SI232: Subspace Learning
Homework 2: Structure from Motion via Subspace Learning

Instructor: Prof. Manolis C. Tsakiris

Due 12/15/2019, 11.59PM, GMT +8

Recall from the class that if imaged points $\boldsymbol{x}_i, \boldsymbol{x}_i'$ satisfy homography relation, i.e., $\boldsymbol{x}_i' \sim \boldsymbol{H}\boldsymbol{x}_i$, then it follows that $\boldsymbol{x}_i' \times \boldsymbol{H}\boldsymbol{x}_i = \mathbf{0}$. This gives 3 equations, each of which is linear in entries of $\boldsymbol{H}$. In a similar fashion to fundamental matrix estimation, we would like to rewrite the equations to have the form $\boldsymbol{v}_j(\boldsymbol{x}_i, \boldsymbol{x}_i')^\top \boldsymbol{h} = 0, \forall j \in [3]$, where $\boldsymbol{v}_j : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^9$ embeds $\boldsymbol{x}_i'$ and $\boldsymbol{x}_i$ to a higher dimensional space, and $\boldsymbol{h} \in \mathbb{R}^9$ is in one-to-one correspondence with $\boldsymbol{H} \in \mathbb{R}^3 \times \mathbb{R}^3$.

Problem 1. Write down the expression for $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ in terms of $\boldsymbol{x}_i, \boldsymbol{x}_i'$ and $\boldsymbol{h}$ in terms of $\boldsymbol{H}$. Are $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ linearly independent? As such, how many correspondences are needed at least to determine an up-to-scale unique $\boldsymbol{h}$? Justify your answer.

Recall again from the class that in fundamental matrix estimation, each correspondence $\boldsymbol{x}_i, \boldsymbol{x}_i'$ gives only *one* equation in $\boldsymbol{F}$, or equivalently only *one* fundamental embedding. When the true correspondences are general, their fundamental embeddings are expected to span a hyperplane $\mathrm{Span}(\boldsymbol{f})^\perp$. Conversely, the fundamental embeddings of mismatched correspondences may not lie in this hyperplane. As such, we proposed in class to robustly estimate $\boldsymbol{f}$ via a non-convex non-smooth optimization problem called Dual Principal Component Pursuit [1, 2, 3]:

$$\min_{\boldsymbol{f} \in \mathbb{R}^9} \|\tilde{\mathbf{X}}^\top \boldsymbol{f}\|_1 \ \text{ s.t. } \ \|\boldsymbol{f}\|_2 = 1. \tag{1}$$

, where the columns of $\tilde{\mathbf{X}} \in \mathbb{R}^{9\times(N+M)}$ are fundamental embeddings of N true matches and M mismatches. This problem can be solved by an iteratively-reweighted-least-squares (IRLS) algorithm [1, Algorithm 2].

Problem 2. Explain the intuition of the DPCP optimization problem (1). Moreover, write the objective function of (1) in the form of a sum, which will help you for the next problem.

On the other hand, in homography matrix estimation, each correspondence gives *three* equations in $\boldsymbol{H}$, or equivalently *three* homographic embeddings. In that case, *all* three homographic embeddings lie in the hyperplane $\mathrm{Span}(\boldsymbol{h})^\perp$. Another way to think about this is the subspace spanned by the homographic embeddings is in the hyperplane $\mathrm{Span}(\boldsymbol{h})^\perp$.

Problem 3. Modify the DPCP objective function in equation (1) to robustly estimate $\boldsymbol{h}$. In particular, this objective function should consider the *distance* between the hyperplane $\mathrm{Span}(\boldsymbol{h})^\perp$ and the *subspace* spanned by the homographic embeddings for each correspondence. Hint: an orthonormal basis for the latter must be found. Design an IRLS algorithm to solve the modified problem.

Problem 4. Based on your previous answers, implement the functions given in the folder. Test your implementation on the given data. What do you observe? How do you interpret the results?

Submission instructions. Send email to *dingtj@shanghaitech.edu.cn* with subject *SI232:HW2* and attachment *firstname-lastname-hw2-sl19.zip* or *firstname-lastname-hw2-sl19.tar.gz*. The attachment should include the following content:

1. A file called *hw2.pdf* containing your answers to each one of the analytical questions. If at all possible, you should generate this file using latex. If not possible, you may use another editor, or scan your handwritten solutions. But note that you must submit a single PDF file with all your answers.

2. For coding questions, submit *SfM_Homo_Code.m* along with the other utility functions that you use in your code. Do *not* submit data.

The TA will run your scripts to generate the results. Thus, your script should include all needed plotting commands so that figures pop up automatically. Please make sure that the figure numbers match those you describe in *hw2.pdf*. In writing your code, you should assume that the TA will place the input data in the directory that is *relevant* to the question solved by your script. Also, make sure to comment your code properly.

References

[1] M. C. Tsakiris and R. Vidal, "Dual principal component pursuit," *Journal of Machine Learning Research*, vol. 19, no. 18, pp. 1–50, 2018.

[2] Z. Zhu, Y. Wang, D. Robinson, D. Naiman, R. Vidal, and M. Tsakiris, "Dual principal component pursuit: Improved analysis and efficient algorithms," *Neural Information Processing Systems*, 2018.

[3] T. Ding, Z. Zhu, T. Ding, Y. Yang, D. Robinson, M. Tsakiris, and R. Vidal, "Noisy dual principal component pursuit," in *International Conference on Machine Learning*, pp. 1617–1625, 2019.