ECE 320 Fall 2013: Linearity and Time-Invariance

Proving Time-Invariance

1. Write the output $y[n]$ in terms of the input $x[n]$.
2. Assume $f[n]$ is a different input with output $g[n]$. Write the output $g[n]$ in terms of $f[n]$.
3. If $f[n] = x[n - n_0]$, is the output $g[n] = y[n - n_0]$?
4. To answer this question, first write down $y[n - n_0]$ in terms of $x[n]$.
5. Next, assume that $f[n] = x[n - n_0]$ and then write $g[n]$ in terms of $x[n]$.
6. Are the expressions you wrote for $y[n - n_0]$ and $g[n]$ in terms of $x[n]$ the same for any choice of n_0? If so, the system is time-invariant. If $g[n] \neq y[n - n_0]$ for even one value of n_0, it is not time-invariant. See if you can find a simple input signal demonstrating that delaying the input by a fixed amount does not delay the output by the same amount. This kind of example proving something is not true is called a counter-example.

To prove a system is linear, we must prove **both** the scaling and additivity property.

Proving Scaling

1. Write the output $y[n]$ in terms of the input $x[n]$.
2. Assume $f[n]$ is a different input with output $g[n]$. Write $g[n]$ in terms of $f[n]$.
3. If $f[n] = ax[n]$, is $g[n] = ay[n]$?
4. To answer this question, first write $ay[n]$ in terms of $x[n]$.
5. Next, assume that $f[n] = ax[n]$, and write $g[n]$ in terms of $x[n]$.
6. Are the expressions you wrote for $ay[n]$ and $g[n]$ in terms of $x[n]$ equal for any choice of a? If so, the system satsfies scaling. If $g[n] \neq ay[n]$, for even one value of a, the system is not linear. See if you can find a simple counter-example demonstrating that scaling the input by a fixed amount does not scale the output by the same amount.

Proving Additivity

1. Write the output $y_1[n]$ in terms of the input $x_1[n]$.
2. Write the output $y_2[n]$ in terms of the input $x_2[n]$.
3. Assume $f[n]$ is a different input with output $g[n]$. Write $g[n]$ in terms of $f[n]$.
4. If $f[n] = x_1[n] + x_2[n]$, is $g[n] = y_1[n] + y_2[n]$?
5. To answer this question, first write $y_1[n] + y_2[n]$ in terms of $x_1[n]$ and $x_2[n]$.
6. Next, assume that $f[n] = x_1[n] + x_2[n]$ and then write write $g[n]$ in terms of $x_1[n]$ and $x_2[n]$.
7. Are the expressions you wrote for $y_1 + y_2[n]$ and $g[n]$ equal for any possible signals $x_1[n] + x_2[n]$? If so, the system satisfies additivity. If $g[n] \neq y_1[n] + y_2[n]$, for even one choice of $x_1[n]$ and $x_2[n]$, the system is not linear. See if you can find a simple counter-example demonstrating that adding two inputs does not produce an output which is the sum of their individual outputs.