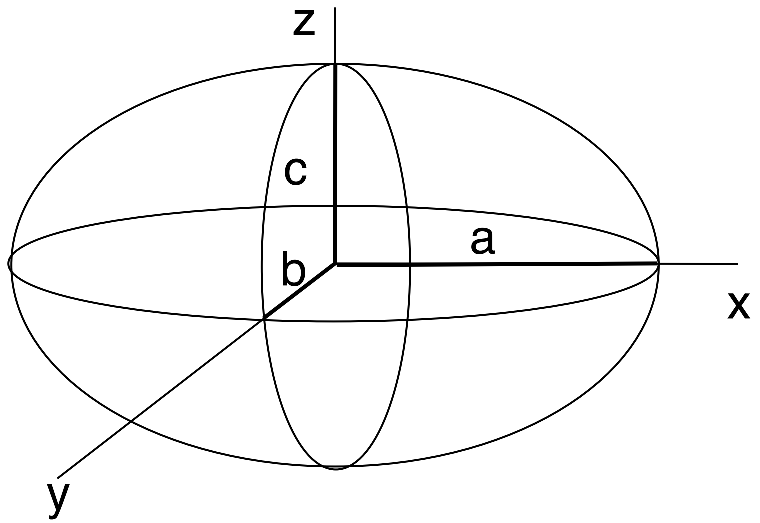
Solid Ellipsoid

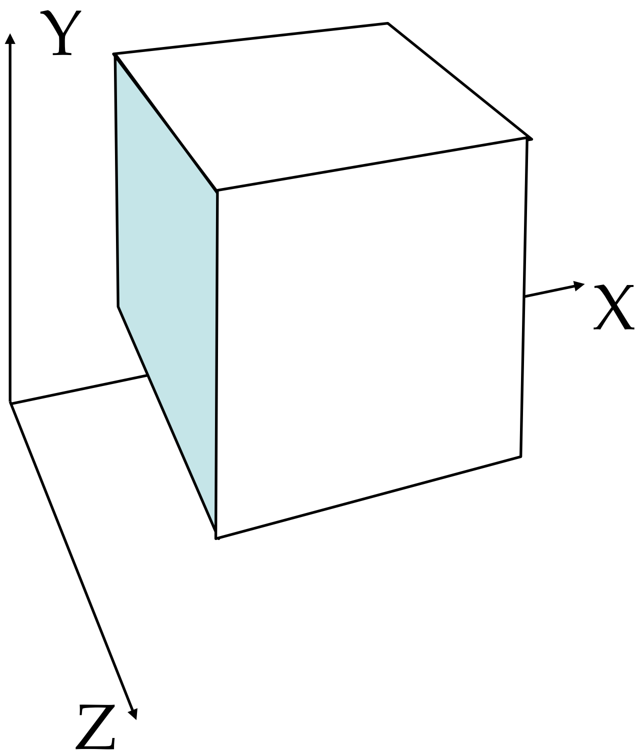
- Parametric



$$x = k * a * \cos\varphi \cos\theta$$
$$y = k * b * \cos\varphi \sin\theta$$
$$z = k * c * \sin\varphi$$
$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$
$$-\pi \le \theta \le \pi$$
$$0 \le k \le 1$$

Three parameters !

Parametrically-defined solid box



$$x = u$$

$$y = v$$

$$z = w$$

$$u_1 \le u \le u_2$$

$$v_1 \le v \le v_2$$

$$w_1 \le w \le w_2$$

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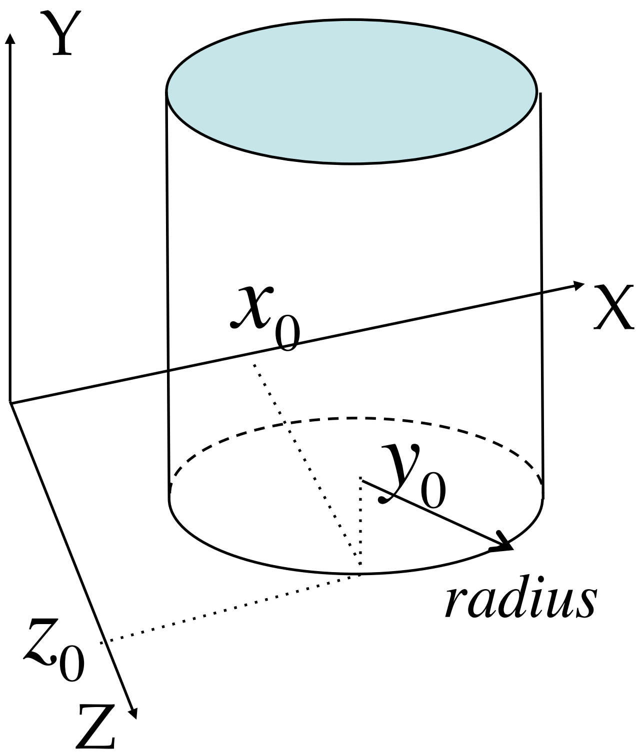
http://www.ntu.edu.sg/home/assourin/FVRML.htm

Parametrically-defined solid cylinder



$$x = u \cdot radius \cdot \sin(2\pi v) + x_0$$

$$y = w \cdot height + y_0$$

$$z = u \cdot radius \cdot \cos(2\pi v) + z_0$$

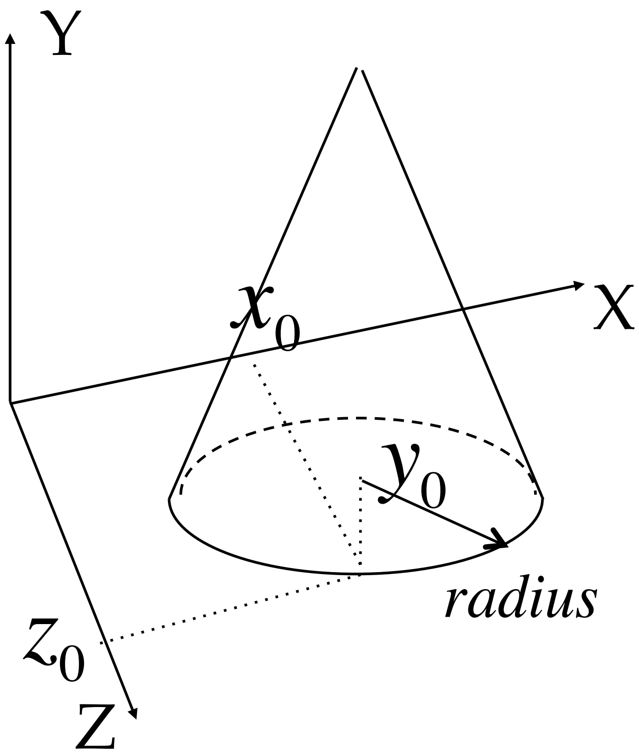
$$0 \le u \le 1$$

$$0 \le v \le 1$$

$$0 \le w \le 1$$

Parametrically-defined solid cone

$$cone\_radius(\tau) = radius + \tau(0 - radius) = radius(1-\tau), \tau = [0,1]$$



$$x = u \cdot radius \cdot (1-w) \cdot \sin(2\pi v) + x_0$$

$$y = w \cdot height + y_0$$

$$z = u \cdot radius \cdot (1-w) \cdot \cos(2\pi v) + z_0$$

$$0 \le u \le 1$$

$$0 \le v \le 1$$

$$0 \le w \le 1$$

Lab Experiment 3

- *Download file **surface.wrl** and **solid.wrl** from the course-site. Use them as templates for the following exercises.*
- *Define parametrically in separate files*
  - *3D plane,*
  - *3D triangle,*
  - *bilinear surface,*
  - *sphere,*
  - *ellipsoid,*
  - *cone.*
- *Explore how the shapes change when their sampling resolution is changed.*
- *Define parametrically in separate files*
  - *solid box,*
  - *solid sphere,*
  - *solid cylinder,*
  - *solid cone.*
- *Consider conversion of any closed surface into a solid object by introducing the third parameter. For example, convert a cylindrical surface into a solid cylinder.*
- *Study how to make surfaces and solids using the concept of translational and rotational sweeping.*
- *Use curve y=sin(x) for making a solid by applying rotational and translational sweepings together.*
- *Create a folder with name Lab3 and copy there all the FVRML files you have experimented with.*
- *Write a brief report explaining what each file defines and also copy it to Lab3 folder.*

Solid Objects

- Voxels (volume elements)
- Parametric representation
- Explicit (variant of implicit) representation

Using explicit functions for defining solid objects

Let’s change in any implicit function “=“ to “≤” or “≥”

$$f(x, y) = 0 \qquad g = f(x,y) \leq 0 \qquad \text{or} \qquad g = f(x,y) \geq 0$$

$$f(x,y,z) = 0 \qquad g = f(x,y,z) \leq 0 \qquad \text{or} \qquad g = f(x,y,z) \geq 0$$

It becomes an explicit function in +1 dimension (scientific name Frep), i.e. $g = f(x,y,z) \geq 0$ evaluates some coordinate or value in the dimension other than x, y, z.

In this course, we will ONLY use $\geq \mathbf{0}$:

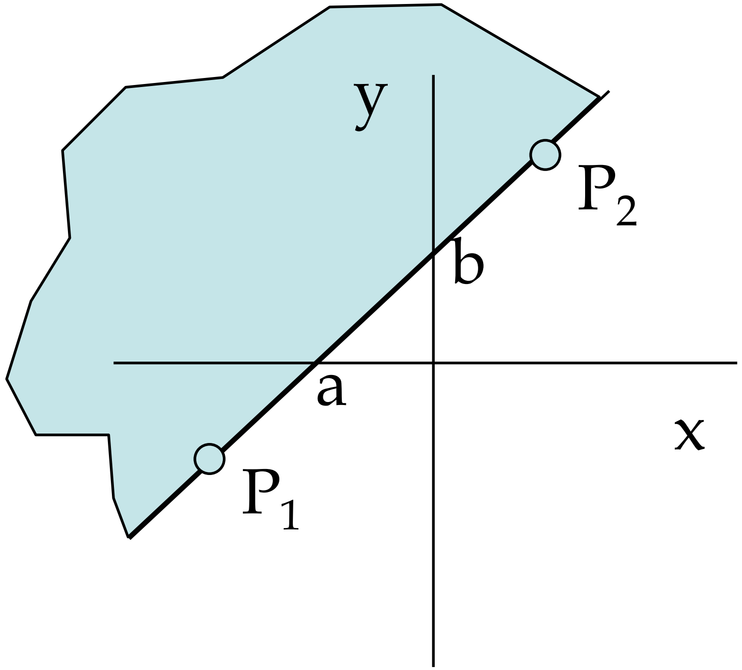
$\boldsymbol{g = f(x, y) \geq 0}$ and $\boldsymbol{g = f(x, y, z) \geq 0}$

to be consistent with the rendering algorithm and other mathematics used in the remaining part of this module.

Half-plane
Implicit Representation

$$Ax + By + C \geq 0$$

$$\frac{x}{a} + \frac{y}{b} - 1 \geq 0$$



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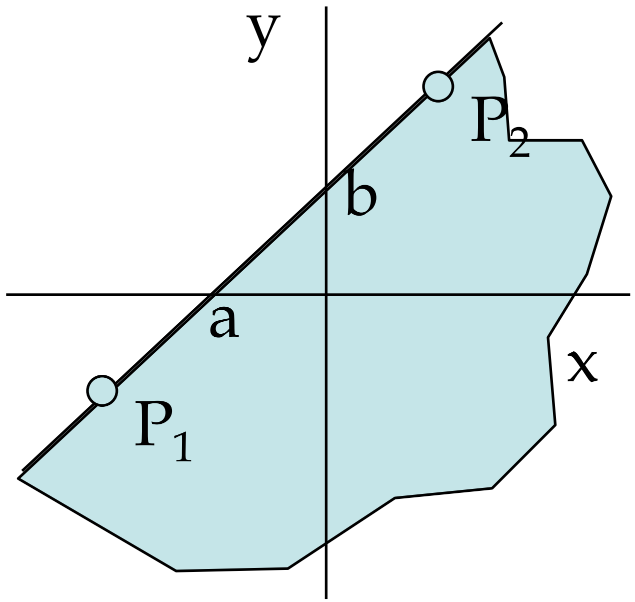
Half-plane
Implicit Representation

- Implicit

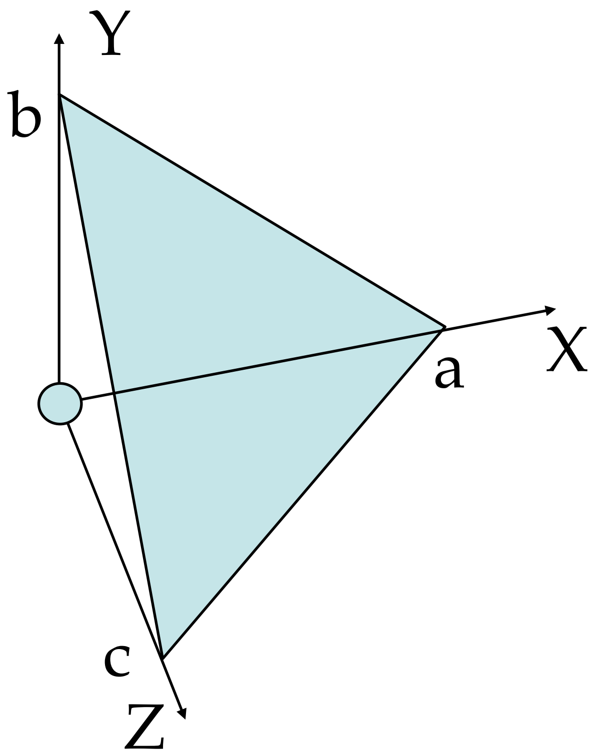
$$-(Ax + By + C) \geq 0$$

$$-\frac{x}{a} - \frac{y}{b} + 1 \geq 0$$



Plane-bounded Half-space

- Below the plane

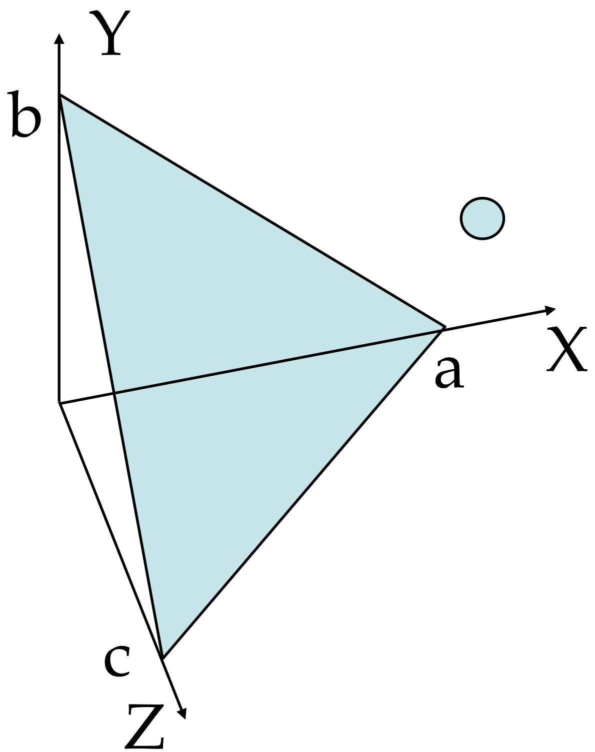


$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1=0 \Rightarrow$$

$$1-\frac{x}{a}-\frac{y}{b}-\frac{z}{c}\geq 0$$

Plane-bounded Half-space

- Above the plane

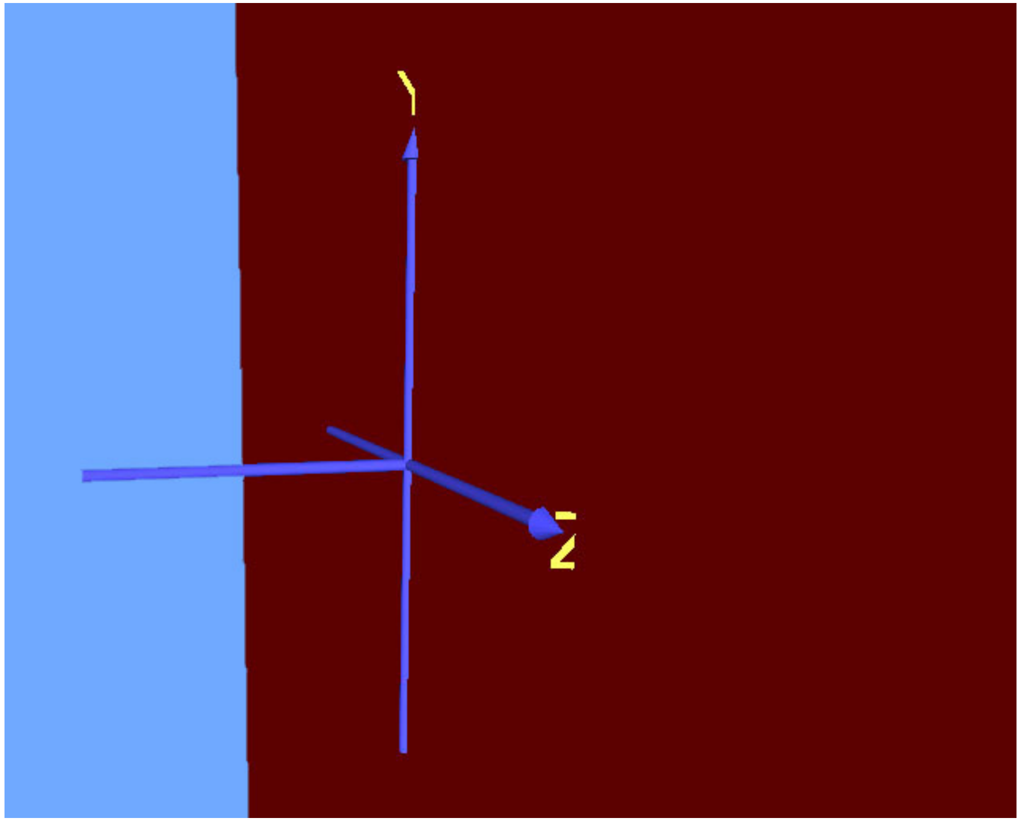


$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1=0 \Rightarrow$$

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1 \geq 0$$

Plane-bounded Half-space

- $x \geq 0$



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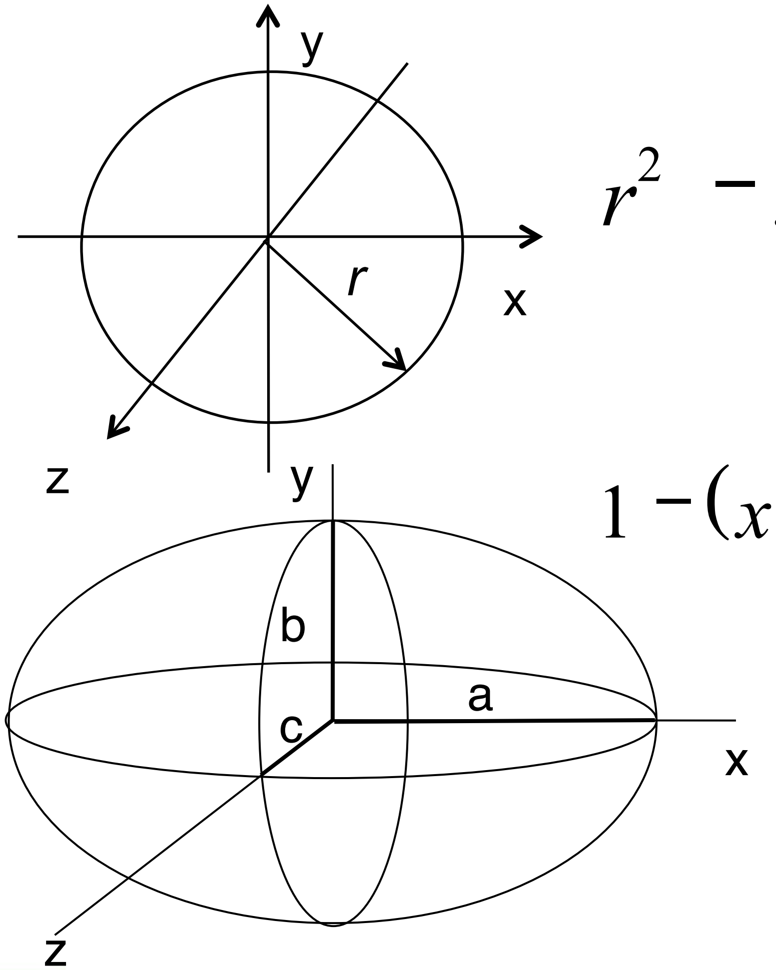
Solid Sphere and Ellipsoid



$$r^2 - x^2 - y^2 - z^2 \geq 0$$

$$1 - (x/a)^2 - (y/b)^2 - (z/c)^2 \geq 0$$

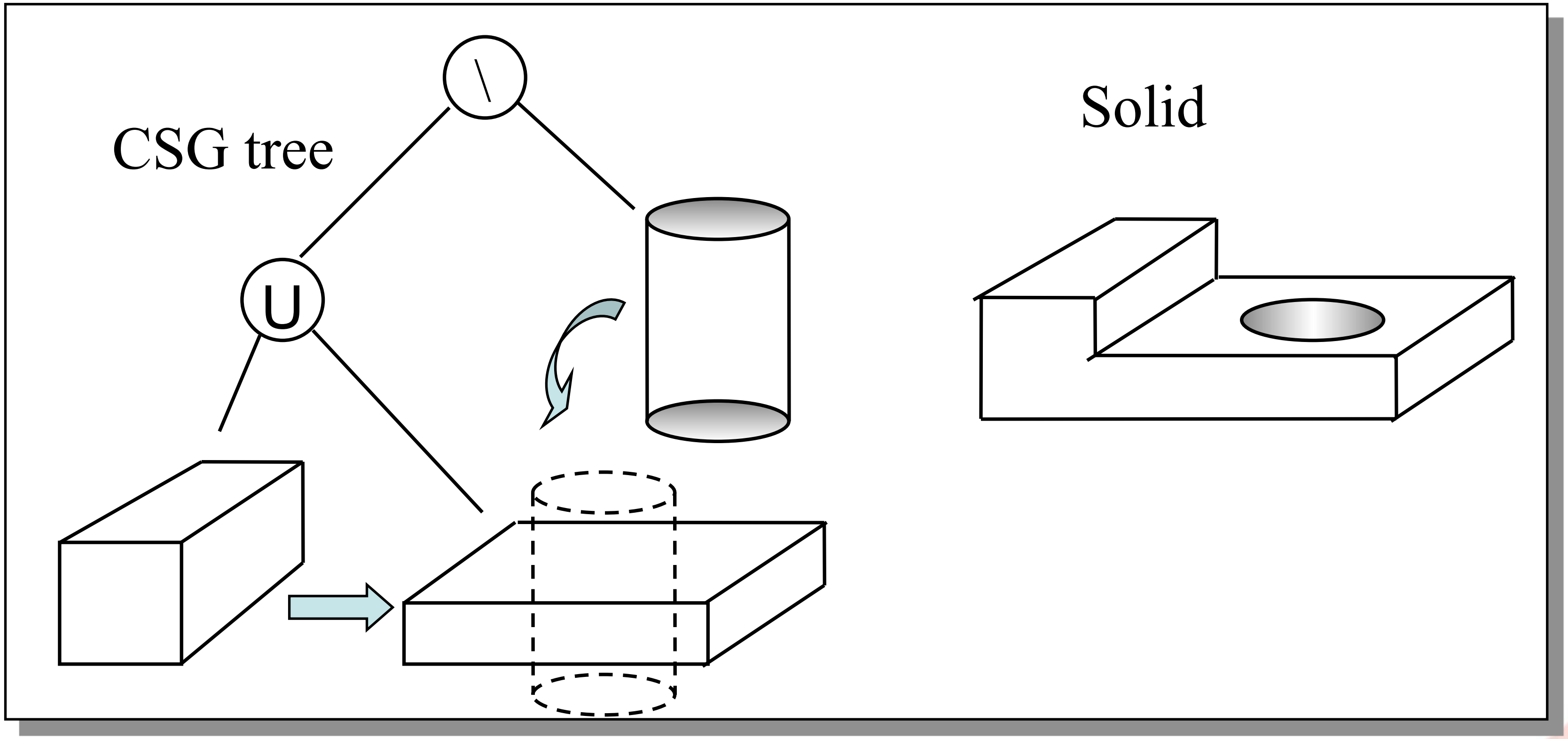
Creating Complex Shapes from Simple Shapes

- Constructive Solid Geometry (CSG)
- Sweeping

Constructive Solid Geometry (CSG)

- CSG is a family of schemes introduced for representing rigid solids as Boolean constructions and combinations of solid components.
- The three basic operators *union* U, *intersection* ∩, and *difference* \ are applied to primitive objects.
- In CSG, objects are represented as binary trees, called *CSG trees*. Each leaf is a primitive object and each non-terminal node is either a Boolean operator or a motion (translation, rotation) which operates on the subnodes.

Constructive Solid Geometry



Implicit Representation of CSG

$G: \quad f(x, y, z) \geq 0$

$G_3 = G_1 \cup G_2: \quad f_3 = f_1 \vee f_2 = \max(f_1, f_2)$ Union

$G_3 = G_1 \cap G_2: \quad f_3 = f_1 \wedge f_2 = \min(f_1, f_2)$ Intersection

$G_3 = -G_1: \quad f_3 = -f_1$ Outer part or Complement

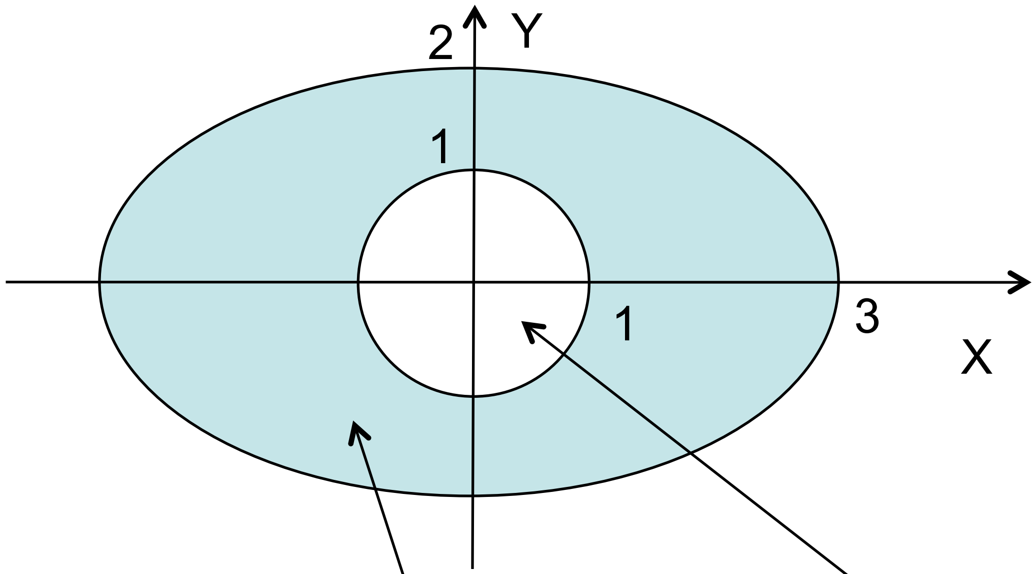
$G_3 = G_1 \setminus G_2: \quad f_3 = f_1 \setminus f_2 = \min(f_1, -f_2)$ Subtraction

Example:

$G_5 = G_1 \cup ((G_2 \cap G_3) \setminus G_4):$

$f_5 = f_1 \vee ((f_2 \wedge f_3) \setminus f_4) = \max(f_1, \min(\min(f_2, f_3), -f_4)) \geq 0$

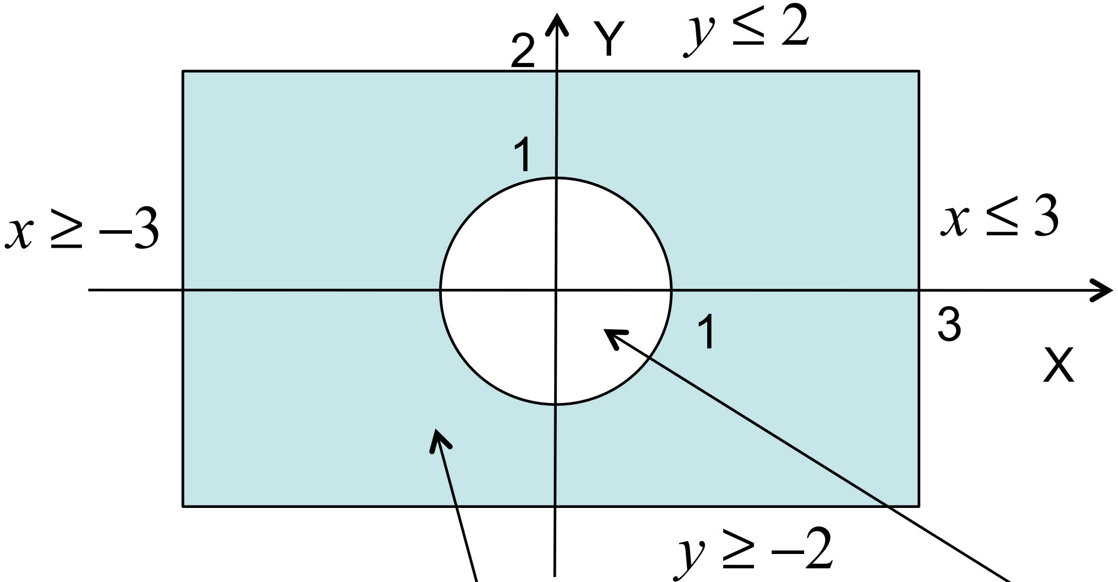
Implicit Representation of Boolean Operations



$$f(x,y)=\min\left(\left(1-\left(\frac{x}{3}\right)^2-\left(\frac{y}{2}\right)^2\right),-\left(1^2-x^2-y^2\right)\right)\geq 0$$

subtraction

Implicit Representation of Boolean Operations



$$f(x,y) = \min\left(\min(3-x, x+3, 2-y, y+2), -\left(1^2 - x^2 - y^2\right)\right) \ge 0$$

intersection

subtraction

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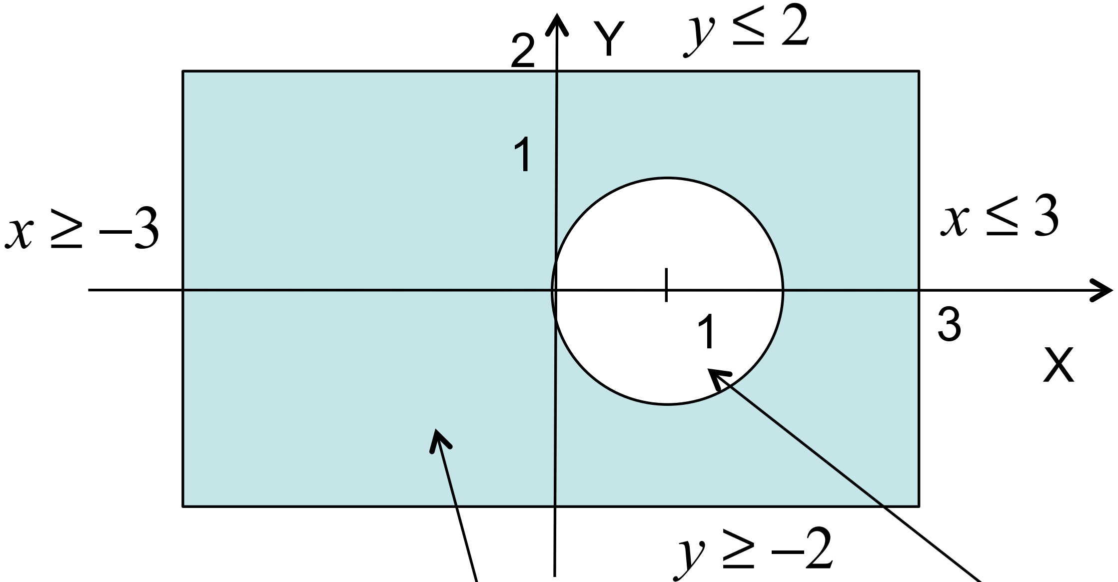
Implicit Representation of Boolean Operations

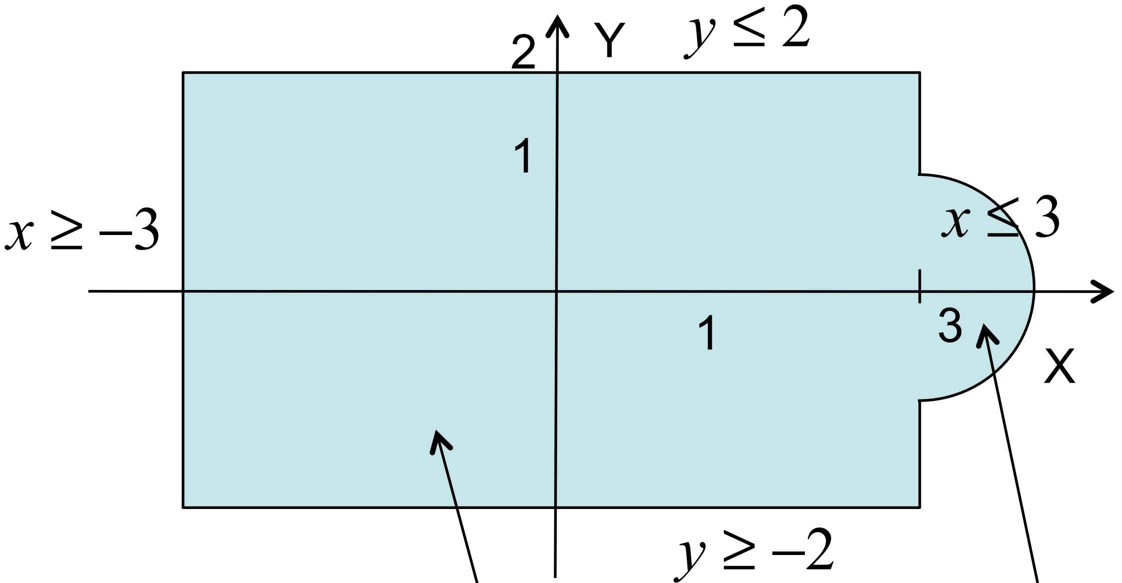


$$f(x, y) = \min\left(\min(3 - x, x + 3, 2 - y, y + 2), -\left(1^2 - (x-1)^2 - y^2\right)\right) \geq 0$$

intersection

subtraction

Implicit Representation of Boolean Operations

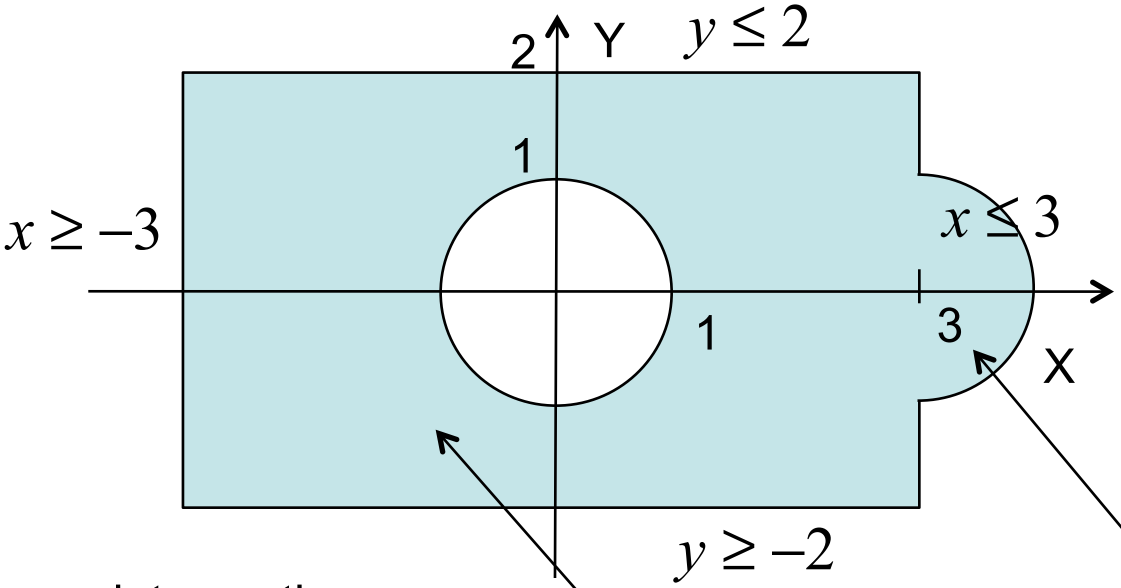


$$f(x,y)=\max\Big(\min(3-x,x+3,2-y,y+2),\big(1^2-(x-3)^2-y^2\big)\Big)\ge 0$$

intersection

union

Implicit Representation of Boolean Operations



subtraction

union

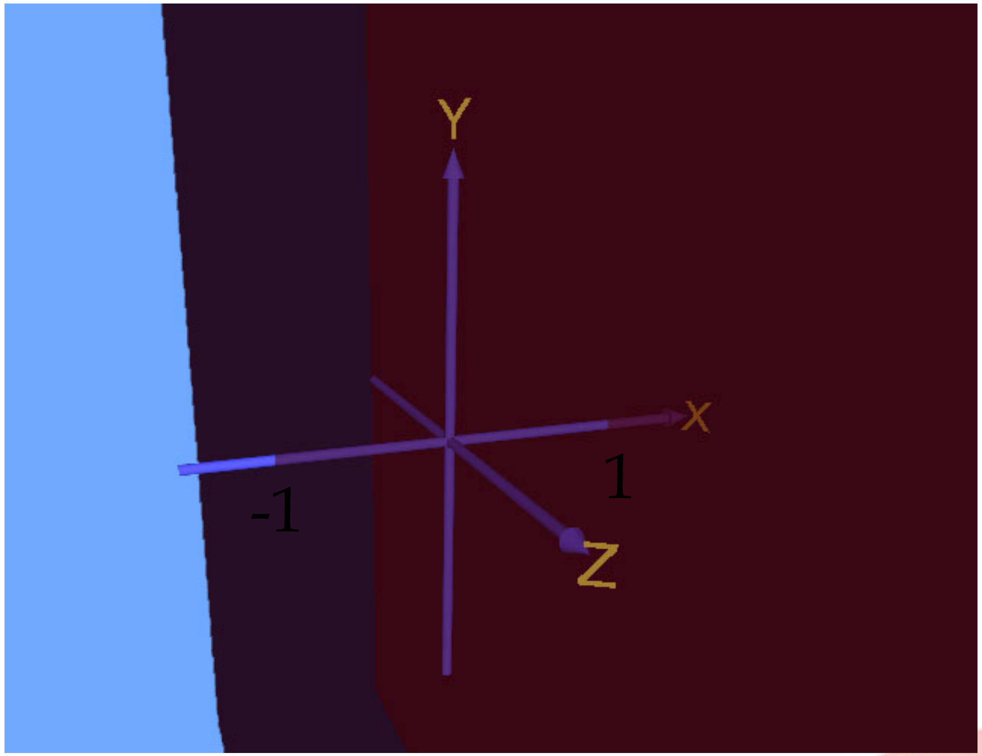
intersection

$$f(x,y) = \min\Big(\max\big(\min(3-x, x+3, 2-y, y+2), \big(1^2-(x-3)^2-y^2\big)\big), -\big(1^2-x^2-y^2\big)\Big) \ge 0$$

CSG with Implicit Functions

- *min(x+1,1-x)* ≥ 0

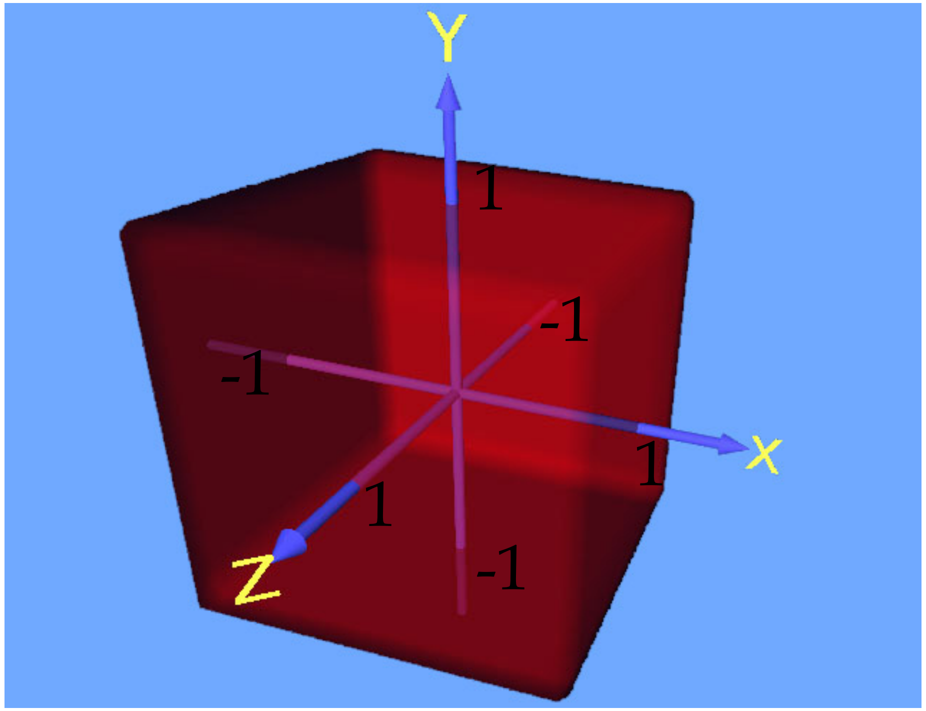
intersection



CSG with Implicit Functions

- *min(x+1,1-x,y+1,1-y,z+1,1-z)* ≥ 0

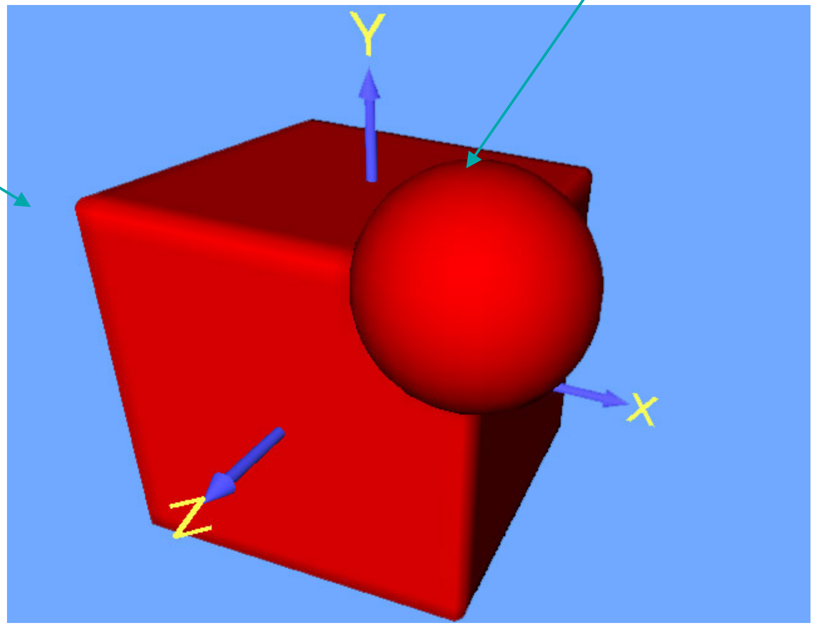
intersection



CSG with Implicit Functions

- $max(\ min(x+1,1-x,y+1,1-y,z+1,1-z),$
  $0.5^2-(x-1)^2-(y-1)^2-(z-1)^2\ \ ) \geq 0$

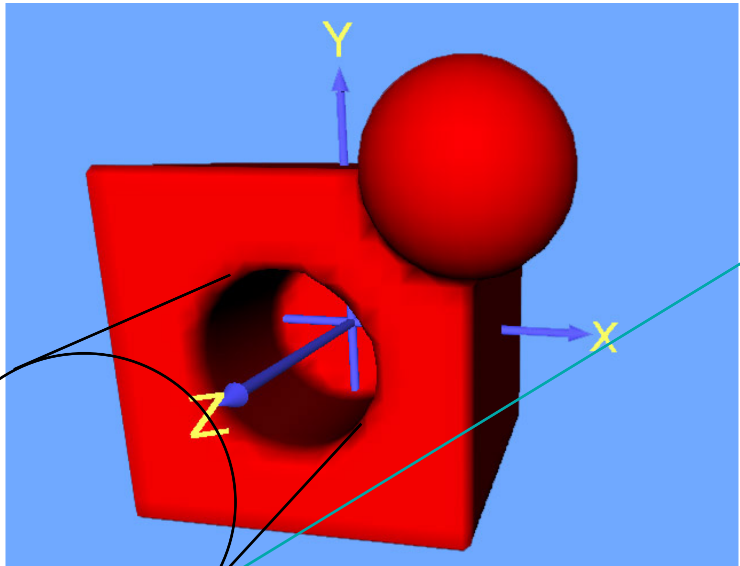
union

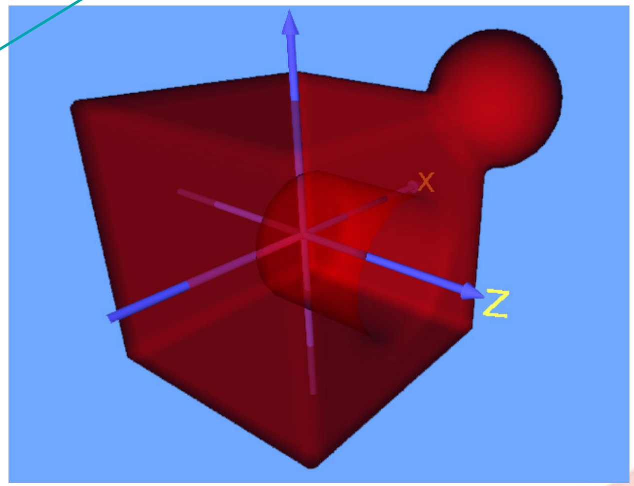


CSG with Implicit Functions

difference

- $min(max(min(x+1,1-x,y+1,1-y,z+1,1-z),$
  $0.5^2-(x-1)^2-(y-1)^2-(z-1)^2), - min(0.5^2-x^2-y^2,z)) \geq 0$





Geometric Shapes: Solid Sweeping

Module 3

Lecture 7

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Summary. Constructive Solid Geometry using Function Reprentations (FReps)

$$G: \quad f(x,y,z) \geq 0$$

$$G_3 = G_1 \cup G_2: \quad f_3 = f_1 \vee f_2 = \max(f_1, f_2)$$ Union

$$G_3 = G_1 \cap G_2: \quad f_3 = f_1 \wedge f_2 = \min(f_1, f_2)$$ Intersection

$$G_3 = -G_1: \quad f_3 = -f_1$$ Outer part or Complement

$$G_3 = G_1 \setminus G_2: \quad f_3 = f_1 \setminus f_2 = \min(f_1, -f_2)$$ Subtraction

Example:

$$G_5 = G_1 \cup ((G_2 \cap G_3) \setminus G_4):$$

$$f_5 = f_1 \vee ((f_2 \wedge f_3) \setminus f_4) = \max(f_1, \min(\min(f_2, f_3), -f_4)) \geq 0$$

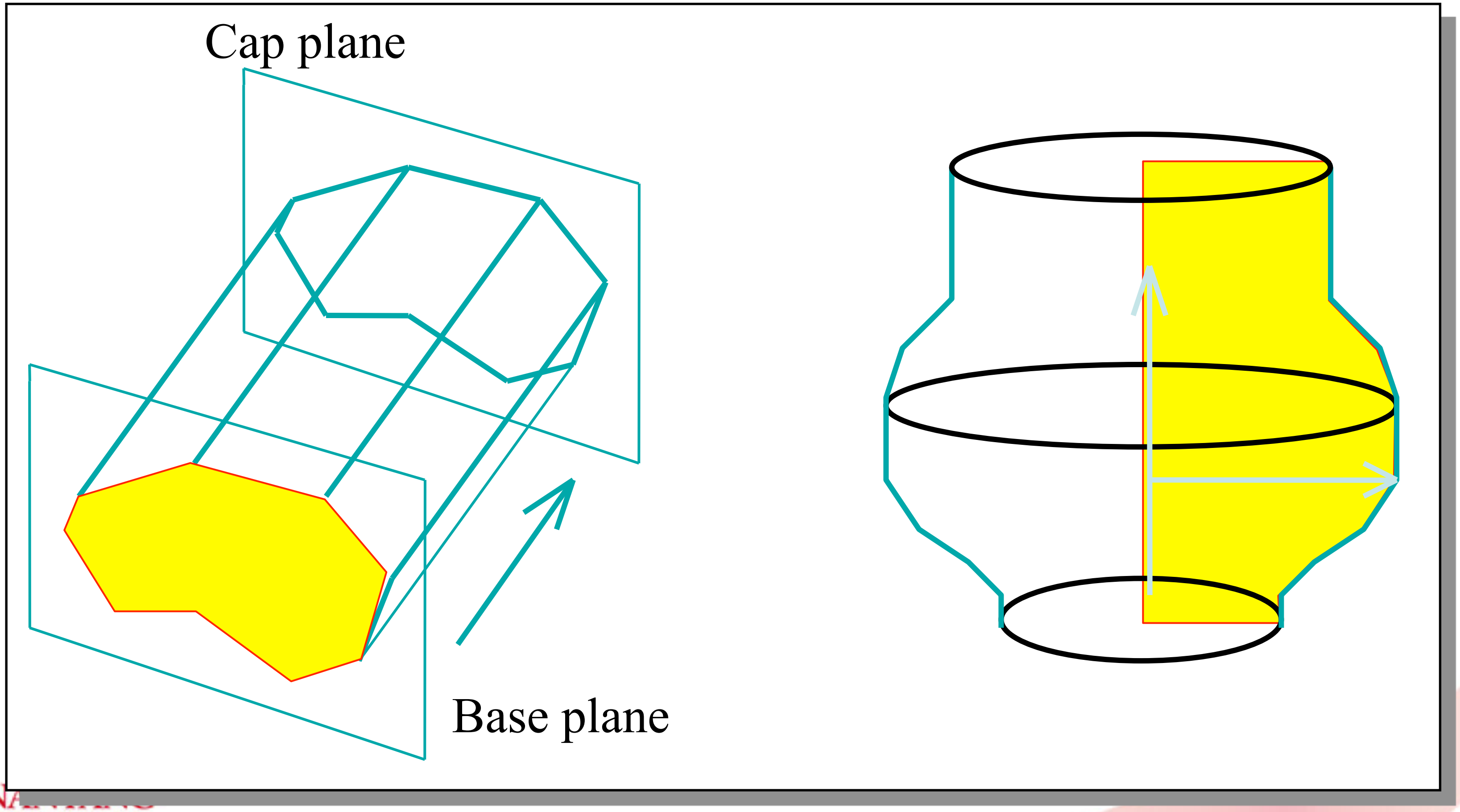
Lab Experiment 4

- *Download file **CSGsolid.wrl** from the course-site and display the shape. Use it as a template for the following exercises.*
- *Define a complex CSG solid shape using set-theoretic operations in **min/max** form on at least one plane halfspace, ellipsoid, cylinder, and cone. Using symbols of these operations (| and &) is not allowed. Note that min/max functions can take only two arguments.*
- *Adjust the tight bounding box (nearly touching the shape) and the optimum resolution for your shape to render it within 5 seconds only.*
- *Define in **FMaterial** field a variable diffuse color for the whole shape by writing functions r(u,v,w), g(u,v,w), b(u,v,w) where u=x, v=y, and w=z. Make sure the color values are correct (within [0,1]) on the visible surfaces of the shape and the shape rendering is still interactive.*
- *Create a folder with name Lab4 and copy there all the FVRML files you have experimented with.*
- *Write a brief report explaining what each file defines and also copy it to Lab4 folder.*

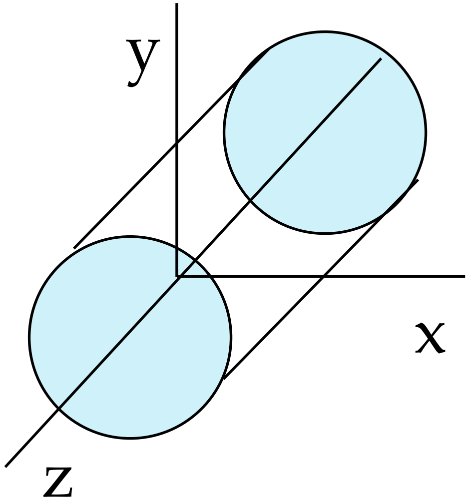
Solid Sweeping

- Shapes are created by moving curve, surface or solid along some path
- Sweeping of curves generates surfaces, sweeping of surfaces produces solids, sweeping of solids creates solids
- Two particular cases of sweeping--*translational* and *rotational sweeping*--can be easily defined parametrically

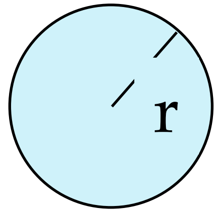
Translational and Rotational Sweeping



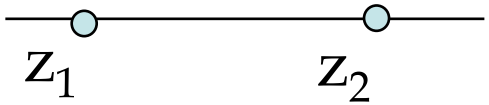
Parametric Representation of Translational Sweeping. Solid Cylinder.



$$x = r\cos\varphi$$
$$y = r\sin\varphi \quad 0 \le r \le R$$
$$z = 0 \quad 0 \le \varphi \le 2\pi$$



$$z = z_1(1-\tau) + z_2\tau$$
$$0 \le \tau \le 1$$



$$x = r\cos\varphi \quad 0 \le r \le R$$
$$y = r\sin\varphi \quad 0 \le \varphi \le 2\pi$$
$$z = z_1(1\text{-}\tau) + z_2\tau \quad 0 \le \tau \le 1$$

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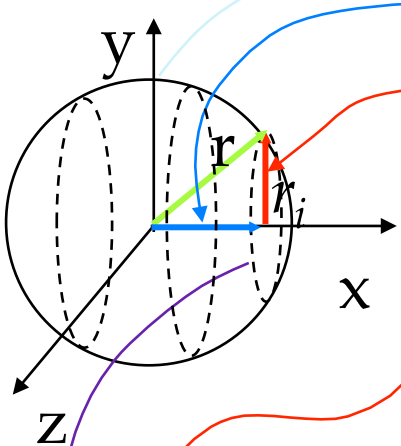
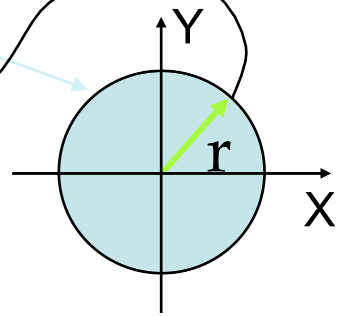
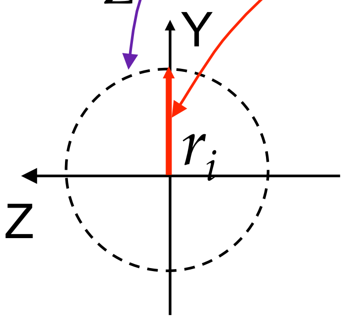
Parametric Representation of Rotational Sweeping. Solid Sphere.



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$0 \le \varphi \le 2\pi$$
$$0 \le r \le R$$





$$y = r_i\cos\theta$$
$$z = r_i\sin\theta$$
$$0 \le \theta \le \pi$$

$$x = r\cos\varphi$$
$$y = r\sin\varphi\cos\theta$$
$$z = r\sin\varphi\sin\theta$$
$$0 \le \varphi \le 2\pi$$
$$0 \le r \le R$$
$$0 \le \theta \le \pi$$

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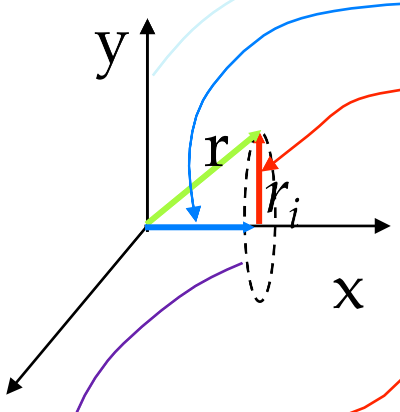
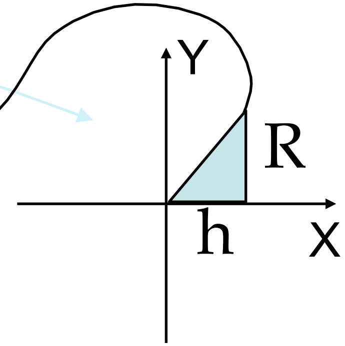
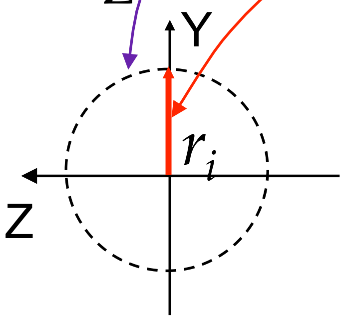
Parametric Representation of Rotational Sweeping. Solid Cone



$$x = h * u$$
$$y = R * v * u$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$

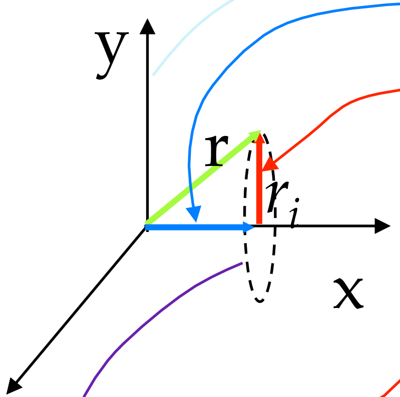




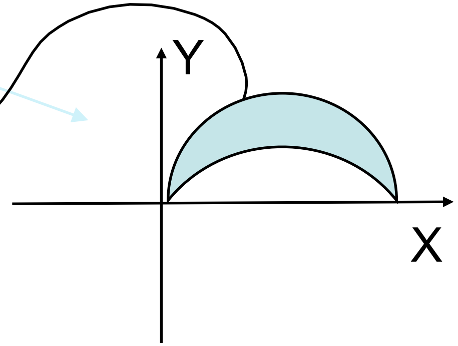
$$y = r_i \cos\theta$$
$$z = r_i \sin\theta$$
$$0 \le \theta \le 2\pi$$

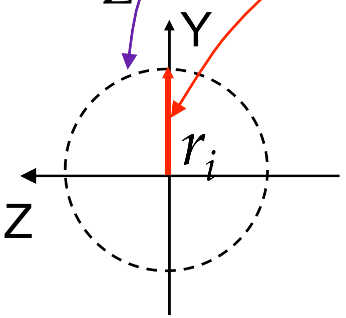
$$x = h * u$$
$$y = R * v * u \cos\theta$$
$$z = R * v * u \sin\theta$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$
$$0 \le \theta \le 2\pi$$

Parametric Representation of Rotational Sweeping. Solid Anything



$$
\begin{aligned}
x &= f_x(u,v)\\
y &= f_y(u,v)\\
0 &\le u \le 1\\
0 &\le v \le 1
\end{aligned}
$$





$$
\begin{aligned}
y &= r_i \cos\theta\\
z &= r_i \sin\theta\\
0 &\le \theta \le 2\pi
\end{aligned}
$$

$$
\begin{aligned}
x &= f_x(u,v)\\
y &= f_y(u,v)\cos\theta\\
z &= f_y(u,v)\sin\theta\\
0 &\le u \le 1\\
0 &\le v \le 1\\
0 &\le \theta \le 2\pi
\end{aligned}
$$

Experimenting with Sweeping

x=wcos(u)

y=wsin(u)

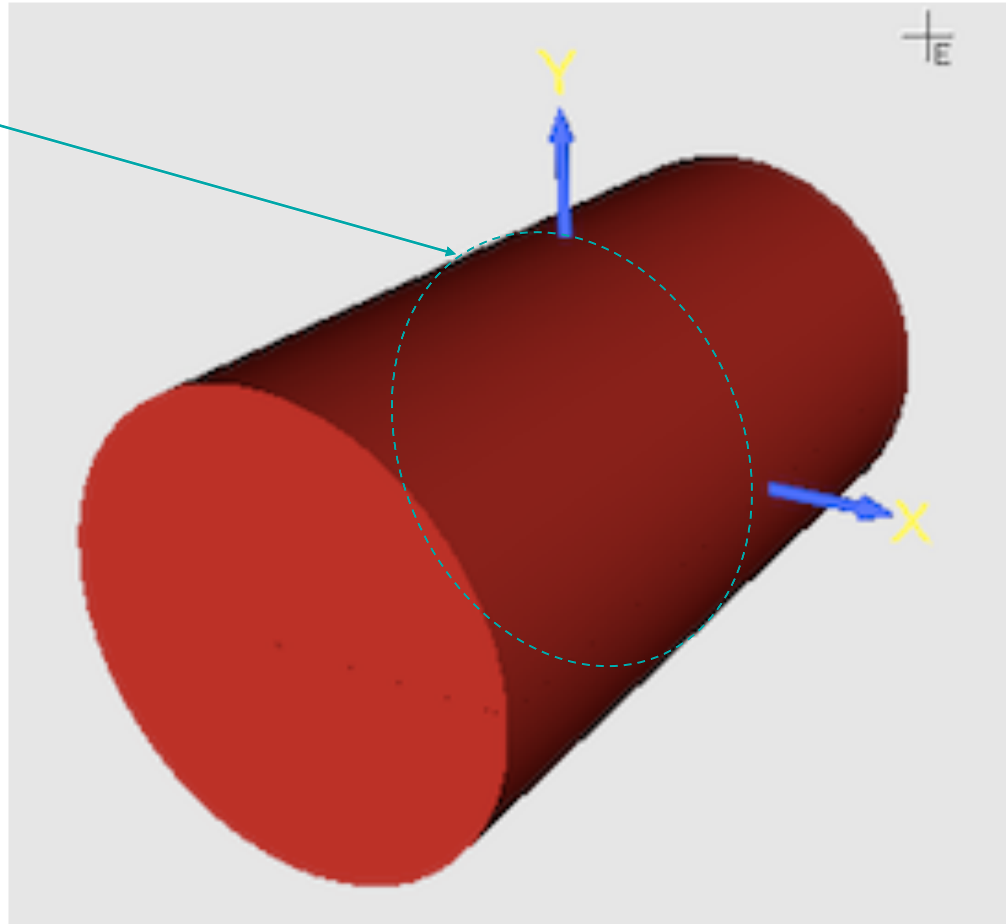
z=v

u=[0, 2p]

v=[-1, 1]

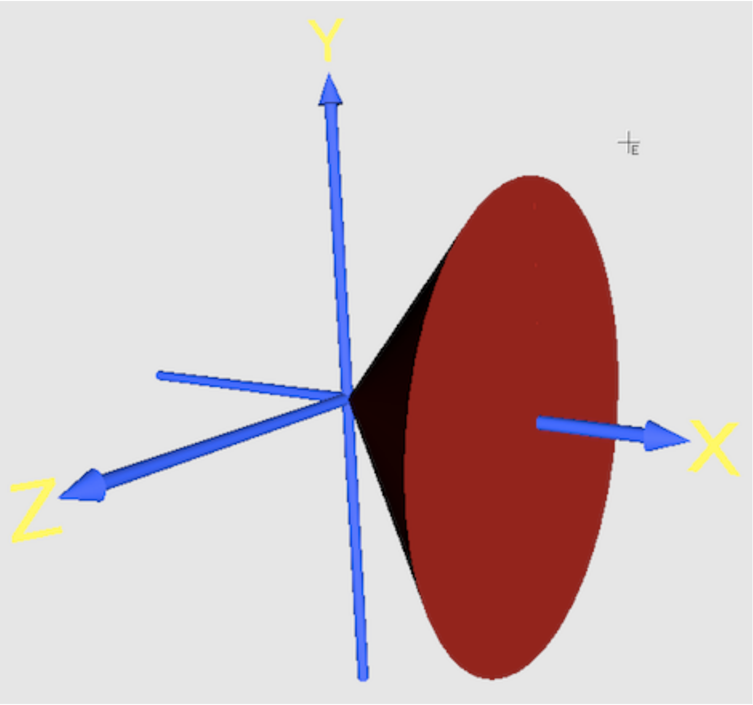
w=[0,1]

Translational sweeping



Experimenting with Sweeping

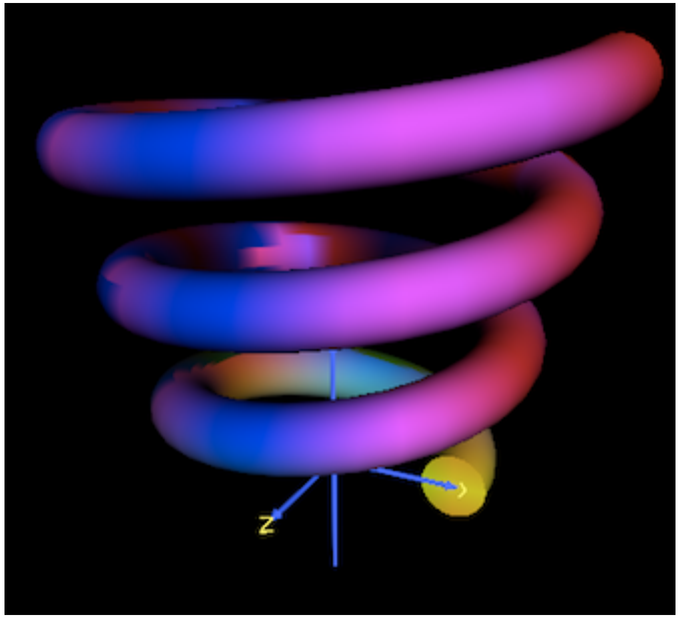
$$x = u$$
$$y = w * u \cos v$$
$$z = w * u \sin v$$
$$0 \le u \le 1$$
$$0 \le v \le 2\pi$$
$$0 \le w \le 1$$



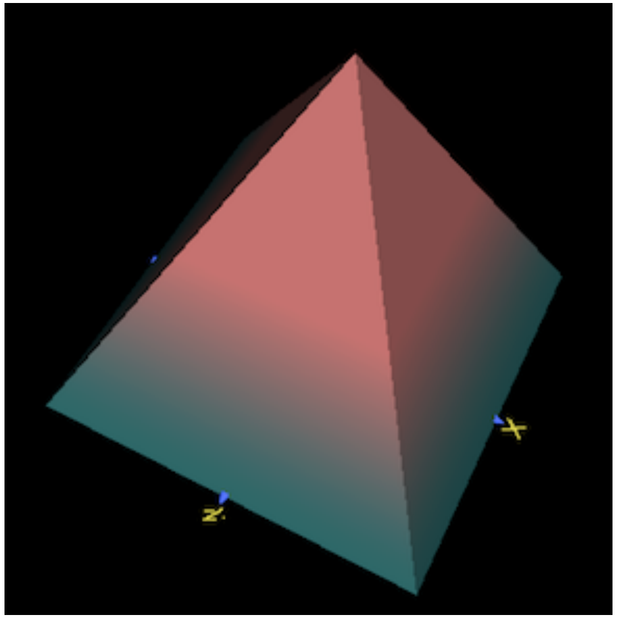
Rotational sweeping

Experimenting with Sweeping

x=(u*cos(v*2*pi)+0.3*w+1)*cos(w*2*pi)
y= u*sin(v*2*pi)+w
z=-(u*cos(v*2*pi)+0.3*w+1)*sin(w*2*pi)
u=[0,0.25], v=[0,1], w=[0,3]



x=-1*(1-v)+2*(1-v)*u
y=2*v
z=-1*(1-v)+2*(1-v)*w
u=[0,1], v=[0,1], w=[0,1]



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More examples are at:
http://www3.ntu.edu.sg/home/assourin/fvrml.htm

Summary

- Solid objects can be defined by
  - Changing implicit equations of surfaces into inequalities ≤ 0 or ≥ 0
  - Parametric equations with 3 parameters (3D solids)
- Sweeping can be easily formulated by using parametric functions for the cases of translational and rotational sweeping
- Boolean or Set-theoretic operations can be defined for implicit functions and implicit inequalities when min/max functions are used; We assume only inequalities ≥ 0 for the functions-arguments and functions-results of the operations.

Analytic Shape Representations. Summary

- Implicit
  - $f(x,y)=0$ 2D curve
  - $f(x,y,z)=0$ 3D surface
- Explicit
  - $y=f(x)$ $x=f(y)$ 2D curves
  - $z=f(x,y)$ $y=f(x,z)$ $x=f(y,z)$ 3D surfaces
  - $g=f(x,y,z)\ge 0$ solid objects

    $\cup$ $\max(f_1, f_2) \ge 0$ $\cap$ $\min(f_1, f_2) \ge 0$ $\setminus$ $\min(f_1, -f_2) \ge 0$
- Parametric
  - 2D/3D Curves: 1 parameter
  - Surfaces: 2 parameters
  - 3D Solids: 3 parameters

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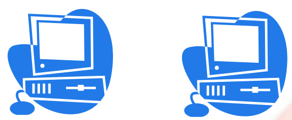
Geometric Shapes: Practical (Programming) Revision

Module 3

Lecture 8

Lab. Experiment 1

1. *Display a simple polygon mesh as it is illustrated in Fig. 3 (download* ***polygons.wrl****)*
2. *Explore different Graphics Modes of the VRML browser (Wireframe, Vertices, Flat). Make sure OpenGL is selected in Settings/Renderer when you right-click at the VRML browser window (See Fig. 1b).*
3. *Examine how the color of the shape defined in* ***diffuseColor*** *field can be changed. Note that the color values must be real numbers between 0 and 1. See what happens if the color values are less than 0 or greater than 1.*
4. *Change the displayed polygon mesh (a pyramid) to anything else by adding new vertices and polygons. Make a six-sided polygon (hexagon) and a cube.*
5. *Notice how the order of vertices changes the visible side of polygons.*
6. *Create a folder with name Lab1 and copy there all the FVRML files you have experimented with.*
7. *Write a brief report explaining what each file defines and also copy it to Lab1 folder.*



Labs. Experiment 2

- *Download file **curve.wrl** from the course-site (Fig. 3). Use it as a template for the following exercises.*
- *Define parametrically in different files*
  - *straight line segment,*
  - *circle and its arc,*
  - *ellipse and its arcs,*
  - *2D spiral,*
  - *3D helix.*
- *Convert the explicitly defined curve y=sin(x) to parametric representation x(u), y(u) and define it in FVRML file.*
- *Explore what happens when you change the curves resolutions to as little as 2 and see how the shape of the curves changes.*
- *Change the curves parameter domain to see how they elongate or shorten.*
- *Create a folder with name Lab2 and copy there all the FVRML files you have experimented with.*
- *Write a brief report explaining what each file defines and also copy it to Lab2 folder.*

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FVRML. Curves

```
FShape {
polygonizer       "analytical_curve"
geometry FGeometry {
definition "      x=1*(cos(2*pi*u))^3;
                  y=1*(sin(2*pi*u))^3;
                  z=0;"
parameters [0 1]
resolution [100]
 }
appearance FAppearance {
material FMaterial { diffuseColor "r=1; g=0; b=0;"   }   }
}
```

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Lab Experiment 3

- *Download file __surface.wrl__ and __solid.wrl__ from the course-site. Use them as templates for the following exercises.*
- *Define parametrically in separate files*
  - *3D plane,*
  - *3D triangle,*
  - *bilinear surface,*
  - *sphere,*
  - *ellipsoid,*
  - *cone.*
- *Explore how the shapes change when their sampling resolution is changed.*
- *Define parametrically in separate files*
  - *solid box,*
  - *solid sphere,*
  - *solid cylinder,*
  - *solid cone.*
- *Consider conversion of any closed surface into a solid object by introducing the third parameter. For example, convert a cylindrical surface into a solid cylinder.*
- *Study how to make surfaces and solids using the concept of translational and rotational sweeping.*
- *Use curve y=sin(x) for making a solid by applying rotational and translational sweepings together.*
- *With reference to the lecture slides of Module 4, study the problem of degenerate normals on the example of a sphere. By changing parameter ranges and resolution, avoid calculation of the degenerate normal on the polar points of the sphere.*
- *Create a folder with name Lab3 and copy there all the FVRML files you have experimented with.*
- *Write a brief report explaining what each file defines and also copy it to Lab3 folder.*

FVRML. Parametric Surfaces

```
FShape {

geometry FGeometry {
definition "      x=u;
                  y=v;
                  z=0;"
parameters [0 1 0 1]
resolution [75 75]
 }
appearance FAppearance {
material FMaterial { diffuseColor "r=0; g=1; b=0;"   }    }
}
```

FVRML. Parametric Solids

```
FShape {

geometry FGeometry {
definition "      x=u;
                  y=v;
                  z=w;"
parameters [0 1 0 1 0 1]
resolution [75 75 75]
 }
appearance FAppearance {
material FMaterial { diffuseColor "r=0; g=1; b=0;"   }    }
}
```

Lab Experiment 4

- *Download file CSGsolid.wrl from the course-site and display the shape. Use it as a template for the following exercises.*
- *Define a complex CSG solid shape using set-theoretic operations in **min/max** form on at least one plane halfspace, ellipsoid, cylinder, and cone. Using symbols of these operations (| and &) is not allowed. Note that min/max functions can take only two arguments.*
- *Adjust the tight bounding box (nearly touching the shape) and the optimum resolution for your shape to render it within 5 seconds only.*
- *Define in **FMaterial** field a variable diffuse color for the whole shape by writing functions r(u,v,w), g(u,v,w), b(u,v,w) where u=x, v=y, and w=z. Make sure the color values are correct (within [0,1]) on the visible surfaces of the shape and the shape rendering is still interactive.*
- *Create a folder with name Lab4 and copy there all the FVRML files you have experimented with.*
- *Write a brief report explaining what each file defines and also copy it to Lab4 folder.*

FVRML. Implicit Solids

```
FShape {

geometry FGeometry {
definition "      function frep(x,y,z,t){
                  shape1=0.7^6-x^6-y^6-z^6; shape2=0.25^2-x^2-y^2;
                  final=min(shape1, -shape2);   return final;}"
bboxCenter 0 0 0   bboxSize 2 2 2
resolution [100 100 100]
 }
appearance FAppearance {
material FMaterial { diffuseColor "r=1; g=(v+1)/2; b=0;"   }   }
}
```

Conclusion

- E-learning week
- Discussion group in the course site (labs and general questions)
- Revision lecture on 8 November (LT11)
- Final mark Exam 60%, Labs 30%, Tutorials 10%
- Invitation to research in virtual reality, human-computer interaction, and shape modelling

Geometric Shapes: Blobby Shapes

Module 3

Lecture 8 (e-week lecture)

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Blobby Shapes

- Blobby function

$$g = f(x, y, z) = ae^{-r}$$

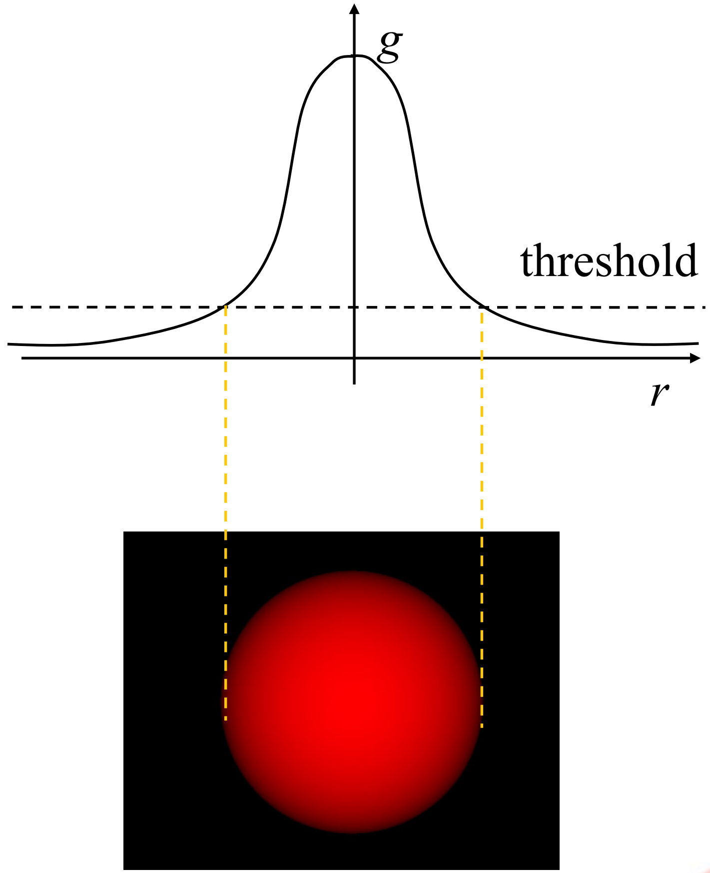
where

$$r = \pm\sqrt{(x - x_b)^2 + (y - y_b)^2 + (z - z_b)^2}$$

- Solid blob

$$g = f(x, y, z) = ae^{-r} - c \geq 0$$

Animated threshold



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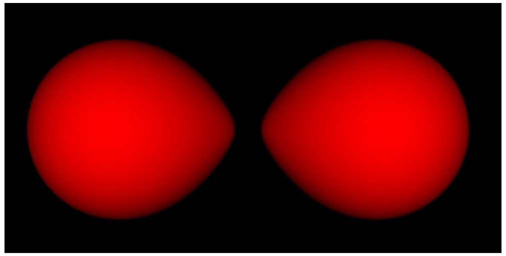
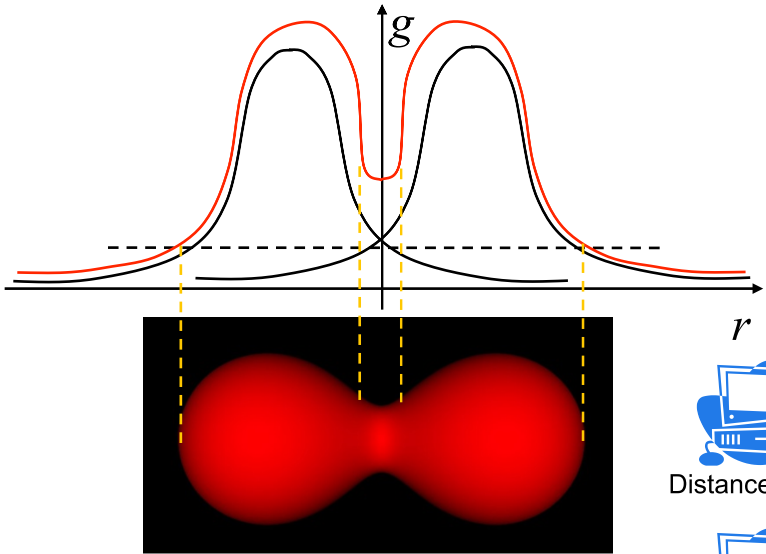
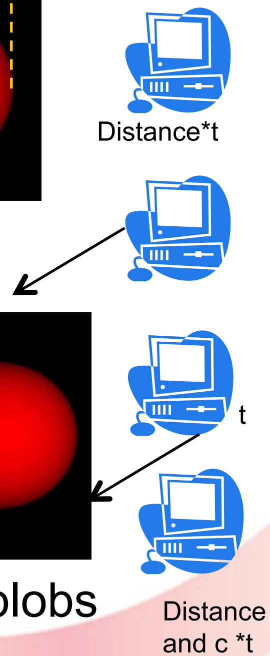
Blobby Shapes

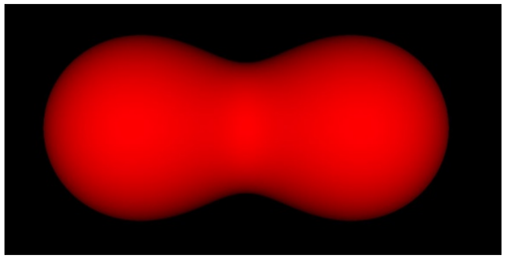
- Multiple blobs

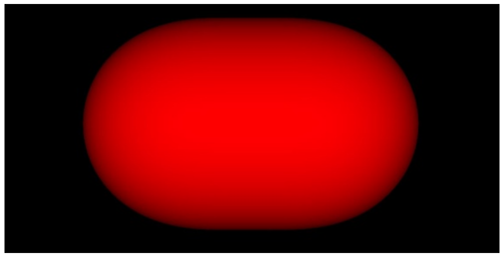
$$g = f(x,y,z) = \sum_i a_i e^{-r_i} - c \geq 0$$

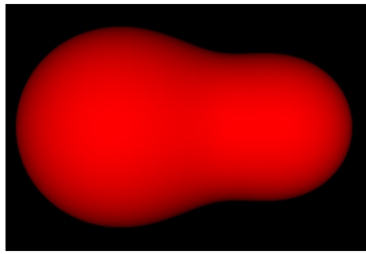








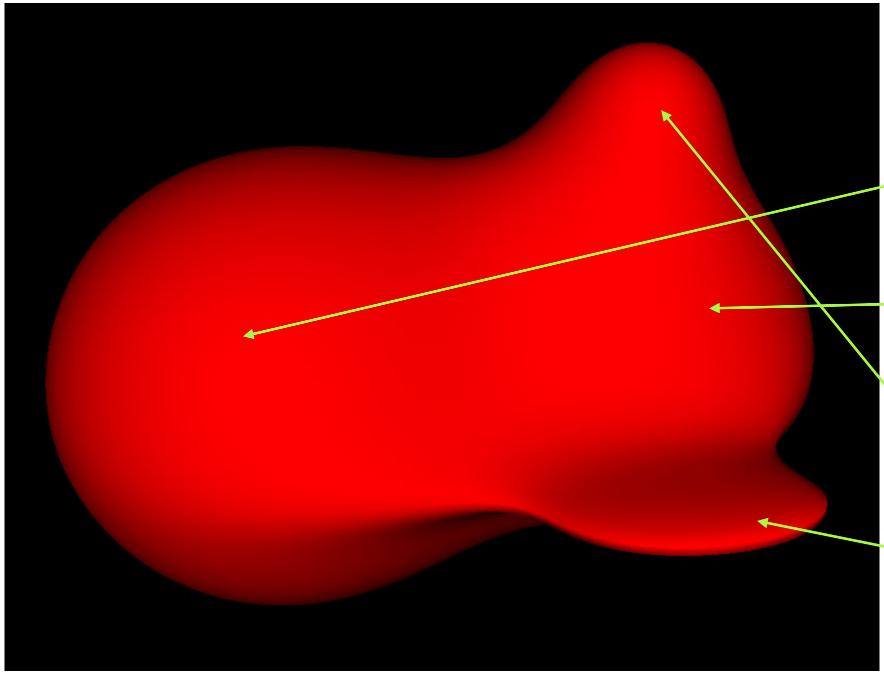




Changing centres of the blobs

Scaling the blobs

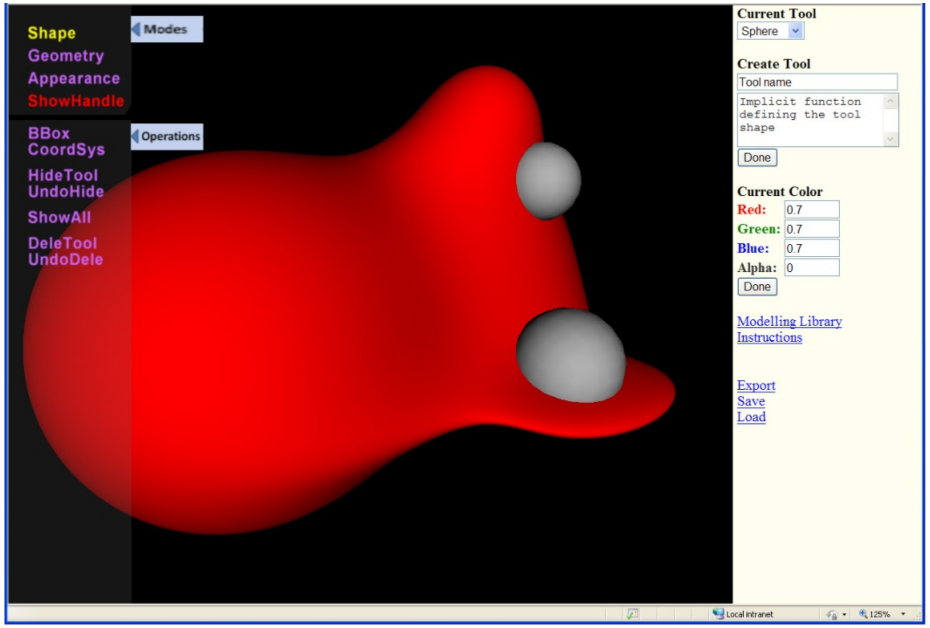
Blobby Shapes

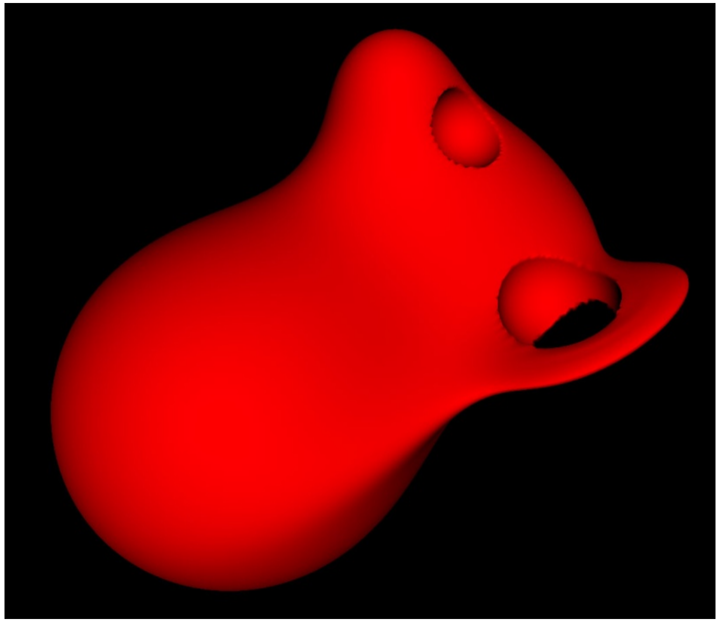


$$g = f(x, y, z) =$$
$$2e^{-\sqrt{(x+1.8)^2+y^2+z^2}} +$$
$$e^{-\sqrt{(x-1.8)^2+y^2+z^2}} +$$
$$0.3e^{-\sqrt{(x-1.8)^2+(y-2.5)^2+z^2}} +$$
$$0.3e^{-\sqrt{(0.5(x-1.8))^2+(3y)^2+(z-3)^2}} - 0.25 \geq 0$$

Blobby shapes

- Applying CSG operations to the blobby shape





Geometric Shapes: How to draw curves, surfaces and solids.

Module 3

Lecture 9 (e-week lecture)

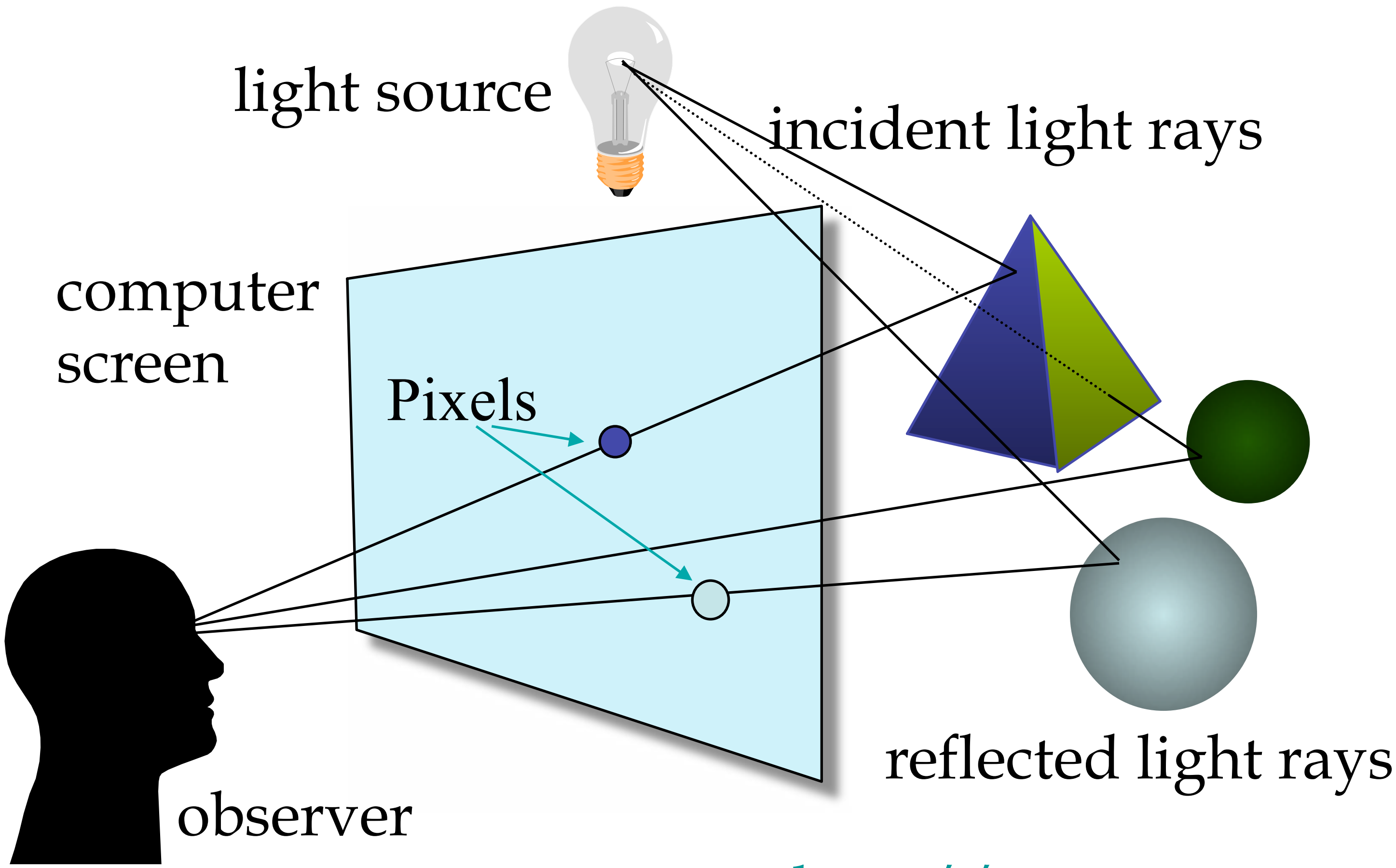
How to Draw a Curve?

- Linear interpolation
- Parametric representations are the most suitable for it

How to Draw a Surface?

- Ray Tracing
- Wire-frame Representation
- Polygonization

Ray Tracing

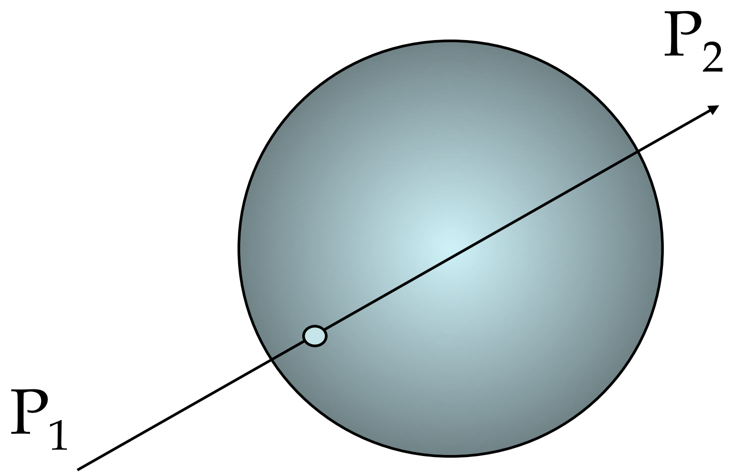


http://www.povray.org

Ray Tracing

- Intersection with a sphere

$$\begin{cases} r^2 - x^2 - y^2 - z^2 = 0 \\ x = x_1 + (x_2 - x_1)t \\ y = y_1 + (y_2 - y_1)t \\ z = z_1 + (z_2 - z_1)t \end{cases}$$



Wire-frame of a Parametric Surface

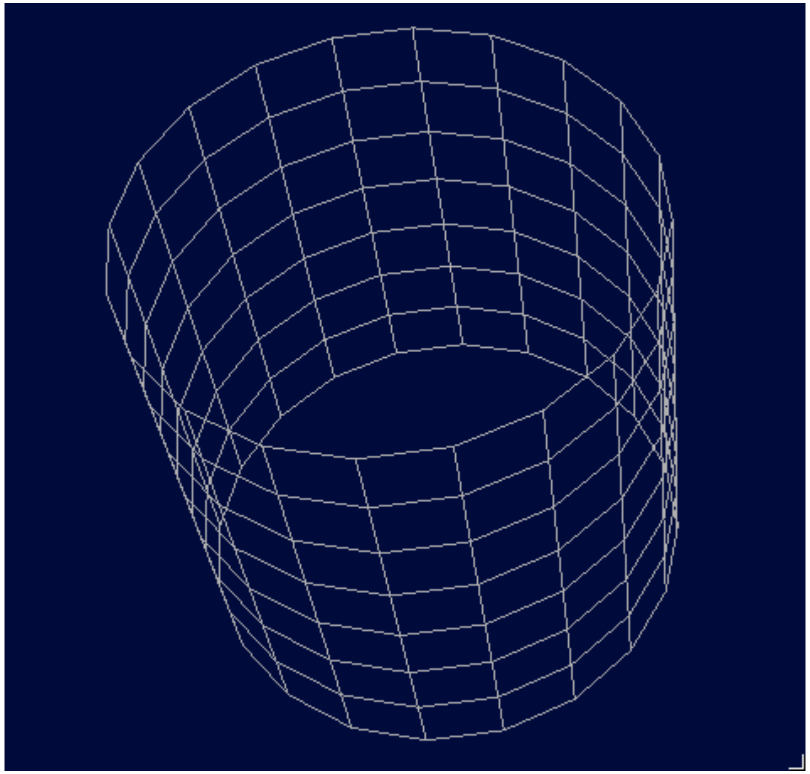
$$x = x(u, v)$$

$$y = y(u, v)$$

$$z = z(u, v)$$

$$u_1 \leq u \leq u_2$$

$$v_1 \leq v \leq v_2$$



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Wire-frame of a Sphere

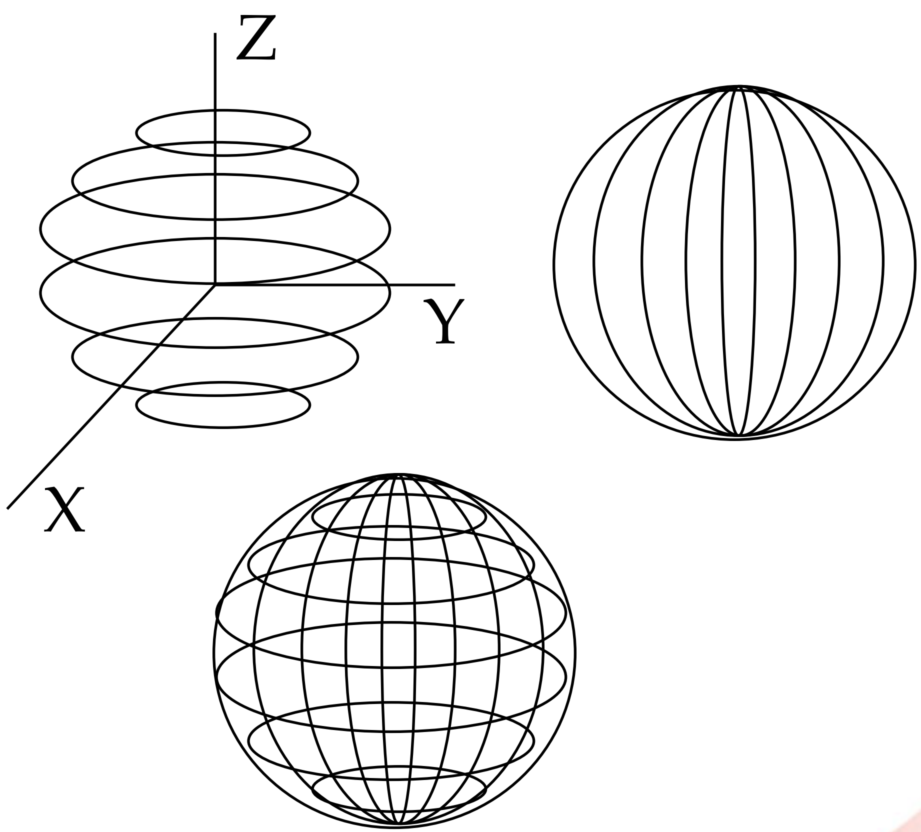
$$x = r\cos u \cos v$$

$$y = r\sin u \cos v$$

$$z = r\sin u$$

$$-\pi \le u \le \pi$$

$$-\pi/2 \le v \le \pi/2$$



Polygonization of a Sphere

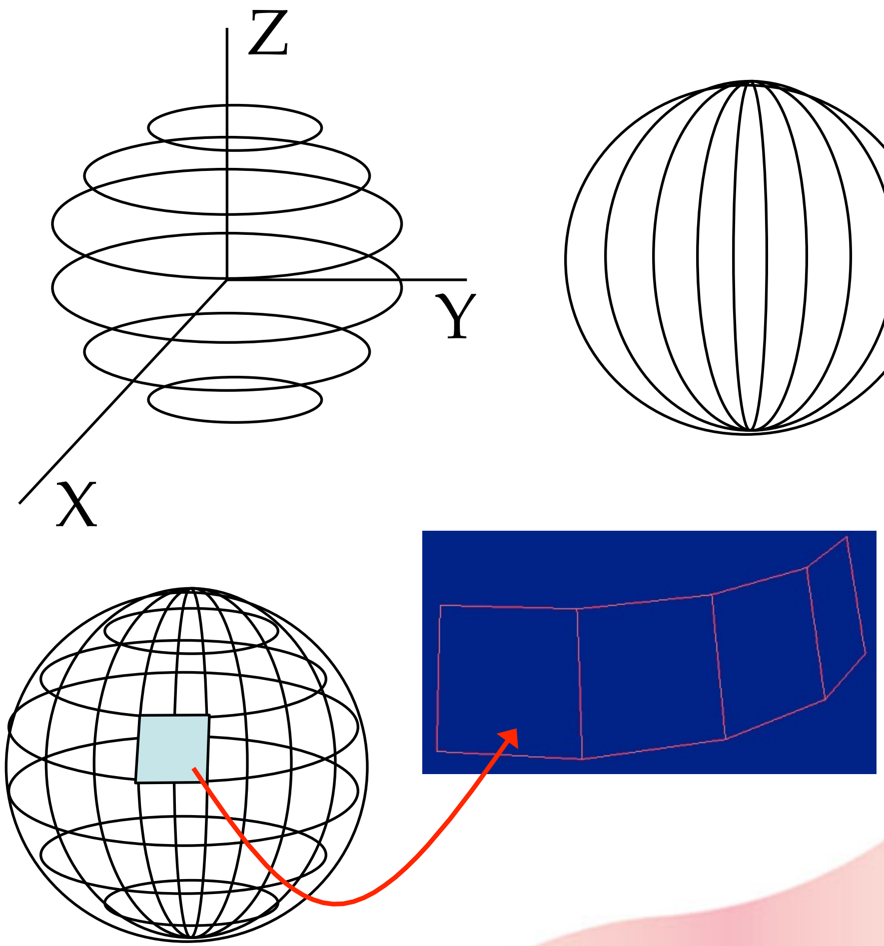
$$x = r\cos u \cos v$$

$$y = r\sin u \cos v$$

$$z = r\sin v$$

$$-\pi \le u \le \pi$$

$$-\pi/2 \le v \le \pi/2$$



How to Draw a Solid Shape?

- Voxel graphics
- Visible surface rendering
- Usually based on sampling and requires longer processing time

Summary

- Line interpolation and polygonization dominate in visualization
- Point-based rendering and voxel (volume) graphics evolve rapidly