Chapter 1

Voltage, Current, Power, and Sources

1.1 Introduction

The seven base units of the International System of Units (SI), along with derived units relevant to electrical and computer engineering, are presented in this chapter. The definitions of the terms *voltage, current*, and *power* are given as well.

A voltage source with voltage V_s provides a constant potential difference to the circuit connected between the positive terminal and the negative terminal. A current source with current I_s provides a constant current of I_s amperes to the circuit connected to the two terminals. If the voltage from the voltage source is constant with time, the voltage source is called the *direct current (dc) source*. Likewise, if the current from the current source is constant with time, the current source is called the *dc source*. If the voltage from the voltage source is a sinusoid, the voltage source is called *alternating current (ac) voltage source*. Likewise, if the current from the current source is a sinusoid, the current source is called the *ac current source*.

The voltage or current on the dependent sources depends solely on the controlling voltage or controlling current. Dependent sources are introduced along with circuit symbols.

The elementary signals that are useful throughout the text are introduced next. The elementary signals are *Dirac delta function, step function, ramp function, rectangular pulse, triangular pulse*, and *exponential decay*.

1.2 International System of Units

The International System of Units (SI) is the modern form of the metric system derived from the meter-kilogram-second (MKS) system. The SI system is founded on seven base units for the seven quantities assumed to be mutually independent. Tables 1.1–1.6, which give information on the SI system, come from the NIST Reference on Constants, Units, and Uncertainty (http://physics.nist.gov/cuu/Units/units.html), the official reference of the National Institute of Standards and Technology.

A **meter** is defined as the length of a path traveled by light in a vacuum during a time interval of 1/299,792,458 [$(\approx 1/(3 \times 10^8)$)] of a second.

A **kilogram** is equal to the mass of the international prototype of the kilogram.

TABLE 1.1 SI Base Units.

Base Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of a substance	mole	mol
Luminous intensity	candela	cd

TABLE 1.2 Examples of SI Derived Units.

Derived Quantity	Name	Symbol
Area	square meter	m^2
Volume	cubic meter	m^3
Speed, velocity	meter per second	m/s
Acceleration	meter per second squared	m/s^2
Wave number	reciprocal meter	m^{-1}
Mass density	kilogram per cubic meter	kg/m^3
Specific volume	cubic meter per kilogram	m^3/kg
Current density	ampere per square meter	A/m^2
Magnetic field strength	ampere per meter	A/m
Luminance	candela per square meter	cd/m^2

TABLE 1.3 SI Derived Units with Special Names and Symbols.

Derived Quantity	Name	Symbol	Expression in terms of other SI units
Plane angle	radian	rad	—
Solid angle	steradian	sr	—
Frequency	hertz	Hz	—
Force	newton	N	—
Pressure, stress	pascal	Pa	N/m^2
Energy, work, quantity of heat	joule	J	$N \cdot m$
Power, radiant flux	watt	W	J/s
Electric charge, quantity of electricity	coulomb	C	—
Electric potential difference, electromotive force	volt	V	W/A
Capacitance	farad	F	C/V
Electric resistance	ohm	Ω	V/A
Electric conductance	siemens	S	A/V
Magnetic flux	weber	Wb	$V \cdot s$
Magnetic flux density	tesla	T	Wb/m^2
Inductance	henry	H	Wb/A
Celsius temperature	degrees Celsius	°C	—
Luminous flux	lumen	lm	$cd \cdot sr$
Illuminance	lux	lx	lm/m^2

TABLE 1.4 Examples of SI Derived Units with Names and Symbols (Including Special Names and Symbols.)

Derived Quantity	Name	Symbol
Dynamic viscosity	Pascal second	$Pa \cdot s$
Moment of force	newton meter	$N \cdot m$
Surface tension	newton per meter	N/m
Angular velocity	radian per second	rad/s
Angular acceleration	radian per second squared	rad/s^2
Heat flux density, irradiance	watt per square meter	W/m^2
Thermal conductivity	watt per meter kelvin	$W/(m \cdot K)$
Energy density	joule per cubic meter	J/m^3
Electric field strength	volt per meter	V/m
Electric charge density	coulomb per cubic meter	C/m^3
Electric flux density	coulomb per square meter	C/m^2
Permittivity	farad per meter	F/m
Permeability	henry per meter	H/m
Exposure (X- and γ-rays)	coulomb per kilogram	C/kg

TABLE 1.5 Metric Prefixes.

Prefix	Symbol	Magnitude
yocto	y	10^{-24}
zepto	z	10^{-21}
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10^{1}
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}
peta	P	10^{15}
exa	E	10^{18}
zetta	Z	10^{21}
yotta	Y	10^{24}

TABLE 1.6 Units Outside the SI That Are Accepted for Use with the SI System.

Name	Symbol	Value in SI Units
Minute (time)	min	$1\ min = 60\ s$
Hour	h	$1\ h = 60\ min = 3600\ s$
Day	d	$1\ d = 24\ h = 86{,}400\ s$
Degree (angle)	$^\circ$	$1^\circ = (\pi/180)\ rad$
Minute (angle)	$'$	$1' = (1/60)^\circ = (\pi/10{,}800)\ rad$
Second (angle)	$''$	$1'' = (1/60)' = (\pi/648{,}000)\ rad$
Liter	L	$1\ L = 1\ dm^3 = 10^{-3}\ m^3$
Metric ton	t	$1\ t = 1000\ kg$
Neper	Np	$1\ Np = 20 \log_{10}(e)\ dB = 20/\ln(10)\ dB$
Bel	B	$1\ B = (1/2) \ln(10)\ Np, 1\ dB = 0.1\ B$
Electronvolt	eV	$1\ eV = 1.60218 \times 10^{-19}\ J$
Unified atomic mass unit	u	$1\ u = 1.66054 \times 10^{-27}\ kg$
Astronomical unit	ua	$1\ ua = 1.49598 \times 10^{11}\ m$

A **second** is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

An **ampere** is the constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newtons per meter of length.

A **kelvin**, is 1/273.16 of the thermodynamic temperature of the triple point of water.

A **mole** is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is **mol**. When the mole is used, the elementary entities must be specified; they may be atoms, molecules, ion, electrons, other particles, or specified groups of such particles.

The **candela** is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz (Hz) and that has the radiant intensity in that direction of 1/683 watt per steradian.

1.3 Charge, Voltage, Current, and Power

1.3.1 ELECTRIC CHARGE

Atoms are the basic building blocks of matter. The nucleus of atoms consists of protons and neutrons. Electrons orbit around the nucleus. Protons are positively charged, and electrons are negatively charged, while neutrons are electrically neutral. The amount of charge on the proton is given by

$$e = 1.60217662 \times 10^{-19}\ C$$

Here, the unit for charge is in coulombs (C).

$$-e = -1.60217662 \times 10^{-19}\ C$$

Notice that the charge is quantized as the integral multiple of e. Since there are equal numbers of protons and electrons in an atom, it is electrically neutral. When a plastic is rubbed by fur, some electrons from the fur are transferred to the plastic. Since the fur lost electrons and the plastic gained them, the former is positively charged and the latter negatively charged. When the fur and the plastic are placed close together, they attract each other. Opposite charges attract, and like charges repel. However, since the electrons and protons are not destroyed, the total amount of charge remains the same. This is called the *conservation of charge*.

1.3.2 ELECTRIC FIELD

According to Coulomb's law, the magnitude of force between two charged bodies is proportional to the charges Q and q and inversely proportional to the distance squared; that is,

$$F = \frac{1}{4\pi\varepsilon}\frac{Qq}{r^2} \tag{1.1}$$

Here, ε is permittivity of the medium. The permittivity of free space, ε_0, is given by

$$\varepsilon_0 = \frac{1}{4\pi c^2 10^{-7}}\,(\mathrm{F/m}) = 8.8541878176 \times 10^{-12}\,(\mathrm{F/m}) \tag{1.2}$$

Here, c is the speed of light in the vacuum, given by $c = 299{,}792{,}458$ m/s $\approx 3 \times 10^8$ m/s. The unit for permittivity is farads per meter (F/m). The direction of the force coincides with the line connecting the two bodies. If the charges have the same polarity, the two bodies repel each other. On the other hand, if the charges have the opposite polarity, they attract each other.

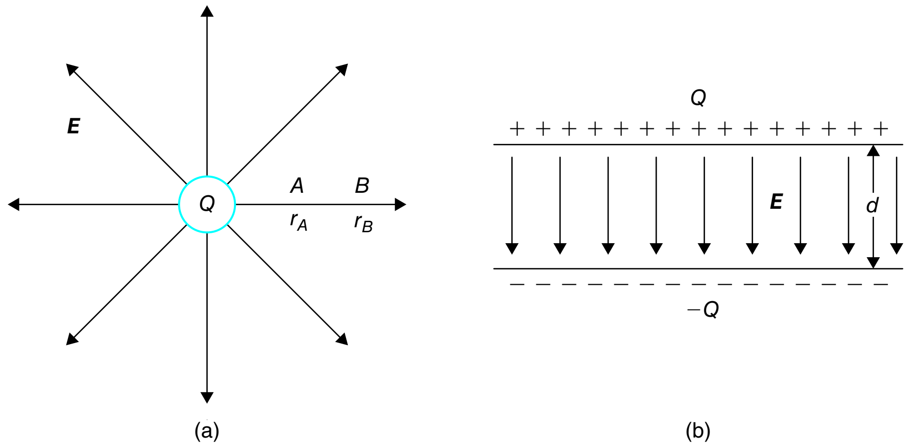
If a positive test charge with magnitude q is brought close to a positive point charge with magnitude Q, the test charge will have a repulsive force. The magnitude of the force is inversely proportional to the distance squared between the point charge and the test charge. The presence of the point charge creates a field around it, where charged particles experience force. This is called an **electric field**, which is defined as the force on a test charge q as the charge q decreases to zero; that is,

$$\boldsymbol{E} = \lim_{q \to 0} \frac{\boldsymbol{F}}{q} \quad (\text{V/m}) \tag{1.3}$$

The electric field is a force per unit charge. The electric field $\boldsymbol{E}$ is a vector quantity whose direction is the same as that of the force. Figure 1.1 shows the electric field for a positive point charge and charged parallel plates.

FIGURE 1.1

Electric field for (a) a point charge and (b) parallel plates.



If an object with charge q is placed in the presence of electric field $\boldsymbol{E}$, the object will experience a force as follows:

$$\boldsymbol{F} = q\boldsymbol{E} \tag{1.4}$$

For a positive point charge Q, the electric field is given by

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\boldsymbol{a}_r \tag{1.5}$$

where $\boldsymbol{a}_r$ is a unit vector in the radial direction from the positive point charge Q. For parallel plates with area S per plate, distance d between the plates, the electric field is constant within the plates and the magnitude of the electric field is given by

$$E = \frac{Q}{\varepsilon S} \tag{1.6}$$

The direction of the field is from the plate with positive charges to the plate with negative charges, as shown in Figure 1.1(b).

1.3.3 VOLTAGE

If a positive test charge dq is moved against the electric field created by a positive charge, an external agent must apply work to the test charge. Let dw_{AB} be the amount of the work needed to move the test charge from B (initial) to A (final). Here, dw_{AB} is the potential energy in joules. Then, the potential difference between points A and B is defined as the work done per unit charge against the force; that is,

$$v_{AB} = v_A - v_B = \frac{dw_{AB}}{dq} \quad \text{(J/C)} \tag{1.7}$$

The unit for the potential difference is joules per coulomb, which is also called a *volt* (V):

$$1\ \text{V} = 1\ \text{J/C}$$

The potential difference between A and B is called *voltage*. The potential difference between points A and B is given by

$$v_{AB} = v_A - v_B = -\int_B^A \boldsymbol{E} \cdot d\ell \tag{1.8}$$

The negative sign implies that moving against the electric field increases the potential. For a positive point charge Q at origin with an electric field given by Equation (1.5), the potential difference between two points A and B with distances r_A and r_B, respectively, from Q is given by

$$v_{AB} = v_A - v_B = -\int_{r_B}^{r_A} \frac{1}{4\pi\varepsilon} \frac{Q}{r^2} dr = -\frac{Q}{4\pi\varepsilon}\left(\frac{-1}{r}\right)\Bigg|_{r_B}^{r_A} = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \text{V} \tag{1.9}$$

Notice that the integral of $1/r^2$ is $-1/r$. If r_B is infinity, the potential difference is

$$v_{AB} = v_A - v_B = v_A = \frac{Q}{4\pi\varepsilon r_A} \text{ V} \tag{1.10}$$

The potential is zero at infinity. This is a reference potential. For the parallel plates shown in Figure 1.1(b), the potential difference between A and B is

$$v = Ed = \frac{Q}{\varepsilon S} d \tag{1.11}$$

If the potential at B is set at zero ($v_B = 0$), the potential at point A is given by

$$v_A = \frac{dw_A}{dq} \quad \text{(J/C)} \tag{1.12}$$

or simply

$$v = \frac{dw}{dq} \quad \text{(J/C)} \tag{1.13}$$

The potential difference v is called *voltage*. A **battery** is a device that converts chemical energy to electrical energy. When a positive charge is moved from the negative terminal to the positive terminal through the 12-V battery, the battery does 12 joules of work on each unit charge. The potential energy of the charge increases by 12 joules. The battery provides energy to the rest of the circuit.

1.3.4 CURRENT

In the absence of an electric field, the free electrons in the conduction band of conductors such as copper wire make random movements. The number of electrons crossing a cross-sectional area of the copper wire from left to right will equal the number of electrons crossing the same cross-sectional area from right to left. The net number of electrons crossing this area will be zero. When an electric field is applied along the copper wire, the negatively charged electrons will move toward the direction of higher potential. The current is defined as the total amount of charge q passing through a cross-sectional area in t seconds; that is,

$$I = \frac{q}{t} \tag{1.14}$$

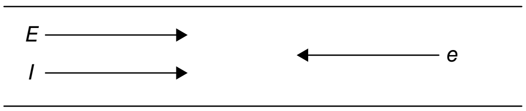
The unit for the current is coulombs per second (C/s) or amperes (A). If the amount of charge crossing the area changes with time, the current is defined as

$$i(t) = \frac{dq(t)}{dt} \tag{1.15}$$

The direction of current is defined as the direction of positive charges. Since the charge carriers inside the conductors are electrons, the direction of electrons is opposite to the direction of the current. Figure 1.2 shows the directions of the electric field, current, and electron inside a conductor.

FIGURE 1.2

The directions of E, I, and e.



The charge transferred between time t_1 and t_2 can be obtained by integrating the current from t_1 and t_2; that is,

$$q = \int_{t_1}^{t_2} i(\lambda)d\lambda \tag{1.16}$$

EXAMPLE 1.1

The charge flowing into a circuit element for $t \geq 0$ is given by

$$q(t) = 2 \times 10^{-3}(1 - e^{-1000t}) \text{ coulomb}$$

Find the current flowing into the element for $t \geq 0$.

$$i(t) = \frac{dq(t)}{dt} = 2 \times 10^{-3} \times 1000e^{-1000t} \quad A = 2e^{-1000t} \quad A \text{ for } t \geq 0$$

Exercise 1.1

The charge flowing into a circuit element for $t \geq 0$ is given by

$$q(t) = 4 \times 10^{-3} e^{-2000t} \text{ coulomb}$$

Find the current flowing into the element for $t \geq 0$.

Answer:

$$i(t) = \frac{dq(t)}{dt} = -8e^{-2000t} \ A \text{ for } t \geq 0$$

EXAMPLE 1.2

The current flowing into a circuit element is given by

$$i(t) = 5 \sin(2\pi 10t) \text{ mA}$$

for $t \geq 0$. Find the charge flowing into the device for $t \geq 0$. Also, find the total charge entered into the device at $t = 0.05$ s.

$$q(t) = \int_0^t i(\lambda)d\lambda = \frac{5 \times 10^{-3}}{2\pi 10}[1 - \cos(2\pi 10t)]$$

$$= 7.9577 \times 10^{-5}[1 - \cos(2\pi 10t)] \text{ coulomb}$$

At $t = 0.05$ s, we have

$$q(0.05) = 1.5915 \times 10^{-4}[1 - \cos(2\pi 10 \times 0.05)] = 1.5915 \times 10^{-4} \text{ coulombs}$$

Exercise 1.2

The current flowing into a circuit element is given by

$$i(t) = 5 \cos(2\pi 10t) \text{ mA}$$

for $t \geq 0$. Find the charge flowing into the device for $t \geq 0$. Also, find the total charge entered into the device at $t = 0.0125$ s.

Answer:

$$q(t) = \int_0^t i(\lambda)d\lambda = \frac{5 \times 10^{-3}}{2\pi 10} \sin(2\pi 10t) = 7.9577 \times 10^{-5} \sin(2\pi 10t) \text{ coulombs}$$

$$q(0.0125) = 7.9577 \times 10^{-5} \sin(2\pi 10 \times 0.0125) = 5.6270 \times 10^{-5} \text{ coulombs}$$

1.3.5 POWER

The battery provides a constant potential difference (voltage) of v volts from the negative terminal to the positive terminal. When a positive charge dq is moved from the negative terminal to the positive terminal through the battery, the potential energy is increased by $dq\,v = dw$. When the positive charge dq moves through the rest of the circuit from the positive terminal to the negative terminal, the potential energy is decreased by the same amount $(dq\,v)$. The rate of potential energy loss is given by

$$p = \frac{dw}{dt} = \frac{dq\,v}{dt} = iv \tag{1.17}$$

The rate of energy loss is defined as *power*. Equation (1.17) can be rewritten as

$$dw = dq\,v = p\,dt \tag{1.18}$$

The energy is the product of power and time. If Equation (1.18) is integrated as a function of time, we get

$$w(t) = \int_{-\infty}^{t} p(\lambda)d\lambda \tag{1.19}$$

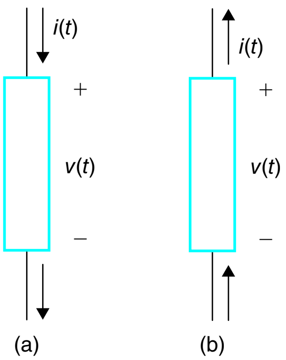
According to Equation (1.19), the energy is the integral of power. As shown in Equation (1.17), power is the derivative of energy. Taking the derivative of Equation (1.19), we obtain

$$p(t) = \frac{dw(t)}{dt} \tag{1.20}$$

If the voltage and the current are time-varying, the power is also time-varying. If the voltage and current are expressed as a function of time, Equation (1.17) can be written as

$$p(t) = i(t)v(t) \tag{1.21}$$

FIGURE 1.3
(a) Power is positive.
(b) Power is negative.



The power given by Equation (1.21) is called *instantaneous power*. According to Equation (1.21), instantaneous power is the product of current and voltage as a function of time. In the **passive sign convention**, if the direction of current is from the positive terminal of a device, through the device, and to the negative terminal of the device [as shown in Figure 1.3(a)], the power is positive. On the other hand, if the current leaves the positive terminal of a device, flows through the rest of the circuit, and enters the negative terminal of the device [as shown in Figure 1.3(b)], the power is negative.

If power is positive [i.e., $p(t) > 0$], the element is absorbing power. On the other hand, if power is negative, the element is delivering (supplying) power. In a given circuit, the total absorbed power equals the total delivered or supplied power. This is called *conservation of power*.

EXAMPLE 1.3

Let the voltage across an element be $v(t) = 100\cos(2\pi 60t)$ V, and the current though the element from positive terminal to negative terminal be $i(t) = 5\cos(2\pi 60t)$ A for $t \geq 0$. Find the instantaneous power $p(t)$ and plot $p(t)$.

continued

Example 1.3 continued

$$p(t) = i(t)\,v(t) = 5\cos(2\pi 60t) \times 100\cos(2\pi 60t) = 500\cos^2(2\pi 60t)$$
$$= 250 + 250\cos(2\pi \times 120t)\ \text{W}$$

The power $p(t)$ is shown in Figure 1.4. Since $p(t) \geq 0$ for all t, the element is not delivering power any time. On average, the element absorbs 250 W of power.

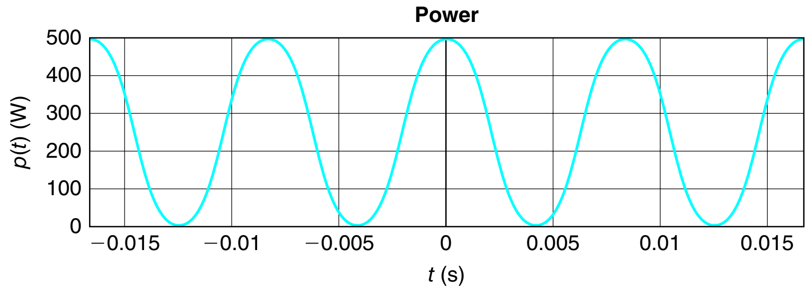


FIGURE 1.4

Plot of $p(t)$.

Exercise 1.3

Let the voltage across an element be $v(t) = 100\cos(2\pi 60t)$ V and the current though the element from positive terminal to negative terminal be $i(t) = 6\sin(2\pi 60t)$ A for $t \geq 0$. Find the instantaneous power $p(t)$ and plot $p(t)$.

$$p(t) = i(t)\,v(t) = 6\sin(2\pi 60t) \times 100\cos(2\pi 60t) = 300\sin(2\pi 120t)\ \text{W}.$$

The power $p(t)$ is shown in Figure 1.5. Since $p(t) > 0$ half of the time and $p(t) < 0$ the other half of the time, the element absorbs power for 1/240 s, then delivers power for the next 1/240 s, and then repeats the cycle. On average, the element does not absorb any power.

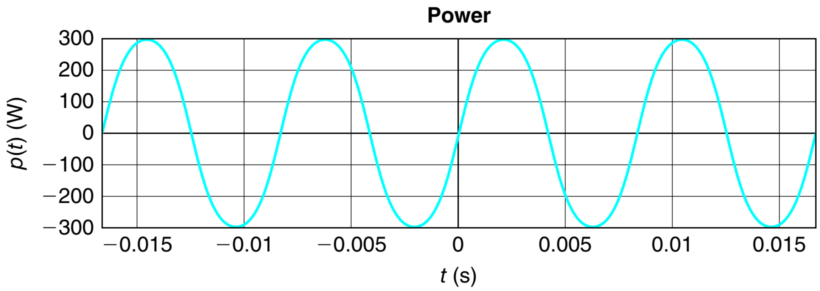


FIGURE 1.5

Power $p(t)$.

1.4 Independent Sources

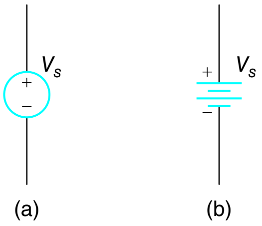


FIGURE 1.6

Circuit symbols for voltage sources.

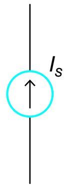
A voltage source with voltage V_s provides a constant potential difference to the circuit connected between the positive terminal and the negative terminal. The circuit notations for the voltage source are shown in Figure 1.6.

If a positive charge Δq is moved from the negative terminal to the positive terminal through the voltage source, the potential energy of the charge is increased by $\Delta q V_s$. If a negative charge with magnitude Δq is moved from the positive terminal to the negative terminal through the voltage source, the potential energy of the charge is increased by $\Delta q V_s$. A battery is an example of a voltage source.

A current source with current I_s provides a constant current of I_s amperes to the circuit connected to the two terminals. The circuit notation for the current source is shown in Figure 1.7.

FIGURE 1.7

A circuit symbol for the current source.



1.4.1 DIRECT CURRENT SOURCES AND ALTERNATING CURRENT SOURCES

If the voltage from the voltage source is constant with time, the voltage source is called the *direct current (dc) source*. Likewise, if the current from the current source is constant with time, the current source is called the *direct current (dc) source*.

If the voltage from the voltage source is a sinusoid, as shown in Figure 1.8, the voltage source is called *alternating current (ac) voltage source*. Likewise, if the current from the current source is a sinusoid, the current source is called *alternating current (ac) current source*. A detailed discussion of ac signals is given in Chapter 9. The circuit notation for an ac voltage source and ac current source are shown in Figure 1.9. The phase is given in degrees. The circuit notation for dc voltage shown in Figure 1.6(a) and the circuit notation for dc current shown in Figure 1.7 are also used for ac voltage and ac current, respectively.

FIGURE 1.8

Plot of a cosine wave with period T, amplitude V_m, and phase zero.

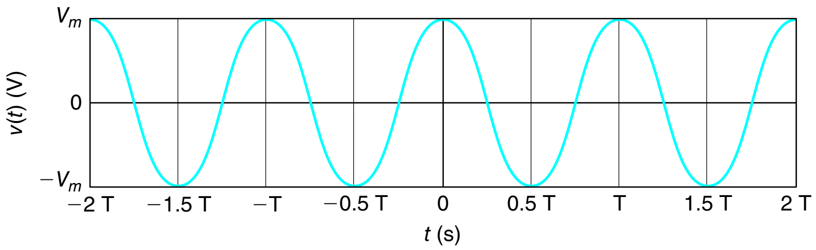


FIGURE 1.9

Circuit symbols for (a) ac voltage source; (b) ac current source.

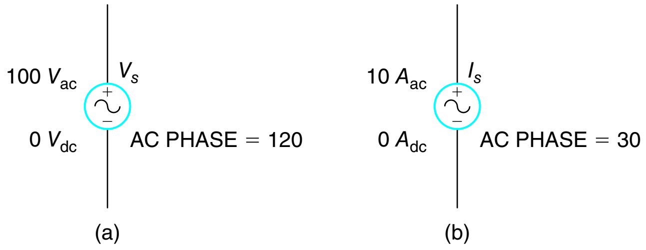
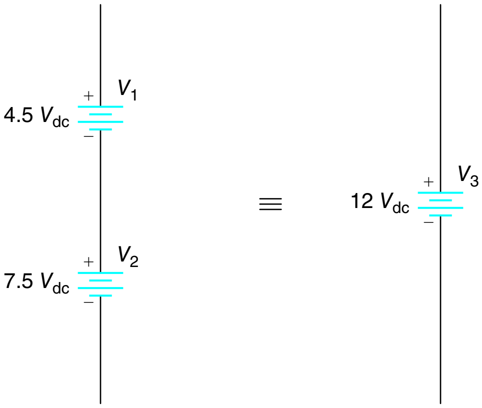


FIGURE 1.10

An equivalent voltage source.

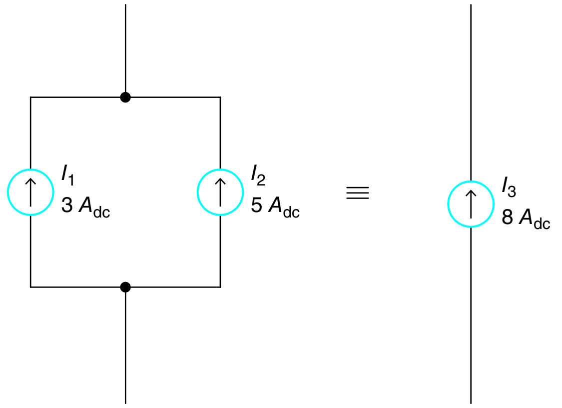


When dc voltage sources are connected in series, they can be combined into a single equivalent dc voltage source, as shown in Figure 1.10, where $V_3 = V_1 + V_2 = 4.5\text{ V} + 7.5\text{ V} = 12\text{ V}$. If there are other components, such as the resistors between V_1 and V_2 in the circuit shown in Figure 1.10, the voltage sources can be combined, so long as all the components are connected in series. Resistors are discussed further in Chapter 2.

When dc current sources are connected in parallel, they can be combined into a single equivalent dc current source, as shown in Figure 1.11, where $I_3 = I_1 + I_2 = 3\text{ A} + 5\text{ A} = 8\text{ A}$. If other components such as resistors are connected in parallel to I_1 and I_2 in the circuit shown in Figure 1.11, the current sources can be combined, so long as all the components are connected in parallel between the same points.

FIGURE 1.11

An equivalent current source.

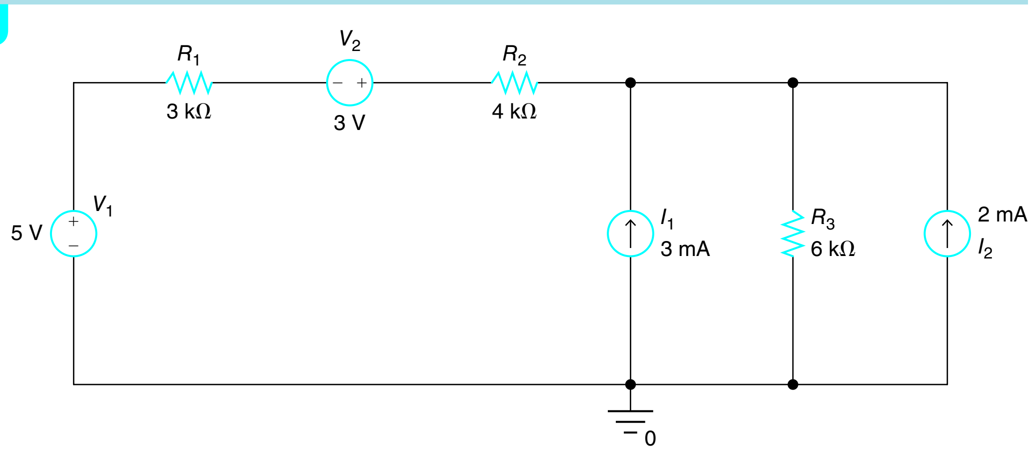


EXAMPLE 1.4

Redraw the circuit shown in Figure 1.12 with one voltage source and one current source, without affecting the voltages across and currents through the resistors in the circuit.

FIGURE 1.12

Circuit for EXAMPLE 1.4.



Since V_1 and V_2 are part of a single wire, they can be combined into the single voltage source V_3. Since V_2 has the same polarity as V_1, the value of V_3 is given by

$$V_3 = V_1 + V_2 = 5\text{ V} + 3\text{ V} = 8\text{ V}$$

Since I_1 and I_2 are connected between the same points in the circuit, they can be combined into the single current source I_3. Since I_2 has the same polarity as I_1, the value of I_3 is given by

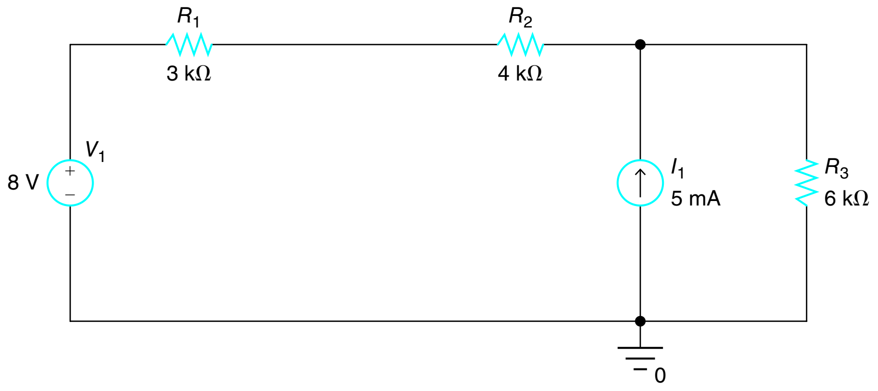
$$I_3 = I_1 + I_2 = 3\text{ mA} + 2\text{ mA} = 5\text{ mA}$$

The equivalent circuit, with one voltage source and one current source, is shown in Figure 1.13.

continued

Example 1.4 continued

FIGURE 1.13

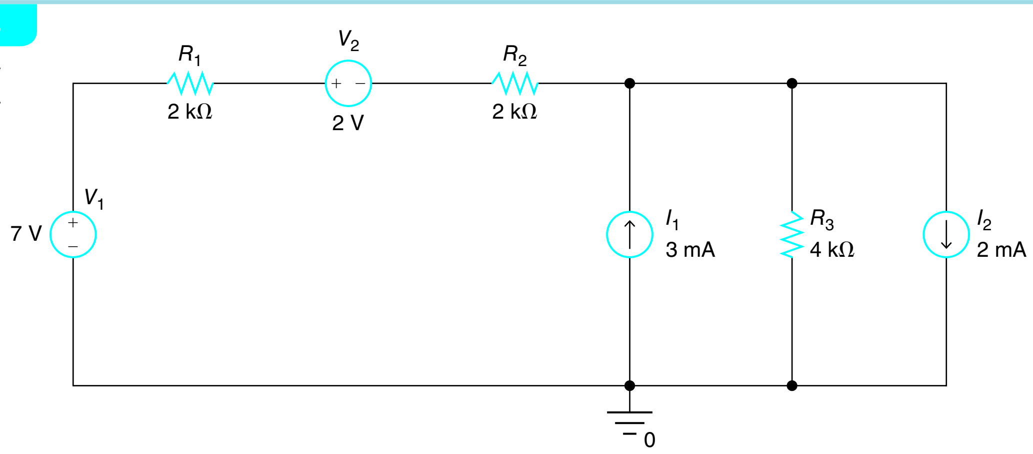


A circuit with one current source and one voltage source.

Exercise 1.4

Redraw the circuit shown in Figure 1.14 with one voltage source and one current source, without affecting the voltages across and currents through the resistors in the circuit.

FIGURE 1.14

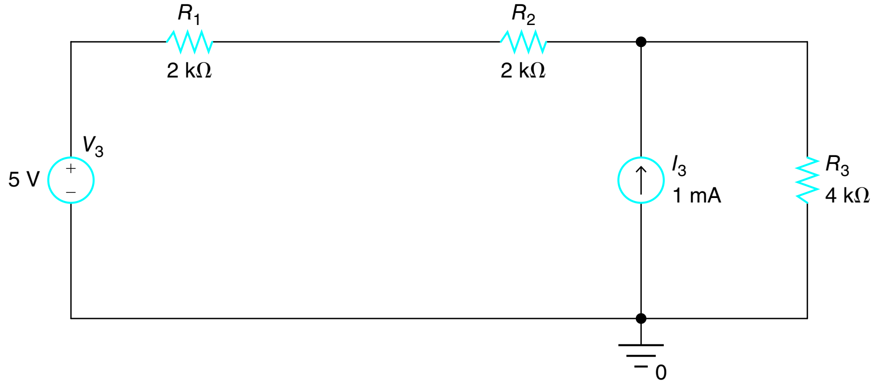


Circuit for EXERCISE 1.4.

Answer:

The equivalent circuit with one voltage source and one current source is shown in Figure 1.15.

FIGURE 1.15



A circuit with one current source and one voltage source.

An ac voltage waveform can be represented as

$$v(t) = V_m \cos\left(\frac{2\pi t}{T} + \phi\right) \text{ V} \tag{1.22}$$

Here, V_m is the amplitude (peak value) of the cosine wave, T is the period of the cosine wave, and ϕ is the phase of the cosine wave. The peak-to-peak amplitude is $2V_m$. The cosine wave repeats itself every T seconds. The number of periods per second, called *frequency* and denoted by f, is given by

$$f = \frac{1}{T} \text{ Hz} \tag{1.23}$$

The unit for the frequency is 1/s and is called *hertz* (Hz). In terms of the frequency in hertz, the ac voltage waveform can be written as

$$v(t) = V_m \cos(2\pi f t + \phi) \text{ V} \tag{1.24}$$

Since the angle changes by 2π radians in one period, and there are f periods in 1 second, the changes in angle in 1 second is given by

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{1.25}$$

The parameter ω is called the *angular velocity* of the cosine wave and has a unit of radians per second (rad/s). In terms of radian frequency ω, the cosine wave becomes

$$v(t) = V_m \cos(\omega t + \phi) \tag{1.26}$$

The ac current waveform can be written as

$$i(t) = I_m \cos\left(\frac{2\pi t}{T} + \phi\right) = I_m \cos(2\pi f t + \phi) = I_m \cos(\omega t + \phi) \text{ A} \tag{1.27}$$

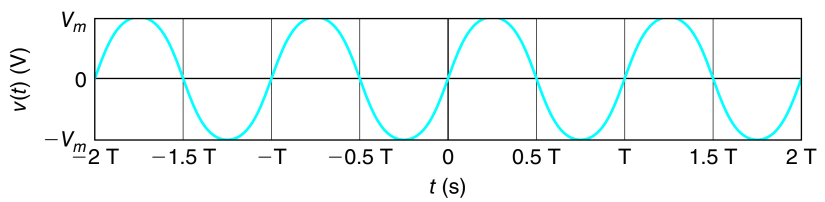
If the cosine wave shown in Figure 1.8 is shifted to the right by $T/4$, we get

$$\begin{aligned} v(t) &= V_m \cos\left[\frac{2\pi\left(t - \frac{T}{4}\right)}{T}\right] = V_m \cos\left(\frac{2\pi}{T}t - \frac{\pi}{2}\right) = V_m \sin\left(\frac{2\pi}{T}t\right) \\ &= V_m \sin(2\pi f t) = V_m \sin(\omega t) \end{aligned} \tag{1.28}$$

The sine wave given by Equation (1.28) is shown in Figure 1.16.

FIGURE 1.16

Plot of a sine wave with period T and amplitude V_m.



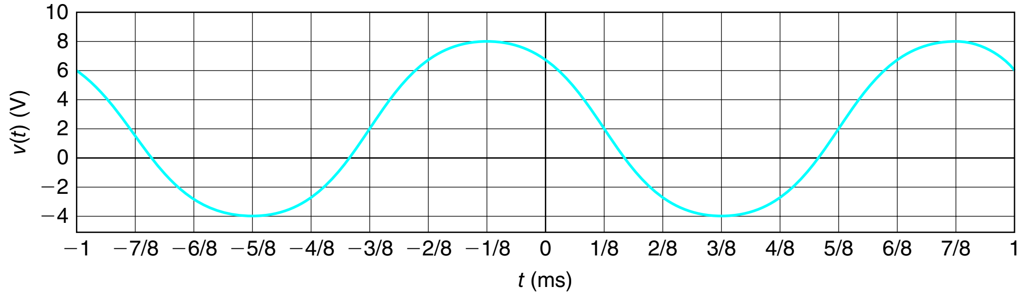
Notice that if the phase of the cosine is shifted by −90 degrees, the cosine wave becomes a sine wave.

EXAMPLE 1.5

Find the equation of the sinusoidal signal shown in Figure 1.17.

FIGURE 1.17

A sinusoid for EXAMPLE 1.5.



Since the period is $T = 1$ ms, the frequency is $f = 1/T = 1/1\text{ms} = 1000$ Hz $= 1$ kHz. The radian frequency is $2\pi f = 2\pi 1000 = 6283.1853$ rad/s. The difference between the maximum and minimum is $8 - (-4) = 12$ V, which is the peak-to-peak amplitude. The peak value of the amplitude is $V_m = 12\text{V}/2 = 6$ V. The average amplitude is $(8 - 4)/2 = 2$ V, which is the dc component. The cosine wave is shifted to the left by $T/8$ ms, which is $\pi/4$ rad $= 45°$. Therefore, the equation is given by

$$v(t) = 2 + 6\cos(2\pi 1000t + 45°)\text{ V}$$

Exercise 1.5

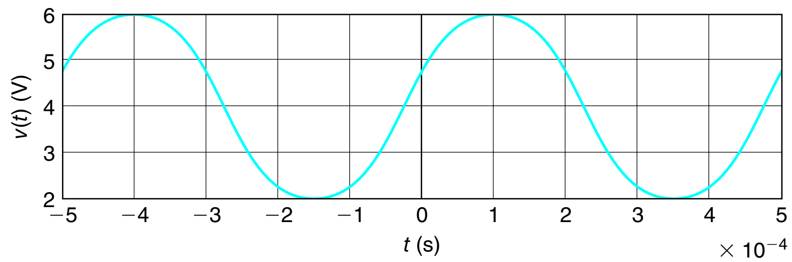
Plot $v(t) = 4 + 2\cos(2\pi 2000t - 72°)$ V.

Answer:

The signal $v(t)$ is shown in Figure 1.18.

FIGURE 1.18

The plot of $v(t)$.



1.5 Dependent Sources

The voltage sources and current sources discussed previously are called **independent sources** because they are stand-alone sources that provide power to the external circuit connected to the sources. Usually, the independent sources convert one form of energy to electrical energy. For example, a battery converts chemical energy into electrical energy.

A solar cell converts energy from the Sun into electrical energy. A wind turbine converts wind energy into electrical energy. The amount of energy supplied from the source to the circuit per unit time is the power of the source. Dependent sources do not have the ability to convert one form of energy into electrical energy. The voltage or current of the dependent sources depend solely on the controlling voltage or controlling current. The dependent sources are used to model integrated circuit (IC) devices.

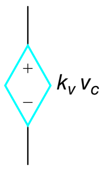
Depending on whether the dependent source is a voltage source or a current source, and whether the dependent source is controlled by a voltage or a current, there are four different dependent sources. The four types of dependent sources are:

Voltage-controlled voltage source (VCVS)
Voltage-controlled current source (VCCS)
Current-controlled voltage source (CCVS)
Current-controlled current source (CCCS)

These four types of dependent sources are discussed next.

FIGURE 1.19

Circuit symbol for VCVS.



1.5.1 VOLTAGE-CONTROLLED VOLTAGE SOURCE (VCVS)

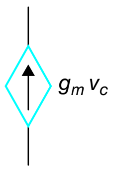
The voltage on the VCVS is proportional to the controlling voltage, which is the voltage in another part of the circuit. For example, the controlling voltage can be the voltage across a circuit element in another part of the circuit. Let v_d be the controlled voltage and v_c be the controlling voltage. Then, we have

$$v_d = k_v v_c$$

where k_v is the unitless (V/V) proportionality constant. Figure 1.19 shows the circuit symbol for VCVS.

FIGURE 1.20

Circuit symbol for VCCS.



1.5.2 VOLTAGE-CONTROLLED CURRENT SOURCE (VCCS)

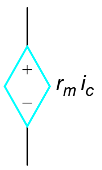
The current on the VCCS is proportional to the controlling voltage. Let i_d be the controlled current and v_c be the controlling voltage. Then, we have

$$i_d = g_m v_c$$

where g_m is the conductance in siemens (S). Figure 1.20 shows the circuit symbol for VCCS.

FIGURE 1.21

Circuit symbol for CCVS.



1.5.3 CURRENT-CONTROLLED VOLTAGE SOURCE (CCVS)

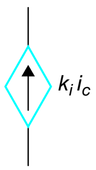
The voltage on the CCVS is proportional to the controlling current, the current in another part of the circuit. For example, the controlling current can be the current through a circuit element in another part of the circuit. Let v_d be the controlled voltage and i_c be the controlling current. Then, we have

$$v_d = r_m i_c$$

where r_m is the resistance in ohms (Ω). Figure 1.21 shows the circuit symbol for CCVS.

FIGURE 1.22

Circuit symbol for CCCS.



1.5.4 CURRENT-CONTROLLED CURRENT SOURCE (CCCS)

The current on the CCCS is proportional to the controlling current. Let i_d be the controlled current and i_c be the controlling current. Then, we have

$$i_d = k_i i_c$$

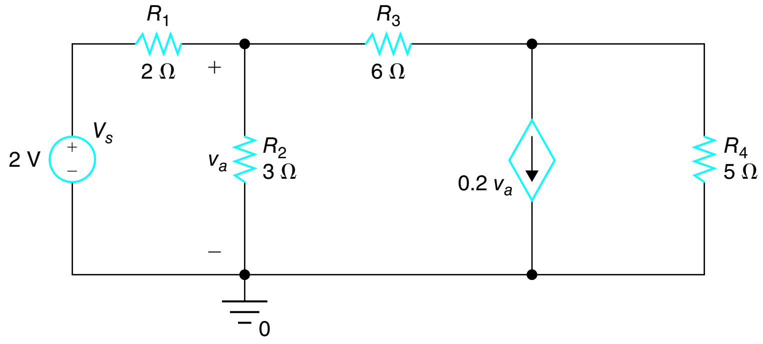
where k_i is the unitless (A/A) proportionality constant. Figure 1.22 shows the circuit symbol for CCCS.

EXAMPLE 1.6

In the circuit shown in Figure 1.23, the controlling voltage, which is the voltage across R_2, is $v_a = 0.9851$ V. Find the controlled current through the VCCS.

FIGURE 1.23

Circuit for EXAMPLE 1.6.



The current through the VCCS in the direction indicated in Figure 1.23 (↓) is

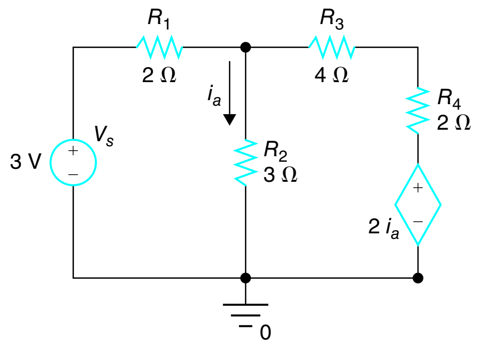
$$0.2\, v_a = 0.2\ (\text{A/V}) \times 0.9851\ \text{V} = 0.1970\ \text{A}$$

Exercise 1.6

In the circuit shown in Figure 1.24, the controlling current, which is the current through R_2, is $i_a = 0.5625$ A. Find the controlled voltage across the CCVS.

FIGURE 1.24

Circuit for EXERCISE 1.6.



Answer:

$2\, i_a = 1.125$ V.

1.6 Elementary Signals

Several elementary signals that will be useful in later chapters are presented in this section.

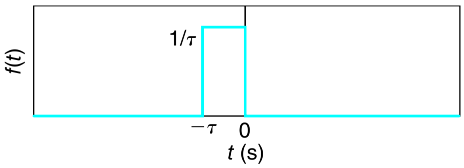
1.6.1 DIRAC DELTA FUNCTION

A rectangular pulse with height $1/\tau$ and width τ is shown in Figure 1.25. The pulse is centered at $-\tau/2$ and the area of the pulse is 1. The rectangular pulse can be written as

$$f(t) = \frac{1}{\tau} rect\left(\frac{t + \frac{\tau}{2}}{\tau}\right) \tag{1.29}$$

FIGURE 1.25

A rectangular pulse.



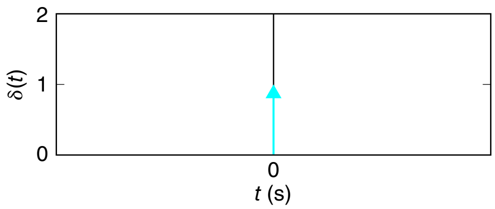
If the pulse width τ is decreased to zero, the height of the pulse is increased to infinity while maintaining the area at 1. The limiting form of a rectangular pulse shown in Figure 1.25 as $\tau \to 0$ is defined as the *Dirac delta function* (or *delta function*) and is denoted by $\delta(t)$; that is,

$$\delta(t) = \lim_{\tau \to 0} \frac{1}{\tau} rect\left(\frac{t + \frac{\tau}{2}}{\tau}\right) \tag{1.30}$$

The mathematical symbol for the Dirac delta function is shown in Figure 1.26.

FIGURE 1.26

Symbol for the Dirac delta function.



If the rectangular pulse given by Equation (1.29) is shifted to the right by $\tau/2$, it becomes

$$g(t) = \frac{1}{\tau} rect\left(\frac{t}{\tau}\right) \tag{1.31}$$

The Dirac delta function can also be defined as

$$\delta(t) = \lim_{\tau \to 0} \frac{1}{\tau} rect\left(\frac{t}{\tau}\right) \tag{1.32}$$

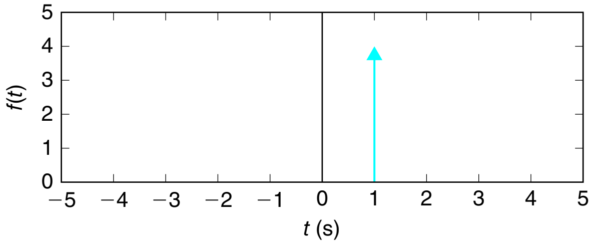
EXAMPLE 1.7

Plot $f(t) = 4\,\delta(t - 1)$.

The Dirac delta function is located at $t = 1$ and has an area of 4. The signal $f(t)$ is shown in Figure 1.27.

FIGURE 1.27

Plot of $f(t)$.



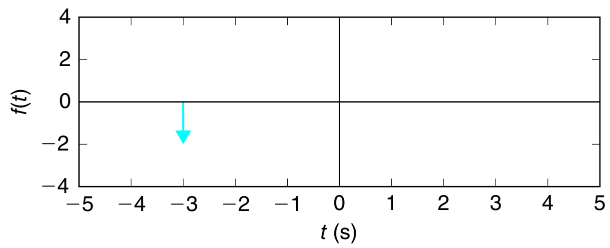
Exercise 1.7

Plot $f(t) = -2\,\delta(t + 3)$.

Answer:
$f(t)$ is shown in Figure 1.28.

FIGURE 1.28

Plot of $f(t)$.



When a continuous signal $f(t)$ is multiplied by $\delta(t - \text{a})$ and integrated from $-\infty$ to ∞, we obtain $f(a)$; that is,

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a) \tag{1.33}$$

This result is called the **sifting property** of the delta function because it sifts out a single value of $f(t)$, $f(a)$, at the location of the delta function ($t = \text{a}$). To prove the sifting property, we replace $\delta(t - \text{a})$ with

$$\delta(t-a) = \lim_{\tau\to 0} \frac{1}{\tau} rect\left(\frac{t-a}{\tau}\right)$$

Then, the integral becomes

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = \lim_{\tau\to 0} \frac{1}{\tau}\int_{-\infty}^{\infty} f(t) rect\left(\frac{t-a}{\tau}\right)dt = \lim_{\tau\to 0} \frac{1}{\tau}\int_{a-\frac{\tau}{2}}^{a+\frac{\tau}{2}} f(t) rect\left(\frac{t-a}{\tau}\right)dt$$

As $\tau \to 0$, $f(t) \to f(a)$ for $(a - \tau/2) < t < (a + \tau/2)$. Thus, the integral becomes

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = \lim_{\tau\to 0} \frac{1}{\tau}\int_{a-\frac{\tau}{2}}^{a+\frac{\tau}{2}} f(a) \times 1\,dt = \lim_{\tau\to 0} \frac{1}{\tau} f(a)\tau = f(a)$$

1.6.2 STEP FUNCTION

The unit step function $u(t)$ is the integral of the Dirac delta function $\delta(t)$. If Equation (1.29) is integrated, we obtain

$$\int_{-\infty}^{t} f(\lambda)d\lambda = \frac{1}{\tau}\int_{-\infty}^{t} rect\left(\frac{\lambda + \frac{\tau}{2}}{\tau}\right)d\lambda = \begin{cases} 0, & t < -\tau \\ \frac{t}{\tau} + 1, & -\tau \le t < 0 \\ 1, & 0 \le t \end{cases} \tag{1.34}$$

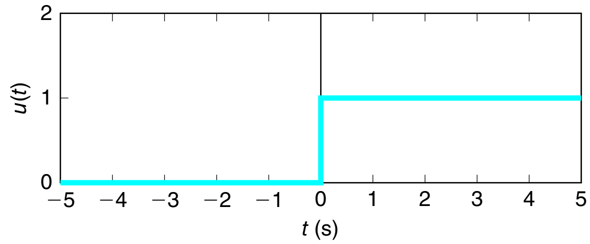
The unit step function is defined as the limiting form of Equation (1.34). In the limit as $\tau \to 0$, Equation (1.34) becomes

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 \le t \end{cases} \tag{1.35}$$

Notice that at $t = 0$, $u(t) = 1$. The unit step function defined by Equation (1.35) is shown in Figure 1.29.

FIGURE 1.29

A unit step function.



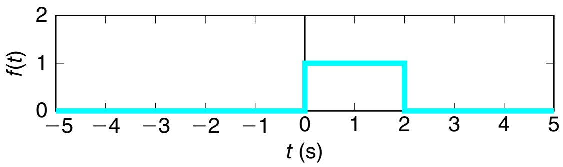
EXAMPLE 1.8

Plot $f(t) = u(t) - u(t - 2)$.

Notice that $u(t) = 1$ for $t \ge 0$ and zero for $t < 0$, and $u(t - 2) = 1$ for $t \ge 2$ and zero for $t < 2$. Thus, $u(t) - u(t - 2) = 0$ for $t \ge 2$, and $u(t) - u(t - 2) = 1$ for $0 \le t < 2$, and zero for $t < 0$. The signal $f(t)$ is shown in Figure 1.30.

FIGURE 1.30

Plot of $f(t)$.



Exercise 1.8

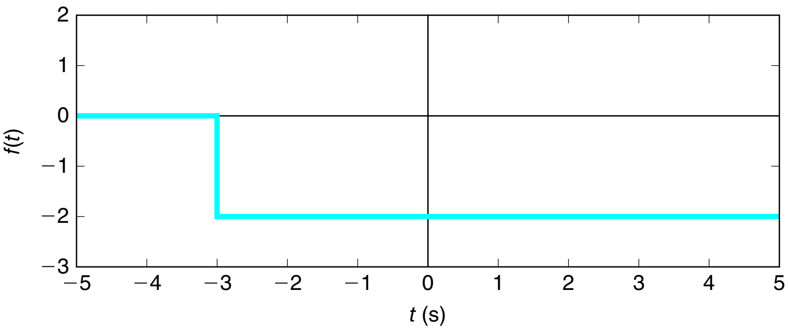
Plot $f(t) = -2\,u(t + 3)$.

Answer:

$f(t)$ is shown in Figure 1.31.

FIGURE 1.31

Plot of $f(t)$.



If Equation (1.31) is integrated, we obtain

$$\int_{-\infty}^{t} g(\lambda)d\lambda = \frac{1}{\tau}\int_{-\infty}^{t} rect\left(\frac{\lambda}{\tau}\right)d\lambda = \begin{cases} 0, & t < \frac{-\tau}{2} \\ \frac{t}{\tau} + \frac{1}{2}, & \frac{-\tau}{2} \le t < \frac{\tau}{2} \\ 1, & \frac{\tau}{2} \le t \end{cases} \tag{1.36}$$

If $u(t)$ is defined as the limiting form of Equation (1.36) as $\tau \to 0$, we obtain

$$u(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{2}, & t = 0 \\ 1, & 0 \le t \end{cases} \tag{1.37}$$

In this text, the definition of $u(t)$ given by Equation (1.35) is used. Since $u(0) = 1$, it does include voltages and currents at $t = 0$.

1.6.3 RAMP FUNCTION

A unit ramp function is defined by

$$r(t) = t\,u(t) \tag{1.38}$$

The unit ramp function is shown in Figure 1.32. The unit ramp function is the integral of the unit step function:

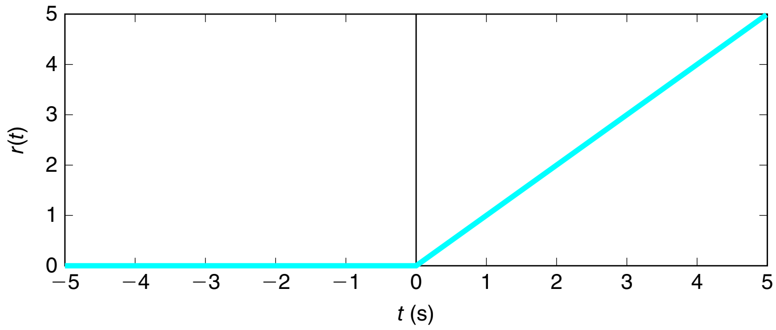
$$r(t) = \int_{-\infty}^{t} u(\lambda)d\lambda \tag{1.39}$$

The derivative of the unit ramp function is the unit step function.

$$u(t) = \frac{dr(t)}{dt} \tag{1.40}$$

FIGURE 1.32

A unit ramp function.



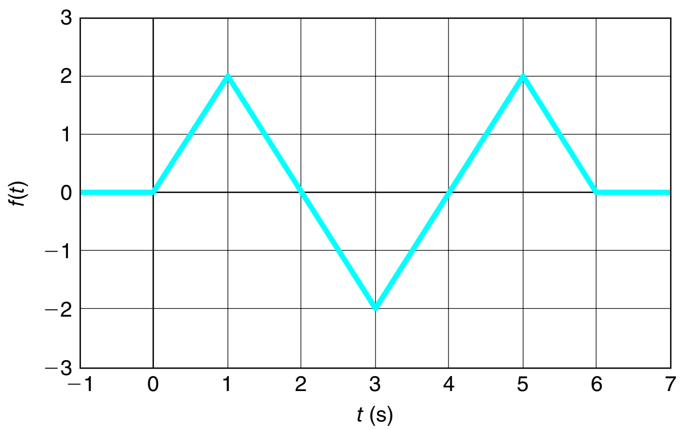
EXAMPLE 1.9

Plot $f(t) = 2tu(t) - 4(t-1)u(t-1) + 4(t-3)u(t-3) - 4(t-5)u(t-5) + 2(t-6)u(t-6)$.

For $t < 0, f(t) = 0$.
For $0 \le t < 1, f(t)$ is a linear line with slope of 2.
For $1 \le t < 3, f(t)$ is a linear line with slope of -2.
For $3 \le t < 5, f(t)$ is a linear line with slope of 2.
For $5 \le t < 6, f(t)$ is a linear line with slope of -2.
For $6 \le t, f(t) = 0$.
The waveform $f(t)$ is shown in Figure 1.33.

FIGURE 1.33

Waveform $f(t)$.



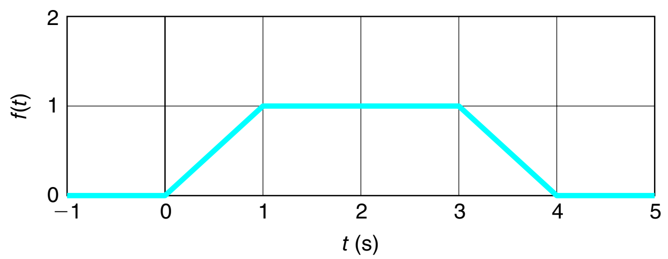
Exercise 1.9

Plot $f(t) = tu(t) - (t-1)u(t-1) - (t-3)u(t-3) + (t-4)u(t-4)$.

Answer:
The waveform is shown in Figure 1.34.

FIGURE 1.34

Waveform $f(t)$.



EXAMPLE 1.10

Find the equation of the waveform shown in Figure 1.35.

continued

Example 1.10 continued

For $t < 0, f(t) = 0$.

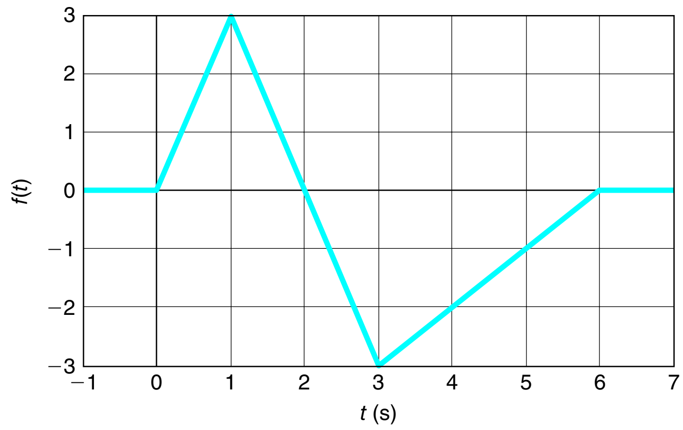
For $0 \le t < 1$, $f(t)$ is a linear line with slope of 3. Thus, $f(t) = 3tu(t)$.

For $1 \le t < 3$, $f(t)$ is a linear line with slope of -3. To change the slope from 3 to -3, we need to add $-6(t-1)u(t-1)$. At this point, we have $f(t) = 3tu(t) - 6(t-1)u(t-1)$.

For $3 \le t < 6$, $f(t)$ is a linear line with slope of 1. To change the slope from -3 to 1, we need to add $4(t-3)u(t-3)$. At this point, we have $f(t) = 3tu(t) - 6(t-1)u(t-1) + 4(t-3)u(t-3)$.

FIGURE 1.35

Waveform $f(t)$.



For $6 \le t$, $f(t) = 0$. To change the slope from 1 to 0, we need to add $-(t-6)u(t-6)$. Thus, we have the final equation given by

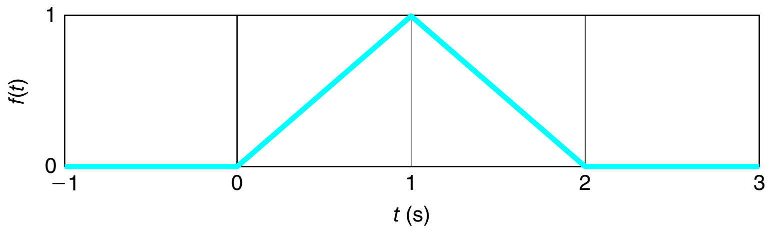
$$f(t) = 3tu(t) - 6(t-1)u(t-1) + 4(t-3)u(t-3) - (t-6)u(t-6).$$

Exercise 1.10

Find the equation of the waveform shown in Figure 1.36.

FIGURE 1.36

Waveform for EXERCISE 1.10.



Answer:
$f(t) = tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2)$.

1.6.4 EXPONENTIAL DECAY

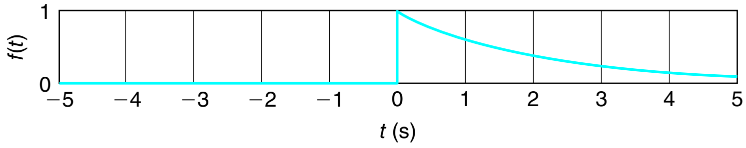
A signal that decays exponentially can be written as

$$f(t) = e^{-at}u(t), \quad a > 0 \tag{1.41}$$

The signal $f(t)$ for $a = 0.5$ is shown in Figure 1.37.

FIGURE 1.37

$f(t) = e^{-at}u(t), a = 0.5.$



A damped cosine and damped sine, respectively, can be written as

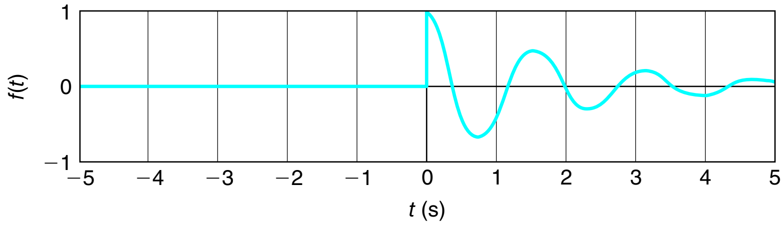
$$f(t) = e^{-at}\cos(bt)u(t), \qquad a > 0 \tag{1.42}$$

$$f(t) = e^{-at}\sin(bt)u(t), \qquad a > 0 \tag{1.43}$$

A damped cosine signal is shown in Figure 1.38 for $a = 0.5$ and $b = 4$.

FIGURE 1.38

A damped cosine signal.

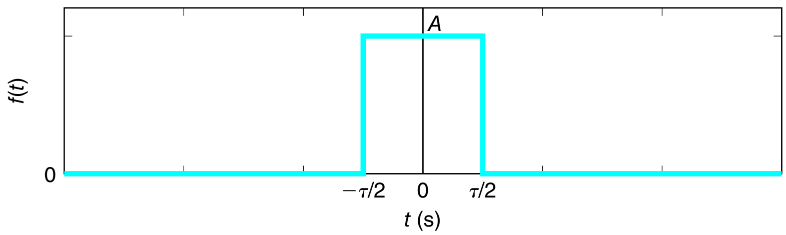


1.6.5 RECTANGULAR PULSE AND TRIANGULAR PULSE

A rectangular pulse with amplitude A and pulse width τ is shown in Figure 1.39. The center of the pulse is at $t = 0$.

FIGURE 1.39

A rectangular pulse.



The rectangular pulse shown in Figure 1.39 is denoted by

$$f(t) = A\,rect\left(\frac{t}{\tau}\right)$$

EXAMPLE 1.11

Plot $f(t) = rect\left(\frac{t+1}{2}\right) + 3\,rect\left(\frac{t-1}{2}\right) - 2\,rect\left(\frac{t-3.5}{3}\right)$.

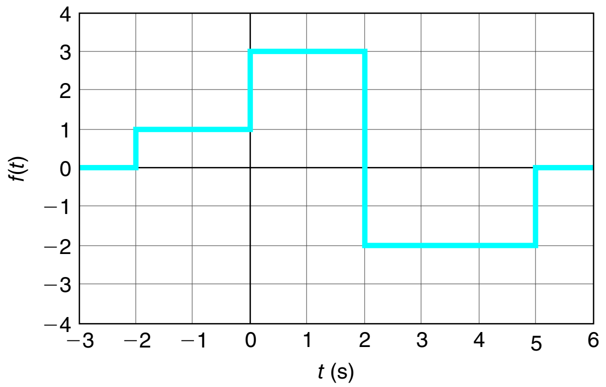
The first rectangle is centered at $t = -1$ and has a height of 1 and width of 2. The second rectangle is centered at $t = 1$ and has a height of 3 and width of 2. The third rectangle is centered at $t = 3.5$ and has a height of -2 and width of 3. The waveform $f(t)$ is shown in Figure 1.40.

continued

Example 1.11 continued

FIGURE 1.40

Waveform $f(t)$.



Exercise 1.11

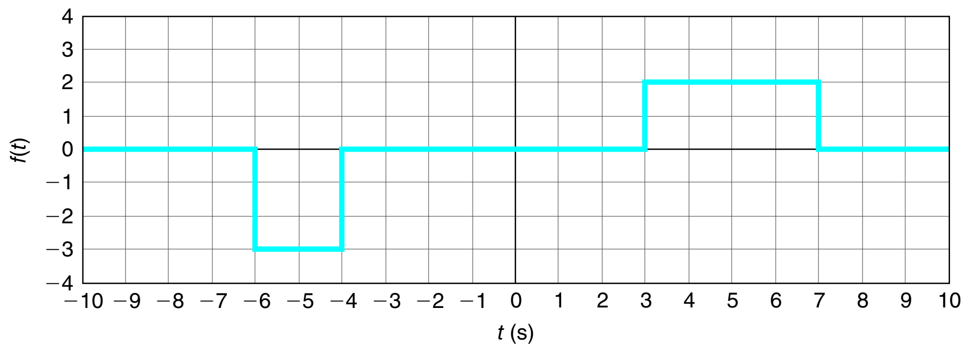
Plot $f(t) = -3\,rect\left(\frac{t+5}{2}\right) + 2\,rect\left(\frac{t-5}{4}\right)$.

Answer:

The waveform $f(t)$ is shown in Figure 1.41.

FIGURE 1.41

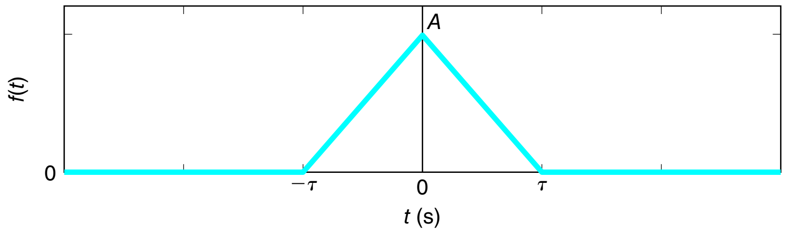
Waveform $f(t)$.



A triangular pulse with amplitude A and base 2τ is shown in Figure 1.42. The center of the pulse is at $t = 0$.

FIGURE 1.42

A triangular pulse.



The triangular pulse shown in Figure 1.42 is denoted by

$$f(t) = A\, tri\left(\frac{t}{\tau}\right)$$

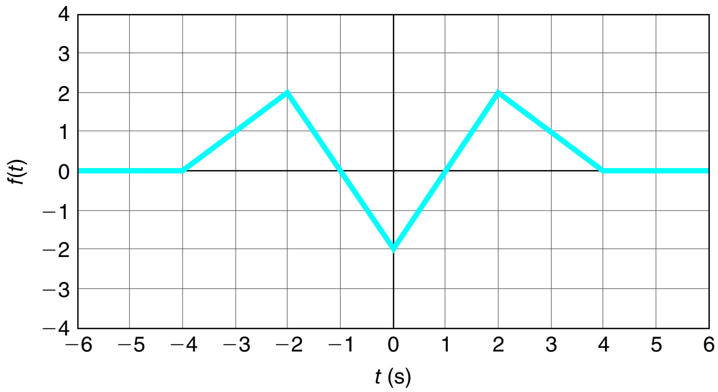
EXAMPLE 1.12

Plot $f(t) = 2\,tri\left(\frac{t+2}{2}\right) - 2\,tri\left(\frac{t}{2}\right) + 2\,tri\left(\frac{t-2}{2}\right)$.

The first triangle is centered at $t = -2$ and has a height of 2 and base of 4. The second triangle is centered at $t = 0$ and has a height of -2 and base of 4. The third triangle is centered at $t = 2$ and has a height of 2 and base of 4. The waveform $f(t)$ is shown in Figure 1.43.

FIGURE 1.43

Waveform $f(t)$.

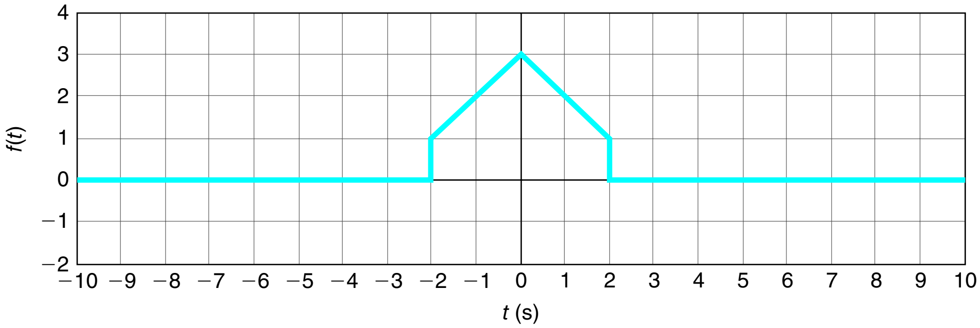


Exercise 1.12

Find the equation of the waveform $f(t)$ shown in Figure 1.44.

FIGURE 1.44

Waveform for EXERCISE 1.12.



Answer:

$$f(t) = rect\left(\frac{t}{4}\right) + 2tri\left(\frac{t}{2}\right)$$

SUMMARY

In this chapter, the seven base units of the International System of Units (SI), along with derived units relevant to electrical and computer engineering, are presented. The definitions of *voltage, current,* and *power*, among other terms, are given. The potential difference per unit charge between A and B is called *voltage between A and B,* $v_{AB} = w_{AB}/q$, where w_{AB} is the amount of the work needed to move the test charge from B to A.

Current is defined as the rate of change of charge:

$$i(t) = \frac{dq(t)}{dt}$$

Power is the product of current and voltage:

$$p(t) = i(t)v(t)$$

Energy is the integral of power:

$$w(t) = \int_{-\infty}^{t} p(\lambda)d\lambda$$

The four types of dependent sources are:

Voltage-controlled voltage source (VCVS)
Voltage-controlled current source (VCCS)
Current-controlled voltage source (CCVS)
Current-controlled current source (CCCS)

PROBLEMS

1.1 Find the current flowing through an element if the charge flowing through the element is given by

$$q(t) = \begin{cases} 0.002t, \text{ C} & t \geq 0 \\ 0, & t < 0 \end{cases}$$

1.2 Find the current flowing through an element if the charge flowing through the element is given by

$$q(t) = \begin{cases} 5e^{-0.2t}, \text{ C} & t \geq 0 \\ 0, & t < 0 \end{cases}$$

1.3 Find the current flowing through an element if the charge flowing through the element is given by

$$q(t) = \begin{cases} 8(1 - e^{-0.003t}), \text{ C} & t \geq 0 \\ 0, & t < 0 \end{cases}$$

1.4 Find the current flowing through an element if the charge flowing through the element is given by

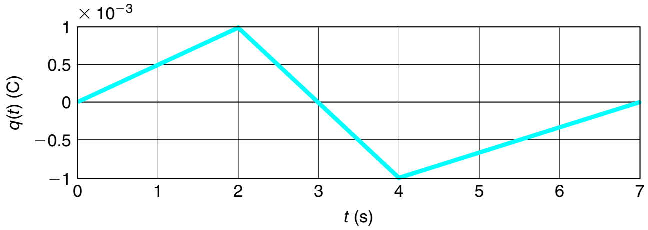
$$q(t) = \begin{cases} 7te^{-0.003t}, \text{ C} & t \geq 0 \\ 0, & t < 0 \end{cases}$$

1.5 Find the current flowing through an element if the charge flowing through the element is given by

$$q(t) = \begin{cases} 8 \times 10^{-6} \sin(2\pi \times 1000t), \text{ C} & t \geq 0 \\ 0, & t < 0 \end{cases}$$

1.6 The charge entering an element is shown in Figure P1.6. Plot the current through the element for $0 \leq t < 7$ s.

FIGURE P1.6



1.7 Find the total charge passing through an element at one cross section over the time interval $0 \leq t \leq 5$ s if the current through the same cross section is given by

$$i(t) = 5 \text{ mA}$$

1.8 Find the total charge passing through an element at one cross section over the time interval $0 \le t \le 5$ s if the current through the same cross section is given by

$$i(t) = \begin{cases} 5e^{-0.2t}\ \mu A, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

1.9 Find the total charge passing through an element at one cross section over the time interval $0 \le t \le 5$ s if the current through the same cross section is given by

$$i(t) = \begin{cases} 3(1 - e^{-0.5t})\ A, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

1.10 Find the total charge passing through an element at one cross section over the time interval $0 \le t \le 5$ s if the current through the same cross section is given by

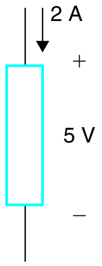
$$i(t) = \begin{cases} 2te^{-3t}\ A, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

1.11 Find the total charge passing through an element at one cross section over the time interval $0 \le t \le 5$ s if the current through the same cross section is given by

$$i(t) = \begin{cases} 7\sin(\pi t/5)\ A, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

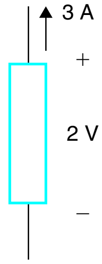
1.12 Find the power in the circuit element shown in Figure P1.12 and state whether the element is absorbing power or delivering power.

FIGURE P1.12



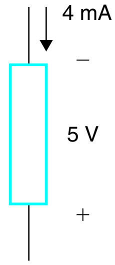
1.13 Find the power in the circuit element shown in Figure P1.13 and state whether the element is absorbing power or delivering power.

FIGURE P1.13



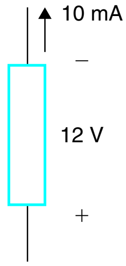
1.14 Find the power in the circuit element shown in Figure P1.14 and state whether the element is absorbing power or delivering power.

FIGURE P1.14



1.15 Find the power in the circuit element shown in Figure P1.15 and state whether the element is absorbing power or delivering power.

FIGURE P1.15



1.16 Find the power $p(t)$ on the element when the current through the element $i(t)$ from positive terminal to negative terminal and voltage $v(t)$ across the element are given by

$$i(t) = 2\text{ mA}, \quad v(t) = 5\text{ V}$$

1.17 Find the power $p(t)$ on the element when the current through the element $i(t)$ from positive terminal to negative terminal and voltage $v(t)$ across the element are given by

$$i(t) = 25\cos(2\pi 1000t)\text{ mA},$$

$$v(t) = 5\sin(2\pi 1000t)\text{ V}$$

1.18 Find the power $p(t)$ on the element when the current through the element $i(t)$ from positive terminal to negative terminal and voltage $v(t)$ across the element are given by

$$i(t) = 60\ \mathrm{e}^{-0.07t}\ u(t)\ \mathrm{mA},$$

$$v(t) = 7\ \mathrm{e}^{-0.08t}\ u(t)\ \mathrm{V}$$

1.19 Find the power $p(t)$ on the element when the current through the element $i(t)$ from positive terminal to negative terminal and voltage $v(t)$ across the element are given by

$$i(t) = 8 \cos(2\pi 100t)\ \mathrm{mA},$$

$$v(t) = 3 \cos(2\pi 100t)\ \mathrm{V}$$

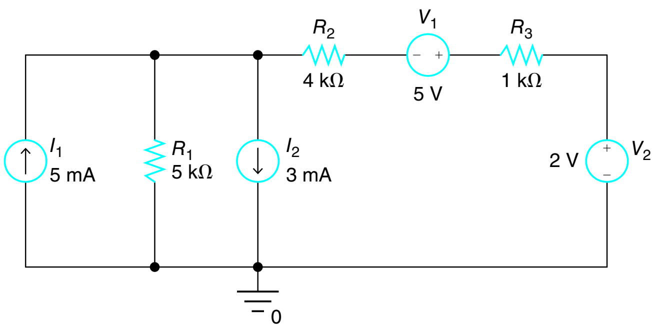
1.20 Find the power $p(t)$ on the element when the current through the element $i(t)$ from positive terminal to negative terminal and voltage $v(t)$ across the element are given by

$$i(t) = 6 \sin(2\pi 100t)\ \mathrm{mA},$$

$$v(t) = 2 \sin(2\pi 100t)\ \mathrm{V}$$

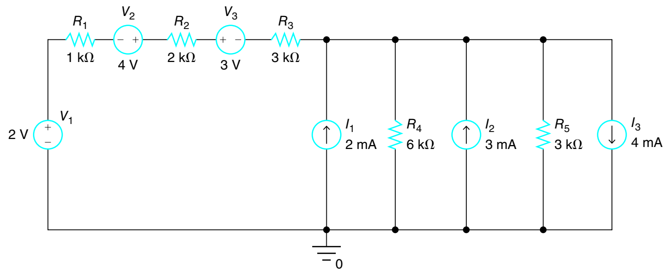
1.21 Redraw the circuit shown in Figure P1.21 with one voltage source and one current source, without affecting the voltages across and currents through the resistors in the circuit.

FIGURE P1.21



1.22 Redraw the circuit shown in Figure P1.22 with one voltage source and one current source, without affecting the voltages across and currents through the resistors in the circuit.

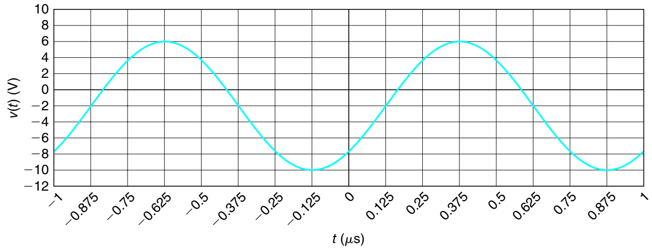
FIGURE P1.22



1.23 Plot $v(t) = -2 + 6 \cos(2\pi 5000t - 90°)$ V.

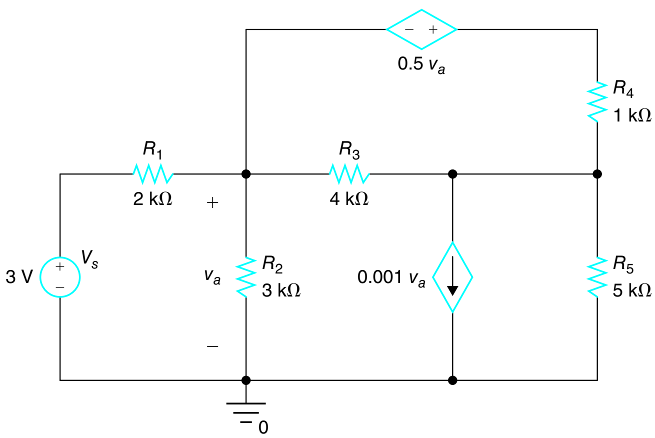
1.24 Find the equation of the sinusoid shown in Figure P1.24.

FIGURE P1.24



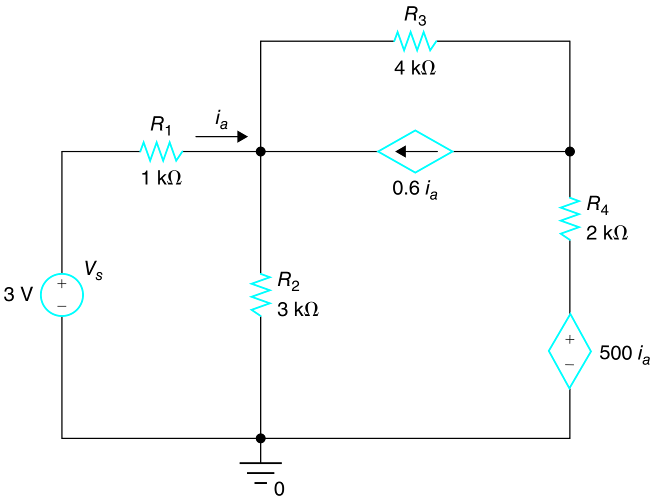
1.25 In the circuit shown in Figure P1.25, the controlling voltage, which is the voltage across R_2, is $v_a = 1.2908$ V. Find the controlled voltage across the VCVS and the controlled current through the VCCS.

FIGURE P1.25



1.26 In the circuit shown in Figure P1.26, the controlling current, which is the current through R_1, is $i_a = 0.8714$ mA. Find the controlled voltage across the CCVS and the controlled current through the CCCS.

FIGURE P1.26



1.27 Plot

$$f(t) = u(t) - 3\,u(t-2) + 6\,u(t-5) - 4\,u(t-8)$$

1.28 Plot

$$f(t) = -5\,\delta(t+2) + 7\,\delta(t-6)$$

1.29 Plot

$$f(t) = 2t\,u(t) - 4(t-1)\,u(t-1) + 3\,(t-3) \times u(t-3) - (t-5)\,u(t-5)$$

1.30 Plot

$$f(t) = -2t\,u(t) + 6(t-2)\,u(t-2) - 5(t-3) \times u(t-3) + (t-5)\,u(t-5)$$

1.31 Plot

$$f(t) = 2\,rect\left(\frac{t+3}{4}\right)$$

1.32 Plot

$$f(t) = 2\,tri\left(\frac{t+4}{2}\right)$$