single

Q1 $\rightarrow V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = -10 \text{ cm}^3/\text{sec}$$

$$\rightarrow \frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$$

$$-10 = \frac{4}{3}\pi(3(20)^2)\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-10}{4\pi(20)^2}$$

$$\frac{drm}{dt} = -\frac{1}{16\pi} \approx -0.002 \text{ cm}^3/\text{sec}$$

Q2 a

~~$\lim_{x \to x_0} f(x)$ for $x > x_0$~~

~~$\lim_{x \to x_0} f(x)$ for $x < x_0$~~

$$\frac{1+x^2}{1-x^2}$$

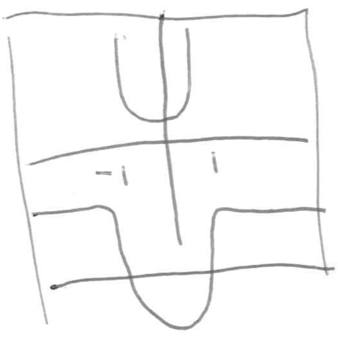
$1 - x^2 = 0$

$x^2 = 1$

$x = \sqrt{1}$

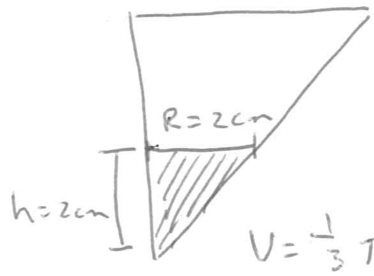
$x = 1$

$x = -1$



Q3

$$\frac{dV}{dt} = \frac{3\,cm^3}{1\ sec}$$



$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi h^3 = \frac{\pi h^3}{3}$$

$$\frac{dV}{dt} = \frac{\pi}{3}\frac{d}{dt}\left[h(t)^3\right]$$

$$\frac{dV}{dt} = \frac{\pi}{3}\cdot 3\,(h(t))^2\cdot\frac{dh}{dt}$$

$$\frac{dv}{dt} = \frac{\pi}{3}\cdot 3h^2\cdot\frac{dh}{dt}$$

$$3\,\cancel{\frac{cm^3}{1 sec}} = \frac{\pi}{3}\cdot 3\,\cancel{h^2}\cdot 4\cdot\frac{dh}{dt}$$

$$3\,\cancel{\frac{cm^3}{1sec}} = \cancel{\frac{3\pi}{3}}\cdot 4\ \ 4\pi\cdot\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{3}{4\pi}\ \frac{cm}{sec}$$

$b = \pi r^2 \leftarrow$ [water evaporates at surface]

$$\frac{dV}{dt} = -\pi r^2\cdot b$$

$$\frac{dh}{dt} = -\frac{3}{4\pi}\cdot\pi r^2\cdot b$$

$$\frac{dh}{dt} = -\frac{3br^2}{4}$$

Single

Q4 $$\int_0^1 \frac{x\,dx}{\sqrt{1+3x^2}}$$

$$\int_1^4 \frac{1}{\sqrt{u}} \frac{1}{6} du$$

$$\int_1^4 \frac{1}{6\sqrt{u}} du$$

$$\frac{1}{6} \int_1^4 \frac{1}{\sqrt{u}} du$$

$$\frac{1}{6} \int_1^4 u^{-\frac{1}{2}} du$$

$$\cancel{\frac{1}{6}\left[2u^{\frac{1}{2}}\right]_1^4} \quad \frac{1}{6}\left[2u^{\frac{1}{2}}\right]_1^4$$

$$\frac{1}{3}$$

Q5

$$\cancel{\int_0^T k \cdot e^{r(T-t)} dt}$$

$$\int_0^1 k \cdot e^{r(1-t)} dt$$

interest rate at end of year

daily deposit times interest rate at end of year, discounted by time $1-t$.

Q6

$y = e^x$

$C = 2\pi \cancel{x} = 2\pi x$

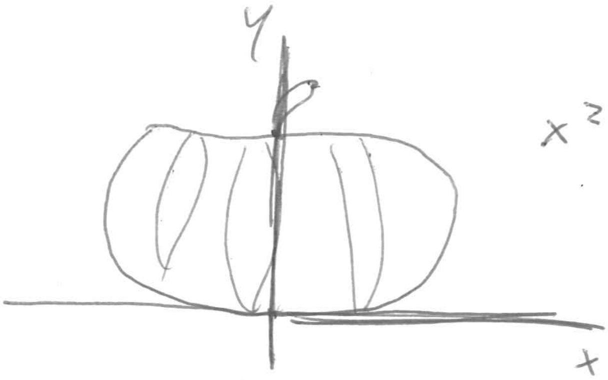
$H = f(x)$

$S = (2\pi x) f(x) = 2\pi x \cancel{\ln} e^x$

shell $V = 2\pi x \, f(x) \, dx$

$$V = 2\pi \int_0^1 x \cdot e^x \, dx$$

Q7



$x^2 + y^4 = 1$

$C = 2\pi r = 2\pi x$

$H = f(x)$

$S = (2\pi x) f(x) = 2\pi x \sqrt[4]{-x^2+1}$

shell $v = 2\pi \cdot x \cdot f(x) \cdot dx$

$$V = 2\pi \int_0^1 x \cdot \sqrt[4]{-x^2+1} \, (4\, dx)$$

$$\cancel{V = 2\pi \int_0^1 x^2 \cdot \sqrt[4]{-x+1} \, dx}$$

multi

Q1

$\cancel{(1,1,1)}$ $\cancel{x \quad y \quad z}$ $\cancel{x^2+2y^2+2z^2=5}$

$\cancel{1x^2+2\cdot 1y^2+2\cdot 1z^2=5}$

$\cancel{1^2+2\cdot 1^2+2\cdot 1^2=5}$

$\cancel{1+2+2=5}$

$\cancel{5=5}$

$$\boxed{x^2+2y^2+2z^2=5}$$

$$\cancel{\frac{z=5-x^2-2z^2}{2}}$$

$$\boxed{z=\sqrt{\frac{2(-x^2-2y^2+5)}{2}}}$$

1

$$= \underbrace{A}_{\text{slope of } x}(x-\underbrace{a}_{x \text{ coordinate}})+\underbrace{B}_{\text{slope of } y}(y-\underbrace{b}_{y \text{ coordinate}})+C(z-\underbrace{c}_{z \text{ coordinate}})=0$$

$$z=\sqrt{\frac{2(-1^2-2+5)}{2}}$$

$$A(x-1)+B(y-1)+C(z-1)=0$$

$$P(1,1,1)$$

$$a(x-x_0)+b(y-y_0)+z_0$$

$$L(x,y)=a(x-1)+b(y-1)+c(z-1)$$

$$a=\frac{\partial f}{\partial x}(1,1) \qquad b=\frac{\partial f}{\partial y}(1,1)$$

$$\frac{\partial f}{\partial x}=x^2 \qquad \frac{\partial f}{\partial y}=2\cdot 1x^2$$

$$=1 \qquad\qquad =2$$

$$2(z-1)$$

$$L(x,y)=1(x-1)+2(y-1)+\cancel{\sqrt{\frac{2(-1-2+5)}{2}}}$$

$$1(\cancel{x}-1)+\cancel{2}(y-1)+\cancel{\frac{\sqrt{4}}{2}}$$

$$x-1+(2y-2)+1$$

$$x+2y+2z=5$$

Multi

Q2 $\quad P = (\overset{x}{2}, \overset{y}{1}) \quad w = x^2 - xy^3$

$A = 3i + 4j$

$$\nabla_A P = 3\frac{\partial P}{\partial x} + 4\frac{\partial P}{\partial y}$$

$$\nabla P = \begin{bmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial P}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x}(x^2 - xy^3) \\ \frac{\partial}{\partial y}(x^2 - xy^3) \end{bmatrix} = \begin{bmatrix} -y^3 + 2x \\ -3y^2 x \end{bmatrix}$$

$$\nabla w P(2,1) = \begin{bmatrix} -1^3 + 2(2) \\ -3(1^2) \cdot 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -6 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= 3(3) + (-6)(4)$$

$$= -15$$

$h = 0.01$

$$h\vec{w} = \begin{bmatrix} 3h \\ -6h \end{bmatrix}$$

$$= \begin{bmatrix} 0.03 \\ -0.06 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= 0.03(3) + (-0.06)(4)$$

$$= -0.15$$

multi

Q 3

$$\text{Plane} = 2x + y - z = 6$$

$$\lambda = x^2 + y^2 + z^2$$

$$2x = 2\lambda$$

$$2y = \lambda$$

$$2z = -\lambda$$

$$2\lambda + \frac{\lambda}{2} \cancel{4} - \frac{-\lambda}{2} = 6$$

$$\underset{x}{\frac{2\lambda \cdot 2}{2}} + \underset{y}{\frac{\lambda}{2}} + \underset{z}{\frac{\lambda}{2}} = 6$$

$$\frac{6\lambda}{2} = 6$$

$$3\lambda = 6$$

$$\lambda = 2$$

$$P = (2, \cancel{2}1, 1)$$

multi

Q4

$$\begin{array}{cc} & \begin{array}{ccc} & A & \end{array} \\ A = & \left[\begin{array}{ccc} 1 & 0 & 3 \\ -2 & 1 & -1 \\ -1 & 1 & \boxed{2} \; a \end{array}\right] \begin{array}{c} 4 \\ -2 \\ 2 \end{array} \\ & \begin{array}{ccc} -2 & 2 & 4 \end{array} \end{array} \quad \overset{X}{\begin{bmatrix} x_x \\ x_y \\ x_z \end{bmatrix}} = \overset{u}{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}} =$$

$1x = 3$
$-2x + y = -1$
$-1x + y = 2$

$$\begin{bmatrix} 1x + 0y + 3z \\ -2x + 1y + -1z \\ -1x + 1y + 2z \end{bmatrix} \begin{array}{c} = 4 \\ = -2 \\ = 2 \end{array}$$

$x = 3$
$y = 5$
$z = 1$

Q4

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \left[\begin{array}{ccc} 1 & 0 & 3 \\ -2 & 1 & -1 \\ -1 & 1 & \boxed{2} \end{array}\right] \begin{array}{c} 4 \\ -2 \\ 2 \end{array} \\ & \begin{array}{ccc} -2 & 2 & 4 \end{array} \end{array}$$

Row C = row A − row B,
so matrix is linearly dependent

$1x = 3z$
$-2x + 1y = -1z$
$-1x + 1y =$

$A = a_1 i + a_2 j + a_3 k$
$B = b_1 i + b_2 j + b_3 k$

$C = A \times B =$

1	0	3
-2	1	-1
-1	1	1

$1x + 0y + 3z = 4$
$-2x + 1y + -1z = -2$
$-1x + 1y + 1z = 1$

$1x = 3$
$y = 5$

multi

Q4

$$A = \overset{A}{\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & -1 \\ -1 & 1 & 2 \end{bmatrix}} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \overset{Az}{\begin{bmatrix} 1x & 0y & 3z \\ -2x & 1y & -1z \\ -1x & 1y & 2z \end{bmatrix}}$$

$1x = 3z$

$-2x + 1y = -1z$

$-1x + 1y = 2z$

$x = 3z$

$y = 5z$

$z = 1$

$x = 3$

$y = 5$

$c(3, 5, 1)$

$$\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x & x & x \\ -3 & p & 5 \\ x & x & x \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$1 \cdot -3 = -3$

$-2 \cdot p = -8$

$-1 \cdot 5 = -5$

$-3 - 5 = -8$

$-8 - 2p = -4$

multi

Q5 $f(x,y) = x + 4y + \frac{2}{xy}$

$\frac{\partial f}{\partial x} = 1 - \frac{2}{x^2 y} = 0$ | $\frac{2}{x^2 y} = 1$ | $\frac{2}{x^2} = y$ | $y = 8y^4 = \frac{1}{2}$

$\frac{\partial f}{\partial y} = 4 - \frac{2}{y^2 x} = 0$ | $\frac{2}{y^2 x} = 4$ | $\frac{1}{2y^2} = x$ | $x = \frac{x^4}{8} = 2$

$\frac{\partial f}{\partial xx} = \frac{4}{x^3 y}$

$\frac{\partial f}{\partial yy} = \frac{4}{y^3 x}$

$\frac{\partial f}{\partial xy} = \frac{2}{x^2 y^2}$ | $f_{xx} = 1$, $f_{yy} = 16$, $f_{xy} = 2$ | $1 \cdot 16 - 2^2 > 0$, $1 > 0$

Q6 $F(x,y,z) = (y + y^2 z)i + (x - z + 2xyz)j + (-y + xy^2)k$

	∂x	∂y	∂z
∂i	0	$1 + 2yz$	y^2
∂j	$1 + 2yz$	$2xz$	$-1 + 2xy$
∂k	y^2	$-1 + 2xy$	0

$\frac{\partial k}{\partial x} = \frac{\partial i}{\partial z} = y^2$

$\frac{\partial j}{\partial x} = \frac{\partial i}{\partial y} = 1 + 2yz$

$\left[\frac{\partial i}{\partial x} = \frac{\partial k}{\partial z} = 0\right]$

$\frac{\partial k}{\partial y} = \frac{\partial j}{\partial z} = -1 + 2xy$

$\int_C F \cdot dr = 3 - 10 = -7$

$\frac{\partial f}{\partial x} = (y + y^2 z)x + g(y,z) = F \to xy + xy^2 z$

$\frac{\partial f}{\partial y} = (x - z)y + xzy^2 + g(x,z) = F \to -2y$

$\frac{\partial f}{\partial z} = (-y + xy^2)z + g(x,y) = F \to + C$

$F = xy + xy^2 z - 2y + C$

lin algebra

Q1

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-1 & -1 \\ -1 & \lambda+1 \end{bmatrix}\right)$$

$$(\lambda-1)(\lambda+1) - 1 = 0$$

$$\lambda = \sqrt{2}, -\sqrt{2}$$

Q2

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

$$\lambda = 4, 8$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-2 & -3 \\ -x & \lambda-y \end{bmatrix}\right)$$

$$(\lambda-2)(\lambda-y) + 3x = 0$$

$$2(8-y) = -3x \rightarrow 16 - 2y = -3x$$

$$6(4-y) = -3x \rightarrow 24 - 6y = -3x$$

$$6 - 2y = -x$$

$$x = -4$$

$$y = 2$$

Q3

$$-x + 5y = -1$$
$$x - y = 2$$
$$x + 3y = 3$$

$$= \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 3 & 3 \\ 0 & -4 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$$x + 3y = 3$$
$$-4y = -1$$

$$y = \frac{1}{4}$$

$$x = \frac{9}{4}$$

one solution: $x, y = \frac{9}{4}, \frac{1}{4}$

Q4 $A = \begin{bmatrix}1&-1&2\\0&1&0\\1&2&1\end{bmatrix}$

$\lambda = 1$

$E_{\lambda=1} = N\left(\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} - \begin{bmatrix}1&-1&2\\0&1&0\\1&2&1\end{bmatrix}\right) = N\left(\begin{bmatrix}0&-1&2\\0&0&0\\1&2&0\end{bmatrix}\right)$

$(A-\lambda I)v = 0$

$v_1 + 2v_2 = 0 \qquad v_1 = -2v_2$

$-v_2 + 2v_3 = 0 \qquad v_3 = \frac{v_2}{2}$

$E_{\lambda=1} = \text{span}\left(\begin{bmatrix}-2\\1\\1/2\end{bmatrix}\right)$

Q5 $A^{-1}_v = \frac{1}{\lambda_v}$

$A^{-1}_{v_1} = \frac{1}{1}$

$A^{-1}_{v_2} = \frac{1}{2}$

$A^{-1}_{v_3} = \frac{1}{4}$

$\det(A^{-1}) = 1 \cdot \left(\frac{1}{2} \cdot \frac{1}{4}\right) = \frac{1}{8}$

Q6

$$\begin{array}{ccccc} l_2 & l_2 & A & & U \\ \begin{bmatrix}1&0&0\\0&1&0\\0&-1&1\end{bmatrix} & \begin{bmatrix}1&0&0\\-3&1&0\\-2&0&1\end{bmatrix} & \begin{bmatrix}1&2&4\\3&8&14\\2&6&13\end{bmatrix} & \to \begin{bmatrix}1&2&4\\0&-2&-2\\0&-2&-5\end{bmatrix} = & \begin{bmatrix}1&2&4\\0&-2&-2\\0&0&-3\end{bmatrix}\end{array}$$

$L = l_1^{-1} l_2^{-1} \begin{bmatrix}1&0&0\\3&1&0\\2&1&1\end{bmatrix}$