1.5. CONCEPT OF IMPEDANCE AND ADMITTANCE

Impedance (Z) – *represents the total opposition to the flow of alternating current, expressed in ohms (Ω).*

- *represents the passive elements R, L, C and their combination in the frequency domain*

Admittance (Y) – *the reciprocal of impedance, expressed in siemens (S)*

*If **I** = $I_m \angle\theta_I$ is the resulting current drawn by a passive, linear RLC circuit from source a voltage*
V = $V_m\angle\theta_V$, *then*

$$Z = \frac{V}{I} = \frac{V_m\angle\theta_V}{I_m\angle\theta_I} = Z\angle\theta = Z\cos\theta + jZ\sin\theta = R + jX = \sqrt{R^2 + X^2}\angle\tan^{-1}\frac{X}{R}$$

where $Z = \frac{V_m}{I_m} = \sqrt{R^2 + X^2} =$ *magnitude of the impedance*

$\theta = \theta_V - \theta_I = \tan^{-1}\frac{X}{R} =$ *phase angle of the impedance*

$R = Z\cos\theta =$ *active or real component of the impedance*

$X = Z\sin\theta =$ *reactive or quadrature component of impedance*

Similarly,

$$Y = \frac{I_m}{V_m}\angle\theta_I - \theta_V = Y\angle\theta_y = Y\cos\theta_y + jY\sin\theta_y = G + jB = \sqrt{G^2 + B^2}\angle\tan^{-1}\frac{B}{G}$$

where

$$Y = \frac{I_m}{V_m} = \sqrt{G^2 + B^2} = \frac{1}{Z} =$$ *magnitude of the admittance*

$$\theta_y = \theta_I - \theta_V = -\theta = \tan^{-1}\frac{B}{G}$$ = *phase angle of the admittance*

$$G = Y\cos\theta_y =$$ *conductive (or conductance) component*

$$B = Y\sin\theta_y =$$ *susceptive (or susceptance) component*

For a ***resistive*** *element,* ***the current through it is in phase with the voltage across it***.

$$Z = R \text{ and } Y = 1/R = G$$

For a ***purely inductive*** *element,* ***the current through it lags the voltage across it by 90°.***

$$Z = jX_L \text{ and } Y = -jB_L$$

$$\text{where } X_L = \omega L = 2\pi fL \text{ and } B_L = 1/X_L$$

For a ***purely capacitive*** *element,* ***the current through it leads the voltage across it by 90°.***

$$Z = -jX_C \text{ and } Y = jB_C$$

$$\text{where } X_C = 1/\omega C = 1/2\pi fC \text{ and } B_c = 1/X_C$$

For the ***inductive*** *circuit (series RL or parallel RL), the current* ***lags*** *the voltage by an angle less than 90° (equal to the angle of the equivalent impedance).*

$$Z = R + jX$$

and

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$$

*For the **capacitive** circuit (series RC or parallel RC), the current **leads** the voltage by an angle less than 90° (equal to the angle of the equivalent impedance).*

$$Z = R - jX \quad \textbf{and} \quad Y = \frac{1}{Z} = \frac{1}{R - jX} = \frac{R + jX}{R^2 + X^2}$$

1.6. FREQUENCY DOMAIN ANALYSIS OF SIMPLE RLC SYSTEMS

Series circuits

$$I = I_1 = I_2 = I_3 = \ldots\ldots$$
$$V = V_1 + V_2 + V_3 + \ldots\ldots$$
$$Z = Z_1 + Z_2 + Z_3 + \ldots\ldots$$

Voltage Division Rule:

In a series circuit, the ratio of any two voltages is also the ratio of the corresponding impedances.

Parallel circuits

$$V = V_1 = V_2 = V_3 = \ldots\ldots$$
$$I = I_1 + I_2 + I_3 + \ldots\ldots$$
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \mathrm{K} \quad = Y = Y_1 + Y_2 + Y_3 + \ldots\ldots$$

Current Division Rule:

In a parallel circuit, the ratio of any two currents is also the ratio of the corresponding admittances or the inverse ratio of the corresponding impedances.

1.7. DELTA-WYE (OR WYE-DELTA) TRANSFORMATION

- involves the conversion of a network with the □ (or delta) configuration into a network with the T (or wye) configuration, and vice versa, in order to facilitate network analysis.

Delta to wye conversion:

$$Z_{wye} = \frac{\text{product of adjacent impedances in delta}}{\text{sum of impedances in delta}}$$

Wye to delta conversion:

$$Z_{delta} = \frac{\text{sum of product of adjacent impedances in wye}}{\text{opposite impedance in wye}}$$

Note: *For balanced delta or wye*, $Z_{wye} = Z_{delta}/3$ *and* $Z_{delta} = 3\,Z_{wye}$