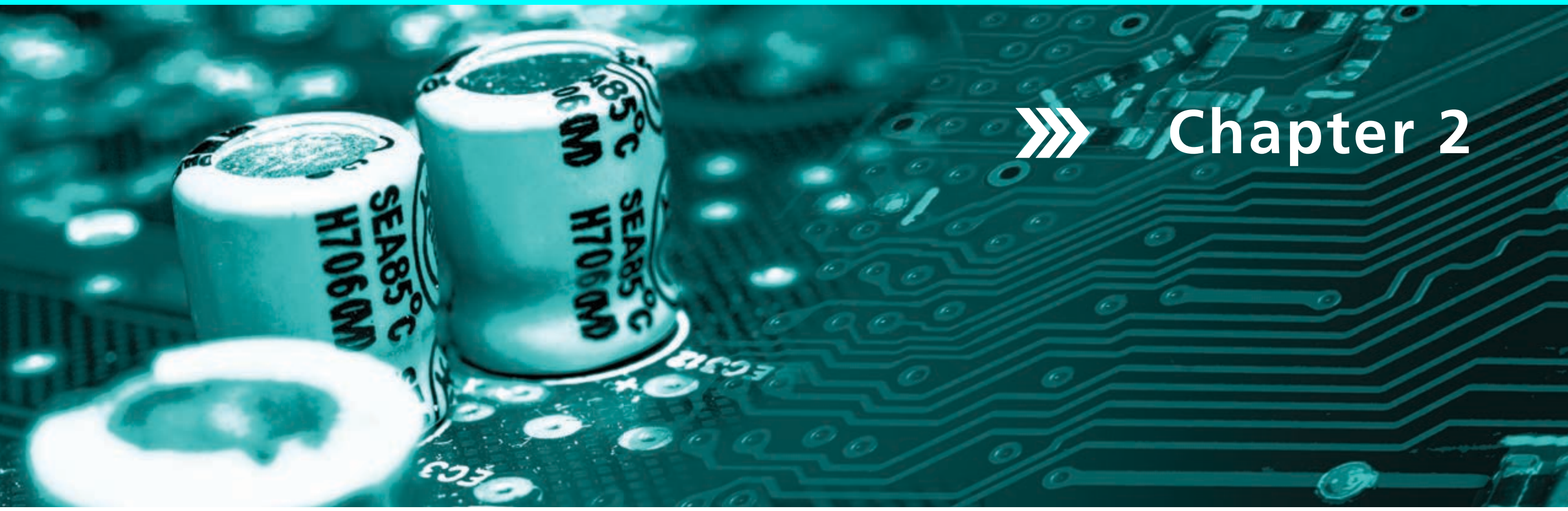


Chapter 2

Circuit Laws

2.1 Introduction

Nodes, branches, loops, and meshes are defined in this chapter. The equation of resistance of a conductor is expressed as a function of conductivity (or resistivity), and the dimension of the conductor. Ohm's law is introduced.

Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) are presented in this chapter. These two Kirchhoff's laws provide the theoretical basis for the nodal analysis and mesh analysis discussed in the next chapter and applied in circuit analysis in the rest of the chapters.

Finding the equivalent resistance of series and parallel connection of resistors are discussed. Simple circuit rules can be applied to analyze circuits after simplifying the circuits using equivalent resistances.

The voltage divider rule and the current divider rule are useful tools to analyze circuits without too much effort.

If a circuit contains resistors in wye (Y) shape, it can be changed to delta (Δ) shape. On the other hand, if a circuit contains resistors in delta shape, it can be changed to wye shape. The transformation from wye to delta and delta to wye may make it easier to simplify the circuit.

2.2 Circuit

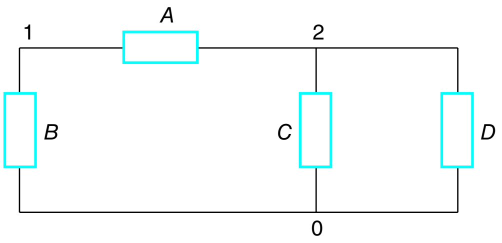
A **circuit** is an interconnection of elements, which can be voltage sources, current sources, resistors, capacitors, inductors, coupled coils, transformers, op amps, etc. A **node** is a point in a circuit where two or more elements are joined. A **simple node** is a node that connects two elements. A **path** in a circuit is a series of connected elements from a node to another node that does not go to the same node more than once. A **branch** is a path in a circuit consisting of a single element. The voltage of a node measured with respect to a **reference node** is called **node voltage**. The **ground node** where the voltage is at ground level is usually taken to be the reference node. A **loop** of a circuit is a closed path starting from a node and returning to the same node. The loop must have minimum of two branches in its closed path. A **mesh** is a loop that does not contain another loop inside it.

EXAMPLE 2.1

Find all the nodes, loops, and meshes for the circuit shown in Figure 2.1.

FIGURE 2.1

Circuit for EXAMPLE 2.1.



In the circuit shown in Figure 2.1, there are three nodes: 0, 1, and 2. Elements B, C, and D are joined at node 0. Node 1 connects elements A and B, and node 2 connects elements A, C, and D. If node 0 is the ground node, then the potential is set to zero at node 0. The voltages at node 1 and node 2 are measured with respect to node 0.

There are four branches in the circuit shown in Figure 2.1: A, B, C, and D. There are three loops in the circuit shown in Figure 2.1:

0-B-1-A-2-D-0
0-B-1-A-2-C-0
0-C-2-D-0

There are two meshes in the circuit shown in Figure 2.1:

0-B-1-A-2-C-0
0-C-2-D-0

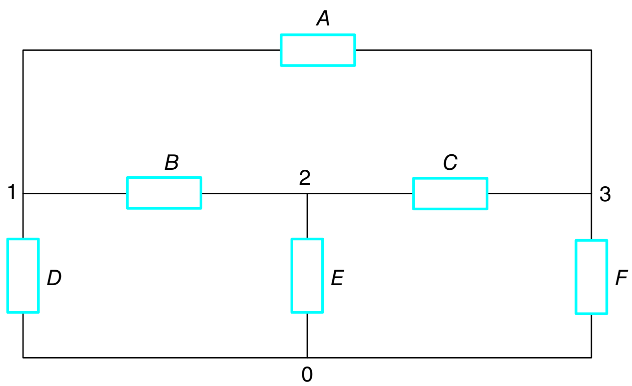
The loop 0-B-1-A-2-D-0 contains two meshes: 0-B-1-A-2-C-0 and 0-C-2-D-0.

Exercise 2.1

Find all the nodes, meshes, and loops for the circuit shown in Figure 2.2.

FIGURE 2.2

Circuit for EXERCISE 2.1.



Answer:
Nodes: 0, 1, 2, 3
Meshes:
1-A-3-C-2-B-1
0-D-1-B-2-E-0
0-E-2-C-3-F-0
Loops: In addition to the meshes, we have
0-D-1-A-3-F-0
0-D-1-B-2-C-3-F-0
0-D-1-A-3-C-2-E-0
0-E-2-B-1-A-3-F-0

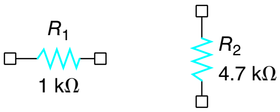
2.3 Resistor

A *resistor* is a circuit component that regulates the flow of current. The resistance of a resistor measures its ability to limit the current. When the resistance value is large, the amount of current flow through the resistor is small. On the other hand, if the resistance value is small, the amount of current flow through the resistor is large. The resistance value of a resistor is determined by the resistivity of the material used to make it, as well as its dimensions.

Low-power resistors can be made from carbon composition material made of fine granulated graphite mixed with clay. For high power, wire-wound resistors can be used. The wire-wound resistors are constructed by twisting a wire made of nichrome or similar material around a ceramic core. The circuit symbol for a resistor is shown in Figure 2.3.

FIGURE 2.3

Circuit symbol for a resistor.



The *current density* is defined as the amount of current through the unit area. If A is the cross-sectional area of a conductor that carries a constant current I, the current density is given by

$$J = \frac{I}{A} \tag{2.1}$$

The current is obtained by integrating the current density through the area. Thus, we have

$$I = \int \mathbf{J} \cdot d\mathbf{A} \tag{2.2}$$

It can be shown that the current density is proportional to the electric field intensity; that is,

$$J = \sigma E \tag{2.3}$$

where σ is the **conductivity** of the material. The unit for conductivity is siemens per meter (S/m). Equation (2.3) is called the *microscopic Ohm's law.*

Let the length of a cylindrical conductor with the cross-sectional area A be ℓ. Let the potential difference between the ends of the conductor be V. This potential difference generates a constant electric field E inside the conductor. The potential difference V is related to the electric field through

$$V = E\ell \tag{2.4}$$

Thus, we have $E = V/\ell$. Substituting this result into Equation (2.3), we get

$$J = \sigma E = \sigma\left(\frac{V}{\ell}\right) \tag{2.5}$$

Since $J = I/A$, Equation (2.5) becomes

$$\frac{I}{A} = \sigma\left(\frac{V}{\ell}\right) \tag{2.6}$$

Solving for V in Equation (2.6), we get

$$V = \frac{\ell}{\sigma A} I = RI \tag{2.7}$$

where R is defined as the resistance of the conductor. This is called the macroscopic Ohm's law. The SI unit for the resistance R is the ohm (Ω = V/A). The resistance of a material is given by

$$R = \frac{\ell}{\sigma A} \tag{2.8}$$

The resistance is proportional to the length and inversely proportional to the conductivity and the cross-sectional area. The **resistivity** ρ of a material is defined as the reciprocal of the conductivity σ. Thus, we have

$$\rho = \frac{1}{\sigma} \tag{2.9}$$

The SI unit for resistivity is $\Omega \cdot \mathrm{m}$. In terms of the resistivity, the resistance R can be written as

$$R = \frac{\rho \ell}{A} \tag{2.10}$$

The resistance is proportional to the resistivity and length and inversely proportional to the cross-sectional area. The inverse of resistance is called **conductance** and is denoted by G. The unit for conductance is S (siemens). Notice that $\mathrm{S} = \Omega^{-1} = \mathrm{A/V}$.

$$G = \frac{1}{R} \tag{2.11}$$

With resistivity (ρ) and cross-sectional area (A) fixed, let R_1 be the resistance when the length is ℓ_1, R_2 be the resistance when the length is ℓ_2, and R be the resistance when the length is $\ell = \ell_1 + \ell_2$. Then, we have

$$R = \frac{\rho \ell}{A} = \frac{\rho(\ell_1 + \ell_2)}{A} = \frac{\rho \ell_1}{A} + \frac{\rho \ell_2}{A} = R_1 + R_2 \tag{2.12}$$

Equation (2.12) says that when two resistors are connected in series, the equivalent resistance is the sum of the two resistances. In general, as shown in section 2.7 later in this chapter, if n resistors, $R_1, R_2, \ldots, R_n$, are connected in series, the equivalent resistance is given by

$$R_{\mathrm{eq}} = R_1 + R_2 + \cdots + R_n \tag{2.13}$$

With resistivity (ρ) and length (ℓ) fixed, let R_1 be the resistance when the cross-sectional area is A_1, R_2 be the resistance when the cross-sectional area is A_2, and R be the resistance when the cross-sectional area is $A = A_1 + A_2$. Then, we have

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{A_1 + A_2} = \frac{1}{\dfrac{A_1}{\rho \ell} + \dfrac{A_2}{\rho \ell}} = \frac{1}{\dfrac{1}{\dfrac{\rho \ell}{A_1}} + \dfrac{1}{\dfrac{\rho \ell}{A_2}}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \tag{2.14}$$

Equation (2.14) says that when two resistors are connected in parallel, the equivalent resistance is given by $R_1 R_2/(R_1 + R_2)$. In general, as shown in section 2.7 later in this chapter, if n resistors, $R_1, R_2, \ldots, R_n$, are connected in parallel, the equivalent resistance is given by

$$R_{\mathrm{eq}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}} \tag{2.15}$$

2.4 Ohm's Law

As shown in Equation (2.7), the voltage-current relation across the resistor is given by

$$V = RI \tag{2.16}$$

Equation (2.16) states that the voltage across the resistor is proportional to the current flowing through the resistor. The proportionality constant of this linear equation is resistance R. Equation (2.16) can be rewritten as

$$I = \frac{V}{R} \tag{2.17}$$

Equation (2.17) states that the current through the resistor is proportional to the voltage applied across the resistor and inversely proportional to resistance R.

Equation (2.16) can be rewritten as

$$R = \frac{V}{I} \tag{2.18}$$

Equation (2.18) states that the resistance is the ratio of voltage to current. Notice that the SI unit for voltage is volt (V), the SI unit for current is ampere (A), and the SI unit for resistance is ohm (Ω), as discussed in Chapter 1. According to Equations (2.16)–(2.18), we have

$$\mathrm{V} = \Omega \cdot \mathrm{A}$$
$$\mathrm{A} = \mathrm{V}/\Omega$$
$$\Omega = \mathrm{V}/\mathrm{A}$$

FIGURE 2.4

A circuit with a voltage source and a resistor.

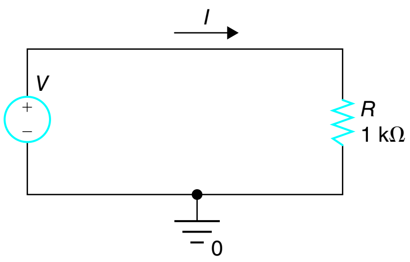


Figure 2.4 shows a circuit consisting of a voltage source and a resistor with resistance 1 $k\Omega$. Figure 2.5 shows the current I through the resistor as the voltage is swept from 0 V to 10 V. The current increases linearly from 0 mA to 10 mA. The slope of the I-V characteristic is given by 10 mA/10 V = 0.001 S. The slope is the conductance. The inverse of the conductance is the resistance, given by

$$R = \frac{10\ \mathrm{V}}{10\ \mathrm{mA}} = \frac{10\ \mathrm{V}}{0.01\ \mathrm{A}} = 1000\,\Omega = 1\ k\Omega$$

The power absorbed by a resistor is given by the product of current and voltage. Thus, we have

$$P = IV = VI\ (\mathrm{W}) \tag{2.19}$$

Substitution of V by RI from Equation (2.16) into Equation (2.19) yields

$$P = RI \times I = RI^2\ (\mathrm{W}) \tag{2.20}$$

Substitution of I by V/R from Equation (2.17) into Equation (2.19) yields

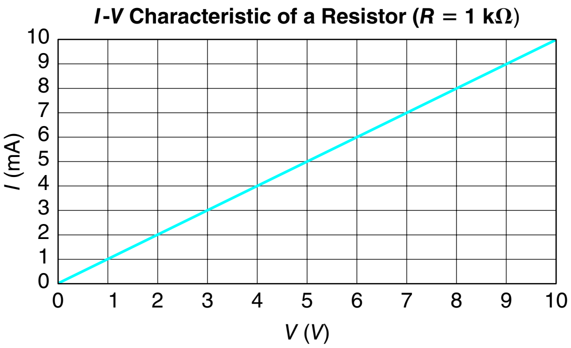
$$P = \frac{V^2}{R}\ (\mathrm{W}) \tag{2.21}$$

The power on the voltage source in Figure 2.4 is

$$P_s = V(-I) = -IV \tag{2.22}$$

FIGURE 2.5

I-V characteristic of a resistor.



The direction of the current through the voltage source from the positive terminal to the negative terminal is negative. When the power is negative, the circuit element (in this case the voltage source) is delivering power to the rest of the circuit. The power absorbed by the resistor in Figure 2.4 is IV. The total power on the circuit shown in Figure 2.4 is

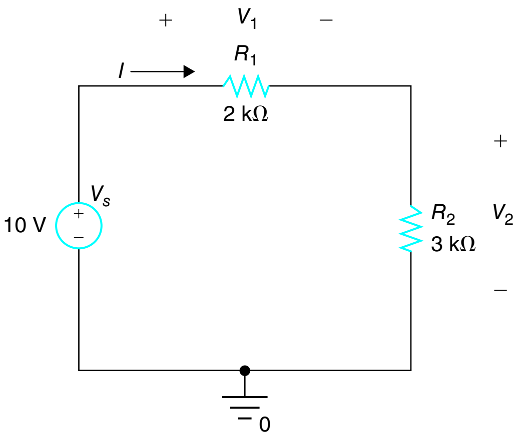
$$-IV + IV = 0$$

In general, the total power delivered equals the total power absorbed. This is called *conservation of power.* The energy is the integral of the power. If the power is constant, the energy is the power times the time. The energy spent on the resistor in Figure 2.4 in T seconds is

$$W = PT = IVT = I^2RT = \frac{V^2}{R}T \qquad (2.23)$$

FIGURE 2.6

Circuit with two resistors and a voltage source.



Consider a circuit shown in Figure 2.6. The current I around the mesh in the circuit shown in Figure 2.6 is given by $I = 2$ mA. We are interested in finding the voltage V_1 across R_1, voltage V_2 across R_2, and powers absorbed by R_1, R_2, and delivered by V_s.

Since the current through R_1 is 2 mA, according to Ohm's law, the voltage across R_1 is given by

$$V_1 = R_1 \times I = 2000\ \Omega \times 0.002\ A = 4\ \text{V}$$

Similarly, the voltage across R_2 is given by

$$V_2 = R_2 \times I = 3000\ \Omega \times 0.002\ A = 6\ \text{V}$$

The power absorbed by R_1 is given by

$$P_{R_1} = I \times V_1 = 0.002 \times 4 = 0.008\ \text{W} = 8\ \text{mW}.$$

Similarly, the power absorbed by R_2 is given by

$$P_{R_2} = I \times V_2 = 0.002 \times 6 = 0.012\ \text{W} = 12\ \text{mW}$$

The power from V_s is given by

$$P_{Vs} = (-I) \times V_s = -0.002 \times 10 = -0.020\ \text{W} = -20\ \text{mW}$$

According to passive sign convention, the power is defined as the product of the voltage and the current through the device. Since the current through the voltage source is −2 mA, the power is negative. When the power is negative, the device delivers power to the rest of the circuit. Since the power is positive for R_1 and R_2, the power is absorbed by the resistors. The total power absorbed is 8 mW + 12 mW = 20 mW. The power absorbed is equal to the power delivered by the source, which is 20 mW. This confirms the principle of conservation of power.

EXAMPLE 2.2

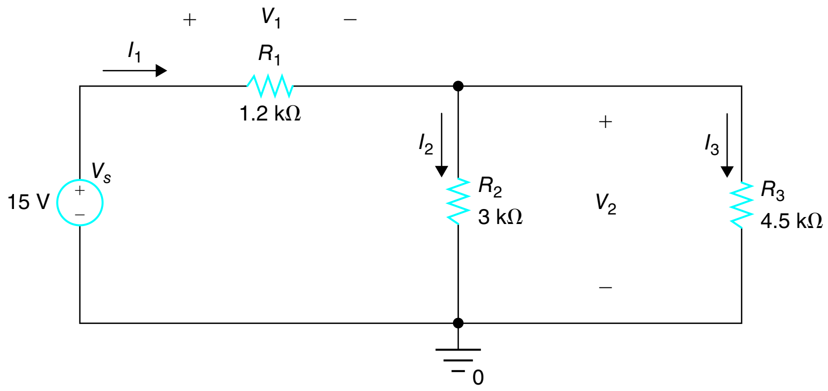
Assume that the voltage across R_2, which is also the voltage across R_3, is given by $V_2 = 9$ V in the circuit shown in Figure 2.7. Find I_2, I_3, V_1, I_1, and powers absorbed or delivered by R_1, R_2, R_3, and V_s.

continued

Example 2.2 continued

FIGURE 2.7

Circuit for EXAMPLE 2.2.



The current through R_2 can be found from Equation (2.17):

$$I_2 = \frac{V_2}{R_2} = \frac{9\text{ V}}{3\ k\Omega} = \frac{9\text{ V}}{3 \times 10^3\ \Omega} = 3 \times 10^{-3} A = 3\text{ mA}$$

Similarly, the current through R_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{9\text{ V}}{4.5\ k\Omega} = \frac{9\text{ V}}{4.5 \times 10^3\ \Omega} = 2 \times 10^{-3} A = 2\text{ mA}$$

The potential difference across R_1 is given by

$$V_1 = V_s - V_2 = 15\text{ V} - 9\text{ V} = 6\text{ V}$$

The current through R_1 is given by

$$I_1 = \frac{V_1}{R_1} = \frac{6\text{ V}}{1.2\ k\Omega} = \frac{6\text{ V}}{1.2 \times 10^3\ \Omega} = 5 \times 10^{-3} A = 5\text{ mA}$$

The power absorbed by R_1, R_2, and R_3, respectively, are given by

$$P_{R_1} = I_1V_1 = 5\text{ mA} \times 6\text{ V} = 5 \times 10^{-3}A \times 6\text{ V} = 30 \times 10^{-3}\,W = 30\text{ mW}$$

$$P_{R_2} = I_2V_2 = 3\text{ mA} \times 9\text{ V} = 3 \times 10^{-3}A \times 9\text{ V} = 27 \times 10^{-3}\,W = 27\text{ mW}$$

$$P_{R_3} = I_3V_2 = 2\text{ mA} \times 9\text{ V} = 2 \times 10^{-3}A \times 9\text{ V} = 18 \times 10^{-3}\,W = 18\text{ mW}$$

The power from the voltage source is

$$P_s = (-I_1)V_s = -5\text{ mA} \times 15\text{ V} = -5 \times 10^{-3}A \times 15\text{ V} = -75 \times 10^{-3}\,W = -75\text{ mW}$$

The current from the positive terminal to the negative terminal of the voltage source is $-I_1$. Since the power is negative, the voltage source delivers 75 mW of power to the rest of the circuit. The sum of powers absorbed by three resistors is

$$P_{R_1} + P_{R_2} + P_{R_3} = 30\text{ mW} + 27\text{ mW} + 18\text{ mW} = 75\text{ mW}$$

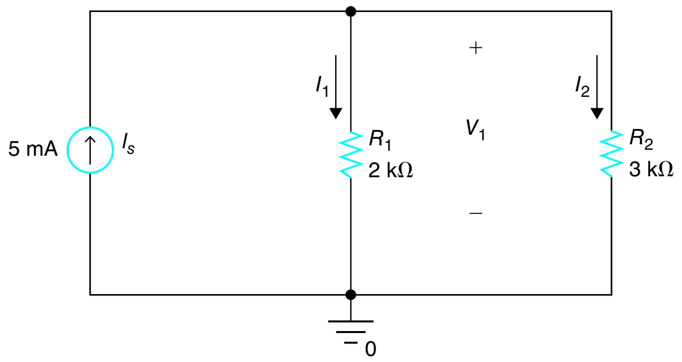
matching the power supplied by the voltage source.

Exercise 2.2

The voltage V_1 in the circuit shown in Figure 2.8 is given by 6 V. Find I_1, I_2, and powers on R_1, R_2, and I_s.

FIGURE 2.8

Circuit for EXERCISE 2.2.



Answer:

$I_1 = 3\text{ mA}, I_2 = 2\text{ mA}, P_{R_1} = 18\text{ mW}, P_{R_2} = 12\text{ mW}, P_{I_s} = -30\text{ mW}.$

2.5 Kirchhoff's Current Law (KCL)

As defined in section 2.2, a node is a point in a circuit where two or more elements are connected. It is part of wires that interconnect elements. A node cannot store or destroy electric charges. What comes into a node must leave the same node. From this fact, we have the following theorem, called **Kirchhoff's current law (KCL):**

The sum of currents entering a node equals the sum of currents leaving the same node.

Another way to describe KCL is:

The sum of currents leaving a node is zero.

Notice that for this statement to be true, at least one of the currents leaving the node must be negative (meaning that the current actually enters the node).

Still another way to describe KCL is:

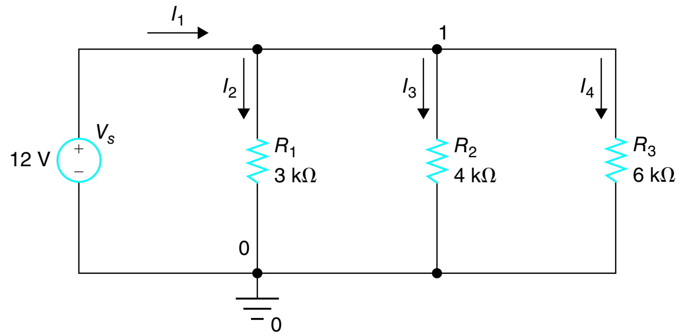
The sum of currents entering a node is zero.

Notice that for this statement to be true, at least one of the currents entering the node must be negative (meaning that the current actually leaves the node).

Consider a circuit shown in Figure 2.9. We are interested in finding currents $I_1, I_2, I_3,$ and I_4.

FIGURE 2.9

Circuit with three resistors and a voltage source.



The voltage at the ground node (node 0) is 0 V. The voltage at node 1, connecting V_s, R_1, R_2, and R_3 is 12 V. Thus, the voltages across R_1, R_2, and R_3 are all 12 V. Applying Ohm's law to each resistor, we have

$$I_2 = \frac{V_s}{R_1} = \frac{12\text{ V}}{3\ k\Omega} = 4\text{ mA}$$

$$I_3 = \frac{V_s}{R_2} = \frac{12\text{ V}}{4\ k\Omega} = 3\text{ mA}$$

$$I_4 = \frac{V_s}{R_3} = \frac{12\text{ V}}{6\ k\Omega} = 2\text{ mA}$$

According to KCL, the current entering node 1 (I_1) must equal the currents leaving node 1 ($I_2 + I_3 + I_4$). Thus, we have

$$I_1 = I_2 + I_3 + I_4 = 4\text{ mA} + 3\text{ mA} + 2\text{ mA} = 9\text{ mA}$$

Notice that the sum of the currents leaving node 1 is zero:

$$-I_1 + I_2 + I_3 + I_4 = -9\text{ mA} + 4\text{ mA} + 3\text{ mA} + 2\text{ mA} = 0\text{ mA}$$

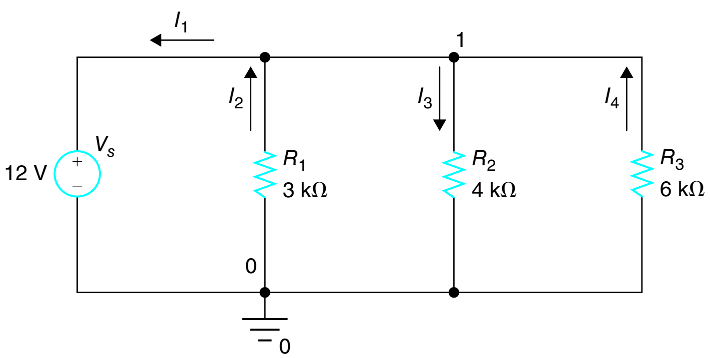
Notice that the sum of the current entering node 1 is also zero:

$$I_1 - I_2 - I_3 - I_4 = 9\text{ mA} - 4\text{ mA} - 3\text{ mA} - 2\text{ mA} = 0\text{ mA}$$

In addition, notice that the initial assignment of the direction of currents can be arbitrary. If the actual current flows in the opposite direction, the value of the current will be negative. Figure 2.10 shows the circuit shown in Figure 2.9, but with a different current assignment.

FIGURE 2.10

Circuit shown in Figure 2.9 with a different current assignment.



Applying Ohm's law to each resistor, we have

$$I_2 = \frac{0\text{ V} - 12\text{ V}}{3\ k\Omega} = -4\text{ mA}$$

$$I_3 = \frac{12\text{ V} - 0\text{ V}}{4\ k\Omega} = 3\text{ mA}$$

$$I_4 = \frac{0\text{ V} - 12\text{ V}}{6\ k\Omega} = -2\text{ mA}$$

According to KCL, the sum of currents entering node 1 ($I_2 + I_4$) must equal the sum of currents leaving node 1 ($I_1 + I_3$). Thus,

$$I_2 + I_4 = I_1 + I_3$$

Substituting $I_2 = -4$ mA, $I_3 = 3$ mA, $I_4 = -2$ mA, we have

$$-4\text{ mA} - 2\text{ mA} = I_1 + 3\text{ mA}$$

Thus, $I_1 = -9$ mA.
Notice that the sum of currents leaving node 1 is zero:

$$I_1 - I_2 + I_3 - I_4 = -9\text{ mA} + 4\text{ mA} + 3\text{ mA} + 2\text{ mA} = 0\text{ mA}$$

Notice that the sum of current entering node 1 is also zero:

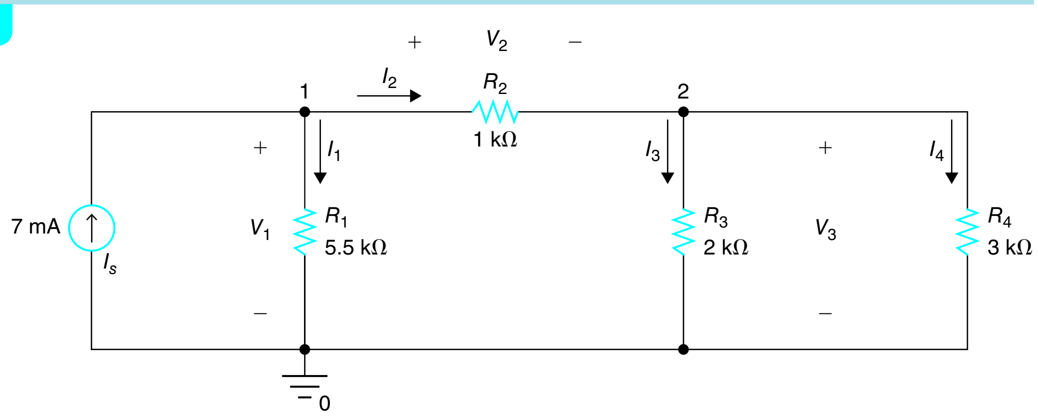
$$-I_1 + I_2 - I_3 + I_4 = 9\text{ mA} - 4\text{ mA} - 3\text{ mA} - 2\text{ mA} = 0\text{ mA}$$

EXAMPLE 2.3

Assume that the current through R_3 in the circuit shown in Figure 2.11 is given by $I_3 = 3$ mA. Find V_3, I_4, I_2, V_2, I_1, and V_1.

FIGURE 2.11

Circuit for EXAMPLE 2.3.



According to Ohm's law, the voltage across R_3 is given by

$$V_3 = R_3 I_3 = 2\ k\Omega \times 3\text{ mA} = 2000\ \Omega \times 0.003\ A = 6\text{ V}$$

Since R_3 and R_4 share the same nodes, the voltage across R_4 is V_3. According to Ohm's law, the current through R_4 is given by

$$I_4 = \frac{V_3}{R_4} = \frac{6\text{ V}}{3\ k\Omega} = 2\text{ mA}$$

According to KCL, the current entering node 2, I_2, equals the sum of currents leaving node 2, $I_3 + I_4$. Therefore, we have

$$I_2 = I_3 + I_4 = 3\text{ mA} + 2\text{ mA} = 5\text{ mA}$$

According to Ohm's law, the voltage across R_2 is given by

$$V_2 = R_2 I_2 = 1\ k\Omega \times 5\text{ mA} = 5\text{ V}$$

According to KCL, the current entering node 1, I_s, equals to the sum of currents leaving node 1, $I_1 + I_2$. Therefore, we have

$$I_s = I_1 + I_2$$

Thus, the current through R_1 is given by

$$I_1 = I_s - I_2 = 7\text{ mA} - 5\text{ mA} = 2\text{ mA}$$

continued

Example 2.3 continued

According to Ohm's law, the voltage across R_1, which is also the voltage across I_s, is given by

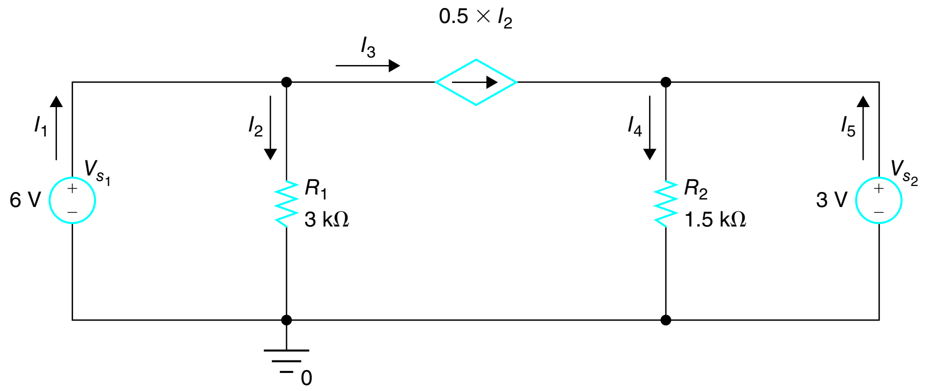
$$V_1 = R_1 I_1 = 5.5\ k\Omega \times 2\ \text{mA} = 11\ \text{V}$$

Exercise 2.3

Find I_2, I_3, I_1, I_4, and I_5 in the circuit shown in Figure 2.12.

FIGURE 2.12

Circuit for EXERCISE 2.3.



Answer:

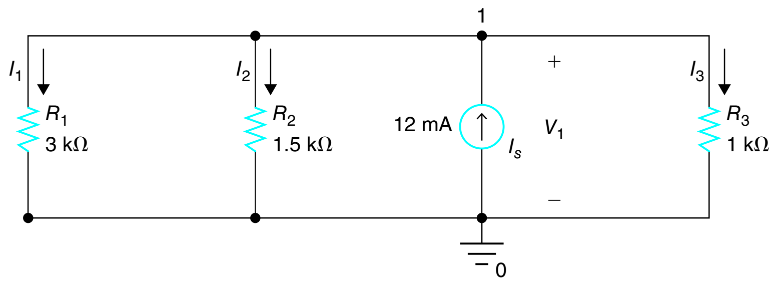
$I_2 = 2$ mA, $I_3 = 1$ mA, $I_1 = 3$ mA, $I_4 = 2$ mA, and $I_5 = 1$ mA.

EXAMPLE 2.4

In the circuit shown in Figure 2.13, find the voltage V_1 and currents I_1, I_2, and I_3.

FIGURE 2.13

Circuit for EXAMPLE 2.4.



According to KCL, the current entering node 1 must equal the sum of currents leaving node 1; that is,

$$I_s = I_1 + I_2 + I_3 \tag{2.24}$$

The current entering node 1 from the current source is $I_s = 12$ mA. From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V_1}{R_1}, \tag{2.25}$$

continued

Example 2.4 continued

where V_1 is the voltage across R_1, as shown in Figure 2.13. Notice that V_1 is also the voltage across I_s, R_2, and R_3. Similarly, the currents I_2 and I_3 are given, respectively, by

$$I_2 = \frac{V_1}{R_2} \tag{2.26}$$

$$I_3 = \frac{V_1}{R_3} \tag{2.27}$$

Substitution of Equations (2.25)–(2.27) into Equation (2.24) yields

$$I_s = \frac{V_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1}{R_3} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_1 \tag{2.28}$$

Solving Equation (2.28) for V_1, we obtain

$$V_1 = \frac{I_s}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{12 \times 10^{-3}}{\frac{1}{3000} + \frac{1}{1500} + \frac{1}{1000}}\text{ V} = \frac{36}{1+2+3}\text{ V} = 6\text{ V} \tag{2.29}$$

Substitution of $V_1 = 6$ V into Equations (2.25)−(2.27) results in

$$I_1 = \frac{V_1}{R_1} = \frac{6\text{ V}}{3000\,\Omega} = 2\text{ mA} \tag{2.30}$$

$$I_2 = \frac{V}{R_2} = \frac{6\text{ V}}{1500\,\Omega} = 4\text{ mA} \tag{2.31}$$

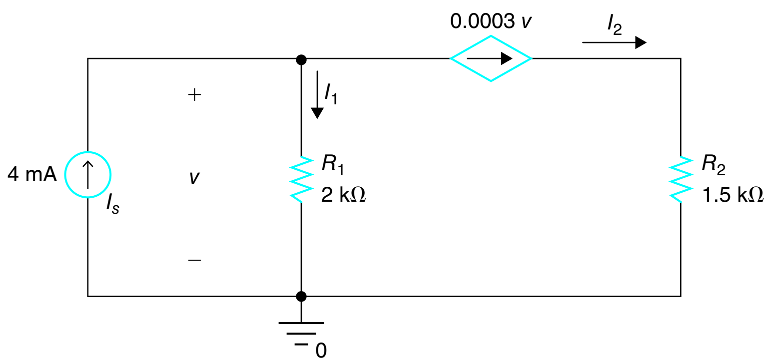
$$I_3 = \frac{V}{R_3} = \frac{6\text{ V}}{1000\,\Omega} = 6\text{ mA} \tag{2.32}$$

Exercise 2.4

Find v, I_1, and I_2 in the circuit shown in Figure 2.14.

FIGURE 2.14

Circuit for EXERCISE 2.4.



Answer:

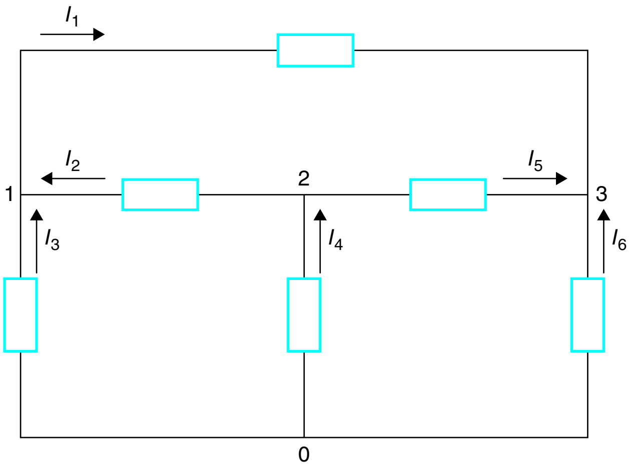
$v = 5$ V, $I_1 = 2.5$ mA, $I_2 = 1.5$ mA.

EXAMPLE 2.5

In the circuit shown in Figure 2.15, let $I_1 = 3$ A, $I_3 = 10$ A, and $I_6 = -8$ A. Find I_2, I_4, and I_5.

FIGURE 2.15

Circuit for EXAMPLE 2.5.



Summing the currents leaving node 0, we obtain

$$I_3 + I_4 + I_6 = 0$$

Solving for I_4, we get

$$I_4 = -I_3 - I_6 = -10 - (-8) = -10 + 8 = -2 \text{ A}$$

Summing the currents leaving node 1, we obtain

$$I_1 - I_2 - I_3 = 0$$

Solving for I_2, we get

$$I_2 = I_1 - I_3 = 3 - 10 = -7 \text{ A}$$

Summing the currents leaving node 3, we obtain

$$-I_1 - I_5 - I_6 = 0$$

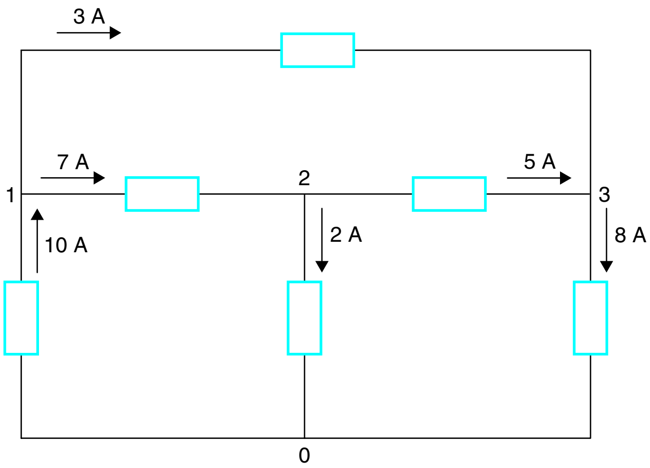
Solving for I_5, we get

$$I_5 = -I_1 - I_6 = -3 - (-8) = -3 + 8 = 5 \text{ A}$$

When the current is negative, the current actually flows in the opposite direction. Figure 2.16 shows the circuit with the actual direction (positive direction) of current.

FIGURE 2.16

Circuit for EXAMPLE 2.5 with positive current direction.

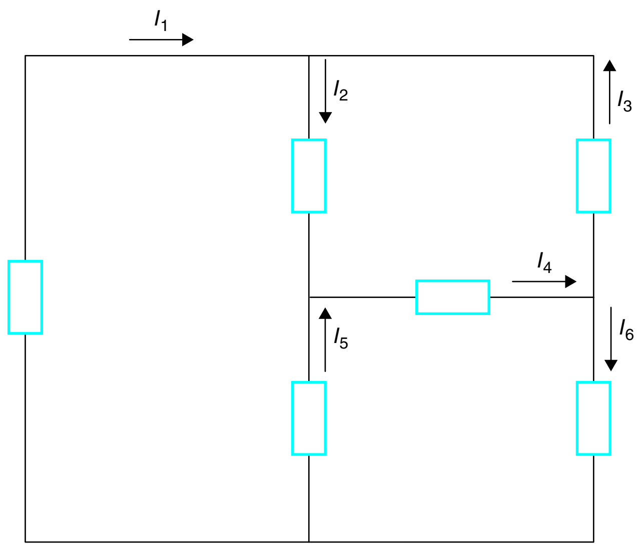


Exercise 2.5

Let $I_1 = 9$ mA, $I_2 = 6$ mA, $I_4 = 2$ mA in the circuit shown in Figure 2.17. Find I_3, I_5, and I_6.

FIGURE 2.17

Circuit for EXERCISE 2.5.



Answer:

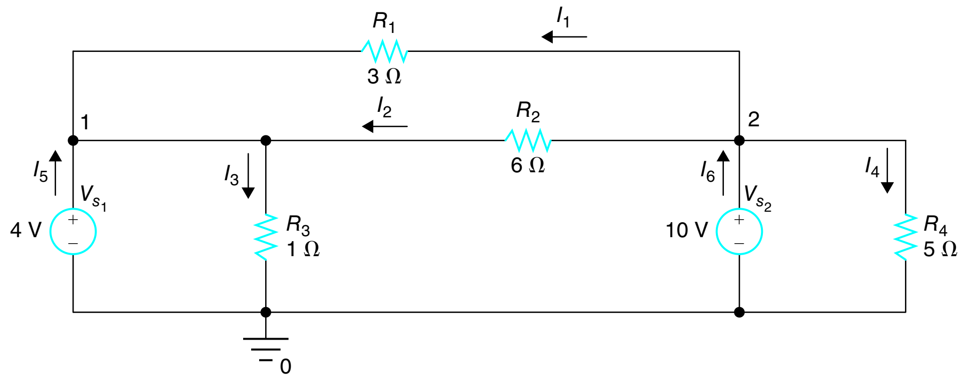
$I_3 = -3$ mA, $I_5 = -4$ mA, $I_6 = 5$ mA.

EXAMPLE 2.6

Find I_1, I_2, I_3, I_4, I_5, and I_6 in the circuit shown in Figure 2.18.

FIGURE 2.18

Circuit for EXAMPLE 2.6.



The voltage across R_1 is $V_{s_2} - V_{s_1} = 10 - 4 = 6$ V. The current I_1 through R_1 is given by

$$I_1 = \frac{V_{s_2} - V_{s_1}}{R_1} = \frac{10 - 4}{3} = \frac{6\text{ V}}{3\,\Omega} = 2\,A$$

continued

Example 2.6 continued

from Ohm's law. Similarly, the current I_2 through R_2 is given by

$$I_2 = \frac{V_{s_2} - V_{s_1}}{R_2} = \frac{10 - 4}{6} = \frac{6\ \text{V}}{6\ \Omega} = 1\ A$$

from Ohm's law. Application of Ohm's law on R_3 and R_4 yields, respectively,

$$I_3 = \frac{V_{s_1}}{R_3} = \frac{4\ \text{V}}{1\ \Omega} = 4\ A$$

$$I_4 = \frac{V_{s_2}}{R_4} = \frac{10\ \text{V}}{5\ \Omega} = 2\ A$$

Summing the current leaving node 1, we obtain

$$-I_1 - I_2 + I_3 - I_5 = -2 - 1 + 4 - I_5 = 0$$

from KCL. Solving for I_5, we obtain

$$I_5 = -2 - 1 + 4 = 1\ A$$

Summing the current leaving node 2, we obtain

$$I_1 + I_2 + I_4 - I_6 = 2 + 1 + 2 - I_6 = 0$$

from KCL. Solving for I_6, we obtain

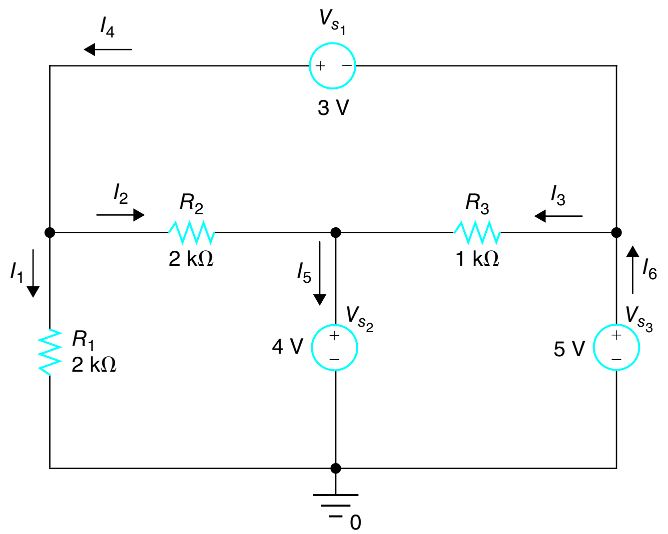
$$I_6 = 2 + 1 + 2 = 5\ A$$

Exercise 2.6

Find I_1, I_2, I_3, I_4, I_5, and I_6 in the circuit shown in Figure 2.19.

FIGURE 2.19

Circuit for EXERCISE 2.6.



continued

Exercise 2.6 continued

Answer:
$I_1 = 4$ mA, $I_2 = 2$ mA, $I_3 = 1$ mA, $I_4 = 6$ mA, $I_5 = 3$ mA, $I_6 = 7$ mA.

2.6 Kirchhoff's Voltage Law (KVL)

As defined in section 2.2, a node is a point in a circuit where two or more elements are connected. It is part of wires that interconnect elements. The voltage of a node must be unique, and the voltage for any node cannot have two different values. We have the following theorem, called **Kirchhoff's voltage law (KVL):**

The sum of voltage drops around a loop equals the sum of voltage rises of the same loop.

Another way to describe KVL is:

The sum of voltage drops around a loop is zero.

Notice that for this statement to be true, at least one of the voltage drops around the loop must be negative (meaning that the voltage actually rises on the branch).

Still another way to describe KVL is:

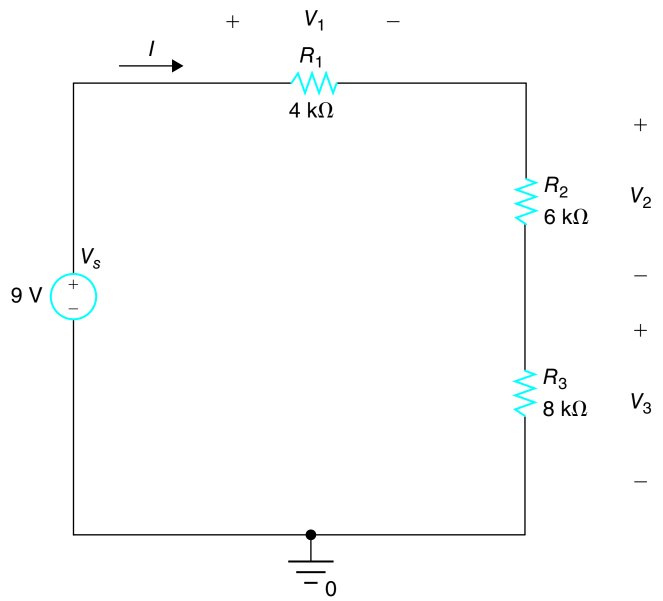
The sum of voltage rises around a loop is zero.

Notice that at least one of the voltage rises around the loop must be negative (meaning that the voltage actually drops on the branch) for this statement to be true. Since a mesh is also a loop, the KVL applies to mesh as well.

Consider a circuit shown in Figure 2.20. We are interested in finding the voltages across the resistors R_1, R_2, and R_3 and the current through them.

FIGURE 2.20

A circuit with three resistors and a voltage source.



Let the current through the circuit be I. Let the voltage across R_1, R_2, and R_3 be V_1, V_2, and V_3, respectively. According to Ohm's law, the voltage V_1 across R_1 is given by

$$V_1 = R_1 I$$

Similarly, the voltages V_2 and V_3 are given respectively by

$$V_2 = R_2 I$$

$$V_3 = R_3 I$$

According to KVL, the sum of voltage drops around the circuit equals zero. When the circuit is traversed in the clockwise direction starting from the negative terminal of the voltage source, through the voltage source to the positive terminal of the voltage source and through the resistors R_1, R_2, and R_3 in that order, and coming back to the negative terminal of the voltage source, the sum of voltage drops is zero:

$$-V_s + R_1 I + R_2 I + R_3 I = 0$$

Solving this equation for I, we have

$$I = \frac{V_s}{R_1 + R_2 + R_3} = \frac{9\text{ V}}{18\ k\Omega} = 0.5\text{ mA}$$

From this current, we can find the voltages V_1, V_2, and V_3 using Ohm's law. Thus,

$$V_1 = R_1 I = 4\ k\Omega \times 0.5\text{ mA} = 2\text{ V}$$

$$V_2 = R_2 I = 6\ k\Omega \times 0.5\text{ mA} = 3\text{ V}$$

$$V_3 = R_3 I = 8\ k\Omega \times 0.5\text{ mA} = 4\text{ V}$$

Notice that multiplication of $k\Omega$ and mA results in V; that is,

$$1\ k\Omega \times 1\text{ mA} = 10^3\ \Omega \times 10^{-3}\ A = 1\text{ V}$$

Also, division of V by $k\Omega$ yields mA, and division of V by mA yields $k\Omega$, as shown here:

$$\frac{1\text{ V}}{1\ k\Omega} = \frac{1\text{ V}}{10^3\ \Omega} = 10^{-3} A = 1\text{ mA}$$

$$\frac{1\text{ V}}{1\text{ mA}} = \frac{1\text{ V}}{10^{-3}\ A} = 10^3\ \Omega = 1\ k\Omega$$

For more details on prefixes, see Chapter 1.

EXAMPLE 2.7

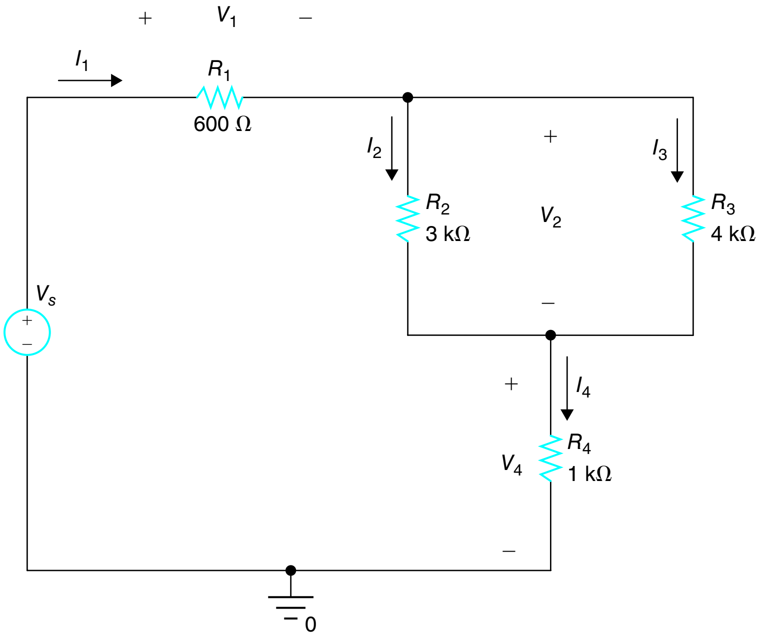
Suppose that $V_2 = 6$ V in the circuit shown in Figure 2.21. Find I_2, I_3, I_4, V_4, I_1, V_1, and V_s.

continued

Example 2.7 continued

FIGURE 2.21

Circuit for EXAMPLE 2.7.



According to Ohm's law, the current through R_2 is given by

$$I_2 = \frac{V_2}{R_2} = \frac{6\text{ V}}{3\ k\Omega} = 2\text{ mA}$$

Similarly, the current through R_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{6\text{ V}}{4\ k\Omega} = 1.5\text{ mA}$$

From KCL, the current through R_4 is the sum of I_2 and I_3. Thus, we have

$$I_4 = I_2 + I_3 = 2\text{ mA} + 1.5\text{ mA} = 3.5\text{ mA}$$

Notice that the current through R_1 is also the sum of I_2 and I_3. Thus, we have

$$I_1 = I_2 + I_3 = 2\text{ mA} + 1.5\text{ mA} = 3.5\text{ mA}$$

According to Ohm's law, the voltage across R_4 is given by

$$V_4 = R_4 I_4 = 1\ k\Omega \times 3.5\text{ mA} = 1 \times 10^3\ \Omega \times 3.5 \times 10^{-3} A = 3.5\text{ V}$$

Similarly, the voltage across R_1 is given by

$$V_1 = R_1 I_1 = 600\ \Omega \times 3.5\text{ mA} = 600\ \Omega \times 3.5 \times 10^{-3} A = 2.1\text{ V}$$

According to KVL, the sum of voltage drops around the mesh on the left side equals zero. Summing the voltage drops starting from the voltage source in the clockwise direction, we obtain

$$-V_s + V_1 + V_2 + V_4 = 0$$

continued

Example 2.7 continued

Solving for V_s, we get

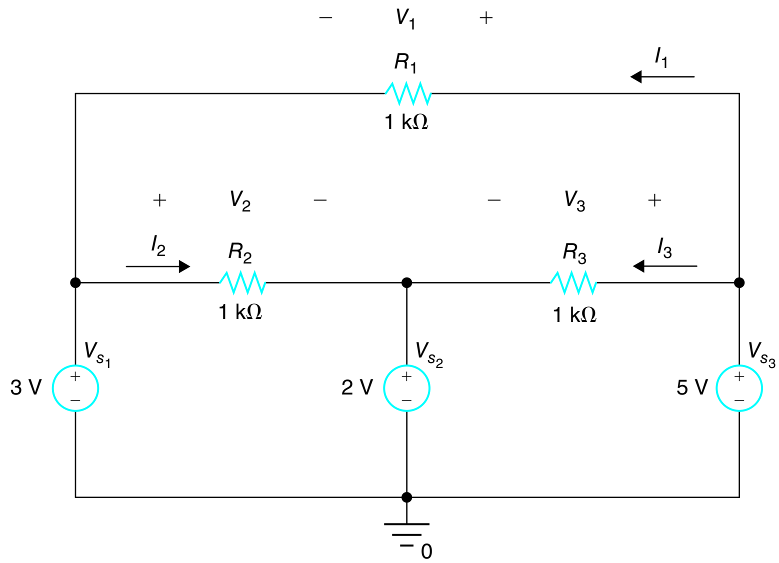
$$V_s = V_1 + V_2 + V_4 = 2.1\text{ V} + 6\text{ V} + 3.5\text{ V} = 11.6\text{ V}$$

Exercise 2.7

Find the voltages V_1, V_2, and V_3, and currents I_1, I_2, and I_3 in the circuit shown in Figure 2.22.

FIGURE 2.22

Circuit for EXERCISE 2.7.



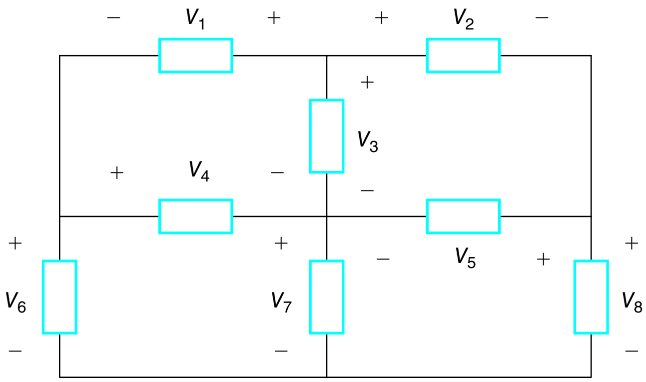
Answer:
$V_1 = 2\text{ V}, V_2 = 1\text{ V}, V_3 = 3\text{ V}, I_1 = 2\text{ mA}, I_2 = 1\text{ mA}, I_3 = 3\text{ mA}.$

EXAMPLE 2.8

Let $V_1 = 6$ V, $V_5 = 5$ V, $V_6 = 3$ V, and $V_7 = 7$ V in the circuit shown in Figure 2.23. Find V_2, V_3, V_4, and V_8.

FIGURE 2.23

Circuit for EXAMPLE 2.8.



Collecting the voltage drops around the mesh at the lower left of the circuit in the clockwise direction (KVL), we obtain

$$-V_6 + V_4 + V_7 = 0$$

Thus,

$$V_4 = V_6 - V_7 = 3 - 7 = -4\text{ V}$$

Collecting the voltage drops around the mesh at the upper left of the circuit in the clockwise direction (KVL), we obtain

$$-V_1 + V_3 - V_4 = 0$$

continued

Example 2.8 continued

Thus,

$$V_3 = V_1 + V_4 = 6 - 4 = 2\text{ V}$$

Collecting the voltage drops around the mesh at the upper right of the circuit in the clockwise direction (KVL), we obtain

$$-V_3 + V_2 + V_5 = 0$$

Thus,

$$V_2 = V_3 - V_5 = 2 - 5 = -3\text{ V}$$

Collecting the voltage drops around the mesh at the lower right of the circuit in the clockwise direction (KVL), we obtain

$$-V_7 - V_5 + V_8 = 0$$

Thus,

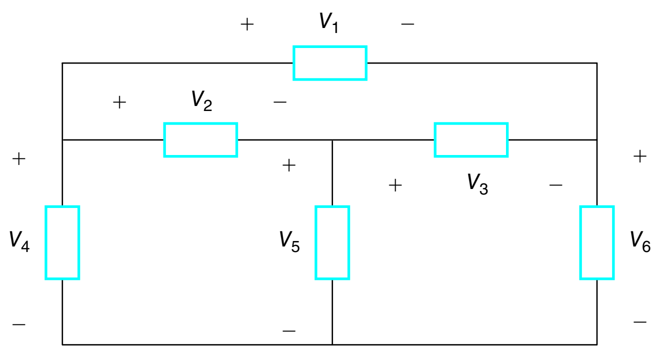
$$V_8 = V_7 + V_5 = 7 + 5 = 12\text{ V}$$

Exercise 2.8

Let $V_1 = 7$ V, $V_3 = 4$ V, and $V_5 = 5$ V in the circuit shown in Figure 2.24. Find V_2, V_4, and V_6.

FIGURE 2.24

Circuit for EXERCISE 2.8.



Answer:

$V_2 = 3$ V, $V_4 = 8$ V, $V_6 = 1$ V.

EXAMPLE 2.9

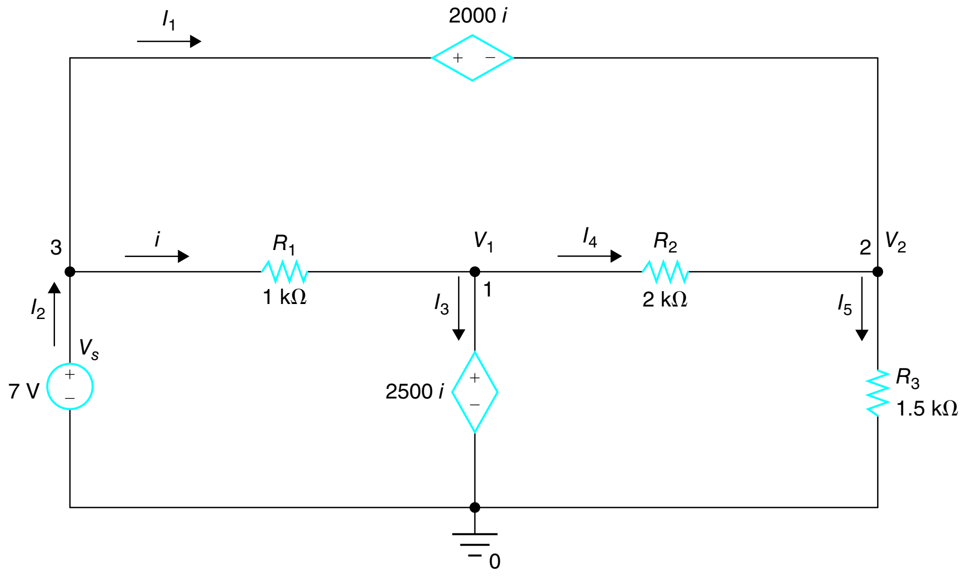
Find i, V_1, V_2, I_2, I_3, I_4, I_5, I_6 in the circuit shown in Figure 2.25.

continued

Example 2.9 continued

FIGURE 2.25

Circuit for EXAMPLE 2.9.



Collecting the voltage drops around the mesh (KVL) in the lower-left mesh of the circuit, we obtain

$$-7 + 1000i + 2500i = 0$$

Solving for i, we get

$$i = \frac{7}{3500}\,A = 0.002\,A = 2\text{ mA}$$

The voltage drop across R_1 is given by

$$V_{R_1} = iR_1 = 2\text{ mA} \times 1\,k\Omega = 2 \times 10^{-3} \times 1000\text{ V} = 2\text{ V}$$

The voltage V_1 is given by

$$V_1 = V_s - V_{R_1} = 7 - 2 = 5\text{ V}$$

The voltage V_2 is given by

$$V_2 = V_s - 2000i = 7 - 2000 \times 0.002 = 7 - 4 = 3\text{ V}$$

The current through R_2 is given by

$$I_4 = \frac{V_1 - V_2}{R_2} = \frac{5\text{ V} - 3\text{ V}}{2000\,\Omega} = \frac{2\text{ V}}{2000\,\Omega} = 1\text{ mA}$$

The current through R_3 is given by

$$I_5 = \frac{V_2}{R_3} = \frac{3\text{ V}}{1500\,\Omega} = 2\text{ mA}$$

continued

Example 2.9 continued

Summing the currents leaving node 1 (KCL), we obtain

$$-i + I_3 + I_4 = -2\text{ mA} + I_3 + 1\text{ mA} = 0$$

Thus, we have

$$I_3 = 1\text{ mA}$$

Summing the currents leaving node 2 (KCL), we obtain

$$-I_1 - I_4 + I_5 = -I_1 - 1\text{ mA} + 2\text{ mA} = 0$$

Thus, we have

$$I_1 = 1\text{ mA}$$

Summing the currents leaving node 3 (KCL), we obtain

$$i + I_1 - I_2 = 2\text{ mA} + 1\text{ mA} - I_2 = 0$$

Thus, we have

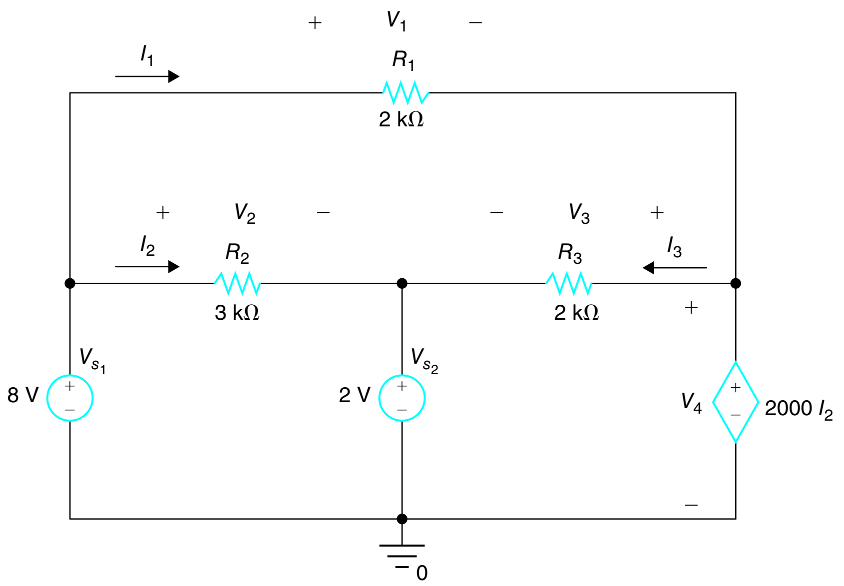
$$I_2 = 3\text{ mA}$$

Exercise 2.9

Find I_2, V_4, V_1, V_2, V_3, I_1, and I_3 in the circuit shown in Figure 2.26.

FIGURE 2.26

Circuit for EXERCISE 2.9.



Answer:
$I_2 = 2\text{ mA}, V_4 = 4\text{ V}, V_1 = 4\text{ V}, V_2 = 6\text{ V}, V_3 = 2\text{ V}, I_1 = 2\text{ mA}, I_3 = 1\text{ mA}.$

2.7 Series and Parallel Connection of Resistors

In this section, the equivalent resistance is found when the resistors are connected in series and in parallel. Reducing the circuit with fewer resistors makes it easier to analyze the circuit. Also, the nonstandard resistor values can be obtained by combining two or more resistors.

2.7.1 SERIES CONNECTION OF RESISTORS

Consider a circuit shown in Figure 2.27(a). In this circuit, two resistors with resistances R_1 and R_2 are connected in series. Let the voltage across R_1 be V_1, the voltage across R_2 be V_2, voltage across both R_1 and R_2 be V, and the current through the resistors be I. Then, the voltage across R_1 is

$$V_1 = R_1 I$$

and the voltage across R_2 is

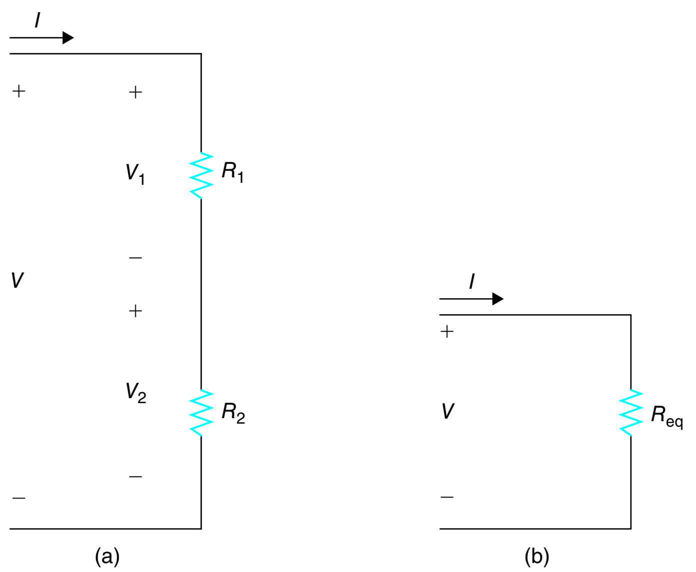
$$V_2 = R_2 I$$

since the current through R_1 and the current through R_2 are identical. There cannot be two different currents on one wire. Thus, we have

$$V = V_1 + V_2 = R_1 I + R_2 I = (R_1 + R_2)I = R_{eq} I$$

FIGURE 2.27

(a) Series connection of two resistors.
(b) Equivalent resistor.



where

$$R_{eq} = R_1 + R_2$$

is the **equivalent resistance** of the series connection of R_1 and R_2. We can replace the series connection of R_1 and R_2 by a single resistor with resistance

$$R_{eq} = R_1 + R_2 \tag{2.33}$$

as shown in Figure 2.27(b), without changing the voltage V and the current I.

If n resistors with resistances $R_1, R_2, \ldots, R_n$ are connected in series, as shown in Figure 2.28(a), the equivalent resistance R_{eq} is

$$R_{eq} = R_1 + R_2 + \cdots + R_n \tag{2.34}$$

Let $V_1, V_2, \ldots, V_n$ be the voltages across $R_1, R_2, \ldots, R_n$, respectively, and V be the voltage across all the resistors, and I be the current through all the resistors. Then, from Ohm's law, we have

$$V_1 = R_1 I,\ V_2 = R_2 I,\ \ldots,\ V_n = R_n I$$

and

$$\begin{aligned} V &= V_1 + V_2 + \cdots + V_n = R_1 I + R_2 I + \cdots + R_n I \\ &= (R_1 + R_2 + \cdots + R_n) I = R_{eq} I \end{aligned}$$

where

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

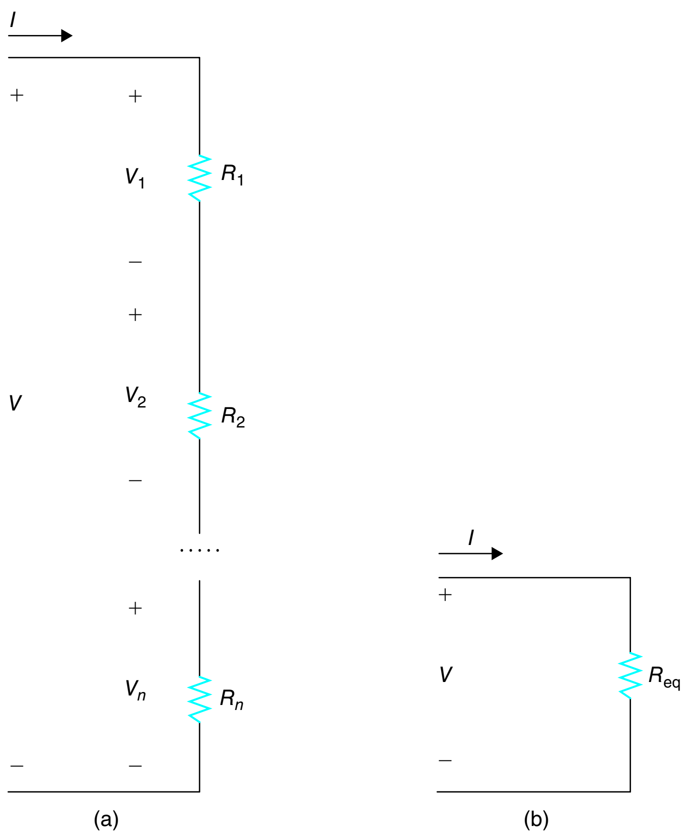
is the **equivalent resistance** of the series connection of the n resistors. We can replace the series connection of the n resistors by a single resistor with resistance

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

as shown in Figure 2.28(b) without changing the voltage V and the current I.

FIGURE 2.28

(a) Series connection of n resistors.
(b) Equivalent resistor.



Since the conductance is the inverse of the resistance ($G = 1/R$), the equivalent conductance of two resistors connected in series is, from $R_{eq} = R_1 + R_2$,

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2}$$

Solving for G_{eq}, we have

$$G_{eq} = \frac{1}{\dfrac{1}{G_1} + \dfrac{1}{G_2}} \tag{2.35}$$

or

$$G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$$

The equivalent resistance is given by

$$R_{eq} = \frac{1}{G_1} + \frac{1}{G_2} = \frac{G_1 + G_2}{G_1 G_2} \tag{2.36}$$

In terms of R_1 and R_2, the equivalent conductance is given by

$$G_{eq} = \frac{1}{R_1 + R_2} \tag{2.37}$$

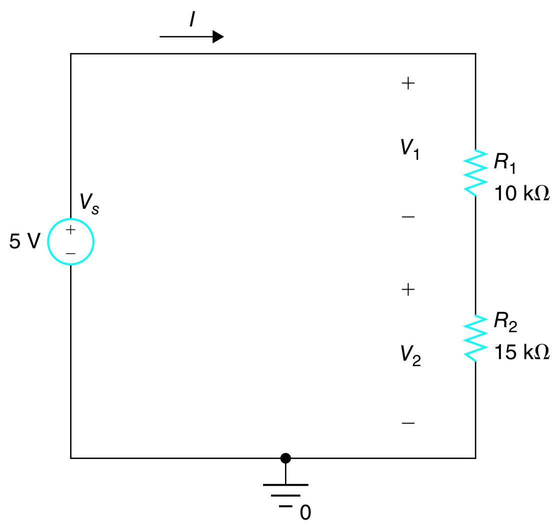
In general, when n resistors are connected in series, the equivalent conductance is given by

$$G_{eq} = \frac{1}{\dfrac{1}{G_1} + \dfrac{1}{G_2} + \cdots + \dfrac{1}{G_n}} = \frac{1}{R_1 + R_2 + \cdots + R_n} \tag{2.38}$$

Consider a circuit shown in Figure 2.29. We are interested in finding I, V_1, and V_2.

FIGURE 2.29

Circuit with series connection of resistors.



The equivalent resistance of R_1 and R_2 is given by

$$R_{eq} = R_1 + R_2 = 10\ k\Omega + 15\ k\Omega = 25\ k\Omega$$

The circuit with the equivalent resistance is shown in Figure 2.30.

The voltage across the equivalent resistance is the same as the voltage of the voltage source since they share the same nodes. The current I through the equivalent resistance is given by

$$I = \frac{V}{R_{eq}} = \frac{5\ \text{V}}{25\ k\Omega} = 0.2\ \text{mA}$$

According to Ohm's law, the voltage across R_1 is given by

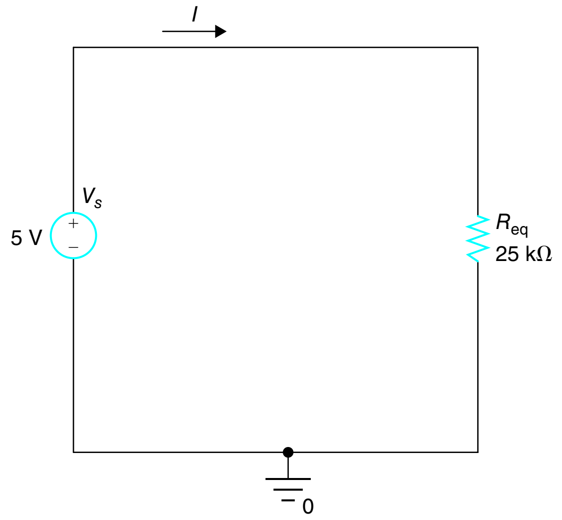
$$V_1 = R_1 I = 10\ k\Omega \times 0.2\ \text{mA} = 2\ \text{V}$$

Similarly, the voltage across R_2 is given by

$$V_2 = R_2 I = 15\ k\Omega \times 0.2\ \text{mA} = 3\ \text{V}$$

FIGURE 2.30

Circuit with the equivalent resistance.

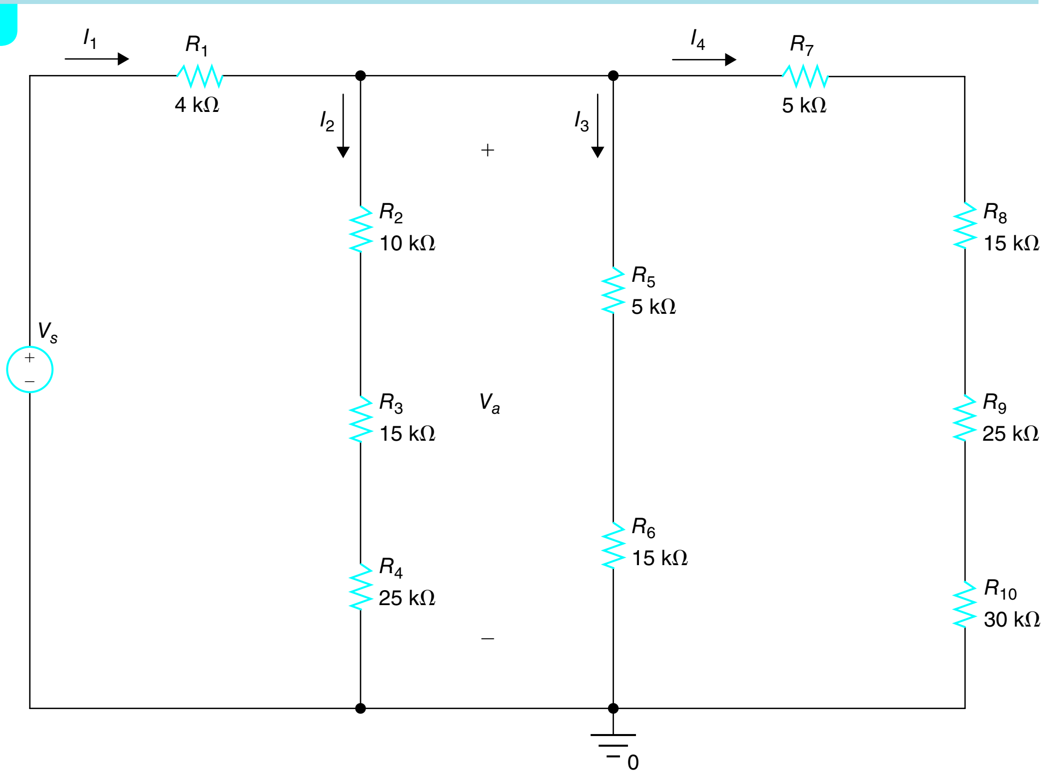


EXAMPLE 2.10

Suppose that $I_3 = 750\ \mu A$ in the circuit shown in Figure 2.31. Find V_a, I_2, I_4, I_1, and V_s.

FIGURE 2.31

Circuit for EXAMPLE 2.10.



The equivalent resistance of the series connection of R_2, R_3, and R_4 is given by

$$R_a = R_2 + R_3 + R_4 = 50\ k\Omega$$

continued

Example 2.10 continued

The equivalent resistance of the series connection of R_5 and R_6 is given by

$$R_b = R_5 + R_6 = 20\ k\Omega$$

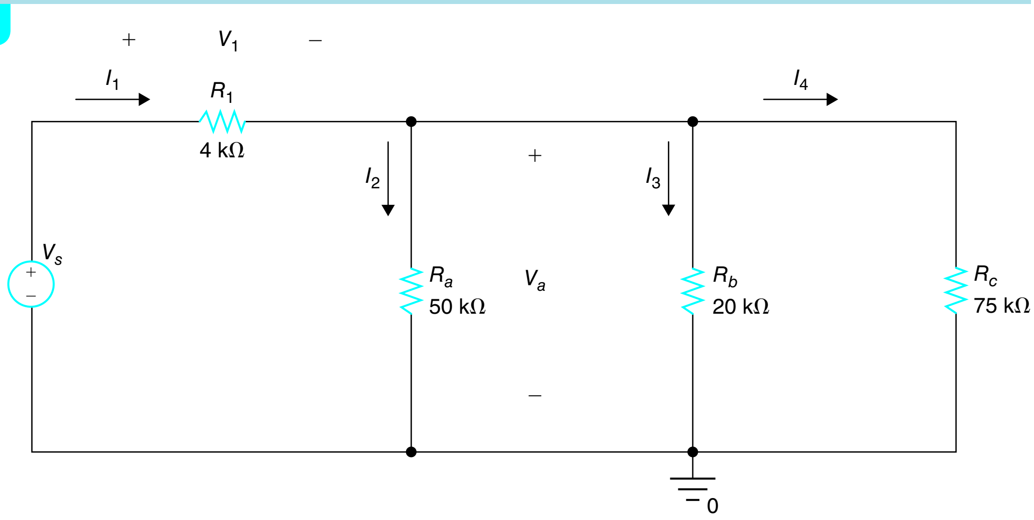
The equivalent resistance of the series connection of R_7, R_8, R_9, and R_{10} is given by

$$R_c = R_7 + R_8 + R_9 + R_{10} = 75\ k\Omega$$

The circuit shown in Figure 2.32 is equivalent to the circuit shown in Figure 2.31.

FIGURE 2.32

Circuit with equivalent resistances.



According to Ohm's law, the voltage across R_b is given by

$$V_a = R_b I_3 = 20\ k\Omega \times 0.75\ \text{mA} = 15\ \text{V}$$

The voltage across R_a and R_c is also V_a. From Ohm's law, the current I_2 is given by

$$I_2 = \frac{V_a}{R_a} = \frac{15\ \text{V}}{50\ k\Omega} = 300\ \mu\text{A}$$

Similarly, the current through R_c is given by

$$I_4 = \frac{V_a}{R_c} = \frac{15\ \text{V}}{75\ k\Omega} = 200\ \mu\text{A}$$

From KCL, we have

$$I_1 = I_2 + I_3 + I_4 = 300\ \mu\text{A} + 750\ \mu\text{A} + 200\ \mu\text{A} = 1250\ \mu\text{A} = 1.25\ \text{mA}$$

The voltage across R_1 is

$$V_1 = R_1 I_1 = 4\ k\Omega \times 1.25\ \text{mA} = 5\ \text{V}$$

From KVL, we have

$$V_s = V_1 + V_a = 5\ \text{V} + 15\ \text{V} = 20\ \text{V}$$

Exercise 2.10

Find I, V_1, V_2, V_3, and V_4 in the circuit shown in Figure 2.33.

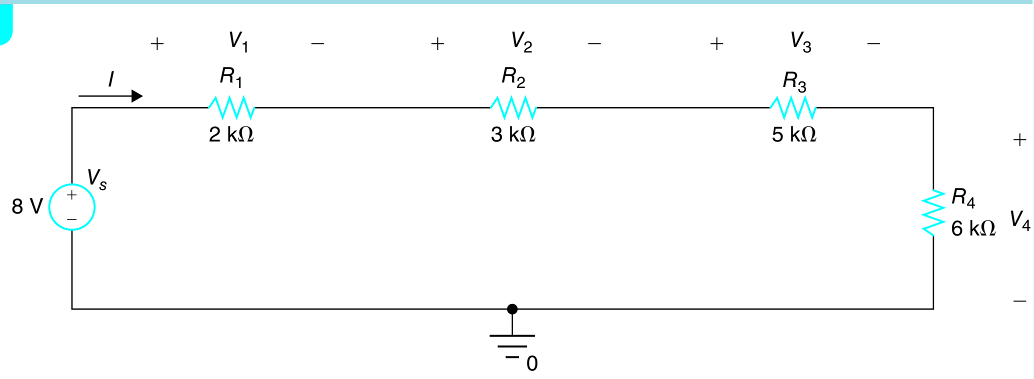


FIGURE 2.33 Circuit for EXERCISE 2.10

Answer:
$I = 0.5$ mA, $V_1 = 1$ V, $V_2 = 1.5$ V, $V_3 = 2.5$ V, $V_4 = 3$ V.

2.7.2 PARALLEL CONNECTION OF RESISTORS

Consider a parallel connection of two resistors with resistances R_1 and R_2, respectively, as shown in Figure 2.34(a).

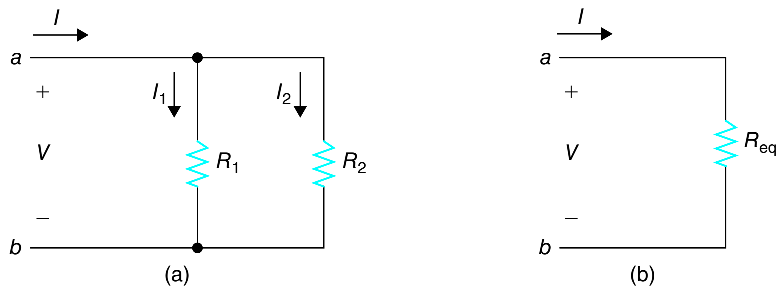


FIGURE 2.34 (a) Parallel connection of two resistors. (b) Equivalent resistance.

Let the voltage across the resistors be V, the current through the resistor R_1 be I_1, the current through the resistor R_2 be I_2, the current from a to b be I. Then, from Ohm's law, the current through R_1 is given by

$$I_1 = \frac{V}{R_1}$$

and the current through R_2 is given by

$$I_2 = \frac{V}{R_2}$$

Since the resistors are connected between the same points, the voltage across R_1 is identical to the voltage across R_2. The current I is given by the sum of I_1 and I_2:

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V = \frac{V}{R_{eq}} \tag{2.39}$$

where R_{eq} is the equivalent resistance of R_1 and R_2. From the last two terms of the Equation (2.39), we obtain

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \tag{2.40}$$

Thus, the equivalent resistance of R_1 connected in parallel to R_2 is given by

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

We can replace the parallel connection of R_1 and R_2 by a single resistor with resistance

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

as shown in Figure 2.34(b) without changing the voltage V and the current I. The equivalent resistance of two resistors in parallel is denoted by $R_1 \| R_2$. Thus, we have

$$R_{eq} = R_1 \| R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

Notice that $\frac{R_2}{R_1 + R_2} < 1$. Multiplication of R_1 on both sides yields $\frac{R_1 R_2}{R_1 + R_2} < R_1$. Thus, we have $R_{eq} < R_1$. Similarly, multiplication of R_2 on both sides of $\frac{R_1}{R_1 + R_2} < 1$ yields $\frac{R_1 R_2}{R_1 + R_2} < R_2$. Thus, we have $R_{eq} < R_2$. This result shows that the equivalent resistance R_{eq} of the parallel connection of two resistors R_1 and R_2 is smaller than R_1 and smaller than R_2.

If $R_1 = 0$ and $R_2 \neq 0$, $R_{eq} = R_1 \| R_2 = 0$. If a resistor is connected in parallel with a short circuit, all currents flow through the short circuit.

If $R_1 < \infty$ and $R_2 = \infty$, $R_{eq} = R_1 \| R_2 = R_1$.

If $R_1 \ll R_2$, $R_{eq} = R_1 \| R_2 \cong R_1$. If R_2 is significantly greater than R_1, the equivalent resistance R_{eq} of the parallel connection of two resistors R_1 and R_2 is close to R_1 (slightly smaller). For example, the equivalent resistance R_{eq} of two resistors $R_1 = 1\ k\Omega$ and $R_2 = 1\ \mathrm{M}\Omega$ is $999.001\ \Omega = 0.999001\ k\Omega$. If we take $R_1 = 1\ k\Omega$ as the equivalent resistance, the error is 0.1%, which is within the tolerance of most resistors.

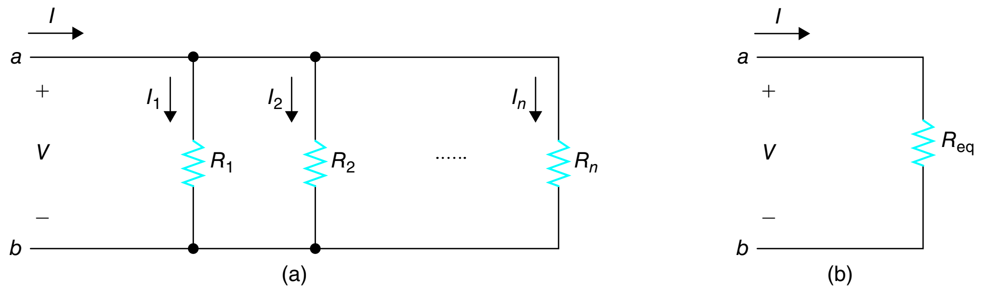
If $R_1 < R_2$, more currents flow through R_1 than R_2 ($V/R_1 > V/R_2$).

If n resistors with resistances $R_1, R_2, \ldots, R_n$, respectively, are connected in parallel, as shown in Figure 2.35(a), the equivalent resistance R_{eq} satisfies the equation

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

FIGURE 2.35

(a) Parallel connection of n resistors. (b) Equivalent resistance.



from which we have

$$R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}} \tag{2.41}$$

The proof of this result is similar to the one for two resistors. Let the current through R_i, $i = 1, 2, \ldots, n$, be I_i, the voltage across the resistors be V, and the current from a to b be I. Then, from Ohm's law, we have

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ \ldots,\ I_n = \frac{V}{R_n}$$

$$\begin{aligned} I = I_1 + I_2 + \cdots + I_n &= \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_n} \\ &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right) V = \frac{V}{R_{eq}} \end{aligned}$$

Thus,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

from which we obtain Equation (2.41). The equivalent circuit is shown in Figure 2.35(b).

Since conductance is the inverse of resistance ($G = 1/R$), the equivalent conductance of two resistors connected in parallel is, from $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$,

$$G_{eq} = G_1 + G_2 \tag{2.42}$$

The equivalent resistance R_{eq} is given by

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1 + G_2} \tag{2.43}$$

In general, when n resistors are connected in parallel, the equivalent conductance is given by

$$G_{eq} = G_1 + G_2 + \cdots + G_n \tag{2.44}$$

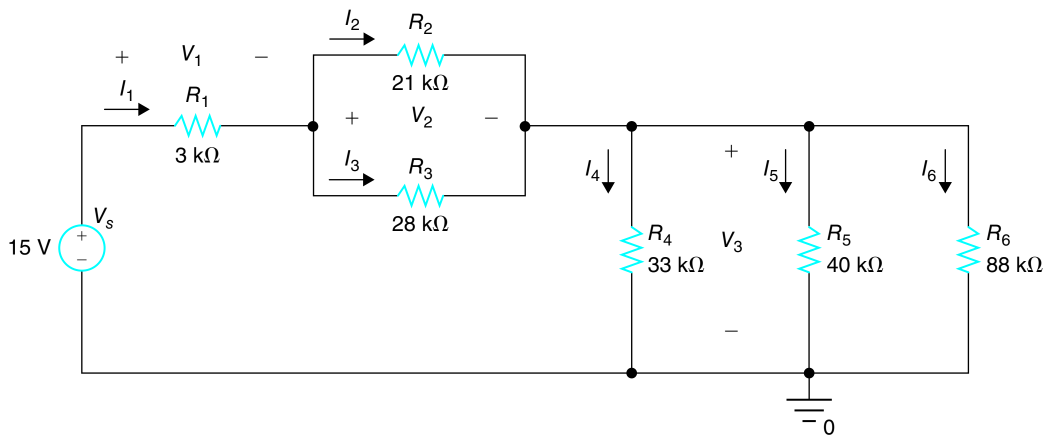
The equivalent resistance is given by

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1 + G_2 + \cdots + G_n} \tag{2.45}$$

Consider a circuit shown in Figure 2.36. We are interested in finding I_1, V_1, V_2, I_2, I_3, V_3, I_4, I_5, and I_6.

FIGURE 2.36

Circuit to be analyzed.



The equivalent resistance of the parallel connection of R_2 and R_3 is given by

$$R_a = R_2 \| R_3 = \frac{21\ k\Omega \times 28\ k\Omega}{21\ k\Omega + 28\ k\Omega} = \frac{588}{49}\ k\Omega = 12\ k\Omega$$

The equivalent resistance of the parallel connection of R_4, R_5, and R_6 is given by

$$R_b = R_4 \| R_5 \| R_6 = \frac{1}{\frac{1}{33} + \frac{1}{40} + \frac{1}{88}}\ k\Omega = \frac{1}{0.0666667}\ k\Omega = 15\ k\Omega$$

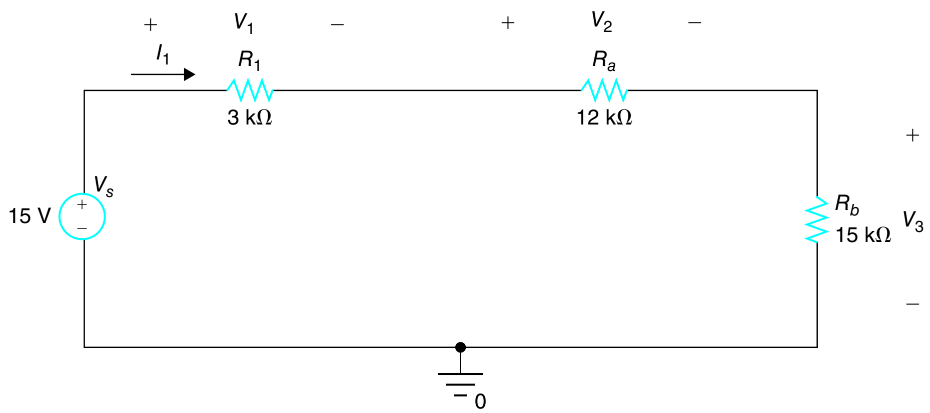
The circuit shown in Figure 2.36 can be redrawn using R_1, R_a, and R_b, as shown in Figure 2.37.

Resistors R_1, R_a, and R_b are connected in series. The total resistance seen from the voltage source is

$$R_{eq} = R_1 + R_a + R_b = 3\ k\Omega + 12\ k\Omega + 15\ k\Omega = 30\ k\Omega$$

FIGURE 2.37

Equivalent circuit with R_1, R_a, R_b.



The current I_1 flowing out of the positive terminal of the voltage source is given by

$$I_1 = \frac{V_s}{R_{eq}} = \frac{15\ \text{V}}{30\ k\Omega} = 0.5\ \text{mA}$$

Notice that the current through R_a and R_b is also I_1. The voltage drop across R_1 is given by

$$V_1 = R_1 I_1 = 3\ k\Omega \times 0.5\ \text{mA} = 1.5\ \text{V}$$

The voltage drop across R_a (also across R_2 and R_3) is given by

$$V_2 = R_a I_1 = 12\ k\Omega \times 0.5\ \text{mA} = 6\ \text{V}$$

Similarly, the voltage drop across R_b (also across R_4, R_5, and R_6) is given by

$$V_3 = R_b I_1 = 15\ k\Omega \times 0.5\ \text{mA} = 7.5\ \text{V}$$

From Ohm's law, the currents I_2, I_3, I_4, I_5, and I_6 are given, respectively, by

$$I_2 = \frac{V_2}{R_2} = \frac{6\ \text{V}}{21\ k\Omega} = 0.2857\ \text{mA}$$

$$I_3 = \frac{V_2}{R_3} = \frac{6\ \text{V}}{28\ k\Omega} = 0.2143\ \text{mA}$$

$$I_4 = \frac{V_3}{R_4} = \frac{7.5\ \text{V}}{33\ k\Omega} = 0.2273\ \text{mA}$$

$$I_5 = \frac{V_3}{R_5} = \frac{7.5\ \text{V}}{40\ k\Omega} = 0.1875\ \text{mA}$$

$$I_6 = \frac{V_3}{R_6} = \frac{7.5\ \text{V}}{88\ k\Omega} = 0.08523\ \text{mA}$$

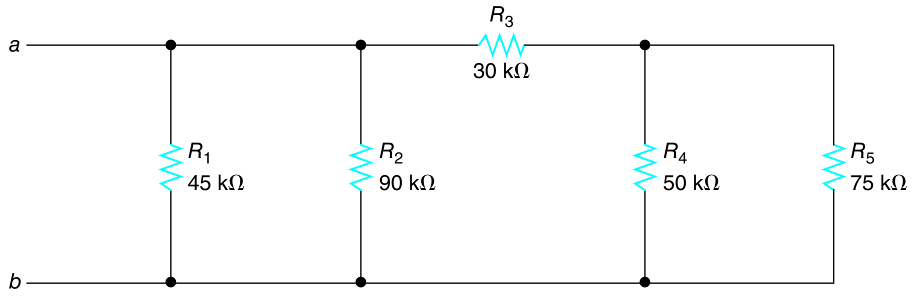
Notice that $I_2 + I_3 = I_1 = 0.5$ mA and $I_4 + I_5 + I_6 = I_1 = 0.5$ mA.

EXAMPLE 2.11

Find the equivalent resistance between terminals *a* and *b* for the circuit shown in Figure 2.38.

FIGURE 2.38

Circuit for EXAMPLE 2.11.



continued

Example 2.11 continued

Let R_6 be the equivalent resistance of the parallel connection of R_4 and R_5. Then, from Equation (2.40), we have

$$R_6 = \frac{R_4 R_5}{R_4 + R_5} = \frac{50\,k\Omega \times 75\,k\Omega}{50\,k\Omega + 75\,k\Omega} = \frac{3750}{125}\,k\Omega = 30\,k\Omega$$

Let R_7 be the equivalent resistance of the series connection of R_3 and R_6. Then, from Equation (2.33), we have

$$R_7 = R_3 + R_6 = 30\,k\Omega + 30\,k\Omega = 60\,k\Omega$$

The equivalent resistance R_{eq} between a and b is the equivalent resistance of the parallel connection of R_1, R_2, and R_7. Application of Equation (2.41) yields

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_7}} = \frac{1}{\frac{1}{45\,k\Omega} + \frac{1}{90\,k\Omega} + \frac{1}{60\,k\Omega}}$$

$$= \frac{180}{\frac{180}{45} + \frac{180}{90} + \frac{180}{60}}\,k\Omega = \frac{180}{9}\,k\Omega = 20\,k\Omega$$

MATLAB® can be used to facilitate the calculation of the equivalent resistance. Create a function named P given here, and save the file in the current folder. The function P calculates the equivalent resistance of resistors connected in parallel.

```
function [Req] = P(x)
% Equivalent Resistance of Parallel Connection.
% Req = 1/(1/R1 + 1/R2 + ... + 1/Rn).
Req=1/sum(1./x);
end
```

The input is a vector of resistance values. As an example, we can find the equivalent resistance of parallel connection of 3 $k\Omega$ resistor and 6 $k\Omega$ resistor by calling the function P:

```
>>Req=P([3000,6000])
Req =
      2000
```

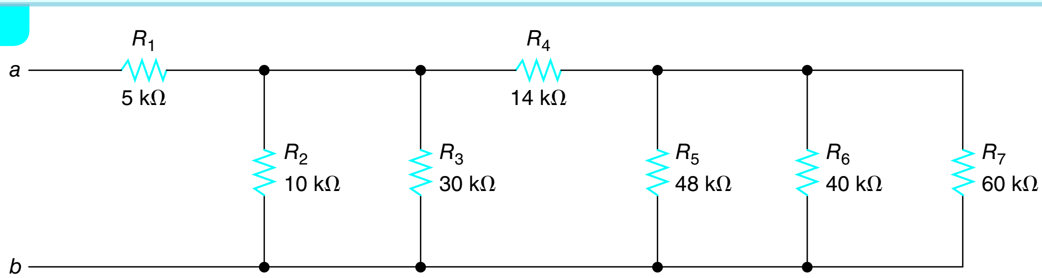
The answer is $3 \times 6/(3 + 6)\,k\Omega = 18/9\,k\Omega = 2\,k\Omega$. The equivalent resistance for the circuit shown in Figure 2.38 can be found using MATLAB:

```
>>R1=45000;R2=90000;R3=30000;R4=50000;R5=75000;
>> Req=P([R1,R2,R3+P([R4,R5])])
Req =
      20000
```

Exercise 2.11

Find the equivalent resistance between terminals *a* and *b* for the circuit shown in Figure 2.39.

FIGURE 2.39



Circuit for EXERCISE 2.11.

Answer:

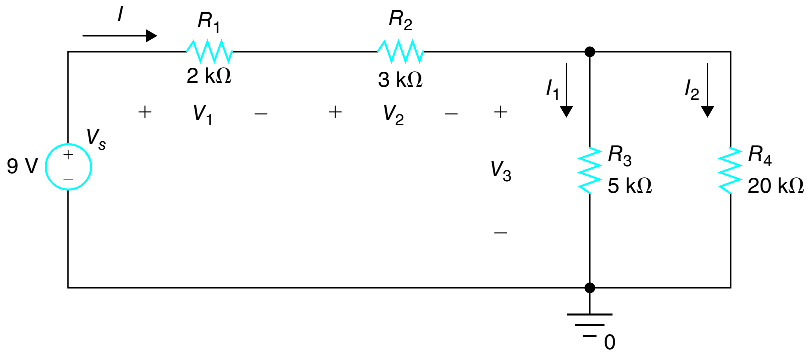
$R_{eq} = 11\ k\Omega$.

```
>> R1=5000;R2=10000;R3=30000;R4=14000;R5=48000;R6=40000;R7=60000;
>> Req=R1+P([R2,R3,R4+P([R5,R6,R7])])
Req =
      11000
```

EXAMPLE 2.12

For the circuit shown in Figure 2.40, find the equivalent resistance of R_1, R_2, R_3, and R_4. Also, find the currents I, I_1, and I_2, and voltages across R_1, R_2, R_3, and R_4 and powers absorbed by R_1, R_2, R_3, and R_4.

FIGURE 2.40



Circuit for EXAMPLE 2.12.

The equivalent resistance of R_1, R_2, R_3, and R_4 is given by

$$R_{eq} = R_1 + R_2 + \frac{R_3 R_4}{R_3 + R_4} = 2\ k\Omega + 3\ k\Omega + \frac{5\ k\Omega \times 20\ k\Omega}{5\ k\Omega + 20\ k\Omega} = 9\ k\Omega$$

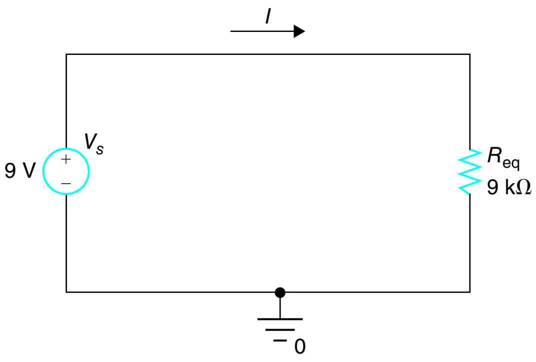
With the equivalent resistance, the circuit shown in Figure 2.40 can be redrawn as the one shown in Figure 2.41.

continued

Example 2.12 continued

FIGURE 2.41

Circuit with the equivalent resistance.



The current I from the positive terminal of the voltage source into the equivalent resistor is

$$I = \frac{V_s}{R_{eq}} = \frac{9\text{ V}}{9\ k\Omega} = 1\text{ mA}$$

From Ohm's law, the voltage drop across R_1 is given by

$$V_1 = IR_1 = 1\text{ mA} \times 2\ k\Omega = 2\text{ V}$$

The polarity of V_1 is shown in Figure 2.40. Similarly, the voltage drop across R_2 is given by

$$V_2 = IR_2 = 1\text{ mA} \times 3\ k\Omega = 3\text{ V}$$

From KVL, the sum of voltage drops around the circuit is zero. Starting from the negative terminal of the voltage source, the sum of voltage drops is

$$-9 + V_1 + V_2 + V_3 = -9 + 2 + 3 + V_3 = 0$$

Thus, $V_3 = 4$ V. This result can also be obtained by simply subtracting V_1 and V_2 from V_s; that is,

$$V_3 = 9 - V_1 - V_2 = 9 - 2 - 3 = 4\text{ V}$$

From Ohm's law, the current through R_3 is given by

$$I_1 = \frac{V_3}{R_3} = \frac{4\text{ V}}{5\ k\Omega} = 0.8\text{ mA}$$

Similarly, the current through R_4 is given by

$$I_2 = \frac{V_3}{R_4} = \frac{4\text{ V}}{20\ k\Omega} = 0.2\text{ mA}$$

Notice that I_2 can also be found from KCL. Since the sum of the currents leaving a node (connecting R_2, R_3, and R_4) is zero, we have

$$-1\text{ mA} + 0.8\text{ mA} + I_2 = 0$$

or $I_2 = 0.2$ mA. The power absorbed by R_1 is found to be

$$P_1 = I^2R_1 = (1\text{ mA})^2 \times 2\ k\Omega = 2\text{ mW}$$

Similarly, the power absorbed by R_2, R_3, and R_4 is given, respectively, by

$$P_2 = I^2R_2 = (1\text{ mA})^2 \times 3\ k\Omega = 3\text{ mW}$$

$$P_3 = I_1^2R_3 = (0.8\text{ mA})^2 \times 5\ k\Omega = 3.2\text{ mW}$$

$$P_4 = I_2^2R_4 = (0.2\text{ mA})^2 \times 20\ k\Omega = 0.8\text{ mW}$$

continued

Example 2.12 continued

The sum of powers absorbed by the four resistors R_1, R_2, R_3, and R_4 is given by

$$P_1 + P_2 + P_3 + P_4 = 2\text{ mW} + 3\text{ mW} + 3.2\text{ mW} + 0.8\text{ mW} = 9\text{ mW}$$

Notice that the power delivered by the voltage source is

$$P_s = -IV_s = (-1\text{ mA}) \times 9\text{ V} = -9\text{ mW}$$

The current through the voltage source V_s is $-I$, which is the current from the positive terminal of the voltage source, through the voltage source, to the negative terminal of the voltage source. The negative power implies that the voltage source delivers power to the rest of the circuit. The sum of all the powers on the circuit is

$$P_s + P_1 + P_2 + P_3 + P_4 = 0 \quad \textbf{(2.46)}$$

This result is called *conservation of power.* Equation (2.46) can be rewritten as

$$-P_s = P_1 + P_2 + P_3 + P_4 \quad \textbf{(2.47)}$$

The power on the left side is the power supplied by the source, and the power on the right side is the power absorbed by all the resistors. In general, the power supplied equals the power absorbed in a circuit. This is another description of the conservation of power.

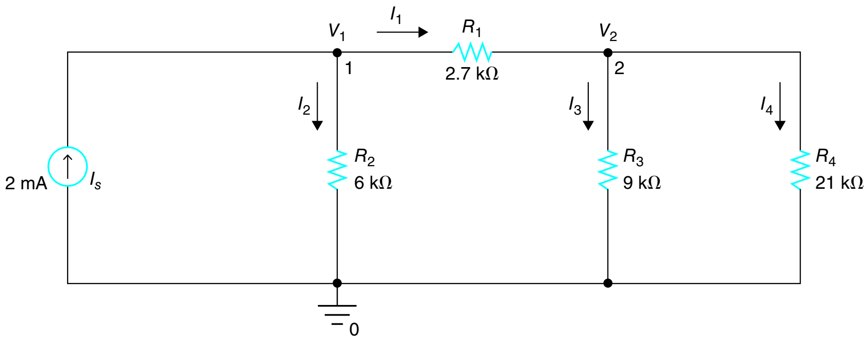
Instead of using $P = I^2R$ to calculate the power absorbed by each resistor, we can use $P = IV$ or $P = \dfrac{V^2}{R}$. The result will be identical to those given previously.

Exercise 2.12

For the circuit shown in Figure 2.42, find the equivalent resistance R_{eq} seen from the current source. Also, find the currents I_1, I_2, I_3, and I_4, voltages V_1 and V_2, voltage across R_1, and powers absorbed or supplied by R_1, R_2, R_3, R_4, and I_s.

FIGURE 2.42

Circuit for EXERCISE 2.12.



Answer:

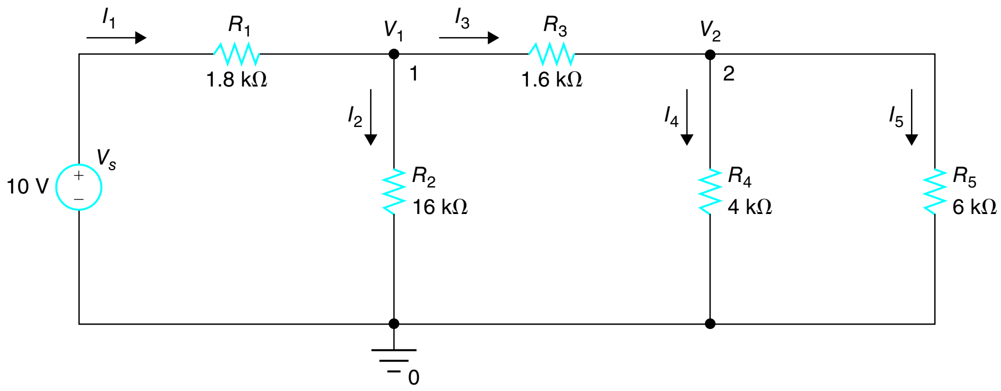
$R_{eq} = 3.6\ k\Omega$, $I_1 = 0.8$ mA, $I_2 = 1.2$ mA, $I_3 = 0.56$ mA, $I_4 = 0.24$ mA, $V_1 = 7.2$ V, $V_2 = 5.04$ V, $V_{R_1} = 2.16$ V, $P_{R_1} = 1.728$ mW, $P_{R_2} = 8.64$ mW, $P_{R_3} = 2.8224$ mW, $P_{R_4} = 1.2096$ mW, $P_{I_s} = -14.4$ mW, $P_{R_1} + P_{R_2} + P_{R_3} + P_{R_4} = 14.4$ mW.

EXAMPLE 2.13

For the circuit shown in Figure 2.43, find the equivalent resistance R_{eq} seen from the voltage source. Also, find the currents I_1, I_2, I_3, I_4, and I_5, voltages V_1 and V_2 and voltages across R_1, R_2, R_3, R_4, and R_5 and powers absorbed or supplied by R_1, R_2, R_3, R_4, R_5, and V_s.

FIGURE 2.43

Circuit for EXAMPLE 2.13.



Let R_a be the equivalent resistance of the parallel connection of R_4 and R_5. Then, we have

$$R_a = R_4 \| R_5 = \frac{R_4 \times R_5}{R_4 + R_5} = \frac{4\,k\Omega \times 6\,k\Omega}{4\,k\Omega + 6\,k\Omega} = \frac{4 \times 6}{4 + 6} k\Omega = \frac{24}{10} k\Omega = 2.4\,k\Omega$$

Let R_b be the sum of R_3 and R_a. Then, we have

$$R_b = R_3 + R_a = 1.6\,k\Omega + 2.4\,k\Omega = 4\,k\Omega$$

Let R_c be the equivalent resistance of the parallel connection of R_2 and R_b. Then, we have

$$R_c = R_2 \| R_b = \frac{R_2 \times R_b}{R_2 + R_b} = \frac{16\,k\Omega \times 4\,k\Omega}{16\,k\Omega + 4\,k\Omega} = \frac{16 \times 4}{16 + 4} k\Omega = \frac{64}{20} k\Omega = 3.2\,k\Omega$$

The equivalent resistance R_{eq} is the sum of R_1 and R_c. Thus, we get

$$R_{eq} = R_1 + R_c = 1.8\,k\Omega + 3.2\,k\Omega = 5\,k\Omega$$

FIGURE 2.44

The circuit with equivalent resistance.

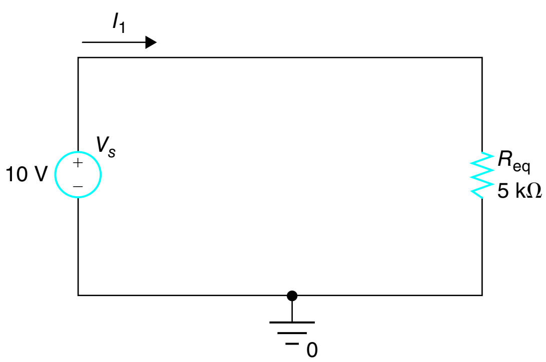


Figure 2.44 shows the circuit with the voltage source and the equivalent resistance.

The current I_1, flowing out of the positive terminal of the voltage source, is given by

$$I_1 = \frac{V_s}{R_{eq}} = \frac{10\text{ V}}{5\,k\Omega} = 2\text{ mA}$$

The voltage drop across R_1 is given by (from Ohm's law)

$$V_{R_1} = R_1 I_1 = 1.8\,k\Omega \times 2\text{ mA} = 1.8 \times 10^3 \times 2 \times 10^{-3}\text{ V} = 3.6\text{ V}$$

continued

Example 2.13 continued

The voltage V_1 at node 1 is obtained by subtracting V_{R_1} from V_s:

$$V_1 = V_s - V_{R_1} = 10\text{ V} - 3.6\text{ V} = 6.4\text{ V}$$

The voltage across R_2 is V_1. Thus,

$$V_{R_2} = V_1 = 6.4\text{ V}$$

The current I_2 through R_2 is given by

$$I_2 = \frac{V_1}{R_2} = \frac{6.4\text{ V}}{16\ k\Omega} = 0.4\text{ mA}$$

Application of KCL at node 1 results in

$$I_3 = I_1 - I_2 = 2\text{ mA} - 0.4\text{ mA} = 1.6\text{ mA}$$

The voltage drop across R_3 is given by (from Ohm's law)

$$V_{R_3} = R_3 I_3 = 1.6\ k\Omega \times 1.6\text{ mA} = 1.6 \times 10^3 \times 1.6 \times 10^{-3}\text{ V} = 2.56\text{ V}$$

The voltage V_2 at node 2 is obtained by subtracting V_{R_3} from V_1:

$$V_2 = V_1 - V_{R_3} = 6.4\text{ V} - 2.56\text{ V} = 3.84\text{ V}$$

The voltage across R_4 and R_5 is V_2. Thus,

$$V_{R_4} = V_2 = 3.84\text{ V}$$

$$V_{R_5} = V_2 = 3.84\text{ V}$$

The current I_4 through R_4 is given by

$$I_4 = \frac{V_2}{R_4} = \frac{3.84\text{ V}}{4\ k\Omega} = 0.96\text{ mA}$$

The current I_5 through R_5 is given by

$$I_5 = \frac{V_2}{R_5} = \frac{3.84\text{ V}}{6\ k\Omega} = 0.64\text{ mA}$$

Notice that the sum of I_2, I_4, and I_5 is 2 mA, confirming KCL at node 0. The powers at the resistors and V_s are:

$$P_{R_1} = I_1 \times V_{R_1} = 2\text{ mA} \times 3.6\text{ V} = 7.2\text{ mW}$$

$$P_{R_2} = I_2 \times V_{R_2} = 0.4\text{ mA} \times 6.4\text{ V} = 2.56\text{ mW}$$

$$P_{R_3} = I_3 \times V_{R_3} = 2\text{ mA} \times 2.56\text{ V} = 4.096\text{ mW}$$

$$P_{R_4} = I_4 \times V_{R_4} = 0.96\text{ mA} \times 3.84\text{ V} = 3.6864\text{ mW}$$

continued

Example 2.13 continued

$$P_{R_5} = I_5 \times V_{R_5} = 0.64 \text{ mA} \times 3.84 \text{ V} = 2.4576 \text{ mW}$$

$$P_{Vs} = -I_1 \times V_s = -2 \text{ mA} \times 10 \text{ V} = -20 \text{ mW}$$

The power from the voltage source is negative, indicating that power is delivered from the voltage source. The total absorbed power (20 mW) by five resistors equals the power delivered (−20 W) by the voltage source.

MATLAB

```
%EXAMPLE 2.13
%Function P.m should be in the same folder as this file.
clear all;format long;
Vs=10;
R1=1800;R2=16000;R3=1600;R4=4000;R5=6000;
Ra=P([R4,R5])
Rb=R3+Ra
Rc=P([R2,Rb])
Req=R1+Rc
I1=Vs/Req
VR1=R1*I1
V1=Vs-VR1
I2=V1/R2
I3=I1-I2
VR3=R3*I3
V2=V1-VR3
I4=V2/R4
I5=V2/R5
PR1=I1^2*R1
PR2=I2^2*R2
PR3=I3^2*R3
PR4=I4^2*R4
PR5=I5^2*R5
PVs=-I1*Vs
PSum=PR1+PR2+PR3+PR4+PR5+PVs

Answers:
Ra =
     2400
Rb =
     4000
Rc =
     3200
Req =
      5000
I1 =
   0.002000000000000
VR1 =
   3.600000000000000
V1 =
   6.400000000000000
I2 =
     4.000000000000000e-04
I3 =
   0.001600000000000
```

continued

Example 2.13 continued

MATLAB continued

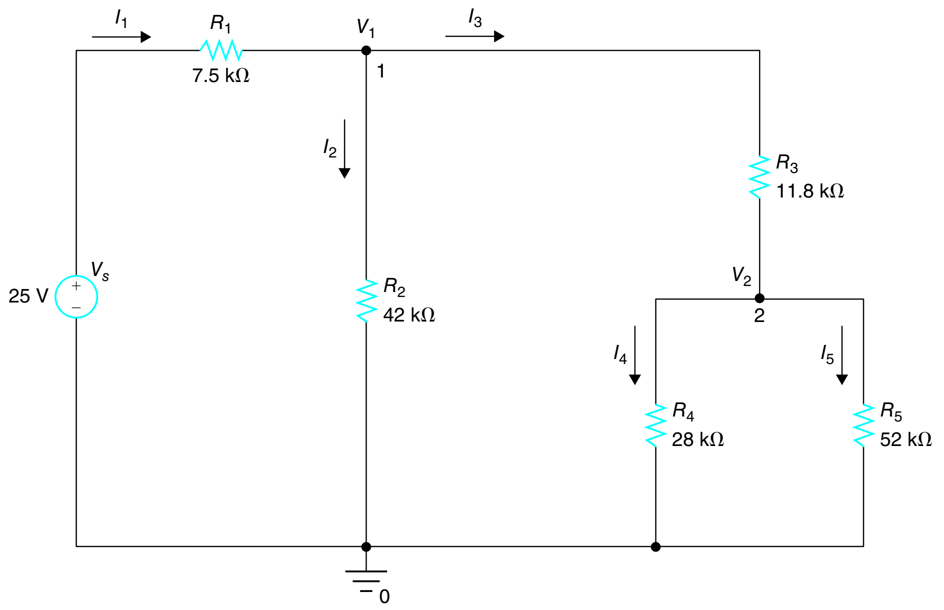
```
VR3 =
   2.560000000000000
V2 =
   3.840000000000000
I4 =
     9.600000000000000e-04
I5 =
     6.400000000000001e-04
PR1 =
   0.007200000000000
PR2 =
   0.002560000000000
PR3 =
   0.004096000000000
PR4 =
   0.003686400000000
PR5 =
   0.002457600000000
PVs =
  -0.020000000000000
PSum =
     0
```

Exercise 2.13

For the circuit shown in Figure 2.45, find the equivalent resistance R_{eq} seen from the voltage source. Also, find the currents I_1, I_2, I_3, I_4, and I_5, voltages V_1 and V_2 and voltages across R_1 and R_3, and powers absorbed or supplied by R_1, R_2, R_3, R_4, R_5, and V_s.

FIGURE 2.45

Circuit for EXERCISE 2.13.



continued

Exercise 2.13 continued

Answer:

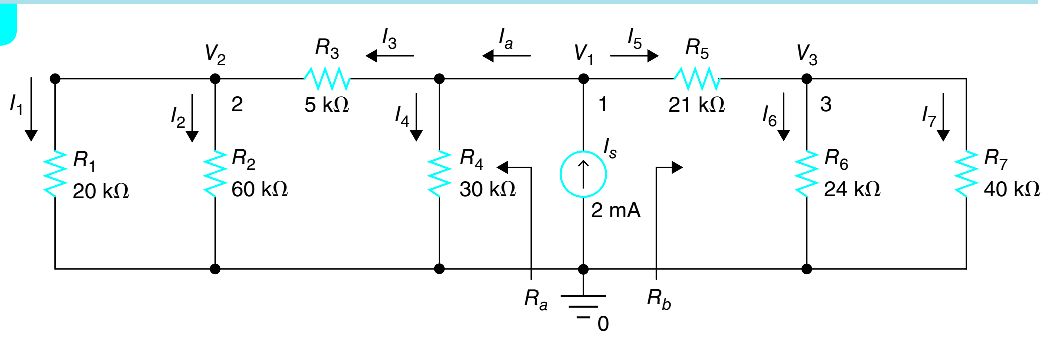
$R_{eq} = 25\ k\Omega$, $I_1 = 1$ mA, $I_2 = 0.416667$ mA, $I_3 = 0.583333$ mA, $I_4 = 0.37916667$ mA, $I_5 = 0.20416667$ mA, $V_1 = 17.5$ V, $V_2 = 10.6167$ V, $V_{R_1} = 7.5$ V, $V_{R_3} = 6.8833$ V, $P_{R_1} = 7.5$ mW, $P_{R_2} = 7.2916667$ mW, $P_{R_3} = 4.01528$ mW, $P_{R_4} = 4.0254861$ mW, $P_{R_5} = 2.167569$, $P_{Vs} = -25$ mW.

EXAMPLE 2.14

For the circuit shown in Figure 2.46, find the equivalent resistance R_{eq} seen from the current source. Also, find the currents I_a, I_1, I_2, I_3, I_4, I_5, I_6, and I_7, and voltages V_1, V_2, and V_3.

FIGURE 2.46

Circuit for EXAMPLE 2.14.



Let R_a be the equivalent resistance to the left of the current source, and R_b be the equivalent resistance to the right of the current source. Let R_8 be the equivalent resistance of the parallel connection of R_1 and R_2. Then, we have

$$R_8 = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20\ k\Omega \times 60\ k\Omega}{20\ k\Omega + 60\ k\Omega} = 15\ k\Omega$$

Let R_9 be the sum of R_3 and R_8. Then we have

$$R_9 = R_3 + R_8 = 5\ k\Omega + 15\ k\Omega = 20\ k\Omega$$

The equivalent resistance R_a is the parallel connection of R_9 and R_4. Thus, we get

$$R_a = \frac{R_9 \times R_4}{R_9 + R_4} = \frac{20\ k\Omega \times 30\ k\Omega}{20\ k\Omega + 30\ k\Omega} = 12\ k\Omega$$

The equivalent resistance R_b is given by

$$R_b = R_5 + (R_6 \| R_7) = R_5 + \frac{R_6 \times R_7}{R_6 + R_7}$$

$$= 21\ k\Omega + \frac{24\ k\Omega \times 40\ k\Omega}{24\ k\Omega + 40\ k\Omega} = 21\ k\Omega + 15\ k\Omega = 36\ k\Omega$$

continued

Example 2.14 continued

The equivalent resistance R_{eq} seen from the current source is the parallel connection of R_a and R_b. Therefore,

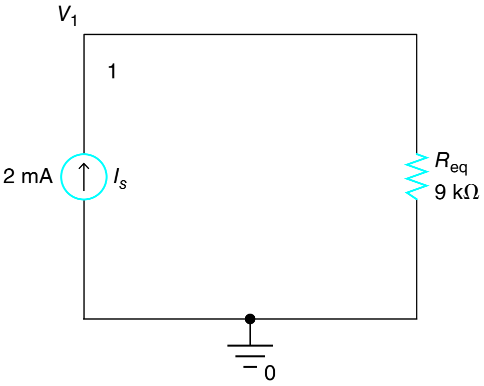
$$R_{eq} = \frac{R_a \times R_b}{R_a + R_b} = \frac{12\ k\Omega \times 36\ k\Omega}{12\ k\Omega + 36\ k\Omega} = 9\ k\Omega$$

The circuit with the current source and the equivalent resistance is shown in Figure 2.47. The voltage across the equivalent resistance, which is also the voltage across the current source, is given by

$$V_1 = I_s \times R_{eq} = 2\text{ mA} \times 9\ k\Omega = 2 \times 10^{-3}\text{A} \times 9000\ \Omega = 18\text{ V}$$

FIGURE 2.47

The current source and the equivalent resistance.



The current to the left side of the current source is given by

$$I_a = \frac{V_1}{R_a} = \frac{18\text{ V}}{12\ k\Omega} = 1.5\text{ mA}$$

The current to the right side of the current source is given by

$$I_5 = \frac{V_1}{R_b} = \frac{18\text{ V}}{36\ k\Omega} = 0.5\text{ mA}$$

The current I_5 can also be obtained from $I_5 = I_s - I_a$. The current through R_4 is

$$I_4 = \frac{V_1}{R_4} = \frac{18\text{ V}}{30\ k\Omega} = 0.6\text{ mA}$$

The current through R_3 is

$$I_3 = I_a - I_4 = 1.5\text{ mA} - 0.6\text{ mA} = 0.9\text{ mA}$$

The voltage across R_3 is

$$V_{R_3} = R_3 \times I_3 = 5\ k\Omega \times 0.9\text{ mA} = 4.5\text{ V}$$

The voltage V_2 is given by

$$V_2 = V_1 - V_{R_3} = 18\text{ V} - 4.5\text{ V} = 13.5\text{ V}$$

The current through R_1 can be found by applying Ohm's law:

$$I_1 = \frac{V_2}{R_1} = \frac{13.5\text{ V}}{20\ k\Omega} = 0.675\text{ mA}$$

Similarly, the current through R_2 is found to be

$$I_2 = \frac{V_2}{R_2} = \frac{13.5\text{ V}}{60\ k\Omega} = 0.225\text{ mA}$$

continued

Example 2.14 continued

The voltage across R_5 is given by

$$V_{R_5} = R_5 \times I_5 = 21\ k\Omega \times 0.5\ \text{mA} = 10.5\ \text{V}$$

The voltage V_3 is

$$V_3 = V_1 - V_{R_5} = 18\ \text{V} - 10.5\ \text{V} = 7.5\ \text{V}.$$

The current through R_6 is given by

$$I_6 = \frac{V_3}{R_6} = \frac{7.5\ \text{V}}{24\ k\Omega} = 0.3125\ \text{mA}$$

The current through R_7 is given by

$$I_7 = \frac{V_3}{R_7} = \frac{7.5\ \text{V}}{40\ k\Omega} = 0.1875\ \text{mA}$$

Notice that

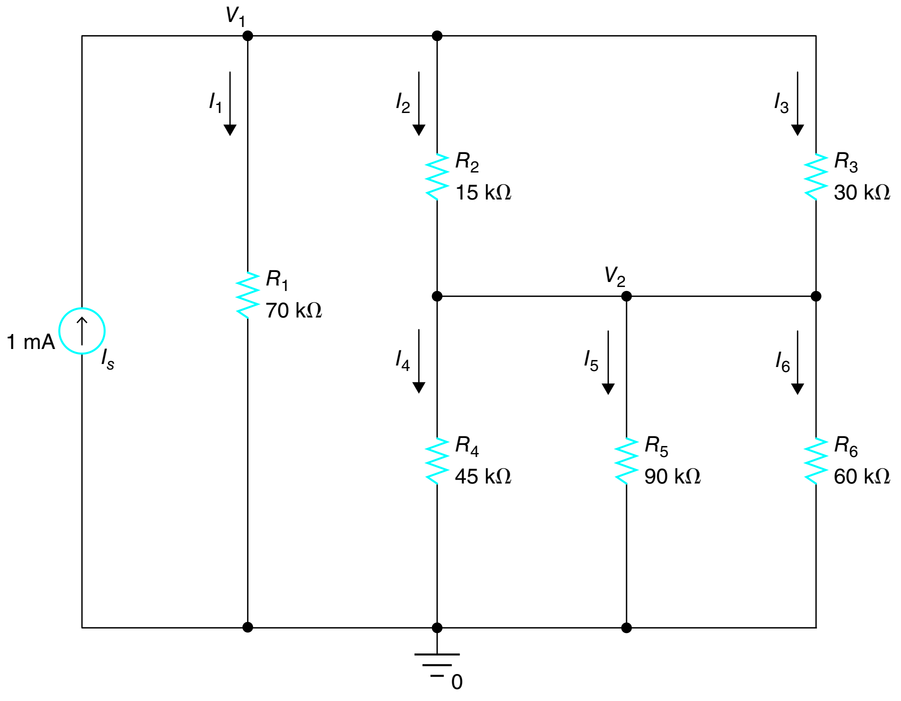
$$I_1 + I_2 + I_4 + I_6 + I_7 = 2\ \text{mA} = I_s$$

Exercise 2.14

Find R_{eq} seen from the current source, V_1, V_2, I_1, I_2, I_3, I_4, I_5, and I_6 in the circuit shown in Figure 2.48.

FIGURE 2.48

Circuit for EXERCISE 2.14.



Answer:

$R_{eq} = 21\ k\Omega$, $V_1 = 21$ V, $V_2 = 14$ V, $I_1 = 0.3$ mA, $I_2 = 0.46667$ mA, $I_3 = 0.23333$ mA, $I_4 = 0.31111$ mA, $I_5 = 0.15556$ mA, $I_6 = 0.23333$ mA

2.8 Voltage Divider Rule

Suppose that two resistors with resistances R_1 and R_2, respectively, are connected in series to a voltage source with voltage V_s volts, as shown in Figure 2.49. The equivalent resistance is $R_1 + R_2$. According to Ohm's law, the current through the mesh is

$$I = \frac{V_s}{R_1 + R_2}$$

Thus, the voltage V_1 across the first resistor with resistance R_1 is

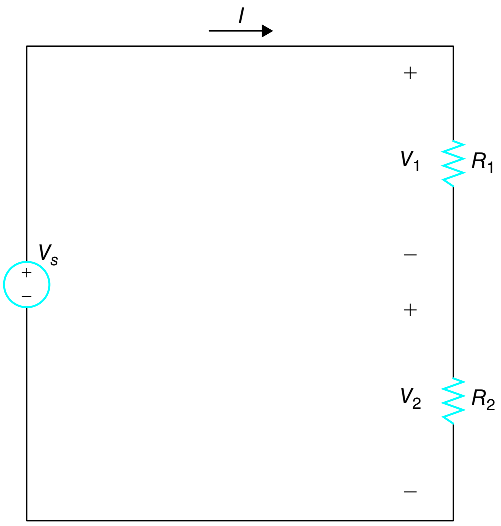
$$V_1 = R_1 I = R_1 \frac{V_s}{R_1 + R_2} = \frac{R_1}{R_1 + R_2} V_s \tag{2.48}$$

and the voltage V_2 across the second resistor with resistance R_2 is

$$V_2 = R_2 I = R_2 \frac{V_s}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} V_s \tag{2.49}$$

FIGURE 2.49

A circuit with two resistors in series.



This result shows that when two resistors are connected in series to a voltage source with voltage V_s volts, the total voltage (V_s volts) is divided between V_1 and V_2; that is,

$$V_s = V_1 + V_2$$

in proportion to resistance values R_1 and R_2. This is referred to as the **voltage divider rule**. If R_1 is greater than R_2, the voltage across R_1 is more than the voltage across R_2. On the other hand, if R_1 is smaller than R_2, the voltage across R_1 is less than the voltage across R_2. In other words, the voltage drop across the resistor with a larger resistance value is more than the voltage drop across the resistor with a smaller resistance value. If $R_1 \gg R_2$, $V_1 \approx V_s$, and $V_2 \approx 0$. If $R_1 \ll R_2$, $V_1 \approx 0$, and $V_2 \approx V_s$.

The voltage divider rule can be generalized to include more than two resistors. If a voltage source with voltage V_s is connected to a series connection of n resistors with resistances $R_1, R_2, \ldots, R_n$, respectively, the voltage across each resistor R_i, $1 \le i \le n$, is given by

$$V_i = \frac{R_i}{R_1 + R_2 + \cdots + R_n} V_s \tag{2.50}$$

The voltage divider rule can be described by using conductance rather than resistance. Suppose that two resistors with resistances R_1 and R_2, respectively, are connected in series to a voltage source with voltage V_s volts. The voltage across R_1 is given by

$$V_1 = \frac{R_1}{R_1 + R_2} V_s = \frac{\frac{1}{G_1}}{\frac{1}{G_1} + \frac{1}{G_2}} V_s = \frac{G_2}{G_1 + G_2} V_s$$

Similarly, the voltage across R_2 is given by

$$V_2 = \frac{R_2}{R_1 + R_2} V_s = \frac{\frac{1}{G_2}}{\frac{1}{G_1} + \frac{1}{G_2}} V_s = \frac{G_1}{G_1 + G_2} V_s$$

Let G be the conductance of a series connection of R_1 and R_2. Then,

$$G = \frac{1}{R_1 + R_2}$$

The current through the resistors is $I = GV_s$. The voltage across the resistor R_1 is

$$V_1 = R_1 I = R_1 G V_s = \frac{G}{G_1} V_s$$

Similarly, the voltage across the resistor R_2 is given by

$$V_2 = R_2 I = R_2 G V_s = \frac{G}{G_2} V_s$$

If a voltage source with voltage V_s is connected to a series connection of n resistors with resistances $R_1, R_2, \ldots, R_n$, respectively, the voltage across each resistor R_i, $1 \le i \le n$, is given by

$$V_i = \frac{R_i}{R_1 + R_2 + \cdots + R_n} V_s = \frac{\dfrac{1}{R_1 + R_2 + \cdots + R_n}}{\dfrac{1}{R_i}} V_s = \frac{G}{G_i} V_s$$

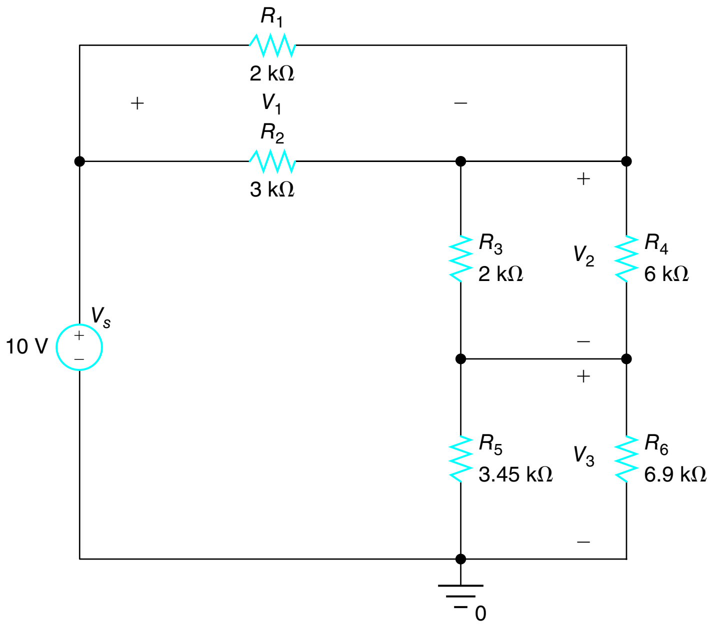
where

$$G = \frac{1}{\dfrac{1}{G_1} + \dfrac{1}{G_2} + \cdots + \dfrac{1}{G_n}} = \frac{1}{R_1 + R_2 + \cdots + R_n}$$

Consider a circuit shown in Figure 2.50. We are interested in finding V_1, V_2, and V_3.

FIGURE 2.50

Circuit to be analyzed by the voltage divider rule.



Let $R_a = R_1 \| R_2$, $R_b = R_3 \| R_4$, $R_c = R_5 \| R_6$. Then, we have

$$R_a = R_1 \| R_2 = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{6}{5}\,k\Omega = 1.2\,k\Omega$$

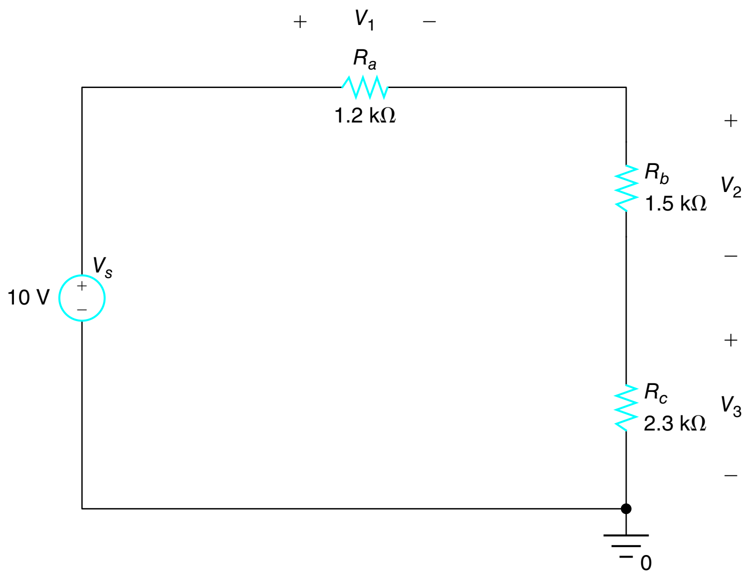
$$R_b = R_3 \| R_4 = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{12}{8}\,k\Omega = 1.5\,k\Omega$$

$$R_c = R_5 \| R_6 = \frac{R_5 \times R_6}{R_5 + R_6} = \frac{23.805}{10.35}\,k\Omega = 2.3\,k\Omega$$

The circuit shown in Figure 2.51 is equivalent to the circuit shown in Figure 2.50.

FIGURE 2.51

A circuit with equivalent resistances.



Resistors R_a, R_b, and R_c are connected in series. The total equivalent resistance seen from the voltage source is given by

$$R_{eq} = R_a + R_b + R_c = 1.2\ k\Omega + 1.5\ k\Omega + 2.3\ k\Omega = 5\ k\Omega$$

Application of the voltage divider rule to the circuit shown in Figure 2.51 yields

$$V_1 = \frac{R_a}{R_a + R_b + R_c} V_s = \frac{1.2\,k\Omega}{5\,k\Omega} \times 10\text{ V} = 2.4\text{ V}$$

$$V_2 = \frac{R_b}{R_a + R_b + R_c} V_s = \frac{1.5\,k\Omega}{5\,k\Omega} \times 10\text{ V} = 3\text{ V}$$

$$V_3 = \frac{R_c}{R_a + R_b + R_c} V_s = \frac{2.3\,k\Omega}{5\,k\Omega} \times 10\text{ V} = 4.6\text{ V}$$

Notice that

$$V_1 + V_2 + V_3 = 2.4\text{ V} + 3\text{ V} + 4.6\text{ V} = 10\text{ V} = V_s$$

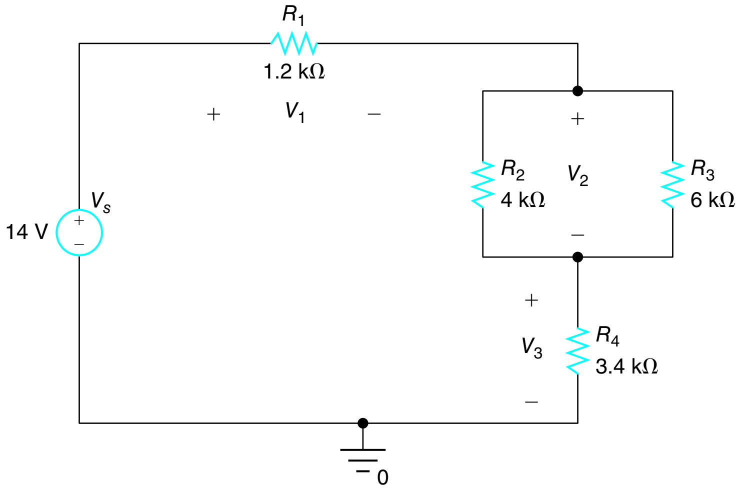
confirming KVL.

EXAMPLE 2.15

Use the voltage divider rule to find the voltages V_1, V_2, and V_3 for the circuit shown in Figure 2.52.

FIGURE 2.52

Circuit for EXAMPLE 2.15.



The equivalent resistance of the parallel connection of R_2 and R_3 is

$$R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{4 \times 6}{4 + 6} k\Omega = 2.4\ k\Omega$$

The total resistance of the circuit seen from the voltage source V_s is

$$R_1 + (R_2 \| R_3) + R_4 = 1.2\ k\Omega + 2.4\ k\Omega + 3.4\ k\Omega = 7\ k\Omega$$

Thus, from the voltage divider rule, the voltages V_1, V_2, and V_3 are given, respectively, by

$$V_1 = \frac{R_1}{R_1 + (R_2 \| R_3) + R_4} V = \frac{1.2\ k\Omega}{7\ k\Omega} \times 14\ \text{V} = 2.4\ \text{V}$$

$$V_2 = \frac{R_2 \| R_3}{R_1 + (R_2 \| R_3) + R_4} V = \frac{2.4\ k\Omega}{7\ k\Omega} \times 14\ \text{V} = 4.8\ \text{V}$$

$$V_3 = \frac{R_4}{R_1 + (R_2 \| R_3) + R_4} V = \frac{3.4\ k\Omega}{7\ k\Omega} \times 14\ \text{V} = 6.8\ \text{V}$$

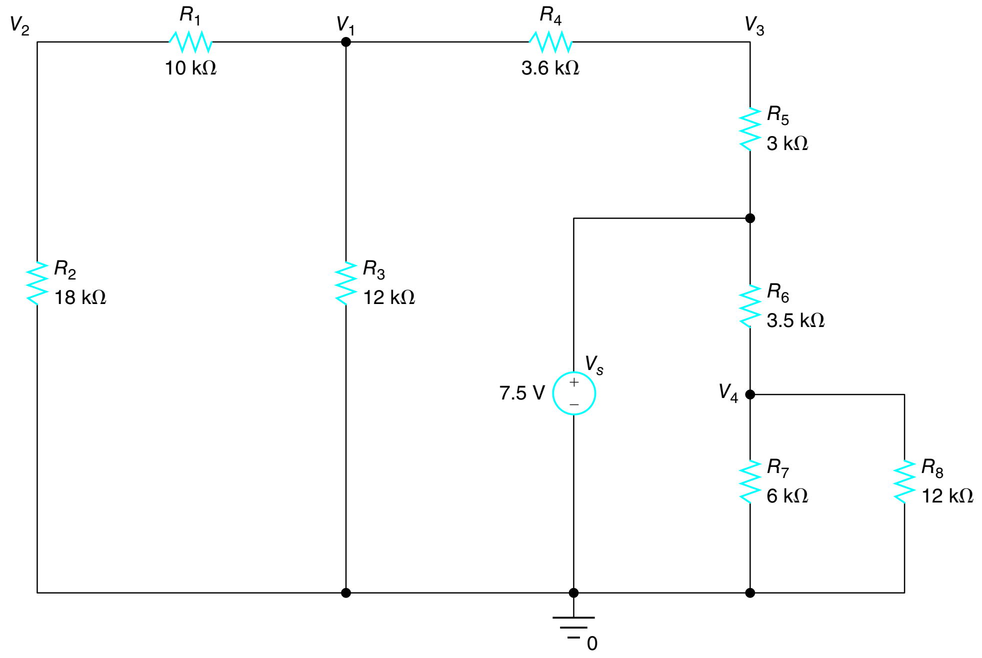
Notice that the sum of V_1, V_2, and V_3 equals the voltage from the voltage source, V_s, confirming KVL.

Exercise 2.15

Find V_1, V_2, V_3, and V_4 in the circuit shown in Figure 2.53.

FIGURE 2.53

Circuit for EXERCISE 2.15.



Answer:

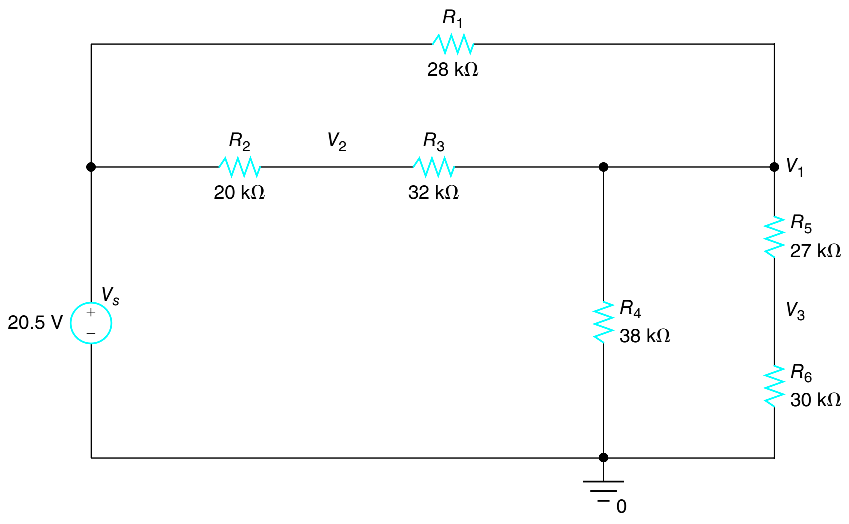
$V_1 = 4.2$ V, $V_2 = 2.7$ V, $V_3 = 6$ V, and $V_4 = 4$ V.

EXAMPLE 2.16

Find V_1, V_2, and V_3 in the circuit shown in Figure 2.54.

FIGURE 2.54

Circuit for EXAMPLE 2.16.



continued

Example 2.16 continued

Let R_a be the equivalent resistance of R_1, R_2, and R_3, and R_b be the equivalent resistance of R_4, R_5, and R_6. Then we have

$$R_a = R_1 \| (R_2 + R_3) = 28\ k\Omega \| 52\ k\Omega = \frac{28\ k\Omega \times 52\ k\Omega}{28\ k\Omega + 52\ k\Omega} = \frac{1456}{80}\ k\Omega = 18.2\ k\Omega$$

$$R_b = R_4 \| (R_5 + R_6) = 38\ k\Omega \| 57\ k\Omega = \frac{38\ k\Omega \times 57\ k\Omega}{38\ k\Omega + 57\ k\Omega} = \frac{2166}{95}\ k\Omega = 22.8\ k\Omega$$

Since R_a and R_b are connected in series, the voltage V_1 across R_b is given as follows (from the voltage divider rule):

$$V_1 = V_s \times \frac{R_b}{R_a + R_b} = 20.5\ \text{V} \times \frac{22.8\ k\Omega}{18.2\ k\Omega + 22.8\ k\Omega} = 20.5\ \text{V} \times \frac{22.8}{41} = 11.4\ \text{V}$$

The voltage across the R_2–R_3 path is $V_s - V_1 = 20.5\ \text{V} - 11.4\ \text{V} = 9.1\ \text{V}$. This voltage is split across R_2 and R_3 in proportion to the resistance values. Thus, we have

$$V_2 = V_1 + (V_s - V_1) \times \frac{R_3}{R_2 + R_3} = 11.4\ \text{V} + (20.5\ \text{V} - 11.4\ \text{V}) \times \frac{32\ k\Omega}{20\ k\Omega + 32\ k\Omega} = 17\ \text{V}$$

The voltage across the R_5-R_6 path is $V_1 = 11.4$ V. This voltage is split across R_5 and R_6 in proportion to the resistance values. Thus, we have

$$V_3 = V_1 \times \frac{R_6}{R_5 + R_6} = 11.4\ \text{V} \times \frac{30\ k\Omega}{27\ k\Omega + 30\ k\Omega} = 6\ \text{V}$$

MATLAB

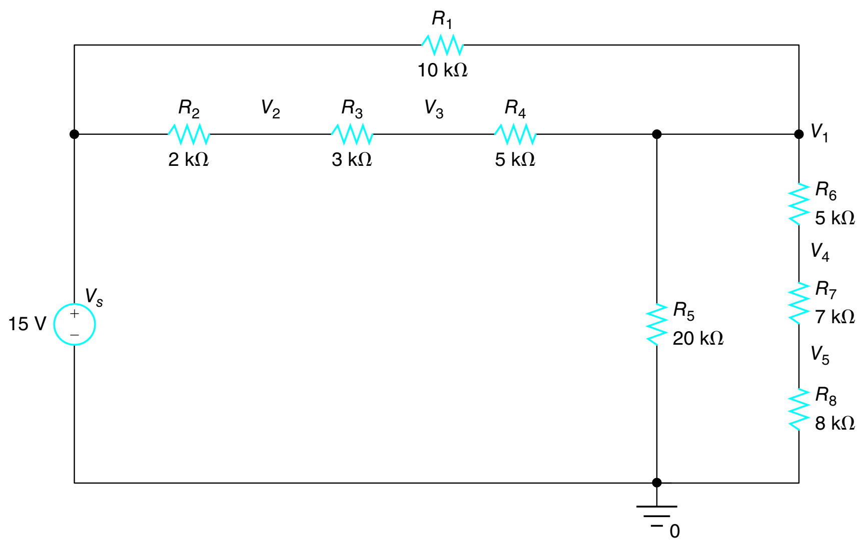
```
%EXAMPLE 2.16
%Function P.m should be in the same folder as this file.
clear all;format long;
Vs=20.5;R1=28000;R2=20000;R3=32000;R4=38000;R5=27000;R6=30000;
Ra=P([R1,R2+R3])
Rb=P([R4,R5+R6])
V1=Vs*Rb/(Ra+Rb)
VR1=Vs-V1
V2=V1+VR1*R3/(R2+R3)
V3=V1*R6/(R5+R6)

Answers:
Ra =
       18200
Rb =
     2.280000000000000e+04
V1 =
  11.399999999999999
VR1 =
   9.100000000000001
V2 =
    17
V3 =
   5.999999999999999
```

Exercise 2.16

Find V_1, V_2, V_3, V_4, and V_5 in the circuit shown in Figure 2.55.

FIGURE 2.55

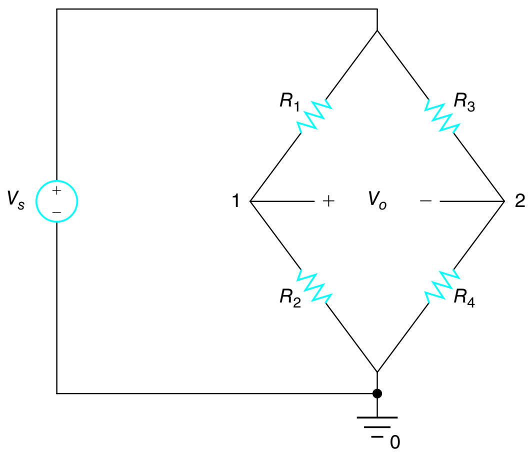
Circuit for EXERCISE 2.16.



Answer:
$V_1 = 10$ V, $V_2 = 14$ V, $V_3 = 12.5$ V, $V_4 = 7.5$ V, and $V_5 = 4$ V.

FIGURE 2.56

A Wheatstone bridge circuit.



2.8.1 WHEATSTONE BRIDGE

A Wheatstone bridge circuit is shown in Figure 2.56.

According to the voltage divider rule, the voltage V_s is divided between R_1 and R_2 in proportion to the resistance values. Thus, the voltage at node 1, which is the voltage across R_2, is given by

$$V_1 = \frac{R_2}{R_1 + R_2} \times V_s$$

Similarly, voltage V_s is divided between R_3 and R_4 in proportion to the resistance values. Thus, the voltage at node 2, which is the voltage across R_4, is given by

$$V_2 = \frac{R_4}{R_3 + R_4} \times V_s$$

The output voltage V_o is the difference between voltage V_1 and voltage V_2. Therefore, we have

$$V_o = V_1 - V_2 = \frac{R_2}{R_1 + R_2} \times V_s - \frac{R_4}{R_3 + R_4} \times V_s = \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) \times V_s \quad (2.51)$$

The bridge is called **balanced** if V_o is zero. If the bridge is balanced, we have

$$\frac{R_2}{R_1 + R_2} = \frac{R_4}{R_3 + R_4}$$

or

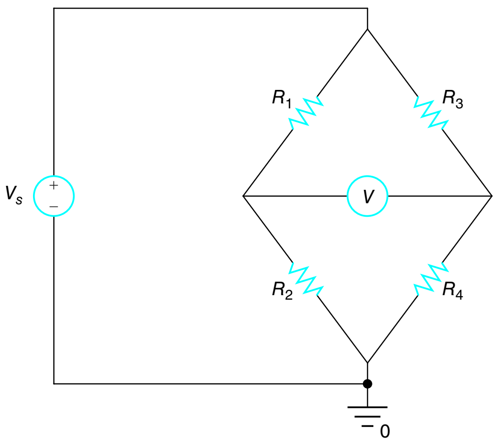
$$R_2R_3 + R_2R_4 = R_1R_4 + R_2R_4$$

or

$$R_1R_4 = R_2R_3 \tag{2.52}$$

FIGURE 2.57

A Wheatstone bridge with a voltmeter.



If three of the four resistance values are known, we can find the fourth resistance value. The Wheatstone bridge can be used to find the value of unknown resistance. A voltmeter is placed at the output, as shown in Figure 2.57. Let R_4 be the unknown resistance to be determined. The resistance values of R_1 and R_2 are fixed, and R_3 is a precision variable resistor. R_3 is varied until the voltmeter reading is zero, which is called the *null condition*. Then, the unknown resistance value is given by

$$R_4 = \frac{R_2R_3}{R_1} \tag{2.53}$$

The Wheatstone bridge can be used to measure small changes in resistance due to changes in the physical condition of sensors, such as temperature sensors, light sensors, and strain gauges. In this application, resistor R_4 is replaced by a sensor and R_4 represents the resistance value under certain conditions. For example, R_4 may be the resistance when no force is applied to a strain gauge, or R_4 may be the resistance when no light is applied to a light sensor. R_3 is adjusted to create balanced condition with R_4. When the physical condition changes, R_4 changes by a small amount. Let the amount of change be $\pm\,\Delta R$. Then, R_4 becomes $R_4 \pm \Delta R$, and the bridge is no longer balanced and V_o is no longer zero. The output voltage given by Equation (2.51) becomes

$$\begin{aligned}
V_o = V_1 - V_2 &= \left(\frac{R_2}{R_1 + R_2} - \frac{R_4 \pm \Delta R}{R_3 + R_4 \pm \Delta R}\right) \times V_s \\
&= \left[\frac{R_2}{R_1 + R_2} - \frac{R_4\left(1 \pm \dfrac{\Delta R}{R_4}\right)}{(R_3 + R_4)\left(1 \pm \dfrac{\Delta R}{R_3 + R_4}\right)}\right] \times V_s \\
&= \frac{R_2}{R_1 + R_2}\left(1 - \frac{1 \pm \dfrac{\Delta R}{R_4}}{1 \pm \dfrac{\Delta R}{R_3 + R_4}}\right) \times V_s = \frac{R_2}{R_1 + R_2}\left(\frac{1 \pm \dfrac{\Delta R}{R_3 + R_4} - 1 \mp \dfrac{\Delta R}{R_4}}{1 \pm \dfrac{\Delta R}{R_3 + R_4}}\right) \times V_s \\
&= \frac{R_2}{R_1 + R_2}\left(\frac{\pm\Delta R\left(\dfrac{1}{R_3 + R_4} - \dfrac{1}{R_4}\right)}{1 \pm \dfrac{\Delta R}{R_3 + R_4}}\right) \times V_s \\
&= \frac{R_2}{R_1 + R_2} \times \frac{R_3}{(R_3 + R_4)R_4}\left(\frac{\mp\Delta R}{1 \pm \dfrac{\Delta R}{R_3 + R_4}}\right) \times V_s
\end{aligned} \tag{2.54}$$

Assume $\Delta R \ll (R_3 + R_4)$. Then, $\Delta R/(R_3 + R_4) \approx 0$ and Equation (2.54) can be approximated by

$$V_o = \frac{R_2}{R_1 + R_2} \times \frac{R_3}{(R_3 + R_4)R_4} \times (\mp\,\Delta R)V_s \tag{2.55}$$

If $R_1 = R_2 = R_3 = R_4 = R$, Equation (2.55) becomes

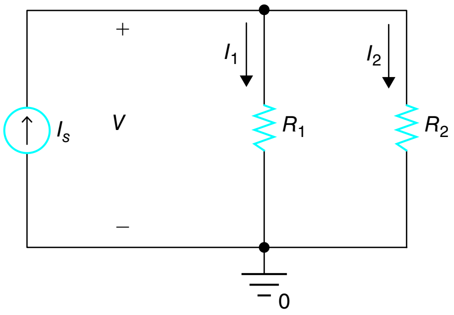
$$V_o = \mp\frac{\Delta R}{4R}V_s \tag{2.56}$$

The output voltage of the bridge is proportional to the change of the resistance ΔR with sign inversion. Notice that $-V_o = V_2 - V_1 = \pm\Delta R \times V_s/(4R)$. This signal can be amplified by an instrumentation amplifier, as discussed in Chapter 5.

2.9 Current Divider Rule

FIGURE 2.58

A circuit with two resistors in parallel.



Suppose that a current source with I_s amperes of current is connected in parallel to a pair of resistors with resistances R_1 and R_2, respectively, as shown in Figure 2.58.

The equivalent resistance value is

$$R = R_1 \| R_2 = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{R_1R_2}{R_1 + R_2}$$

Thus, the voltage across the resistors is, from Ohm's law,

$$V = RI_s = \frac{I_s}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{R_1R_2}{R_1 + R_2}I_s$$

The current I_1 through R_1 is

$$I_1 = \frac{V}{R_1} = \frac{\dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}}{R_1}I_s = \frac{\dfrac{1}{R_1}}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}I_s = \frac{R_2}{R_1 + R_2}I_s \tag{2.57}$$

and the current through R_2 is

$$I_2 = \frac{V}{R_2} = \frac{\dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}}{R_2}I_s = \frac{\dfrac{1}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}I_s = \frac{R_1}{R_1 + R_2}I_s \tag{2.58}$$

From KCL, the sum of I_1 and I_2 is I_s; that is,

$$I_s = I_1 + I_2$$

This result suggests that the current through R_1 is given by I_s times the ratio of R_2 to $R_1 + R_2$, and the current through R_2 is given by I_s times the ratio of R_1 to $R_1 + R_2$. If R_1 is greater than R_2, the current through R_1 is smaller than the current through R_2. On the other hand, if R_1 is smaller than R_2, the current through R_1 is more than the current through R_2. In other words, more current flows through the resistor with smaller resistance value than through the resistor with larger resistance value. This is referred to as the **current divider rule**. If $R_1 \gg R_2$, $I_1 \approx 0$ and $I_2 \approx I_s$. On the other hand, if $R_1 \ll R_2$, $I_1 \approx I_s$ and $I_2 \approx 0$. If $R_1 = 0$ and $R_2 \neq 0$, $I_1 = I_s$ and $I_2 = 0$. If $R_1 \neq 0$ and $R_2 = 0$, $I_1 = 0$ and $I_2 = I_s$.

The current divider rule can be generalized to include more than two resistors. If a current source with current I_s is connected to a parallel connection of n resistors with resistances $R_1, R_2, \ldots, R_n$, respectively, then the voltage across the resistors is given by

$$V = R_{eq}I_s = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} I_s$$

The current through resistor with resistance R_i, $i = 1, 2, \ldots, n$, is given by

$$I_i = \frac{V}{R_i} = \frac{\frac{1}{R_i}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} I_s \tag{2.59}$$

The current divider rule can be described by using conductance rather than resistance. Suppose that two resistors with resistances R_1 and R_2, respectively, are connected in parallel to a current source with current I_s amperes. The current through R_1 is given by

$$I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} I_s = \frac{G_1}{G_1 + G_2} I_s$$

Similarly, the current through R_2 is given by

$$I_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} I_s = \frac{G_2}{G_1 + G_2} I_s$$

Let G_{eq} be the equivalent conductance of the parallel connection of R_1 and R_2. Then,

$$G_{eq} = G_1 + G_2$$

The voltage across the resistors is $V = I_s/G_{eq}$. The current through the resistor R_1 is

$$I_1 = \frac{V}{R_1} = \frac{I_s}{G_{eq}R_1} = \frac{G_1}{G_{eq}} I_s$$

Similarly, the current through the resistor R_2 is given by

$$I_2 = \frac{V}{R_2} = \frac{I_s}{G_{eq}R_2} = \frac{G_2}{G_{eq}} I_s$$

If a current source with current I_s is connected to a parallel connection of n resistors with resistances $R_1, R_2, \ldots, R_n$, respectively, the current through each resistor R_i, $1 \le i \le n$, is given by

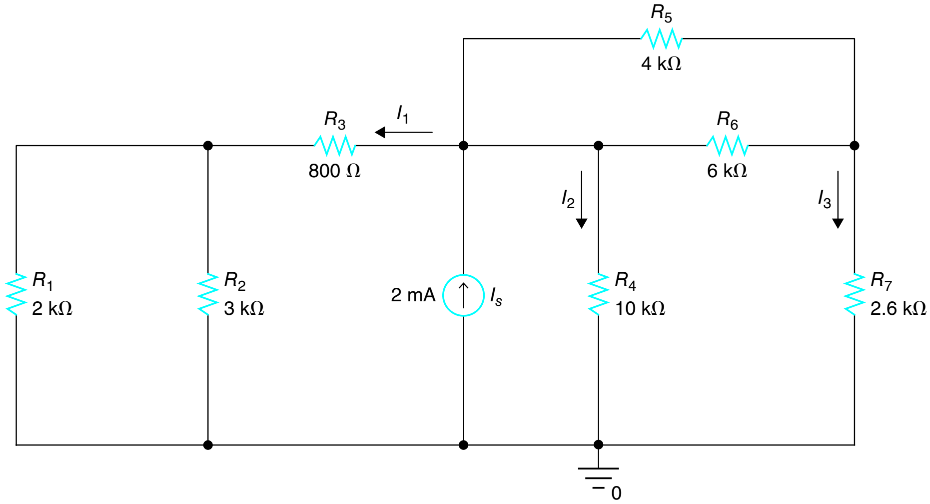
$$I_i = \frac{\frac{1}{R_i}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} I_s = \frac{G_i}{G_1 + G_2 + \cdots + G_n} I_s$$

where $G_i = 1/R_i$.

Consider a circuit shown in Figure 2.59. We are interested in finding I_1, I_2, and I_3.

FIGURE 2.59

A circuit to be analyzed by the current divider rule.



Let $R_a = R_3 + (R_1 \| R_2)$, $R_b = R_7 + (R_5 \| R_6)$. Then, we have

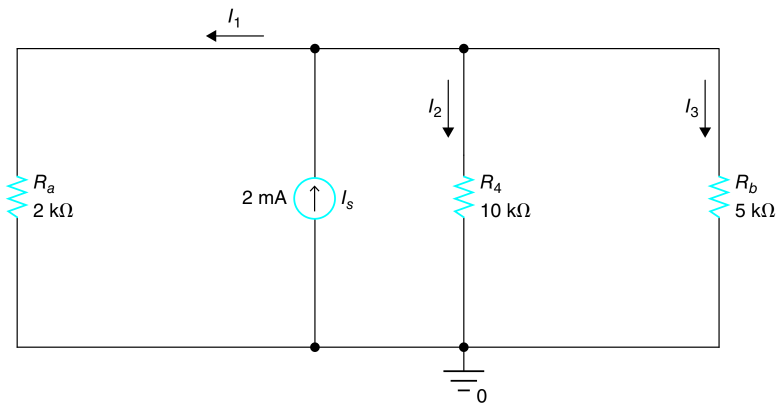
$$R_a = R_3 + (R_1 \| R_2) = R_3 + \frac{R_1 \times R_2}{R_1 + R_2} = 0.8\, k\Omega + \frac{6}{5}\, k\Omega = 2\, k\Omega$$

$$R_b = R_7 + (R_5 \| R_6) = R_7 + \frac{R_5 \times R_6}{R_5 + R_6} = 2.6\, k\Omega + \frac{24}{10}\, k\Omega = 5\, k\Omega$$

The circuit shown in Figure 2.60 is equivalent to the circuit shown in Figure 2.59.

FIGURE 2.60

A circuit with equivalent resistances.



Notice that R_a, R_b, and R_4 are connected in parallel. Application of the current divider rule to the circuit shown in Figure 2.60 yields

$$I_1 = \frac{\frac{1}{R_a}}{\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_4}} I_s = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{5} + \frac{1}{10}} \times 2 \text{ mA} = \frac{5}{5 + 2 + 1}$$

$$\times 2 \text{ mA} = \frac{5}{8} \times 2 \text{ mA} = 1.25 \text{ mA}$$

$$I_2 = \frac{\frac{1}{R_4}}{\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_4}} I_s = \frac{\frac{1}{10}}{\frac{1}{2} + \frac{1}{5} + \frac{1}{10}} \times 2 \text{ mA} = \frac{1}{5 + 2 + 1} \times 2 \text{ mA}$$

$$= \frac{1}{8} \times 2 \text{ mA} = 0.25 \text{ mA}$$

$$I_3 = \frac{\frac{1}{R_b}}{\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_4}} I_s = \frac{\frac{1}{5}}{\frac{1}{2} + \frac{1}{5} + \frac{1}{10}} \times 2 \text{ mA} = \frac{2}{5 + 2 + 1} \times 2 \text{ mA}$$

$$= \frac{2}{8} \times 2 \text{ mA} = 0.5 \text{ mA}$$

Notice that

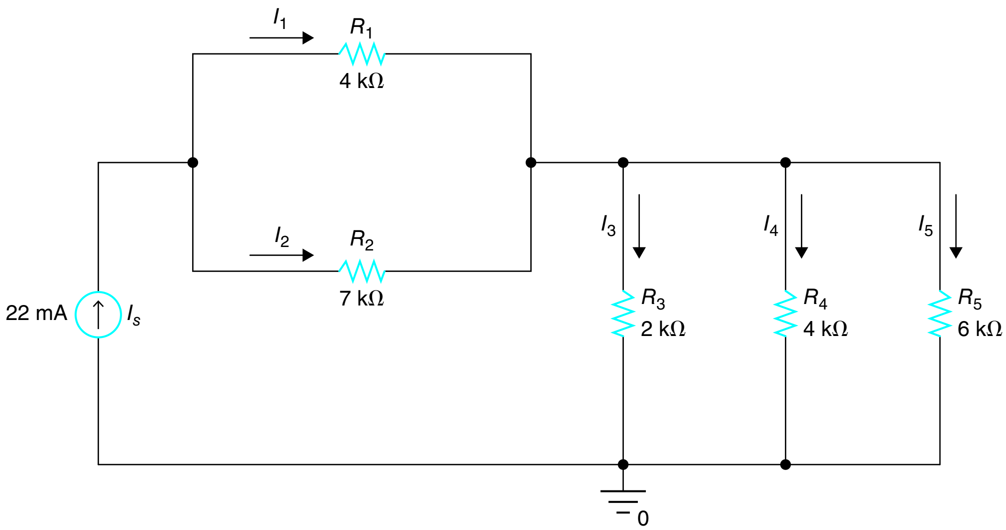
$$I_1 + I_2 + I_3 = 1.25 \text{ mA} + 0.25 \text{ mA} + 0.5 \text{ mA} = 2 \text{ mA} = I_s$$

EXAMPLE 2.17

In the circuit shown in Figure 2.61, use the current divider rule to find the currents I_1, I_2, I_3, I_4, and I_5.

FIGURE 2.61

Circuit for EXAMPLE 2.17



The current from the current source I_s is split between I_1 and I_2. From the current divider rule, we have

$$I_1 = \frac{R_2}{R_1 + R_2} I_s = \frac{7}{4 + 7} \times 22 \text{ mA} = 14 \text{ mA}$$

$$I_2 = \frac{R_1}{R_1 + R_2} I_s = \frac{4}{4 + 7} \times 22 \text{ mA} = 8 \text{ mA}$$

continued

Example 2.17 continued

Notice that once I_1 is found, I_2 can be obtained using KCL. The current from the current source I_s is split between I_3, I_4, and I_5. From the current divider rule, we have

$$I_3 = \frac{\dfrac{1}{R_3}}{\dfrac{1}{R_3}+\dfrac{1}{R_4}+\dfrac{1}{R_5}} I_s = \frac{\dfrac{1}{2000}}{\dfrac{1}{2000}+\dfrac{1}{4000}+\dfrac{1}{6000}} \times 22 \text{ mA}$$

$$= \frac{\dfrac{12{,}000}{2000}}{\dfrac{12{,}000}{2000}+\dfrac{12{,}000}{4000}+\dfrac{12{,}000}{6000}} \times 22 \text{ mA} = \frac{6}{6+3+2} \times 22 \text{ mA} = 12 \text{ mA}$$

$$I_4 = \frac{\dfrac{1}{R_4}}{\dfrac{1}{R_3}+\dfrac{1}{R_4}+\dfrac{1}{R_5}} I_s = \frac{\dfrac{1}{4000}}{\dfrac{1}{2000}+\dfrac{1}{4000}+\dfrac{1}{6000}}$$

$$\times 22 \text{ mA} = \frac{3}{6+3+2} \times 22 \text{ mA} = 6 \text{ mA}$$

$$I_5 = \frac{\dfrac{1}{R_5}}{\dfrac{1}{R_3}+\dfrac{1}{R_4}+\dfrac{1}{R_5}} I_s = \frac{\dfrac{1}{6000}}{\dfrac{1}{2000}+\dfrac{1}{4000}+\dfrac{1}{6000}}$$

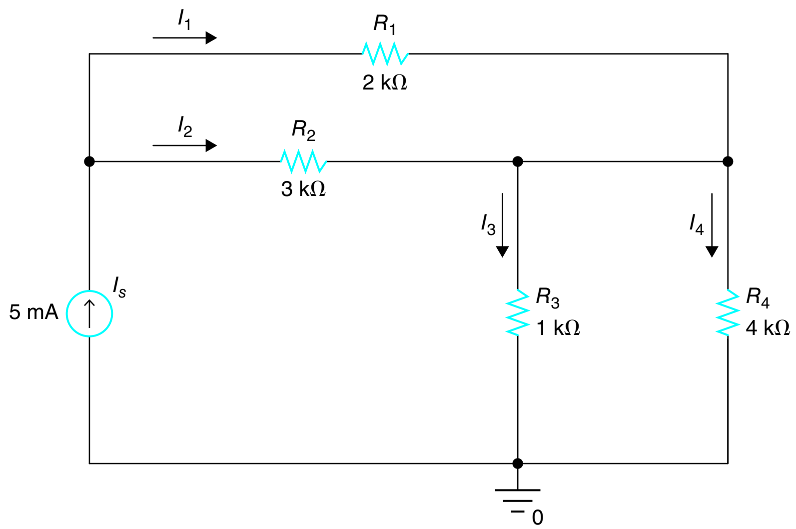
$$\times 22 \text{ mA} = \frac{2}{6+3+2} \times 22 \text{ mA} = 4 \text{ mA}$$

Exercise 2.17

Find I_1, I_2, I_3, and I_4 in the circuit shown in Figure 2.62.

FIGURE 2.62

Circuit for EXERCISE 2.17



Answer:

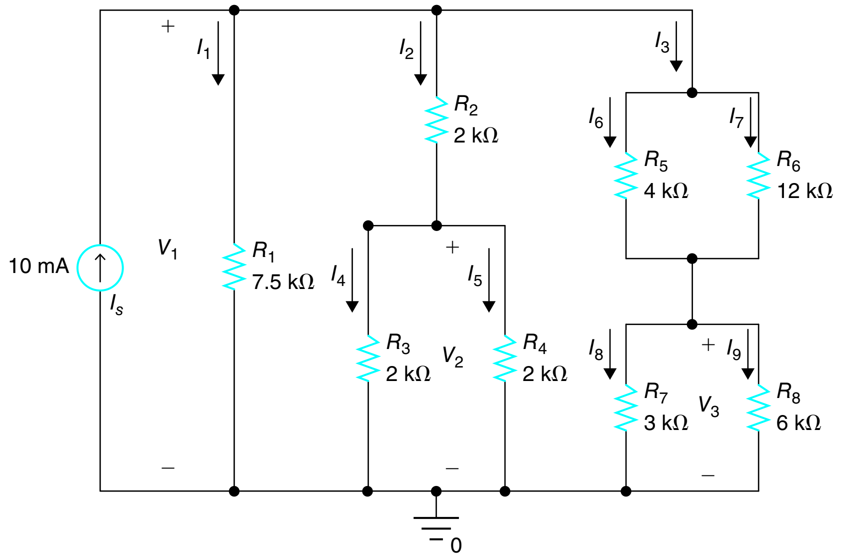
$I_1 = 3$ mA, $I_2 = 2$ mA, $I_3 = 4$ mA, and $I_4 = 1$ mA.

EXAMPLE 2.18

Find I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, V_1, V_2, and V_3 in the circuit shown in Figure 2.63.

FIGURE 2.63

Circuit for EXAMPLE 2.18.



Let R_a be the equivalent resistance of R_2, R_3, and R_4. Then, we have

$$R_a = R_2 + (R_3 \| R_4) = R_2 + \frac{R_3 \times R_4}{R_3 + R_4} = 2\,k\Omega + \frac{2\,k\Omega \times 2\,k\Omega}{2\,k\Omega + 2\,k\Omega}$$
$$= 2\,k\Omega + 1\,k\Omega = 3\,k\Omega$$

Let R_b be the equivalent resistance of R_5, R_6, R_7, and R_8. Then, we have

$$R_b = (R_5 \| R_6) + (R_7 \| R_8) = \frac{R_5 \times R_6}{R_5 + R_6} + \frac{R_7 \times R_8}{R_7 + R_8} = \frac{4 \times 12}{4 + 12}\,k\Omega + \frac{3 \times 6}{3 + 6}\,k\Omega$$
$$= 3\,k\Omega + 2\,k\Omega = 5\,k\Omega$$

Application of the current divider rule results in

$$I_1 = I_s \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_a} + \frac{1}{R_b}} = 10\text{ mA} \times \frac{\frac{1}{7500}}{\frac{1}{7500} + \frac{1}{3000} + \frac{1}{5000}}$$
$$= 10\text{ mA} \times \frac{\frac{15{,}000}{7500}}{\frac{15{,}000}{7500} + \frac{15{,}000}{3000} + \frac{15{,}000}{5000}} = 10\text{ mA} \times \frac{2}{2 + 5 + 3} = 2\text{ mA}$$

continued

Example 2.18 continued

$$I_2 = I_s \frac{\frac{1}{R_a}}{\frac{1}{R_1} + \frac{1}{R_a} + \frac{1}{R_b}} = 10 \text{ mA} \times \frac{\frac{1}{3000}}{\frac{1}{7500} + \frac{1}{3000} + \frac{1}{5000}}$$

$$= 10 \text{ mA} \times \frac{\frac{15{,}000}{3000}}{\frac{15{,}000}{7500} + \frac{15{,}000}{3000} + \frac{15{,}000}{5000}} = 10 \text{ mA} \times \frac{5}{2 + 5 + 3} = 5 \text{ mA}$$

$$I_3 = I_s \frac{\frac{1}{R_b}}{\frac{1}{R_1} + \frac{1}{R_a} + \frac{1}{R_b}} = 10 \text{ mA} \times \frac{\frac{1}{5000}}{\frac{1}{7500} + \frac{1}{3000} + \frac{1}{5000}}$$

$$= 10 \text{ mA} \times \frac{\frac{15{,}000}{5000}}{\frac{15{,}000}{7500} + \frac{15{,}000}{3000} + \frac{15{,}000}{5000}} = 10 \text{ mA} \times \frac{3}{2 + 5 + 3} = 3 \text{ mA}$$

Similarly, application of the current divider rule results in

$$I_4 = I_2 \frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_4}} = 5 \text{ mA} \times \frac{\frac{1}{2000}}{\frac{1}{2000} + \frac{1}{2000}} = 5 \text{ mA} \times \frac{\frac{2000}{2000}}{\frac{2000}{2000} + \frac{2000}{2000}}$$

$$= 5 \text{ mA} \times \frac{1}{2} = 2.5 \text{ mA}$$

$$I_5 = I_2 \frac{\frac{1}{R_4}}{\frac{1}{R_3} + \frac{1}{R_4}} = 5 \text{ mA} \times \frac{\frac{1}{2000}}{\frac{1}{2000} + \frac{1}{2000}} = 5 \text{ mA} \times \frac{\frac{2000}{2000}}{\frac{2000}{2000} + \frac{2000}{2000}}$$

$$= 5 \text{ mA} \times \frac{1}{2} = 2.5 \text{ mA}$$

$$I_6 = I_3 \frac{\frac{1}{R_5}}{\frac{1}{R_5} + \frac{1}{R_6}} = 3 \text{ mA} \times \frac{\frac{1}{4000}}{\frac{1}{4000} + \frac{1}{12{,}000}} = 3 \text{ mA} \times \frac{\frac{12{,}000}{4000}}{\frac{12{,}000}{4000} + \frac{12{,}000}{12{,}000}}$$

$$= 3 \text{ mA} \times \frac{3}{4} = 2.25 \text{ mA}$$

$$I_7 = I_3 \frac{\frac{1}{R_6}}{\frac{1}{R_5} + \frac{1}{R_6}} = 3 \text{ mA} \times \frac{\frac{1}{12{,}000}}{\frac{1}{4000} + \frac{1}{12{,}000}} = 3 \text{ mA} \times \frac{\frac{12{,}000}{12{,}000}}{\frac{12{,}000}{4000} + \frac{12{,}000}{12{,}000}}$$

$$= 3 \text{ mA} \times \frac{1}{4} = 0.75 \text{ mA}$$

continued

Example 2.18 continued

$$I_8 = I_3\frac{\dfrac{1}{R_7}}{\dfrac{1}{R_7}+\dfrac{1}{R_8}} = 3\text{ mA}\times\frac{\dfrac{1}{3000}}{\dfrac{1}{3000}+\dfrac{1}{6000}} = 3\text{ mA}\times\frac{\dfrac{6000}{3000}}{\dfrac{6000}{3000}+\dfrac{6000}{6000}}$$

$$= 3\text{ mA}\times\frac{2}{3} = 2\text{ mA}$$

$$I_9 = I_3\frac{\dfrac{1}{R_8}}{\dfrac{1}{R_7}+\dfrac{1}{R_8}} = 3\text{ mA}\times\frac{\dfrac{1}{6000}}{\dfrac{1}{3000}+\dfrac{1}{6000}} = 3\text{ mA}\times\frac{\dfrac{6000}{6000}}{\dfrac{6000}{3000}+\dfrac{6000}{6000}}$$

$$= 3\text{ mA}\times\frac{1}{3} = 1\text{ mA}$$

Notice that

$$I_1 + I_4 + I_5 + I_8 + I_9 = 2\text{ mA} + 2.5\text{ mA} + 2.5\text{ mA} + 2\text{ mA} + 1\text{ mA} = 10\text{ mA} = I_s$$

The voltages are given by

$$V_1 = R_1I_1 = 15\text{ V}$$

$$V_2 = R_3I_4 = 5\text{ V}$$

$$V_3 = R_7I_8 = 6\text{ V}$$

MATLAB

```
%EXAMPLE 2.18
%Function P.m should be in the same folder as this file.
clear all;format long;
Is=10e-3;
R1=7500;R2=2000;R3=2000;R4=2000;R5=4000;R6=12000;R7=3000;R8=6000;
Ra=R2+P([R3,R4])
Rb=P([R5,R6])+P([R7,R8])
I1=Is*(1/R1)/(1/R1+1/Ra+1/Rb)
I2=Is*(1/Ra)/(1/R1+1/Ra+1/Rb)
I3=Is*(1/Rb)/(1/R1+1/Ra+1/Rb)
I4=I2*R4/(R3+R4)
I5=I2*R3/(R3+R4)
I6=I3*R6/(R5+R6)
I7=I3*R5/(R5+R6)
I8=I3*R8/(R7+R8)
I9=I3*R7/(R7+R8)
V1=R1*I1
V2=R3*I4
V3=R7*I8

Answers:
Ra =
        3000
```

continued

Example 2.18 continued
MATLAB continued

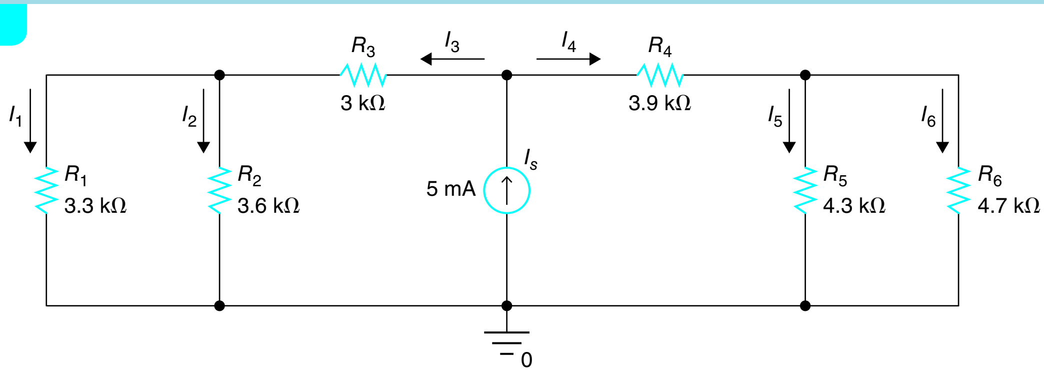
```
Rb =
        5000
I1 =
   0.002000000000000
I2 =
   0.005000000000000
I3 =
   0.003000000000000
I4 =
   0.002500000000000
I5 =
   0.002500000000000
I6 =
   0.002250000000000
I7 =
     7.500000000000001e-04
I8 =
   0.002000000000000
I9 =
     1.000000000000000e-03
V1 =
  15
V2 =
     5
V3 =
   6.000000000000002
```

Exercise 2.18

Find I_1, I_2, I_3, and I_4 in the circuit shown in Figure 2.64.

FIGURE 2.64

Circuit for EXERCISE 2.18.



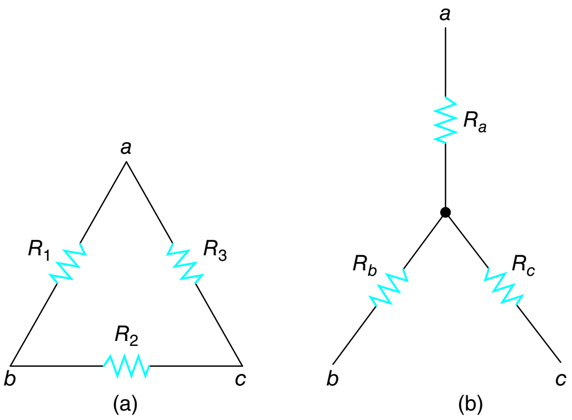
Answer:

$I_1 = 1.4752$ mA, $I_2 = 1.3523$ mA, $I_3 = 2.8275$ mA, $I_4 = 2.1725$ mA, $I_5 = 1.1345$ mA, and $I_6 = 1.0380$ mA.

2.10 Delta-Wye (Δ-Y) Transformation and Wye-Delta (Y-Δ) Transformation

FIGURE 2.65

(a) delta configuration. (b) wye configuration.



The three resistors R_1, R_2, and R_3 in Figure 2.65(a) are configured in triangular shape. This configuration is called *delta* (Δ). If a delta configuration appears in a circuit, it may be difficult to reduce the circuit directly using series or parallel connection of resistors. It may be easier to work with the wye (Y) configuration shown in Figure 2.65(b), where the three resistors R_a, R_b, and R_c are arranged in a Y-shape. In other cases, the delta form may be easier to work with than the Y form.

The transformations from delta to wye and wye to delta can be found by equating the resistances seen from the two terminals of wye and delta. Let us find the resistance for terminals a and b, R_{ab}, with terminal c open. From the wye circuit, we have $R_{ab} = R_a + R_b$. From the delta circuit, we have $R_{ab} = R_1 \| (R_2 + R_3)$. These two are equal. Thus, we have

$$R_{ab} = R_a + R_b = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \tag{2.60}$$

Similarly, R_{bc} and R_{ca} are given, respectively, by

$$R_{bc} = R_b + R_c = \frac{R_2(R_1 + R_3)}{R_2 + R_1 + R_3} \tag{2.61}$$

$$R_{ca} = R_c + R_a = \frac{R_3(R_1 + R_2)}{R_3 + R_1 + R_2} \tag{2.62}$$

When Equation (2.61) is subtracted from the sum of Equations (2.60) and (2.62), we have

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} \tag{2.63}$$

When Equation (2.62) is subtracted from the sum of Equations (2.60) and (2.61), we have

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} \tag{2.64}$$

When Equation (2.60) is subtracted from the sum of Equations (2.61) and (2.62), we have

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} \tag{2.65}$$

Equations (2.63)–(2.65) provide resistance values for a wye configuration from a delta configuration.

The sum of the products of Equations (2.63) and (2.64), (2.64) and (2.65), and (2.63) and (2.65) [(2.63) × (2.64) + (2.64) × (2.65) + (2.63) × (2.65)] results in

$$R_aR_b + R_bR_c + R_aR_c = \frac{R_1R_2R_3}{R_1 + R_2 + R_3} \tag{2.66}$$

When Equation (2.66) is divided by Equation (2.65), we get

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_c} \tag{2.67}$$

When Equation (2.66) is divided by Equation (2.63), we get

$$R_2 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_a} \tag{2.68}$$

When Equation (2.66) is divided by Equation (2.64), we get

$$R_3 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b} \tag{2.69}$$

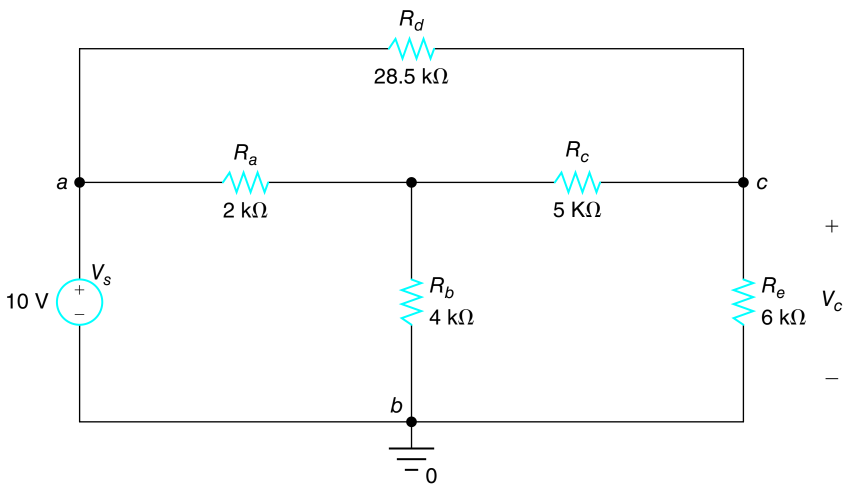
Equations (2.67)–(2.69) provide resistance values for a delta configuration from a wye configuration.

EXAMPLE 2.19

Find the voltage V_c in the circuit shown in Figure 2.66.

FIGURE 2.66

Circuit for EXAMPLE 2.19.



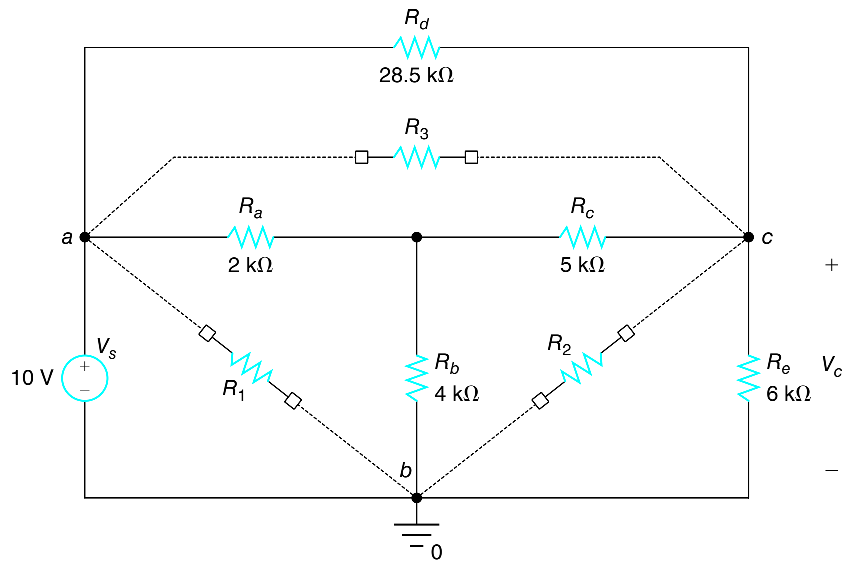
The three resistors R_a, R_b, and R_c form a wye configuration. The wye configuration can be transformed to a delta configuration, as shown in Figure 2.67.

continued

Example 2.19 continued

FIGURE 2.67

Wye to delta transformation.



The resistor values for a delta configuration are given by

$$R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_c}$$
$$= \frac{2\ k\Omega \times 4\ k\Omega + 4\ k\Omega \times 5\ k\Omega + 2\ k\Omega \times 5\ k\Omega}{5\ k\Omega} = 7.6\ k\Omega \qquad \textbf{(2.70)}$$

$$R_2 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_a}$$
$$= \frac{2\ k\Omega \times 4\ k\Omega + 4\ k\Omega \times 5\ k\Omega + 2\ k\Omega \times 5\ k\Omega}{2\ k\Omega} = 19\ k\Omega \qquad \textbf{(2.71)}$$

$$R_3 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b}$$
$$= \frac{2\ k\Omega \times 4\ k\Omega + 4\ k\Omega \times 5\ k\Omega + 2\ k\Omega \times 5\ k\Omega}{4\ k\Omega} = 9.5\ k\Omega \qquad \textbf{(2.72)}$$

After the conversion to the delta configuration, R_d and R_3 are in parallel, and R_e and R_2 are in parallel. Let $R_f = R_d \| R_3$ and $R_g = R_e \| R_2$. Then, we have

$$R_f = R_d \| R_3 = \frac{R_d \times R_3}{R_d + R_3} = \frac{28.5\ k\Omega \times 9.5\ k\Omega}{28.5\ k\Omega + 9.5\ k\Omega} = 7.125\ k\Omega$$

$$R_g = R_e \| R_2 = \frac{R_e \times R_2}{R_e + R_2} = \frac{6\ k\Omega \times 19\ k\Omega}{6\ k\Omega + 19\ k\Omega} = 4.56\ k\Omega$$

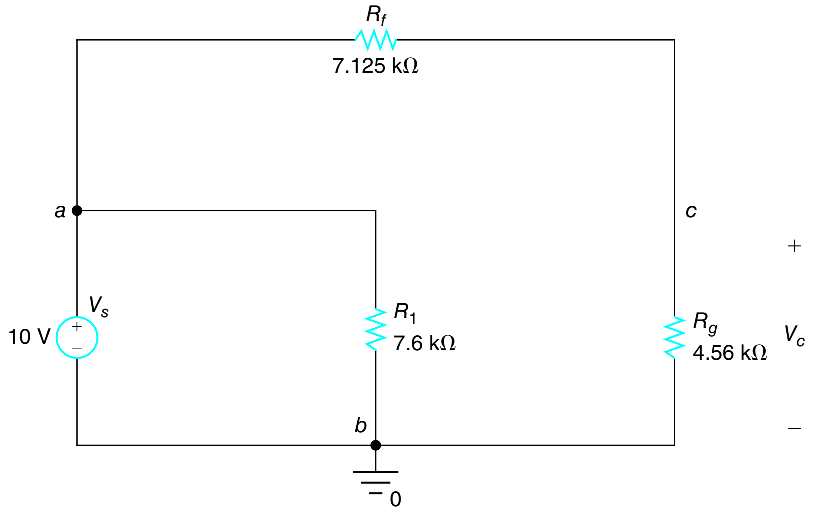
continued

Example 2.19 continued

The circuit reduces to the one shown in Figure 2.68.

FIGURE 2.68

The circuit after simplification.



The voltage V_c is obtained by applying the voltage divider rule:

$$V_c = V_s \times \frac{R_g}{R_f + R_g} = 10\text{ V} \times \frac{4.56\ k\Omega}{7.125\ k\Omega + 4.56\ k\Omega} = 3.9024\text{ V}$$

A MATLAB function Y2D changes a wye configuration to a delta configuration:

```
% Wye to delta conversion
function [R1 R2 R3]=Y2D(x)
R1=(x(1)*x(2)+x(2)*x(3)+x(1)*x(3))/x(3);
R2=(x(1)*x(2)+x(2)*x(3)+x(1)*x(3))/x(1);
R3=(x(1)*x(2)+x(2)*x(3)+x(1)*x(3))/x(2);
end
```

The transformation of a wye configuration with $R_a = 2\ k\Omega$, $R_b = 4\ k\Omega$, and $R_c = 5\ k\Omega$ (shown in Figure 2.66) to a delta configuration with $R_1 = 7.6\ k\Omega$, $R_2 = 19\ k\Omega$, and $R_3 = 9.5\ k\Omega$ can be achieved using Y2D:

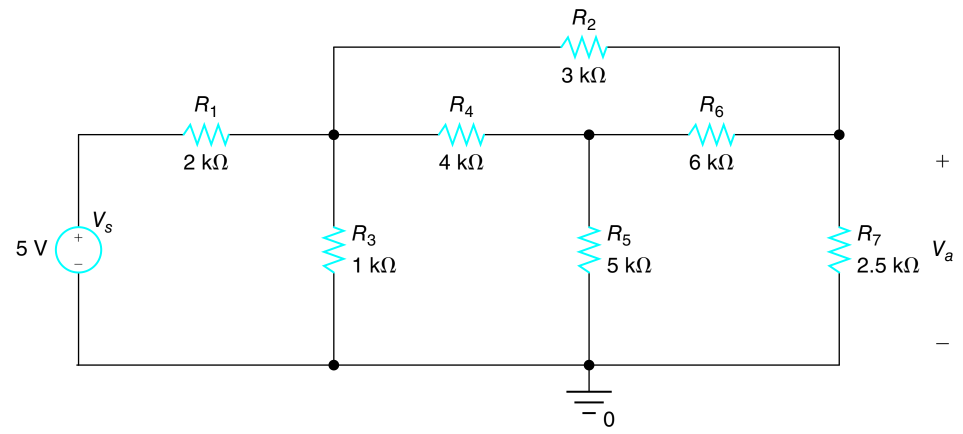
```
>> [R1 R2 R3]=Y2D([2000,4000,5000])
R1 =
   7600
R2 =
  19000
R3 =
   9500
```

Exercise 2.19

Find V_a in the circuit shown in Figure 2.69.

FIGURE 2.69

Circuit for EXERCISE 2.19.



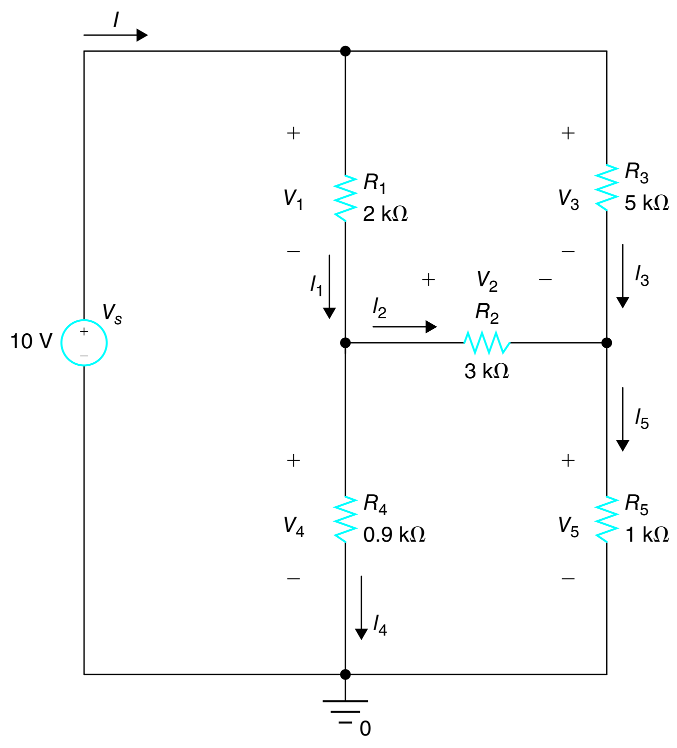
Answer:
$V_a = 0.6535$ V.

EXAMPLE 2.20

Find the voltages V_1, V_2, V_3, V_4, and V_5, and currents I_1, I_2, I_3, I_4, I_5, and I in the circuit shown in Figure 2.70.

FIGURE 2.70

Circuit for EXAMPLE 2.20.



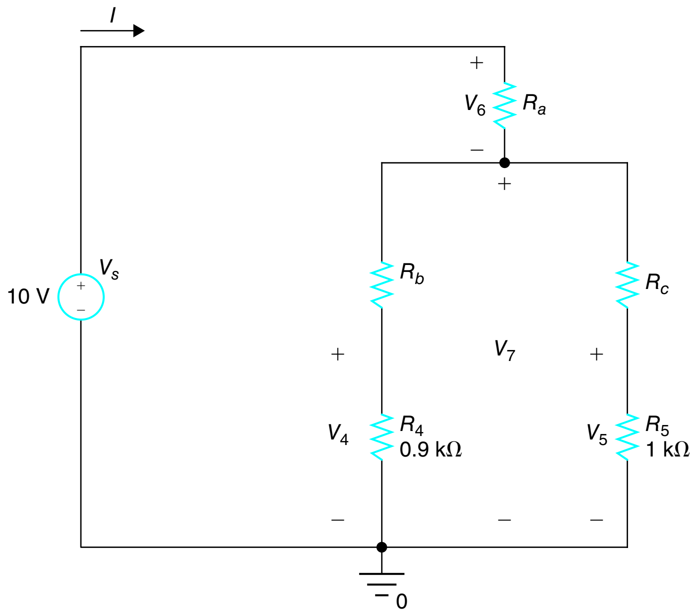
continued

Example 2.20 continued

When the delta consisting of R_1, R_2, and R_3 is transformed to a wye consisting of R_a, R_b, and R_c, we obtain the circuit shown in Figure 2.71.

FIGURE 2.71

The circuit after the delta to wye conversion.



The values of R_a, R_b, and R_c are found from Equations (2.63)–(2.65):

$$R_a = \frac{R_1R_3}{R_1 + R_2 + R_3} = \frac{2\ k\Omega \times 5\ k\Omega}{2\ k\Omega + 3\ k\Omega + 5\ k\Omega} = 1\ k\Omega$$

$$R_b = \frac{R_1R_2}{R_1 + R_2 + R_3} = \frac{2\ k\Omega \times 3\ k\Omega}{2\ k\Omega + 3\ k\Omega + 5\ k\Omega} = 0.6\ k\Omega$$

$$R_c = \frac{R_2R_3}{R_1 + R_2 + R_3} = \frac{3\ k\Omega \times 5\ k\Omega}{2\ k\Omega + 3\ k\Omega + 5\ k\Omega} = 1.5\ k\Omega$$

The equivalent resistance seen from the voltage source is given by

$$R_{eq} = R_a + (R_b + R_4)\|(R_c + R_5) = R_a + \frac{(R_b + R_4)(R_c + R_5)}{R_b + R_4 + R_c + R_5}$$
$$= 1\ k\Omega + \frac{(0.6\ k\Omega + 0.9\ k\Omega)(1.5\ k\Omega + 1\ k\Omega)}{0.6\ k\Omega + 0.9\ k\Omega + 1.5\ k\Omega + 1\ k\Omega} = 1.9375\ k\Omega$$

The current I from the voltage source is given by

$$I = \frac{V_s}{R_{eq}} = \frac{10\text{ V}}{1.9375\ k\Omega} = 5.16129\text{ mA}$$

From Ohm's law, the voltage drop V_6 across R_a is given by

$$V_6 = R_aI = 1\ k\Omega \times 5.16129\text{ mA} = 5.16129\text{ V}$$

continued

Example 2.20 continued

The voltage V_7 in the middle of the wye is given by

$$V_7 = V_s - V_6 = 10\,\text{V} - 5.016129\,\text{V} = 4.83871\,\text{V}$$

Applying the voltage divider rule, we get

$$V_4 = \frac{R_4}{R_b + R_4} V_7 = \frac{0.9\,k\Omega}{0.6\,k\Omega + 0.9\,k\Omega} 4.8387\text{ V} = 2.903226\text{ V}$$

$$V_5 = \frac{R_5}{R_c + R_5} V_7 = \frac{1\,k\Omega}{1.5\,k\Omega + 1\,k\Omega} 4.8387\text{ V} = 1.935484\text{ V}$$

The voltage V_1 is obtained by subtracting V_4 from V_s; that is,

$$V_1 = V_s - V_4 = 10\text{ V} - 2.903226\text{ V} = 7.096774\text{ V}$$

Similarly, the voltage V_3 is obtained by subtracting V_5 from V_s; that is,

$$V_3 = V_s - V_5 = 10\text{ V} - 1.935484\text{ V} = 8.064516\text{ V}$$

The voltage V_2 is obtained by subtracting V_5 from V_4; that is,

$$V_2 = V_4 - V_5 = 2.903226\text{ V} - 1.935484\text{ V} = 0.967742\text{ V}$$

The currents I_1, I_2, I_3, I_4, and I_5 are obtained by applying Ohm's law to each branch:

$$I_1 = \frac{V_1}{R_1} = \frac{7.096774\text{ V}}{2\,k\Omega} = 3.548387\text{ mA}$$

$$I_2 = \frac{V_2}{R_2} = \frac{0.967742\text{ V}}{3\,k\Omega} = 0.322581\text{ mA}$$

$$I_3 = \frac{V_3}{R_3} = \frac{8.064516\text{ V}}{5\,k\Omega} = 1.612903\text{ mA}$$

$$I_4 = \frac{V_4}{R_4} = \frac{2.903226\text{ V}}{0.9\,k\Omega} = 3.225806\text{ mA}$$

$$I_5 = \frac{V_5}{R_5} = \frac{1.935484\text{ V}}{1\,k\Omega} = 1.935484\text{ mA}$$

A MATLAB function D2Y changes a delta configuration to a wye configuration:

```
%Delta to wye conversion
function [Ra Rb Rc]=D2Y(x)
Ra=x(1)*x(3)/sum(x);
Rb=x(1)*x(2)/sum(x);
Rc=x(2)*x(3)/sum(x);
end
```

continued

Example 2.20 continued

The transformation of the delta configuration with $R_1 = 2\ k\Omega$, $R_2 = 3\ k\Omega$, and $R_3 = 5\ k\Omega$ (shown in Figure 2.70) to the wye configuration with $R_a = 1\ k\Omega$, $R_b = 600\ \Omega$, and $R_c = 1.5\ k\Omega$ (shown in Figure 2.71) can be achieved using D2Y:

```
>> [Ra Rb Rc]=D2Y([2000,3000,5000])
Ra =
   1000
Rb =
600
Rc =
   1500
```

MATLAB

```
%EXAMPLE 2.20
%Functions D2Y.m and P.m should be in the same folder as this file.
clear all;format long;
Vs=10;
R1=2000;R2=3000;R3=5000;R4=900;R5=1000;
[Ra,Rb,Rc]=D2Y([R1,R2,R3])
Req=Ra+P([Rb+R4,Rc+R5])
I=Vs/Req
V6=Ra*I
V7=Vs-V6
V4=V7*R4/(Rb+R4)
V5=V7*R5/(Rc+R5)
V1=Vs-V4
V3=Vs-V5
V2=V4-V5
I1=V1/R1
I2=V2/R2
I3=V3/R3
I4=V4/R4
I5=V5/R5

Answers:
Ra =
        1000
Rb =
   600
Rc =
        1500
Req =
     1.937500000000000e+03
I =
   0.005161290322581
V6 =
   5.161290322580645
V7 =
   4.838709677419355
V4 =
   2.903225806451613
```

continued

Example 2.20 continued

MATLAB continued

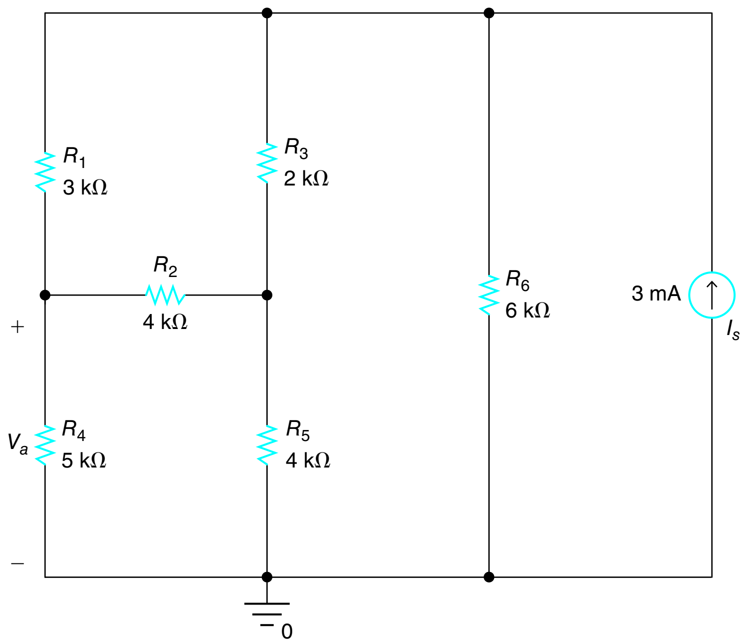
```
V5 =
    1.935483870967742
V1 =
    7.096774193548387
V3 =
    8.064516129032258
V2 =
    0.967741935483871
I1 =
    0.003548387096774
I2 =
      3.225806451612903e-04
I3 =
    0.001612903225806
I4 =
    0.003225806451613
I5 =
    0.001935483870968
```

Exercise 2.20

Find V_a in the circuit shown in Figure 2.72.

FIGURE 2.72

Circuit for EXERCISE 2.20.



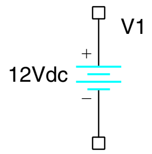
Answer:

$V_a = 4.1597$ V.

2.11 PSpice and Simulink

FIGURE 2.73

Placement of a voltage source.



PSpice® is an application for PCs that can be used to simulate circuits. In this text, PSpice is introduced using the OrCAD demo software. Click on the Place Part icon to start. If not already done, click on Add Library and add the following libraries:

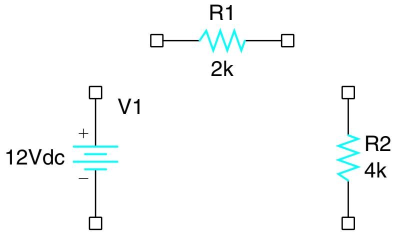
analog, analog_p, breakout, eval, source, special

Other libraries can be added if needed. Select all the libraries and enter *vdc* inside Part. Click Enter and move the cursor to the left side of the window and place the dc voltage source by clicking. Right-click and choose the End mode to stop placing dc voltage sources. Double-click on 0Vdc and change the voltage to 12Vdc, as shown in Figure 2.73.

Enter R inside Part. Place a resistor R_1 as shown in Figure 2.74. Click Ctrl + R to rotate the resistor in the vertical direction and place R_2 to the right side, as shown in Figure 2.74. Right-click and choose the End mode. Change the values of the resistors to 2k and 4k.

FIGURE 2.74

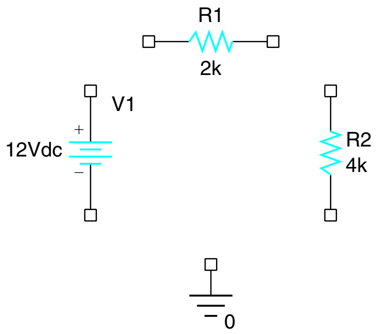
Placement of resistors.



Click on the Place Ground icon. Click on 0/SOURCE and OK. Place the ground at the bottom of the circuit, as shown in Figure 2.75.

FIGURE 2.75

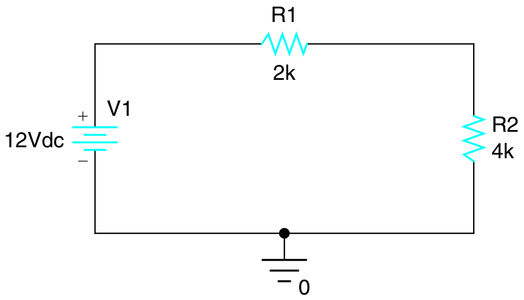
Placement of ground.



Click on the Place Wire icon. Place the cursor at the beginning of the wire, drag the mouse to the end of the wire, and click. To change direction, click and move in a different direction. Repeat this procedure to complete the wiring. When finished, right-click and choose End Wire. Figure 2.76 shows the finished schematic.

FIGURE 2.76

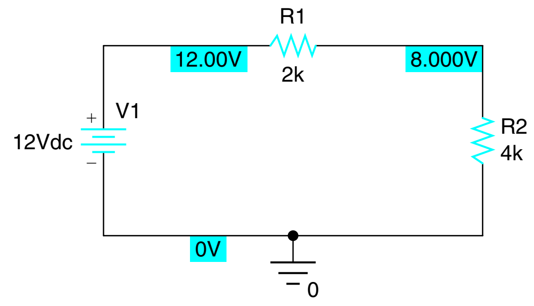
The wiring schematic.



Click on New Simulation Profile icon or select PSpice → New Simulation Profile. Enter a name and select Bias Point as the Analysis type. Click on Run PSpice icon or select PSpice → Run. Click on Enable Bias Voltage Display (V). The positive voltage for each element is displayed as shown in Figure 2.77.

FIGURE 2.77

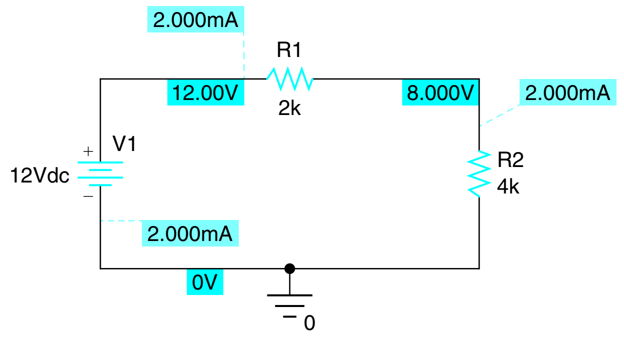
Voltage display.



You can move the voltage display by clicking on it and moving it to the location that you want. Click on Enable Bias Current Display (I). The current values are displayed as shown in Figure 2.78. The current entering each element, 2 mA, is displayed.

FIGURE 2.78

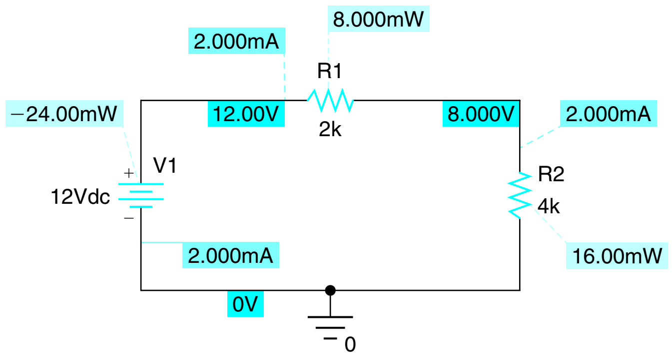
Current display.



Click on Enable Bias Power Display (W). The power is displayed for each element as shown in Figure 2.79. The power on the voltage source is −24 mW, the power on R_1 is 8 mW, and the power on R_2 is 16 mW. The power delivered, 24 mW, is equal to the power absorbed: 8 mW + 16 mW = 24 mW.

FIGURE 2.79

Power display.



More detailed information on the simulation is available in the output file (to view, select PSpice→View Output File or click on the View Simulation Output File icon).

EXAMPLE 2.21

Build the circuit shown in Figure 2.80 and simulate it using PSpice. Find the voltages and currents everywhere.

The part name for voltage source is VSRC and the part name for current source is ISRC. For both sources, enter the values for dc only. The results of the simulation are shown in Figure 2.81.

FIGURE 2.80

Circuit for EXAMPLE 2.21.

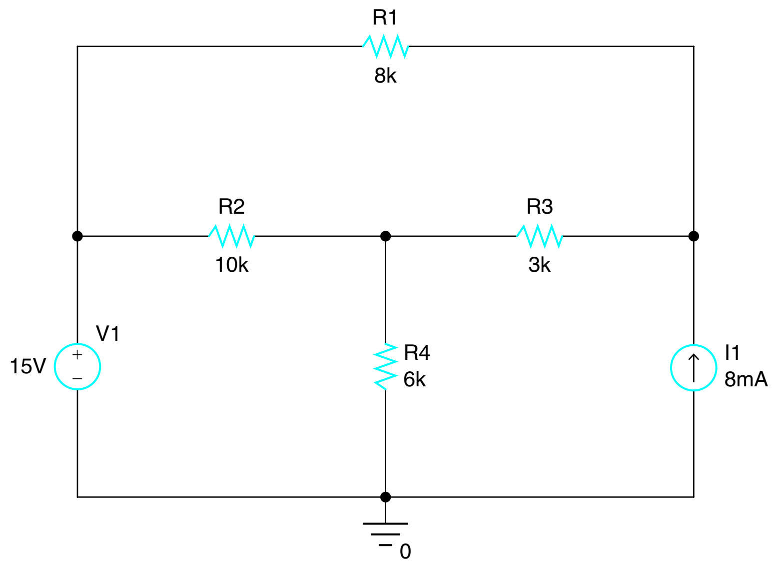
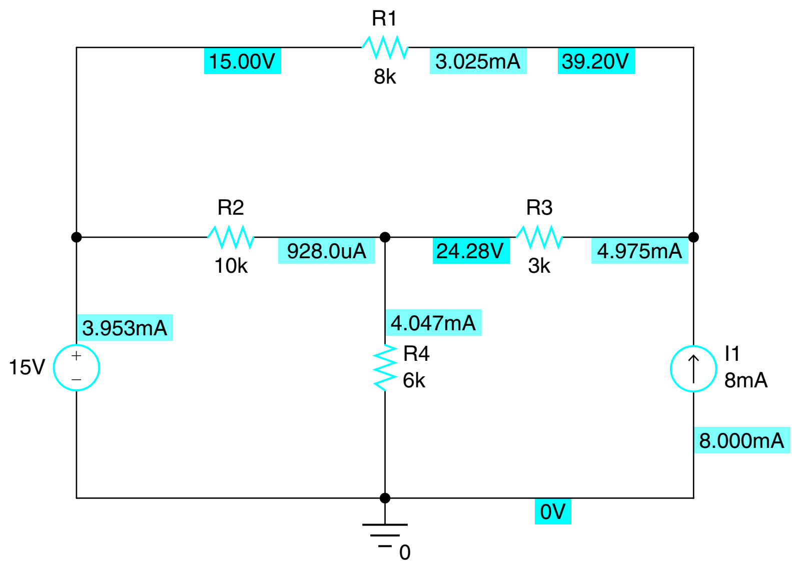


FIGURE 2.81

Results of the simulation.

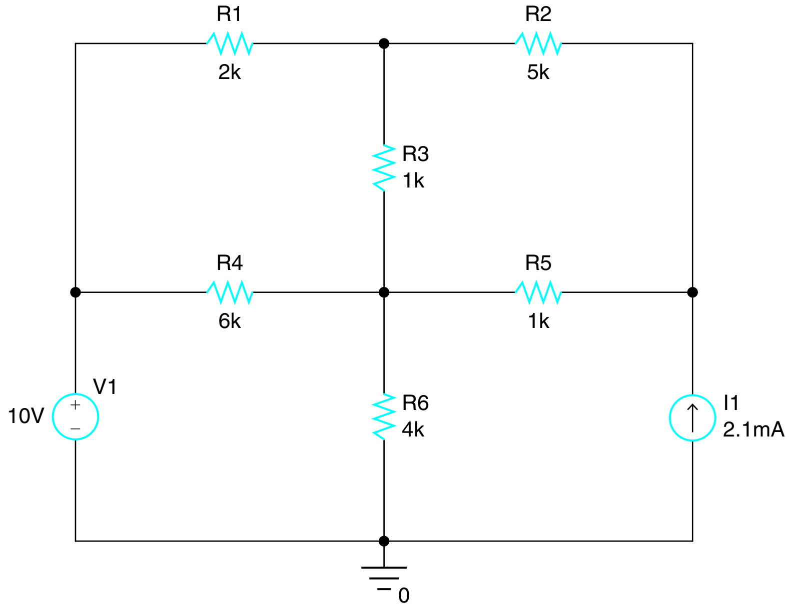


Exercise 2.21

Build the circuit shown in Figure 2.82 and simulate it using PSpice. Find the voltages at every node in the circuit.

FIGURE 2.82

Circuit for EXERCISE 2.21.

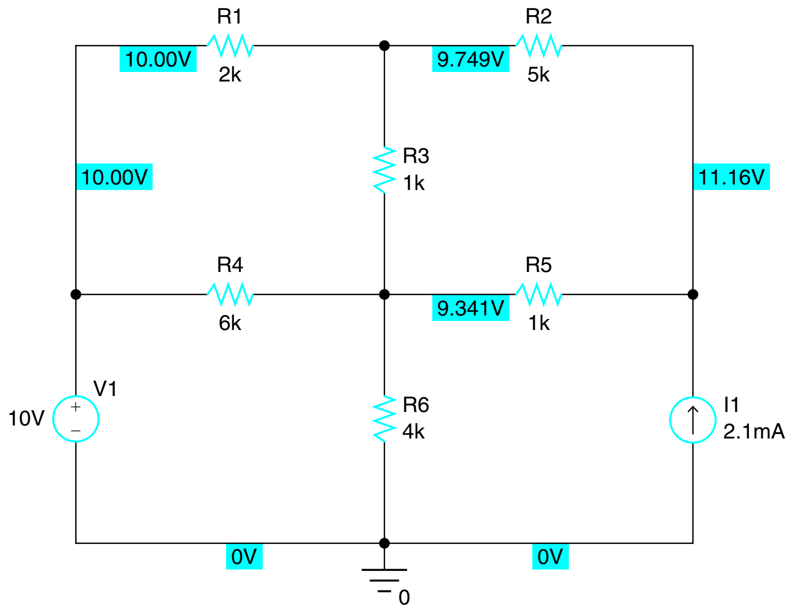


Answer:

Figure 2.83 shows the result of the simulation.

FIGURE 2.83

Circuit with node voltages.



2.11.1 SIMULINK

Simscapes of Simulink® make it possible to simulate electric circuits, as shown in Figure 2.84. The blocks are available in the Simscape Library. PS-Simulink Converter converts physical signals to Simulink output signals for further processing in Simulink. Figure 2.85 displays the voltage across the $2k\Omega$ resistor on the scope.

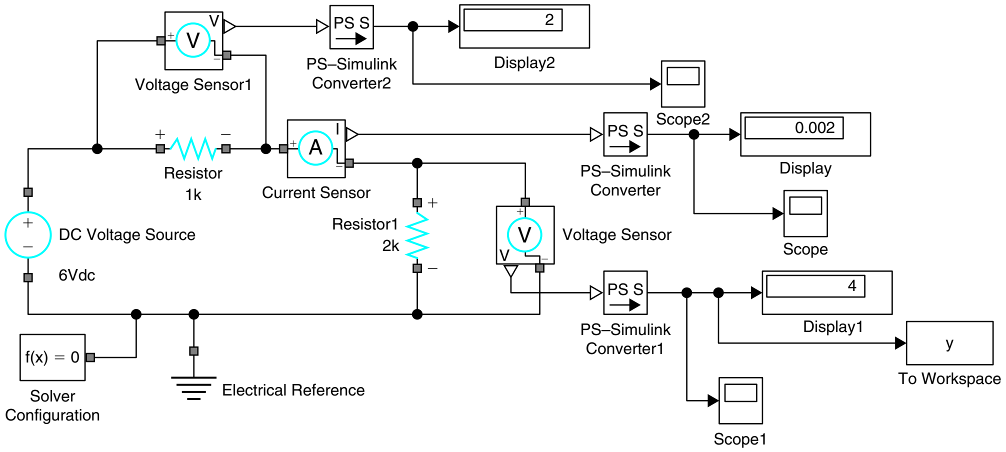


FIGURE 2.84 A Simulink model for a circuit with dc voltage source and two resistors.

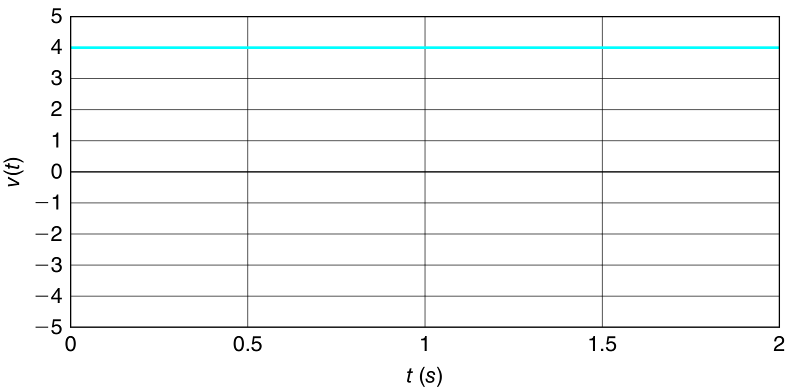


FIGURE 2.85 The voltage across the 2-$k\Omega$ resistor.

SUMMARY

According to Ohm's law, the voltage across the resistor is proportional to the current through the resistor. The proportionality constant is called the *resistance.* For the given voltage, the amount of current flowing through the resistor decreases as the resistance value increases.

KCL says that the sum of currents entering a node equals the sum of currents leaving the same node. Stated another way, the sum of currents leaving a node is zero. KCL is the basis in setting up node equations that lead to node voltages.

KVL says that the sum of voltage drops around a mesh equals the sum of voltage rises around the same mesh. Stated another way, the sum of voltage drops around a mesh is zero. KVL is the basis in setting up mesh equations that lead to mesh currents.

The voltage divider rule says that if resistors are connected to a voltage source in series, the voltage from the voltage source is divided among the resistors in proportion to the resistance value. For certain circuits, the voltage divider rule makes it possible to calculate voltages across resistors with minimal effort.

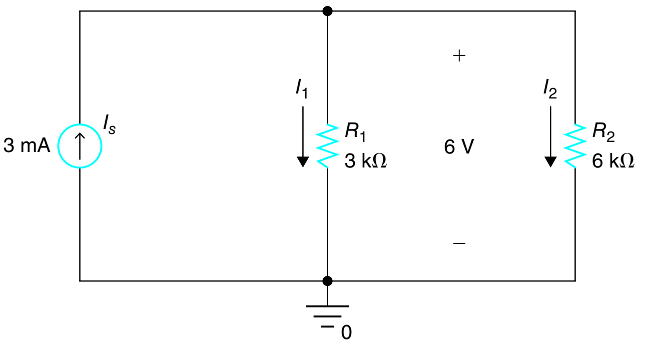
The current divider rule says that if resistors are connected to a current source in parallel, the current from the current source is divided among the resistors in proportion to the conductance value. For certain circuits, the current divider rule makes it possible to calculate currents through resistors with minimal effort.

PROBLEMS

Ohm's Law

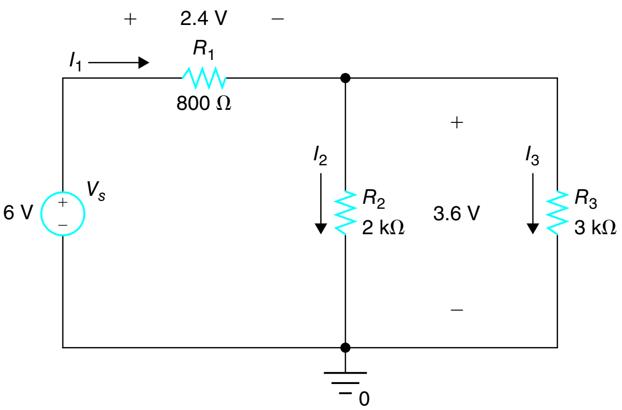
2.1 In the circuit shown in Figure P2.1, the voltage across resistors R_1 and R_2 is 6 V. Find the current I_1 through R_1, and the current I_2 through R_2.

FIGURE P2.1



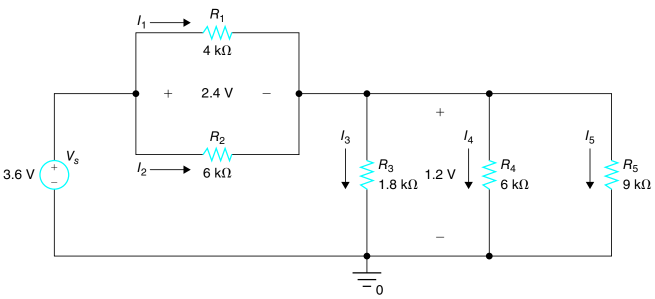
2.2 In the circuit shown in Figure P2.2, the voltage across the resistor R_1 is 2.4 V, and the voltage across the resistors R_2 and R_3 is 3.6 V. Find the currents I_1, I_2, and I_3.

FIGURE P2.2



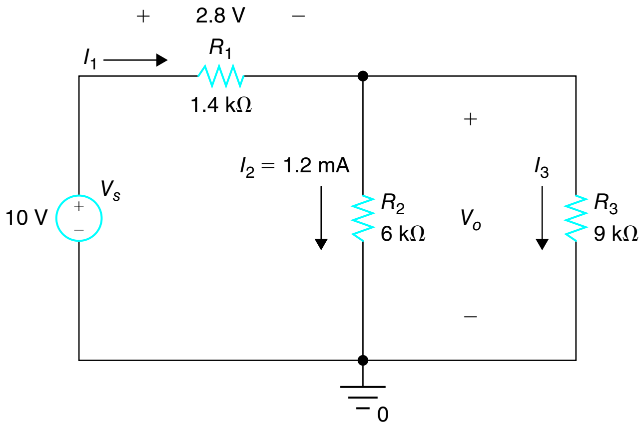
2.3 In the circuit shown in Figure P2.3, the voltage across the resistor R_1 and R_2 is 2.4 V, and the voltage across the resistors R_3, R_4, and R_5 is 1.2 V. Find the currents I_1, I_2, I_3, I_4, and I_5.

FIGURE P2.3



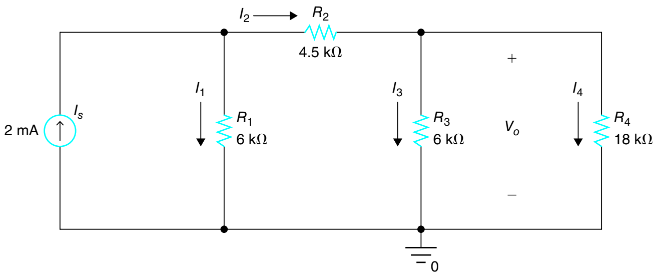
2.4 In the circuit shown in Figure P2.4, the current through the resistor R_2 is $I_2 = 1.2$ mA, and the voltage across the resistor R_1 is 2.8 V. Find the voltage V_o and currents I_1, and I_3.

FIGURE P2.4



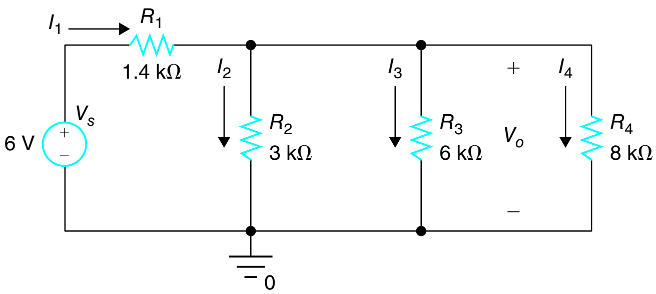
2.5 In the circuit shown in Figure P2.5, the current through the resistor R_4 is $I_4 = 0.2$ mA. Find the voltage V_o and current I_3.

FIGURE P2.5



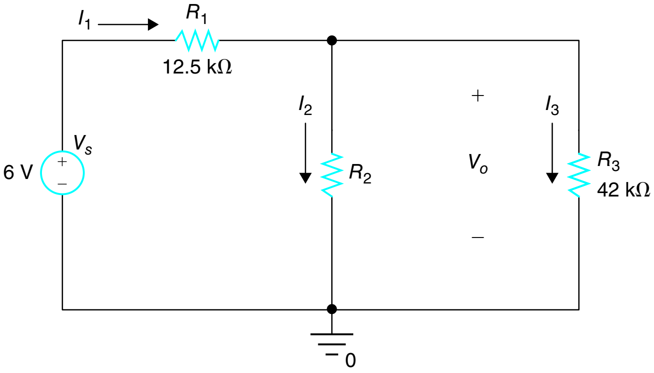
2.6 In the circuit shown in Figure P2.6, the current through the resistor R_4 is $I_4 = 0.4$ mA. Find the voltage V_o and currents I_2 and I_3.

FIGURE P2.6



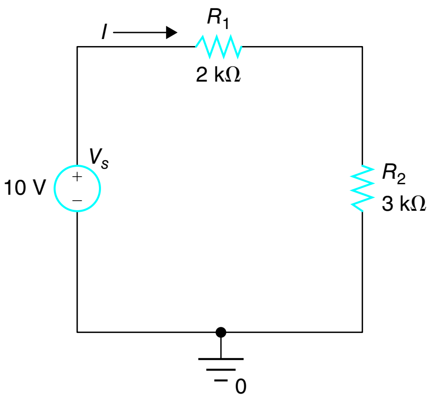
2.7 In the circuit shown in Figure P2.7, the current through the resistor R_2 is $I_2 = 7/60$ mA, and the current through the resistor R_3 is $I_3 = 1/12$ mA. Find the voltage V_o and the resistance value of R_2.

FIGURE P2.7



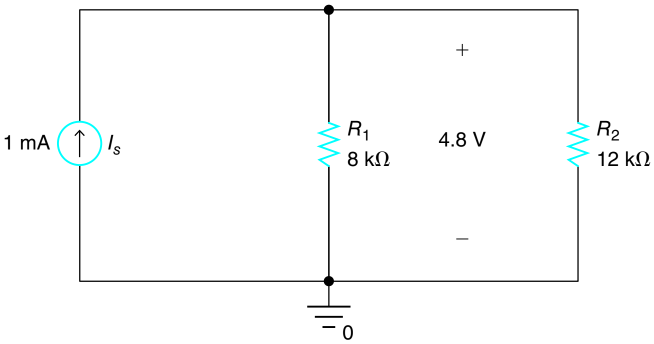
2.8 In the circuit shown in Figure P2.8, the current through the mesh is $I = 2$ mA. Find the power values on R_1, R_2, and V_s and state whether the power is absorbed or delivered.

FIGURE P2.8



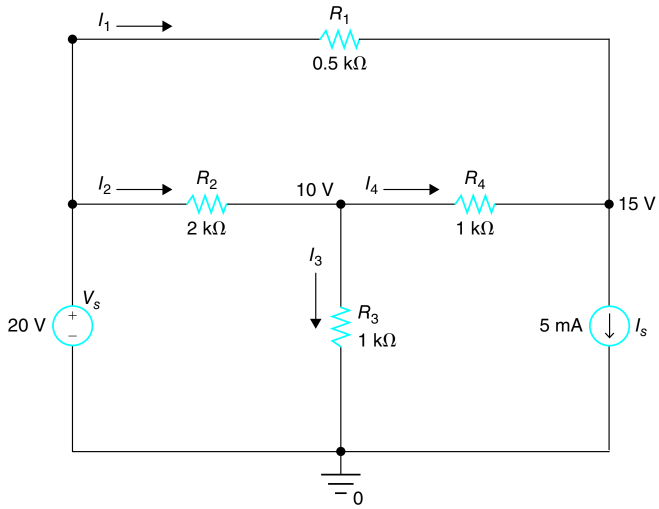
2.9 In the circuit shown in Figure P2.9, the voltage across I_s, R_1, and R_2 is $V_o = 4.8$ V. Find the power values on R_1, R_2, and I_s and state whether the power is absorbed or delivered.

FIGURE P2.9



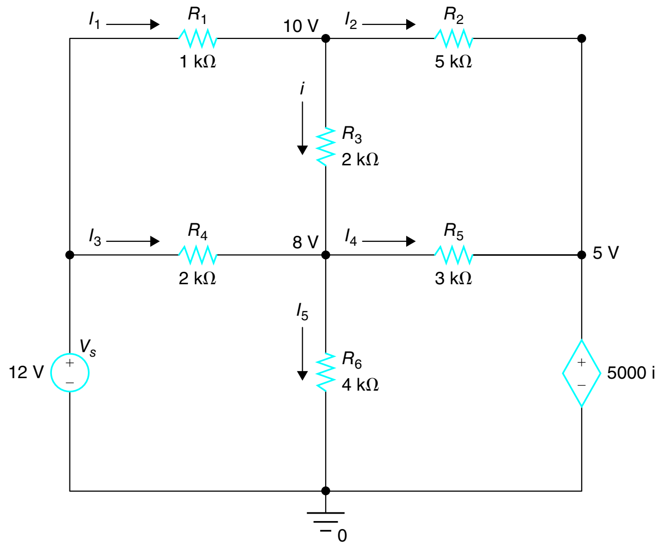
2.10 In the circuit shown in Figure P2.10, all the node voltages are given. Find the values of currents I_1, I_2, I_3, and I_4.

FIGURE P2.10



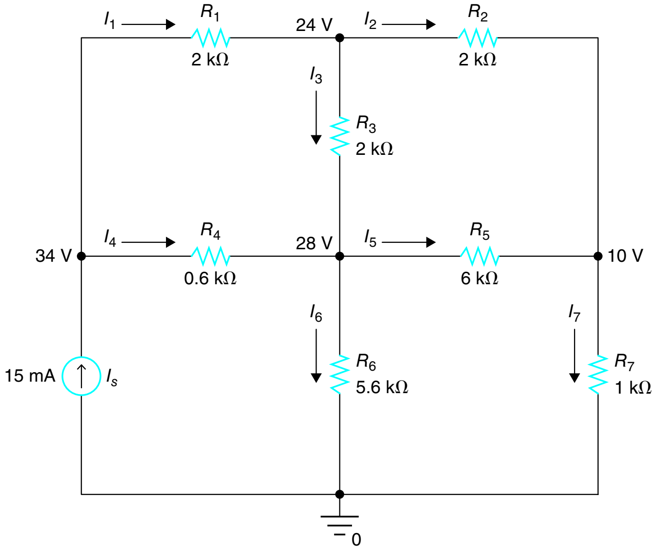
2.11 In the circuit shown in Figure P2.11, find i, I_1, I_2, I_3, I_4, and I_5.

FIGURE P2.11



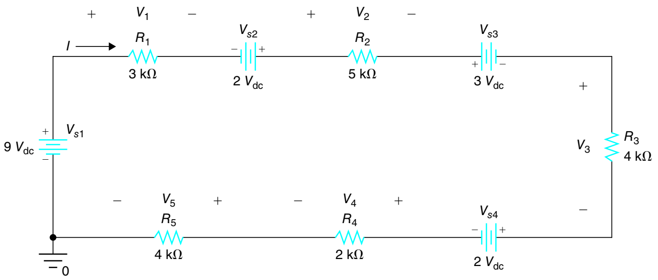
2.12 In the circuit shown in Figure P2.12, find I_1, I_2, I_3, I_4, I_5, I_6, and I_7.

FIGURE P2.12



2.13 Find I, V_1, V_2, V_3, and V_4 in the circuit shown in Figure P2.13.

FIGURE P2.13



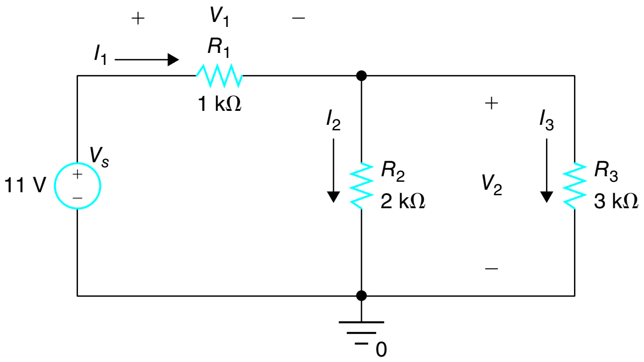
2.14 The diameter of a 26 AWG copper (conductivity $= \sigma = 5.69 \times 10^7$ S/m) wire is 0.405 mm. Find the resistance of the copper wire when the length of the wire is

a. $\ell = 20$ m
b. $\ell = 200$ m
c. $\ell = 2$ km
d. $\ell = 20$ km

Kirchhoff's Current Law (KCL)

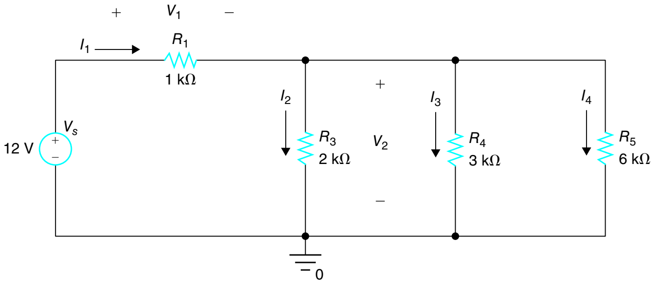
2.15 In the circuit shown in Figure P2.15, the current through R_2 is given as $I_2 = 3$ mA. Find V_2, I_3, I_1, and V_1.

FIGURE P2.15



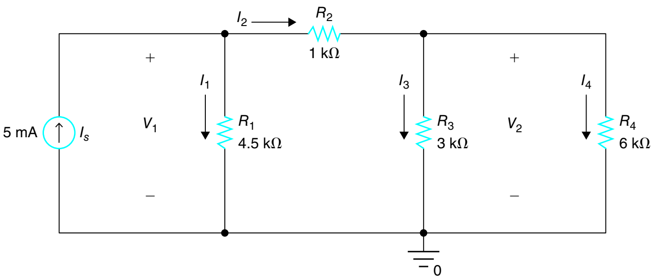
2.16 In the circuit shown in Figure P2.16, voltage V_2 is given as $V_2 = 6$ V. Find I_2, I_3, I_4, I_1, and V_1.

FIGURE P2.16



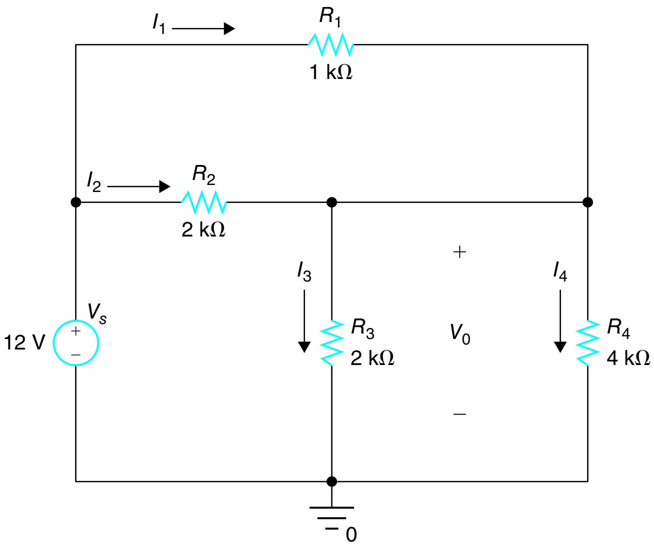
2.17 In the circuit shown in Figure P2.17, the current I_4 is given as $I_4 = 1$ mA. Find V_2, I_3, I_2, I_1, and V_1.

FIGURE P2.17



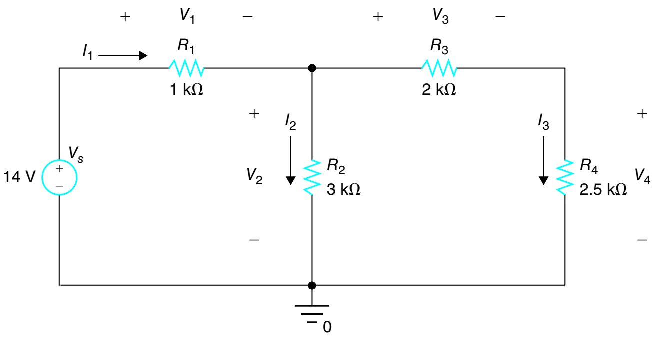
2.18 In the circuit shown in Figure P2.18, the voltage V_o is given as $V_o = 8$ V. Find I_3, I_4, I_1, and I_2.

FIGURE P2.18



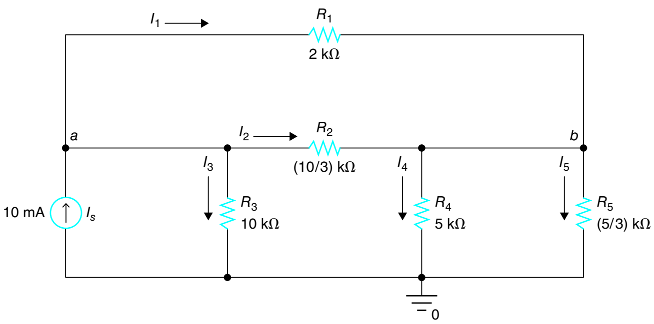
2.19 In the circuit shown in Figure P2.19, the voltage V_4 is given as $V_4 = 5$ V. Find I_3, V_3, V_2, I_2, I_1 and V_1.

FIGURE P2.19



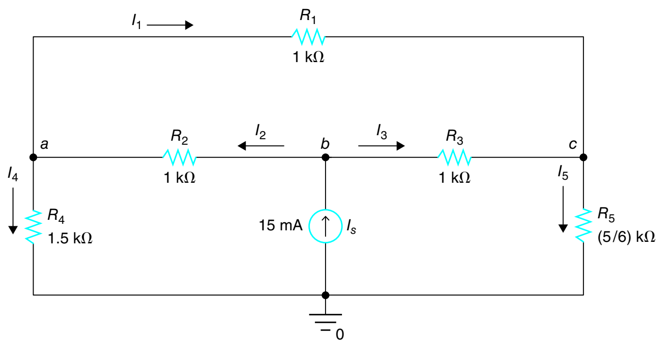
2.20 Let $I_1 = 5$ mA, $I_3 = 2$ mA, and $I_4 = 2$ mA in the circuit shown in Figure P2.20. Find I_2 and I_5.

FIGURE P2.20



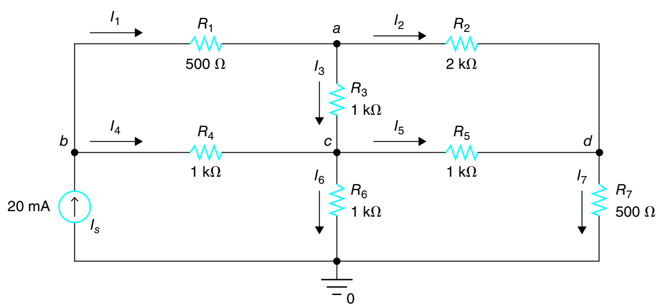
2.21 Let $I_1 = 2$ mA, and $I_3 = 10$ mA in the circuit shown in Figure P2.21. Find I_2, I_4, and I_5.

FIGURE P2.21



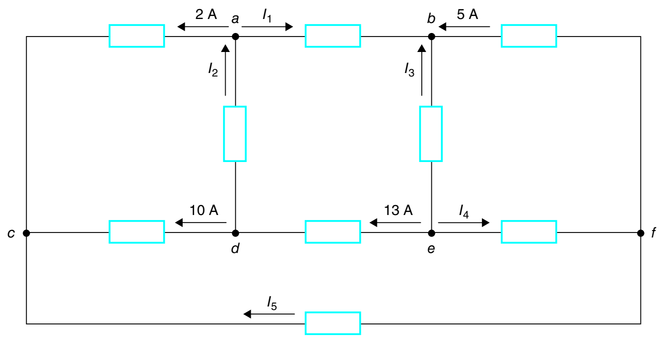
2.22 Let $I_3 = 5$ mA, $I_4 = 10$ mA, and $I_5 = 5$ mA in the circuit shown in Figure P2.22. Find I_1, I_2, I_6, and I_7.

FIGURE P2.22



2.23 Find I_1, I_2, I_3, I_4, and I_5 in the circuit shown in Figure P2.23.

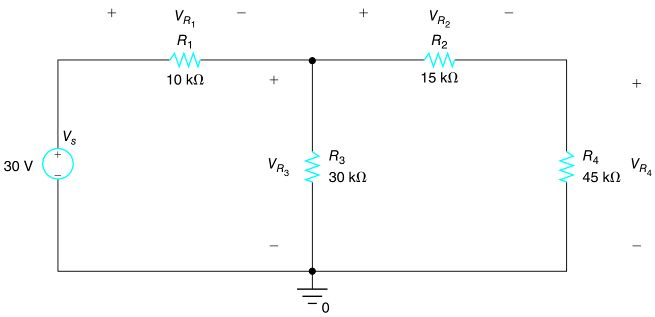
FIGURE P2.23



Kirchhoff's Voltage Law (KVL)

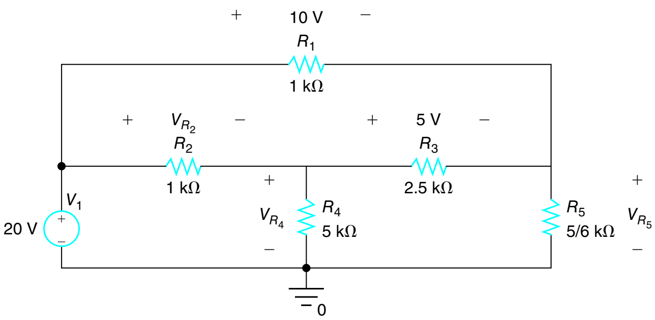
2.24 Let $V_{R_1} = 10$ V and $V_{R_4} = 15$ V in the circuit shown in Figure P2.24. Find V_{R_3} and V_{R_2}.

FIGURE P2.24



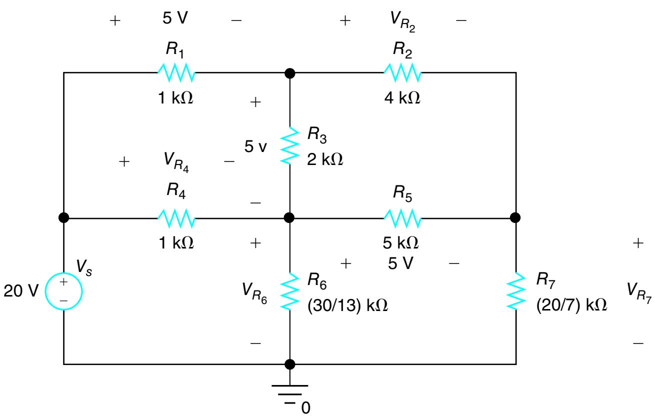
2.25 Find V_{R_2}, V_{R_4}, and V_{R_5} in the circuit shown in Figure P2.25.

FIGURE P2.25



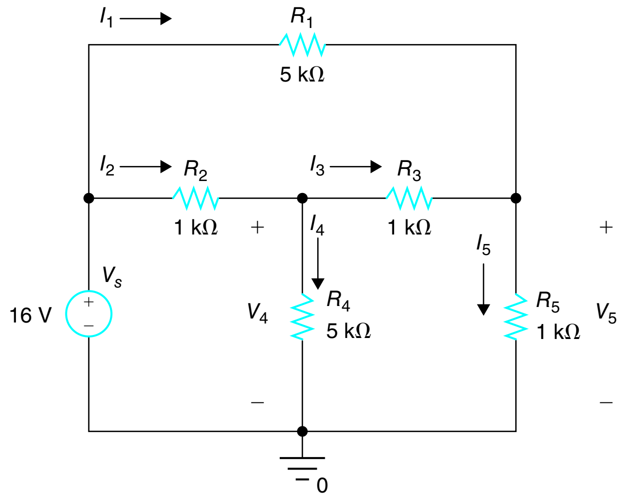
2.26 Find V_{R_2}, V_{R_4}, V_{R_6}, and V_{R_7} in the circuit shown in Figure P2.26.

FIGURE P2.26



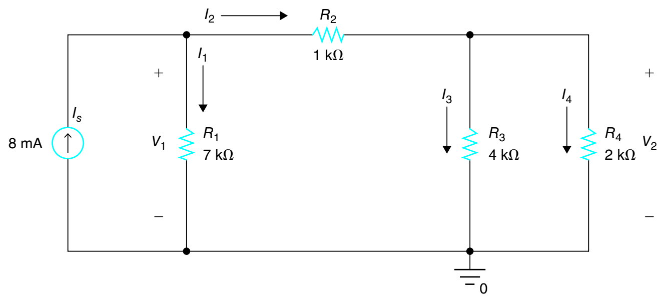
2.27 In the circuit shown in Figure P2.27, let V_5 be 6 V. Find I_5, I_1, I_3, V_4, I_4, and I_2.

FIGURE P2.27



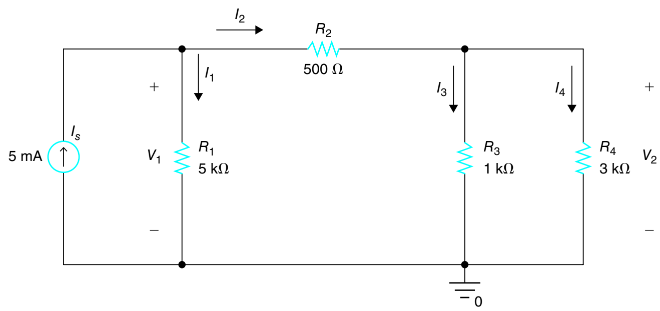
2.28 In the circuit shown in Figure P2.28, the current through R_3 is $I_3 = 2$ mA. Find V_2, I_4, I_2, I_1, and V_1.

FIGURE P2.28



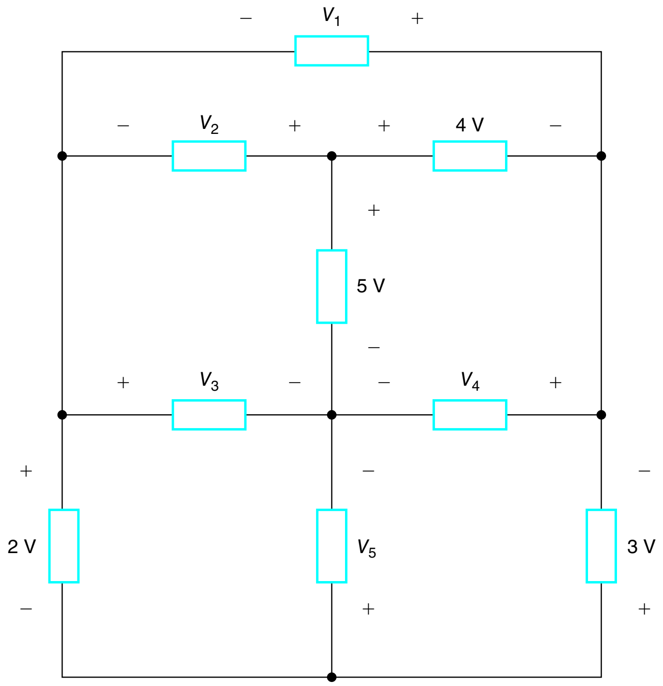
2.29 In the circuit shown in Figure P2.29, the current through R_1 is $I_1 = 1$ mA. Find V_1, I_2, V_2, I_3, and I_4.

FIGURE P2.29



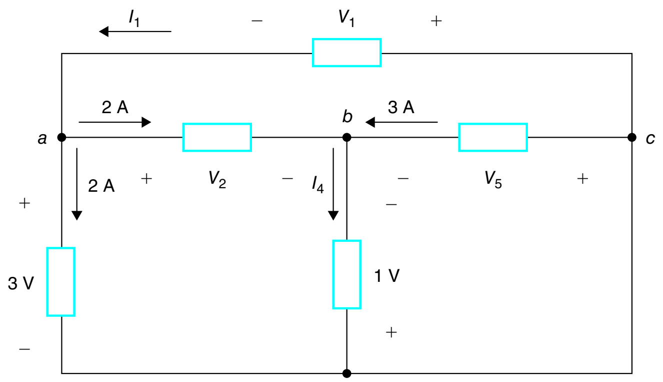
2.30 Find V_1, V_2, V_3, V_4, and V_5 in the circuit shown in Figure P2.30.

FIGURE P2.30



2.31 Find V_1, V_2, V_5, I_1, and I_4 in the circuit shown in Figure P2.31.

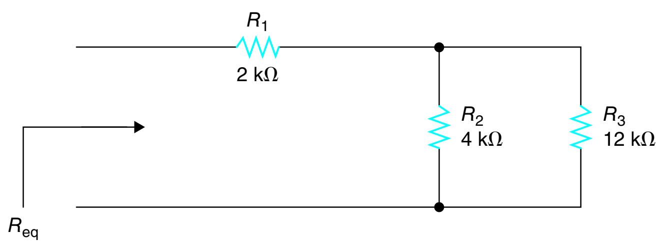
FIGURE P2.31



Equivalent Resistance

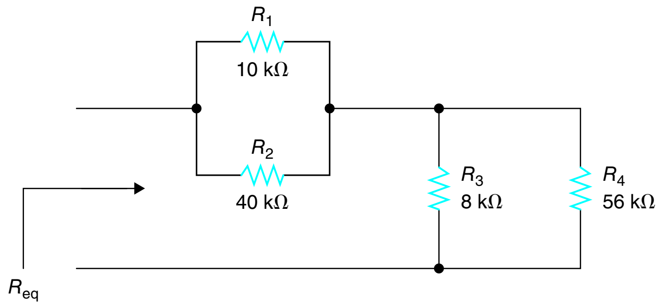
2.32 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.32.

FIGURE P2.32



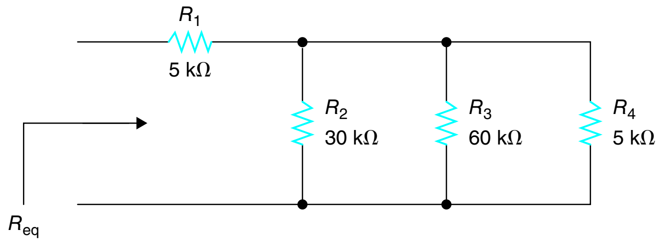
2.33 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.33.

FIGURE P2.33



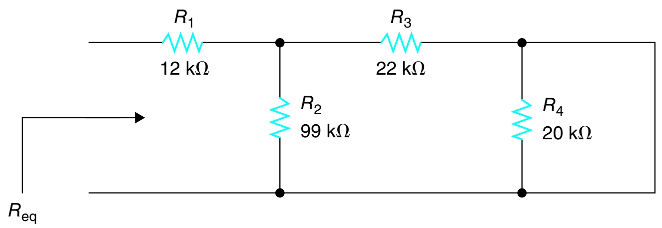
2.34 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.34.

FIGURE P2.34



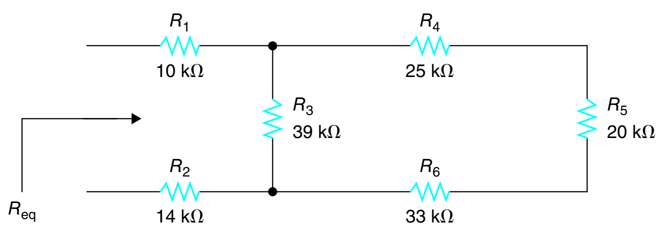
2.35 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.35.

FIGURE P2.35



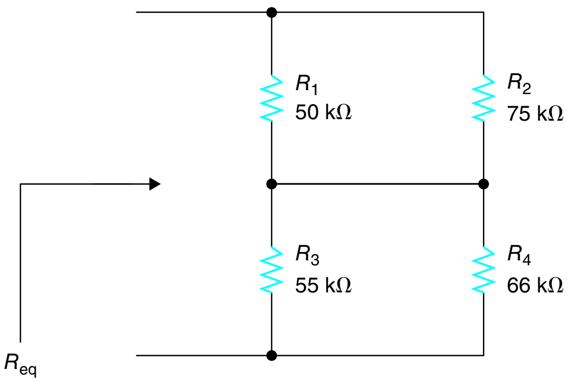
2.36 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.36.

FIGURE P2.36



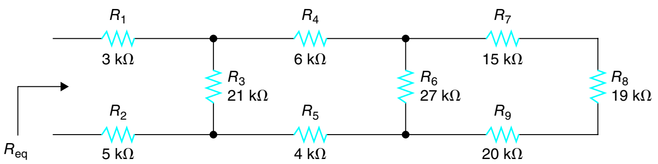
2.37 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.37.

FIGURE P2.37



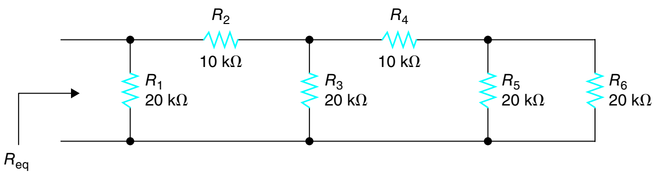
2.38 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.38.

FIGURE P2.38



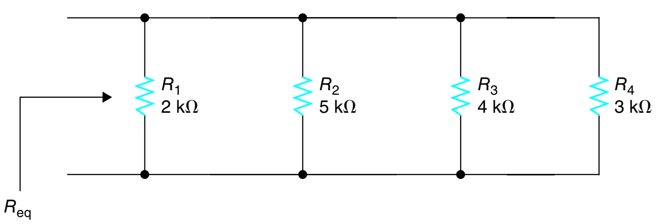
2.39 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.39.

FIGURE P2.39



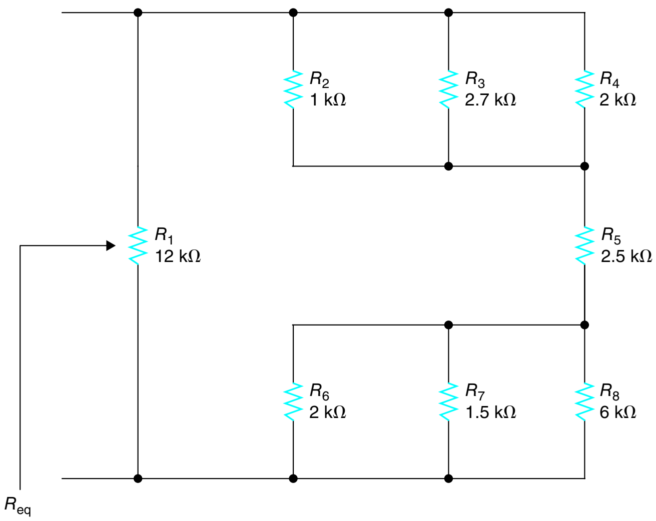
2.40 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.40.

FIGURE P2.40



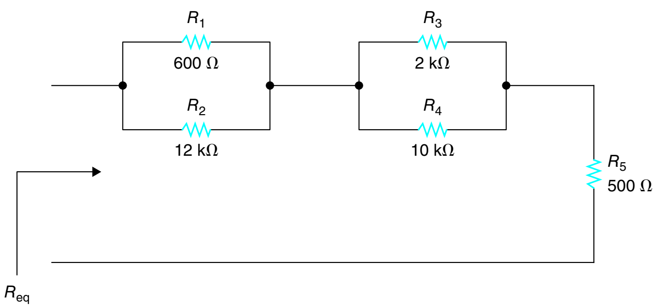
2.41 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.41.

FIGURE P2.41



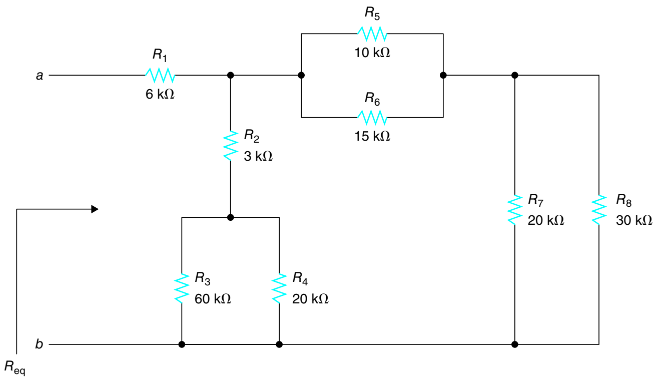
2.42 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.42.

FIGURE P2.42



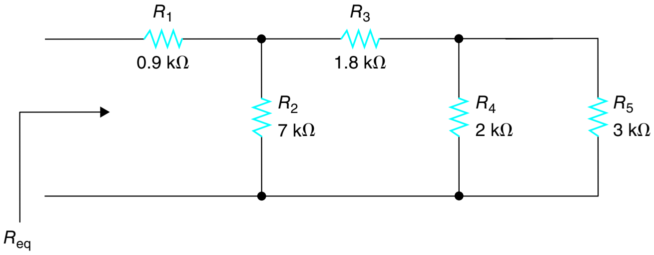
2.43 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.43.

FIGURE P2.43



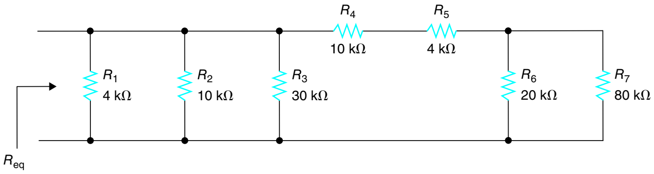
2.44 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.44.

FIGURE P2.44



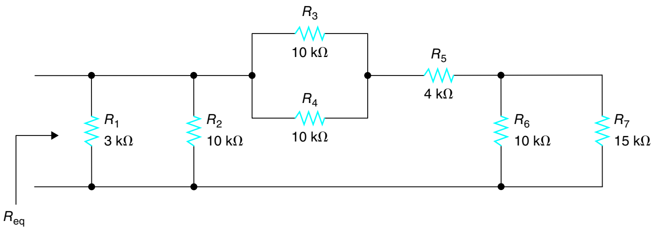
2.45 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.45.

FIGURE P2.45



2.46 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.46.

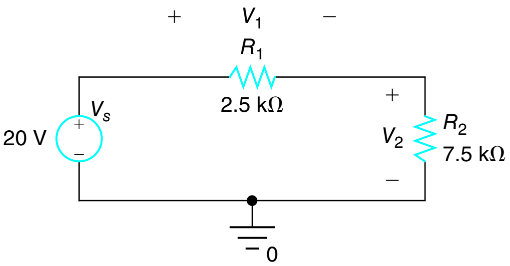
FIGURE P2.46



Voltage Divider Rule

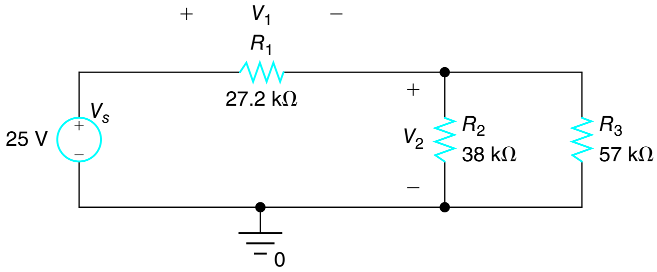
2.47 Use the voltage divider rule to find voltages V_1 and V_2 in the circuit shown in Figure P2.47.

FIGURE P2.47



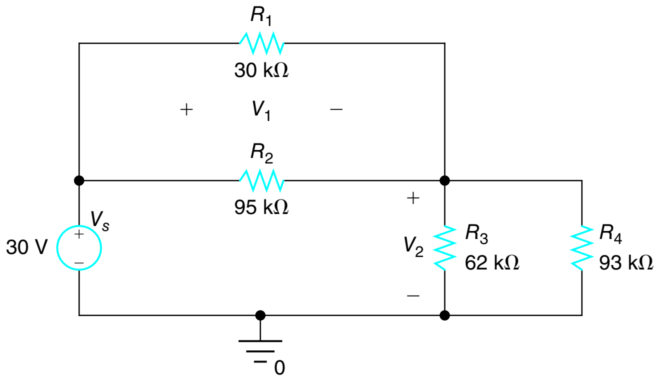
2.48 Use the voltage divider rule to find voltages V_1 and V_2 in the circuit shown in Figure P2.48.

FIGURE P2.48



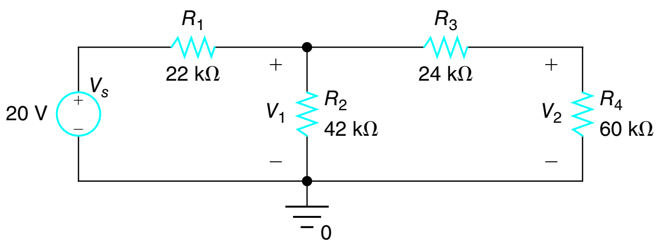
2.49 Use the voltage divider rule to find voltages V_1 and V_2 in the circuit shown in Figure P2.49.

FIGURE P2.49



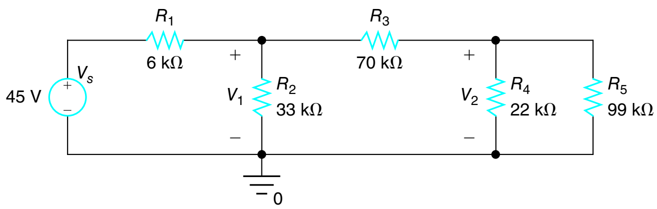
2.50 Use the voltage divider rule to find voltages V_1 and V_2 in the circuit shown in Figure P2.50.

FIGURE P2.50



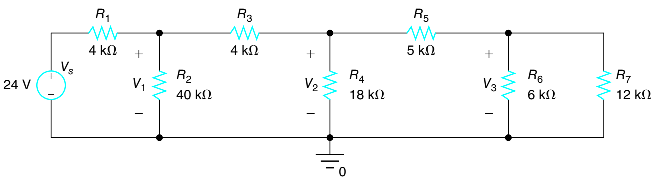
2.51 Use the voltage divider rule to find voltages V_1 and V_2 in the circuit shown in Figure P2.51.

FIGURE P2.51



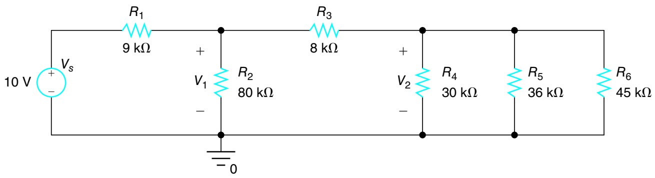
2.52 Use the voltage divider rule to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P2.52.

FIGURE P2.52



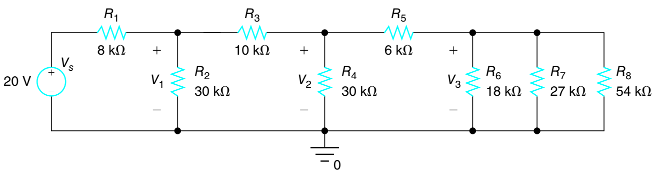
2.53 Use the voltage divider rule to find voltages V_1 and V_2 in the circuit shown in Figure P2.53.

FIGURE P2.53



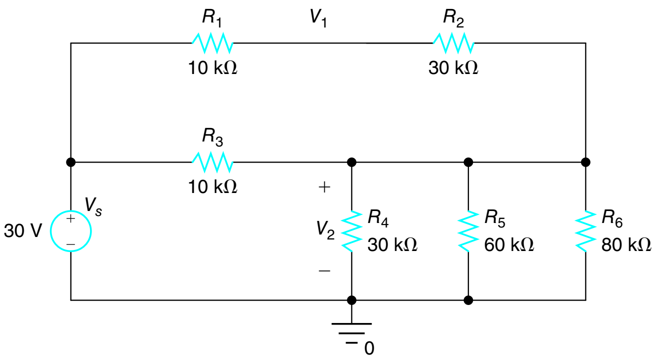
2.54 Use the voltage divider rule to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P2.54.

FIGURE P2.54



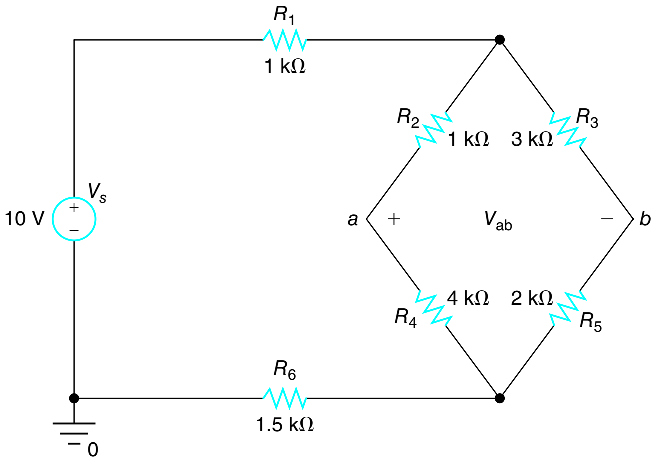
2.55 Use the voltage divider rule to find voltages V_1 and V_2 in the circuit shown in Figure P2.55.

FIGURE P2.55



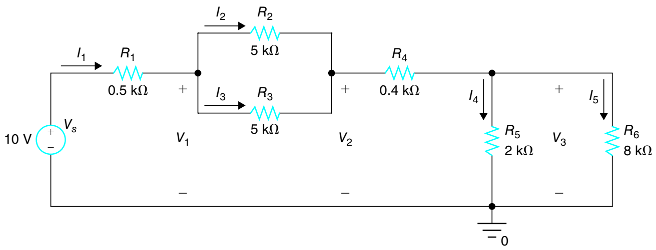
2.56 Use the voltage divider rule to find voltage V_{ab} in the circuit shown in Figure P2.56.

FIGURE P2.56



2.57 Use the voltage divider rule to find I_1, I_2, I_3, I_4, I_5, V_1, V_2, and V_3 in the circuit shown in Figure P2.57.

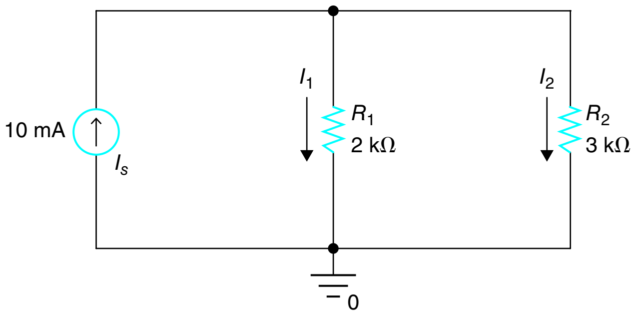
FIGURE P2.57



Current Divider Rule

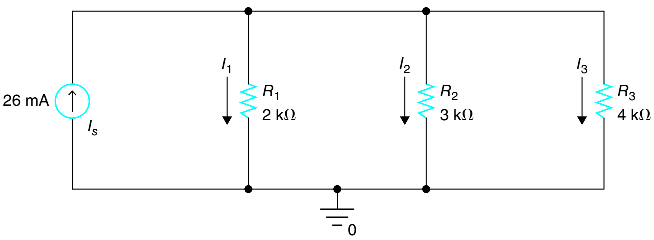
2.58 Use the current divider rule to find currents I_1 and I_2 in the circuit shown in Figure P2.58.

FIGURE P2.58



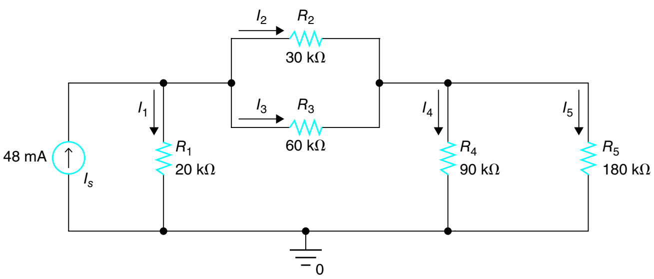
2.59 Use the current divider rule to find currents I_1, I_2, and I_3 in the circuit shown in Figure P2.59.

FIGURE P2.59



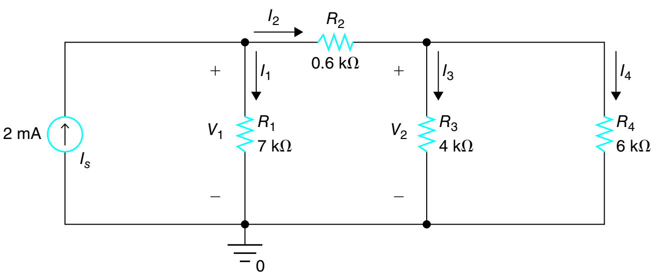
2.60 Use the current divider rule to find I_1, I_2, I_3, I_4, and I_5 in the circuit shown in Figure P2.60.

FIGURE P2.60



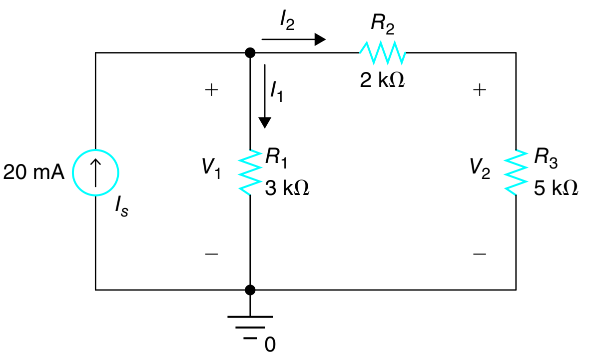
2.61 Use the current divider rule to find I_1, I_2, I_3, I_4, V_1, and V_2 in the circuit shown in Figure P2.61.

FIGURE P2.61



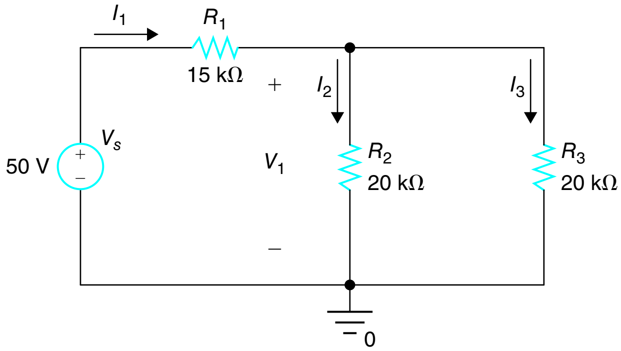
2.62 Find I_1, I_2, V_1, and V_2 in the circuit shown in Figure P2.62.

FIGURE P2.62



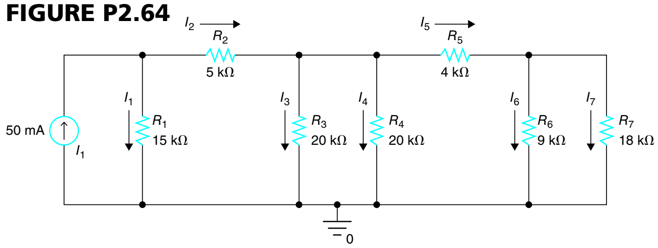
2.63 Find V_1, I_1, I_2, and I_3 in the circuit shown in Figure P2.63.

FIGURE P2.63



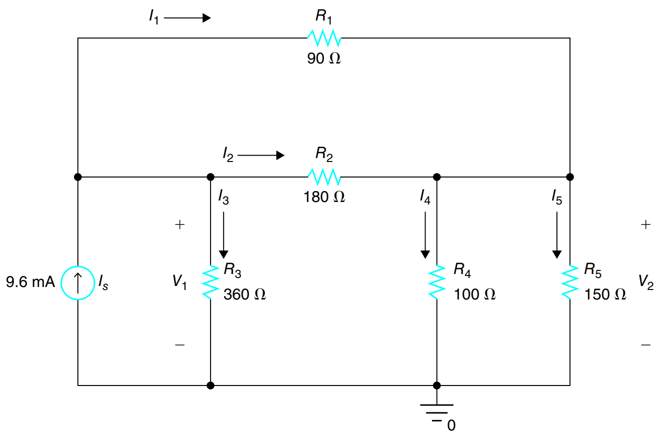
2.64 Use the current divider rule to find I_1, I_2, I_3, I_4, I_5, I_6, and I_7 in the circuit shown in Figure P2.64.

FIGURE P2.64



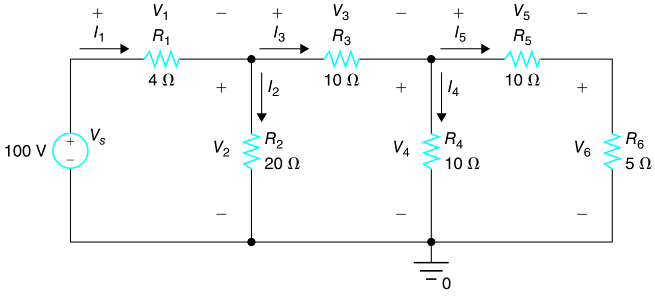
2.65 Find I_1, I_2, I_3, I_4, I_5, V_1, and V_2 in the circuit shown in Figure P2.65.

FIGURE P2.65



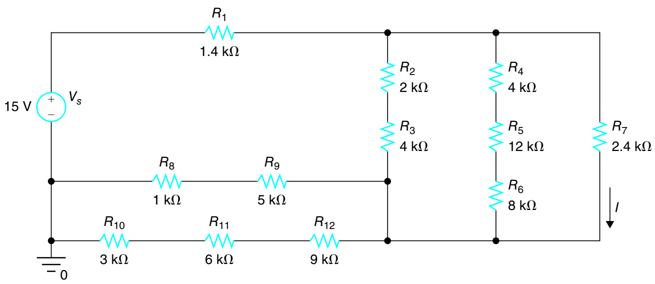
2.66 Find I_1, V_1, V_2, I_2, I_3, V_3, V_4, I_4, I_5, V_5, and V_6 in the circuit shown in Figure P2.66.

FIGURE P2.66



2.67 Find I in the circuit shown in Figure P2.67.

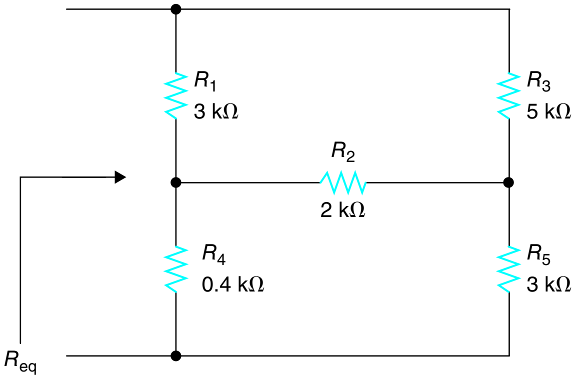
FIGURE P2.67



Delta-Wye and Wye-Delta

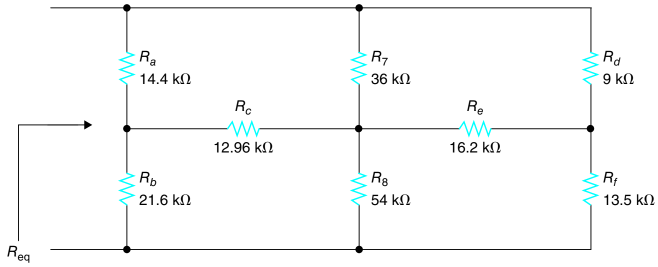
2.68 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.68.

FIGURE P2.68



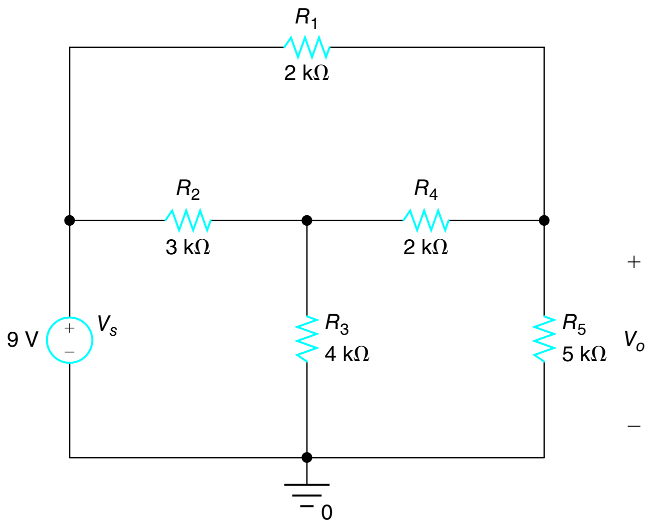
2.69 Find the equivalent resistance R_{eq} of the circuit shown in Figure P2.69.

FIGURE P2.69



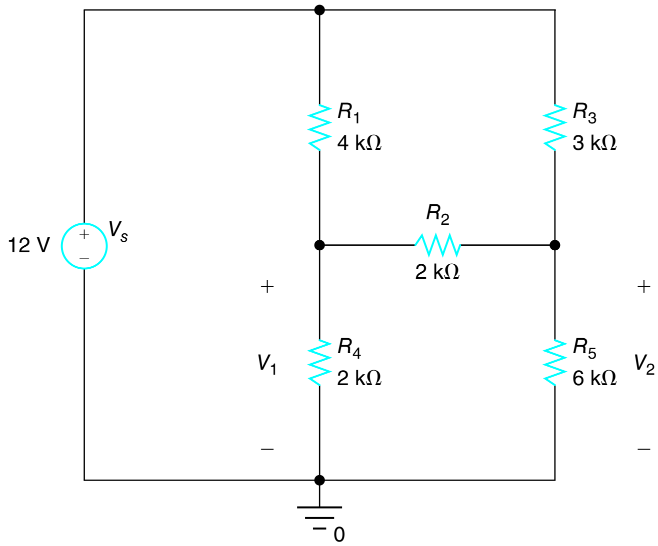
2.70 Find V_o in the circuit shown in Figure P2.70.

FIGURE P2.70



2.71 Find V_1 and V_2 in the circuit shown in Figure P2.71.

FIGURE P2.71



2.72 Find V_1 in the circuit shown in Figure P2.72.

FIGURE P2.72