lin algebra

Q1

$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\right) = 0$

$\det\left(\begin{bmatrix} \lambda-1 & -1 \\ -1 & \lambda+1 \end{bmatrix}\right)$

$(\lambda-1)(\lambda+1) - 1 = 0$

$\lambda = \sqrt{2}, -\sqrt{2}$

Q2

$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$

$\lambda = 4, 8$

$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}\right) = 0$

$\det\left(\begin{bmatrix} \lambda-2 & -3 \\ -x & \lambda-y \end{bmatrix}\right)$

$(\lambda-2)(\lambda-y) + 3x = 0$

$2(8-y) = -3x \rightarrow 16 - 2y = -3x$

$6(4-y) = -3x \rightarrow 24 - 6y = -3x$

$6 - 2y = -x$

$x = -4$

$y = 2$

Q3

$-x + 5y = -1$

$x - y = 2$

$x + 3y = 3$

$= \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 3 & 3 \\ 0 & -4 & -1 \\ 0 & 0 & 0 \end{array}\right]$

$x + 3y = 3$

$-4y = -1$

$y = \frac{1}{4}$

$x = \frac{9}{4}$

one solution: $\bar{x}, x = \frac{9}{4}, \frac{1}{4}$

lin algebra

Q4 $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

$\lambda = 1$

$E_{\lambda=1} = N\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}\right) = N\left(\begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}\right)$

$(A - \lambda I)v = 0$

$v_1 + 2v_2 = 0 \qquad v_1 = -2v_2$

$-v_2 + 2v_3 = 0 \qquad v_3 = \frac{v_2}{2}$

$E_{\lambda=1} = \text{span}\left(\begin{bmatrix} -2 \\ 1 \\ 1/2 \end{bmatrix}\right)$

Q5 $A^{-1}_v = \frac{1}{\lambda v}$

$A^{-1}{}_{v_1} = \frac{1}{1}$

$A^{-1}{}_{v_2} = \frac{1}{2}$

$A^{-1}{}_{v_3} = \frac{1}{4}$

$\det(A^{-1}) = 1 \cdot \left(\frac{1}{2} \cdot \frac{1}{4}\right) = \frac{1}{8}$

Q6

$\underset{l_2}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}} \underset{l_2}{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}} \rightarrow \begin{bmatrix} 1 & 2 & 4 \\ 0 & -2 & -2 \\ 0 & -2 & -5 \end{bmatrix} = \underset{U}{\begin{bmatrix} 1 & 2 & 4 \\ 0 & -2 & -2 \\ 0 & 0 & -3 \end{bmatrix}}$

$L = l_1^{-1} l_2^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$