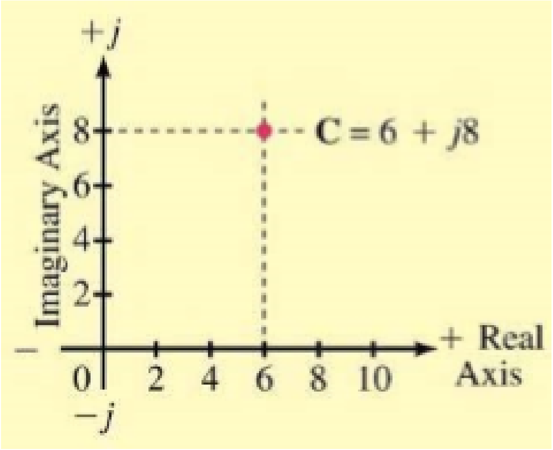
COMPLEX NUMBER

A **complex number** is a number of the form $\mathbf{C} = a + jb$ where a and b are real and $j = \sqrt{-1}$. a is the **real** part of C and b is the **imaginary** part. Note that the letter j is used in electrical engineering to represent the imaginary component since the letter i already has heavy use as the symbol for current (i).

<u>**Geometric Representation**</u> We represent complex numbers geometrically in two different forms.

In the rectangular form, the x-axis serves as the *real* axis and the y-axis serves as the *imaginary* axis. So, for example, the complex number C = 6 + j8 can be plotted in rectangular form as:



$\mathbf{C} = 6 + j8$
(rectangular form)

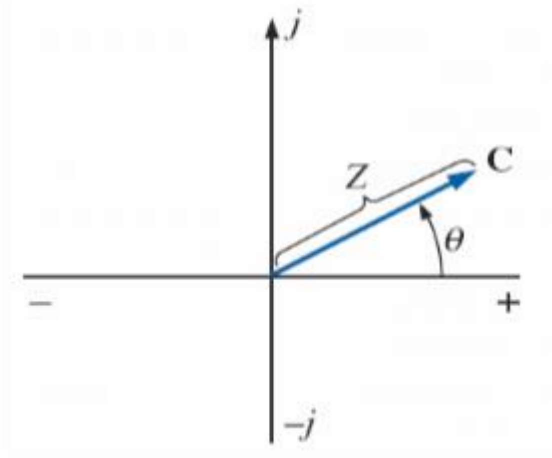
Properties of j

$$j = \sqrt{-1}$$

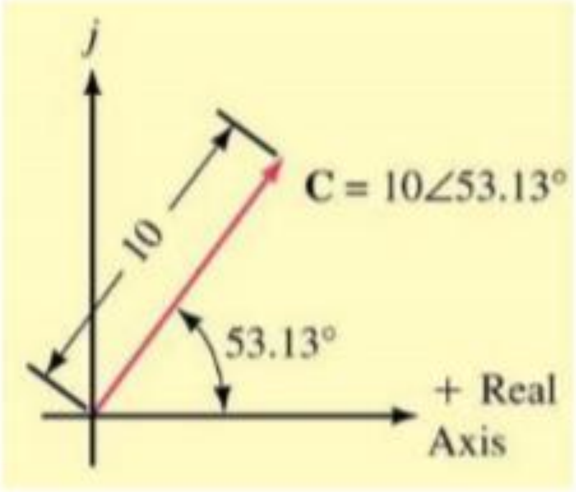
$$j^2 = (\sqrt{-1})(\sqrt{-1}) = -1$$

$$\frac{1}{j} = \frac{1}{j} \times \left(\frac{j}{j} \right) = \frac{j}{j^2} = -j$$

The alternative geometric representation for complex numbers is the polar form. Since a complex number can be represented as a point in the real-imaginary plane, and points in this plane can also be represented in polar coordinates, a complex number can be represented in polar form by $C = Z\angle\theta$ where Z is the distance, or magnitude, from the origin, and θ is the angle measured counterclockwise from the positive real axis.



So, for example, $C = 10\angle 53.13^\circ$ would be plotted as:



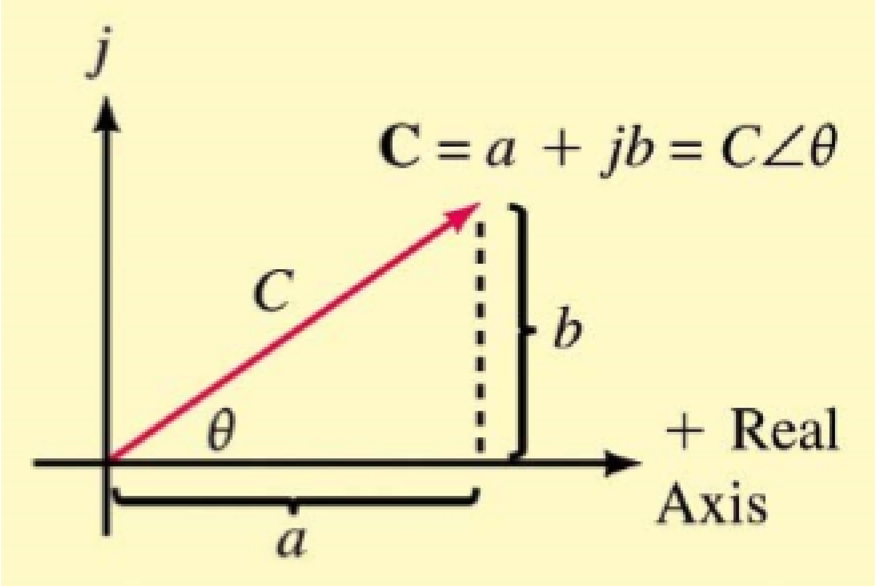
C = $10\angle 53.13°$
(polar form)

Conversion Between Forms We often need to convert between rectangular and polar forms.

To convert between forms where

$$\mathbf{C} = a + jb \qquad \text{(rectangular form)}$$

$$\mathbf{C} = C\angle\theta \qquad \text{(polar form)}$$



apply the following relations

$$a = C\cos\theta$$

$$b = C\sin\theta$$

$$C = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

Addition and Subtraction of Complex Numbers

This is easiest to perform in rectangular form.

We simply add/subtract the real and imaginary parts separately.

For example,

$$(6 + j12) \; + \; (7 + j2) = (6 + 7) \; + j(12 + 2) = 13 + j14$$

$$(6 + j12) \; - \; (7 + j2) = (6 - 7) \; + j(12 - 2) = -1 + j10$$

Multiplication and Division of Complex Numbers

This is easiest to perform in polar form.

For multiplication: multiply magnitudes and add the angles

$$(6\angle 70)\cdot(2\angle 30) = 6\cdot 2\angle(70+30) = 12\angle 100$$

For division: Divide the magnitudes and subtract the angles

$$\frac{(6\angle 70)}{(2\angle 30)} = \frac{6}{2}\angle(70-30) = 3\angle 40$$

The RECIPROCAL and CONJUGATE of a COMPLEX NUMBER

The ***reciprocal*** of $\mathbf{C} = C\angle\theta$, is $\dfrac{1}{C\angle\theta} = \dfrac{1}{C}\angle -\theta$

The ***conjugate*** of $\mathbf{C}$ is denoted $\mathbf{C}^*$, and has the same real value but the opposite imaginary part:

$$\mathbf{C} = a + jb = C\angle\theta$$

$$\mathbf{C}^* = a - jb = C\angle -\theta$$