Neighbourhood Processing

Neighbourhood

$(x-1, y+1)$ $(x, y+1)$ $(x+1, y+1)$

$(x-1, y)$ (x, y) $(x+1, y)$

$(x-1, y-1)$ $(x, y-1)$ $(x+1, y-1)$

** 4-neighbours of P, $N_4(P)$

*** Diagonal neighbours of P, $N_D(P)$

} All these together called: 8-neighbors of P, $N_8(P)$

Distance Measures

$P(x,y)$, $q(s,t)$

- Euclidean Distance: $D_e(P,q) = \sqrt{(x-s)^2 + (y-t)^2}$

- D_4 distance (city block distance): $D_4(P,q) = |x-s| + |y-t|$
  ↳ Pixels with $D_4 = 1$

- D_8 distance (chessboard distance): $D_8(P,q) = \max(|x-s|, |y-t|)$
  ↳ Pixels with $D_8 = 1$

Linear vs Non-Linear operations

Conditions of Linear operations:

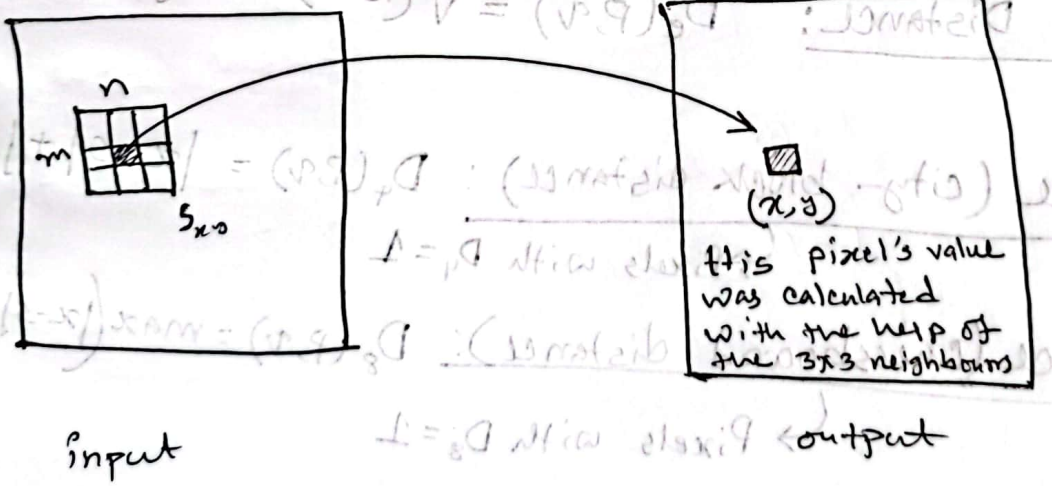
① Additivity property: $H[f(x_1, y_1) + f(x_2, y_2)]$

$= H[f(x_1, y_1)] + H[f(x_2, y_2)]$

② Homogeneity property:

$H[a f(x, y)] = a H[f(x, y)]$

Neighbourhood operations



Neighbourhood Operation : Mechanism

** Neighbourhood processing is also called Spetial filtering.

*** Special filtering consists of:

① A neighbourhood (defined by the filter kernel or Mask)

② A predefined Operation
- Linear Operation
- Non-Linear Operation

*** A processed image is generated as the centers of the filter kernel visits each pixel in the input image.

Linear Special Filtering

Formula:

- Kernel shape : $m \times n$
- Number of pixels padded : p
- Stride : s
- Input image shape : $M \times N$
- Output image shape : $H \times W$

$$H = \text{floor}\left(\frac{M + 2p - m}{s} + 1\right)$$

$$W = \text{floor}\left(\frac{N + 2p - n}{s} + 1\right)$$

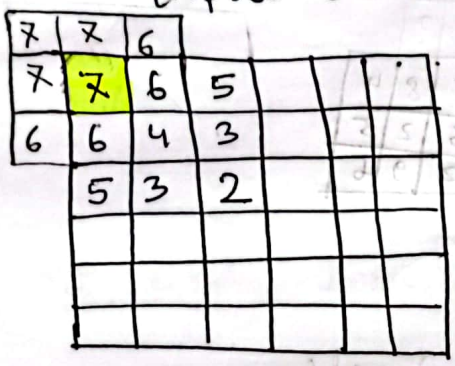
if we want input and output to be same shape:

No. of padding pixels, $p = \frac{m-1}{2}$

$$= \frac{n-1}{2}$$

Math Demo:

input (6×6)



Kernel (3×3)

0	-1	0
-1	5	-1
0	-1	0

- Kernel shape : 3×3 ($m \times n$)
- $S = 1$
- Padding = Nearest ; $P = 1$
- input image = 6×6 ($M \times N$)
- Out image = $H \times W$

$7 \cdot 0$
$7 \cdot -1$
$6 \cdot 0$
$7 \cdot -1$
$7 \cdot 5$
$6 \cdot -1$
$6 \cdot 0$
$6 \cdot -1$
$4 \cdot 0$
9

$$H = \text{floor}\left(\frac{M + 2p - m}{S} + 1\right)$$

$$= \text{floor}\left(\frac{6 + 2 \cdot 1 - 3}{1} + 1\right)$$

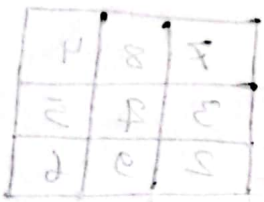
$$= 6$$

$$W = \text{floor}\left(\frac{N + 2p - n}{S} + 1\right)$$

$$= \text{floor}\left(\frac{6 + 2 \cdot 1 - 3}{1} + 1\right)$$

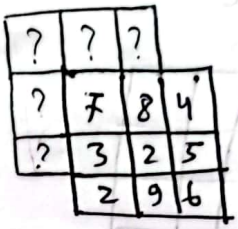
$$= 6$$

*** So, input ও যেভাবে $(x, y) = x$ নিবে, output ও $(x, y) = 9$ হবে।



Padding

Kernel stride করার সময় border pixel এ problem হয়



Solution:

Solution-(I): Don't visite border pixels.
- image shrinks.

Solution-(II): Do Padding

Linear Padding:
Nearest করে then বসাতে
~~বসাতে~~ 0.

Padding Type:

(I) Zero Padding: extend করে pixel value = 0

(II) Constant Padding: image extend করে pixel value = some constant

(IV) Mirror Padding: Mirror the edge pixels

(III) Clamp / Edge / Nearest: Repeat edge pixels.

Mirror Padding →

		7	8	4
4	3	3	4	5
9	2	2	9	6
2	2	2	9	
2	2	3	4	

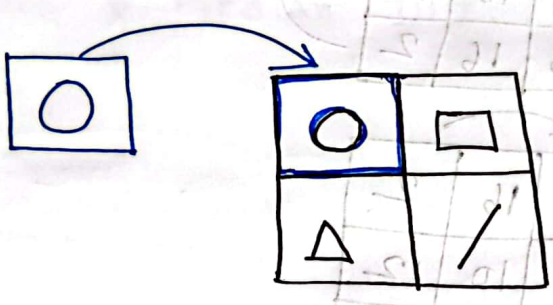
Nearest padding →

		7	8	4
3	3	3	4	5
2	2	2	9	6
2	2	2	9	
2	2	2	9	

Correlation

[**] Sliding dot product.

[**] It is the measure of similarity of the two signals as a function of the displacement of one function relative to other.



[**] একটা weight matrix থাকে (filter kernel), যেটাকে slide করায় original image এর উপর। যেখান থেকে similarity বের করা হয়।

[**] Linier Spetial filtering is basically a 2D signal correlation operation.

Convolution

[**] If we rotate the original filter w(x,y) 180° and then find the correlation, it's called convolution.

original filter :

4	5	6
9	10	12
13	16	2

90° rotate :

13	16	2
9	10	12
4	5	6

180° rotate :

2	16	13
12	10	9
6	5	4

[**] for symmetric filters convolution and correlation gives same result.

Smoothing Spetial Filters

- Linear
  - Box / avegage filtering
  - Gaussian filtering

- Non-Linear
  - Median filtering

Linear Smoothing Filters

Average Fter:

→ जो pixel का average

$$w(x,y) = \frac{1}{mn}$$

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

Gaussian Filter:

→ weighted

$$w(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Applications

① Blurring

② Sharpening the edges

③ Denoising

④ Low pass Filtering (getting rid of high variations)

Ex For the same kernel size the gaussian blur preserves more information than avegrage filtering.

↳ edge better preserve করে।

Non-Linear smoothing filtering / order statistic filtering

[***] Kernel নামের একটা

[**] neighbourhood এর value গুলোকে sort করে Statistical operation চালিয়ে value বের করবে।

(I) Median filtering: মাঝখানের value (after sorting)

(II) Max and Min filtering.

(III) Midpoint filtering

(IV) Alpha-trimmed mean filtering.

Median filtering

(**) Gaussian / Box filtering ভালো smoothing করে, Median filtering-ও ভালো smoothing করে but Median filtering better.

Box > Gaussian > Median filtering

Linear | Non-Linear

Unsharp Masking & High Boost Filtering

① Blur the original image:

$$f'(x,y) = \text{Blur}[f(x,y)]$$

②
$$g_{mask}(x,y) = f(x,y) - f'(x,y)$$

③
$$g(x,y) = f(x,y) + g_{mask}(x,y)$$