Theoretical Computer Science
Winter semester 2021/22
Prof. Dr. Georg Schied

Assignment 1

Deadline: Wednesday, 13 October 2021

- You have to submit solutions for all exercises that are marked as obligatory.
- The exercises can be solved and handed in in teams of 2.
- To pass, 50% of the points must be achieved (10 out of 20)

Exercise 1.1 - obligatory (4 points)

a) Let A and B be two finite sets. How many elements contains $\wp(A \times B)$ dependent on the cardinality of A and B?

b) Let M = {1, 2,..., 8} be the set of natural numbers between 1 and 8. Describe the is-divisor-of relation as a set of pairs, according to definition 2.3.

c) Which of the following relationships can be modeled as a function, which as a relation?

  (1) child-mother relationship
  (2) is-uncle-of relationship
  (3) relationship between students and enrolment numbers
  (4) relationship between students and courses they take

Exercise 1.2 - obligatory (6 points)

Let G be the following graph:
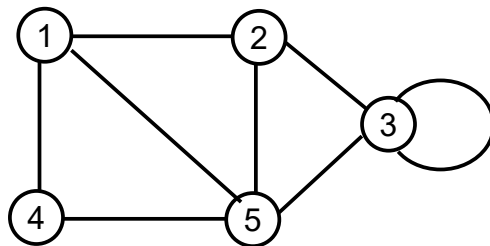
a) Describe G formally as a set V of vertices and a set E of edges.

b) How would G be stored using an adjecency matrix?

c) How would G be stored using adjacency lists?

d) How can you compute the degree of a vertex from the adjacency matrix representation?

e) How can you compute the degree of a vertex from the adjacency lists representation?

Exercise 1.3 - obligatory (6 points)

Let $G = (V, E)$ be a directed graph, where
$V = \{1, 2, 3, 4\}$
$E = \{ (1,2), (1,4), (2,2), (2,3), (2,4), (3,1), (4,3) \}$

a) Draw G as a diagram.

b) Determine the *indegree* and the *outdegree* of vertex 2.

c) Is G *planar*?

d) Is G *strongly connected*?

e) Is G *acyclic*?

Exercise 1.4

Anna, Bruno, Cindy, Dave, and Emma are the attendants of a party. Anna knows one of the other persons, Bruno knows two of the persons, Cindy knows three persons and Dave knows four persons. How many persons knows Emma? Depict a graph that represents the situation.

Hint: The relation "knows" is considered here as a symmetric relation, i.e. if person X knows person Y, then person Y also knows person X.

Exercise 1.5 - obligatory (4 points)

a) Let be *G* the following graph :
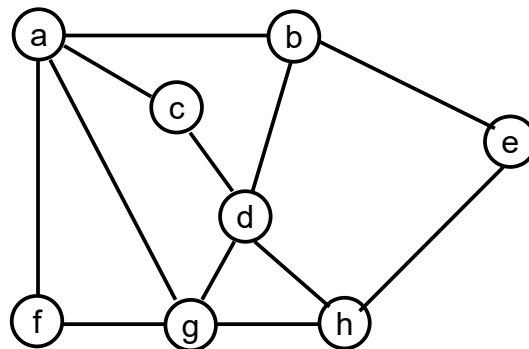
Has *G* an Eulerian Cycle (Euler tour) or an Eulerian path? Give a short explanation for your answer. If possible, give an example of an Eulerian tour or path.

b) If G_n is a *complete graph* with *n* vertices, does there exist an Euler tour or an Eulerian path for G_n?

Exercise 1.6

Devise a (non-planar) graph that requires 5 colors to be colored admissibly.