single

Q1 → $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = -10 \text{ cm}^3/\text{sec}$$

→ $\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$

$-10 = \frac{4}{3}\pi(3(20)^2)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-10}{4\pi(20)^2}$

$\frac{dr}{dt} = -\frac{1}{16\pi} \approx -0.002 \text{ cm}^3/\text{sec}$

Q2 a

$\frac{1+x^2}{1-x^2}$

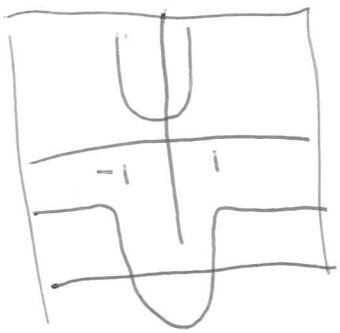
$1 - x^2 = 0$

$x^2 = 1$

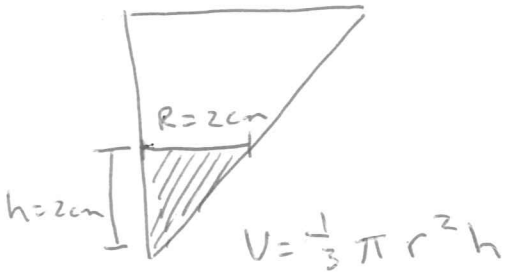
$x = \sqrt{1}$

$x = 1$

$x = -1$



Q3



$$\frac{dV}{dt} = \frac{3\,cm^3}{1\,sec}$$

$$V = \frac{1}{3}\pi h^3 = \frac{\pi h^3}{3}$$

$$\frac{dV}{dt} = \frac{\pi}{3}\frac{d}{dt}\left[h(t)^3\right]$$

$$\frac{dV}{dt} = \frac{\pi}{3} \cdot 3(h(t))^2 \cdot \frac{dh}{dt}$$

$$\frac{dv}{dt} = \frac{\pi}{3} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$3\,\frac{cm^3}{1\,sec} = \frac{\pi}{3} \cdot 3 \cdot 4 \cdot \frac{dh}{dt}$$

$$3\,\frac{cm^3}{1\,sec} = \frac{4\pi}{3} \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{3}{4\pi}\,\frac{cm}{sec}$$

$b = \pi r^2$ ← [water evaporates at surface]

$$\frac{dV}{dt} = -\pi r^2 \cdot b$$

$$\frac{dh}{dt} = -\frac{3}{4\pi} \cdot \pi r^2 \cdot b$$

$$\frac{dh}{dt} = -\frac{3br^2}{4}$$

Single

Q4 $\int_0^1 \frac{x\,dx}{\sqrt{1+3x^2}}$

$\int_1^4 \frac{1}{\sqrt{u}} \frac{1}{6} du$

$\int_1^4 \frac{1}{6\sqrt{u}} du$

$\frac{1}{6} \int_1^4 \frac{1}{\sqrt{u}} du$

$\frac{1}{6} \int_1^4 u^{-\frac{1}{2}} du$

$\frac{1}{6} \left[2u^{\frac{1}{2}} \right]_1^4 \quad \frac{1}{6} \left[2u^{\frac{1}{2}} \right]_1^4$

$\frac{1}{3}$

Q5 $\int_0^1 k \cdot e^{r(1-t)} dt$

$\int_0^1 k \cdot e^{r(1-t)} dt$

interest rate at end of year

daily deposit times interest rate at end of year, discounted by time $1-t$.

Q6

$y = e^x$

$C = 2\pi x = 2\pi x$

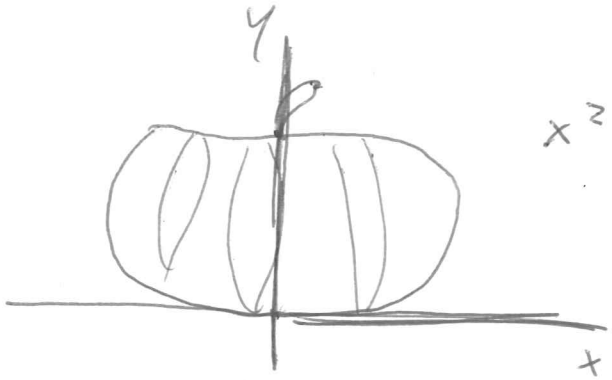
$H = f(x)$

$S = (2\pi x) f(x) = 2\pi x e^x$

shell $V = 2\pi x f(x)\, dx$

$$V = 2\pi \int_0^1 x \cdot e^x \, dx$$

Q7



$x^2 + y^4 = 1$

$C = 2\pi r = 2\pi x$

$H = f(x)$

$S = (2\pi x) f(x) = 2\pi x \sqrt[4]{-x^2+1}$

shell $V = 2\pi \cdot x \cdot f(x) \cdot dx$

$$V = 2\pi \int_0^1 x \cdot \sqrt[4]{-x^2+1}\, dx$$

$$V = 2\pi \int_0^1 x \cdot \sqrt[4]{-x^2+1}\, dx$$