multi

Q1

~~$(1,1,1)$~~ ~~$x\ y\ z$~~ ~~$x^2 + 2y^2 + 2z^2 = 5$~~

~~$1x^2 + 2\cdot 1y^2 + 2\cdot 1z^2 = 5$~~

~~$1^2 + 2\cdot 1^2 + 2\cdot 1^2 = 5$~~

~~$1+2+2=5$~~

~~$5=5$~~

$$\boxed{x^2 + 2y^2 + 2z^2 = 5}$$

~~$2 = \sqrt{\frac{8-x^2-2y^2}{2}}$~~

$$\boxed{z = \sqrt{\frac{2(-x^2-2y^2+5)}{2}}}$$

$$= A(x-a) + B(y-b) + C(z-c) = 0 \quad ①$$

slope of x

x coordinate

slope of y

y coordinate

z coordinate

$$z = \sqrt{\frac{2(-1^2-2+5)}{2}}$$

$$A(x-1) + B(y-1) + C(z-1) = 0$$

$$P(1,1,1)$$

$$a(x-x_0) + b(y-y_0) + z_0$$

$$L(x,y) = a(x-1) + b(y-1) + c(z-1)$$

$$a = \frac{\partial f}{\partial x}(1,1) \qquad b = \frac{\partial f}{\partial y}(1,1)$$

$$\frac{\partial f}{\partial x} = x^2 \qquad \frac{\partial f}{\partial y} = 2\cdot 1y^2$$

$$= 1 \qquad = 2$$

$$2(z-1)$$

$$L(x,y) = 1(x-1) + 2(y-1) + \cancel{\sqrt{\frac{2(-1-2+5)}{2}}}$$

~~$1(x-1) + 2(y-1) + \frac{\sqrt{5}}{2}$~~

~~$x-1 + (2y-2) + 1$~~

$$x + 2y + 2z = 5$$

Multi

Q2 $P = (\overset{x}{2}, \overset{y}{1}) = \quad w = x^2 - xy^3$

$A = 3i + 4j$

$$\nabla_A P = 3\frac{\partial P}{\partial x} + 4\frac{\partial P}{\partial y}$$

$$\nabla_{\vec{w}} P = \begin{bmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial P}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x}(x^2 - xy^3) \\ \frac{\partial}{\partial y}(x^2 - xy^3) \end{bmatrix} = \begin{bmatrix} -y^3 + 2x \\ -3y^2 x \end{bmatrix}$$

$$\nabla_{\vec{w}} P(2,1) = \begin{bmatrix} -1^3 + 2(2) \\ -3(1^2)\cdot 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -6 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= 3(3) + (-6)(4)$$

$$= -15$$

$h = 0.01$

$$h\vec{w} = \begin{bmatrix} 3h \\ -6h \end{bmatrix}$$

$$= \begin{bmatrix} 0.03 \\ -0.06 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= 0.03(3) + (-0.06)(4)$$

$$= -0.15$$

multi

Q3 $\text{Plane} = 2x + y - z = 6$

$\lambda = x^2 + y^2 + z^2$

$2x = 2\lambda$

$2y = \lambda$

$2z = -\lambda$

$2\lambda + \frac{\lambda}{2} \cancel{x} - \frac{-\lambda}{2} = 6$

$\overset{x}{\frac{2\lambda \cdot 2}{2}} + \overset{y}{\frac{\lambda}{2}} + \overset{z}{\frac{\lambda}{2}} = 6$

$\frac{6\lambda}{2} = 6$

$3\lambda = 6$

$\lambda = 2$

$P = (2, \cancel{z}1, 1)$

multi

$$\begin{array}{cccc} & A & X & u \end{array}$$

Q4

$$A = \begin{array}{c} \left[\begin{array}{ccc} 1 & 0 & 3 \\ -2 & 1 & -1 \\ -1 & 1 & \boxed{2}\text{ⓐ} \end{array}\right] \begin{array}{c} 4 \\ -2 \\ 2 \end{array} \\ \begin{array}{ccc} -2 & 2 & 4 \end{array} \end{array} \begin{bmatrix} x_x \\ x_y \\ x_z \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} =$$

$1x = 3$
$-2x + y = -1$
$-1x + y = 2$

$$\begin{bmatrix} 1x + 0y + 3z \\ -2x + 1y + -1z \\ -1x + 1y + 2z \end{bmatrix} \begin{array}{l} = 4 \\ = -2 \\ = 2 \end{array}$$

$x = 3$
$y = 5$
~~$z = 4$~~
$z = 1$

Q4

$$\begin{array}{c|ccc|c} & A & B & C & \\ A & 1 & 0 & 3 & 4 \\ B & -2 & 1 & -1 & -2 \\ C & -1 & 1 & \boxed{2} & 2 \\ & -2 & 2 & 4 & \end{array}$$

Row C = Row A - Row B,
so matrix is linearly dependent

$1x = 3z$
$-2x + 1y = -1z$
$-1x + 1y =$

$A = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$B = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$C = A \times B =$

$$\begin{array}{ccc} 1 & 0 & 3 \\ -2 & 1 & -1 \\ -1 & 1 & 1 \end{array}$$

$1x + 0y + 3z = 4$
$-2x + 1y + -1z = -2$
$-1x + 1y + 1z = 1$

$1x = 3$
$y = 5$

multi

$$\text{Q4} \qquad A = \overset{A}{\begin{bmatrix}1 & 0 & 3\\ -2 & 1 & -1\\ -1 & 1 & 2\end{bmatrix}} \cdot \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \overset{Az}{\begin{bmatrix}1x & 0y & 3z\\ -2x & 1y & -1z\\ -1x & 1y & 2z\end{bmatrix}}$$

$1x = 3z$

$-2x + 1y = -1z$

$-1x + 1y = 2z$

$x = 3z$

$y = 5z$

$z = 1$

$x = 3$

$y = 5$

$c(3, 5, 1)$

$$\begin{bmatrix}1 & 0 & 3\\ -2 & 1 & -1\\ -1 & 1 & 1\end{bmatrix} \cancel{\begin{bmatrix}x\\ y\\ z\end{bmatrix}} \cdot \begin{bmatrix}x & x & x\\ -3 & p & 5\\ x & x & x\end{bmatrix} = \cancel{\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}}$$

$1 \cdot -3 = -3$

$-2 \cdot p = -8$

$-1 \cdot 5 = -5$

$-3 - 5 = -8$

$-8 - 2p = -4$

multi

Q5 $f(x,y) = x + 4y + \frac{2}{xy}$

$\frac{\partial f}{\partial x} = 1 - \frac{2}{x^2y} = 0$ | $\cancel{4}\frac{2}{x^2y} = \cancel{4}1$ | $\frac{2}{x^2} = y$ | $y = 8y^4 = \frac{1}{2}$

$\frac{\partial f}{\partial y} = 4 - \frac{2}{y^2x} = 0$ | $4\frac{2}{y^2x} = 44$ | $\frac{1}{2y^2} = x$ | $x = \frac{x^4}{8} = 2$

$\frac{\partial f}{\partial xx} = \frac{4}{x^3y}$

$\frac{\partial f}{\partial yy} = \frac{4}{y^3x}$

$\frac{\partial f}{\partial xy} = \frac{2}{x^2y^2}$

$f_{xx} = 1$

$f_{yy} = 16$

$f_{xy} = 2$

$1 \cdot 16 - 2^2 > 0$

$1 > 0$

Q6 $F(x,y,z) = (y+y^2z)i + (x-z+2xyz)j + (-y+xy^2)k$

$$\begin{array}{c|ccc} & \partial x & \partial y & \partial z \\ \hline \partial i & 0 & 1+2yz & y^2 \\ \partial j & 1+2yz & 2xz & -1+2xy \\ \partial k & y^2 & -1+2xy & 0 \end{array}$$

$\frac{\partial k}{\partial x} = \frac{\partial i}{\partial z} = y^2$

$\frac{\partial j}{\partial x} = \frac{\partial i}{\partial y} = 1+2yz$

$\left[\frac{\partial i}{\partial x} = \frac{\partial k}{\partial z} = 0\right]$

$\frac{\partial k}{\partial y} = \frac{\partial j}{\partial z} = -1+2xy$

[redacted]

$\int_C F \cdot dr = 3 - 10 = -7$

$\frac{\partial f}{\partial x} = (y+y^2z)x + g(y,z) = F \rightarrow xy + xy^2z$

$\frac{\partial f}{\partial y} = (x-z)y + xzy^2 + g(x,z) = F \rightarrow -2y$

$\frac{\partial f}{\partial z} = (-y+xy^2)z + g(x,y) = F \rightarrow + C$

$F = xy + xy^2z - 2y + C$

lin algebra

Q1

$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\right) = 0$

$\det\left(\begin{bmatrix} \lambda-1 & -1 \\ -1 & \lambda+1 \end{bmatrix}\right)$

$(\lambda-1)(\lambda+1) - 1 = 0$

$\lambda = \sqrt{2}, -\sqrt{2}$

Q2

$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$

$\lambda = 4, 8$

$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}\right) = 0$

$\det\left(\begin{bmatrix} \lambda-2 & -3 \\ -x & \lambda-y \end{bmatrix}\right)$

$(\lambda-2)(\lambda-y) + 3x = 0$

$2(8-y) = -3x \rightarrow 16 - 2y = -3x$

$6(4-y) = -3x \rightarrow 24 - 6y = -3x$

$6 - 2y = -x$

$x = -4$

$y = 2$

Q3

$$\begin{aligned} -x + 5y &= -1 \\ x - y &= 2 \\ x + 3y &= 3 \end{aligned} = \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 3 & 3 \\ 0 & -4 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$x + 3y = 3$

$-4y = -1$

$y = \frac{1}{4}$

$x = \frac{9}{4}$

one solution: $\bar{x}, x = \frac{9}{4}, \frac{1}{4}$