Theoretical Computer Science

Winter semester 2021/22

Prof. Dr. Georg Schied

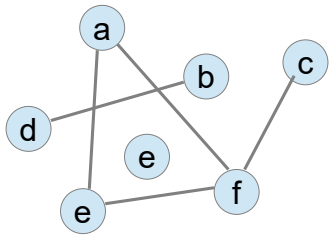
Assignment 2

Deadline: Wednesday, 20 October 2021

- To pass, 10 out of 20 points must be achieved.

Exercise 2.1

What are the *connected components* of the following graph?



Exercise 2.2 - obligatory (6 points)

a) How many *leaves* has a *complete binary tree* of height $h = 9$?

b) How many *nodes* has a *complete binary tree* of height $h = 9$?

c) Which *minimum height* has a binary tree consisting of 100 nodes?

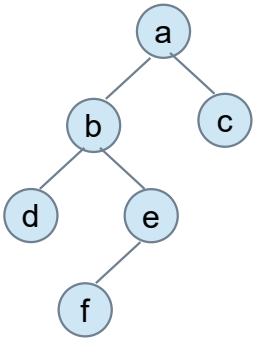
d) Is the following statement true? (give a short justification).

"Every complete binary tree has more internal nodes than leaves."

Exercise 2.3 - obligatory (6 points)

a) Traverse this binary tree in

- Pre-order,
- In-order,
- Post-order.



b) Traversing a binary tree yields this results:

- Pre-order: 3, 2, 1, 4, 5, 6, 7
- In-order: 1, 2, 3, 5, 6, 4, 7
- Post-order: 1, 2, 6, 5, 7, 4, 3

Reconstruct the tree from this traversal sequences.

Exercise 2.4 - obligatory (3 points)

a) Draw an abstract syntax tree of the following expression:

```
x * 3 - (z + 4) / y
```

b) [optional, 3 bonus points] Generate stack-machine code for the evaluation of the expression (see lesson 5).

Exercise 2.5

A simple two player game has the following rules: Player A begins and chooses a number from the set {1, 2}. Subsequently the players B and A choose alternatingly a number from the set {1, 2, 3}, but it is not allowed to select the same number as the adversary in the preceeding move. All of the choosen numbers from both players are summed up. If a player reaches the sum 6 he wins the game. If the sum exceeds 6, the player loses.

Example 1:

A: 1 sum = 1
B: 3 sum = 4
A: 2 sum = 6, Player A wins

Example 2:

A: 2 sum = 2
B: 1 sum = 3
A: 2 sum = 5
B: 1 sum = 6, Player B wins

A so called *decision tree* can be used to depict all possible moves of the game. The nodes of the tree represent states of the game and the child nodes of a node are the states after the next possible moves.

a) Draw a decision tree for this game.

b) Who will win if both players play in an optimal way?

Exercise 2.6 - obligatory (5 points)

Let u_1 = ab and u_2 = bbb be string over alphabet Σ = {a,b}. Compute:

(1) u_2u_1

(2) u_1u_1

(3) u_1^0

(4) $| u_1 \cdot \varepsilon \cdot u_2 |$

(5) $| u_2^5 |$