(i)

$$\text{P8}\;\dfrac{\text{P3}\;\dfrac{\rho[f \mapsto \alpha \mapsto int, x \mapsto int](x) = int}{\rho[f \mapsto \alpha \mapsto int, x \mapsto \alpha]x : int} \qquad \text{P3}\;\dfrac{\rho[f \mapsto \forall\alpha.\alpha \to \alpha](ff) = \forall\alpha.\alpha \to \alpha}{\rho[f \mapsto \forall\alpha.\alpha \to \alpha] \;\vdash\; ff : \alpha \to int}}{\rho \;\vdash\; let\, fx = 1 in\, ff\, end : \alpha \to int}$$

The type of f should be polymorphic since it is used both as a function and as an integer (1). If f is not polymorphic this would be ill-typed.

(ii)
Recursive:

$$\dfrac{\dfrac{\dfrac{\rho[x \mapsto int, f \mapsto int \to int](f) = int \to int}{\rho[x \mapsto int, f \mapsto int \to int]f : int \to int}\,\text{P3} \quad \dfrac{\dfrac{\rho[x \mapsto int, f \mapsto int \to int](n) = int}{\rho[x \mapsto int, f \mapsto int \to int]x : int}\,\text{P3} \quad \dfrac{}{\rho[x \mapsto int, f \mapsto int \to int]1 : int}\,\text{P1}}{\rho[x \mapsto int, f \mapsto int \to int]x + 1 : int}\,\text{P4}}{\rho[x \mapsto int, f \mapsto int \to int]f(x+1) : int}\,\text{P9}$$

Body derivation:

$$\dfrac{\dfrac{\dfrac{\rho[x \mapsto int, f \mapsto int \to int](x) = int}{\rho[x \mapsto int, f \mapsto int \to int] \vdash x : int}\,\text{P3} \quad \dfrac{}{\rho[x \mapsto int, f \mapsto int \to int] \vdash 10 : int}\,\text{P1}}{\rho[x \mapsto int, f \mapsto int \to int] \vdash x < 10 : bool}\,\text{P5} \quad \dfrac{}{\rho[x \mapsto int, f \mapsto int \to int] \vdash 42 : int}\,\text{P1}}{\rho[x \mapsto int, f \mapsto int \to int] \vdash if\ x < 10\ then\ 42\ else\ f(x+1) : int}\,\text{Recursive}$$

$$\dfrac{\text{Body derivation} \qquad \dfrac{\dfrac{\rho[x \mapsto int, f \mapsto int \to int](f) = int \to int}{\rho[x \mapsto int, f \mapsto int \to int]f : int \to int}\,\text{P3} \quad \dfrac{}{\rho[x \mapsto int, f \mapsto int \to int]20 : int}\,\text{P1}}{\rho[x \mapsto int, f \mapsto int \to int]f20 : int \to int}\,\text{P9}}{\rho \vdash let\ fx = if\ x < 10\ then\ 42\ else\ f(x+1)\ in\ f20\ end : int \to int}\,\text{P8}$$

Since there are recursive calls on f, f cannot be polymorphic, as it would result in inconsistencies if f changed type during recursion.