28.09.2024

Neuron 1943

Schwellwertelement

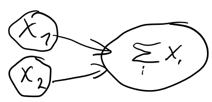
Inputs

x_1
x_2
$\vdots$
x_n

$\sum_{i=1}^{n} x_i$ vgl Schwellwert Θ

binär 0 oder 1

ODER

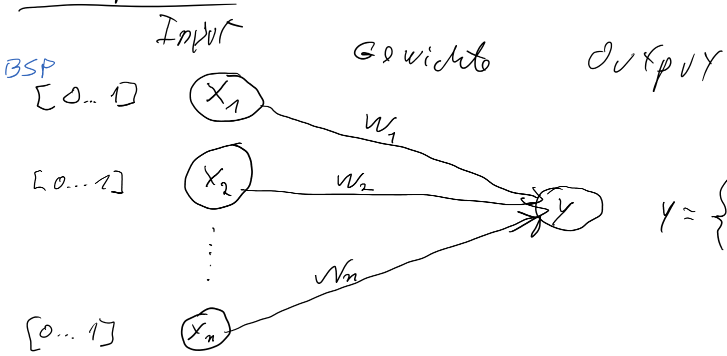


wähle $\Theta = 1$

ODER

x_1	x_2	$x_1 \vee x_2$	Hilfsrechnung $x_1 + x_2$
1	1	1	2
1	0	1	1
0	1	1	1
0	0	0	0

Perzeptron 1957 Frank Rosenblatt



$$y = \begin{cases} 1 & z \geq \Theta \\ 0 & \text{sonst} \end{cases}$$

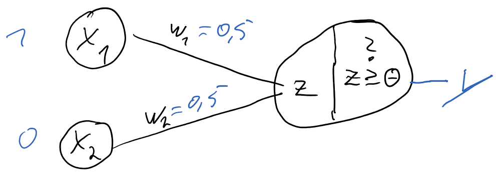
Z: Roheingabe

$$z := w_1 x_1 + w_2 x_2 + \dots + w_n x_n = \sum_{i=1}^{n} w_i x_i$$

Θ : Schwellwert

y : Output

Bsp:



$\Theta = 1$

1. $z = 0{,}5 \cdot 1 + 0{,}5 \cdot 0 = 0{,}5$

2. $z = 0{,}5 < \Theta = 1$

$\Rightarrow$ $y = 0$ da $y = \begin{cases} 1, & z \geq \Theta \\ 0 & \text{sonst} \end{cases}$

Bsp: wähle $x_1 = 1$ $x_2 = 1$ $w_1 = w_2 = 0{,}5$
$\Theta = 1$

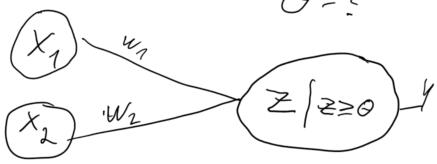
Output $y = 2$ ..... $y = 1$

$y = 1$

Logisches ODER mittels Perzeptron

x_1	x_2	ODER $x_1 \vee x_2$
1	1	1
1	0	1
0	1	1
0	0	0

$\theta = ?$



$$y = \begin{cases} 1, & z \geq \theta \\ 0 & \text{sonst} \end{cases}$$

wähle $w_1 = 1$, $w_2 = 1$, $\theta = 1$

4 Fälle

x_1	x_2		y
1	1	$z = w_1 x_1 + w_2 x_2 = 1 \cdot 1 + 1 \cdot 1 = 2$	1
1	0	$z = \ldots = 1 \cdot 1 + 1 0 = 1$	1
0	1	$z = \ldots = 1 \cdot 0 + 1 \cdot 1 = 1$	1
0	0	$z = \ldots = 1 \cdot 0 + 1 0 = 0$	0

Hausaufgabe UND ... ?

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bisher w_i, θ parametrieren

Vereinfachung

$$w_1 x_1 + w_2 x_2 + \cdots + w_n x_n \geq \theta \qquad \Big| -\theta$$

$$\Leftrightarrow -\theta + w_1 x_1 + w_2 x_2 + \cdots + w_n x_n \geq 0$$

$x_0 := 1$
$w_0 := -\theta$ ≙ "b"

bias

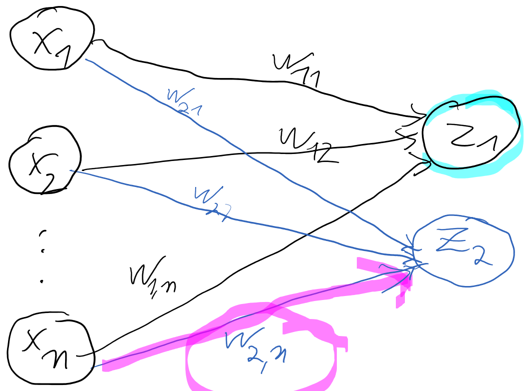
$$\Rightarrow \underbrace{w_0 x_0 + w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}_{=: z} \geq 0$$

$$\Rightarrow y = \begin{cases} 1, & \boxed{z \geq 0} \\ 0 & \text{sonst} \end{cases}$$

Ausblick: Wie lernt ein Perzeptron? → Termin in 14 Tagen

Feed forward Netz (FNN)

Input Gewichte Output



$\boxed{z_1 = w_{11}x_1 + w_{12}x_2 + \dots + w_{1n}x_n}$

$z_2 = w_{21}x_1 + w_{22}x_2 + \dots + w_{2n}x_n$

$\boxed{W_{\text{Ziel, Quelle}}}$

Bsp wähle $n = 3$ $\longrightarrow$

$$\left.\begin{cases} z_1 = w_{11}x_1 + w_{12}x_2 + w_{13}x_3 \\ z_2 = w_{21}x_1 + w_{22}x_2 + w_{23}x_3 \end{cases}\right\}$$

Lineares Gleichungs-system

2 Gleichungen
3 Variablen
6 Parameter

3 Inputvariablen x_1, x_2, x_3
6 Gewichte / Parameter $w_{i,j}$ $\quad i = 1, 2$; $j = 1, 2, 3$
2 Outputs

Alternative Darstellung des Linearen Gleichungssystems

$$\boxed{\begin{aligned} z_1 &= w_{11}x_1 + w_{12}x_2 + w_{13}x_3 \\ z_2 &= w_{21}x_1 + w_{22}x_2 + w_{23}x_3 \end{aligned}} \iff \boxed{W x = Z}$$

$$W := \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix}, \quad X := \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad Z := \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$W x = Z$

$$\underset{(2\times 3)}{\begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix}} \cdot \underset{(3\times 1)}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}} = \underset{(2\times 1)}{\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}}$$

(Zeilen, Spalten)

$$\Rightarrow \begin{pmatrix} w_{11}x_1 + w_{12}x_2 + w_{13}x_3 \\ w_{21}x_1 + w_{22}x_2 + w_{23}x_3 \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

Vektorgleichung

$$\begin{pmatrix} \vdots \\ w_{21}x_1 + w_{22}x_2 + w_{23}x_3 \end{pmatrix} = \begin{pmatrix} \vdots \\ z_2 \end{pmatrix}$$

Vektorgleichung

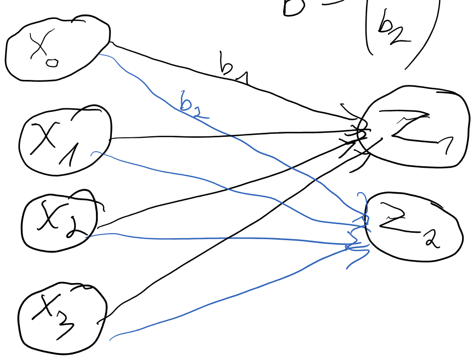
$$\Rightarrow \begin{cases} z_1 = w_{11}x_1 + w_{12}x_2 + w_{13}x_3 \\ z_2 = w_{21}x_1 + w_{22}x_2 + w_{23}x_3 \end{cases}$$

2 Gleichungen eines linearen Gleichungssystems

Modelle: Perzeptron
- FFN

Basis: Lineare Gleichungssysteme
→ Lineare Algebra

Bias



$b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

Rohinput

$$\boxed{Z = Wx + b}$$

$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

$Z \in \mathbb{R}$ da $w_{ij} \in \mathbb{R}$

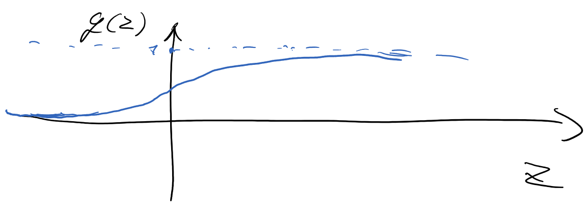
Aktivierungsfunktion a

Problem: $Z \in \mathbb{R}$

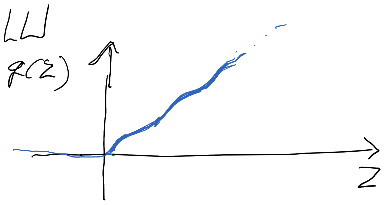
$$a = g(z) = g\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} g(z_1) \\ g(z_2) \end{pmatrix}$$

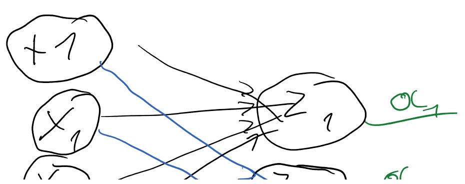
Bsp. g: logistische Funktion

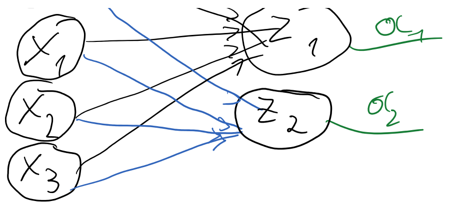
$$g(z) = \frac{1}{1 + e^{-z}}$$



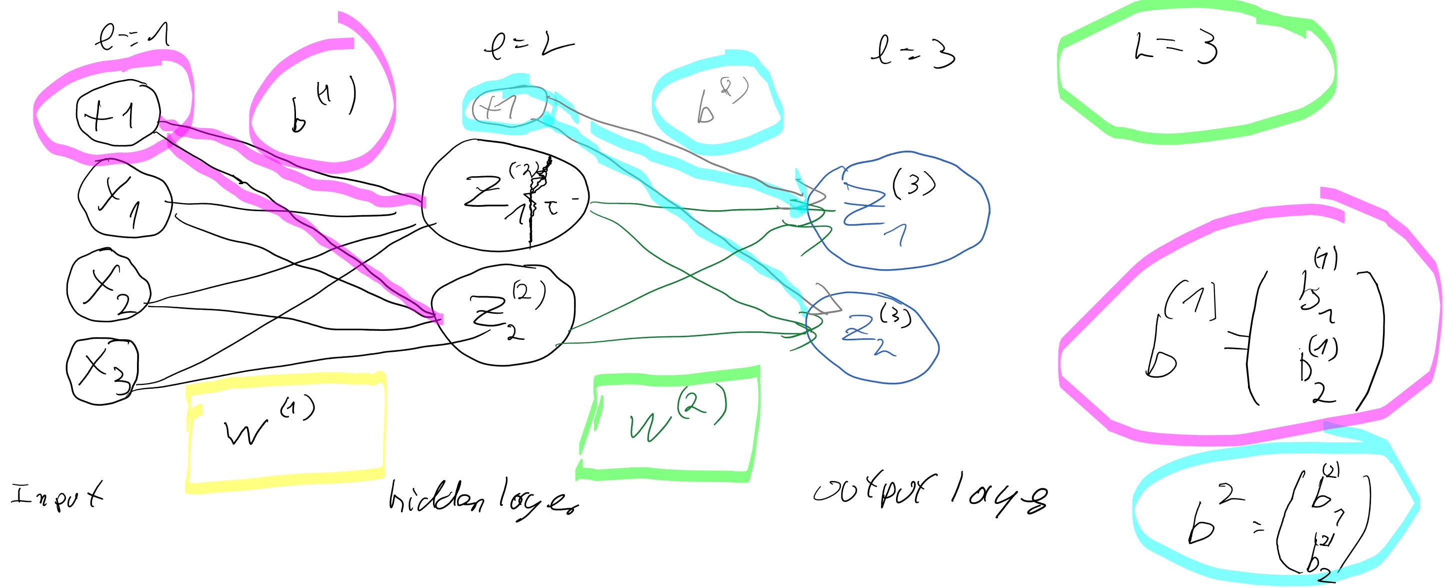
ReLU







Mehrschichtige FNNs



Rohinput

$$z^{(l)} = W^{(l-1)} a^{(l-1)} + b^{(l-1)}$$

Aktivierung

$$a^{(l)} = g\left(z^{(l)}\right)$$

$$a^{(1)} = x$$

berechneter Output $\hat{y} = a^{(l)}$

Glossar

x : Inputvektor eines Neurons bzw. Neuronale Netze $\quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

z : Rohinput " " " " "

$$z = \sum w_i x_i$$

y : Output " " " " "

Θ : Schwellwert

b : Bias

$a = g(z)$ Aktivierungsfunktion

W Matrix der Gewichtungen

Ende 28.09.2024