Chapter 4

Circuit Theorems

4.1 Introduction

In this chapter, several important theorems related to electric circuits are discussed: superposition principle, source transformation, Thévenin's theorem, Norton's theorem, and maximum power transfer.

If a circuit contains more than one source, the circuit can be analyzed by summing the response from each source with all other sources deactivated. This is called the *superposition principle.* Deactivating a voltage source is equivalent to short-circuiting it, and deactivating a current source is equivalent to open-circuiting it. The superposition principle reveals the contribution of each source to the voltages and currents in the circuit. It makes it easier to interpret the response of the circuit because we can trace the source of the response.

A voltage source with a series resistor is interchangeable with a current source in parallel to a resistor. This is called *source transformation.* The resistance value of both circuits is the same. The source transformation can be used to simplify the given circuit to find the desired voltages and currents.

According to Thévenin's theorem, a given circuit is equivalent to a voltage source V_{th} and a series resistor R_{th} between terminals a and b. The Thévenin equivalent voltage V_{th} can be obtained by finding the open-circuit voltage V_{oc} across a and b without modifying the circuit. The Thévenin equivalent resistance R_{th} can be found in one of the three methods. The first method is to find the equivalent resistance seen from a and b after deactivating the independent sources. The first method can be used only when a circuit does not contain dependent sources. The second method is to find the open-circuit voltage V_{oc} across a and b without modifying the circuit and to find the short-circuit current I_{sc} from a and b after connecting a and b by wire (a short-circuit). The Thévenin equivalent resistance R_{th} is the ratio of V_{oc} to I_{sc}; i.e., $R_{th} = V_{oc}/I_{sc}$. The third method is to apply a test voltage between terminals a and b after deactivating the independent sources and measure the current flowing out of the positive terminal of the test voltage source. The Thévenin equivalent resistance R_{th} is the ratio of the test voltage to the current flowing out of the positive terminal of the test voltage source. A test current source can be used instead of a test voltage source.

According to Norton's theorem, a given circuit is equivalent to a current source I_n and a parallel resistor R_n between terminals a and b. The Norton equivalent current I_n can be obtained by finding the short-circuit current I_{sc} from a to b after connecting a and b by wire. The Norton equivalent resistance R_n can be found in one of the three methods. The first method is to find the equivalent resistance seen from terminals a and b after deactivating the independent sources. The first method can be used only when a circuit does not contain dependent sources. The second method is to measure the open-circuit voltage V_{oc} between a and b without modifying the circuit. The Norton equivalent resistance R_n is the ratio of V_{oc} to I_{sc}; i.e., $R_n = V_{oc}/I_{sc}$. The third method is to apply a test voltage between terminals a and b after deactivating the independent sources and measure the current flowing out of the positive terminal of the test voltage source. The Norton equivalent resistance R_n is the ratio of the test voltage to the current flowing out of the positive terminal of the test voltage source. A test current source can be used instead of a test voltage source.

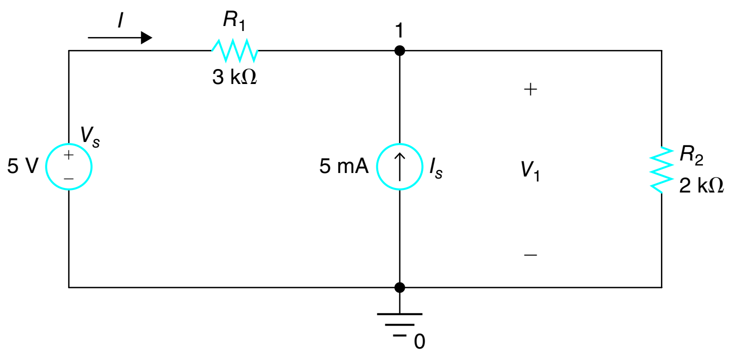
Suppose that a load with resistance R_L is connected to a circuit between terminals a and b. We can find the Thévenin equivalent circuit with respect to the terminals a and b excluding the load resistance. Let V_{th} be the Thévenin equivalent voltage and R_{th} be the Thévenin equivalent resistance. It can be shown that the load resistance R_L that maximizes the power delivered to the load is given by the Thévenin equivalent resistance R_{th}.

4.2 Superposition Principle

Suppose that a circuit has N independent sources with $N \geq 2$. Create N circuits from the original circuit with only one independent source by deactivating the other $N - 1$ independent sources. Deactivating a current source is to open-circuit it and deactivating a voltage source is to short-circuit it. The unknown voltages and currents of the original circuit can be found by adding the voltages and currents from the N circuits with one independent source. This is the superposition principle. The circuit shown in Figure 4.1 contains one voltage source and one current source. It is desired to find voltage V_1 across R_2, which is also the voltage across the current source, using the superposition principle.

FIGURE 4.1

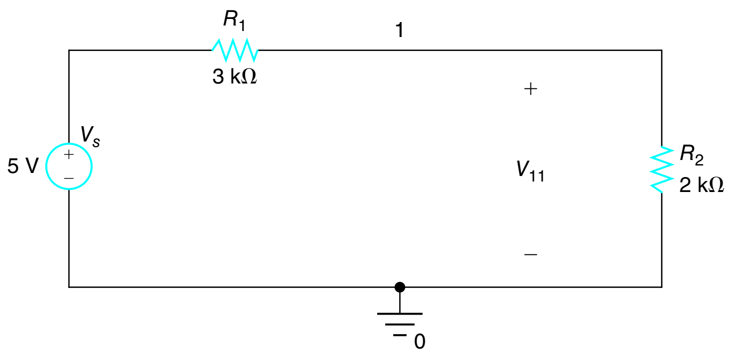
A circuit with a voltage source and a current source.



If the current from the current source is reduced to zero (that is, $I_s = 0$), the current source is open-circuited, and the circuit reduces to the one shown in Figure 4.2.

FIGURE 4.2

Circuit shown in Figure 4.1 with the current source deactivated.



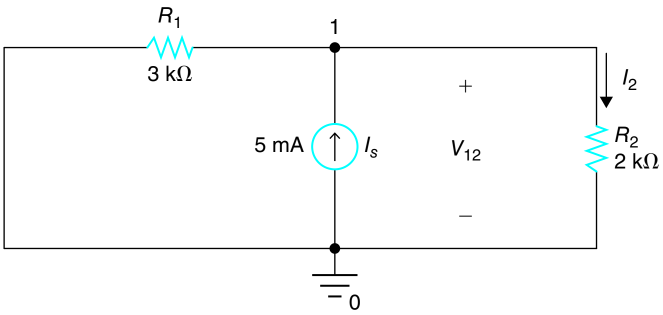
From the voltage divider rule, the voltage at node 1 is given by

$$V_{11} = \frac{R_2}{R_1 + R_2}V_s = \frac{2\ k\Omega}{3\ k\Omega\ + 2\ k\Omega} \times 5\ \text{V} = \frac{2}{5} \times 5\ \text{V} = 0.4 \times 5\ \text{V} = 2\ \text{V}$$

If the voltage from the voltage source is reduced to zero (that is, $V_s = 0$), the voltage source is short-circuited, and the circuit reduces to the one shown in Figure 4.3.

FIGURE 4.3

Circuit shown in Figure 4.1 with the voltage source deactivated.



From the current divider rule, the current through R_2 is given by

$$I_2 = \frac{\dfrac{1}{R_2}}{\dfrac{1}{R_2} + \dfrac{1}{R_1}} I_s = \frac{R_1}{R_1 + R_2} I_s = \frac{3000}{3000 + 2000} \times 5\ \text{mA} = 3\ \text{mA}$$

Thus, the voltage at node 1 is

$$V_{12} = R_2 I_2 = \frac{R_2 R_1}{R_1 + R_2} I_s = \frac{2000 \times 3000}{3000 + 2000}\,\Omega \times 5 \times 10^{-3}\,\text{A} = 1200\,\Omega \times 5 \times 10^{-3}\,\text{A} = 6\ \text{V}$$

The sum of V_{11} and V_{12} is given by

$$V_1 = V_{11} + V_{12} = \frac{R_2}{R_1 + R_2} V_s + \frac{R_2 R_1}{R_1 + R_2} I_s = 0.4 V_s + 1200 I_s = 2\ \text{V} + 6\ \text{V} = 8\ \text{V} \qquad (4.1)$$

As a check, let us find V_1 using nodal analysis. Summing the currents leaving node 1 in the circuit shown in Figure 4.1, we have

$$\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_2} - I_s = 0$$

This equation can be revised as follows:

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1 = \frac{V_s}{R_1} + I_s$$

Solving this equation for V_1, we have

$$V_1 = \frac{\dfrac{V_s}{R_1} + I_s}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{R_2}{R_1 + R_2} V_s + \frac{R_2 R_1}{R_1 + R_2} I_s = \frac{2000}{3000 + 2000} V_s + \frac{2000 \times 3000}{3000 = 2000} I_s \qquad (4.2)$$
$$= 0.4 V_s + 1200\, I_s = 0.4 \times 5\ \text{V} + 1200 \times 5 \times 10^{-3}\ \text{V} = 2\ \text{V} + 6\ \text{V} = 8\ \text{V}$$

Comparison of Equations (4.1) and (4.2) reveals that the answer derived from the superposition principle matches the one from the nodal analysis. This proves that the superposition principle works for the circuit shown in Figure 4.1. In general, if the circuit is linear, the superposition principle holds. Circuits consisting of independent and dependent voltage sources and current sources along with resistors are all linear circuits. Thus, the superposition principle works for these circuits. Equations (4.1) and (4.2) show that the voltage at node 1, V_1, consists of two components. The first component of V_1,

$$V_{11} = \frac{R_2}{R_1 + R_2} V_s$$

is due to the voltage source V_s; and the second component of V_1,

$$V_{12} = \frac{R_2 R_1}{R_1 + R_2} I_s$$

is due to the current source I_s. The voltage at node 1, V_1, is the linear combination of two inputs V_s and I_s. The coefficients a_1 and a_2 in the representation,

$$V_1 = a_1 V_s + a_2 I_s \tag{4.3}$$

are given by

$$a_1 = \frac{R_2}{R_1 + R_2} = 0.4, \quad a_2 = \frac{R_2 R_1}{R_1 + R_2} = 1200\ \Omega \tag{4.4}$$

Equation (4.3) is called linear because the output, V_1, is a linear function of inputs V_s and I_s; that is, V_1 is proportional to V_s and I_s. The proportionality constants are a_1 and a_2. Notice that if I_s is set to zero in Equation (4.3), we obtain V_{11}, and if V_s is set to zero in Equation (4.3), we obtain V_{12}.

Let I be the current from the voltage source, as shown in Figure 4.1. The current I is given by

$$I = \frac{V_s - V_1}{R_1} = \frac{V_s - \dfrac{R_2}{R_1 + R_2} V_s - \dfrac{R_1 R_2}{R_1 + R_2} I_s}{R_1} = \frac{1}{R_1 + R_2} V_s - \frac{R_2}{R_1 + R_2} I_s \tag{4.5}$$

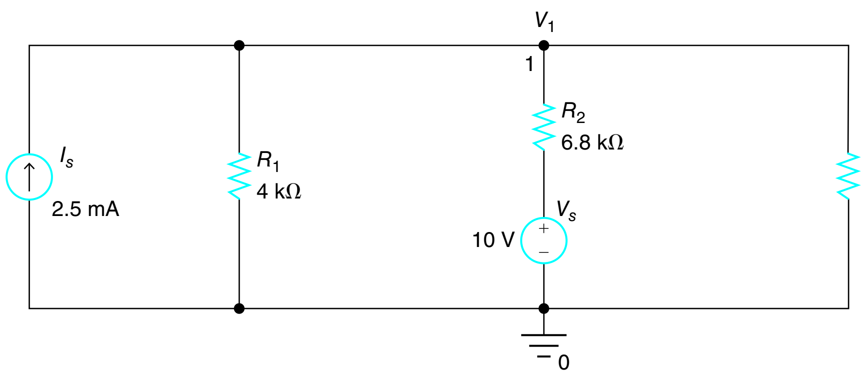
The current I is a linear combination of the two inputs V_s and I_s.

EXAMPLE 4.1

Use the superposition principle to find voltage V_1 in the circuit shown in Figure 4.4.

FIGURE 4.4

Circuit for EXAMPLE 4.1.



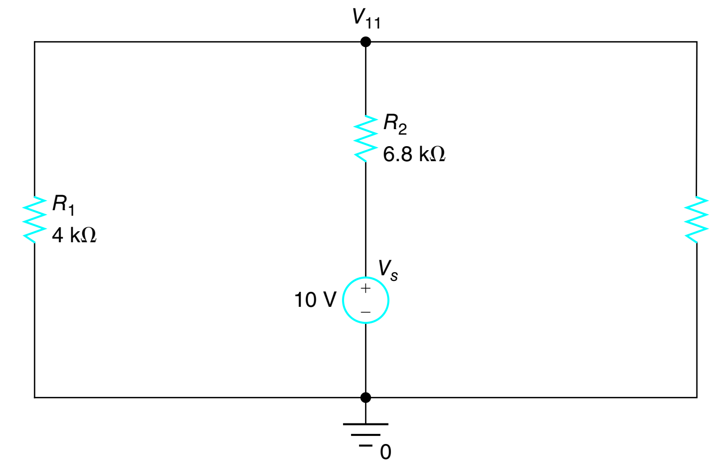
continued

Example 4.1 continued

When the current source is deactivated by open-circuiting it, the circuit shown in Figure 4.4 reduces to the one shown in Figure 4.5.

FIGURE 4.5

The circuit in Figure 4.4, with the current source deactivated by open-circuiting.



The equivalent resistance of the parallel connection of R_1 and R_3 is given by

$$R_a = R_1 \| R_3 = \frac{R_1 R_3}{R_1 + R_3} = \frac{4\ k\Omega \times 16\ k\Omega}{4\ k\Omega + 16\ k\Omega} = \frac{64}{20} k\Omega = 3.2\ k\Omega$$

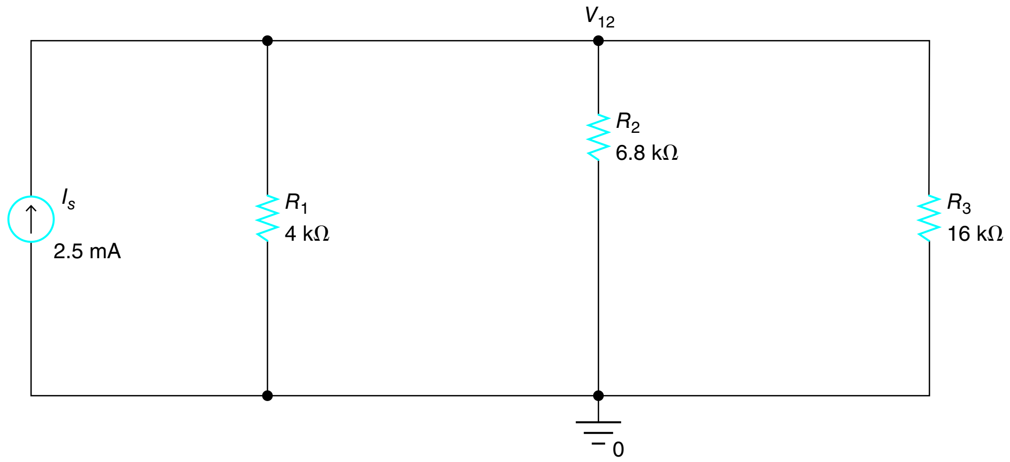
Application of the voltage divider rule yields

$$V_{11} = \frac{R_a}{R_2 + R_a} V_s = \frac{3.2\ k\Omega}{6.8\ k\Omega + 3.2\ k\Omega} \times 10\ \text{V} = \frac{3.2}{10} \times 10\ \text{V} = 3.2\ \text{V}$$

When the voltage source is deactivated by short-circuiting it, the circuit shown in Figure 4.4 reduces to the one shown in Figure 4.6.

FIGURE 4.6

The circuit in Figure 4.4, with the current source deactivated by closed-circuiting.



The equivalent resistance of the parallel connection of R_1, R_3, and R_2 is given by

$$R_b = R_a \| R_2 = \frac{R_a R_2}{R_a + R_2} = \frac{3.2\ k\Omega \times 6.8\ k\Omega}{3.2\ k\Omega + 6.8\ k\Omega} = 2.176\ k\Omega$$

continued

Example 4.1 continued

Voltage V_{12} is the product of R_b and I_s. Thus, we have

$$V_{12} = R_b I_s = 2176\,\Omega \times 2.5 \times 10^{-3}\,\text{A} = 5.44\text{ V}$$

Voltage V_1 is the sum of V_{11} and V_{12}:

$$V_1 = V_{11} + V_{12} = 3.2\text{ V} + 5.44\text{ V} = 8.64\text{ V}$$

As a check, we can find V_1 directly from the circuit shown in Figure 4.4 by applying nodal analysis. Summing the currents leaving node 1, we obtain

$$-2.5 \times 10^{-3} + \frac{V_1}{4000} + \frac{V_1 - 10}{6800} + \frac{V_1}{16{,}000} = 0$$

which can be rearranged as

$$\left(\frac{1}{4000} + \frac{1}{6800} + \frac{1}{16{,}000}\right)V_1 = 2.5 \times 10^{-3} + \frac{10}{6800}$$

Solving for V_1, we obtain

$$V_1 = \frac{2.5 \times 10^{-3} + \dfrac{10}{6800}}{\dfrac{1}{4000} + \dfrac{1}{6800} + \dfrac{1}{16{,}000}} = \frac{3.9705882353 \times 10^{-3}}{4.5955882353 \times 10^{-4}} = 8.64\text{ V}$$

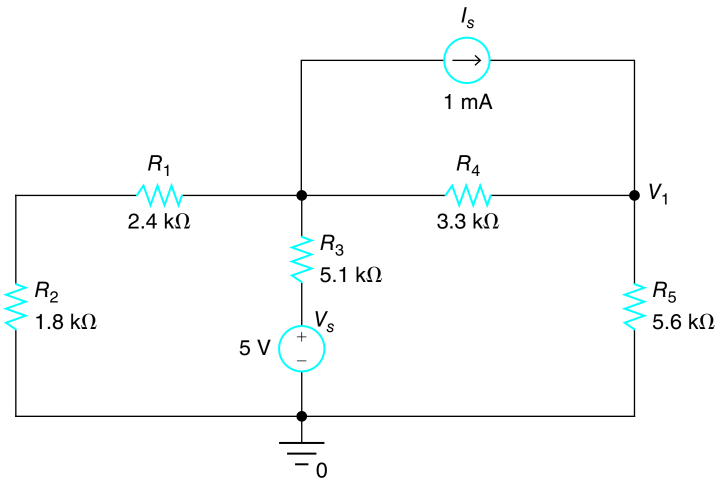
This value is the same as the one obtained from the superposition principle.

Exercise 4.1

Use the superposition principle to find voltage V_1 in the circuit shown in Figure 4.7.

FIGURE 4.7

Circuit for EXERCISE 4.1.



Answer:

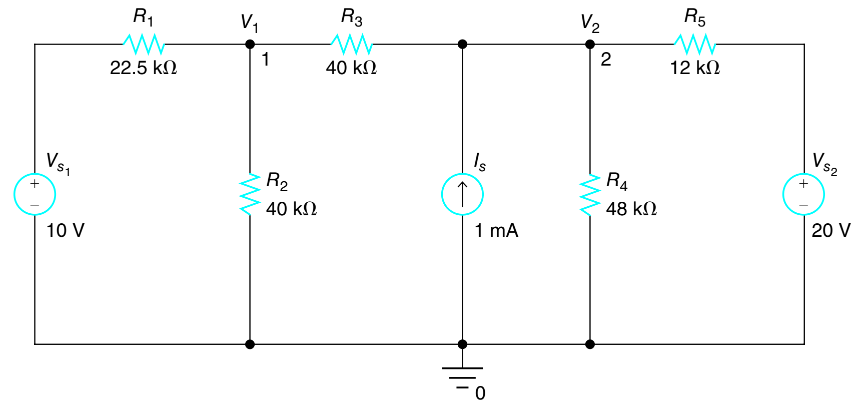
$V_1 = 2.7782$ V, 1.1287 V from V_s, 1.6495 V from I_s.

EXAMPLE 4.2

Use the superposition principle to find the voltage across R_2 in the circuit shown in Figure 4.8.

FIGURE 4.8

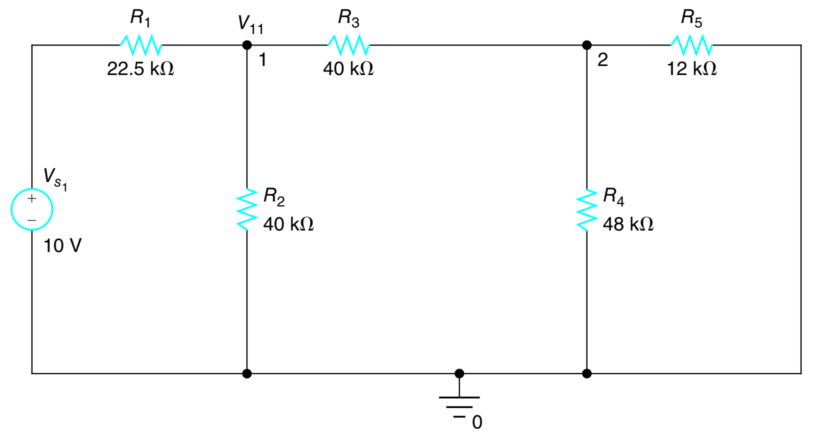
Circuit for EXAMPLE 4.2.



Deactivating the current source from the circuit shown in Figure 4.8 by removing it and also deactivating the voltage source V_{s_2} by short-circuiting it, we obtain the circuit shown in Figure 4.9.

FIGURE 4.9

The circuit shown in Figure 4.8 with I_s and V_{s_2} deactivated.



Let R_a be the equivalent resistance of R_2, R_3, R_4, and R_5 seen from R_1. Then, we have

$$R_a = R_2 \| (R_3 + (R_4 \| R_5)) = 40{,}000 \| \left(40{,}000 + \frac{48{,}000 \times 12{,}000}{48{,}000 + 12{,}000}\right)$$

$$= 40{,}000 \| (40{,}000 + 9600) = 40{,}000 \| 49{,}600 = \frac{40{,}000 \times 49{,}600}{40{,}000 + 49{,}600}$$

$$= \frac{1.984 \times 10^9}{89{,}600} = 22.142857\, k\Omega$$

Applying the voltage divider rule, we get voltage V_{11} at node 1:

$$V_{11} = V_{s_1} \times \frac{R_a}{R_1 + R_a} = 10 \text{ V} \times \frac{22.142857\ k\Omega}{22.5\ k\Omega + 22.142857\ k\Omega} = 4.96 \text{ V} \tag{4.6}$$

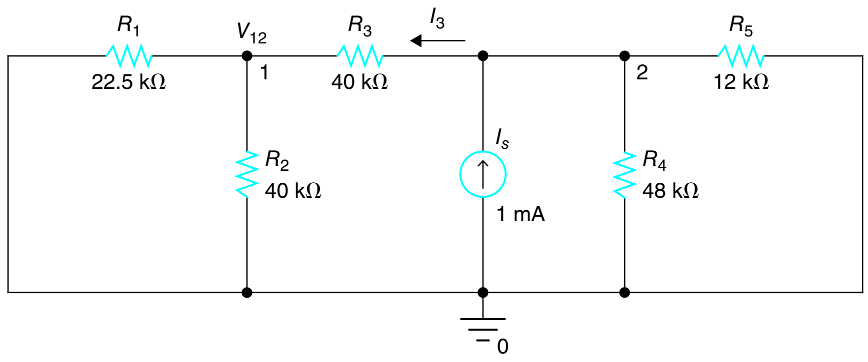
continued

Example 4.2 continued

Deactivating the voltage sources V_{s_1} and V_{s_2} from the circuit shown in Figure 4.8 by short-circuiting both of them, we obtain the circuit shown in Figure 4.10.

FIGURE 4.10

The circuit shown in Figure 4.8 with voltage sources V_{s_1} and V_{s_2} deactivated.



Let R_b be the equivalent resistance of the parallel connection of R_1 and R_2. Then, we have

$$R_b = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{22.5\ k\Omega \times 40\ k\Omega}{22.5\ k\Omega + 40\ k\Omega} = \frac{900}{62.5}\ k\Omega = 14.4\ k\Omega$$

Let R_c be the equivalent resistance of the series connection of R_3 and R_b. Then, we have

$$R_c = R_3 + R_b = 40\ k\Omega + 14.4\ k\Omega = 54.4\ k\Omega$$

Let R_d be the equivalent resistance of the parallel connection of R_4 and R_5. Then, we have

$$R_d = \frac{R_4 \times R_5}{R_4 + R_5} = \frac{48\ k\Omega \times 12\ k\Omega}{48\ k\Omega + 12\ k\Omega} = \frac{576}{60}\ k\Omega = 9.6\ k\Omega$$

From the current divider rule, the current through R_c is given by

$$I_3 = I_s \times \frac{R_d}{R_c + R_d} = 1\ \text{mA} \times \frac{9.6\ k\Omega}{54.4\ k\Omega + 9.6\ k\Omega} = 0.15\ \text{mA}$$

Voltage V_{12} across R_b is

$$V_{12} = I_3 \times R_b = 0.15 \times 10^{-3} \times 14{,}400\ \text{V} = 2.16\ \text{V} \tag{4.7}$$

Deactivating the current source from the circuit shown in Figure 4.8 by removing it and also deactivating the voltage source V_{s_1} by short-circuiting it, we obtain the circuit shown in Figure 4.11.

Let R_e be the equivalent resistance of the parallel connection of R_c and R_4. Then, we have

$$R_e = R_c \| R_4 = 54{,}400 \| 48{,}000 = \frac{54{,}400 \times 48{,}000}{54{,}400 + 48{,}000} = 25.5\ k\Omega$$

Applying the voltage divider rule, we get voltage V_{23} at node 2:

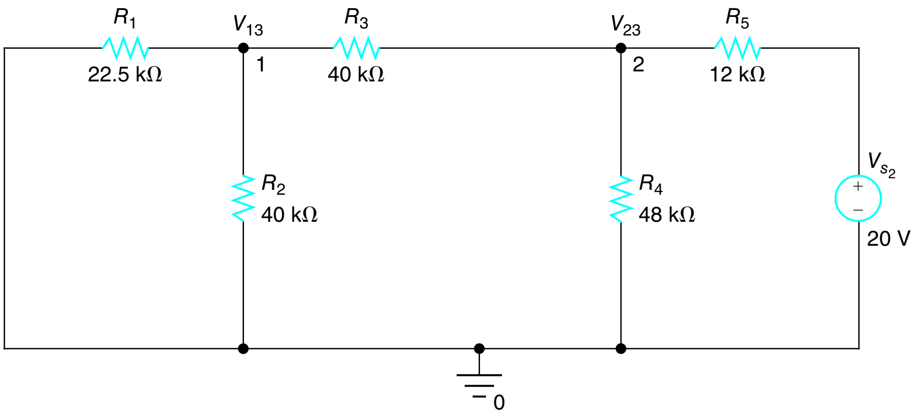
$$V_{23} = V_{s_2} \times \frac{R_e}{R_5 + R_e} = 20\ \text{V} \times \frac{25.5\ k\Omega}{25.5\ k\Omega + 12\ k\Omega} = 13.6\ \text{V}$$

continued

Example 4.2 continued

FIGURE 4.11

The circuit shown in Figure 4.8 with I_s and V_{s_1} deactivated.



Applying the voltage divider rule, we get voltage V_{13} at node 1:

$$V_{13} = V_{23} \times \frac{R_b}{R_c} = 13.6 \text{ V} \times \frac{14.4\ k\Omega}{54.4\ k\Omega} = 3.6 \text{ V} \tag{4.8}$$

Adding the three voltages, we obtain voltage V_1:

$$V_1 = V_{11} + V_{12} + V_{13} = 4.96 \text{ V} + 2.16 \text{ V} + 3.6 \text{ V} = 10.72 \text{ V}$$

To verify this answer, we can find voltage V_1 across R_2 directly from the original circuit shown in Figure 4.8 using nodal analysis, as shown in the MATLAB script given here:

MATLAB

```
%EXAMPLE 4.2
%Function P.m should be in the same folder as this file.
clear all;format long;
R1=22500;R2=40000;R3=40000;R4=48000;R5=12000;
Vs1=10;Vs2=20;Is=1e-3;
%V11 from Vs1
Ra=P([R2,R3+P([R4,R5])])
V11=Vs1*Ra/(Ra+R1)
%V12 from Is
Rb=P([R1,R2])
Rc=R3+Rb
Rd=P([R4,R5])
I3=Is*Rd/(Rc+Rd)
V12=Rb*I3
%V13 from Vs2
Re=P([Rc,R4])
V23=Vs2*Re/(R5+Re)
V13=V23*Rb/Rc
%Sum of V11, V12, V13
V1b=V11+V12+V13
%Check from nodal analysis
syms V1 V2
[V1,V2]=solve((V1-Vs1)/R1+V1/R2+(V1-V2)/R3,...
(V2-V1)/R3-Is+V2/R4+(V2-Vs2)/R5);
V1=vpa(V1,8)
V2=vpa(V2,8)
```

continued

Example 4.2 continued
MATLAB continued

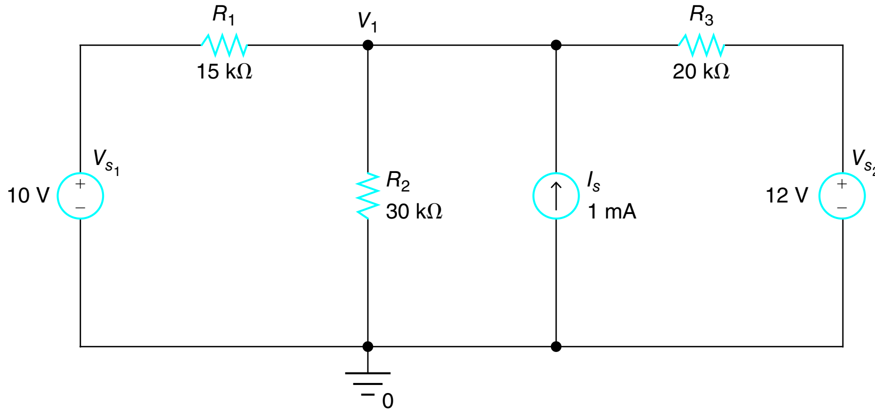
```
Answers:
Ra =
     2.214285714285715e+04
V11 =
   4.960000000000000
Rb =
       14400
Rc =
       54400
Rd =
        9600
I3 =
     1.500000000000000e-04
V12 =
   2.160000000000000
Re =
       25500
V23 =
  13.600000000000000
V13 =
   3.600000000000000
V1b =
  10.719999999999999
V1 =
10.72
V2 =
22.72
```

Exercise 4.2

Use the superposition principle to find voltage V_1 in the circuit shown in Figure 4.12.

FIGURE 4.12

Circuit for EXERCISE 4.2.



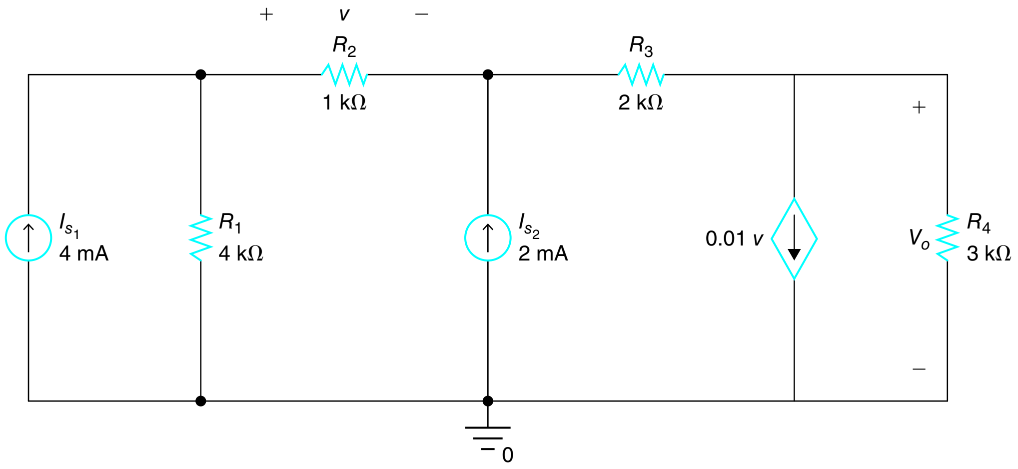
Answer:
$V_1 = 15.1111$ V. 4.4444 V from V_{s_1}, 4 V from V_{s_2}, 6.6667 V from I_s.

EXAMPLE 4.3

Use the superposition principle to find voltage V_0 across R_4 in the circuit shown in Figure 4.13.

FIGURE 4.13

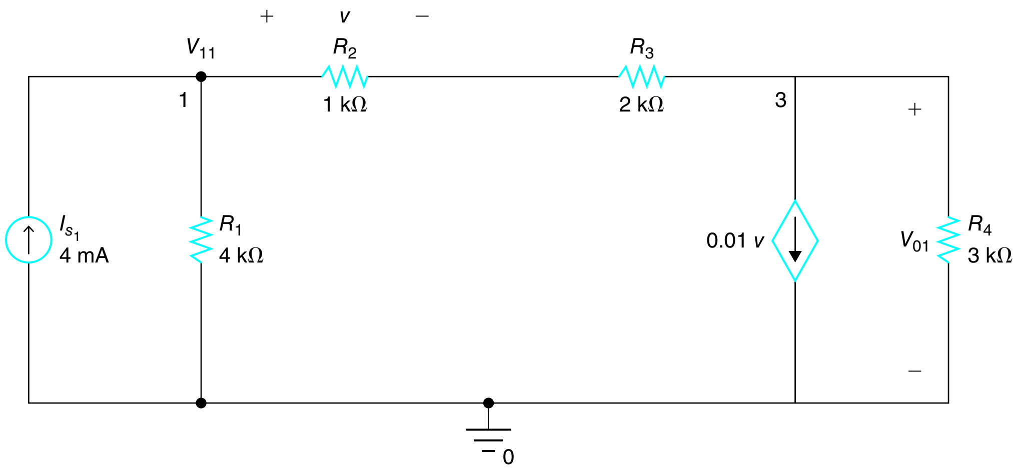
Circuit for EXAMPLE 4.3.



Deactivating the current source I_{s_2} from the circuit shown in Figure 4.13, we obtain the circuit shown in Figure 4.14.

FIGURE 4.14

The circuit shown in Figure 4.13 with the current source I_{s_2} deactivated.



Applying the voltage divider rule, we can find the controlling voltage v to be

$$v = (V_{11} - V_{01})\frac{R_2}{R_2 + R_3} = (V_{11} - V_{01})\frac{1}{3} \tag{4.9}$$

Summing the currents leaving node 1, we obtain

$$-0.004 + \frac{V_{11}}{4000} + \frac{V_{11} - V_{01}}{3000} = 0$$

continued

Example 4.3 continued

Multiplication by 12,000 yields

$$-48 + 3V_{11} + 4(V_{11} - V_{01}) = 0$$

which can be rearranged as

$$7V_{11} - 4V_{01} = 48 \tag{4.10}$$

Summing the currents leaving node 3, we get

$$\frac{V_{01} - V_{11}}{3000} + 0.01\frac{V_{11} - V_{01}}{3} + \frac{V_{01}}{3000} = 0$$

Multiplication by 3000 yields

$$V_{01} - V_{11} + 10(V_{11} - V_{01}) + V_{01} = 0$$

which can be rearranged as

$$9V_{11} - 8V_{01} = 0 \tag{4.11}$$

Solving Equation (4.11) for V_{11}, we obtain

$$V_{11} = \frac{8}{9}V_{01} \tag{4.12}$$

Substitution of Equation (4.12) into Equation (4.10) yields

$$7\frac{8}{9}V_{01} - 4V_{01} = \frac{20}{9}V_{01} = 48$$

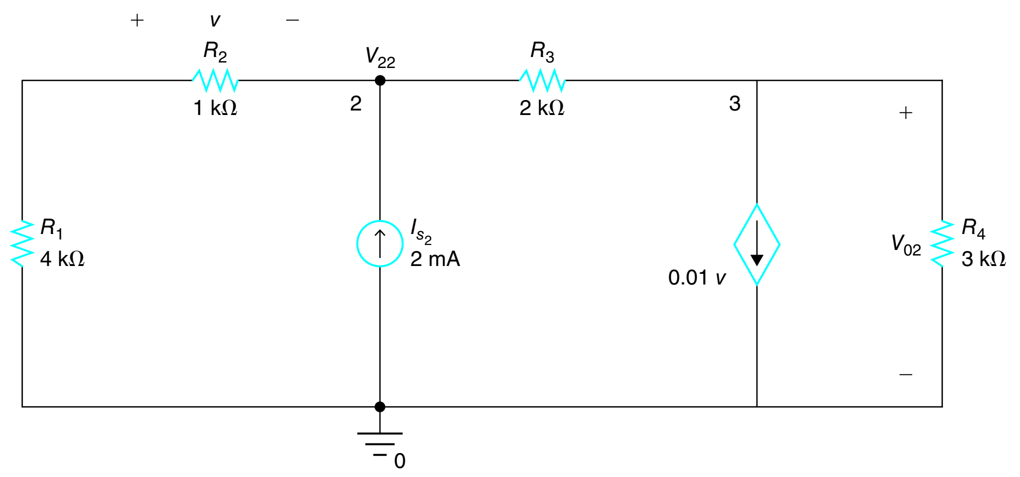
Thus, voltage V_{01} across R_4 from I_{s_1} is given by

$$V_{01} = 48\frac{9}{20} = \frac{108}{5} = 21.6 \text{ V} \tag{4.13}$$

Deactivating the current source I_{s_1} from the circuit shown in Figure 4.13, we obtain the circuit shown in Figure 4.15.

FIGURE 4.15

Circuit shown in Figure 4.13 with the current source I_{s_1} deactivated.



continued

Example 4.3 continued

Applying the voltage divider rule, we can find the controlling voltage v to be

$$v = -V_{22}\frac{R_2}{R_1 + R_2} = \frac{-V_{22}}{5} \tag{4.14}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_{22}}{5000} - 0.002 + \frac{V_{22} - V_{02}}{2000} = 0$$

Multiplication by 10,000 yields

$$2V_{22} - 20 + 5(V_{22} - V_{02}) = 0$$

which can be rearranged as

$$7V_{22} - 5V_{02} = 20 \tag{4.15}$$

Summing the currents leaving node 3, we get

$$\frac{V_{02} - V_{22}}{2000} + 0.01\frac{-V_{22}}{5} + \frac{V_{02}}{3000} = 0$$

Multiplication by 6000 yields

$$3V_{02} - 3V_{22} - 12V_{22} + 2V_{02} = 0$$

which can be rearranged as

$$-15V_{22} + 5V_{02} = 0 \tag{4.16}$$

Solving Equation (4.16) for V_{22}, we obtain

$$V_{22} = \frac{1}{3}V_{02} \tag{4.17}$$

Substitution of Equation (4.17) into Equation (4.15) yields

$$7\frac{1}{3}V_{02} - 5V_{02} = -\frac{8}{3}V_{02} = 20$$

Thus, voltage V_{02} across R_4 from I_{s_2} is given by

$$V_{02} = -20\frac{3}{8} = -\frac{60}{8} = -7.5\text{ V} \tag{4.18}$$

Voltage V_0 across R_4 is the sum of V_{01} and V_{02}. From Equations (4.13) and (4.18), we obtain

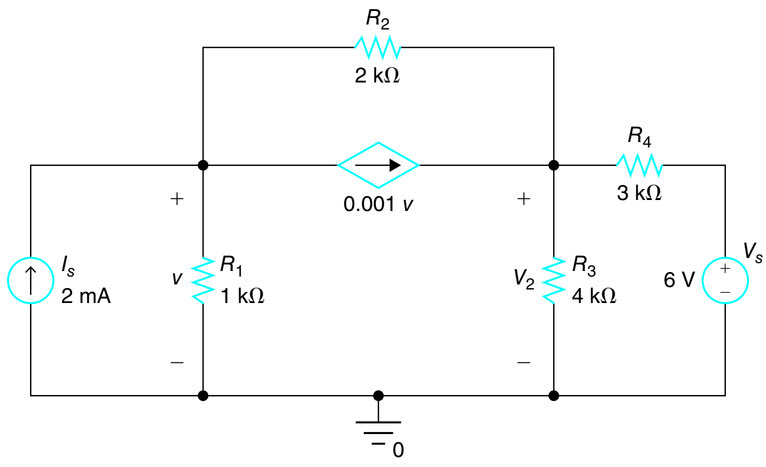
$$V_0 = V_{01} + V_{02} = 14.1\text{ V}$$

Exercise 4.3

Use the superposition principle to find voltage V_2 in the circuit shown in Figure 4.16.

FIGURE 4.16

Circuit for EXERCISE 4.3.



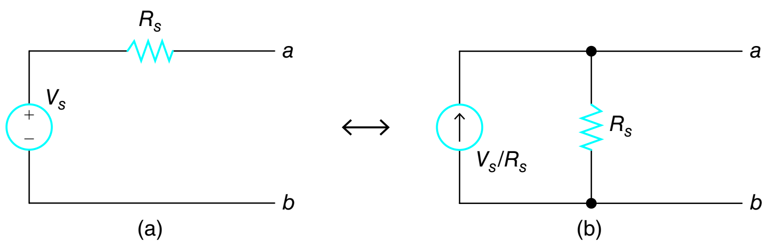
Answer:
$V_2 = 4.0851$ V. 1.5319 V from I_s, 2.5532 V from V_s.

4.3 Source Transformations

A circuit consisting of a voltage source with voltage V_s and a series resistor with resistance R_s, as shown in Figure 4.17(a), is equivalent to a circuit consisting of a current source with current V_s/R_s and a parallel resistor with resistance R_s, as shown in Figure 4.17(b). *Equivalence* means that the circuit shown in Figure 4.17(a) and the circuit shown in Figure 4.17(b) have the same open-circuit voltage across a and b, the same short-circuit current through a and b, and the same resistance looking into the circuit from a and b after deactivating the source. The open-circuit voltage across a and b for the circuit shown in Figure 4.17(a) is V_s, and that for the circuit shown in Figure 4.17(b) is $R_s \times (V_s/R_s) = V_s$. The short-circuit current (with a and b connected by wire) through a and b for the circuit shown in Figure 4.17(a) is V_s/R_s, and that for the circuit shown in Figure 4.17(b) is V_s/R_s based on the current divider rule. After short-circuiting V_s, the resistance across a and b for the circuit shown in Figure 4.17(a) is R_s, and that for the circuit shown in Figure 4.17(b) after open-circuiting the current source is R_s. The circuit shown in Figure 4.17(a) can be replaced by the circuit shown in Figure 4.17(b).

FIGURE 4.17

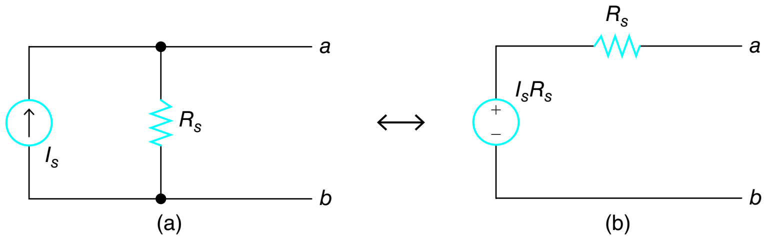
A voltage source and a series resistor are equivalent to a current source and a parallel resistor.



Also, the circuit shown in Figure 4.18(a) is equivalent to the circuit shown in Figure 4.18(b). The circuit shown in Figure 4.18(a) can be replaced by the circuit shown in Figure 4.18(b).

FIGURE 4.18

A current source and a parallel resistor are equivalent to a voltage source and a series resistor.



The source transformations apply to dependent sources as well. Figures 4.19 and 4.20 show the equivalence of a voltage source and a series resistor, and a current source and a parallel resistor.

FIGURE 4.19

A dependent voltage source and a series resistor are equivalent to a dependent current source and a parallel resistor.

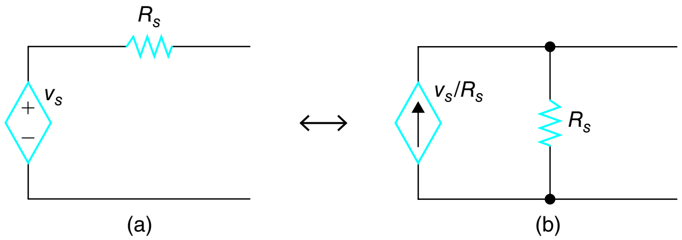


FIGURE 4.20

A dependent current source and a parallel resistor are equivalent to a dependent voltage source and a series resistor.

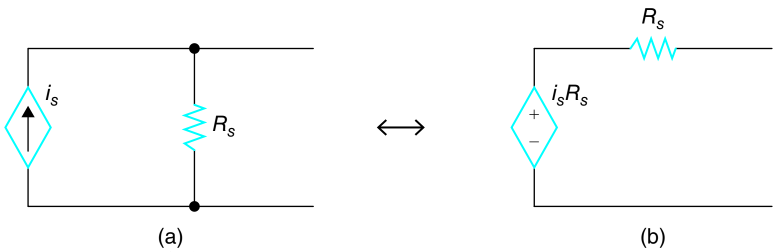


Figure 4.21(a) shows a circuit with a voltage source and a series resistor connected to the rest of the circuit. The voltage v is the voltage at the input of the rest of the circuit, and i is the current into the rest of the circuit. Writing a mesh equation in the clockwise direction, we obtain

$$-v_s + R_s i + v = 0 \tag{4.19}$$

which can be rewritten as

$$v = v_s - R_s i \tag{4.20}$$

Solving Equation (4.20) for i, we get

$$i = \frac{v_s}{R_s} - \frac{v}{R_s} \tag{4.21}$$

The first term, v_s/R_s, on the right side of Equation (4.21) represents a current source with current v_s/R_s, and the second term, v/R_s, on the right side of Equation (4.21) represents a current through a resistor with resistance R_s whose voltage is v. Based on Equation (4.21), we can draw an equivalent circuit, as shown in Figure 4.21(b). Writing a node equation at node 1 of the circuit shown in Figure 4.21(b) as a check, we obtain Equation (4.21). Rearrangement of Equation (4.21) results in Equation (4.20). This shows the equivalence of the circuits shown in Figures 4.21(a–b).

Consider a circuit shown in Figure 4.22. We are interested in finding voltage V_o across R_5 using source transformation.

FIGURE 4.21

Circuit showing proof of equivalence.

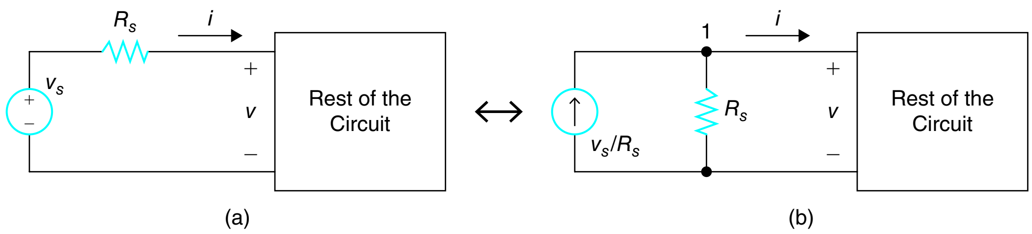
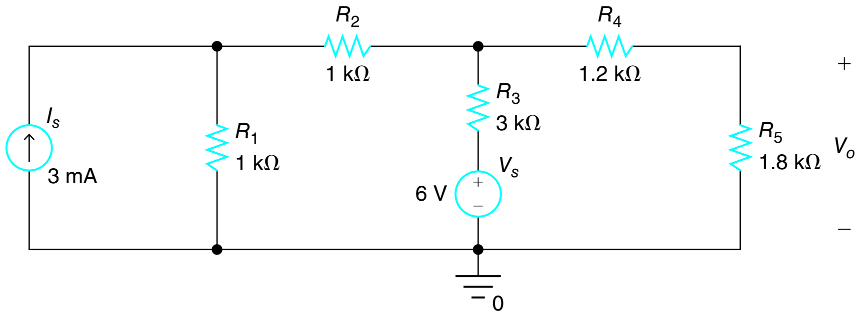


FIGURE 4.22

A circuit used to illustrate source transformation.



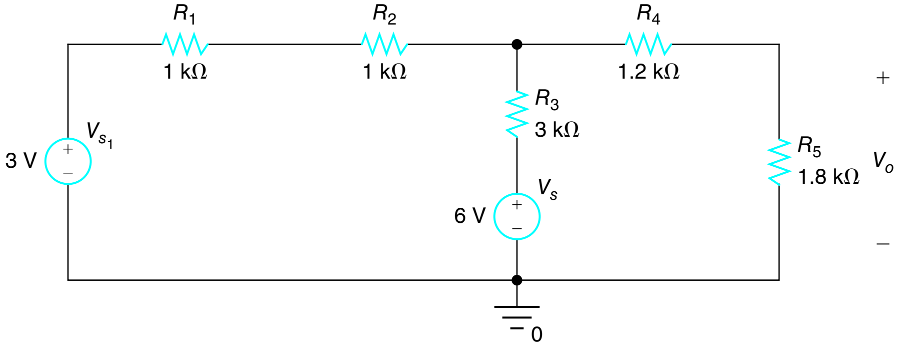
The current source I_s and the parallel resistor R_1 can be transformed into a voltage source with voltage

$$V_{s_1} = R_1 I_s = 1000 \times 3 \times 10^{-3} = 3\text{ V}$$

and a series resistor R_1, as shown in Figure 4.23.

FIGURE 4.23

I_s and R_1 are transformed into V_{s_1} and R_1.



Let R_a be the sum of R_1 and R_2. Then, we have

$$R_a = R_1 + R_2 = 1\ k\Omega + 1\ k\Omega = 2\ k\Omega$$

The voltage source V_{s_1} and the series resistor R_a can be transformed into a current source with current

$$I_{s_1} = \frac{V_{s_1}}{R_a} = \frac{3\text{ V}}{2\ k\Omega} = 1.5\text{ mA}$$

and a parallel resistor R_a, as shown in Figure 4.24. Similarly, the voltage source V_s and the series resistor R_3 can be transformed into a current source with current

$$I_{s_2} = \frac{V_s}{R_3} = \frac{6\text{ V}}{3\ k\Omega} = 2\text{ mA}$$

and a parallel resistor R_3, as shown in Figure 4.24. Notice that the direction of I_{s_2} is identical to the direction of I_{s_1}.

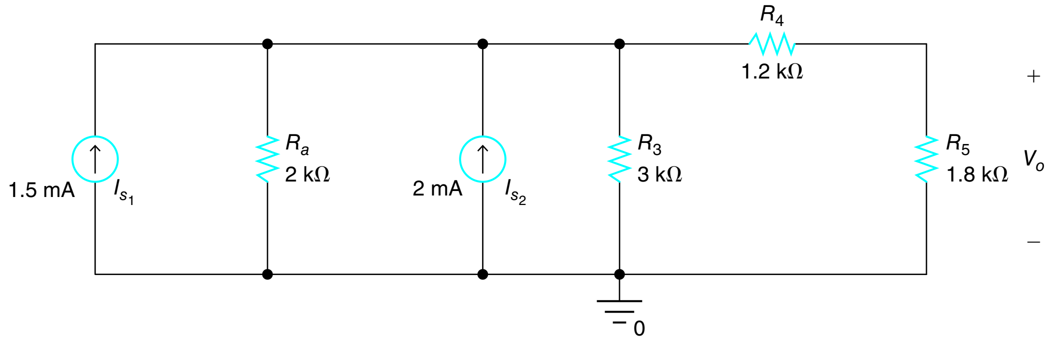


FIGURE 4.24 A circuit after source transformations.

The combined current from the two parallel current sources is

$$I_{s_3} = I_{s_1} + I_{s_2} = 1.5 \text{ mA} + 2 \text{ mA} = 3.5 \text{ mA}$$

The equivalent resistance of the parallel connection of R_a and R_3 is given by

$$R_b = R_a \| R_3 = \frac{R_a \times R_3}{R_a + R_3} = \frac{2000 \times 3000}{2000 + 3000} = 1.2\ k\Omega$$

Replacing the two current sources I_{s_1} and I_{s_2} and two parallel resistors R_a and R_3 by current source I_{s_3} and parallel resistor R_b, we obtain the circuit shown in Figure 4.25.

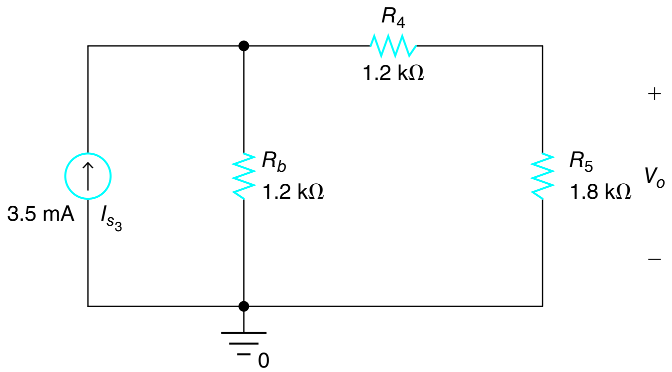


FIGURE 4.25 The circuit in Figure 4.24 with one current source.

The current source I_{s_3} and the parallel resistor R_b can be transformed into a voltage source with voltage

$$V_{s_2} = I_{s_3} \times R_b = 3.5 \times 10^{-3} \times 1200 = 4.2 \text{ V}$$

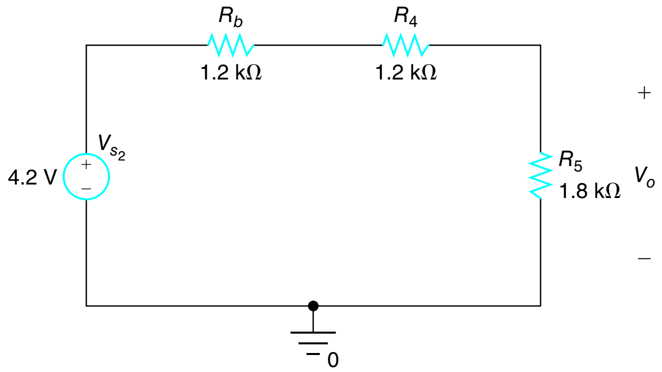
and a series resistor with resistance $R_b = 1.2\ k\Omega$. The circuit shown in Figure 4.25 becomes the one shown in Figure 4.26.

Application of the voltage divider rule to the circuit shown in Figure 4.26 yields

$$V_o = V_{s_2} \times \frac{R_5}{R_b + R_4 + R_5} = 4.2 \text{ V} \times \frac{1.8\ k\Omega}{1.2\ k\Omega + 1.2\ k\Omega + 1.8\ k\Omega}$$
$$= 4.2 \text{ V} \times \frac{1.8}{4.2} = 1.8 \text{ V}$$

FIGURE 4.26

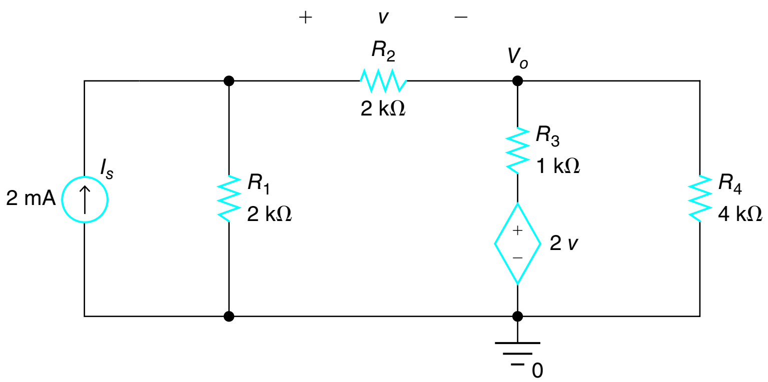
The circuit in Figure 4.25 with one voltage source.



Consider the circuit shown in Figure 4.27. We are interested in finding the voltage V_o using source transformation.

FIGURE 4.27

A circuit with a VCVS.



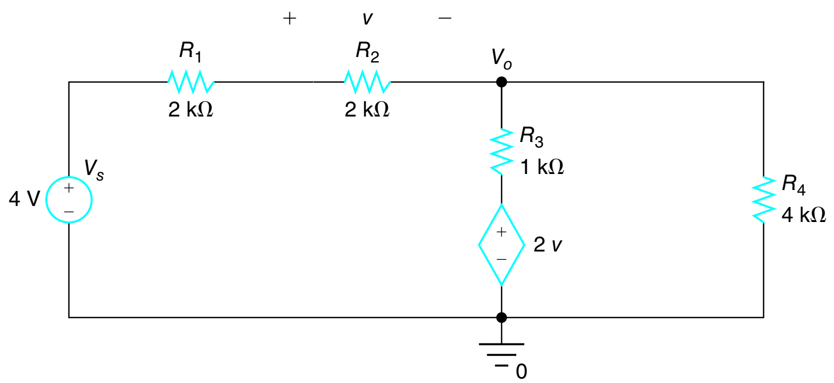
The current source I_s and parallel resistor R_1 can be transformed into a voltage source V_s with voltage

$$V_s = R_1 \times I_s = 2\ k\Omega \times 2\ \text{mA} = 4\ \text{V}$$

and a series resistor R_1, as shown in Figure 4.28.

FIGURE 4.28

I_s and R_1 are transformed into a voltage source V_s and a series resistor R_1.



The voltage-controlled voltage source (VCVS) and R_3 can be transformed into a voltage-controlled current source (VCCS) with current $\dfrac{2v}{R_3}$ and a parallel resistor R_3, as shown in Figure 4.29.

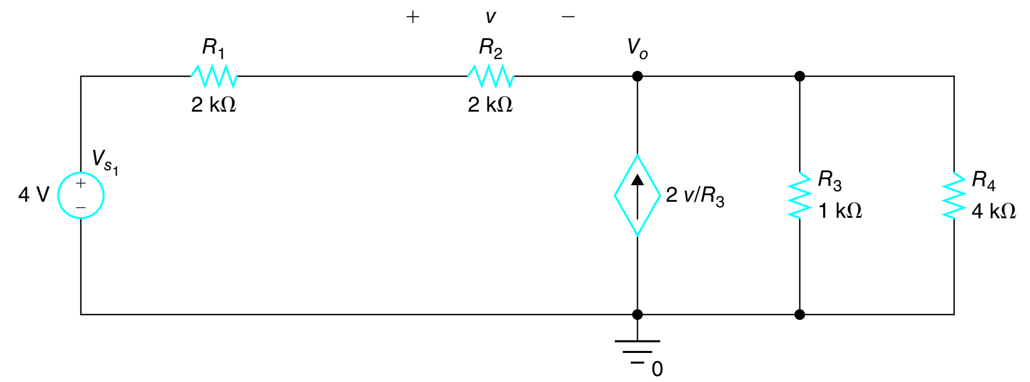


FIGURE 4.29

A VCVS and R_3 are transformed into a VCCS and a parallel resistor R_3.

Let R_a be the equivalent resistance of the parallel connection of R_3 and R_4. Then, we have

$$R_a = R_3 \| R_4 = 1\ k\Omega \| 4\ k\Omega = 0.8\ k\Omega$$

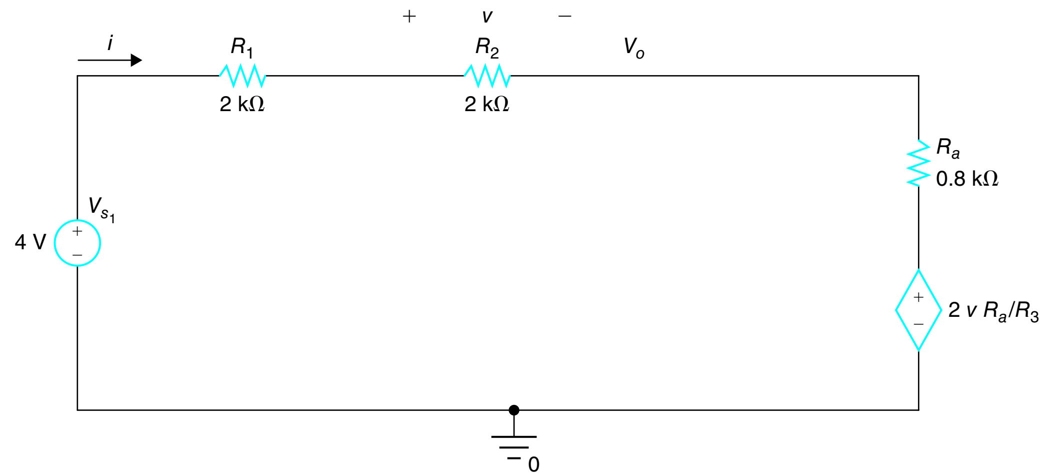


FIGURE 4.30

A VCCS and R_a are transformed into a VCVS and a series resistor R_a.

The VCCS and a parallel resistor R_a can be transformed into a VCVS with voltage $\frac{2vR_a}{R_3}$ and a series resistor R_a, as shown in Figure 4.30.

The original circuit is transformed into a single mesh. Let the mesh current be i. Then, the controlling voltage v is given by

$$v = R_2 i$$

Collecting the voltage drops around the mesh in the clockwise direction, we obtain

$$-R_1 I_s + R_1 i + R_2 i + R_a i + \frac{2R_2R_a}{R_3} i = 0$$

Solving for i, we obtain

$$i = \frac{R_1 I_s}{R_1 + R_2 + R_a + \frac{2R_2R_a}{R_3}} = \frac{4\ \mathrm{V}}{2\ k\Omega + 2\ k\Omega + 0.8\ k\Omega + 3.2\ k\Omega} = 0.5\ \mathrm{mA}$$

Voltage V_o is given by

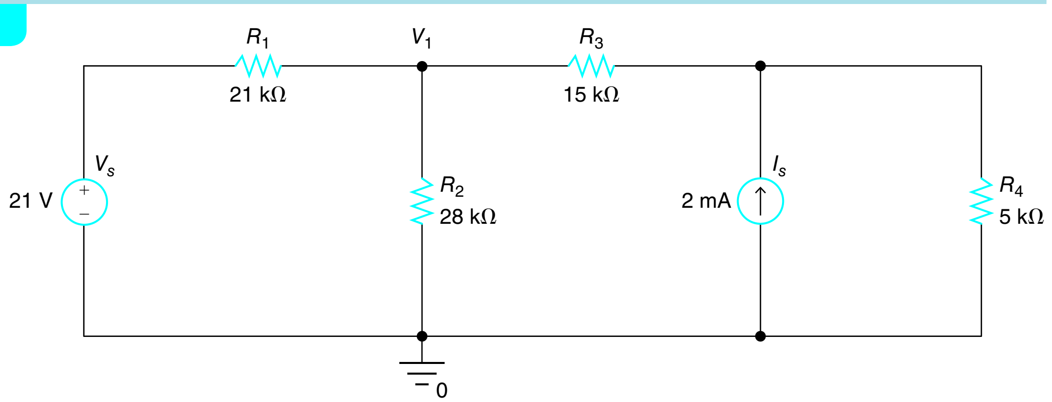
$$V_o = R_a i + \frac{2R_2R_a}{R_3} i = 2 \text{ V}$$

EXAMPLE 4.4

Find voltage V_1 using source transformation for the circuit shown in Figure 4.31.

FIGURE 4.31

Circuit for EXAMPLE 4.4.



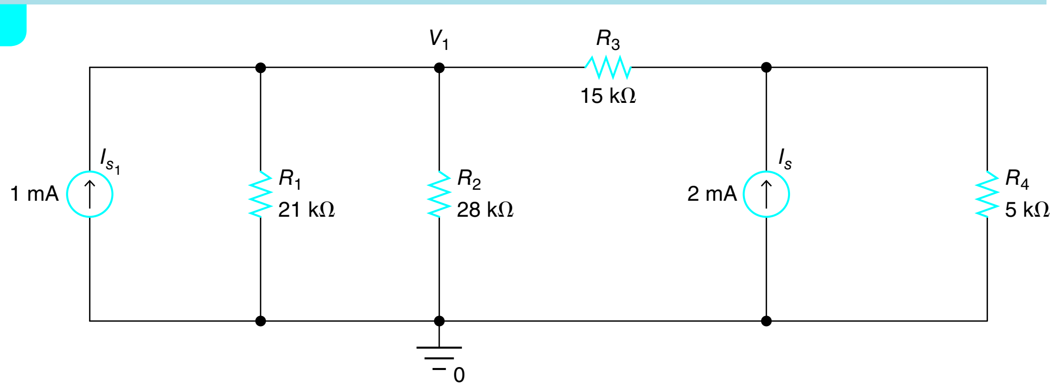
The voltage source V_s and a series resistor R_1 can be transformed into a current source with the current:

$$I_{s_1} = \frac{V_s}{R_1} = \frac{21 \text{ V}}{21\ k\Omega} = 1 \text{ mA}$$

and a parallel resistor R_1, as shown in Figure 4.32.

FIGURE 4.32

The series connection of V_s and R_1 is transformed into a parallel connection of I_{s_1} and R_1.



The equivalent resistance of parallel connection of R_1 and R_2 is given by

$$R_a = R_1 \| R_2 = 21\ k\Omega \| 28\ k\Omega = 12\ k\Omega$$

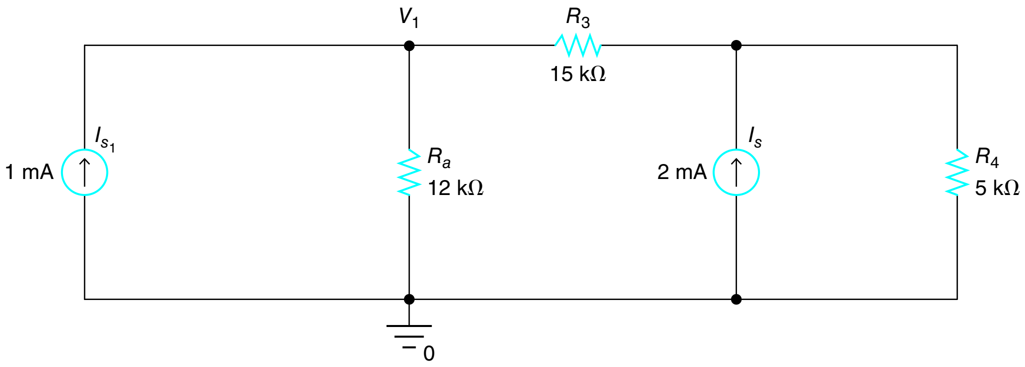
Figure 4.33 shows a circuit with I_{s_1} in parallel with R_a.

continued

Example 4.4 continued

FIGURE 4.33

A circuit with a parallel connection of I_{s_1} and R_a.



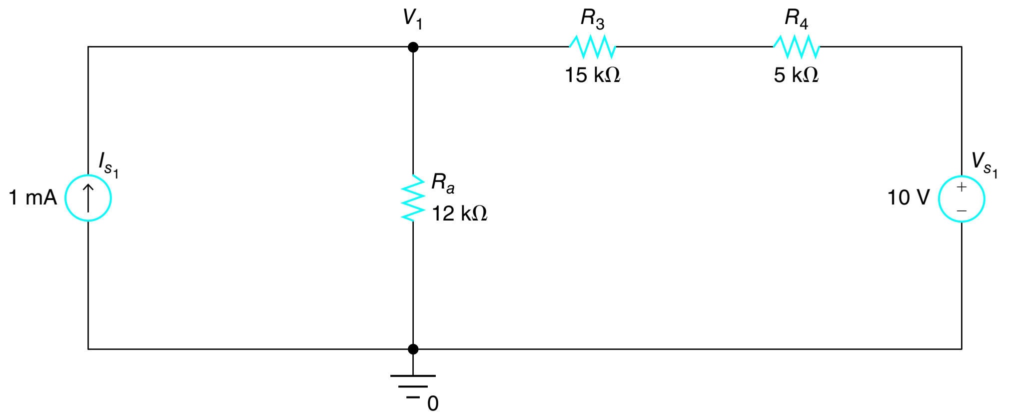
The parallel connection of I_s and R_4 can be transformed into a voltage source with the following voltage:

$$V_{s_1} = R_4 \times I_s = 5\ k\Omega \times 2\ \text{mA} = 10\ \text{V}$$

and a series resistor R_4, as shown in Figure 4.34.

FIGURE 4.34

The circuit in Figure 4.33 with a series connection of V_{s_1} and R_4.



Let R_b be the equivalent resistance of the series connection of R_3 and R_4. Then, we have

$$R_b = R_3 + R_4 = 20\ k\Omega$$

The series connection of V_{s_1} and R_b can be transformed into a parallel connection of a current source with current

$$I_{s_2} = \frac{V_{s_1}}{R_b} = \frac{10\ \text{V}}{20\ k\Omega} = 0.5\ \text{mA}$$

and a parallel resistor R_b, as shown in Figure 4.35.

Let R_c be the equivalent resistance of the parallel connection on R_a and R_b. Then, we have

$$R_c = R_a \| R_b = 12\ k\Omega \| 20\ k\Omega = 7.5\ k\Omega$$

continued

Example 4.4 continued

FIGURE 4.35

The circuit in Figure 4.35 with a parallel connection of I_{s_2} and R_b.

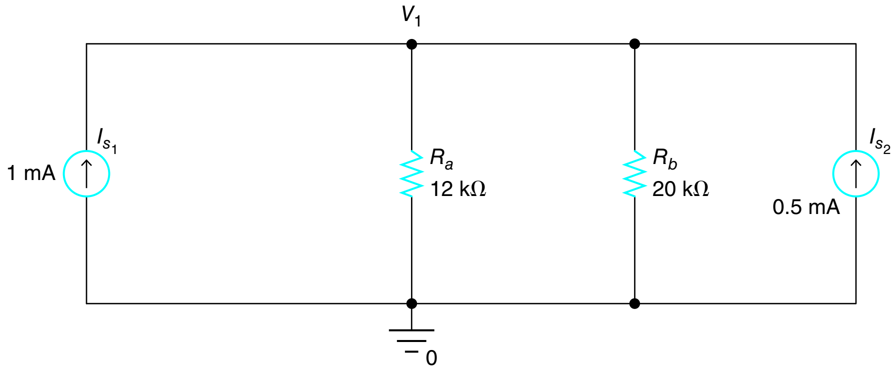
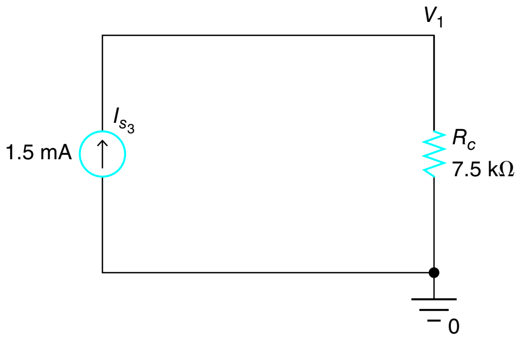


FIGURE 4.36

A circuit with a parallel connection of I_{s_3} and R_c.



The two current sources can be combined into a single current source with the current given by

$$I_{s_3} = I_{s_1} + I_{s_2} = 1 \text{ mA} + 0.5 \text{ mA} = 1.5 \text{ mA}$$

The circuit with a current source I_{s_3} and a parallel resistor R_c is shown in Figure 4.36.

Voltage V_1 is given by

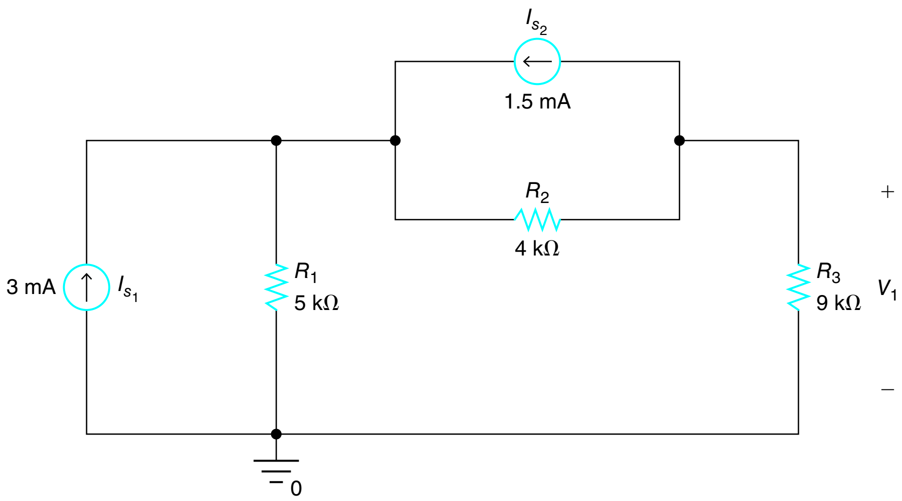
$$V_1 = R_c \times I_{s_3} = 7.5 \text{ k}\Omega \times 1.5 \text{ mA} = 11.25 \text{ V}$$

Exercise 4.4

Find voltage V_1 using source transformation for the circuit shown in Figure 4.37.

FIGURE 4.37

Circuit for EXERCISE 4.4.



Answer:

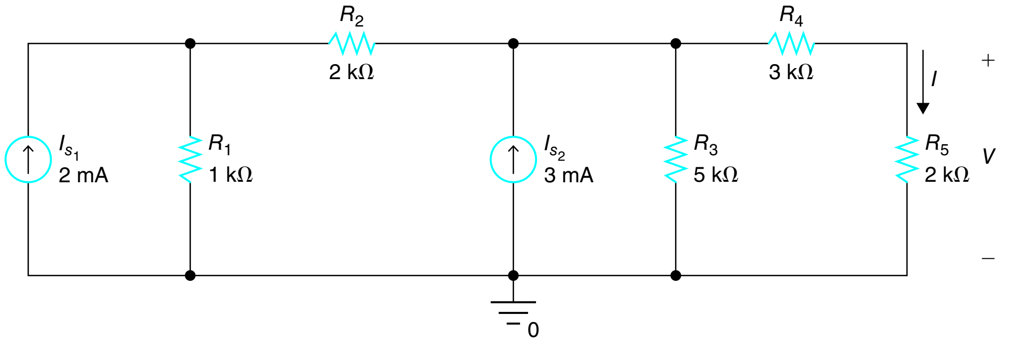
$V_1 = 4.5$ V.

EXAMPLE 4.5

Find voltage V and current I using source transformation for the circuit shown in Figure 4.38.

FIGURE 4.38

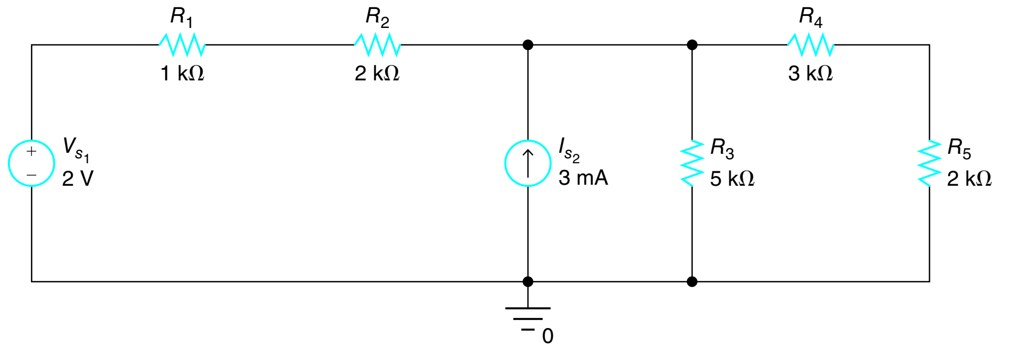
Circuit for EXAMPLE 4.5.



Current source I_{s_1} and resistor R_1 can be transformed into a voltage source with voltage $V_{s_1} = R_1 I_{s_1} = 1\ k\Omega \times 2\text{ mA} = 2\text{ V}$ in series with a resistor R_1 with resistance $1\ k\Omega$, as shown in Figure 4.39.

FIGURE 4.39

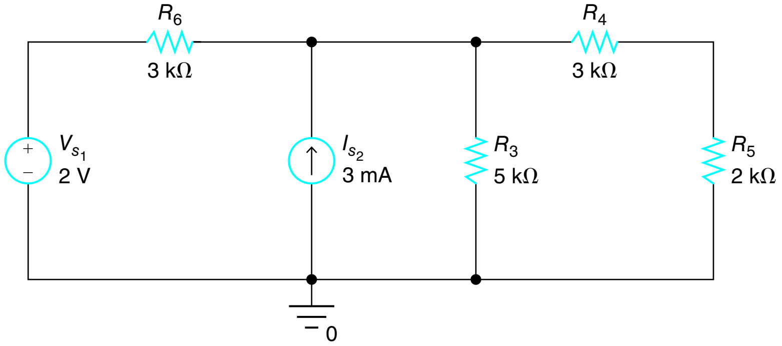
The circuit in Figure 4.38 with I_{s_1} and R_1 replaced by V_{s_1} and R_1.



The equivalent resistance of R_1 and R_2 is given by $R_6 = R_1 + R_2 = 3\ k\Omega$. Figure 4.40 shows the circuit with R_6.

FIGURE 4.40

The circuit in Figure 4.39 with $R_6 = R_1 + R_2$.



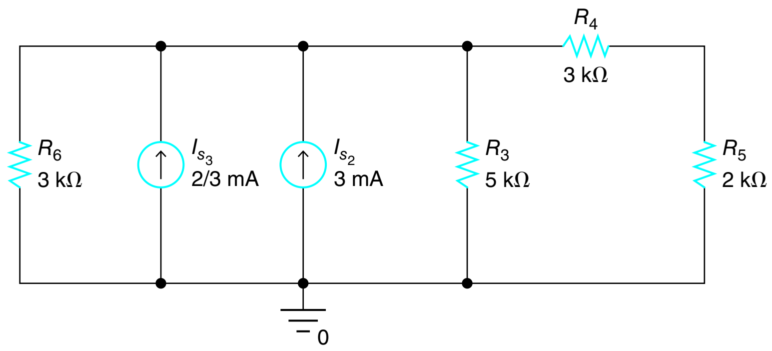
The voltage source V_{s_1} and R_6 can be transformed into a current source with current $2/3$ mA and a resistor R_6, as shown in Figure 4.41.

continued

Example 4.5 continued

FIGURE 4.41

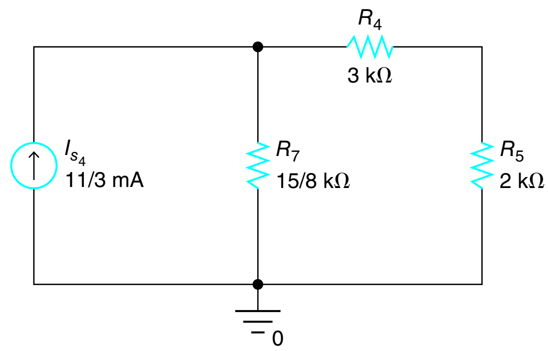
The circuit from Figure 4.40 with I_{s_3} in parallel with R_6.



The current sources I_{s_2} and I_{s_3} are in parallel and can be combined into a single current source I_{s_4} with $2/3 + 3 = 11/3$ mA. The two parallel resistors R_3 and R_6 can be combined into a single resistor R_7 with $3\|5 = 15/8\ k\Omega$. The circuit shown in Figure 4.41 simplifies to the one shown in Figure 4.42.

FIGURE 4.42

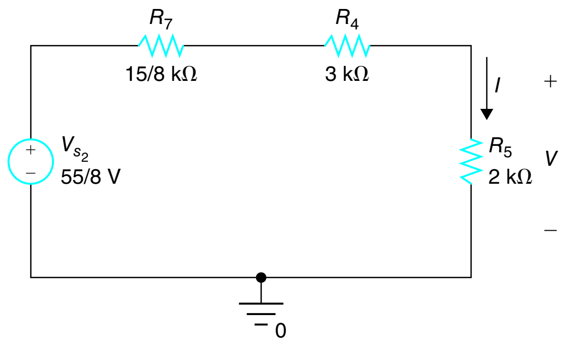
The circuit from Figure 4.41 with I_{s_4} in parallel with R_7.



The current source I_{s_4} and the parallel resistor R_7 can be transformed into a voltage source V_{s_2} with voltage $(11/3 \text{ mA}) \times (15/8\ k\Omega) = 55/8$ V and the series resistor R_7, as shown in Figure 4.43.

FIGURE 4.43

The circuit from Figure 4.42 with V_{s_2} in series with R_7.



The equivalent resistance of R_7, R_4, and R_5 is

$$R = R_7 + R_4 + R_5 = \frac{55}{8}\ k\Omega$$

Thus, the current I through the circuit shown in Figure 4.43 is given by

$$I = \frac{V_{s_2}}{R} = 1 \text{ mA}$$

The voltage V across R_5 is given by

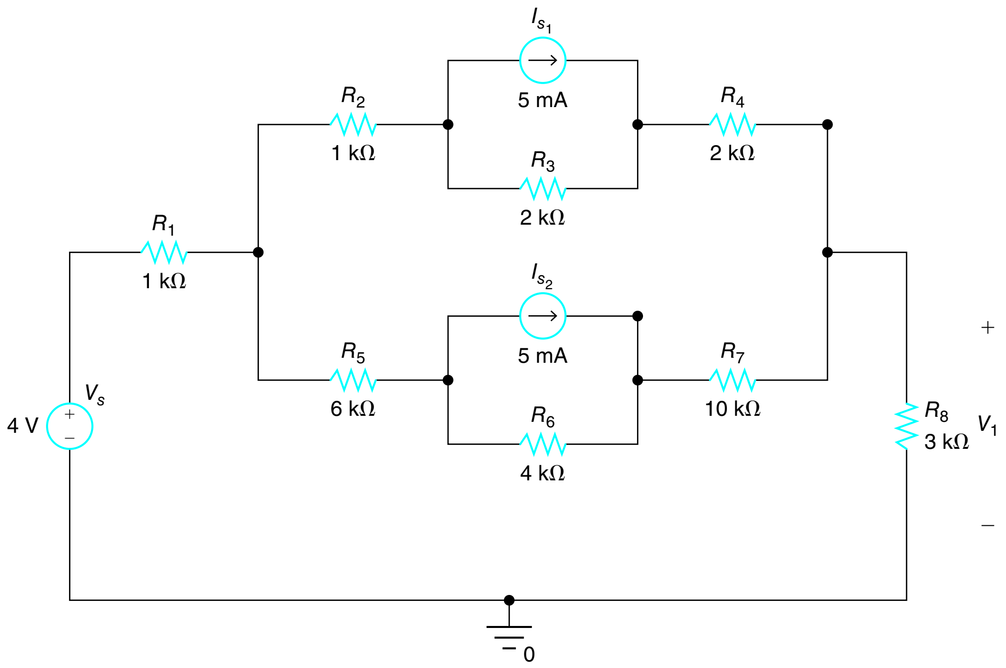
$$V = R_5 I = 2\ k\Omega \times 1 \text{ mA} = 2 \text{ V}$$

Exercise 4.5

Find voltage V_1 using source transformation for the circuit shown in Figure 4.44.

FIGURE 4.44

Circuit for EXERCISE 4.5.



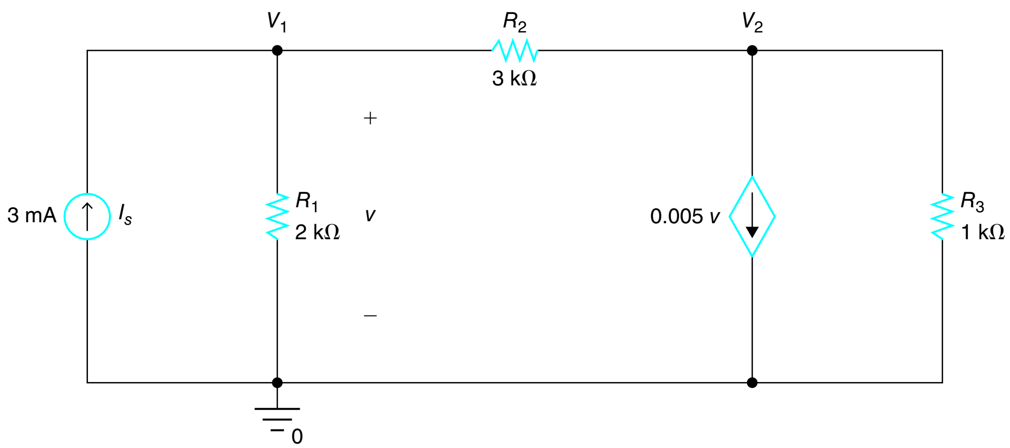
Answer:
$V_1 = 6$ V.

EXAMPLE 4.6

Find voltage V_1 using source transformation for the circuit shown in Figure 4.45.

FIGURE 4.45

Circuit for EXAMPLE 4.6.



continued

Example 4.6 continued

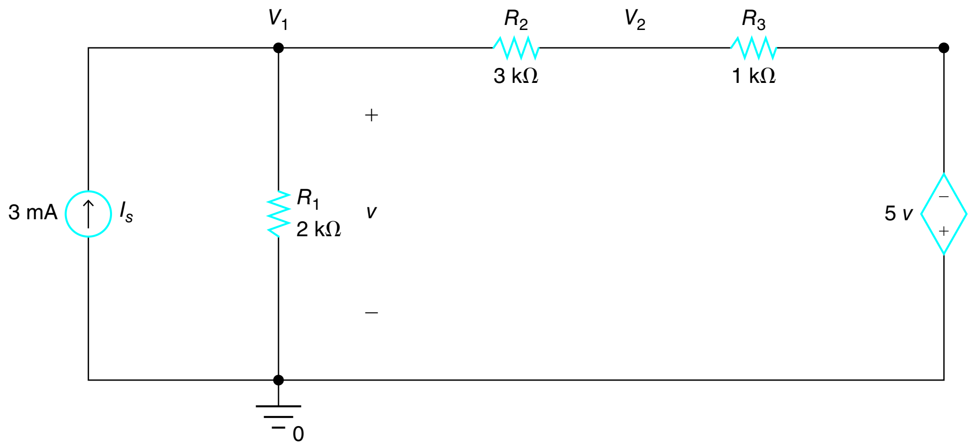
The VCCS and the parallel resistor R_3 can be transformed into a VCVS with voltage

$$0.005v \times 1000 = 5v$$

and a series resistor R_3, as shown in Figure 4.46.

FIGURE 4.46

The circuit in Figure 4.45 with VCVS.



Let R_4 be the sum of R_2 and R_3. Then, we have

$$R_4 = R_2 + R_3 = 4\ k\Omega$$

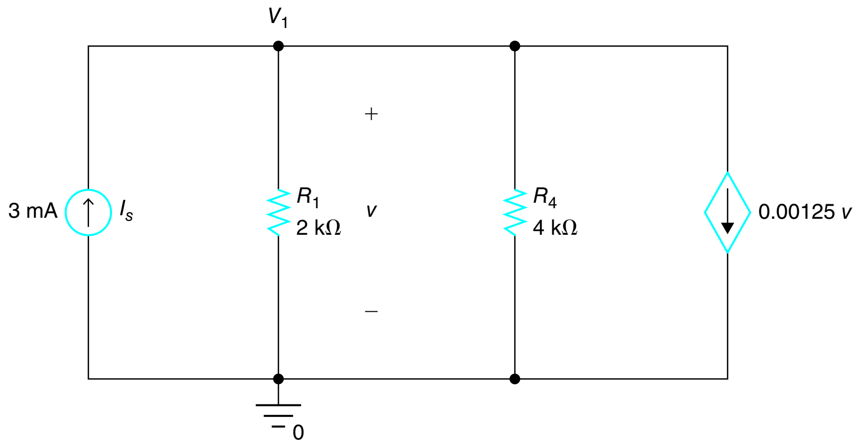
The VCVS and the series resistor R_4 can be transformed into a VCCS with current

$$\frac{-5v}{4000} = -0.00125v$$

and a parallel resistor R_4, as shown in Figure 4.47.

FIGURE 4.47

The circuit in Figure 4.46 with two current sources in parallel and two resistors in parallel.



The sum of currents from the two current sources is given by

$$0.003 - 0.00125v$$

continued

Example 4.6 continued

and the equivalent resistance R_{eq} of the parallel connection of R_1 and R_4 is given by

$$R_{eq} = R_1 \| R_4 = \frac{R_1 \times R_4}{R_1 + R_4} = \frac{2000 \times 4000}{2000 + 4000} = \frac{8000}{6} = 1.3333\ k\Omega$$

The circuit shown in Figure 4.47 reduces to the one shown in Figure 4.48.

The voltage v, which is V_1, is given by, from Ohm's law,

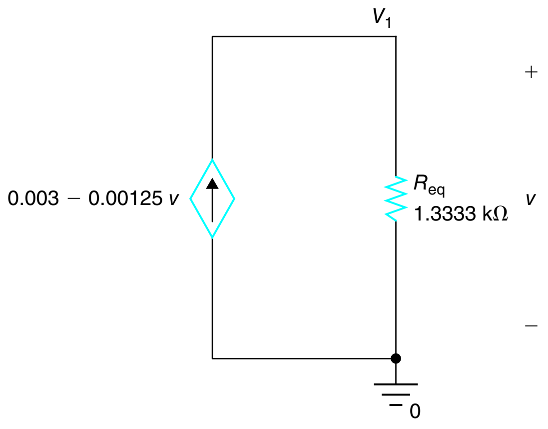
$$v = (0.003 - 0.00125v) \times 1333.3333 = 4 - 1.6667v$$

Solving for v, we obtain

$$v = V_1 = \frac{4}{2.66667}\ \text{V} = 1.5\ \text{V}.$$

FIGURE 4.48

The final reduced circuit.

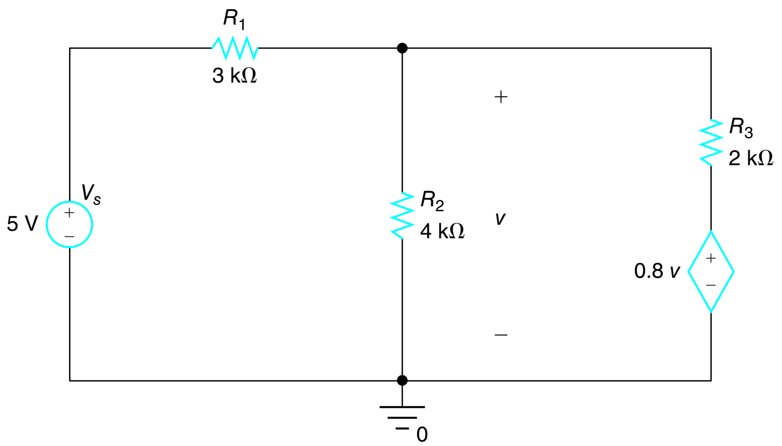


Exercise 4.6

Find voltage v using the source transformation for the circuit shown in Figure 4.49.

FIGURE 4.49

Circuit for EXERCISE 4.6.



Answer:

$v = 2.4390$ V.

4.4 Thévenin's Theorem

A circuit consisting of a voltage source V_{th} and a series resistor R_{th}, representing the original circuit looking from a pair of terminals, is called a **Thévenin equivalent circuit**. The voltage V_{th} is called **Thévenin equivalent voltage**, and the resistance R_{th} is called **Thévenin equivalent resistance**, as shown in Figure 4.50.

FIGURE 4.50

A Thévenin equivalent circuit.

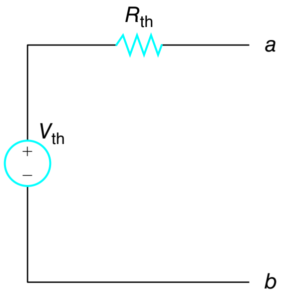
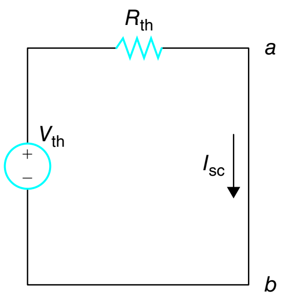


FIGURE 4.51

Short-circuit current.



The Thévenin equivalent circuit can be used to simplify the circuit. When a load resistor is connected between terminals a and b, we can find the effects of the circuit on the load from the Thévenin equivalent circuit. We do not need all the details of the original circuit to find the voltage, current, and power on the load.

Let the voltage across terminals a and b of the Thévenin equivalent circuit be V_{oc}. This voltage is called *open-circuit voltage* because terminals a and b are open (with an infinite resistance between a and b). No current flows on the Thévenin equivalent resistor R_{th}. Thus,

$$V_{oc} = V_{th}$$

If the terminals a and b are short-circuited, as shown in Figure 4.51, the current through the short circuit is given by

$$I_{sc} = \frac{V_{th}}{R_{th}} = \frac{V_{oc}}{R_{th}}$$

If we solve this equation for R_{th}, we have

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

This equation can be used to find the Thévenin equivalent resistance R_{th} from the original circuit.

4.4.1 FINDING THE THÉVENIN EQUIVALENT VOLTAGE V_{th}

Given a circuit and terminals a and b, we can find the Thévenin equivalent voltage V_{th} with respect to terminals a and b by finding the open-circuit voltage V_{oc} across terminals a and b. The open-circuit voltage V_{oc} can be found by utilizing circuit analysis methods such as the voltage divider rule, current divider rule, superposition principle, nodal analysis, and mesh analysis. The Thévenin equivalent voltage V_{th} is found from the original circuit without any changes.

4.4.2 FINDING THE THÉVENIN EQUIVALENT RESISTANCE R_{th}

Given a circuit and terminals a and b, we can find the Thévenin equivalent resistance R_{th} with respect to terminals a and b by using one of the three methods listed next.

Method 1

Deactivate all the independent sources by short-circuiting voltage sources and open-circuiting current sources. Find the equivalent resistance looking into the circuit from terminals a and b. This equivalent resistance is the Thévenin equivalent resistance R_{th}. This method can be used if the circuit does not contain dependent sources.

Method 2

Connect terminals a and b by wire (short-circuit). Find the short-circuit current I_{sc} by utilizing circuit analysis methods such as nodal analysis and mesh analysis. The Thévenin equivalent resistance R_{th} is given by

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

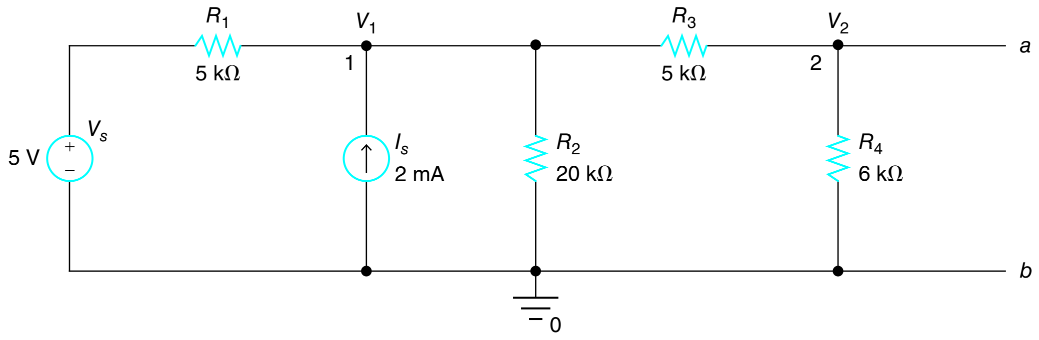
Method 3

Deactivate all the independent sources by open-circuiting current sources and short-circuiting voltage sources. Apply a test voltage of 1 V (or any other value) between terminals a and b with terminal a connected to the positive terminal of the test voltage. Measure the current flowing out of the positive terminal of the test voltage source. The Thévenin equivalent resistance R_{th} is given by the ratio of the test voltage to the current flowing out of the positive terminal of the test voltage source. A test current can be used instead of the test voltage. Apply a test current between terminals *a* and *b* after deactivating the independent sources, and measure the voltage across *a* and *b* of the test current source. The Thévenin equivalent resistance R_{th} is the ratio of the voltage across *a* and *b* to the test current.

Consider a circuit shown in Figure 4.52. We are interested in finding the Thévenin equivalent voltage V_{th} and the Thévenin equivalent resistance R_{th} between terminals *a* and *b*. The Thévenin equivalent voltage V_{th} is the open-circuit voltage V_{oc} between terminals *a* and *b*. The open-circuit voltage is the voltage across the resistor R_4, which is labeled V_2 in the circuit shown in Figure 4.52.

FIGURE 4.52

A circuit with a pair of terminals.



Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 5}{5000} - 0.002 + \frac{V_1}{20{,}000} + \frac{V_1 - V_2}{5000} = 0$$

Multiplication by 20,000 yields

$$4V_1 - 20 - 40 + V_1 + 4V_1 - 4V_2 = 0$$

which can be rearranged as

$$9V_1 - 4V_2 = 60 \tag{4.22}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_2 - V_1}{5000} + \frac{V_2}{6000} = 0$$

Multiplication by 30,000 yields

$$6V_2 - 6V_1 + 5V_2 = 0$$

which can be rearranged as

$$6V_1 = 11V_2$$

or

$$V_1 = \frac{11}{6}V_2 \tag{4.23}$$

Substituting Equation (4.23) into Equation (4.22), we obtain

$$9\frac{11}{6}V_2 - 4V_2 = \frac{75}{6}V_2 = 60$$

Thus, we have

$$V_2 = 4.8 \text{ V}$$

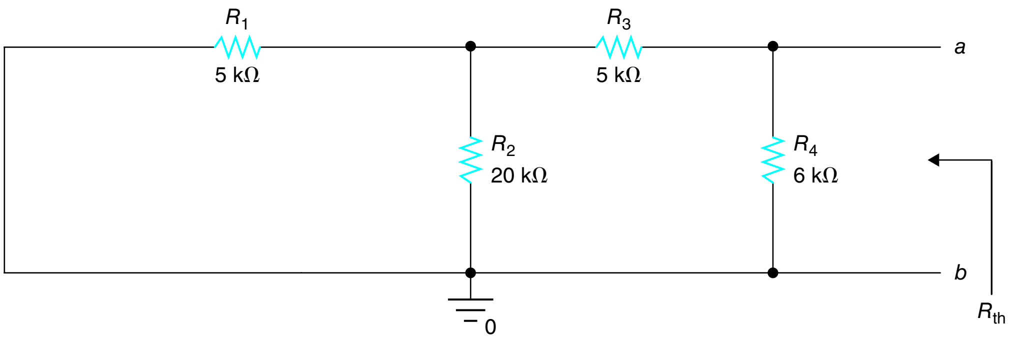
Since V_2 is the open-circuit voltage between terminals a and b, it is the Thévenin equivalent voltage V_{th}. Thus, we have

$$V_{th} = V_{oc} = 4.8 \text{ V}$$

We will try all three methods to find the Thévenin equivalent resistance R_{th}. In method 1, we deactivate all independent sources and find the equivalent resistance looking into the circuit from terminals a and b. Figure 4.53 shows the circuit shown in Figure 4.52 with the voltage source short-circuited and current source open-circuited.

FIGURE 4.53

The circuit from Figure 4.52 with its sources deactivated.



The equivalent resistance of the parallel connection of R_1 and R_2 is

$$R_a = R_1 \| R_2 = 4\ k\Omega$$

The equivalent resistance of the series connection of R_3 and R_a is

$$R_b = R_3 + R_a = R_3 + (R_1 \| R_2) = 5\ k\Omega + 4\ k\Omega = 9\ k\Omega$$

The Thévenin equivalent resistance, R_{th}, is the equivalent resistance of parallel connection of R_b and R_4. Thus, we have

$$R_{th} = R_b \| R_4 = 9\ k\Omega \| 6\ k\Omega = 3.6\ k\Omega$$

In method 2, we find the open-circuit voltage V_{oc} and the short-circuit current I_{sc}. The Thévenin equivalent resistance between terminals a and b is the ratio of V_{oc} to I_{sc}. The open-circuit voltage is found to be

$$V_{oc} = V_2 = 4.8 \text{ V}$$

To find the short-circuit current, we connect terminals a and b by a wire without changing the rest of the circuit, as shown in Figure 4.54.

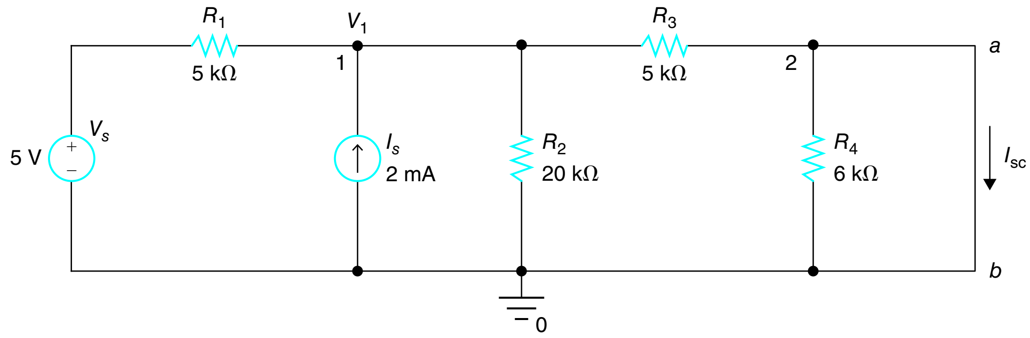


FIGURE 4.54

A circuit with a short-circuit between a and b.

Notice that node 2 is a ground and no current flows through R_4. The short-circuit current I_{sc} is the current through R_3 from left to right (→). Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 5}{5000} - 0.002 + \frac{V_1}{20{,}000} + \frac{V_1}{5000} = 0$$

Multiplication by 20,000 yields

$$4V_1 - 20 - 40 + V_1 + 4V_1 = 0$$

which can be revised as

$$9V_1 = 60$$

The node voltage at node 1 is given by

$$V_1 = \frac{60}{9} = \frac{20}{3}\ \text{V}$$

The current through R_3, which is also I_{sc}, is given by

$$I_{sc} = \frac{V_1}{R_3} = \frac{\frac{20}{3}\ \text{V}}{5\ k\Omega} = \frac{4}{3}\ \text{mA}$$

The Thévenin equivalent resistance is given by

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{4.8\ \text{V}}{\frac{4}{3}\ \text{mA}} = 3.6\ k\Omega$$

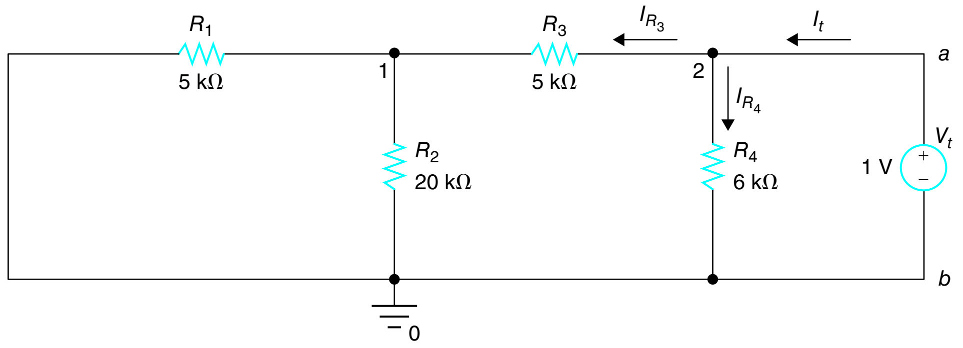
In method 3, we deactivate all independent sources and apply a test voltage across terminals a and b. The Thévenin equivalent resistance is the ratio of the test voltage to the current flowing out of the positive terminal of the test voltage. Figure 4.55 shows a circuit with a test voltage source V_t after deactivating independent sources.

Notice that $R_1 \| R_2 = 4\ k\Omega$ and $R_3 + (R_1 \| R_2) = 9\ k\Omega$. The current through R_3 is

$$I_{R_3} = \frac{V_t}{R_3 + (R_1 \| R_2)} = \frac{1\ \text{V}}{9\ k\Omega} = \frac{1}{9}\ \text{mA}$$

FIGURE 4.55

Circuit with test voltage.



The current through R_4 is

$$I_{R_4} = \frac{V_t}{R_4} = \frac{1\text{ V}}{6\ k\Omega} = \frac{1}{6}\text{ mA}$$

The total current flowing out of the positive terminal of the test voltage source is given by

$$I_t = I_{R_3} + I_{R_4} = \frac{1}{9}\text{ mA} + \frac{1}{6}\text{ mA} = \frac{5}{18}\text{ mA}$$

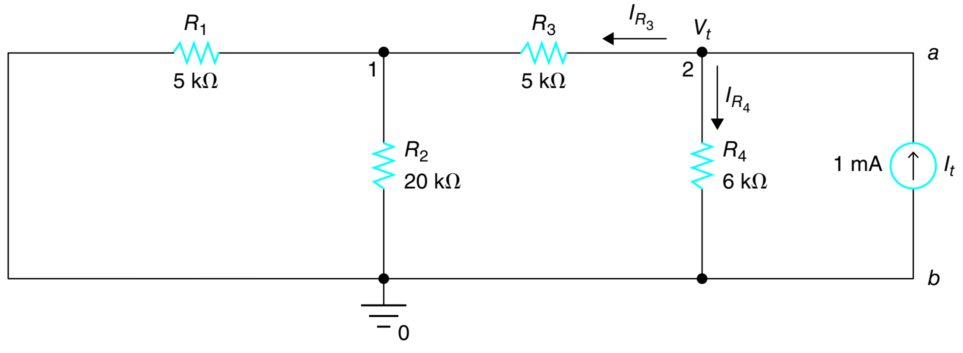
The Thévenin equivalent resistance is given by

$$R_{\text{th}} = \frac{V_t}{I_t} = \frac{1\text{ V}}{\frac{5}{18}\text{ mA}} = \frac{18}{5}\ k\Omega = 3.6\ k\Omega$$

In method 3, instead of a test voltage source, a test current source can be applied after deactivating the independent sources. Figure 4.56 shows a circuit with a test current source I_t after deactivating independent sources.

FIGURE 4.56

Circuit with a test current.



Notice that $R_1 \| R_2 = 4\ k\Omega$ and $R_3 + (R_1 \| R_2) = 9\ k\Omega$. The current through R_4, I_{R_4}, can be obtained by applying the current divider rule:

$$I_{R_4} = I_t \times \frac{9\ k\Omega}{9\ k\Omega + 6\ k\Omega} = 1\text{ mA} \times \frac{9}{15} = \frac{9}{15}\text{ mA}$$

The voltage across R_4, which is also the voltage across the test current source, is given by

$$V_t = R_4 I_{R_4} = 6\ k\Omega \times \frac{9}{15}\text{ mA} = \frac{54}{15}\text{ V} = 3.6\text{ V}$$

The Thévenin equivalent resistance is given by

$$R_{th} = \frac{V_t}{I_t} = \frac{3.6 \text{ V}}{1 \text{ mA}} = 3.6\ k\Omega$$

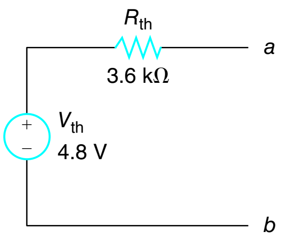


FIGURE 4.57

The Thévenin equivalent circuit.

The Thévenin equivalent circuit is shown in Figure 4.57.

A circuit with VCCS is shown in Figure 4.58. We are interested in finding the Thévenin equivalent voltage V_{th} and the Thévenin equivalent resistance R_{th} between terminals a and b. The Thévenin equivalent voltage V_{th} is the open-circuit voltage V_{oc} between terminals a and b. The open-circuit voltage is the voltage across the resistor R_4, which is labeled V_2 in the circuit shown in Figure 4.58. Notice that the controlling voltage v is equal to V_1.

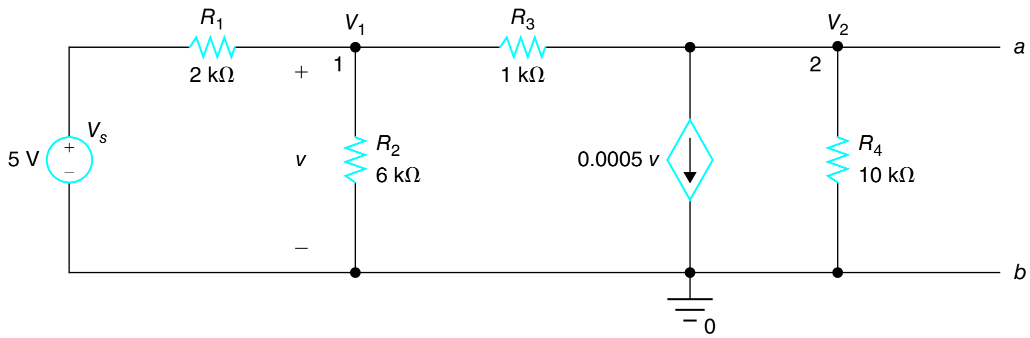


FIGURE 4.58

A circuit with VCCS.

Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 5}{2000} + \frac{V_1}{6000} + \frac{V_1 - V_2}{1000} = 0$$

Multiplication by 6000 yields

$$3V_1 - 15 + V_1 + 6V_1 - 6V_2 = 0,$$

which can be revised as

$$10V_1 - 6V_2 = 15 \tag{4.24}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_2 - V_1}{1000} + 0.0005V_1 + \frac{V_2}{10{,}000} = 0$$

Multiplication by 10,000 yields

$$10V_2 - 10V_1 + 5V_1 + V_2 = 0$$

which can be revised as

$$5V_1 = 11V_2$$

or

$$V_1 = \frac{11}{5}V_2 = 2.2V_2 \tag{4.25}$$

Substituting Equation (4.25) into Equation (4.24), we obtain

$$10 \times 2.2V_2 - 6V_2 = 16V_2 = 15$$

Thus, we have

$$V_2 = \frac{15}{16}\text{ V} = 0.9375\text{ V}$$

Since V_2 is the open-circuit voltage between terminals a and b, it is the Thévenin equivalent voltage V_{th}. Thus, we have

$$V_{th} = 0.9375\text{ V}$$

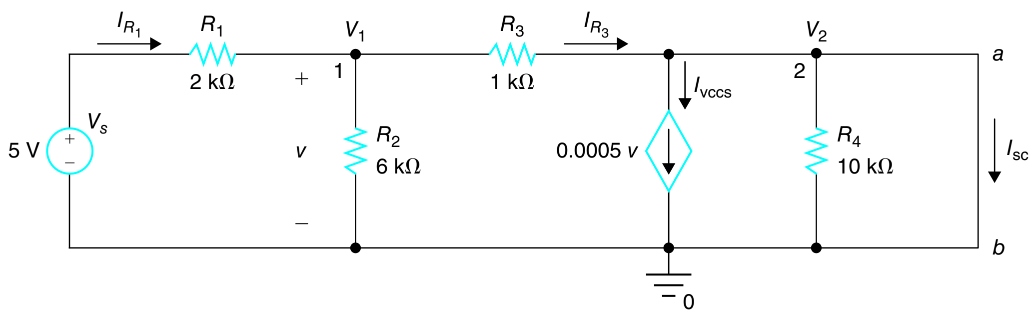
Since the circuit contains a dependent source, method 1 cannot be used to find the Thévenin equivalent resistance R_{th}. We will try method 2 and method 3. In method 2, we find the open-circuit voltage V_{oc} and the short-circuit current I_{sc}. The Thévenin equivalent resistance between terminals a and b is the ratio of V_{oc} to I_{sc}. The open-circuit voltage is found to be

$$V_{oc} = V_2 = 0.9375\text{ V}$$

To find the short-circuit current, we connect terminals a and b by a wire without changing the rest of the circuit, as shown in Figure 4.59.

FIGURE 4.59

A circuit with a short circuit between a and b.



Notice that node 2 is a ground and no current flows through R_4. The short-circuit current I_{sc} is the current through R_3, I_{R_3}, minus the current through VCCS. The equivalent resistance of the parallel connection of R_2 and R_3 is given by

$$R_2 \| R_3 = \frac{6}{7}\,k\Omega = 0.857143\,k\Omega$$

The total resistance seen from the voltage source V_s is given by

$$R_1 + (R_2 \| R_3) = 2\,k\Omega + \frac{6}{7}\,k\Omega = \frac{20}{7}\,k\Omega = 2.857143\,k\Omega$$

The current through R_1 is given by

$$I_{R_1} = \frac{V_s}{R_1 + (R_2 \| R_3)} = \frac{5\text{ V}}{\frac{20}{7}\,k\Omega} = \frac{35}{20}\text{ mA} = 1.75\text{ mA}$$

Voltage V_1 is given by

$$V_1 = V_s - R_1 \times I_{R_1} = 5\text{ V} - 1.75\text{ mA} \times 2\,k\Omega = 1.5\text{ V}$$

Since the controlling voltage v is identical to V_1, we have

$$v = V_1 = 1.5\text{ V}$$

The current through R_3 is given by

$$I_{R_3} = \frac{V_1}{R_3} = \frac{1.5\text{ V}}{1\ k\Omega} = 1.5\text{ mA}$$

The current through VCCS is given by

$$I_{VCCS} = 0.0005 \times V_1 = 0.0005 \times 1.5\text{ A} = 0.75\text{ mA}$$

The short-circuit current is the difference of I_{R_3} and I_{VCCS}. Thus, we have

$$I_{\text{sc}} = I_{R_3} - I_{VCCS} = 1.5\text{ mA} - 0.75\text{ mA} = 0.75\text{ mA}$$

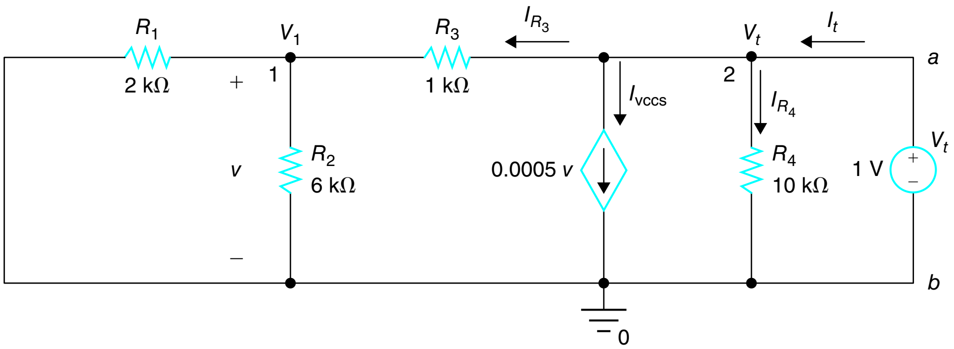
The Thévenin equivalent resistance is given by

$$R_{\text{th}} = \frac{V_{\text{oc}}}{I_{\text{sc}}} = \frac{0.9375\text{ V}}{0.75\text{ mA}} = 1.25\ k\Omega$$

In method 3, we deactivate the independent voltage source and apply a test voltage across terminals a and b. The Thévenin equivalent resistance is the ratio of the test voltage to the current flowing out of the positive terminal of the test voltage source. Figure 4.60 shows a circuit with test voltage source V_t after deactivating the independent source.

FIGURE 4.60

A circuit with test voltage.



Notice that $R_1 \| R_2 = 1.5\ k\Omega$ and $R_3 + (R_1 \| R_2) = 2.5\ k\Omega$. The current through R_3 is

$$I_{R_3} = \frac{V_t}{R_3 + (R_1 \| R_2)} = \frac{1\text{ V}}{2.5\ k\Omega} = 0.4\text{ mA}$$

The voltage at node 1 is given by

$$V_1 = v = V_t - R_3 \times I_{R_3} = 1\text{ V} - 1\ k\Omega \times 0.4\text{ mA} = 1\text{ V} - 0.4\text{ V} = 0.6\text{ V}$$

The current through the VCCS is given by

$$I_{VCCS} = 0.0005v = 0.0005 \times 0.6\text{ V} = 0.0003\text{ A} = 0.3\text{ mA}$$

The current through R_4 is

$$I_{R_4} = \frac{V_t}{R_4} = \frac{1\text{ V}}{10\ k\Omega} = 0.1\text{ mA}$$

The total current flowing out of the positive terminal of the test voltage source is given by

$$I_t = I_{R_3} + I_{VCCS} + I_{R_4} = 0.4 \text{ mA} + 0.3 \text{ mA} + 0.1 \text{ mA} = 0.8 \text{ mA}$$

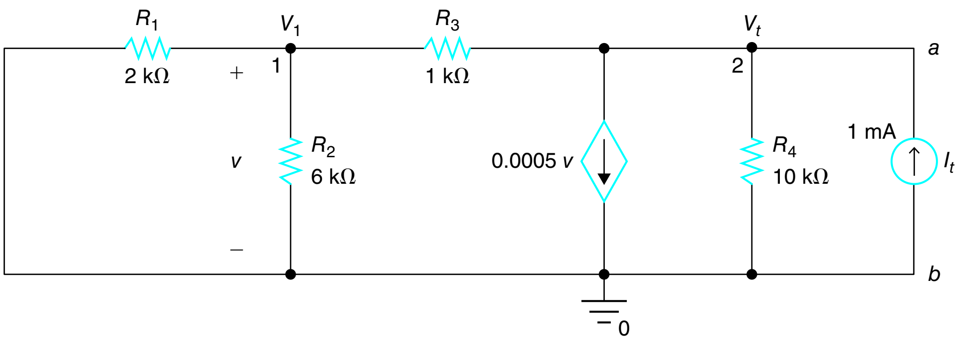
The Thévenin equivalent resistance is given by

$$R_{\text{th}} = \frac{V_t}{I_t} = \frac{1 \text{ V}}{0.8 \text{ mA}} = 1.25 \, k\Omega$$

In method 3, instead of test voltage, a test current can be applied after deactivating the independent voltage source. Figure 4.61 shows a circuit with test current source I_t after deactivating the independent voltage source.

FIGURE 4.61

A circuit with a test current.



Summing the currents leaving node 1, we obtain

$$\frac{V_1}{2000} + \frac{V_1}{6000} + \frac{V_1 - V_t}{1000} = 0$$

Multiplication by 6000 yields

$$3V_1 + V_1 + 6V_1 - 6V_t = 0$$

which can be revised as

$$10V_1 - 6V_t = 0$$

Solving for V_1, we get

$$V_1 = 0.6V_t \tag{4.26}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_t - V_1}{1000} + 0.0005V_1 + \frac{V_t}{10{,}000} - 0.001 = 0$$

Multiplication by 10,000 yields

$$10V_t - 10V_1 + 5V_1 + V_t - 10 = 0$$

which can be revised as

$$-5V_1 + 11V_t = 10 \tag{4.27}$$

Substitution of Equation (4.26) into Equation (4.27) yields

$$-5(0.6V_t) + 11V_t = 8V_t = 10$$

Thus, we have

$$V_t = \frac{10}{8}\ \text{V} = 1.25\ \text{V}$$

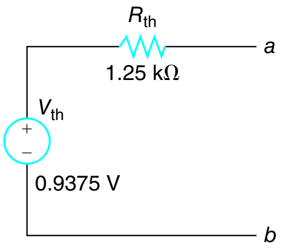
The Thévenin equivalent resistance is given by

$$R_{\text{th}} = \frac{V_t}{I_t} = \frac{1.25\ \text{V}}{1\ \text{mA}} = 1.25\ k\Omega$$

A Thévenin equivalent circuit is shown in Figure 4.62.

FIGURE 4.62

Thévenin equivalent circuit.

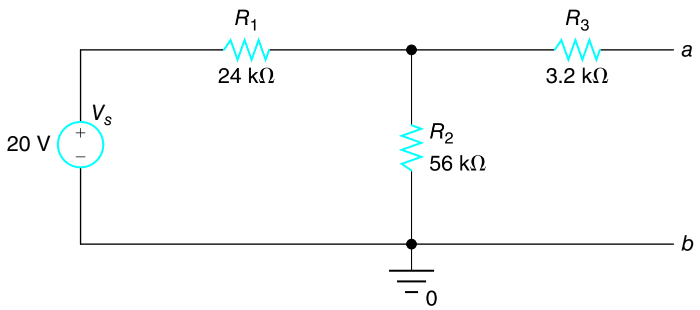


EXAMPLE 4.7

Find the Thévenin equivalent voltage and the Thévenin equivalent resistance between *a* and *b* in the circuit shown in Figure 4.63.

FIGURE 4.63

Circuit for EXAMPLE 4.7.



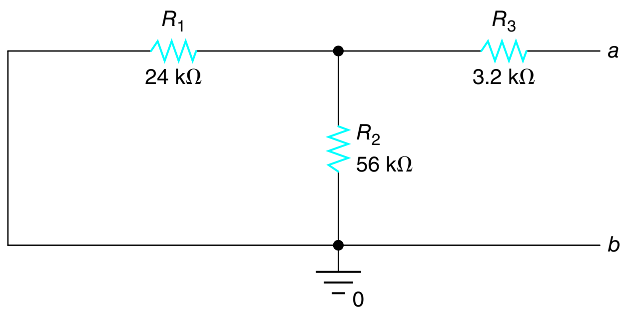
Since a and b is an open circuit, resistance between a and b is infinite. There is no current flowing through R_3. Therefore, there is no voltage drop across R_3. The open-circuit voltage between a and b is identical to the voltage across R_2. From the voltage divider rule, we have

$$V_{\text{th}} = V_{\text{oc}} = V_s \times \frac{R_2}{R_1 + R_2} = 20\ \text{V} \times \frac{56\ k\Omega}{24\ k\Omega + 56\ k\Omega} = 14\ \text{V}$$

To find the Thévenin equivalent resistance, we deactivate the voltage source by short-circuiting it, as shown in Figure 4.64.

FIGURE 4.64

Circuit in Figure 4.63 after deactivating the voltage source.

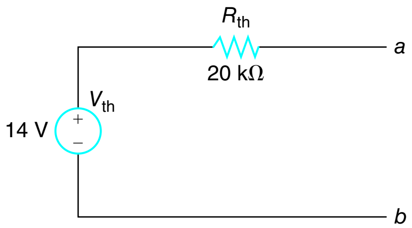


continued

Example 4.7 continued

FIGURE 4.65

The Thévenin equivalent circuit.



The equivalent resistance to the left of *a* and *b* is given by

$$R_{th} = R_3 + (R_1 \| R_2) = R_3 + \frac{R_1 \times R_2}{R_1 + R_2} = 3.2\ k\Omega + \frac{24\ k\Omega \times 56\ k\Omega}{24\ k\Omega + 56\ k\Omega}$$
$$= 3.2\ k\Omega + 16.8\ k\Omega = 20\ k\Omega$$

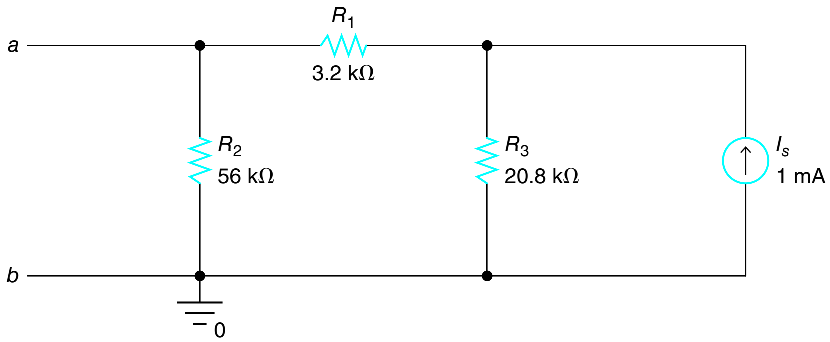
When the circuit is replaced by a Thévenin equivalent circuit, the original circuit shown in Figure 4.63 becomes the circuit shown in Figure 4.65.

Exercise 4.7

Find the Thévenin equivalent circuit between *a* and *b* in the circuit shown in Figure 4.66.

FIGURE 4.66

Circuit for EXERCISE 4.7.



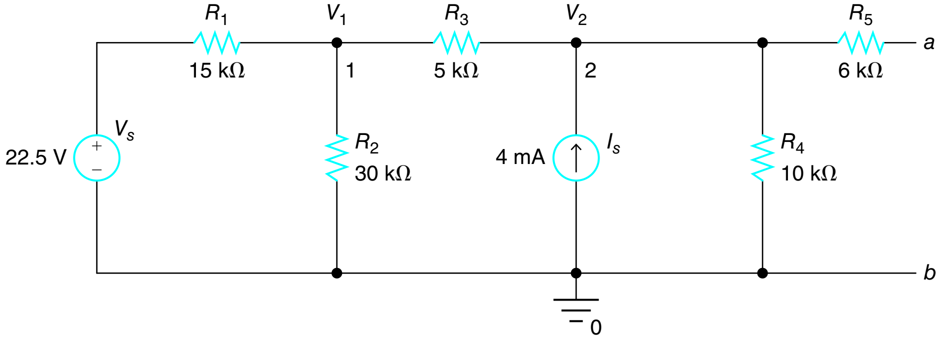
Answer:
$V_{th} = 14.56$ V, $R_{th} = 16.8\ k\Omega$.

EXAMPLE 4.8

Find the Thévenin equivalent voltage and the Thévenin equivalent resistance between *a* and *b* in the circuit shown in Figure 4.67.

FIGURE 4.67

Circuit for EXAMPLE 4.8.



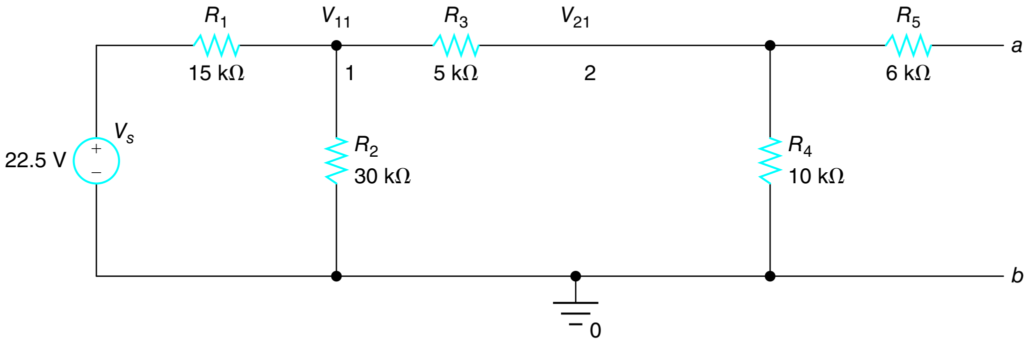
continued

Example 4.8 continued

Since no current flows through R_5, the Thévenin equivalent voltage is the voltage across R_4, which is labeled V_2 in the circuit shown in Figure 4.67. The superposition principle will be applied to find the Thévenin equivalent voltage. When the current source is deactivated, the circuit reduces to the one shown in Figure 4.68.

FIGURE 4.68

The circuit shown in Figure 4.67 with I_s deactivated.



The equivalent resistance of the parallel connection of R_2 and $R_3 + R_4$ is given by

$$R_2 \| (R_3 + R_4) = 30\ k\Omega \| 15\ k\Omega = 10\ k\Omega$$

Application of the voltage divider rule results in

$$V_{11} = V_s \times \frac{R_2 \| (R_3 + R_4)}{R_1 + [R_2 \| (R_3 + R_4)]} = 22.5\ \text{V} \times \frac{10\ k\Omega}{15\ k\Omega + 10\ k\Omega} = 9\ \text{V}$$

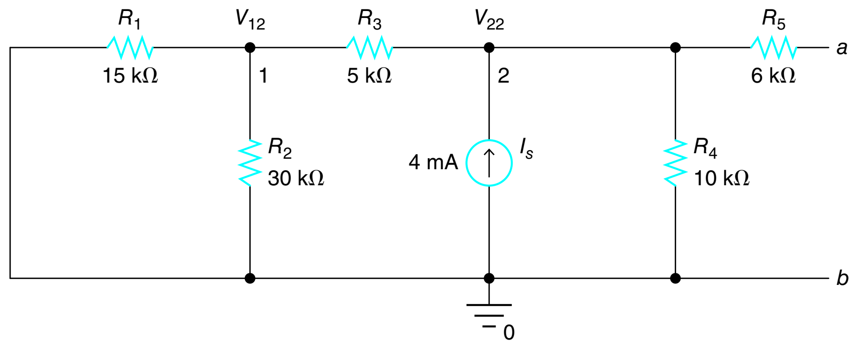
Application of the voltage divider rule to R_3 and R_4 results in

$$V_{21} = V_{11} \times \frac{R_4}{R_3 + R_4} = 9\ \text{V} \times \frac{10\ k\Omega}{5\ k\Omega + 10\ k\Omega} = 6\ \text{V}$$

When the voltage source is deactivated, the circuit shown in Figure 4.67 reduces to the one shown in Figure 4.69.

FIGURE 4.69

The circuit shown in Figure 4.67 with V_s deactivated.



Let R_a be the equivalent resistance of $R_3 + (R_1 \| R_2)$. Then, we have

$$R_a = R_3 + (R_1 \| R_2) = 5\ k\Omega + (15\ k\Omega \| 30\ k\Omega) = 5\ k\Omega + 10\ k\Omega = 15\ k\Omega$$

continued

Example 4.8 continued

From the current divider rule, the current through R_4 is given by

$$I_{R_4} = I_s \times \frac{R_a}{R_a + R_4} = 4\text{ mA} \times \frac{15\ k\Omega}{15\ k\Omega + 10\ k\Omega} = 2.4\text{ mA}$$

The voltage across R_4 is given by

$$V_{22} = R_4 \times I_{R_4} = 10\ k\Omega \times 2.4\text{ mA} = 24\text{ V}$$

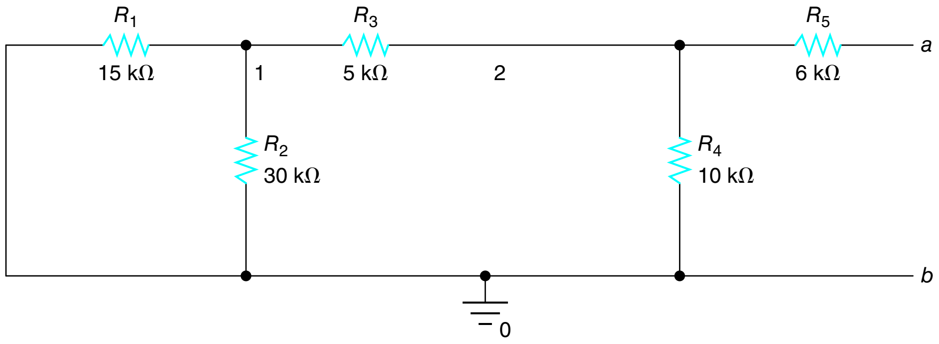
The open-circuit voltage, which is the Thévenin equivalent voltage, is the sum of V_{21} and V_{22}:

$$V_{\text{th}} = V_{\text{oc}} = V_2 = V_{21} + V_{22} = 6\text{ V} + 24\text{ V} = 30\text{ V}$$

To find the Thévenin equivalent resistance R_{th}, we deactivate the sources by short-circuiting the voltage source and open-circuiting the current source, as shown in Figure 4.70.

FIGURE 4.70

The circuit shown in Figure 4.67 after deactivating the sources.



We find the equivalent resistance starting from the left side of the circuit and moving toward terminals *a* and *b*. The equivalent resistance of the parallel connection of R_1 and R_2 is given by

$$R_1 \| R_2 = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{15\ k\Omega \times 30\ k\Omega}{15\ k\Omega + 30\ k\Omega} = \frac{30\ k\Omega}{1 + 2} = 10\ k\Omega$$

$R_1 \| R_2$ is in series with R_3. Thus, we have

$$(R_1 \| R_2) + R_3 = 10\ k\Omega + 5\ k\Omega = 15\ k\Omega.$$

$(R_1 \| R_2) + R_3$ is in parallel with R_4. Thus, we have

$$[(R_1 \| R_2) + R_3] \| R_4 = \frac{15\ k\Omega \times 10\ k\Omega}{15\ k\Omega + 10\ k\Omega} = \frac{30\ k\Omega}{5} = 6\ k\Omega$$

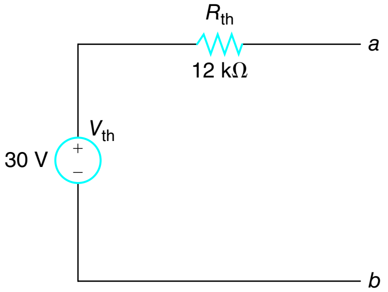
The Thévenin equivalent resistance is the sum of 6 $k\Omega$ and R_5; i.e.,

$$R_{\text{th}} = 6\ k\Omega + 6\ k\Omega = 12\ k\Omega$$

The Thévenin equivalent circuit is shown in Figure 4.71.

FIGURE 4.71

The Thévenin equivalent circuit.

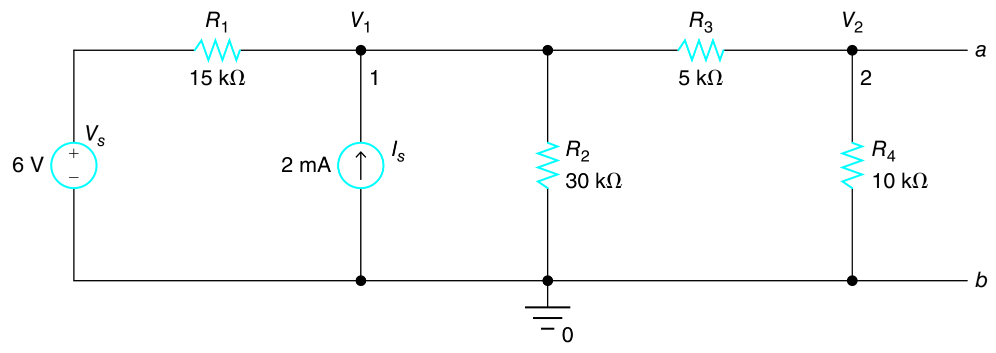


Exercise 4.8

Find the Thévenin equivalent voltage and the Thévenin equivalent resistance between *a* and *b* in the circuit shown in Figure 4.72.

FIGURE 4.72

Circuit for EXERCISE 4.8.



Answer:

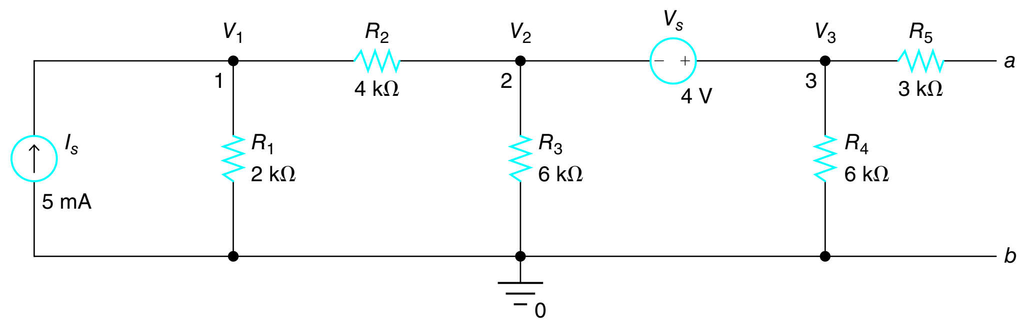
$V_{\text{th}} = 9.6$ V, $R_{\text{th}} = 6\ k\Omega$.

EXAMPLE 4.9

Find the Thévenin equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.73.

FIGURE 4.73

Circuit for EXAMPLE 4.9.



Voltage V_3 at node 3 is 4 V higher than voltage V_2 at node 2. Thus, we have

$$V_3 = V_2 + 4 \tag{4.28}$$

Summing the currents away from node 1, we get

$$-5 \times 10^{-3} + \frac{V_1}{2000} + \frac{V_1 - V_2}{4000} = 0 \tag{4.29}$$

Multiplication of Equation (4.29) by 4000 yields

$$2V_1 + V_1 - V_2 = 20$$

which reduces to

$$3V_1 - V_2 = 20 \tag{4.30}$$

continued

Example 4.9 continued

Summing the currents away from nodes 2 and 3 (i.e., the supernode, as discussed in Chapter 3), we have

$$\frac{V_2 - V_1}{4000} + \frac{V_2}{6000} + \frac{V_3}{6000} = 0 \qquad (4.31)$$

Since one end of R_5 is open, there is no current through R_5. Thus, the voltage drop across R_5 is zero. Substitution of Equation (4.28) into Equation (4.31) results in

$$\frac{V_2 - V_1}{4000} + \frac{V_2}{6000} + \frac{V_2 + 4}{6000} = 0 \qquad (4.32)$$

Multiplication of Equation (4.32) by 12,000 yields

$$3V_2 - 3V_1 + 2V_2 + 2V_2 + 8 = 0$$

which reduces to

$$-3V_1 + 7V_2 = -8 \qquad (4.33)$$

Summing Equations (4.30) and (4.33) results in

$$6V_2 = 12$$

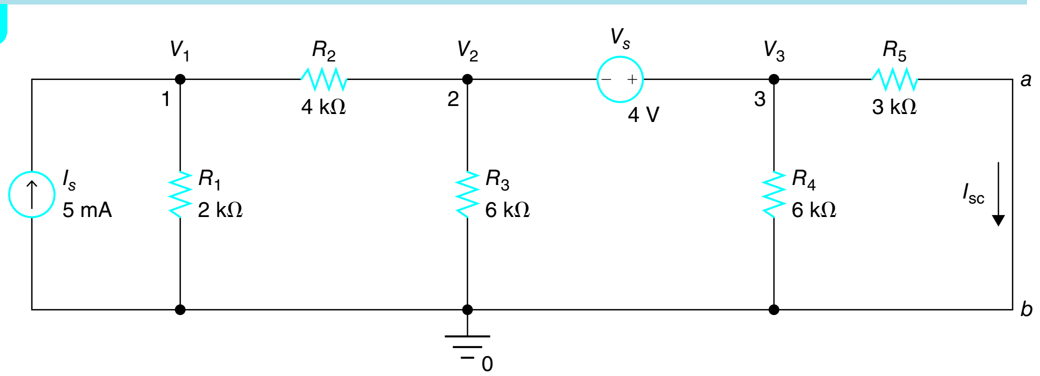
Thus, $V_2 = 2$ V. Substituting this into Equation (4.30), we get $V_1 = 22/3$ V $= 7.3333$ V. The Thévenin voltage is V_3, which is the sum of V_2 and 4 V from the voltage source V_s. Thus,

$$V_{\text{th}} = V_3 = V_2 + 4\text{ V} = 2\text{ V} + 4\text{ V} = 6\text{ V}$$

In method 2 of finding the Thévenin equivalent resistance, the terminals a and b are short-circuited, as shown in Figure 4.74.

FIGURE 4.74

Terminals a and b are short-circuited.



To find the short-circuit current I_{sc} for the circuit shown in Figure 4.46, we modify Equation (4.32) to include the current through the resistor R_5:

$$\frac{V_2 - V_1}{4000} + \frac{V_2}{6000} + \frac{V_3}{6000} + \frac{V_3}{3000} = 0 \qquad (4.34)$$

Substitution of Equation (4.28) into Equation (4.34) yields

$$\frac{V_2 - V_1}{4000} + \frac{V_2}{6000} + \frac{V_2 + 4}{6000} + \frac{V_2 + 4}{3000} = 0 \qquad (4.35)$$

Multiplying Equation (4.35) by 12,000, we get

$$3V_2 - 3V_1 + 2V_2 + 2V_2 + 8 + 4V_2 + 16 = 0$$

continued

Example 4.9 continued

which can be simplified to

$$-3V_1 + 11V_2 = -24 \tag{4.36}$$

Summing Equations (4.30) and (4.36) yields

$$10V_2 = -4$$

from which we get $V_2 = -0.4$ V. Thus, $V_3 = V_2 + 4 = 3.6$ V. The short-circuit current is given by

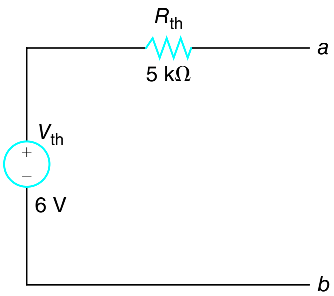
$$I_{sc} = \frac{V_3}{R_3} = \frac{3.6\text{ V}}{3\ k\Omega} = 1.2\text{ mA}$$

Thus, the Thévenin equivalent resistance is given by

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{6\text{ V}}{1.2\text{ mA}} = 5\ k\Omega$$

FIGURE 4.75

The Thévenin equivalent circuit.

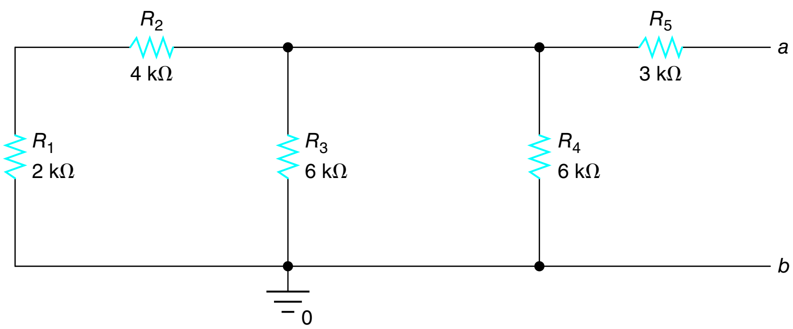


The Thévenin equivalent circuit between terminals a and b is shown in Figure 4.75.

The Thévenin equivalent resistance can also be found using method 1. If current source I_s and voltage source V_s are deactivated (i.e., open-circuit the current source and short-circuit the voltage source) from the circuit shown in Figure 4.73, we obtain the circuit shown in Figure 4.76.

FIGURE 4.76

Circuit shown in Figure 4.73 with the sources deactivated.



The equivalent resistance of the series connection of R_1 and R_2 is $2\ k\Omega + 4\ k\Omega = 6\ k\Omega$. The equivalent resistance of the parallel connection of $R_1 + R_2$ and R_3 is given by $6\ k\Omega \| 6\ k\Omega = 3\ k\Omega$. The equivalent resistance of this result ($3\ k\Omega$) and R_4 is $3\ k\Omega \| 6\ k\Omega = 18\ k\Omega/9 = 2\ k\Omega$. The series connection of this result ($2\ k\Omega$) and R_5 yields $2\ k\Omega + 3\ k\Omega = 5\ k\Omega$. Thus, the Thévenin equivalent resistance is $5\ k\Omega$.

The Thévenin equivalent resistance can also be found using method 3. If a test voltage source with voltage of 1 V is applied between terminals a and b of the circuit shown in Figure 4.73 after deactivating sources, we obtain the circuit shown in Figure 4.77.

Summing the currents away from node 1 of the circuit shown in Figure 4.77, we obtain

$$\frac{V_1}{2000 + 4000} + \frac{V_1}{6000} + \frac{V_1}{6000} + \frac{V_1 - 1}{3000} = 0 \tag{4.37}$$

Multiplication of Equation (4.37) by 6000 results in

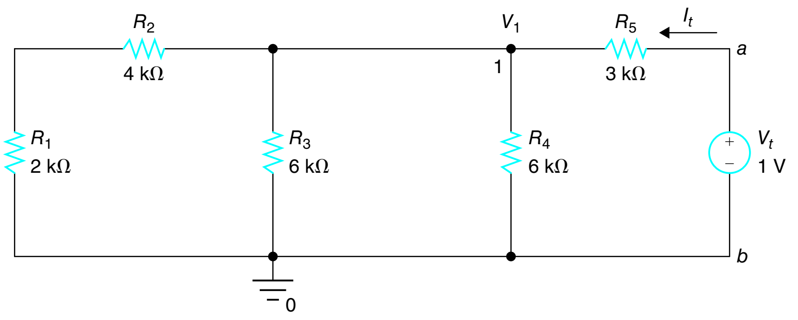
$$5V_1 = 2 \tag{4.38}$$

continued

Example 4.9 continued

FIGURE 4.77

Circuit with a test voltage.



The solution of Equation (4.38) is $V_1 = 2/5\ \text{V} = 0.4\ \text{V}$. Current I_t is given by

$$I_t = \frac{V_t - V_1}{R_5} = \frac{1\ \text{V} - \frac{2}{5}\ \text{V}}{3000} = \frac{1}{5}\ \text{mA} = 0.2\ \text{mA}$$

Thus, the Thévenin equivalent resistance is given by

$$R_{\text{th}} = \frac{V_t}{I_t} = \frac{1\ \text{V}}{\frac{1}{5}\ \text{mA}} = 5\ k\Omega$$

MATLAB

```
%EXAMPLE 4.9
%Function P.m should be in the same folder as this file.
clear all;
Is=5e-3;Vs=4;R1=2000;R2=4000;R3=6000;R4=6000;R5=3000;Vt=1;
syms V1 V2 V3 Va Vb Vc Vd
%Voc = Vth
[V1,V2,V3]=solve(V3==V2+Vs,...
-Is+V1/R1+(V1-V2)/R2,...
(V2-V1)/R2+V2/R3+V3/R4);
Vth=V3;
%Method 2: Rth2 = Voc/Isc = Vth/Isc
[Va,Vb,Vc]=solve(Vc==Vb+Vs,...
-Is+Va/R1+(Va-Vb)/R2,...
(Vb-Va)/R2+Vb/R3+Vc/R4+Vc/R5);
Isc=Vc/R5;
Rth2=Vth/Isc;
%Method 1: Rth1 = Req
Rth1=R5+P([R4,R3,R2+R1]);
%Method 3: Rth3 = Vt/It (test voltage)
Vd=solve(Vd/(R2+R1)+Vd/R3+Vd/R4+(Vd-Vt)/R5);
It=(Vt-Vd)/R5;
Rth3=Vt/It;
%Display results
V1=vpa(V1,7)
V2=vpa(V2,7)
V3=vpa(V3,7)
Va=vpa(Va,7)
Vb=vpa(Vb,7)
Vc=vpa(Vc,7)
Isc=vpa(Isc,7)
Vd=vpa(Vd,7)
```

continued

Example 4.9 continued

MATLAB continued

```
It=vpa(It,7)
Vth=vpa(Vth,10)
Rth2=vpa(Rth3,10)
Rth1=vpa(Rth1,10)
Rth3=vpa(Rth3,10)

Answers:
V1 =
7.333333
V2 =
2.0
V3 =
6.0
Va =
6.533333
Vb =
-0.4
Vc =
3.6
Isc =
0.0012
Vd =
0.4
It =
0.0002
Vth =
6.0
Rth2 =
5000.0
Rth1 =
5000.0
Rth3 =
5000.0
```

Exercise 4.9

Find the Thévenin equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.78.

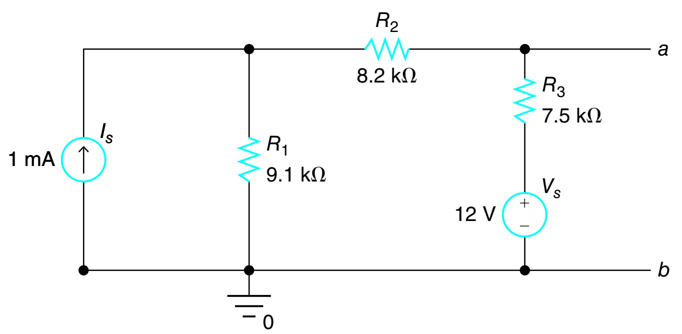


FIGURE 4.78 Circuit for EXERCISE 4.9.

Answer:

$V_{th} = 11.1230$ V, $R_{th} = 5.2319$ $k\Omega$.

EXAMPLE 4.10

Find the Thévenin equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.79.

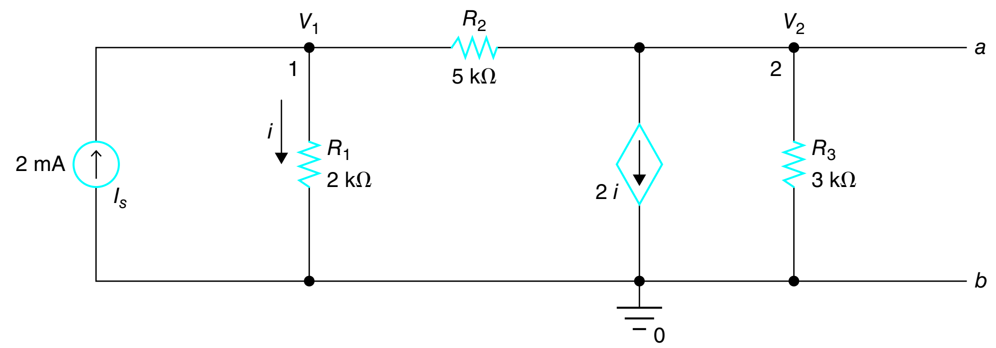


FIGURE 4.79

Circuit with a CCCS.

Notice that the open-circuit voltage between terminals a and b is the voltage across R_3, which is labeled V_2 in the circuit shown in Figure 4.79. Summing the currents away from node 1, we obtain

$$-0.002 + \frac{V_1}{2000} + \frac{V_1 - V_2}{5000} = 0$$

Multiplication of this equation by 10,000 yields

$$-20 + 5V_1 + 2(V_1 - V_2) = 0$$

which can be simplified to

$$7V_1 - 2V_2 = 20 \tag{4.39}$$

Summing the currents away from node 2, we obtain

$$\frac{V_2 - V_1}{5000} + 2\frac{V_1}{2000} + \frac{V_2}{3000} = 0$$

Multiplication by 30,000 yields

$$6V_2 - 6V_1 + 30V_1 + 10V_2 = 0$$

which can be simplified to

$$24V_1 + 16V_2 = 0 \tag{4.40}$$

Solving Equation (4.40) for V_1, we have

$$V_1 = -\frac{2}{3}V_2$$

Substituting this into Equation (4.39), we obtain

$$7V_1 - 2V_2 = 7\left(-\frac{2}{3}V_2\right) - 2V_2 = -\frac{20}{3}V_2 = 20$$

continued

Example 4.10 continued

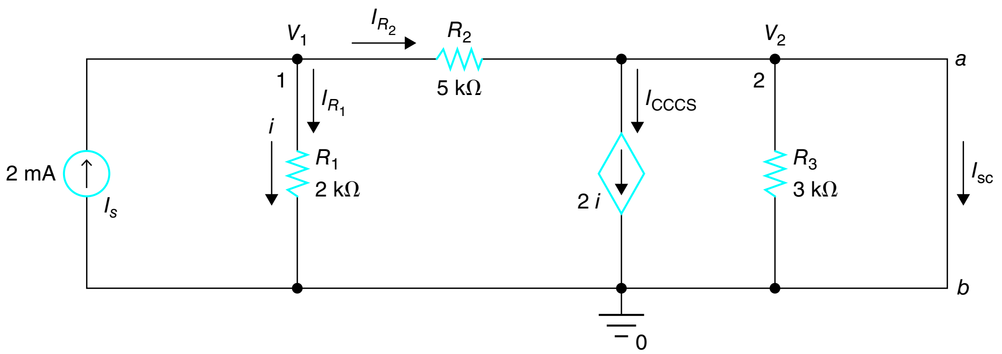
Thus, $V_2 = V_{oc} = -3$ V. The Thévenin equivalent voltage is the open-circuit voltage between a and b, which is V_2. Therefore,

$$V_{th} = V_{oc} = -3\ \text{V}$$

To find the Thévenin equivalent resistance, we first use method 2. The terminals a and b are short-circuited without changing the rest of the circuit, as shown in Figure 4.80.

FIGURE 4.80

A circuit with terminals a and b short-circuited.



Notice that node 2 is connected to ground and no current flows through R_3. From the current divider rule, the current through R_1 is given by

$$I_{R_1} = i = I_s \times \frac{R_2}{R_1 + R_2} = 2\ \text{mA} \times \frac{5\ k\Omega}{2\ k\Omega + 5\ k\Omega} = \frac{10}{7}\ \text{mA}$$

Similarly, the current through R_2 is given by

$$I_{R_2} = I_s \times \frac{R_1}{R_1 + R_2} = 2\ \text{mA} \times \frac{2\ k\Omega}{2\ k\Omega + 5\ k\Omega} = \frac{4}{7}\ \text{mA}$$

The current through a current-controlled current source (CCCS) is given by

$$I_{CCCS} = 2i = 2I_{R_1} = \frac{20}{7}\ \text{mA}$$

Applying KCL at node 2, we obtain

$$I_{R_2} = I_{CCCS} + I_{sc}$$

Solving for I_{sc}, we have

$$I_{sc} = I_{R_2} - I_{CCCS} = \frac{4}{7}\ \text{mA} - \frac{20}{7}\ \text{mA} = -\frac{16}{7}\ \text{mA} = -2.2857\ \text{mA}$$

The Thévenin equivalent resistance is the ratio of the open-circuit voltage to the short-circuit current. Thus, we have

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{-3\ \text{V}}{-\frac{16}{7}\ \text{mA}} = \frac{21}{16}\ k\Omega = 1.3125\ k\Omega$$

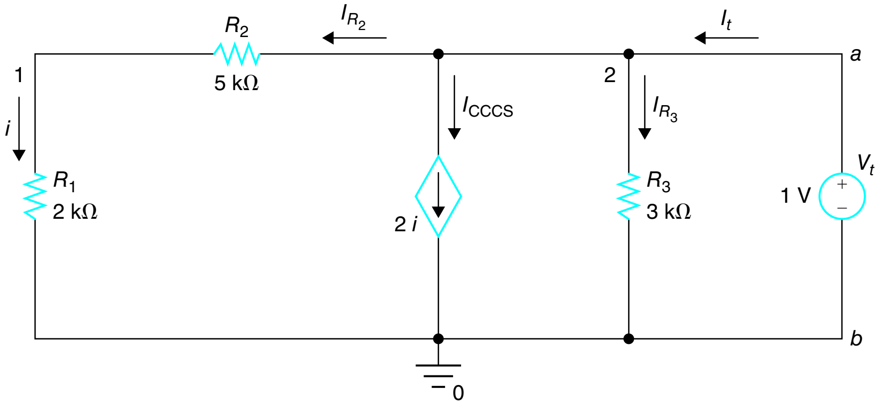
continued

Example 4.10 continued

The Thévenin equivalent resistance can also be found using method 3. After deactivating the current source by removing it from the circuit, a test voltage is applied to the circuit from terminals *a* and *b*, as shown in Figure 4.81.

FIGURE 4.81

A circuit with a test voltage source.



The test voltage V_t is applied across R_3 and $R_2 - R_1$. The current through $R_2 - R_1$, which is also the controlling current, is given by

$$I_{R_2} = i = \frac{V_t}{R_2 + R_1} = \frac{1\text{ V}}{7\ k\Omega} = \frac{1}{7}\text{ mA}$$

The current through R_3 is given by

$$I_{R_3} = \frac{V_t}{R_3} = \frac{1\text{ V}}{3\ k\Omega} = \frac{1}{3}\text{ mA}$$

The current through CCCS is twice the controlling current *i*. Thus,

$$I_{CCCS} = 2i = 2I_{R_2} = \frac{2}{7}\text{ mA}$$

The total current flowing out of the positive terminal of the test voltage source is given by

$$I_t = I_{R_2} + I_{CCCS} + I_{R_3} = \frac{1}{7}\text{ mA} + \frac{2}{7}\text{ mA} + \frac{1}{3}\text{ mA} = \frac{16}{21}\text{ mA} = 0.7619\text{ mA}$$

The Thévenin equivalent resistance is the ratio of V_t to I_t:

$$R_{\text{th}} = \frac{V_t}{I_t} = \frac{1\text{ V}}{\frac{16}{21}\text{ mA}} = \frac{21}{16}\ k\Omega = 1.3125\ k\Omega$$

After deactivating the current source, instead of a test voltage source, a test current source can be applied to the circuit from terminals *a* and *b*, as shown in Figure 4.82.

Summing the currents leaving node 2, we obtain

$$\frac{V_t}{7000} + \frac{2V_t}{7000} + \frac{V_t}{3000} - 0.001 = 0$$

Multiplication by 21,000 yields

$$3V_t + 6V_t + 7V_t = 21$$

continued

Example 4.10 continued

FIGURE 4.82

A circuit with a test current source.

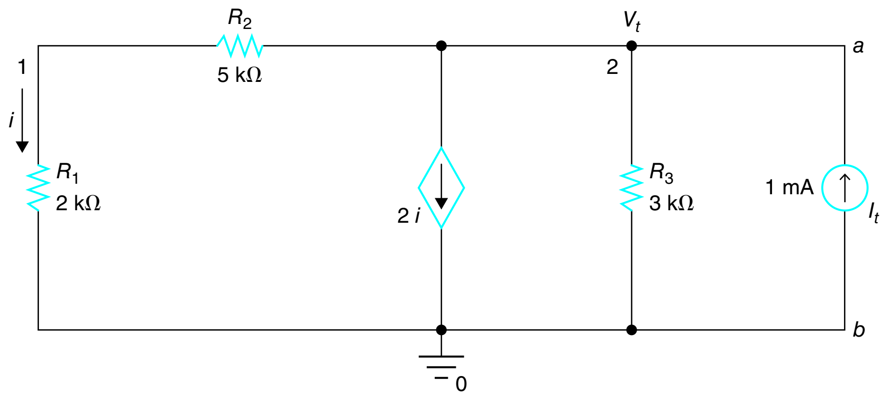
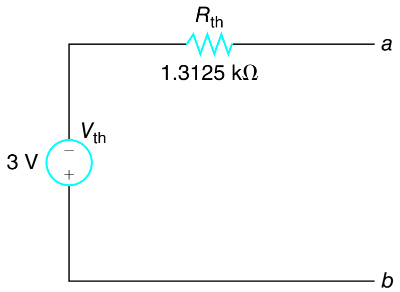


FIGURE 4.83

The Thévenin equivalent circuit.



Solving for V_t, we obtain

$$V_t = \frac{21}{16}\text{ V} = 1.3125\text{ V}$$

The Thévenin equivalent resistance is the ratio of V_t to I_t:

$$R_{\text{th}} = \frac{V_t}{I_t} = \frac{\frac{21}{16}\text{ V}}{1\text{ mA}} = \frac{21}{16}\,k\Omega = 1.3125\,k\Omega$$

The Thévenin equivalent circuit is shown in Figure 4.83.

MATLAB

```
%EXAMPLE 4.10
clear all;
Is=2e-3;R1=2000;R2=5000;R3=3000;ki=2;Vta=1;Itb=1e-3;
syms V1 V2 Va Vtb
%Voc = Vth
[V1,V2]=solve(-Is+V1/R1+(V1-V2)/R2,(V2-V1)/R2+ki*V1/R1+V2/R3);
Vth=V2;
%Method 2: Rth2 = Voc/Isc
Va=solve(-Is+Va/R1+Va/R2);
Isc=Va/R2-ki*Va/R1;
Rth2=Vth/Isc;
%Method 3: Rth3a = Vta/Ita (test voltage)
IR2=Vta/(R2+R1);
Iccs=ki*IR2;
IR3=Vta/R3;
Ita=IR2+Iccs+IR3;
Rth3a=Vta/Ita;
Rth3a=vpa(Rth3a,10);
%Method 3: Rth3b = Vtb/Itb (test current)
Vtb=solve(Vtb/(R2+R1)+ki*Vtb/(R2+R1)+Vtb/R3-Itb);
Rth3b=Vtb/Itb;
Rth3b=vpa(Rth3b,10);
%Display results
V1=vpa(V1,7)
V2=vpa(V2,7)
Va=vpa(Va,7)
```

continued

Example 4.10 continued

MATLAB continued

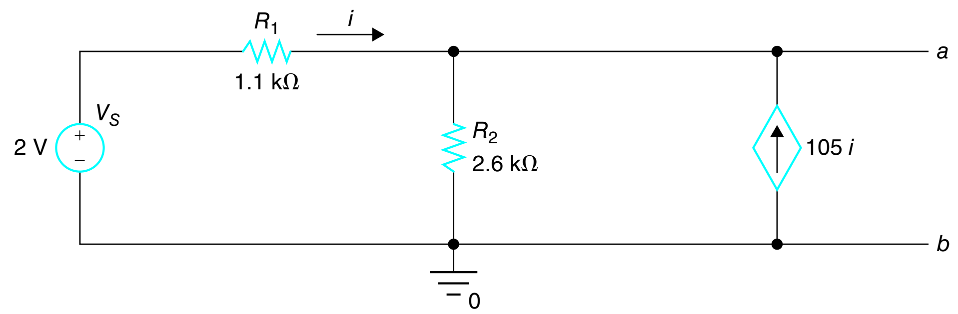
```
Isc=vpa(Isc,7)
Ita=vpa(Ita,7)
Vtb=vpa(Vtb,7)
Vth=vpa(Vth,10)
Rth2=vpa(Rth2,10)
Rth3a=vpa(Rth3a,10)
Rth3b=vpa(Rth3b,10)

Answers:
V1 =
2.0
V2 =
-3.0
Va =
2.857143
Isc =
-0.002285714
Ita =
0.0007619048
Vtb =
1.3125
Vth =
-3.0
Rth2 =
1312.5
Rth3a =
1312.5
Rth3b =
1312.5
```

Exercise 4.10

Find the Thévenin equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.84.

FIGURE 4.84

Circuit for EXERCISE 4.10.



Answer:

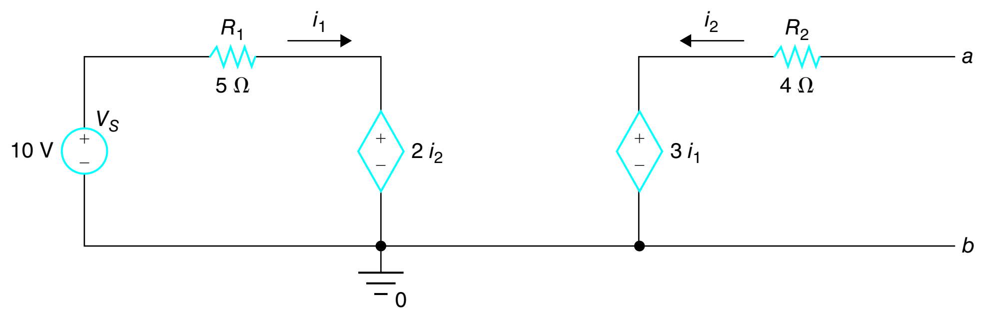
$V_{th} = 1.9920$ V, $R_{th} = 10.3361\ \Omega$.

EXAMPLE 4.11

Find the Thévenin equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.85.

FIGURE 4.85

Circuit for EXAMPLE 4.11.



Since current i_2 equals zero, the voltage across the CCVS on the left side of the circuit is zero. Current i_1 is given by

$$i_1 = \frac{V_s}{R_1} = \frac{10\text{ V}}{5\ \Omega} = 2\text{ A}$$

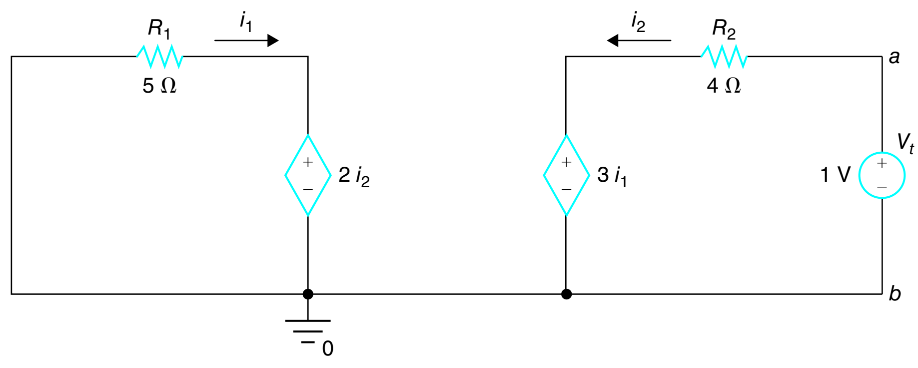
The Thévenin voltage is the voltage across the CCVS on the right side of the circuit. Thus,

$$V_{\text{th}} = 3i_1 = 3 \times 2\text{ V} = 6\text{ V}$$

To find the Thévenin resistance, we deactivate the voltage source by short-circuiting it, and then apply a test voltage V_t of 1 V across a and b, as shown in Figure 4.86.

FIGURE 4.86

A circuit with a test voltage.



Collecting the voltage drops around the mesh on the left side in the clockwise direction, we obtain

$$5i_1 + 2i_2 = 0$$

Solving for i_1, we get

$$i_1 = \frac{-2i_2}{5} = -0.4i_2 \tag{4.41}$$

continued

Example 4.11 continued

Collecting the voltage drops around the mesh on the right side in the clockwise direction, we obtain

$$-3i_1 - 4i_2 + 1 = 0 \tag{4.42}$$

Substitution of Equation (4.41) into Equation (4.42) yields

$$1.2i_2 - 4i_2 + 1 = 0$$

Solving for i_2, we get

$$i_2 = \frac{1}{2.8}\text{ A} = 0.3571\text{ A}$$

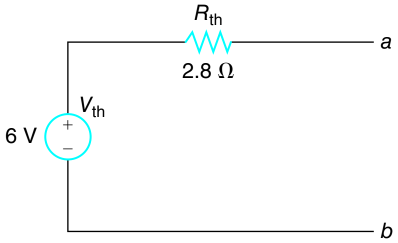
The Thévenin equivalent resistance is given by

$$R_{th} = \frac{V_t}{i_2} = \frac{1\text{ V}}{\frac{1}{2.8}\text{ A}} = 2.8\ \Omega$$

The Thévenin equivalent circuit is shown in Figure 4.87.

FIGURE 4.87

The Thévenin equivalent circuit.

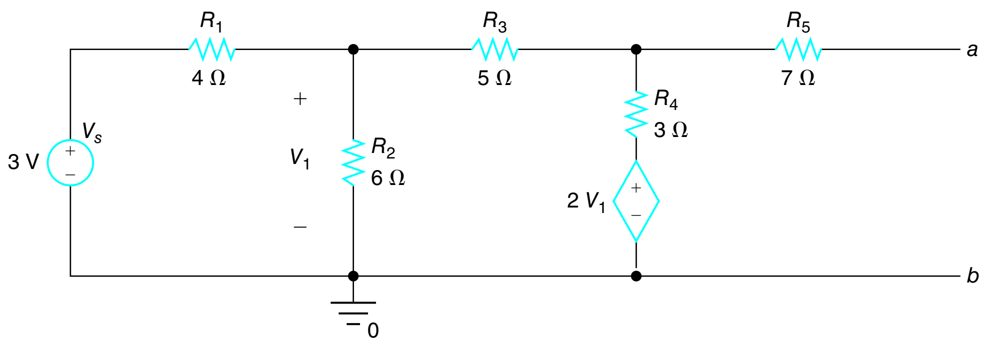


Exercise 4.11

Find the Thévenin equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.88.

FIGURE 4.88

Circuit for EXERCISE 4.11.



Answer:
$V_{th} = 4.1786$ V, $R_{th} = 10.9643\ \Omega$.

EXAMPLE 4.12

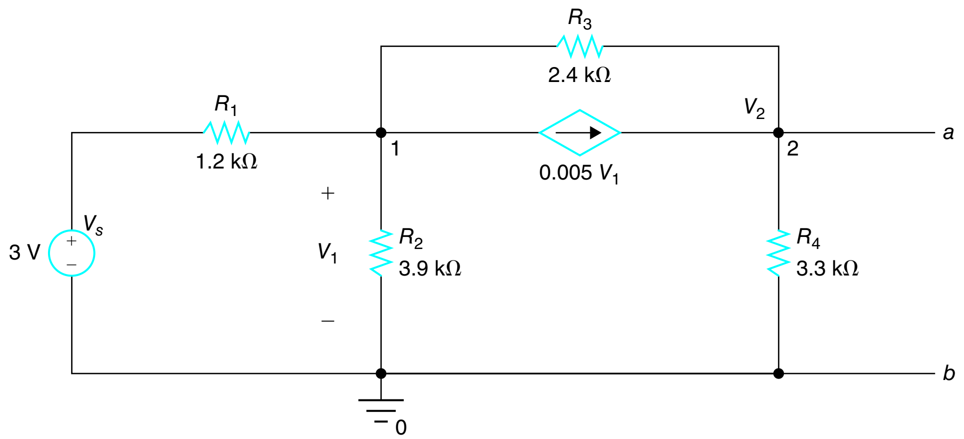
Find the Thévenin equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.89.

continued

Example 4.12 continued

FIGURE 4.89

Circuit for EXAMPLE 4.12.



The Thévenin equivalent circuit between terminals *a* and *b* can be found by finding the open-circuit voltage V_{oc} and the short-circuit current I_{sc}. The open-circuit voltage V_{oc} is V_2, which is the voltage across R_4 in the circuit shown in Figure 4.89. Summing the currents leaving node 1, we obtain

$$\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} + g_m V_1 = 0 \tag{4.43}$$

which can be rearranged as

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + g_m\right)V_1 - \frac{1}{R_3}V_2 = \frac{V_s}{R_1}$$

Substituting the component values, we get

$$\left(\frac{1}{1200} + \frac{1}{3900} + \frac{1}{2400} + 0.005\right)V_1 - \frac{1}{2400}V_2 = \frac{3}{1200}$$

which can be simplified to

$$6.50641 \times 10^{-3}V_1 - 4.16667 \times 10^{-4}V_2 = 2.5 \times 10^{-3}$$

Multiplication by 1000 yields

$$6.50641V_1 - 0.416667V_2 = 2.5 \tag{4.44}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_2 - V_1}{R_3} - g_m V_1 + \frac{V_2}{R_4} = 0 \tag{4.45}$$

which can be revised as

$$-\left(\frac{1}{R_3} + g_m\right)V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_2 = 0$$

Substituting the component values, we get

$$-\left(\frac{1}{2400} + 0.005\right)V_1 + \left(\frac{1}{2400} + \frac{1}{3300}\right)V_2 = 0$$

continued

Example 4.12 continued

which can be simplified to

$$-5.416667 \times 10^{-3} V_1 + 7.19697 \times 10^{-4} V_2 = 0$$

Multiplication by 1000 yields

$$-5.416667 V_1 + 0.719697 V_2 = 0 \tag{4.46}$$

Solving Equation (4.46) for V_2, we obtain

$$V_2 = \frac{5.416667}{0.719697} V_1 = 7.526317 V_1$$

Substituting V_2 into Equation (4.44), we get

$$6.50641 V_1 - 0.416667(7.526317 V_1) = 2.5$$

Thus,

$$V_1 = \frac{2.5}{6.50641 - 0.416667 \times 7.526317} = 0.74174 \text{ V}$$

$$V_2 = 7.526317 V_1 = 5.5826 \text{ V}$$

Alternatively, application of Cramer's rule to Equations (4.44) and (4.46) yields

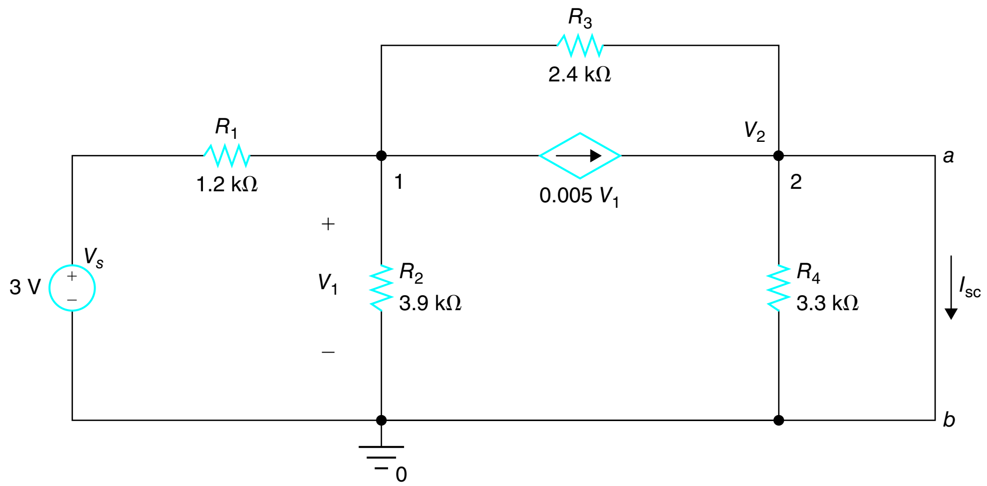
$$V_1 = \frac{\begin{vmatrix} 2.5 & -0.416667 \\ 0 & 0.719697 \end{vmatrix}}{\begin{vmatrix} 6.50641 & -0.416667 \\ -5.416667 & 0.719697 \end{vmatrix}} = \frac{1.7992}{2.4257} = 0.74174 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 6.50641 & 2.5 \\ -5.416667 & 0 \end{vmatrix}}{\begin{vmatrix} 6.50641 & -0.416667 \\ -5.416667 & 0.719697 \end{vmatrix}} = \frac{13.541667}{2.4257} = 5.5826 \text{ V}$$

To find the short-circuit current, we short-circuit a and b, as shown in Figure 4.90.

FIGURE 4.90

The circuit from Figure 4.89 with a and b short-circuited.



continued

Example 4.12 continued

Notice that node 2 is connected to ground and no current flows through R_4. Summing the currents leaving node 1 of the circuit shown in Figure 4.90, we obtain

$$\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_2} + \frac{V_1}{R_3} + g_m V_1 = 0$$

which can be rearranged as

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + g_m\right)V_1 = \frac{V_s}{R_1}$$

Thus, we obtain

$$V_1 = \frac{\dfrac{V_s}{R_1}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + g_m} = \frac{\dfrac{3}{1200}}{\dfrac{1}{1200} + \dfrac{1}{3900} + \dfrac{1}{2400} + 0.005} = 0.38423645 \text{ V}$$

The short-circuit current is given by

$$I_{sc} = \frac{V_1}{R_3} + g_m V_1 = V_1\left(\frac{1}{R_3} + g_m\right)$$

$$= 0.38423645\left(\frac{1}{2400} + 0.005\right) = 2.0813 \text{ mA}$$

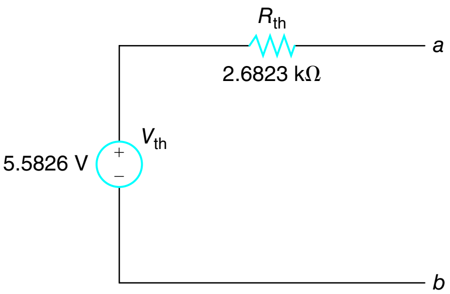
The Thévenin equivalent resistance is given by

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{5.5826 \text{ V}}{2.6823 \text{ mA}} = 2.6823\ k\Omega$$

The Thévenin equivalent circuit is shown in Figure 4.91.

FIGURE 4.91

The Thévenin equivalent circuit.



MATLAB

```
%EXAMPLE 4.12
clear all;
Vs=3;R1=1200;R2=3900;R3=2400;R4=3300;gm=0.005;
syms V1 V2 Va Vb
%Voc = Vth
[V1,V2]=solve((V1-Vs)/R1+V1/R2+(V1-V2)/R3+gm*V1,...
(V2-V1)/R3-gm*V1+V2/R4,V1,V2);
Vth=V2;
%Method 2: Rth = Voc/Isc
Va=solve((Va-Vs)/R1+Va/R2+(Va-0)/R3+gm*Va,Va);
Isc=Va/R3+gm*Va;
Rth=Vth/Isc;
%Display results
V1=vpa(V1,8)
V2=vpa(V2,8)
Va=vpa(Va,8)
Isc=vpa(Isc,8)
Vth=vpa(Vth,8)
Rth=vpa(Rth,8)

Answers:
V1 =
0.74174174
```

continued

Example 4.12 continued

MATLAB continued

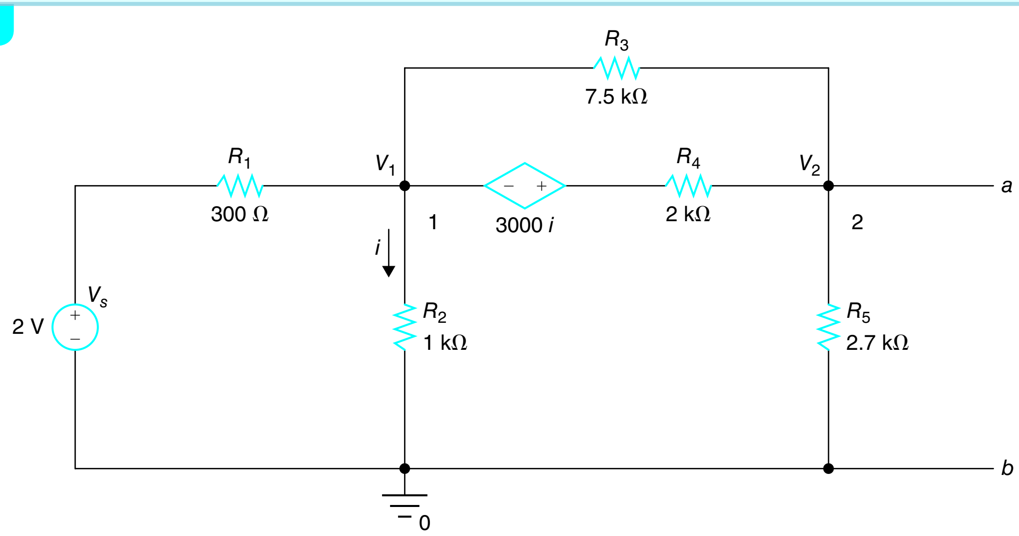
```
V2 =
5.5825826
Va =
0.38423645
Isc =
0.0020812808
Vth =
5.5825826
Rth =
2682.2823
```

Exercise 4.12

Find the Thévenin equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.92.

FIGURE 4.92

Circuit for EXERCISE 4.12.



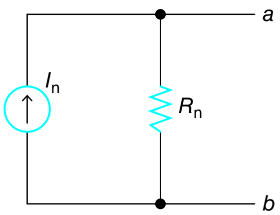
Answer:

$V_{th} = 2.7672$ V, $R_{th} = 1.2582\ k\Omega$.

4.5 Norton's Theorem

FIGURE 4.93

A Norton equivalent circuit.



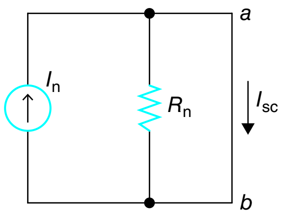
A circuit looking from terminals *a* and *b* can be replaced by a current source with current I_n and a parallel resistor with resistance R_n, as shown in Figure 4.93. This equivalent circuit consisting of a current source and a parallel resistor is called **Norton equivalent circuit**. The current I_n is called **Norton equivalent current** and the resistance R_n is called **Norton equivalent resistance**.

The Norton equivalent circuit can be used to simplify a circuit. When a load resistor is connected between terminals *a* and *b*, we can find the effects of the circuit on the load from the Norton equivalent circuit. We do not need all the details of the original circuit to find the voltage, current, and power on the load.

When the terminals *a* and *b* are short-circuited in the Norton equivalent circuit, as shown in Figure 4.94, the short-circuit current I_{sc} is equal to I_n from the current divider rule. Thus, the Norton equivalent current can be obtained by finding the short-circuit current.

FIGURE 4.94

Short-circuit current.



4.5.1 FINDING THE NORTON EQUIVALENT CURRENT I_n

Given a circuit and terminals *a* and *b*, we can find the Norton equivalent current I_n with respect to terminals *a* and *b* by finding the short-circuit current I_{sc} between terminals *a* and *b*. The short-circuit current I_{sc} can be found by utilizing circuit analysis methods such as the voltage divider rule, current divider rule, superposition principle, nodal analysis, and mesh analysis. The Norton equivalent current I_n is found from the original circuit by connecting a wire between *a* and *b* without any changes for the rest of the circuit.

4.5.2 FINDING THE NORTON EQUIVALENT RESISTANCE R_n

The Norton equivalent resistance can be obtained by applying the three methods used to find the Thévenin equivalent resistance. These three methods are listed next.

Method 1

Deactivate all the independent sources by short-circuiting the voltage sources and open-circuiting the current sources. Find the equivalent resistance looking into the circuit from terminals *a* and *b*. This equivalent resistance is the Norton equivalent resistance R_n. This method can be used if the circuit does not contain dependent sources.

Method 2

Find the open-circuit voltage V_{oc} and the short-circuit current I_{sc} between terminals *a* and *b*. The Norton equivalent resistance R_n is given by

$$R_n = \frac{V_{oc}}{I_{sc}}$$

Method 3

Deactivate all the independent sources by open-circuiting current sources and short-circuiting voltage sources. Apply a test voltage of 1 V (or any other value) between terminals *a* and *b* with terminal *a* connected to the positive terminal of the test voltage. Measure the current flowing out of the positive terminal of the test voltage source. The Norton equivalent resistance R_n is given by the ratio of the test voltage to the current flowing out of the positive terminal of the test voltage source. A test current can be used instead of a test voltage. Apply a test current between terminals *a* and *b* after deactivating the independent sources and measure the voltage across *a* and *b* of the test current source. The Norton equivalent resistance R_n is the ratio of the voltage across *a* and *b* to the test current.

4.5.3 RELATION BETWEEN THE THÉVENIN EQUIVALENT CIRCUIT AND THE NORTON EQUIVALENT CIRCUIT

Application of source transformation to the Norton equivalent circuit shown in Figure 4.95(a) yields the Thévenin equivalent circuit shown in Figure 4.95(b). Notice that the Thévenin equivalent voltage is $V_{th} = I_n R_n$ and the Thévenin equivalent resistance is $R_{th} = R_n$. The source transformation does not change the resistance value. Application of source transformation to the Thévenin equivalent circuit shown in Figure 4.96(a) yields the Norton equivalent

FIGURE 4.95

Transformation from Norton equivalent circuit to Thévenin equivalent circuit.

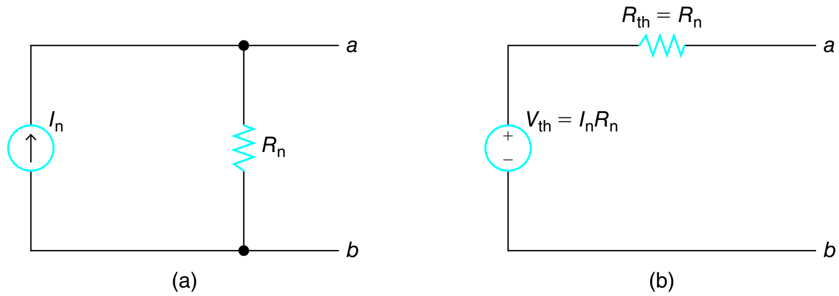
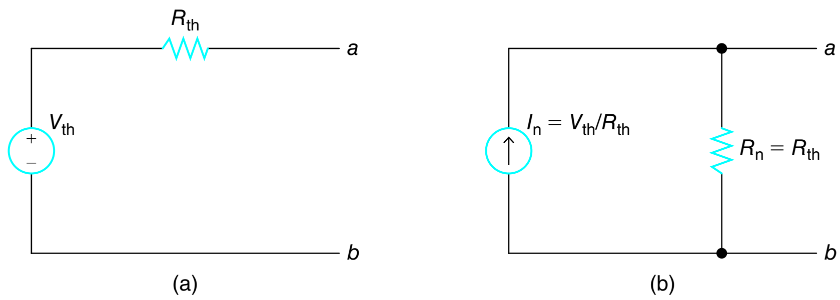


FIGURE 4.96

Transformation from Thévenin equivalent circuit to Norton equivalent circuit.

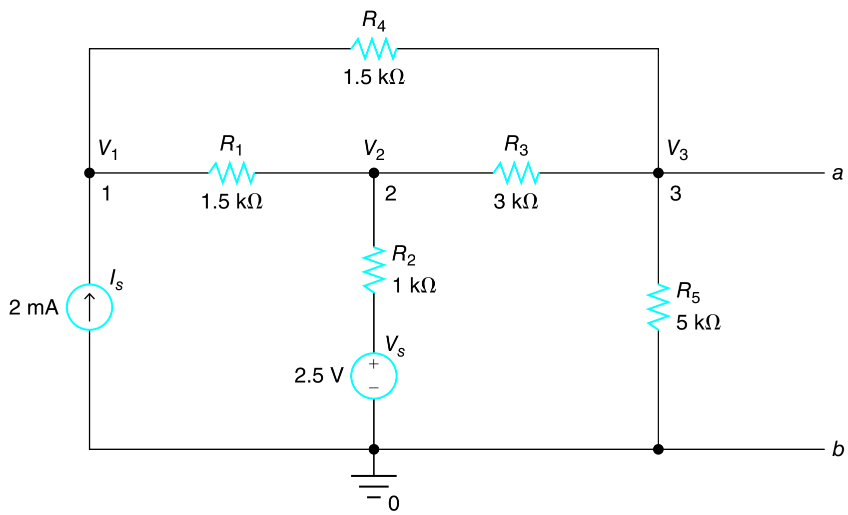


circuit, as shown in Figure 4.96(b). Notice that the Norton equivalent current is $I_n = V_{th}/R_{th}$ and the Norton equivalent resistance is $R_n = R_{th}$. The source transformation does not change the resistance value.

Consider the circuit shown in Figure 4.97. We are interested in finding a Norton equivalent circuit looking into the circuit from terminals *a* and *b*.

FIGURE 4.97

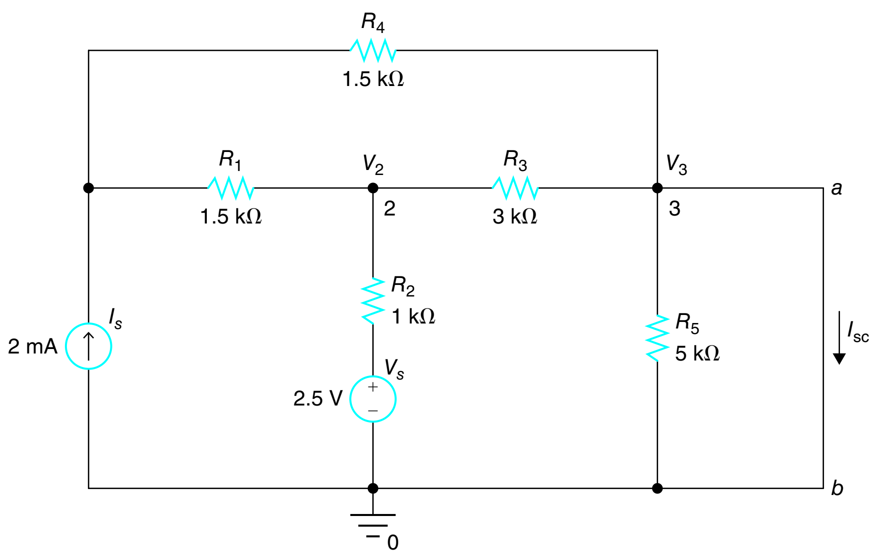
A circuit with two sources.



To find the short-circuit current, we short-circuit *a* and *b*, as shown in Figure 4.98. Node 3 is a ground, and no current flows through R_5.

FIGURE 4.98

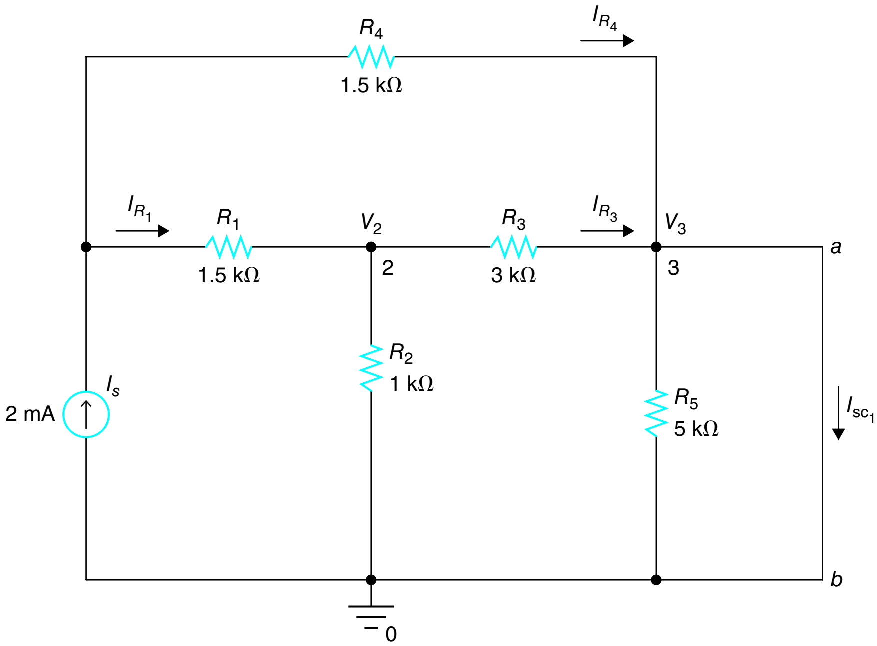
A circuit with *a* and *b* shorted.



We can find the short-circuit current using the superposition principle. First, we deactivate the voltage source by short-circuiting it, as shown in Figure 4.99.

FIGURE 4.99

A circuit with the voltage source deactivated.



Notice that R_2 and R_3 are connected in parallel. Let R_a be the equivalent resistance of $R_1 + (R_2 \| R_3)$. Then, we have

$$R_a = R_1 + (R_2 \| R_3) = 1.5\ k\Omega + (1\ k\Omega \| 3\ k\Omega) = 1.5\ k\Omega + 0.75\ k\Omega = 2.25\ k\Omega$$

Application of the current divider rule yields

$$I_{R_4} = I_s \times \frac{R_a}{R_a + R_4} = 2\ \text{mA} \times \frac{2.25\ k\Omega}{2.25\ k\Omega + 1.5\ k\Omega} = 1.2\ \text{mA}$$

The current through R_1 is given by

$$I_{R_1} = I_s - I_{R_4} = 2\ \text{mA} - 1.2\ \text{mA} = 0.8\ \text{mA}$$

Application of the current divider rule to R_2 and R_3 yields

$$I_{R_3} = I_{R_1} \times \frac{R_2}{R_2 + R_3} = 0.8\ \text{mA} \times \frac{1\ k\Omega}{1\ k\Omega + 3\ k\Omega} = 0.2\ \text{mA}$$

The short-circuit current is given by

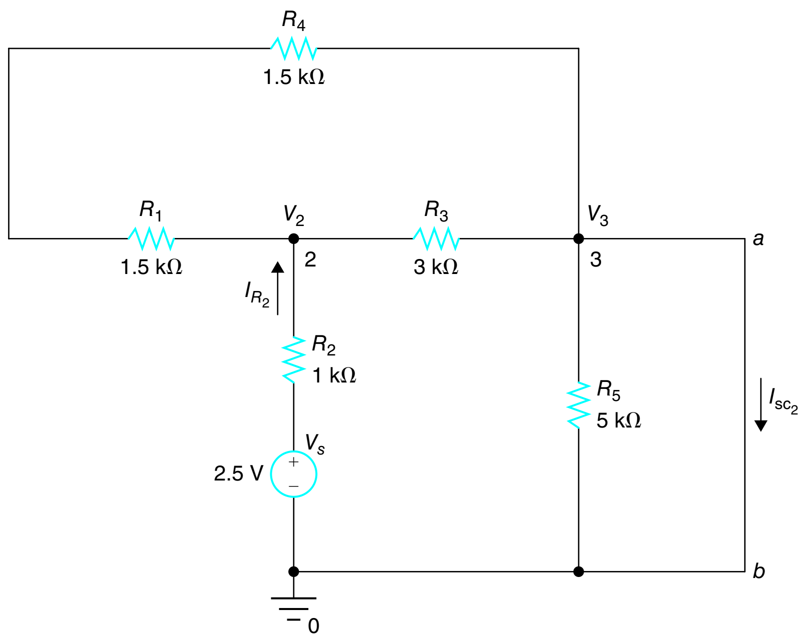
$$I_{sc_1} = I_{R_4} + I_{R_3} = 1.2\ \text{mA} + 0.2\ \text{mA} = 1.4\ \text{mA}$$

Now, we deactivate the current source by open-circuiting it, as shown in Figure 4.100. Let R_b be the equivalent resistance of parallel connection of $R_1 + R_4$ and R_3. Then, we have

$$R_b = (R_1 + R_4) \| R_3 = 3\ k\Omega \| 3\ k\Omega = 1.5\ k\Omega$$

FIGURE 4.100

Circuit with the current source deactivated.



The total resistance seen from the voltage source is $R_2 + R_b$. Thus, the current through R_2, which is also the short-circuit current, is given by

$$I_{R_2} = I_{sc_2} = \frac{V_s}{R_2 + R_b} = \frac{2.5\text{ V}}{1\ k\Omega + 1.5\ k\Omega} = 1\text{ mA}$$

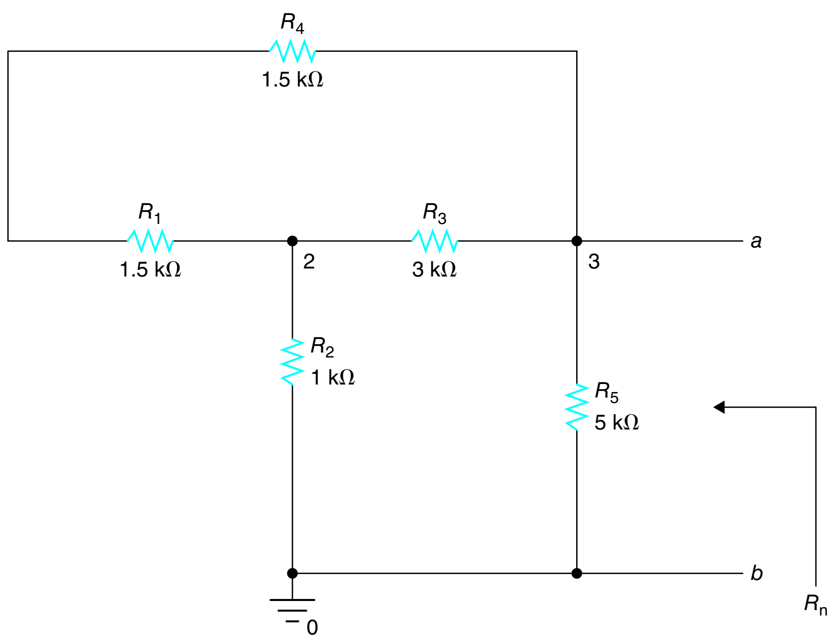
The total short-circuit current from the two sources is

$$I_{sc} = I_{sc_1} + I_{sc_2} = 1.4\text{ mA} + 1\text{ mA} = 2.4\text{ mA}$$

To find the Norton equivalent resistance, we deactivate the two sources in the circuit shown in Figure 4.97 to obtain the circuit shown in Figure 4.101.

FIGURE 4.101

The circuit from Figure 4.97 with sources deactivated.



The Norton equivalent resistance is given by the parallel connection of R_5 and $R_b + R_2$, where $R_b = (R_1 + R_4)\|R_3 = 1.5\ k\Omega$. Thus, we have

$$R_n = R_5\|(R_b + R_2) = 5\ k\Omega\|2.5\ k\Omega = 1.6667\ k\Omega.$$

The Norton equivalent circuit is shown in Figure 4.102.

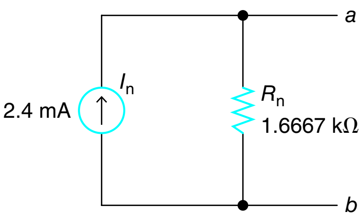


FIGURE 4.102 Norton equivalent circuit.

Consider a circuit with a CCVS shown in Figure 4.103. We are interested in finding the Norton equivalent circuit between terminals a and b.

To find the Norton equivalent current, we short-circuit terminals a and b, as shown in Figure 4.104. Notice that node 2 is connected to ground, and the current through R_4 is zero.

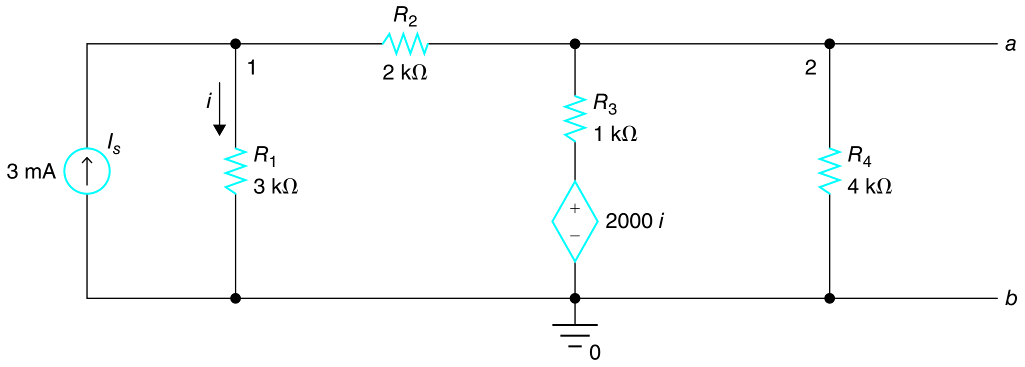


FIGURE 4.103 A circuit with CCVS.

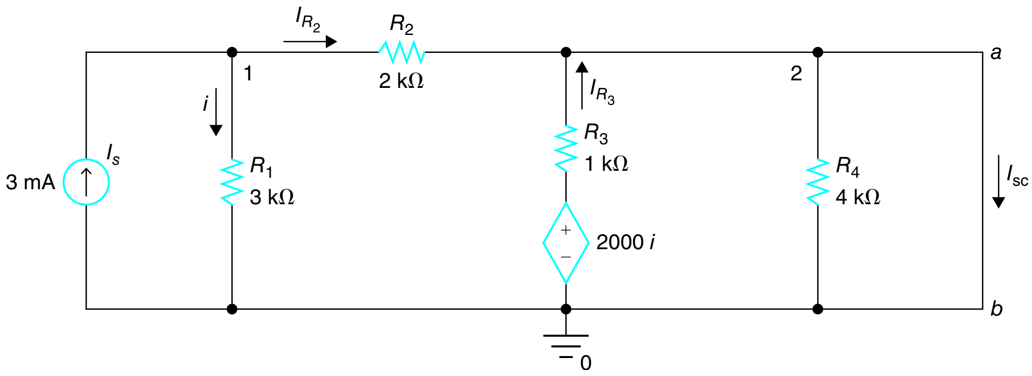


FIGURE 4.104 A circuit with a and b short-circuited.

From the current divider rule, the current through R_2 is given by

$$I_{R_2} = I_s \times \frac{R_1}{R_1 + R_2} = 3\text{ mA} \times \frac{3\ k\Omega}{3\ k\Omega + 2\ k\Omega} = 1.8\text{ mA}$$

From KCL, the current through R_1 is given by

$$i = I_s - I_{R_2} = 3\text{ mA} - 1.8\text{ mA} = 1.2\text{ mA}$$

The voltage across the CCVS is

$$V_{CCVS} = 2000i = 2000\,(\text{V/A}) \times 1.2\text{ mA} = 2.4\text{ V}$$

The current through R_3 is

$$I_{R_3} = \frac{V_{CCVS}}{R_3} = \frac{2.4\text{ V}}{1\ k\Omega} = 2.4\text{ mA}$$

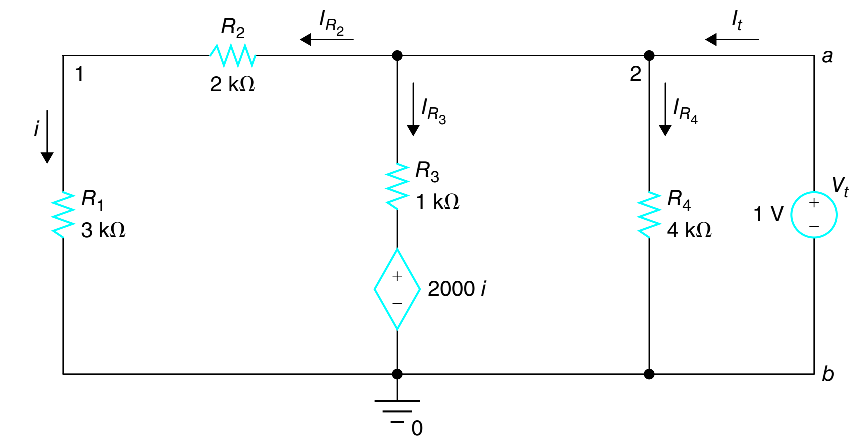
The short-circuit current is the sum of I_{R_2} and I_{R_3}:

$$I_{sc} = I_n = I_{R_2} + I_{R_3} = 1.8 \text{ mA} + 2.4 \text{ mA} = 4.2 \text{ mA}$$

To find the Norton equivalent resistance, after deactivating the current source, we apply a test voltage to the circuit at the terminals a and b, as shown in Figure 4.105.

FIGURE 4.105

A circuit with a test voltage source.



The current through R_2, which is also the controlling current i, is given by

$$I_{R_2} = i = \frac{V_t}{R_2 + R_1} = \frac{1 \text{ V}}{2 \, k\Omega + 3 \, k\Omega} = 0.2 \text{ mA}$$

The voltage of the CCVS is given by

$$V_{CCVS} = 2000i = 2000 \, (\text{V/A}) \times 0.2 \text{ mA} = 0.4 \text{ V}$$

The current through R_3 is given by

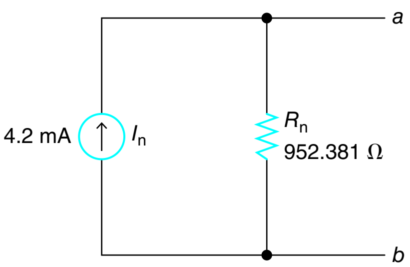
$$I_{R_3} = \frac{V_t - V_{CCVS}}{R_3} = \frac{1 \text{ V} - 0.4 \text{ V}}{1 \, k\Omega} = 0.6 \text{ mA}$$

The current through R_4 is given by

$$I_{R_4} = \frac{V_t}{R_4} = \frac{1 \text{ V}}{4 \, k\Omega} = 0.25 \text{ mA}$$

FIGURE 4.106

Norton equivalent circuit.



The total current flowing out of the positive terminal of the test voltage source is given by

$$I_t = I_{R_2} + I_{R_3} + I_{R_4} = 0.2 \text{ mA} + 0.6 \text{ mA} + 0.25 \text{ mA} = 1.05 \text{ mA}$$

The Norton equivalent resistance is the ratio of V_t to I_t:

$$R_n = \frac{V_t}{I_t} = \frac{1 \text{ V}}{1.05 \text{ mA}} = 952.381 \ \Omega$$

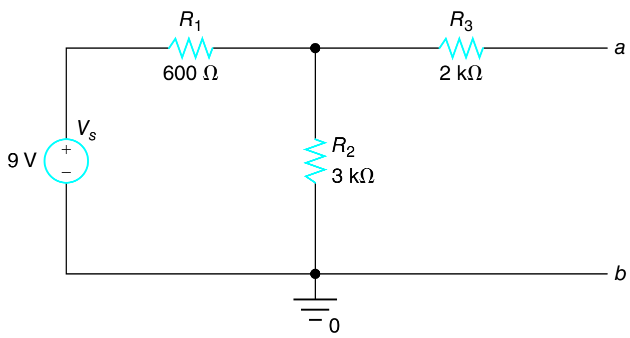
The Norton equivalent circuit is shown in Figure 4.106.

EXAMPLE 4.13

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.107.

FIGURE 4.107

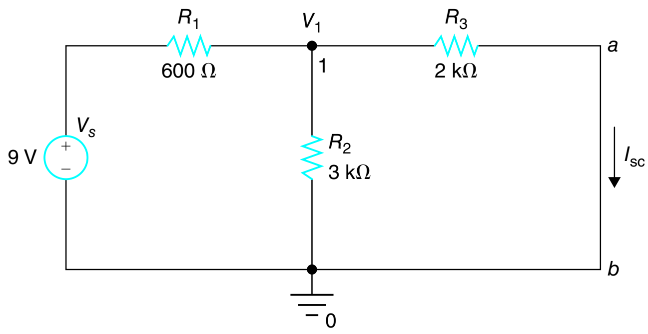
Circuit for EXAMPLE 4.13.



To find the Norton equivalent current, *a* and *b* are short-circuited, as shown in Figure 4.108.

FIGURE 4.108

Circuit shown in Figure 4.107 with *a* and *b* shorted.



Let R_a be the equivalent resistance of the parallel connection of R_2 and R_3. Then, we get

$$R_a = R_2 \| R_3 = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{3\ k\Omega \times 2\ k\Omega}{3\ k\Omega + 2\ k\Omega} = \frac{6}{5}\,k\Omega = 1.2\ k\Omega$$

Application of the voltage divider rule yields

$$V_1 = V_s \times \frac{R_a}{R_1 + R_a} = 9\text{ V} \times \frac{1.2}{0.6 + 1.2} = 9\text{ V} \times \frac{1.2}{1.8} = 6\text{ V}$$

The short-circuit current I_{sc}, which is the Norton equivalent current, is the current through R_3. Thus, we have

$$I_n = I_{sc} = \frac{V_1}{R_3} = \frac{6\text{ V}}{2\ k\Omega} = 3\text{ mA}$$

To find the Norton equivalent resistance, R_n, we deactivate the voltage source by short-circuiting it, as shown in Figure 4.109, and find the equivalent resistance seen from terminals *a* and *b*. The Norton equivalent resistance is given by

continued

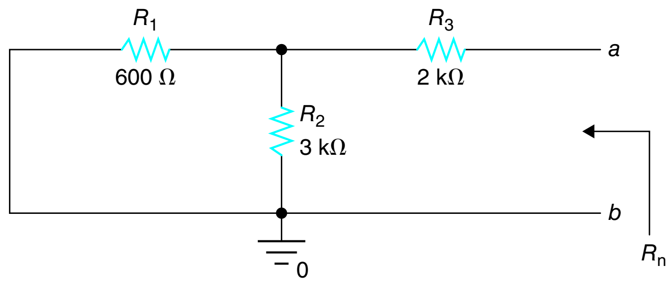
Example 4.13 continued

$$R_n = R_3 + (R_1 \| R_2) = 2\ k\Omega + (0.6\ k\Omega \| 3\ k\Omega) = 2\ k\Omega + \frac{0.6 \times 3}{0.6 + 3} k\Omega$$

$$= 2\ k\Omega + \frac{1.8}{3.6} k\Omega = 2.5\ k\Omega$$

FIGURE 4.109

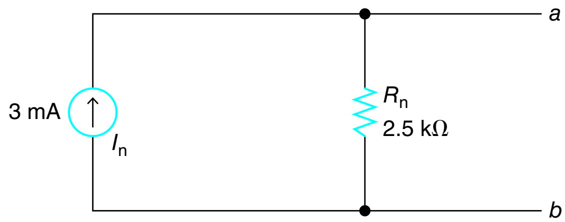
The circuit shown in Figure 4.107 after deactivating the voltage source.



The Norton equivalent circuit is shown in Figure 4.110.

FIGURE 4.110

The Norton equivalent circuit.

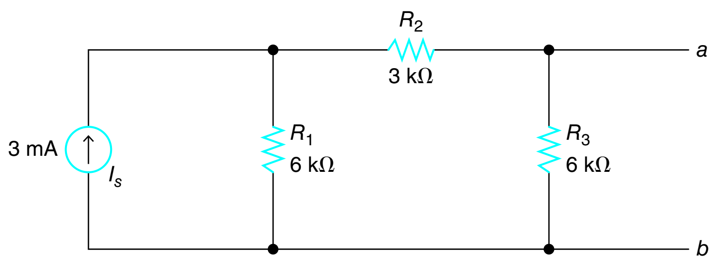


Exercise 4.13

Find the Norton equivalent current I_n and the Norton equivalent resistance R_n between terminals *a* and *b* for the circuit shown in Figure 4.111.

FIGURE 4.111

Circuit for EXERCISE 4.13.



Answer:

$I_n = 2$ mA, $R_n = 3.6\ k\Omega$.

EXAMPLE 4.14

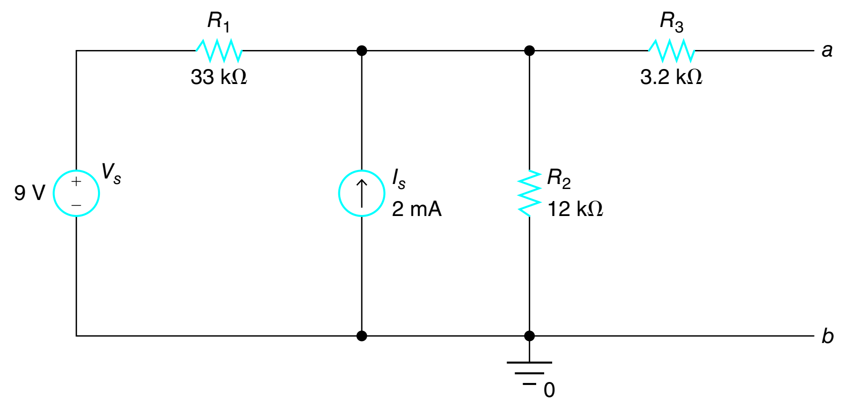
Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.112.

continued

Example 4.14 continued

FIGURE 4.112

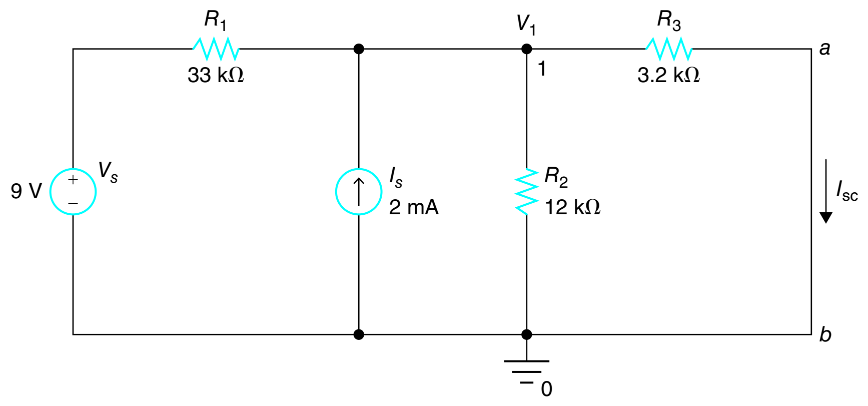
Circuit for EXAMPLE 4.14.



When terminals a and b are short-circuited, we obtain the circuit shown in Figure 4.113. The Norton equivalent current I_n is the short-circuit current I_{sc}. The short-circuit current I_{sc} is the current through R_3. Nodal analysis can be used to find voltage V_1 at node 1.

FIGURE 4.113

The circuit shown in Figure 4.112 with terminals a and b short-circuited.



Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 9}{33{,}000} - 0.002 + \frac{V_1}{12{,}000} + \frac{V_1}{3200} = 0$$

Multiplication by 33,000 yields

$$V_1 - 9 - 66 + 2.75V_1 + 10.3125V_1 = 0$$

which can be simplified to

$$14.0625V_1 = 75$$

Thus, we have

$$V_1 = \frac{75}{14.0625} = 5.3333 \text{ V}$$

The short-circuit current I_{sc} is found to be

$$I_n = I_{sc} = \frac{V_1}{R_3} = \frac{5.3333 \text{ V}}{3200\ \Omega} = 1.6667 \text{ mA}$$

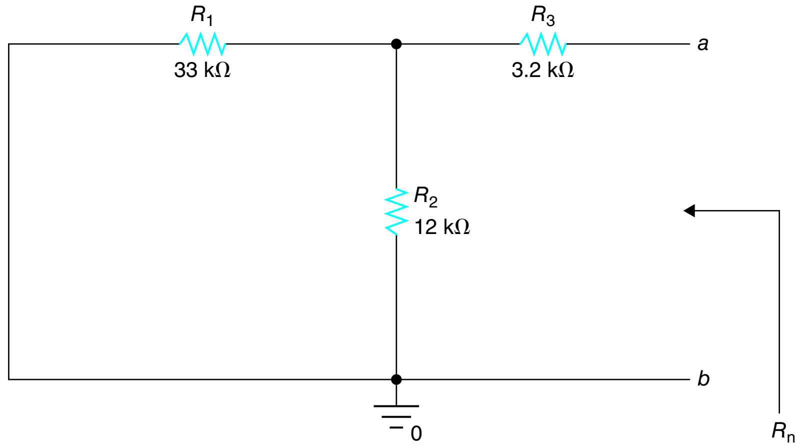
continued

Example 4.14 continued

To find the Norton equivalent resistance R_n, we deactivate the voltage source by short-circuiting it and deactivate the current source by open-circuiting it, as shown in Figure 4.114.

FIGURE 4.114

The circuit shown in Figure 4.113 with sources deactivated.



The Norton equivalent resistance R_n is the equivalent resistance of the circuit shown in Figure 4.114 from terminals *a* and *b*. Thus, we have

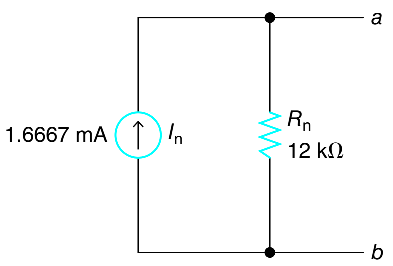
$$R_n = R_3 + (R_1 \| R_2) = R_3 + \frac{R_1 \times R_2}{R_1 + R_2} = 3.2\ k\Omega + \frac{33\ k\Omega \times 12\ k\Omega}{33\ k\Omega + 12\ k\Omega}$$

$$= 3.2\ k\Omega + \frac{396}{45}\ k\Omega = 12\ k\Omega$$

The Norton equivalent circuit is shown in Figure 4.115.

FIGURE 4.115

The Norton equivalent circuit.

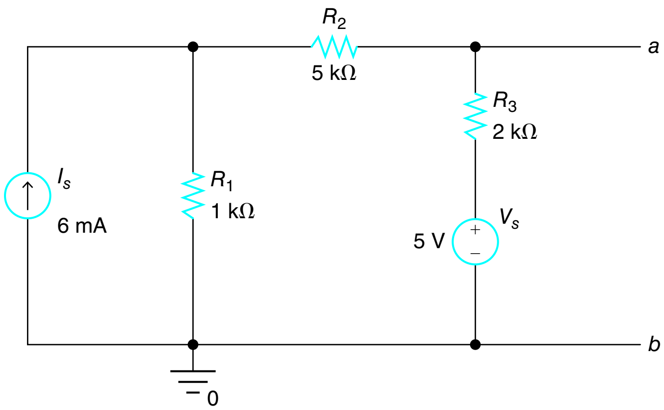


Exercise 4.14

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.116.

FIGURE 4.116

Circuit for EXERCISE 4.14.



Answer:

$I_n = 3.5$ mA, $R_n = 1.5\ k\Omega$.

EXAMPLE 4.15

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.117.

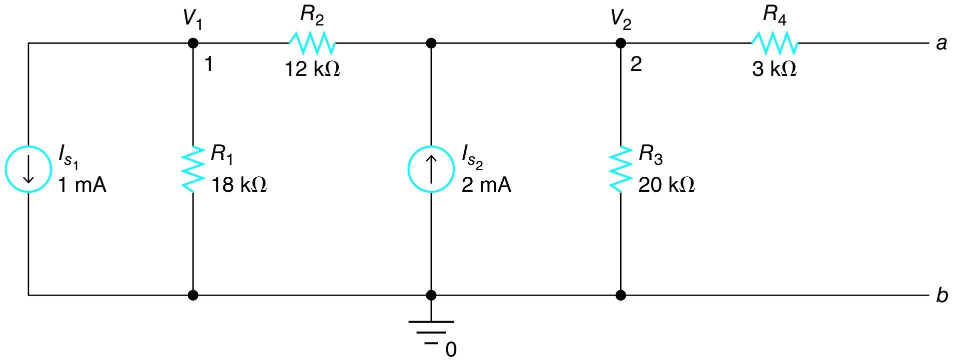


FIGURE 4.117
Circuit for EXAMPLE 4.15.

We will find the open-circuit voltage V_{oc} across a and b, and the short-circuit current I_{sc} through a and b to find the Norton equivalent circuit. Since no current flows through R_4, the open-circuit voltage is voltage V_2 at node 2. Summing the currents leaving node 1, we obtain

$$0.001 + \frac{V_1}{18{,}000} + \frac{V_1 - V_2}{12{,}000} = 0$$

Multiplication by 36,000 yields

$$36 + 2V_1 + 3V_1 - 3V_2 = 0$$

which can be simplified to

$$5V_1 - 3V_2 = -36 \tag{4.47}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_2 - V_1}{12{,}000} - 0.002 + \frac{V_2}{20{,}000} = 0$$

Multiplication by 60,000 yields

$$5V_2 - 5V_1 - 120 + 3V_2 = 0$$

which can be simplified to

$$-5V_1 + 8V_2 = 120 \tag{4.48}$$

Adding Equations (4.47) and (4.48), we obtain

$$5V_2 = 84$$

Thus, we have

$$V_2 = 16.8\ \text{V}$$

continued

Example 4.15 continued

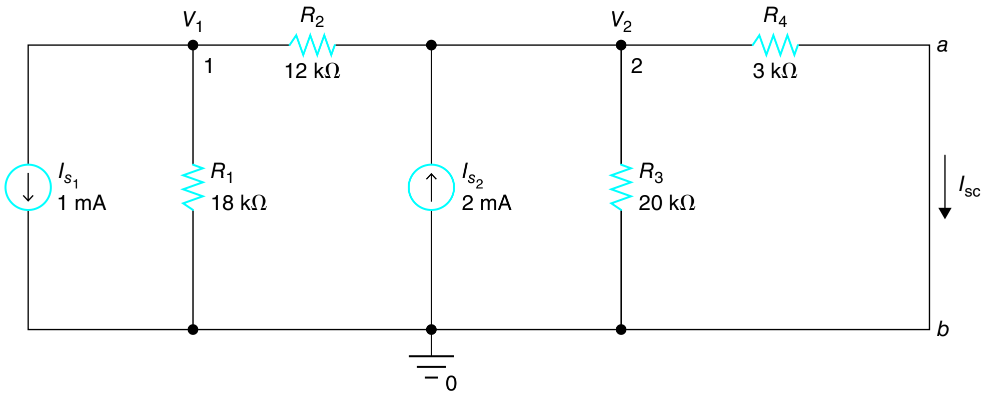
The open-circuit voltage is V_2. Therefore,

$$V_{oc} = V_2 = 16.8 \text{ V}$$

To find the short-circuit current I_{sc}, a and b are short-circuited, as shown in Figure 4.118.

FIGURE 4.118

Circuit with a and b short-circuited.



Equation (4.47) is still valid for the circuit shown in Figure 4.118. Summing the currents leaving node 2, we obtain

$$\frac{V_2 - V_1}{12{,}000} - 0.002 + \frac{V_2}{20{,}000} + \frac{V_2}{3000} = 0$$

Multiplication by 60,000 yields

$$5V_2 - 5V_1 - 120 + 3V_2 + 20V_2 = 0,$$

which can be simplified to

$$-5V_1 + 28V_2 = 120 \tag{4.49}$$

Adding Equations (4.47) and (4.49), we obtain

$$25V_2 = 84$$

Thus, we have

$$V_2 = 3.36 \text{ V}$$

The short-circuit current is the current through R_4. Thus, we have

$$I_{sc} = \frac{V_2}{R_4} = \frac{3.36 \text{ V}}{3 \; k\Omega} = 1.12 \text{ mA}$$

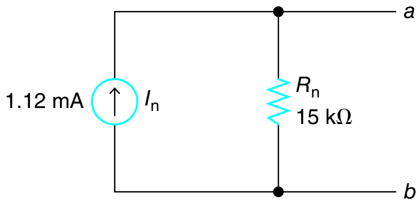
The Norton equivalent resistance is given by

$$R_n = \frac{V_{oc}}{I_{sc}} = \frac{16.8 \text{ V}}{1.12 \text{ mA}} = 15 \; k\Omega$$

The Norton equivalent circuit is shown in Figure 4.119.

FIGURE 4.119

The Norton equivalent circuit.

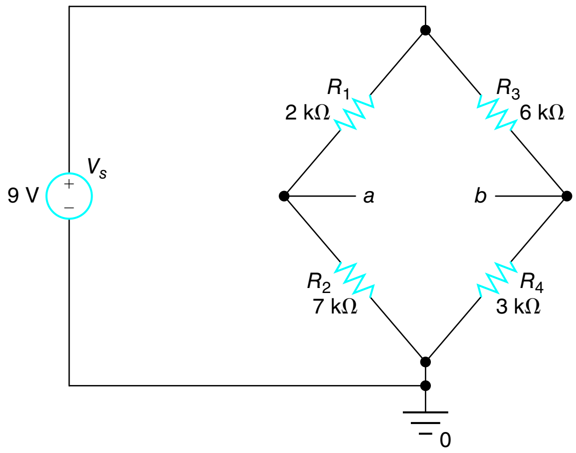


Exercise 4.15

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.120.

FIGURE 4.120

Circuit for EXERCISE 4.15.



Answer:

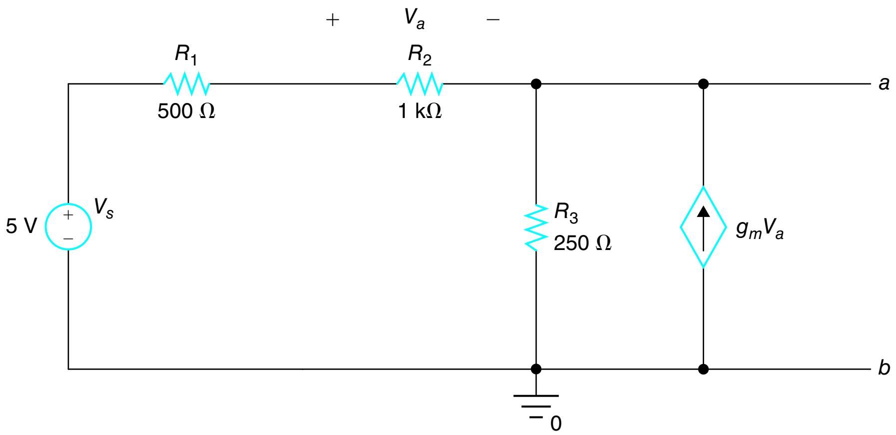
$I_n = 1.125$ mA, $R_n = 3.5556\ k\Omega$.

EXAMPLE 4.16

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.121. Assume that $g_m = 3$ (mA/V).

FIGURE 4.121

Circuit for EXAMPLE 4.16.



To find the Norton equivalent current, we short-circuit *a* and *b* in the circuit shown in Figure 4.121 to get the circuit shown in Figure 4.122.

The current I through R_1 and R_2 is given by

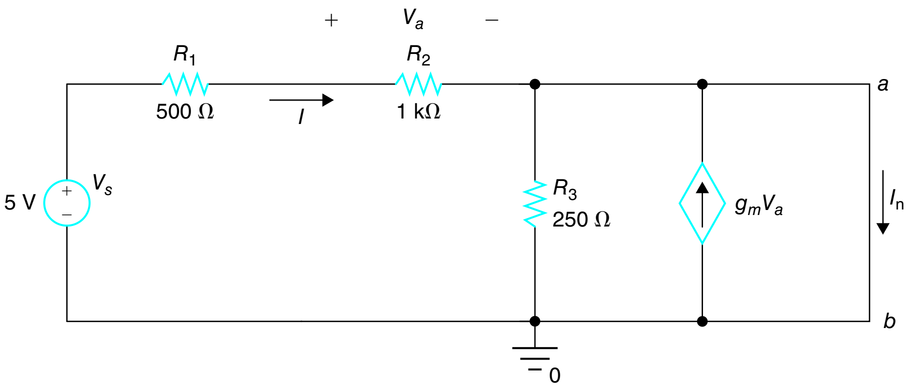
$$I = \frac{5\ \text{V}}{0.5\ k\Omega + 1\ k\Omega} = \frac{10}{3}\ \text{mA}$$

continued

Example 4.16 continued

FIGURE 4.122

The circuit from Figure 4.121 with a and b shorted.



The voltage V_a across R_2 is given by

$$V_a = R_2 I = 1\ k\Omega \times \frac{10}{3}\,\text{mA} = \frac{10}{3}\,\text{V}$$

The current from the VCCS is

$$g_m V_a = 3\left(\frac{\text{mA}}{\text{V}}\right) \times \frac{10}{3}\,(\text{V}) = \frac{30}{3}\,\text{mA} = 10\ \text{mA}$$

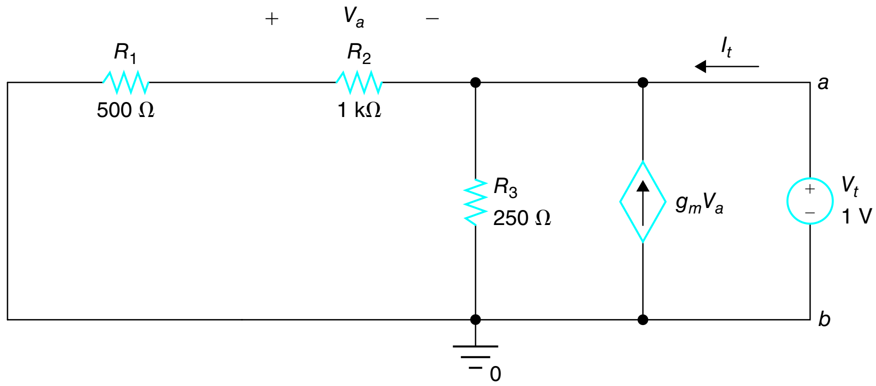
According to the current divider rule, all the currents flowing into node a will flow out through the short circuit. In other words, there is no current through R_3. Thus, we have

$$I_n = I + g_m V_a = \frac{10}{3}\,\text{mA} + \frac{30}{3}\,\text{mA} = \frac{40}{3}\,\text{mA} = 13.3333\ \text{mA}$$

To find the Norton equivalent resistance, we short-circuit the independent voltage source V_s and apply a test voltage of 1 V between terminals a and b, as shown in Figure 4.123.

FIGURE 4.123

Circuit with a test voltage.



From the voltage divider rule, the voltage V_a of the circuit shown in Figure 4.123 is given by

$$V_a = -\frac{R_2}{R_2 + R_1} \times V_t = -\frac{1\ k\Omega}{1\ k\Omega + 0.5\ k\Omega} \times 1 = -\frac{2}{3}\,\text{V}$$

continued

Example 4.16 continued

The current I_t flowing out of the test voltage source V_t is given by

$$I_t = -g_m V_a + \frac{V_t}{R_3} + \frac{V_t}{R_2 + R_1} = (-3 \text{ mA/V}) \times \left(-\frac{2}{3} \text{ V}\right)$$
$$+ \frac{1 \text{ V}}{0.25 \, k\Omega} + \frac{1 \text{ V}}{1.5 \, k\Omega} = \frac{20}{3} \text{ mA}$$

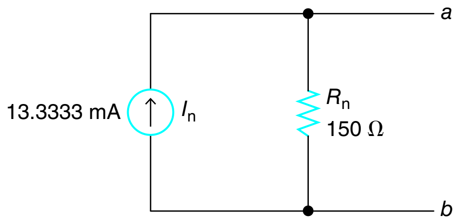
Thus, the Norton equivalent resistance is given by

$$R_n = \frac{V_t}{I_t} = \frac{1 \text{ V}}{\frac{20}{3} \text{ mA}} = \frac{3}{20} k\Omega = 150 \ \Omega$$

The Norton equivalent circuit is shown in Figure 4.124.

FIGURE 4.124

The Norton equivalent circuit.

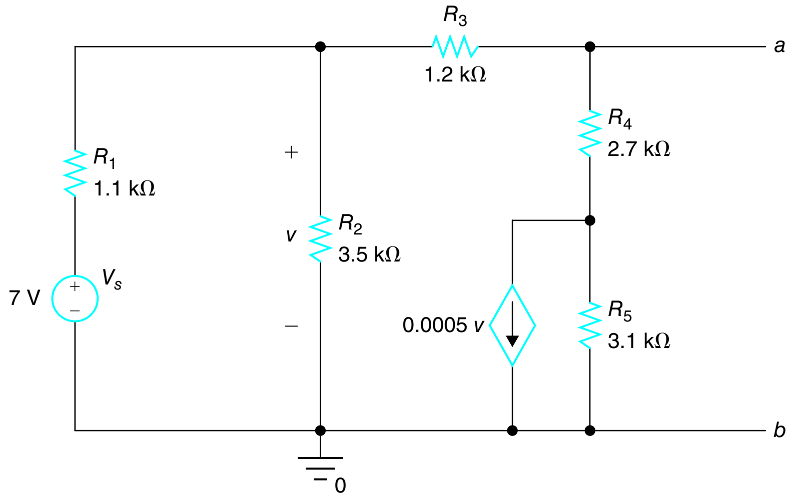


Exercise 4.16

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.125.

FIGURE 4.125

Circuit for EXERCISE 4.16.



Answer:

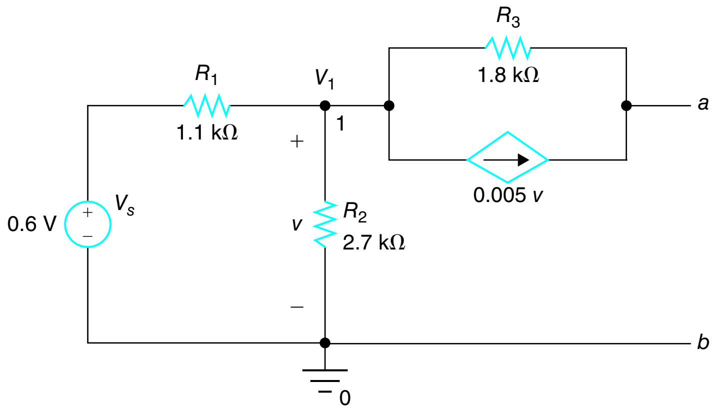
$I_n = 1.7762$ mA, $R_n = 1.2934 \, k\Omega$

EXAMPLE 4.17

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.126.

FIGURE 4.126

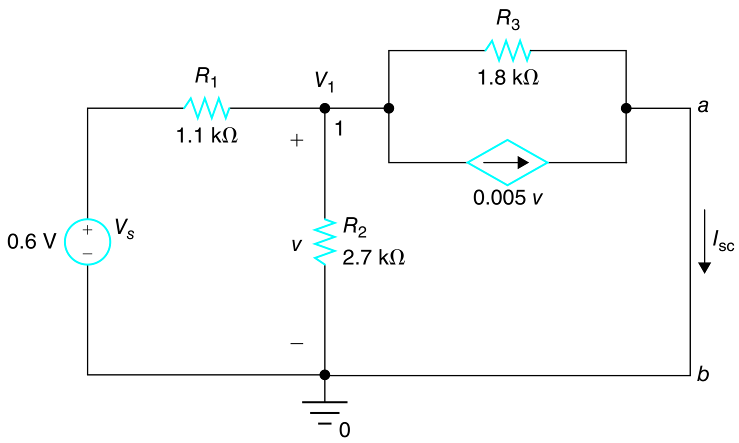
Circuit for EXAMPLE 4.17.



To find the Norton equivalent current, we short-circuit a and b, as shown in Figure 4.127, and find the current through the short circuit.

FIGURE 4.127

Circuit shown in Figure 4.126 with *a* and *b* short-circuited.



Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 0.6}{1100} + \frac{V_1}{2700} + \frac{V_1}{1800} + 0.005 \times V_1 = 0$$

which can be rearranged as

$$\left(\frac{1}{1100} + \frac{1}{2700} + \frac{1}{1800} + 0.005\right)V_1 = \frac{0.6}{1100}$$

Thus, we have

$$V_1 = \frac{\dfrac{0.6}{1100}}{\dfrac{1}{1100} + \dfrac{1}{2700} + \dfrac{1}{1800} + 0.005} = 0.079803 \text{ V}$$

continued

Example 4.17 continued

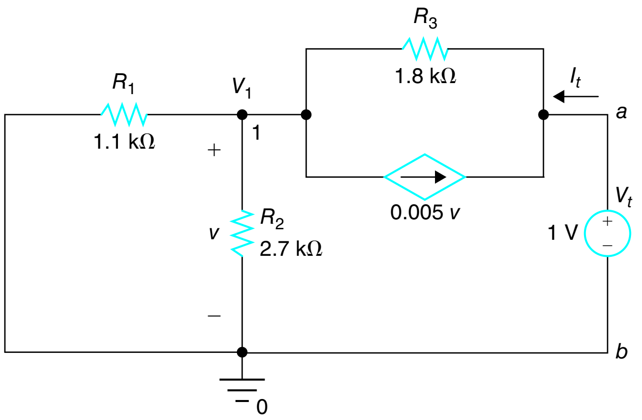
The short-circuit current I_{sc}, which is the Norton equivalent current I_n, is given by

$$I_n = I_{sc} = \frac{V_1}{R_3} + 0.005 \times V_1 = \frac{0.079803}{1800} + 0.005 \times 0.079803 = 443.3498\ \mu\text{A}$$

To find the Norton equivalent resistance R_n, we deactivate the voltage source V_s by short-circuiting it and then apply a test voltage V_t of 1 V between terminals a and b, as shown in Figure 4.128.

FIGURE 4.128

Circuit with a test voltage.



Summing the currents leaving node 1 of the circuit shown in Figure 4.128, we obtain

$$\frac{V_1}{1100} + \frac{V_1}{2700} + \frac{V_1 - 1}{1800} + 0.005 \times V_1 = 0$$

which can be rearranged as

$$\left(\frac{1}{1100} + \frac{1}{2700} + \frac{1}{1800} + 0.005\right)V_1 = \frac{1}{1800}$$

Thus, we have

$$V_1 = \frac{\dfrac{1}{1800}}{\dfrac{1}{1100} + \dfrac{1}{2700} + \dfrac{1}{1800} + 0.005} = 0.0812808\text{ V}$$

The current I_t flowing out of the positive terminal of the test voltage is given by

$$I_t = \frac{V_t - V_1}{R_3} - 0.005 \times V_1 = 103.9956\ \mu\text{A}$$

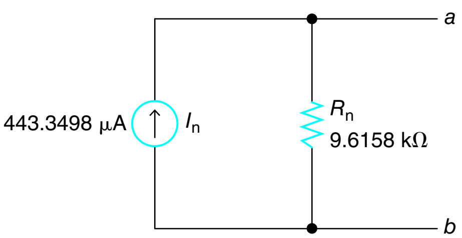
The Norton equivalent resistance is given by

$$R_n = \frac{V_t}{I_t} = \frac{1\text{ V}}{103.9956\ \mu\text{A}} = 9.6158\ k\Omega$$

The Norton equivalent circuit is shown in Figure 4.129.

FIGURE 4.129

The Norton equivalent circuit.

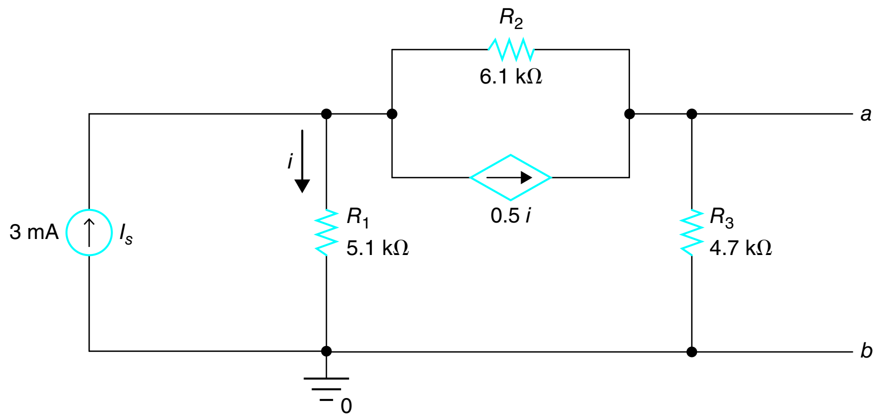


Exercise 4.17

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.130.

FIGURE 4.130

Circuit for EXERCISE 4.17.



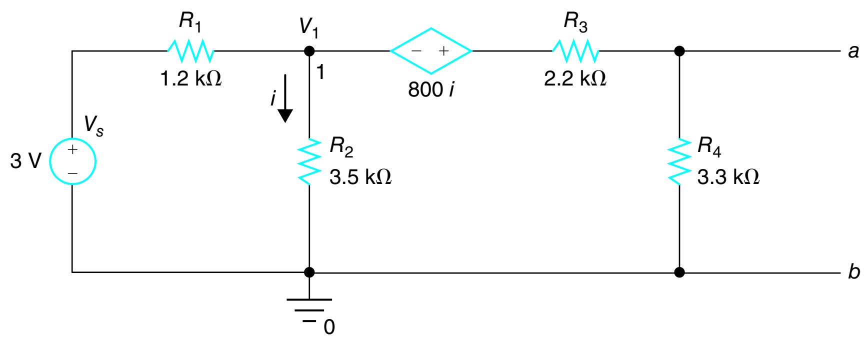
Answer:
$I_n = 1.7158$ mA, $R_n = 3.5343\ k\Omega$

EXAMPLE 4.18

Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.131.

FIGURE 4.131

Circuit for EXAMPLE 4.18.



We can find the open-circuit voltage V_{oc} between a and b. The open-circuit voltage is the voltage across R_4. Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 3}{1200} + \frac{V_1}{3500} + \frac{V_1 + 800 \times \dfrac{V_1}{3500}}{2200 + 3300} = 0$$

which can be rearranged as

$$\left(\frac{1}{1200} + \frac{1}{3500} + \frac{1 + \dfrac{800}{3500}}{5500} \right) V_1 = \frac{3}{1200}$$

continued

Example 4.18 continued

Thus, we have

$$V_1 = \frac{\frac{3}{1200}}{\frac{1}{1200} + \frac{1}{3500} + \frac{1 + \frac{800}{3500}}{5500}} = 1.8623 \text{ V}$$

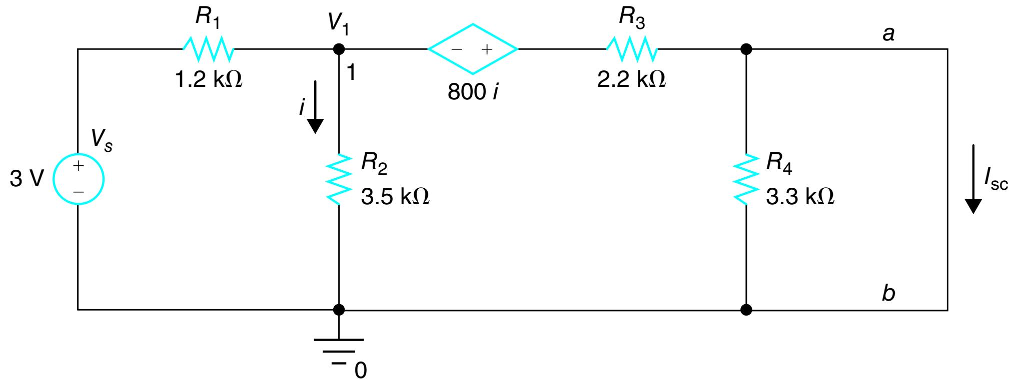
The open-circuit voltage is given by

$$V_{oc} = \left(V_1 + 800 \times \frac{V_1}{3500}\right) \times \frac{R_4}{R_3 + R_4} = 1.8623 \times \left(1 + \frac{800}{3500}\right) \times \frac{3300}{5500} = 1.3728 \text{ V}$$

The Norton equivalent current is the short-circuit current when *a* and *b* are short-circuited, as shown in Figure 4.132.

FIGURE 4.132

Circuit with *a* and *b* short-circuited.



Since the potential difference across R_4 is zero, there is no current through R_4. The short-circuit current is the current through R_3. Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 3}{1200} + \frac{V_1}{3500} + \frac{V_1 + 800 \times \frac{V_1}{3500}}{2200} = 0$$

which can be rearranged as

$$\left(\frac{1}{1200} + \frac{1}{3500} + \frac{1 + \frac{800}{3500}}{2200}\right)V_1 = \frac{3}{1200}$$

Thus, we have

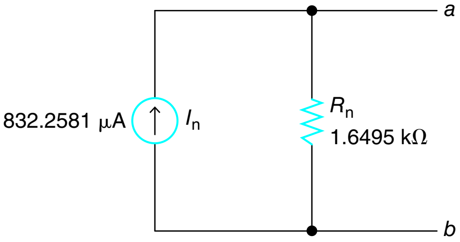
$$V_1 = \frac{\frac{3}{1200}}{\frac{1}{1200} + \frac{1}{3500} + \frac{1 + \frac{800}{3500}}{2200}} = 1.4903 \text{ V}$$

continued

Example 4.18 continued

FIGURE 4.133

The Norton equivalent circuit.



The short-circuit current is given by

$$I_n = I_{sc} = \frac{V_1 + 800 \times \dfrac{V_1}{3500}}{R_3} = \frac{1.8309\ \text{V}}{2200\ \Omega} = 832.2581\ \mu\text{A}$$

The Norton equivalent resistance is given by

$$R_n = \frac{V_{oc}}{I_{sc}} = \frac{1.3728\ \text{V}}{832.2581\ \mu\text{A}} = 1.6495\, k\Omega$$

The Norton equivalent circuit is shown in Figure 4.133.

MATLAB

```
%EXAMPLE 4.18
clear all;format long;
R1=1200;R2=3500;R3=2200;R4=3300;Vs=3;kr=800;
syms V1 Va
%Voc
V1=solve((V1-Vs)/R1+V1/R2+(V1+kr*V1/R2)/(R3+R4));
Voc=(V1+kr*V1/R2)*R4/(R3+R4);
%Isc
Va=solve((Va-Vs)/R1+Va/R2+(Va+kr*Va/R2)/R3);
Isc=(Va+kr*Va/R2)/R3;
Rn=Voc/Isc;
In=Isc;
V1=vpa(V1,7)
Voc=vpa(Voc,7)
Va=vpa(Va,7)
Rn=vpa(Rn,7)
Isc=vpa(Isc,7)
In=vpa(In,7)

Answers:
V1 =
1.862302
Voc =
1.372783
Va =
1.490323
Rn =
1649.468
Isc =
0.0008322581
In =
0.0008322581
```

Exercise 4.18

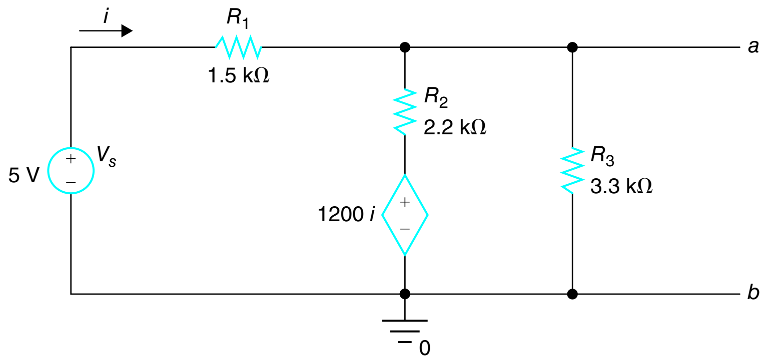
Find the Norton equivalent circuit between terminals *a* and *b* for the circuit shown in Figure 4.134.

continued

Exercise 4.18 continued

FIGURE 4.134

Circuit for EXERCISE 4.18.

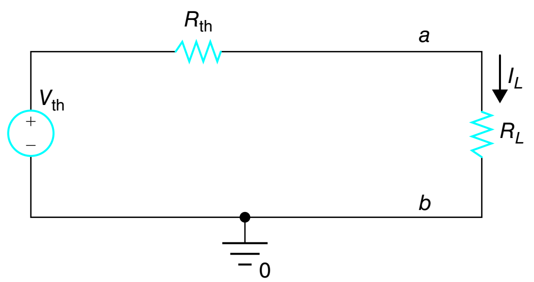


Answer:
$I_n = 5.1515$ mA, $R_n = 559.322\ \Omega$.

4.6 Maximum Power Transfer

FIGURE 4.135

A load connected to the Thévenin equivalent circuit.



Suppose that a load with resistance R_L is connected to a circuit between terminals a and b. We are interested in finding the power P_L delivered to the load and finding the load resistance R_L that maximizes the power delivered to the load. We first find the Thévenin equivalent circuit with respect to the terminals a and b. Let V_{th} be the Thévenin equivalent voltage and R_{th} be the Thévenin equivalent resistance. With the original circuit replaced by the Thévenin equivalent circuit, we obtain the circuit shown in Figure 4.135.

The current through the load resistor is given by

$$I_L = \frac{V_{\text{th}}}{R_{\text{th}} + R_L}$$

and the voltage across the load resistor is given by

$$V_L = R_L I_L = \frac{R_L V_{\text{th}}}{R_{\text{th}} + R_L}$$

Thus, the power delivered to the load is

$$p_L = I_L V_L = \frac{R_L V_{\text{th}}^2}{(R_{\text{th}} + R_L)^2}$$

When $R_L = 0, p_L = 0$; and when $R_L = \infty, p_L = 0$. The power delivered to the load p_L must peak at a certain value. Figure 4.136 shows p_L as a function of R_L for $0 \le R_L \le 5R_{\text{th}}$ ($V_{\text{th}} = 1$ V, $R_{\text{th}} = 1\ \Omega$).

From this figure, we can see that when p_L is at its maximum, the derivative of p_L with respect to R_L is zero; that is, $dp_L/dR_L = 0$. Using

$$\frac{d}{dt}\left(\frac{u(t)}{v(t)}\right) = \frac{v(t)\dfrac{du(t)}{dt} - u(t)\dfrac{dv(t)}{dt}}{v^2(t)},$$

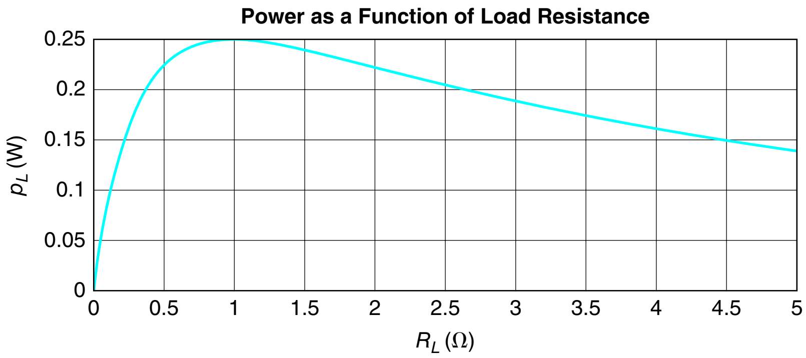


FIGURE 4.136

Plot of the power on the load as a function of load resistance.

we have

$$\frac{dp_L}{dR_L} = \frac{d}{dR_L}\left(\frac{R_L(V_{th})^2}{(R_{th}+R_L)^2}\right) = \frac{(R_{th}+R_L)^2\frac{dR_L}{dR_L} - R_L\frac{d(R_{th}+R_L)^2}{dR_L}}{(R_{th}+R_L)^4}(V_{th})^2$$

$$= \frac{(R_{th}+R_L)^2 1 - R_L 2(R_{th}+R_L)}{(R_{th}+R_L)^4}(V_{th})^2 = \frac{(R_{th}+R_L)(R_{th}-R_L)}{(R_{th}+R_L)^4}(V_{th})^2$$

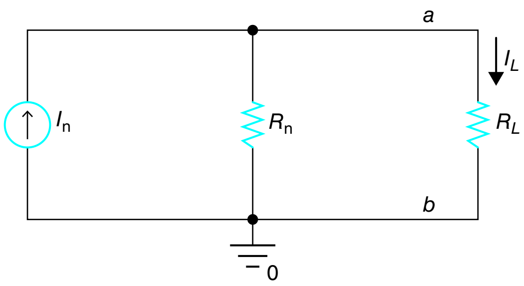
Setting $dp_L/dR_L = 0$, we have two solutions: $R_L = -R_{th}$ or $R_L = R_{th}$. We take the positive answer. Thus, the load resistance R_L that maximizes the power delivered to the load is given by the Thévenin equivalent resistance from terminals a and b.

The maximum power delivered to the load when the load resistance is $R_L = R_{th}$ is given by

$$p_{L,\max} = \frac{R_{th}V_{th}^2}{(R_{th}+R_{th})^2} = \frac{V_{th}^2}{4R_{th}} = \frac{V_{th}^2}{4R_L} \quad (4.50)$$

FIGURE 4.137

A Norton equivalent circuit with load R_L.



For the Norton equivalent circuit with load R_L shown in Figure 4.137, let the current through the load be I_L. Then, from the current divider rule, we have

$$I_L = \frac{R_n}{R_n + R_L} \times I_n$$

The power on the load is given by

$$p_L = I_L^2 R_L = \frac{R_L}{(R_n+R_L)^2}R_n^2 I_n^2$$

$$\frac{dp_L}{dR_L} = \frac{d}{dR_L}\left(\frac{R_L}{(R_n+R_L)^2}(R_nI_n)^2\right) = \frac{(R_n+R_L)^2\frac{dR_L}{dR_L} - R_L\frac{d(R_n+R_L)^2}{dR_L}}{(R_n+R_L)^4}(R_nI_n)^2$$

$$= \frac{(R_n+R_L)^2 1 - R_L 2(R_n+R_L)}{(R_n+R_L)^4}(R_nI_n)^2 = \frac{(R_n+R_L)(R_n-R_L)}{(R_n+R_L)^4}(R_nI_n)^2$$

Setting $dp_L/dR_L = 0$, we have two solutions: $R_L = -R_n$ and $R_L = R_n$. We take the positive answer. Thus, the load resistance R_L that maximizes the power delivered to the load is given by the Norton equivalent resistance from terminals a and b.

The maximum power delivered to the load when the load resistance is $R_L = R_n$ is given by

$$p_{L,\max} = \frac{R_n}{(R_n + R_n)^2} R_n^2 I_n^2 = \frac{I_n^2 R_n}{4} = \frac{I_n^2 R_L}{4} \tag{4.51}$$

We can get the same result using source transformation.

EXAMPLE 4.19

For the circuit shown in Figure 4.138, find the value of the load resistance R_L that maximizes the power delivered to the load. Also, find the maximum power delivered to R_L.

FIGURE 4.138

Circuit for EXAMPLE 4.19.

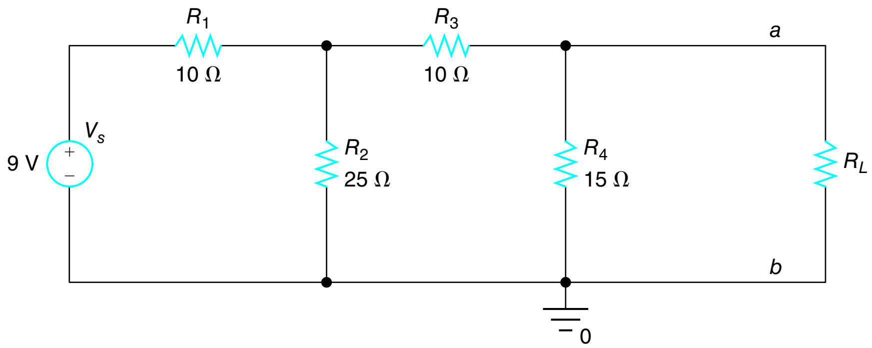
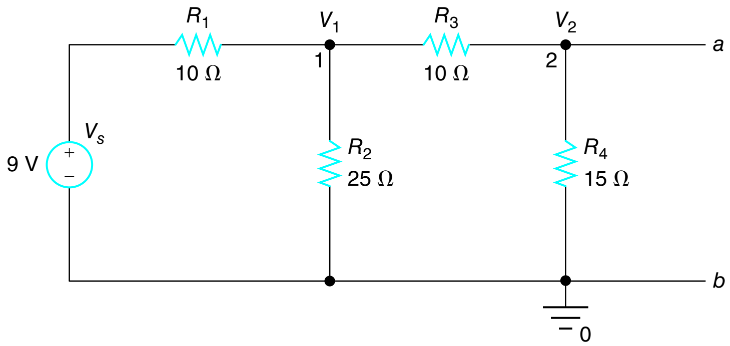


Figure 4.139 shows the circuit in Figure 4.138 without the load resistor.

FIGURE 4.139

Circuit shown in Figure 4.138 without the load resistor.



Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 9}{10} + \frac{V_1}{25} + \frac{V_1 - V_2}{10} = 0 \tag{4.52}$$

Multiplication by 50 yields

$$5V_1 - 45 + 2V_1 + 5V_1 - 5V_2 = 0$$

continued

Example 4.19 continued

which can be simplified to

$$12V_1 - 5V_2 = 45 \tag{4.53}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_2 - V_1}{10} + \frac{V_2}{15} = 0 \tag{4.54}$$

Multiplication by 30 results in

$$3V_2 - 3V_1 + 2V_2 = 0$$

which can be simplified to

$$-3V_1 + 5V_2 = 0 \tag{4.55}$$

Adding Equations (4.53) and (4.55), we get

$$9V_1 = 45$$

Thus, $V_1 = 5$ V. Substituting this into Equation (4.55), we obtain

$$5V_2 = 15$$

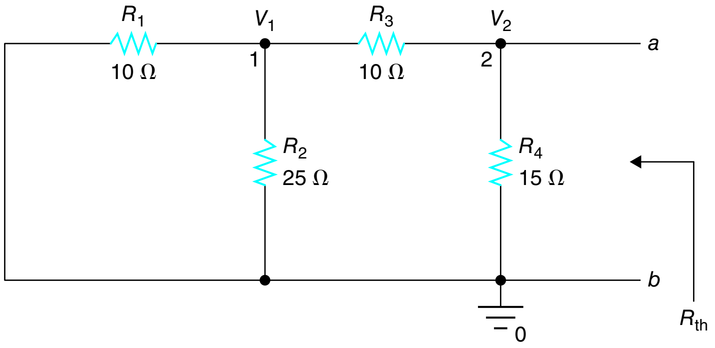
from which we have $V_2 = 3$ V. The Thévenin equivalent voltage between *a* and *b* is voltage V_2. Thus, we have

$$V_{th} = 3 \text{ V}$$

The Thévenin equivalent resistance R_{th} is the equivalent resistance across *a* and *b* after deactivating the voltage source by short-circuiting it, as shown in Figure 4.140.

FIGURE 4.140

The circuit shown in Figure 4.139 with the voltage source deactivated.



Let the equivalent resistance of the parallel connection of R_1 and R_2 be R_a. Then, R_a is

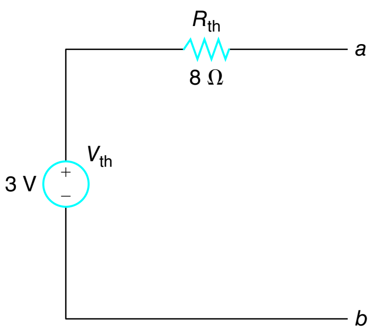
$$R_a = R_1 \| R_2 = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{10 \ \Omega \times 25 \ \Omega}{10 \ \Omega + 25 \ \Omega} = \frac{250}{35} \ \Omega = \frac{50}{7} \ \Omega$$

continued

Example 4.19 continued

FIGURE 4.141

The Thévenin equivalent circuit between a and b.



Let R_b be the equivalent resistance of the series connection of R_3 and R_a. Then, we have

$$R_b = R_3 + R_a = 10\ \Omega + 50/7\ \Omega = 120/7\ \Omega$$

The Thévenin equivalent resistance R_{th} is given by the equivalent resistance of the parallel connection of R_4 and R_b. Thus, we obtain

$$R_{th} = R_4 \| R_b = \frac{R_4 \times R_b}{R_4 + R_b} = \frac{15\ \Omega \times \dfrac{120}{7}\ \Omega}{15\ \Omega + \dfrac{120}{7}\ \Omega} = \frac{1800}{225}\ \Omega = 8\ \Omega$$

The Thévenin equivalent circuit between a and b for the circuit shown in Figure 4.139 is shown in Figure 4.141.

The load resistance for maximum power transfer is $R_L = 8\ \Omega$, and the maximum power delivered to the load is given by

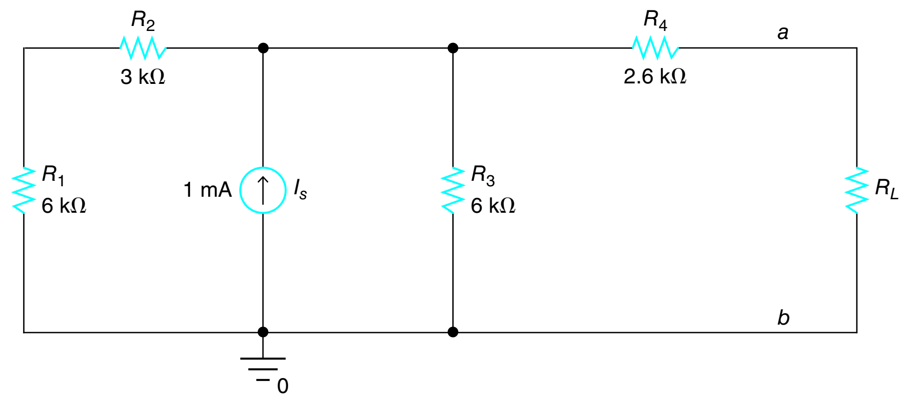
$$p_L = \frac{V_{th}^2}{4R_L} = \frac{3^2}{4 \times 8} = \frac{9}{32}\ \text{W} = 0.28125\ \text{W} = 281.25\ \text{mW}$$

Exercise 4.19

For the circuit shown in Figure 4.142, find the value of the load resistance R_L that maximizes the power delivered to the load. Also, find the maximum power delivered to R_L.

FIGURE 4.142

Circuit for EXERCISE 4.19.



Answer:

$V_{th} = 3.6$ V, $R_{th} = 6.2$ kΩ, $R_L = R_{th} = 6.2$ kΩ, $p_L = 522.5806$ μW.

EXAMPLE 4.20

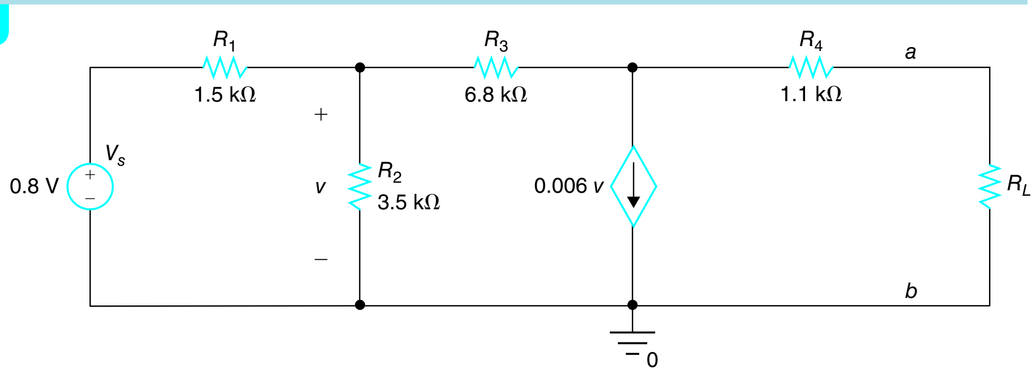
For the circuit shown in Figure 4.143, find the value of the load resistance R_L that maximizes the power delivered to the load. Also, find the maximum power delivered to R_L.

continued

Example 4.20 continued

FIGURE 4.143

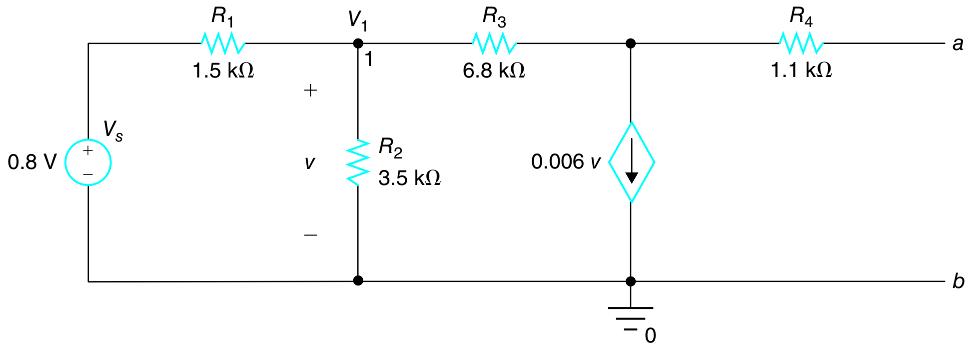
Circuit for EXAMPLE 4.20.



The circuit without the load connected is shown in Figure 4.144.

FIGURE 4.144

The circuit in Figure 4.143 without the load connected.



No current flows through R_4. Summing the currents leaving node 1, we obtain

$$\frac{V_1 - 0.8}{1500} + \frac{V_1}{3500} + 0.006 \times V_1 = 0$$

which can be rearranged as

$$\left(\frac{1}{1500} + \frac{1}{3500} + 0.006\right)V_1 = \frac{0.8}{1500}$$

Thus, we have

$$V_1 = \frac{\frac{0.8}{1500}}{\frac{1}{1500} + \frac{1}{3500} + 0.006} \times \frac{10{,}500}{10{,}500} = \frac{7 \times 0.8}{7 + 3 + 63} = \frac{5.6}{73} = 0.07671233 \text{ V}$$

The open-circuit voltage between a and b is given by

$$V_{\text{th}} = V_{\text{oc}} = V_1 - 0.006V_1 \times R_3 = 0.07671233(1 - 0.006 \times 6800) = -3.05315 \text{ V}$$

To find the short-circuit current, a and b are short-circuited, as shown in Figure 4.145. Summing the currents leaving node 1, we obtain

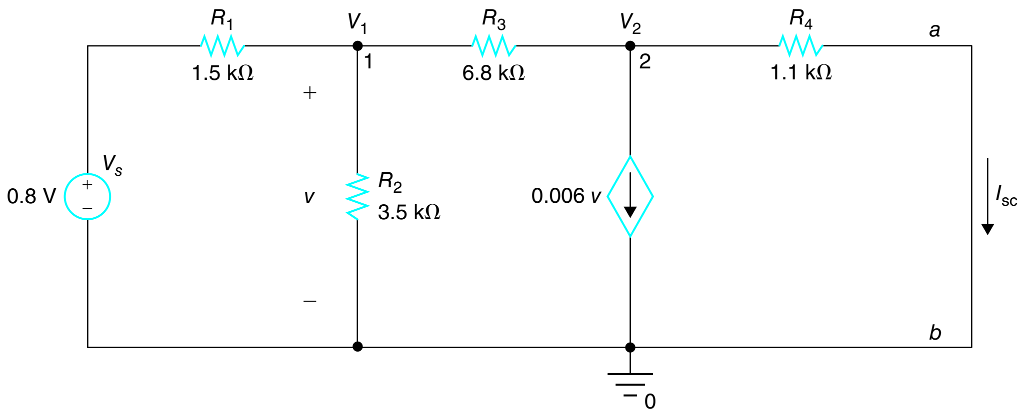
$$\frac{V_1 - 0.8}{1500} + \frac{V_1}{3500} + \frac{V_1 - V_2}{6800} = 0$$

continued

Example 4.20 continued

FIGURE 4.145

The circuit in Figure 4.144 with *a* and *b* shorted.



which can be rearranged as

$$\left(\frac{1}{1500}+\frac{1}{3500}+\frac{1}{6800}\right)V_1-\frac{1}{6800}V_2=\frac{0.8}{1500}$$

or

$$0.00109944V_1-1.470588\times10^{-4}V_2=5.33333\times10^{-4} \tag{4.56}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_2-V_1}{6800}+0.006\times V_1+\frac{V_2}{1100}=0$$

which can be rearranged as

$$\left(0.006-\frac{1}{6800}\right)V_1+\left(\frac{1}{6800}+\frac{1}{1100}\right)V_2=0$$

or

$$0.00585294V_1+0.00105615V_2=0 \tag{4.57}$$

Solving Equation (4.57) for V_2, we obtain

$$V_2=-\frac{0.00585294}{0.00105615}V_1=-5.54177V_1$$

Substituting V_2 into Equation (4.56), we get

$$0.00109944V_1-1.470588\times10^{-4}(-5.54177V_1)=5.333333\times10^{-4}$$

Thus,

$$V_1=\frac{5.333333\times10^{-4}}{0.00109944+1.470588\times10^{-4}\times5.54177}=0.27859\text{ V}$$

$$V_2=-5.54177V_1=-1.54388\text{ V}$$

continued

Example 4.20 continued

Alternatively, application of Cramer's rule to Equations (4.56) and (4.57) yields

$$V_1 = \frac{\begin{vmatrix} 5.33333 \times 10^{-4} & -1.470588 \times 10^{-4} \\ 0 & 0.00105615 \end{vmatrix}}{\begin{vmatrix} 0.00109944 & -1.470588 \times 10^{-4} \\ 0.00585294 & 0.00105615 \end{vmatrix}} = 0.27859 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 0.00109944 & 5.33333 \times 10^{-4} \\ 0.00585294 & 0 \end{vmatrix}}{\begin{vmatrix} 0.00109944 & -1.470588 \times 10^{-4} \\ 0.00585294 & 0.00105615 \end{vmatrix}} = -1.54388 \text{ V}$$

The short-circuit current is given by

$$I_{sc} = \frac{V_2}{R_4} = -0.00140353 \text{ A}$$

The Thévenin equivalent resistance is given by

$$R_{th} = \frac{V_{oc}}{I_{sc}} = 2.1753 \ k\Omega$$

The load resistance that provides maximum power transfer is $R_L = 2.1753 \ k\Omega$. The maximum power transferred is given by

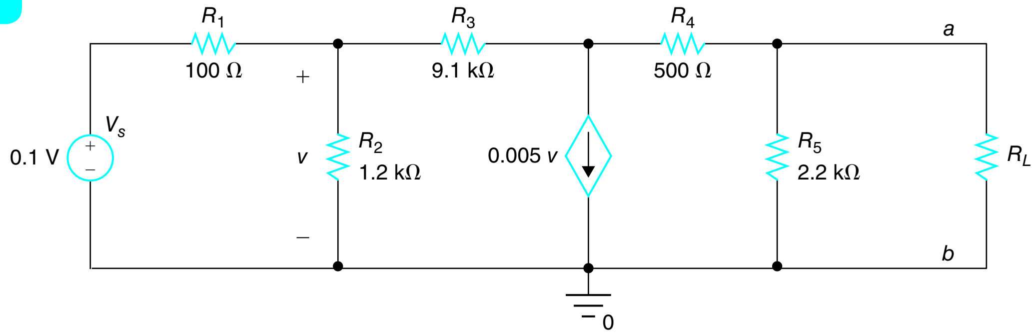
$$p_L = \frac{V_{th}^2}{4R_L} = 1.0713 \text{ mW}$$

Exercise 4.20

For the circuit shown in Figure 4.146, find the value of the load resistance R_L that maximizes the power delivered to the load. Also, find the maximum power delivered to R_L.

FIGURE 4.146

Circuit for EXERCISE 4.20.



Answer:
$R_L = 1.6616 \ k\Omega, p_L = 71.1816 \ \mu\text{W}$.

EXAMPLE 4.21

For the circuit shown in Figure 4.147, find the value of the load resistance R_L that maximizes the power delivered to the load. Also, find the maximum power delivered to R_L.

FIGURE 4.147

Circuit for EXAMPLE 4.21.

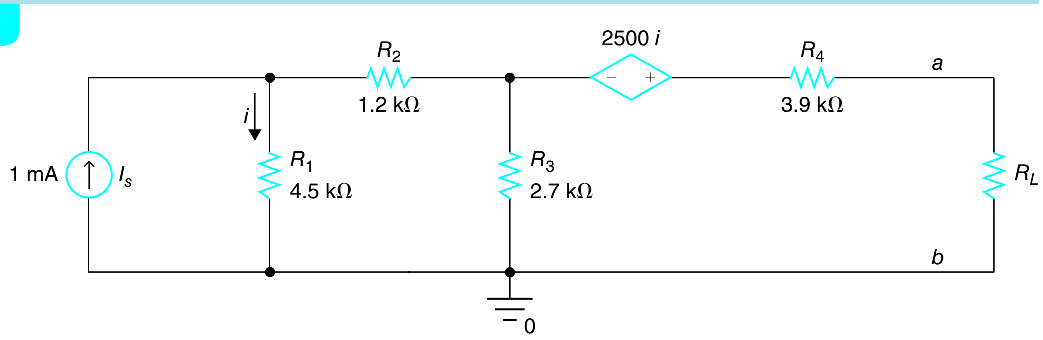
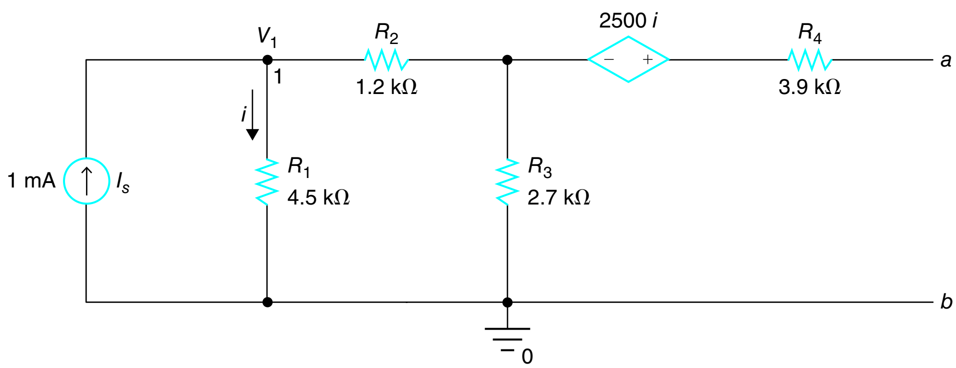


Figure 4.148 shows the circuit without the load resistor. There is no current through R_4.

FIGURE 4.148

The circuit in Figure 4.147 without the load resistor.



Summing the currents leaving node 1, we obtain

$$-0.001 + \frac{V_1}{4500} + \frac{V_1}{1200 + 2700} = 0$$

which can be rearranged as

$$\left(\frac{1}{4500} + \frac{1}{3900}\right)V_1 = 0.001$$

Thus, we have

$$V_1 = \frac{0.001}{\frac{1}{4500} + \frac{1}{3900}} = 2.0893 \text{ V}$$

The open-circuit voltage, which is also the Thévenin voltage, is given by

$$V_{th} = V_{oc} = V_1 \times \frac{R_3}{R_2 + R_3} + 2500 \times \frac{V_1}{R_1}$$

$$= 2.0893 \times \frac{2700}{1200 + 2700} + 2500 \times \frac{2.0893}{4500} = 2.6071 \text{ V}$$

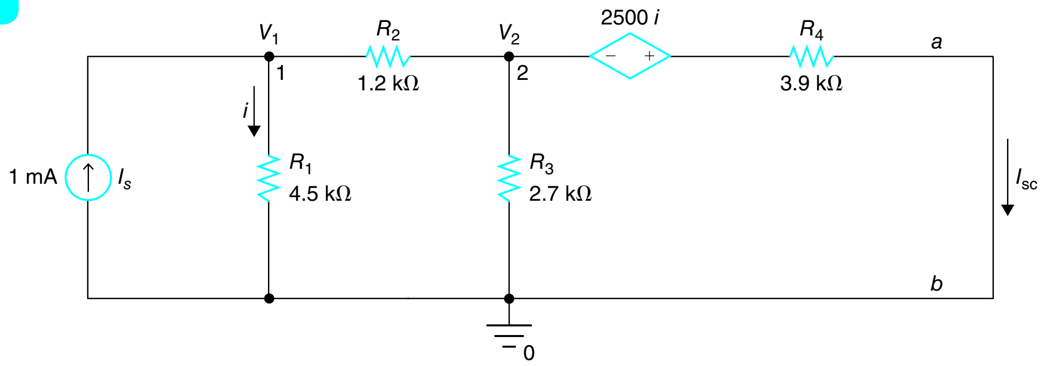
continued

Example 4.21 continued

To find the short-circuit current, terminals a and b are short-circuited, as shown in Figure 4.149.

FIGURE 4.149

The circuit in Figure 4.148 with a and b shorted.



Summing the currents leaving node 1, we obtain

$$-0.001 + \frac{V_1}{4500} + \frac{V_1 - V_2}{1200} = 0$$

which can be rearranged as

$$\left(\frac{1}{4500} + \frac{1}{1200}\right)V_1 - \frac{1}{1200}V_2 = 0.001$$

or

$$0.00105556V_1 - 8.33333 \times 10^{-4}V_2 = 0.001 \tag{4.58}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_2 - V_1}{1200} + \frac{V_2}{2700} + \frac{V_2 + 2500 \times \dfrac{V_1}{4500}}{3900} = 0,$$

which can be rearranged as

$$\left(\frac{-1}{1200} + \frac{\frac{5}{9}}{3900}\right)V_1 + \left(\frac{1}{1200} + \frac{1}{2700} + \frac{1}{3900}\right)V_2 = 0$$

or

$$-6.908832 \times 10^{-4}V_1 + 0.001460114V_2 = 0 \tag{4.59}$$

Solving Equation (4.59) for V_2, we obtain

$$V_2 = \frac{6.908832 \times 10^{-4}}{0.001460114}V_1 = 0.473171V_1$$

Substituting V_2 into Equation (4.58), we get

$$0.00105556V_1 - 8.333333 \times 10^{-4}(0.473171V_1) = 0.001$$

continued

Example 4.21 continued

Thus,

$$V_1 = \frac{0.001}{0.00105556 - 8.333333 \times 10^{-4} \times 0.473171} = 1.5123 \text{ V}$$

$$V_2 = 0.473171 V_1 = 0.7156 \text{ V}$$

Alternatively, application of Cramer's rule to Equations (4.58) and (4.59) yields

$$V_1 = \frac{\begin{vmatrix} 0.001 & -8.33333 \times 10^{-4} \\ 0 & 0.001460114 \end{vmatrix}}{\begin{vmatrix} 0.00105556 & -8.33333 \times 10^{-4} \\ -6.908832 \times 10^{-4} & 0.001460114 \end{vmatrix}} = 1.5123 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 0.00105556 & 0.001 \\ -6.908832 \times 10^{-4} & 0 \end{vmatrix}}{\begin{vmatrix} 0.00105556 & -8.33333 \times 10^{-4} \\ -6.908832 \times 10^{-4} & 0.001460114 \end{vmatrix}} = 0.7156 \text{ V}$$

The short-circuit current is given by

$$I_{sc} = \frac{V_2 + 2500 \times \dfrac{V_1}{R_1}}{R_4} = 0.3989 \text{ mA}$$

The Thévenin equivalent resistance is the ratio of V_{oc} to I_{sc}:

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{V_{th}}{I_{sc}} = 6.5357\, k\Omega$$

The load resistance value that provides the maximum power transfer is $R_L = R_{th} = 6.5357\ k\Omega$. The maximum power delivered to the load is given by

$$p_L = \frac{V_{th}^2}{4R_L} = 0.26 \text{ mW}$$

MATLAB

```
%EXAMPLE 4.21
clear all;format long;
Is=1e-3;R1=4500;R2=1200;R3=2700;R4=3900;
syms V1 V2 Va Vb
%Voc = Vth
[V1,V2]=solve(-Is+V1/R1+(V1-V2)/R2,(V2-V1)/R2+V2/R3);
Vth=V2+2500*V1/R1;
%Isc
[Va,Vb]=solve(-Is+Va/R1+(Va-Vb)/R2,...
(Vb-Va)/R2+Vb/R3+(Vb+2500*Va/R1)/R4);
Isc=(Vb+2500*Va/R1)/R4;
Rth=Vth/Isc;
PL=Vth^2/(4*Rth);
V1=vpa(V1,7)
```

continued

Example 4.21 continued
MATLAB continued

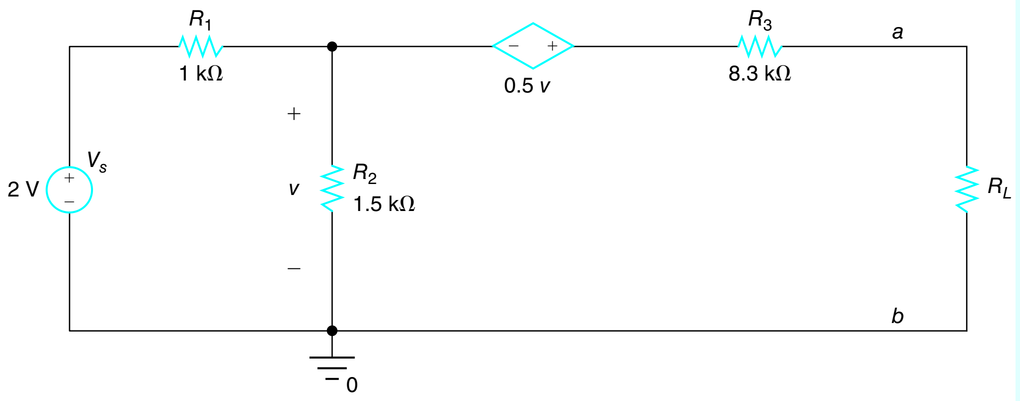
```
V2=vpa(V2,7)
Va=vpa(Va,7)
Vb=vpa(Vb,7)
Isc=vpa(Isc,7)
Vth=vpa(Vth,7)
Rth=vpa(Rth,7)
PL=vpa(PL,7)

Answers:
V1 =
2.089286
V2 =
1.446429
Va =
1.512295
Vb =
0.7155738
Isc =
0.0003989071
Vth =
2.607143
Rth =
6535.714
PL =
0.000260002
```

Exercise 4.21

For the circuit shown in Figure 4.150, find the value of the load resistance R_L that maximizes the power delivered to the load. Also, find the maximum power delivered to R_L.

FIGURE 4.150

Circuit for EXERCISE 4.21.



Answer:

$R_L = 9.2\ k\Omega, p_L = 88.0435\ \mu W.$

4.7 PSpice

In PSpice, the **transfer function** can be used to find the Thévenin equivalent resistance. The Thévenin equivalent voltage can be found by finding the open-circuit voltage. In EXAMPLE 4.7, earlier in this chapter, we showed that the Thévenin equivalent voltage is $V_{th} = 14$ V and the Thévenin equivalent resistance is $R_{th} = 20\ k\Omega$ between a and b for the circuit shown in Figure 4.151. We can verify these values using PSpice. If we try to run the circuit as shown in Figure 4.151, we get an error message that says that there are fewer than two connections at node a. To avoid this error, we can add a resistor with large resistance ($10^{12}\ \Omega$) between a and b, as shown in Figure 4.152. Because of the large resistance value, it is virtually an open circuit between a and b.

FIGURE 4.151

Circuit for EXAMPLE 4.7.

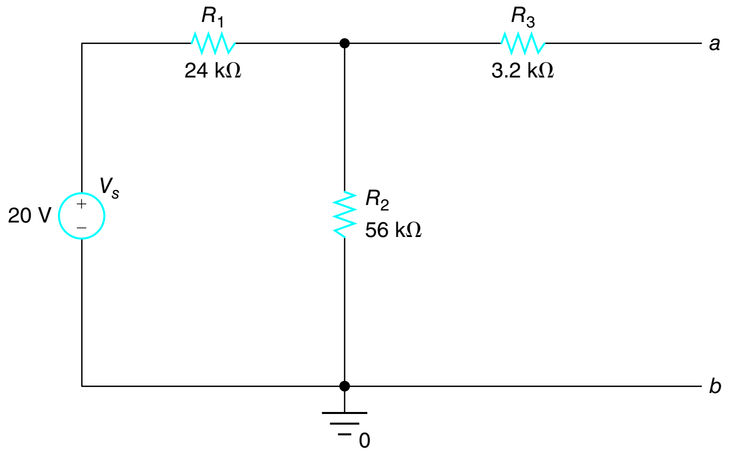
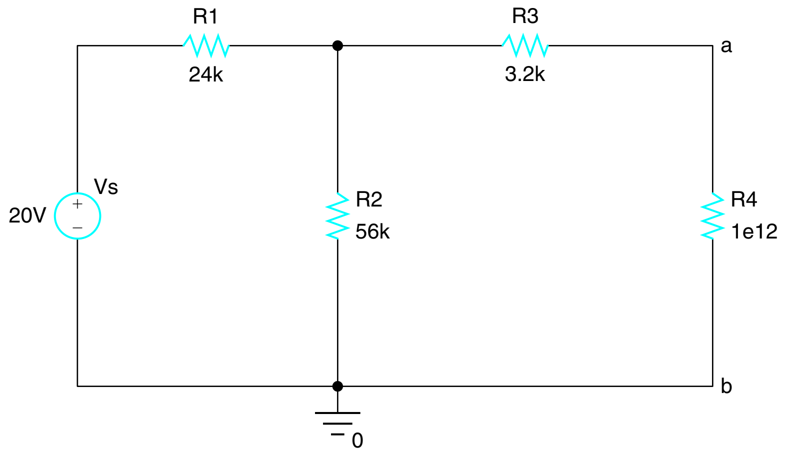


FIGURE 4.152

The circuit shown in Figure 4.151 with a large resistor R_4 between a and b.



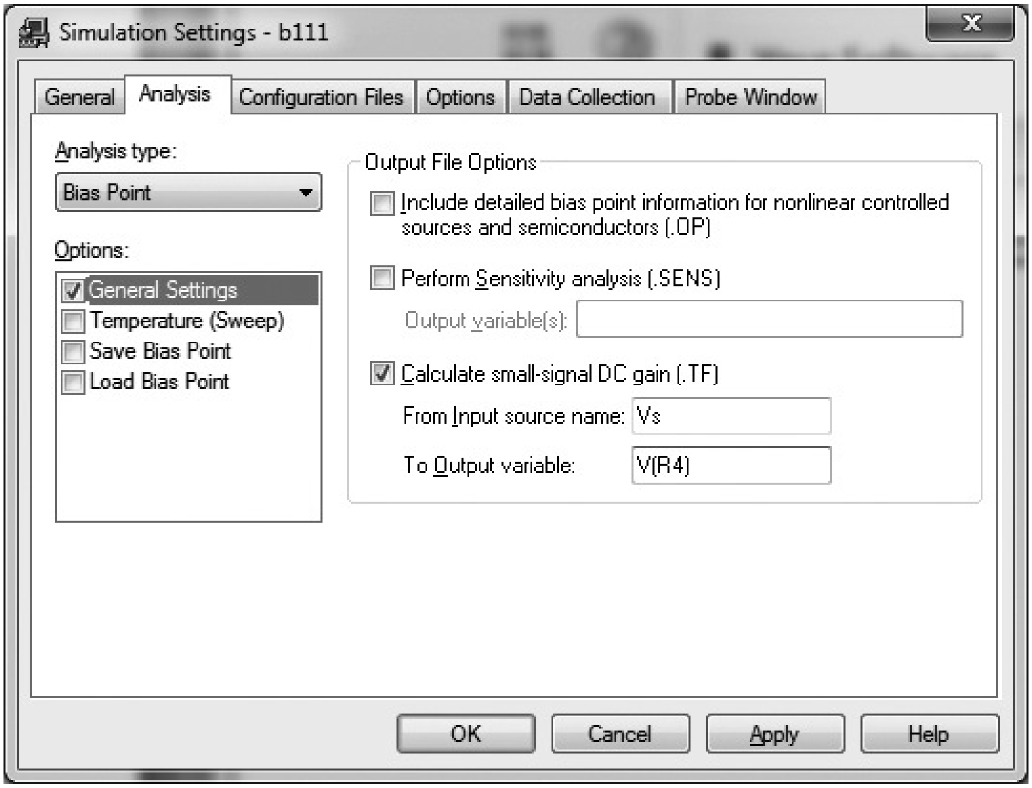
In the Edit Simulation Profile, check Calculate small-signal DC gain (.TF) and enter V_s and $V(R_4)$ respectively for the Input source name and Output variable fields, as shown in Figure 4.153.

After running the simulation, click on the V (Enable Bias Voltage Display) to display the voltages, as shown in Figure 4.154.

The voltage across R_4 is 14 V. This is the open-circuit voltage and the Thévenin equivalent voltage; that is,

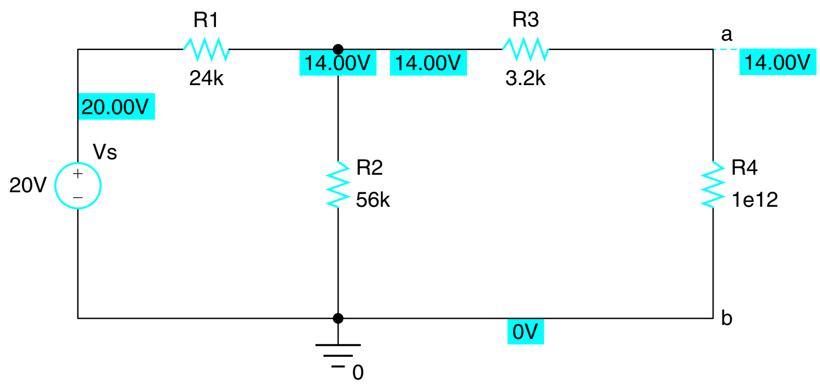
$$V_{th} = V_{oc} = 14\ \text{V}$$

FIGURE 4.153



Setting the transfer function.
(*Source: OrCAD PSpice by Cadence*)

FIGURE 4.154



Node voltages.

If you click on View Simulation Output File ( ) in the SCHEMATIC1 window, or select View → Output File and scroll down, the following results are shown near the end of the output file:

```
****     SMALL-SIGNAL CHARACTERISTICS

V(R_R4)/V_Vs = 7.000E-01
INPUT RESISTANCE AT V_Vs = 8.000E+04
OUTPUT RESISTANCE AT V(R_R4) = 2.000E+04
```

The first line shows the ratio of the voltage across R_4 to the voltage of the voltage source V_s, called transfer function; that is,

$$\frac{V(R_4)}{V_s} = 0.7$$

Thus, the voltage across R_4 is given by

$$V(R_4) = 0.7 \times V_s = 0.7 \times 20 = 14 \text{ V}$$

which is the Thévenin equivalent voltage. The second line shows the input resistance of 80 $k\Omega$ from the source. Notice that

$$R_{in} = R_1 + [R_2 \| (R_3 + R_4)] = 24{,}000\ \Omega + [56{,}000\ \Omega \| (3200\ \Omega + 10^{12}\ \Omega)]$$
$$= 24{,}000\ \Omega + 56{,}000\ \Omega = 80\ k\Omega$$

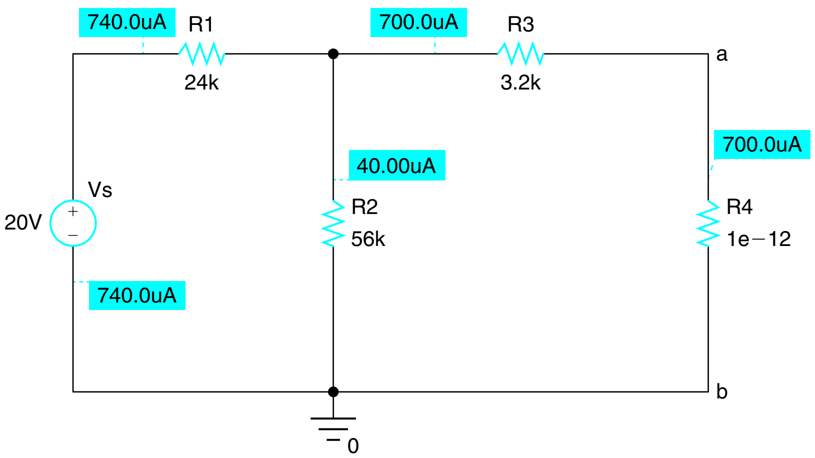
The third line shows the output resistance of 20 $k\Omega$ from the output. Notice that

$$R_{out} = R_{\text{th}} = R_3 + (R_1 \| R_2) = 3.2\ k\Omega + (24\ k\Omega \| 56\ k\Omega)$$
$$= 3.2\ k\Omega + 16.8\ k\Omega = 20\ k\Omega$$

If the value of R_4 is changed to $10^{-12}\ \Omega$, as shown in Figure 4.155, we can find the short-circuit current between a and b.

FIGURE 4.155

A short-circuit current between a and b.



The short-circuit current is $I_{sc} = 700\ \mu$A. The Thévenin equivalent resistance can also be found by taking the ratio of V_{oc} to I_{sc}:

$$R_{\text{th}} = \frac{V_{oc}}{I_{sc}} = \frac{14 \text{ V}}{0.7 \text{ mA}} = 20\ k\Omega$$

The short-circuit current is the Norton equivalent current I_n.

EXAMPLE 4.22

Use PSpice to find the Thévenin equivalent voltage V_{th} and the Thévenin equivalent resistance R_{th} between a and b for the circuit shown in Figure 4.156.

The Thévenin equivalent voltage is $V_{\text{th}} = -3.053$ V, as shown in Figure 4.157. From the transfer function, we obtain the Thévenin equivalent resistance $R_{\text{th}} = 2.175\ k\Omega$, as shown here.

continued

Example 4.22 continued

FIGURE 4.156

Circuit for EXAMPLE 4.22.

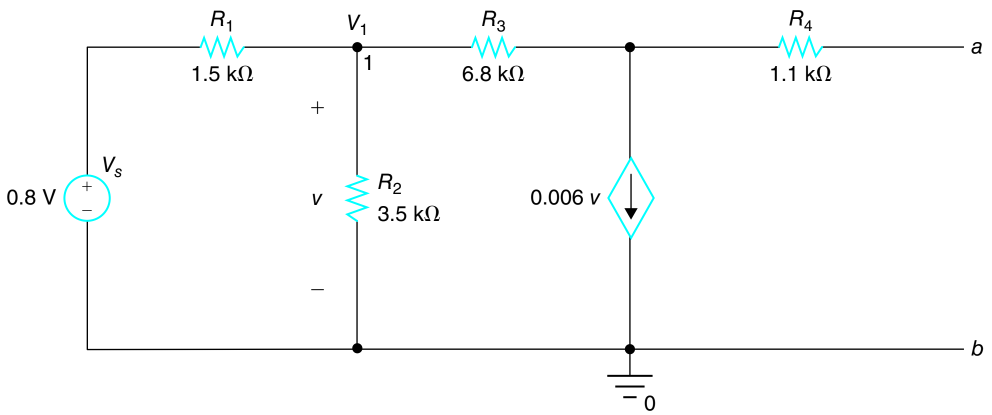
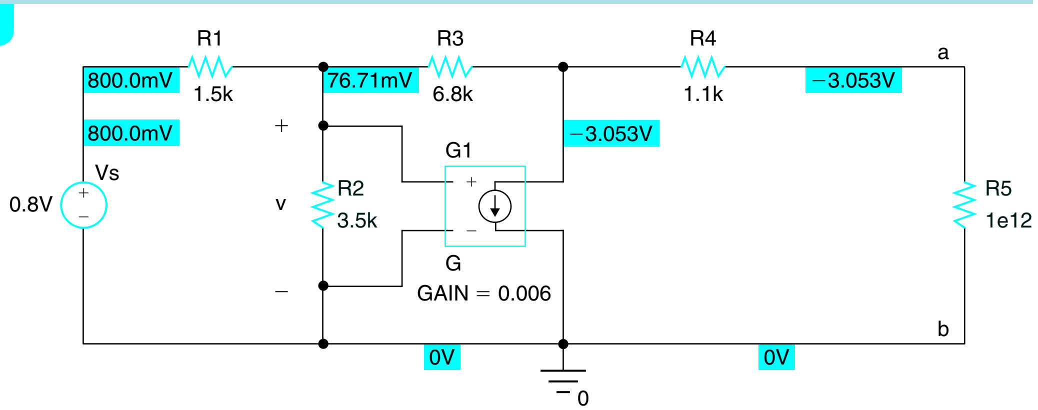


FIGURE 4.157

Node voltages.



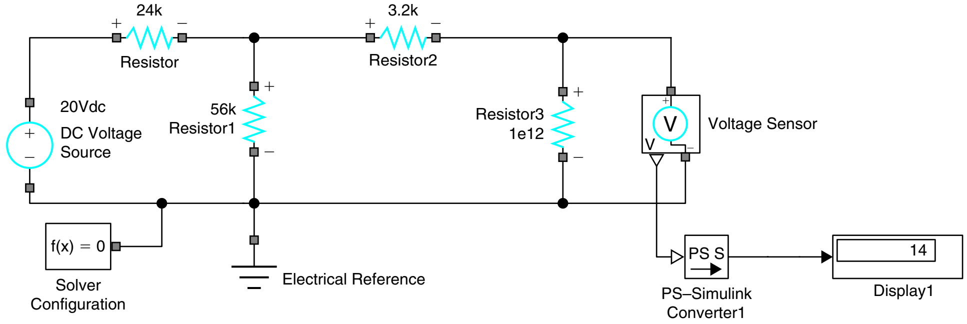

```
****     SMALL-SIGNAL CHARACTERISTICS

V(R_R5)/V_Vs = -3.816E+00
INPUT RESISTANCE AT V_Vs = 1.659E+03
OUTPUT RESISTANCE AT V(R_R5) = 2.175E+03
```

4.7.1 SIMULINK

Figure 4.158 shows a Simulink model to measure the Thévenin voltage for the circuit shown in Figure 4.151. Figure 4.159 shows a Simulink model to measure the Thévenin resistance for the circuit shown in Figure 4.151. A test voltage of 1 V is applied across the terminals *a* and *b* after deactivating the source. The ratio of the test voltage to the current flowing out of the positive terminal of the test source is the Thévenin resistance. The answer from the model is 20 $k\Omega$.

FIGURE 4.158

A Simulink model to measure the Thévenin voltage.



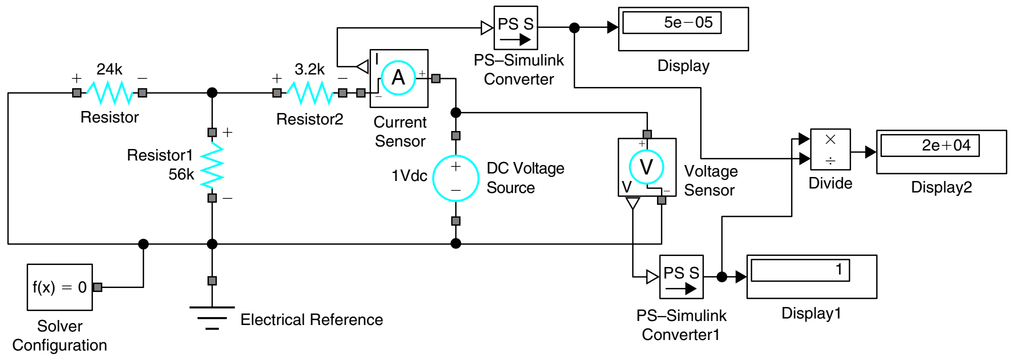


FIGURE 4.159

A Simulink model to measure the Thévenin resistance for the circuit shown in Figure 4.151.

SUMMARY

In this chapter, four important circuit theorems have been presented: the superposition principle, source transformation, Thévenin's theorem, and Norton's theorem. A brief summary is given for each of these theorems.

The *superposition principle* says that if a circuit contains more than one independent source, we can find the response of the circuit from one source at a time while the rest are deactivated and add the responses from individual sources to get the response.

The *source transformation* says that a voltage source with voltage V_s in series with a resistor with resistance R can be replaced by a current source with current V_s/R and a parallel resistor with resistance R. Also, a current source with current I_s in parallel with a resistor with resistance R can be replaced by a voltage source with voltage RI_s and a series resistor with resistance R.

Thévenin's theorem says that given terminals a and b, the circuit looking from the terminals a and b can be represented by a voltage source with voltage V_{th}, called the *Thévenin equivalent voltage*, and a series resistor with resistance R_{th}, called the *Thévenin equivalent resistance.* The Thévenin equivalent voltage V_{th} can be found by calculating the open-circuit voltage between terminals a and b. There are three methods to find the Thévenin equivalent resistance R_{th}. The first method is to find the equivalent resistance looking into the circuit from terminals a and b after deactivating independent sources. The second method is to find the short-circuit current from a to b, and to take the ratio of the open-circuit voltage to the short-circuit current (i.e., $R_{th} = V_{oc}/I_{sc}$). The third method is to apply a test voltage from a to b into the circuit after deactivating the independent sources, measure the current flowing out of the positive terminal of the test voltage source, and then take the ratio of the voltage to current. Alternatively, a test current can be applied to the circuit from terminals a and b after deactivating the independent sources. Then, measure the voltage across the test current source. The Thévenin equivalent resistance is the ratio of the voltage to current.

Norton's theorem says that given terminals a and b, the circuit looking from the terminals a and b can be represented by a current source with current I_n, called the *Norton equivalent current*, and a parallel resistor with resistance R_n, called the *Norton equivalent resistance.* The Norton equivalent current I_n can be found by calculating the short-circuit current from a to b. There are three methods to find the Norton equivalent resistance R_n. The first method is to find the equivalent resistance looking into the circuit from terminals a and b after deactivating independent sources. The second method is to find the open-circuit voltage between terminals a and b, and to take the ratio of the open-circuit voltage to the short-circuit current (i.e., $R_n = V_{oc}/I_{sc}$). The third method is to apply a test voltage from a to b into the circuit after deactivating the independent sources, measure the current flowing out of the positive terminal of the test voltage source, and then take the ratio of the voltage to current. Alternatively, a test current can be applied to the circuit from terminals a and b after deactivating the independent sources. Then, measure the voltage across the test current source. The Norton equivalent resistance is the ratio of the voltage to the current.

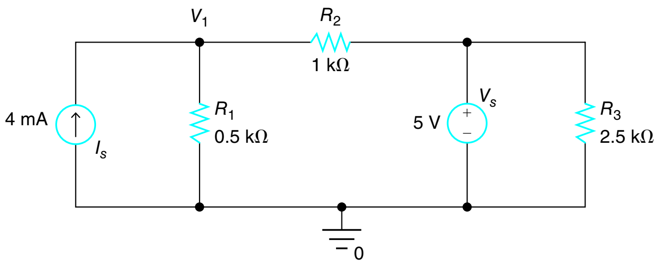
Suppose that a load with resistance R_L is connected to a circuit between terminals a and b. We are interested in finding the load resistance R_L that maximizes the power delivered to the load. We first find the Thévenin equivalent circuit with respect to the terminals a and b. Let V_{th} be the Thévenin equivalent voltage and R_{th} be the Thévenin equivalent resistance. Then, the load resistance R_L that maximizes the power delivered to the load is given by the Thévenin equivalent resistance from terminals a and b.

PROBLEMS

Superposition Principle

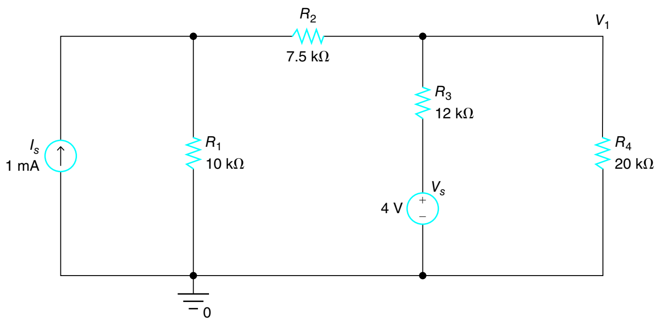
4.1 Use the superposition principle to find voltage V_1 in the circuit shown in Figure P4.1.

FIGURE P4.1



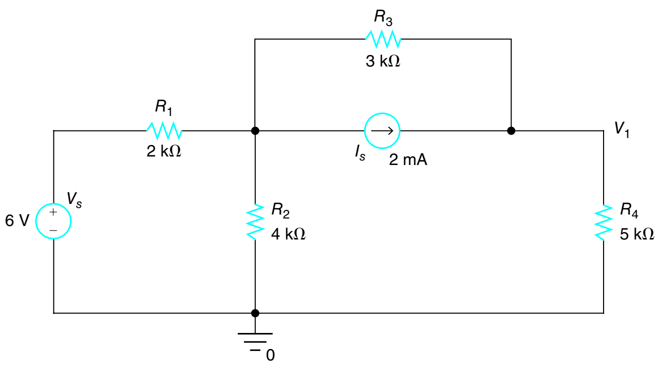
4.2 Use the superposition principle to find voltage V_1 in the circuit shown in Figure P4.2.

FIGURE P4.2



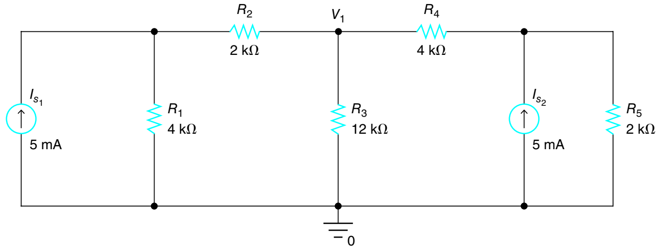
4.3 Use the superposition principle to find voltage V_1 in the circuit shown in Figure P4.3.

FIGURE P4.3



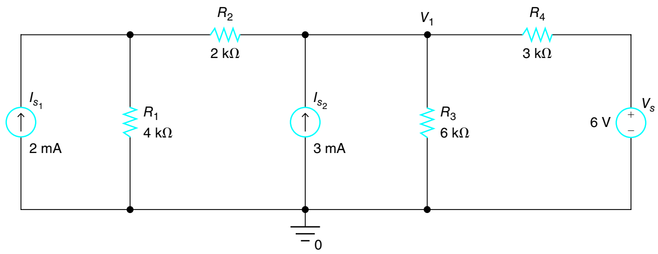
4.4 Use the superposition principle to find voltage V_1 in the circuit shown in Figure P4.4.

FIGURE P4.4



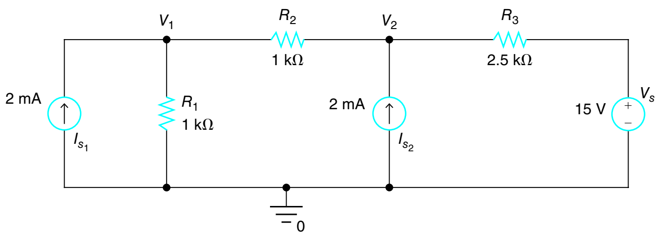
4.5 Use the superposition principle to find voltage V_1 in the circuit shown in Figure P4.5.

FIGURE P4.5



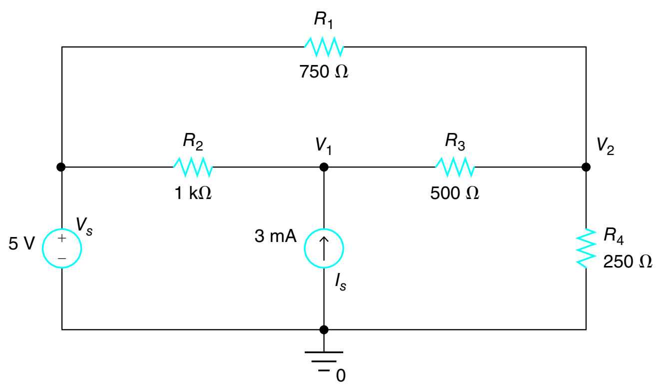
4.6 Use the superposition principle to find voltages V_1 and V_2 in the circuit shown in Figure P4.6.

FIGURE P4.6



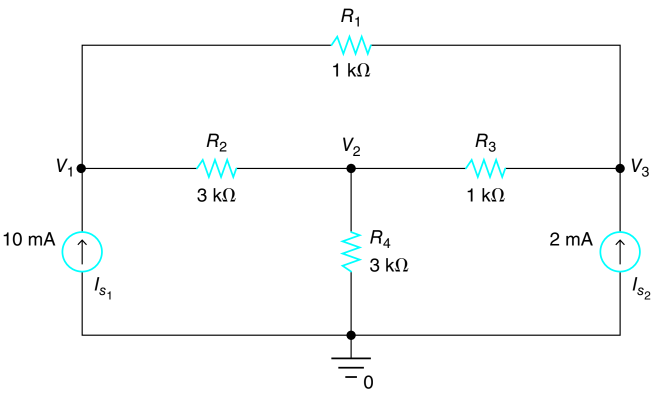
4.7 Use the superposition principle to find voltages V_1 and V_2 in the circuit shown in Figure P4.7.

FIGURE P4.7



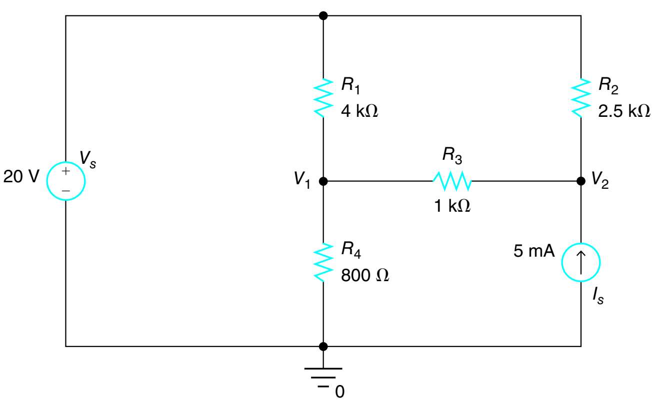
4.8 Use the superposition principle to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.8.

FIGURE P4.8



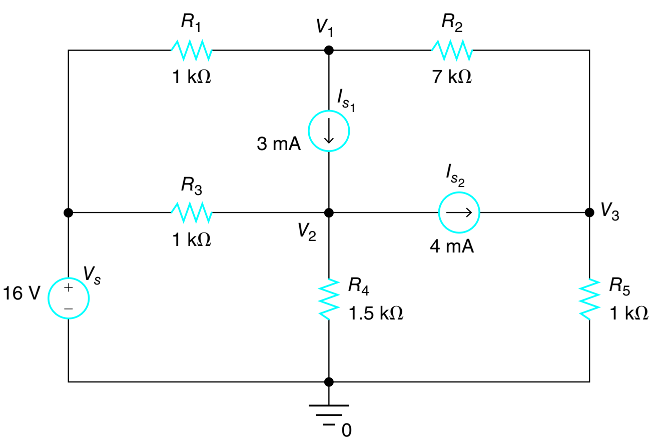
4.9 Use the superposition principle to find voltages V_1 and V_2 in the circuit shown in Figure P4.9.

FIGURE P4.9



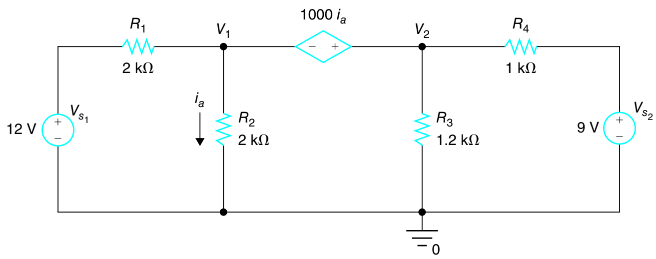
4.10 Use the superposition principle to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.10.

FIGURE P4.10



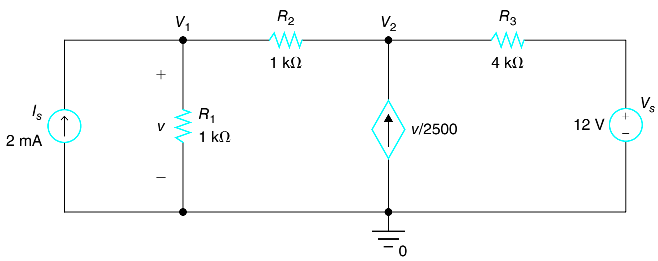
4.11 Use the superposition principle to find voltages V_1 and V_2 in the circuit shown in Figure P4.11.

FIGURE P4.11



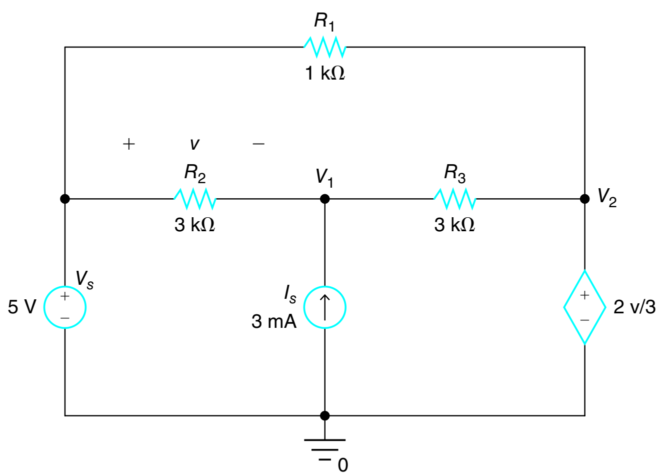
4.12 Use the superposition principle to find voltages V_1 and V_2 in the circuit shown in Figure P4.12.

FIGURE P4.12



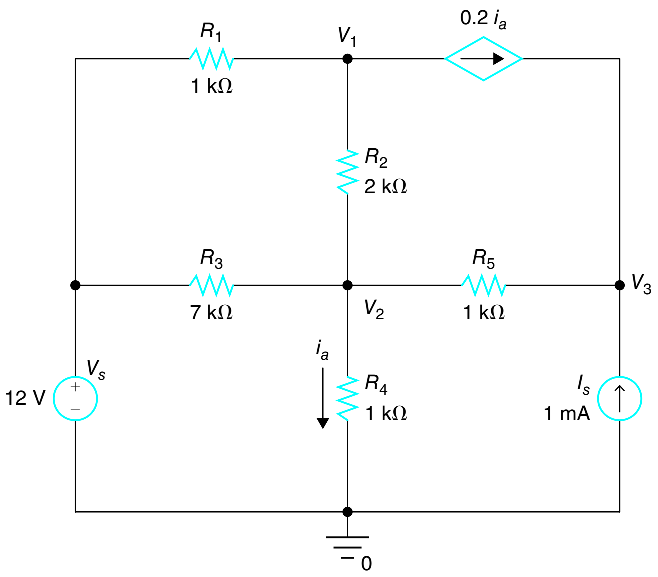
4.13 Use the superposition principle to find voltages V_1 and V_2 in the circuit shown in Figure P4.13.

FIGURE P4.13



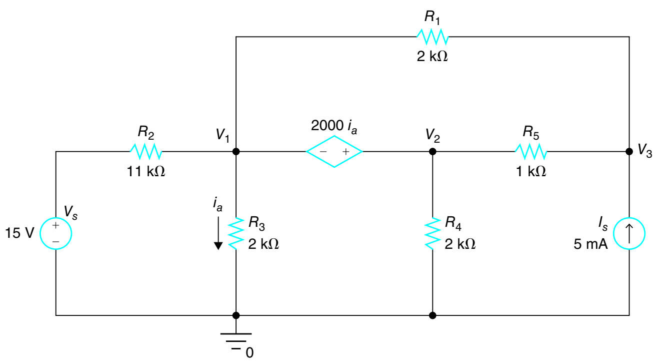
4.14 Use the superposition principle to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.14.

FIGURE P4.14



4.15 Use the superposition principle to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.15.

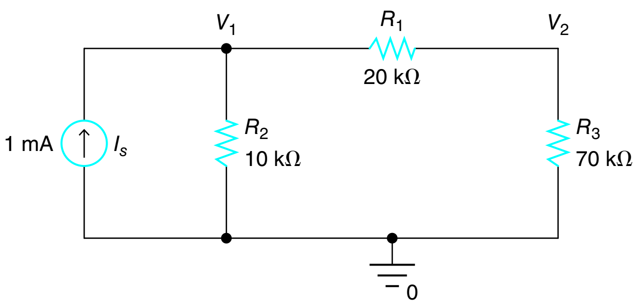
FIGURE P4.15



Source Transformation

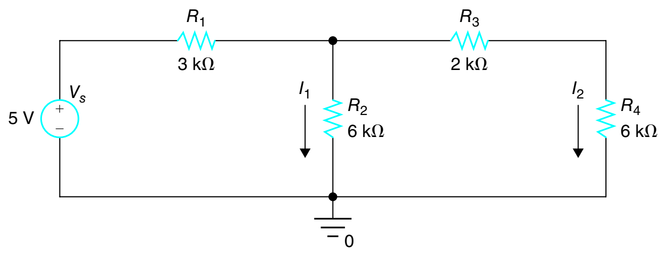
4.16 Use source transformation to find voltages V_1 and V_2 in the circuit shown in Figure P4.16.

FIGURE P4.16



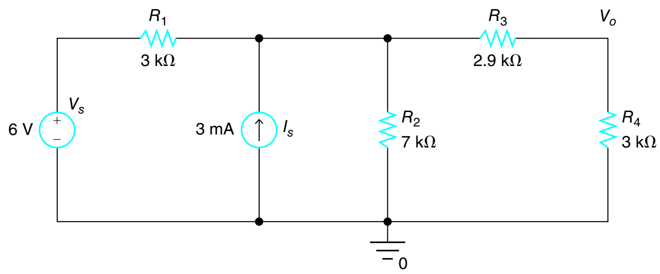
4.17 Use source transformation to find currents I_1 and I_2 in the circuit shown in Figure P4.17.

FIGURE P4.17



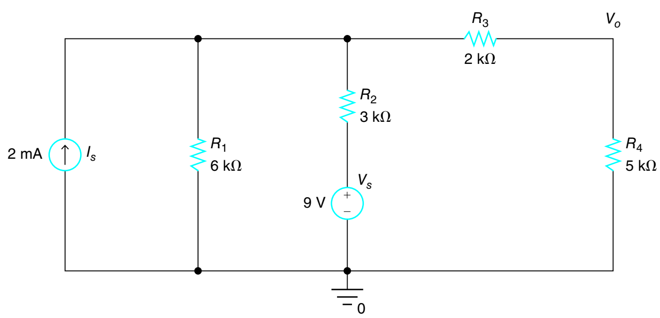
4.18 Use source transformation to find voltage V_o in the circuit shown in Figure P4.18.

FIGURE P4.18



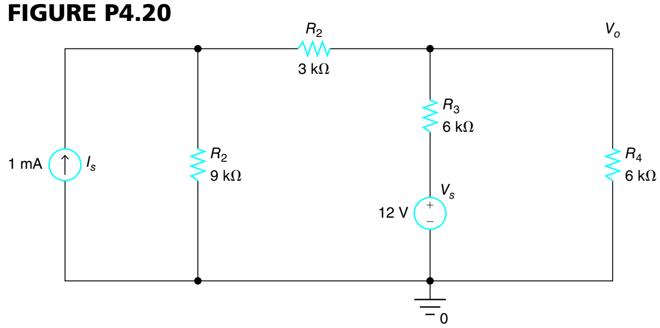
4.19 Use source transformation to find voltage V_o in the circuit shown in Figure P4.19.

FIGURE P4.19



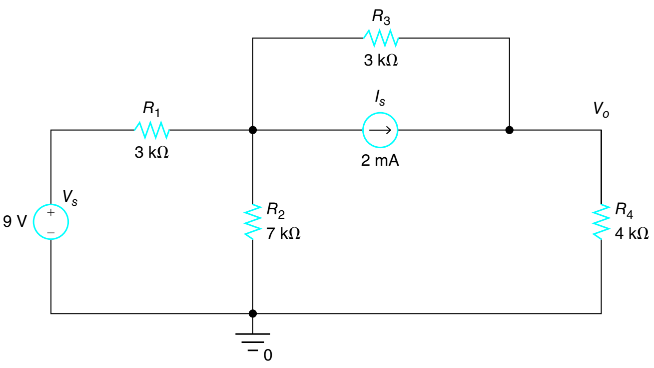
4.20 Use source transformation to find voltage V_o in the circuit shown in Figure P4.20.

FIGURE P4.20



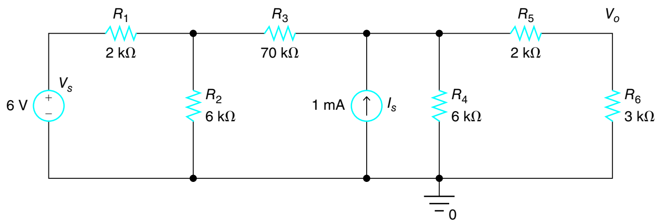
4.21 Use source transformation to find voltage V_o in the circuit shown in Figure P4.21.

FIGURE P4.21



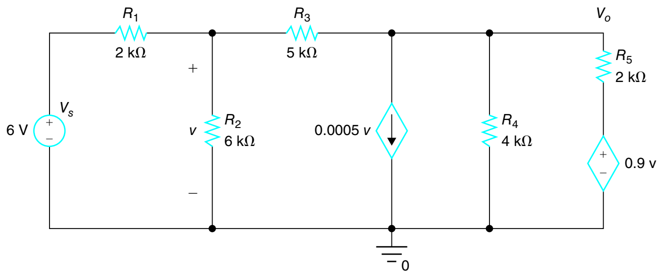
4.22 Use source transformation to find voltage V_o in the circuit shown in Figure P4.22.

FIGURE P4.22



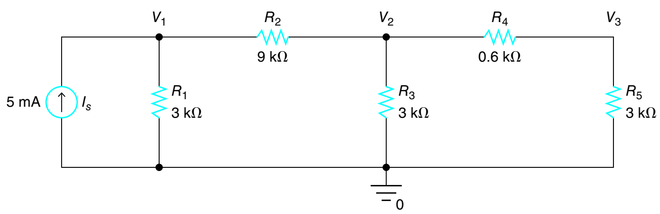
4.23 Use source transformation to find voltage V_o in the circuit shown in Figure P4.23.

FIGURE P4.23



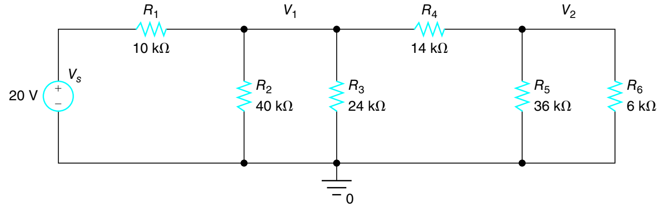
4.24 Use source transformation to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.24.

FIGURE P4.24



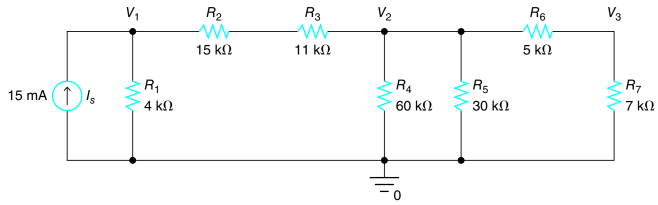
4.25 Use source transformation to find voltages V_1 and V_2 in the circuit shown in Figure P4.25.

FIGURE P4.25



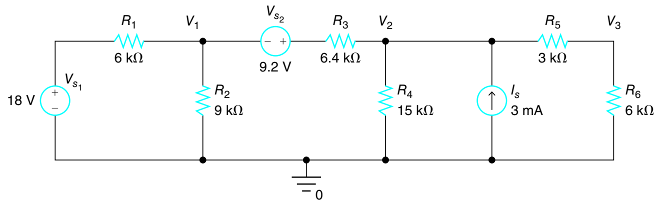
4.26 Use source transformation to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.26.

FIGURE P4.26



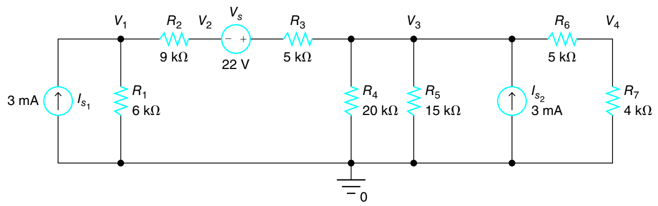
4.27 Use source transformation to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.27.

FIGURE P4.27



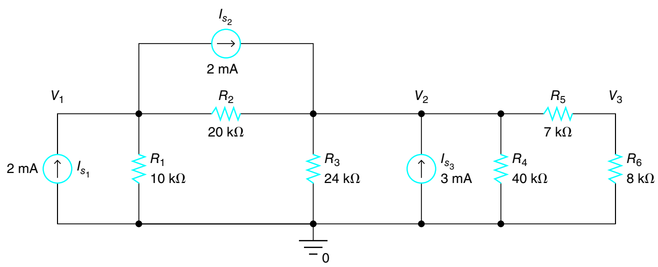
4.28 Use source transformation to find voltages V_1, V_2, V_3, and V_4 in the circuit shown in Figure P4.28.

FIGURE P4.28



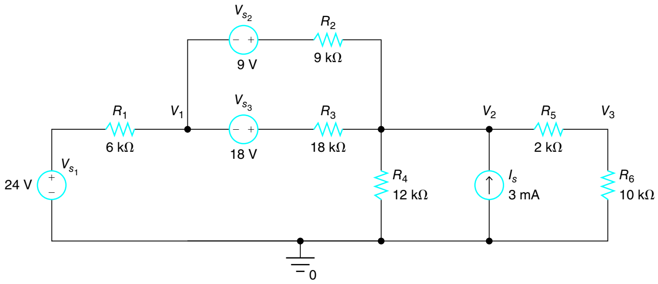
4.29 Use source transformation to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.29.

FIGURE P4.29



4.30 Use source transformation to find voltages V_1, V_2, and V_3 in the circuit shown in Figure P4.30.

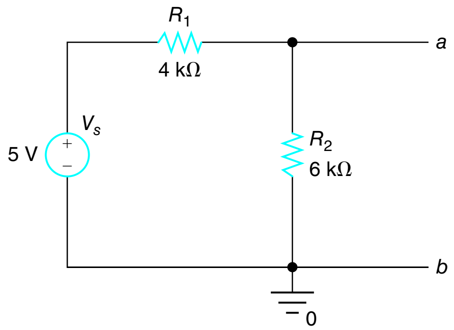
FIGURE P4.30



Thévenin Equivalent Circuit

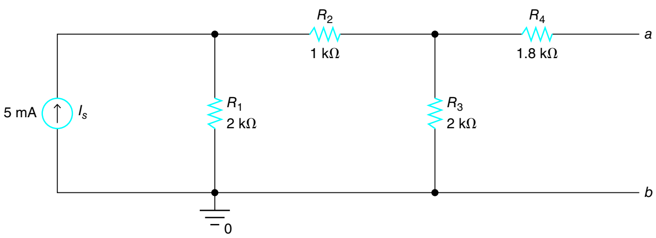
4.31 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.31.

FIGURE P4.31



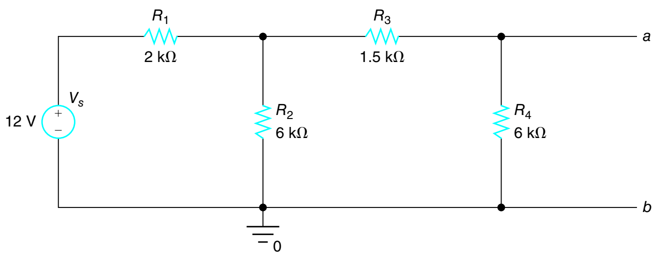
4.32 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.32.

FIGURE P4.32



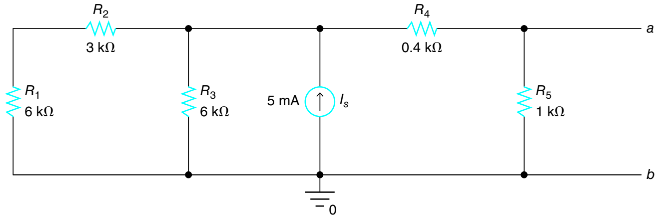
4.33 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.33.

FIGURE P4.33



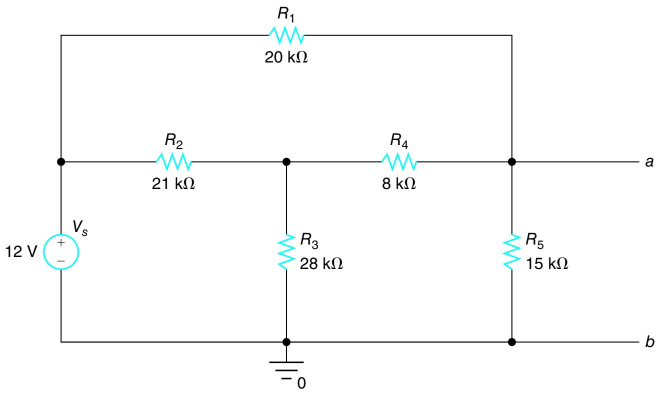
4.34 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.34.

FIGURE P4.34



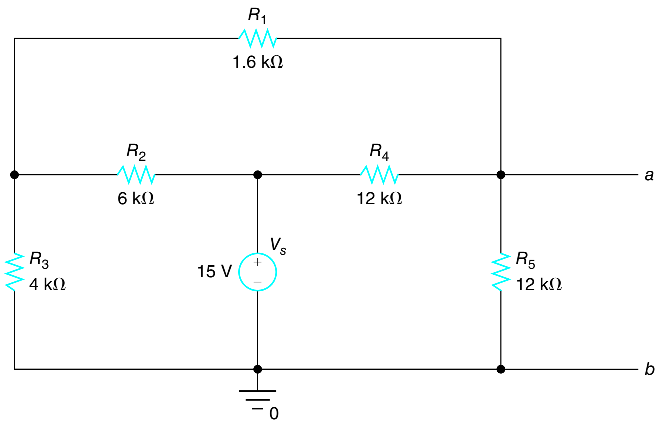
4.35 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.35.

FIGURE P4.35



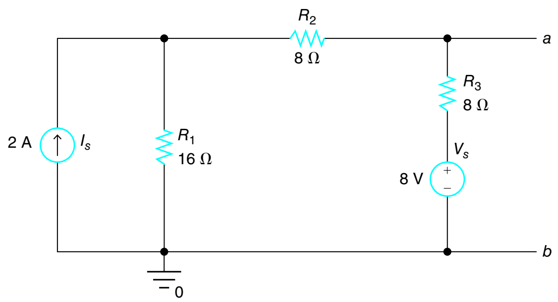
4.36 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.36.

FIGURE P4.36



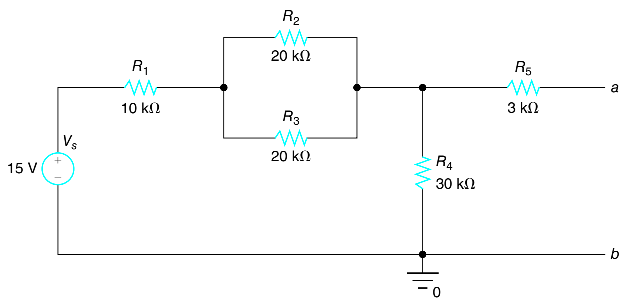
4.37 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.37.

FIGURE P4.37



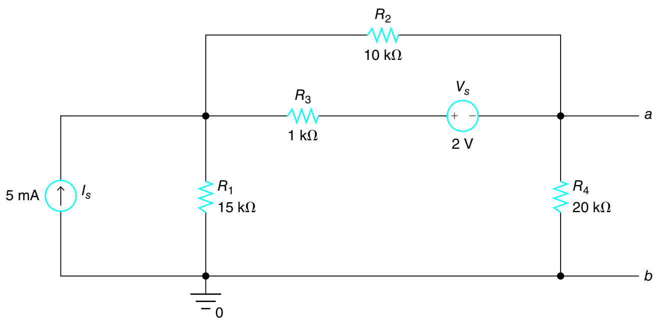
4.38 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.38.

FIGURE P4.38



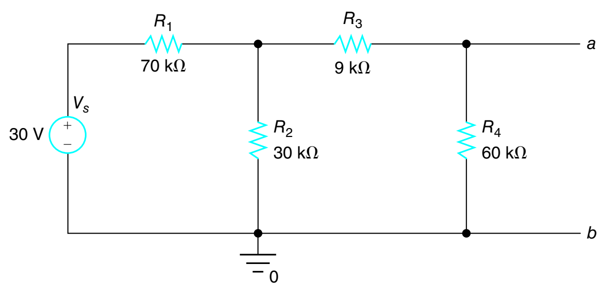
4.39 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.39.

FIGURE P4.39



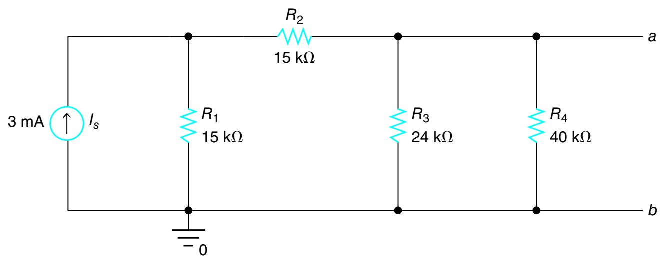
4.40 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.40.

FIGURE P4.40



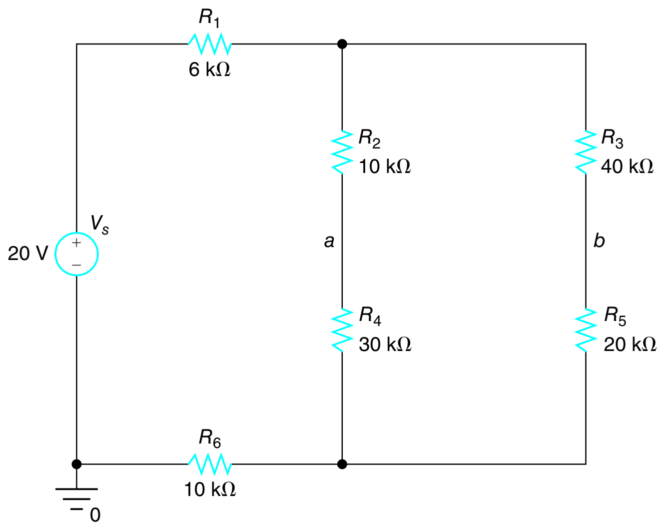
4.41 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.41.

FIGURE P4.41



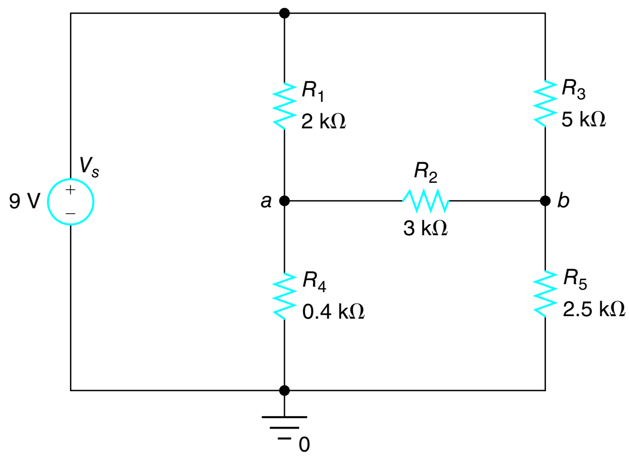
4.42 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.42.

FIGURE P4.42



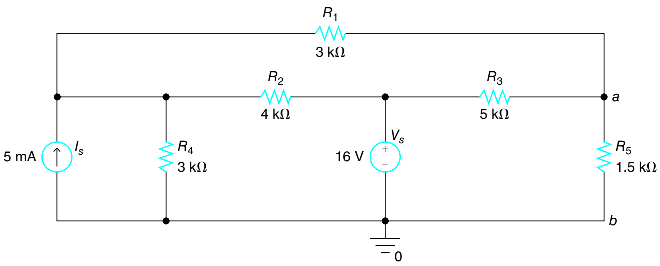
4.43 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.43.

FIGURE P4.43



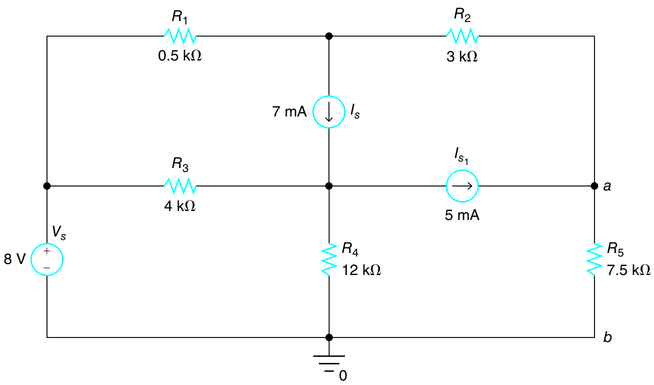
4.44 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.44.

FIGURE P4.44



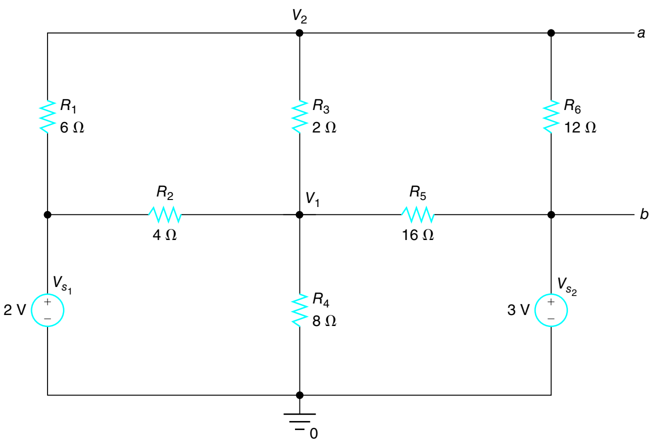
4.45 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.45.

FIGURE P4.45



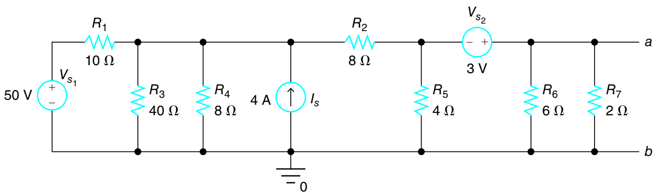
4.46 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.46.

FIGURE P4.46



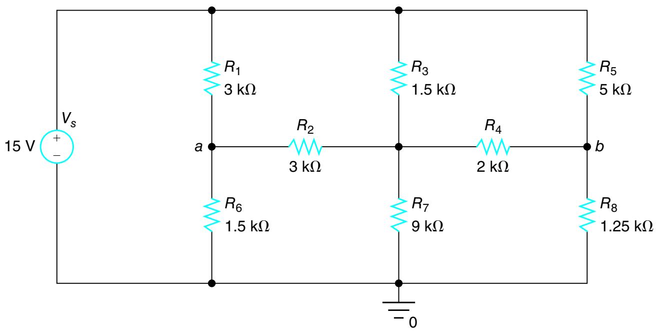
4.47 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.47.

FIGURE P4.47



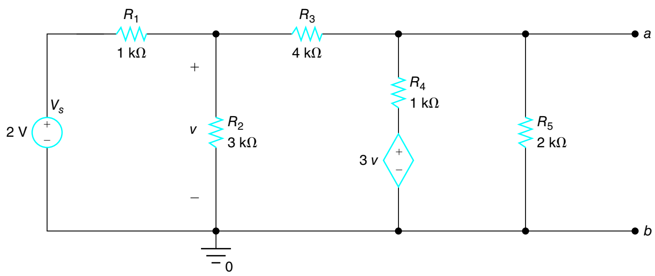
4.48 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.48.

FIGURE P4.48



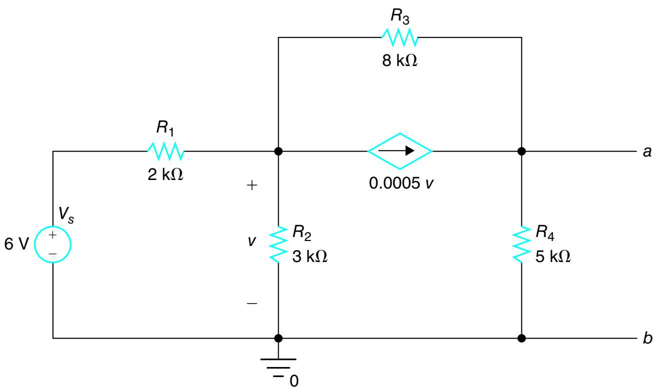
4.49 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.49.

FIGURE P4.49



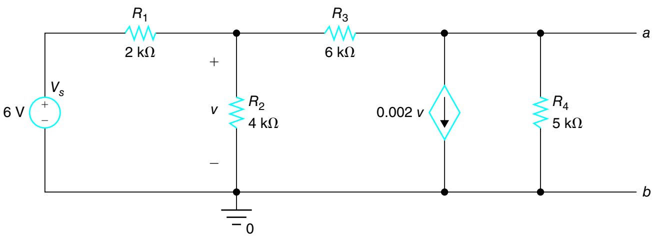
4.50 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.50.

FIGURE P4.50



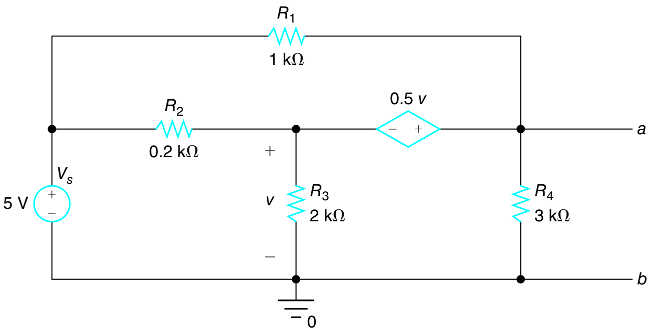
4.51 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.51.

FIGURE P4.51



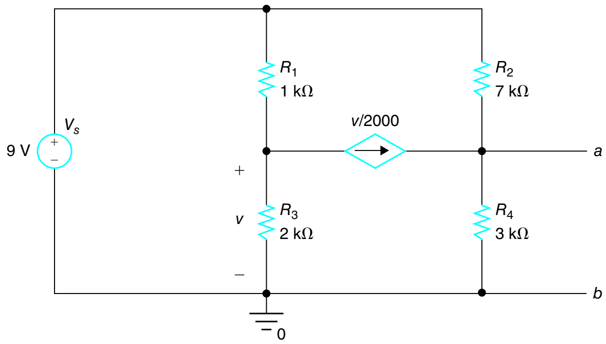
4.52 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.52.

FIGURE P4.52



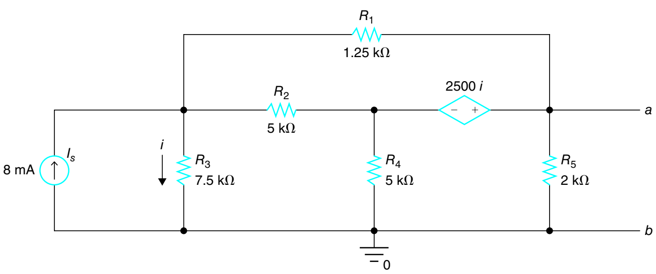
4.53 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.53.

FIGURE P4.53



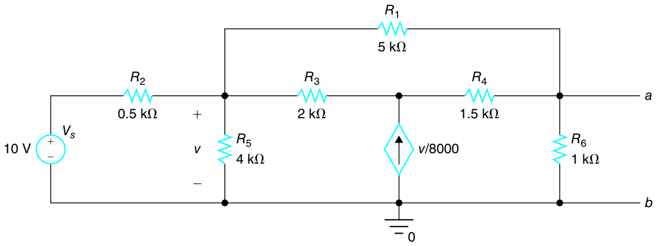
4.54 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.54.

FIGURE P4.54



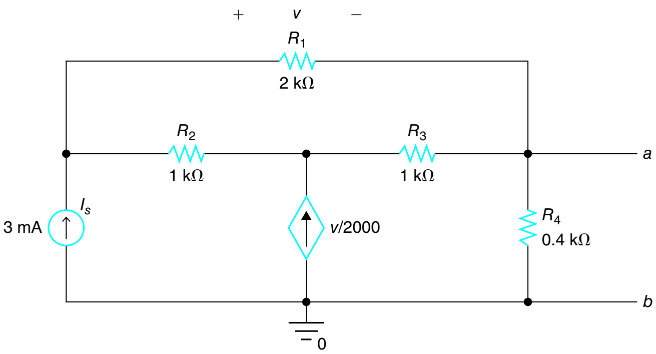
4.55 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.55.

FIGURE P4.55



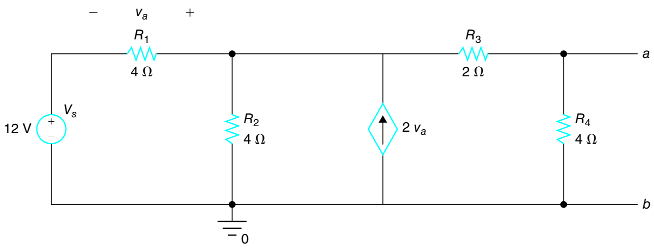
4.56 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.56.

FIGURE P4.56



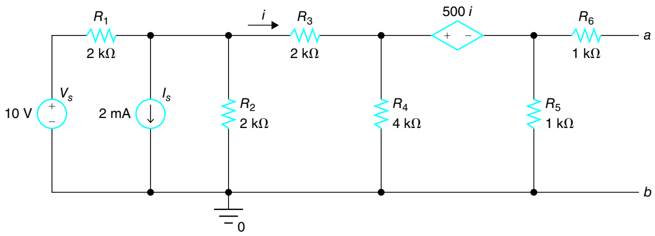
4.57 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.57.

FIGURE P4.57



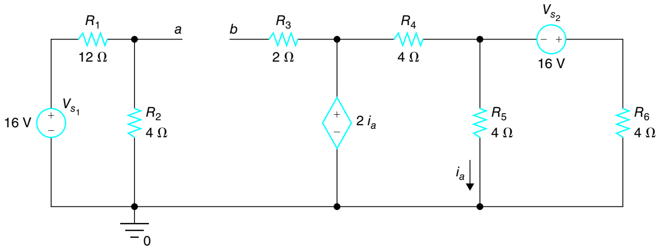
4.58 Find the Thévenin equivalent circuit between *a* nd *b* for the circuit shown in Figure P4.58.

FIGURE P4.58



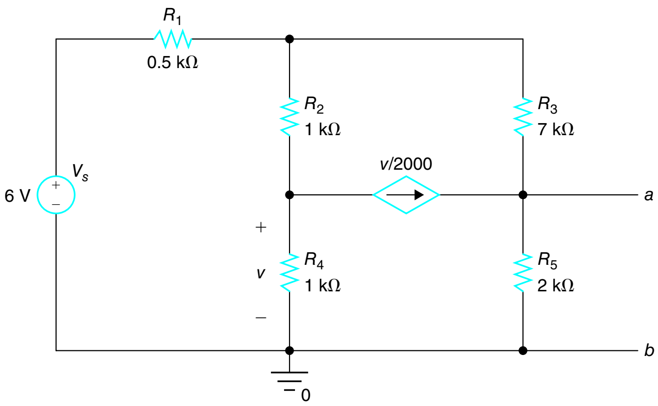
4.59 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.59.

FIGURE P4.59



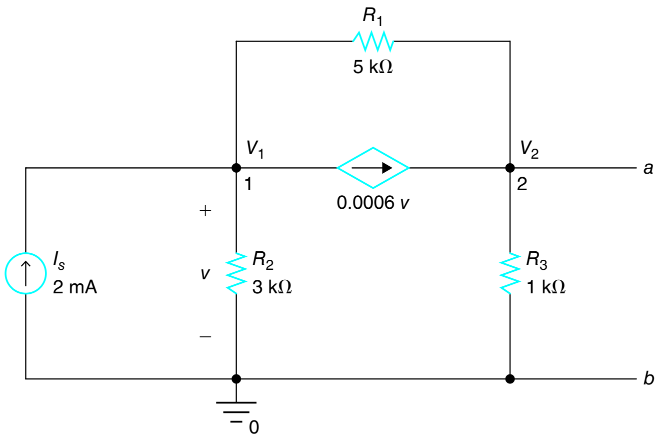
4.60 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.60.

FIGURE P4.60



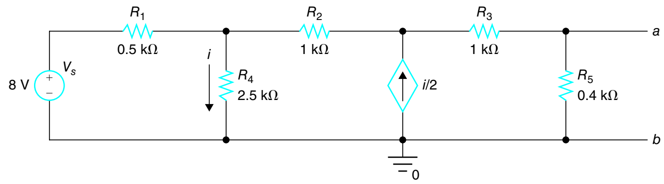
4.61 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.61.

FIGURE P4.61



4.62 Find the Thévenin equivalent circuit between *a* and *b* for the circuit shown in Figure P4.62.

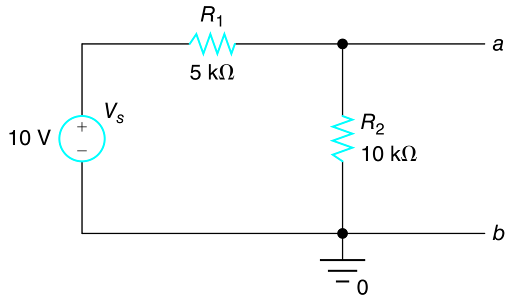
FIGURE P4.62



Norton Equivalent Circuit

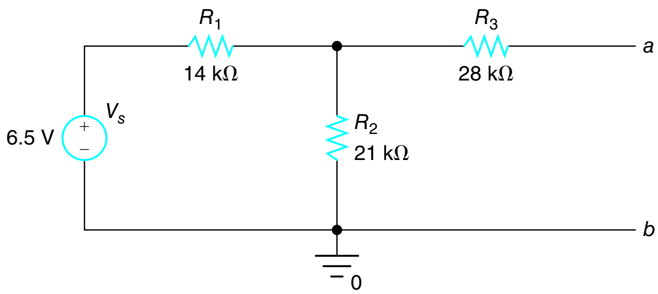
4.63 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.63.

FIGURE P4.63



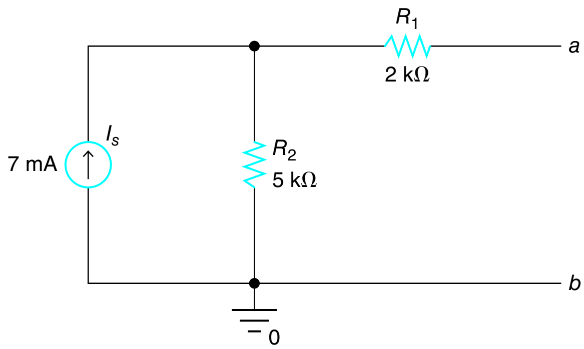
4.64 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.64.

FIGURE P4.64



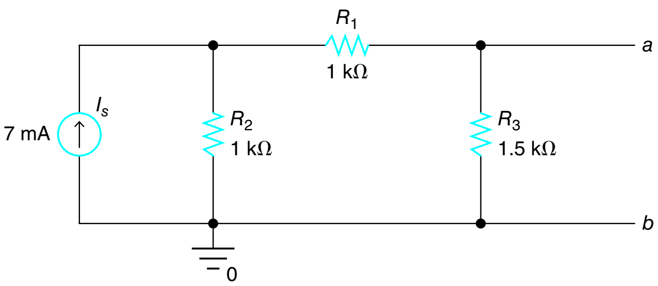
4.65 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.65.

FIGURE P4.65



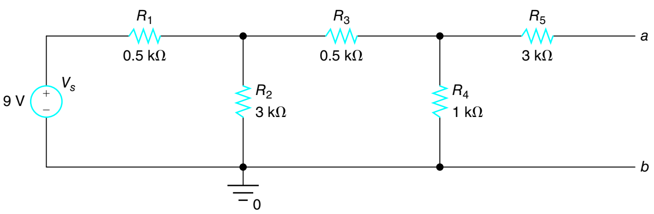
4.66 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.66.

FIGURE P4.66



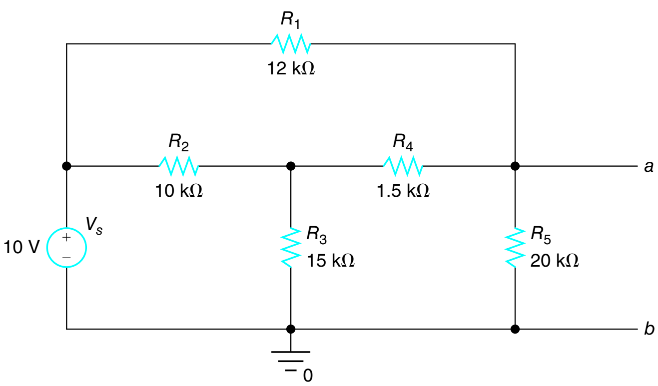
4.67 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.67.

FIGURE P4.67



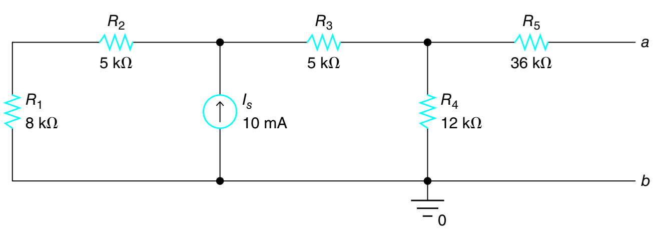
4.68 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.68.

FIGURE P4.68



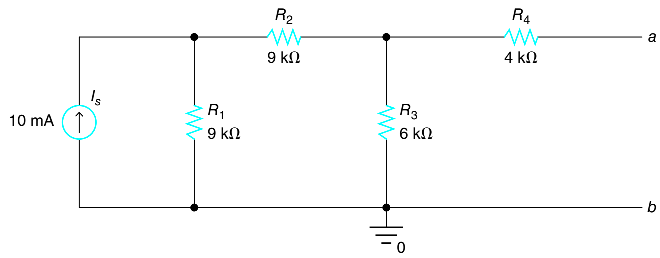
4.69 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.69.

FIGURE P4.69



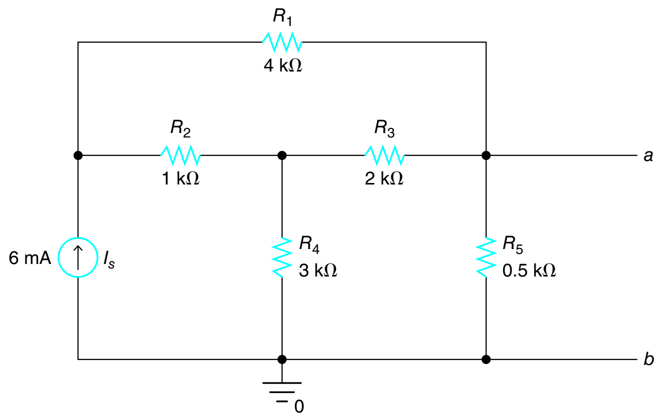
4.70 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.70.

FIGURE P4.70



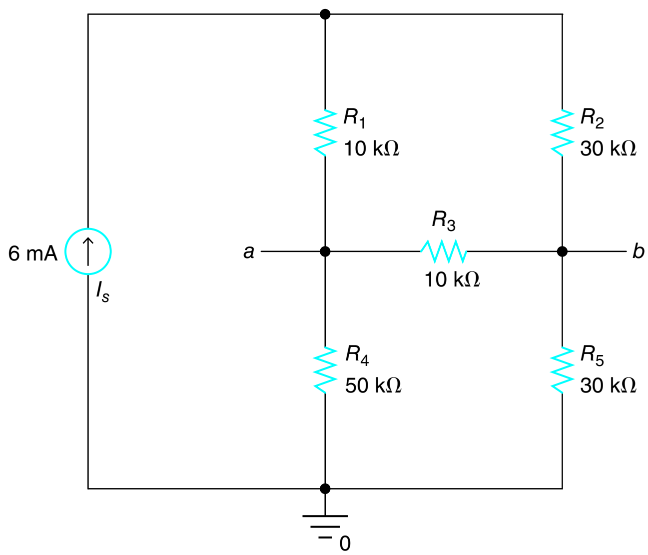
4.71 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.71.

FIGURE P4.71



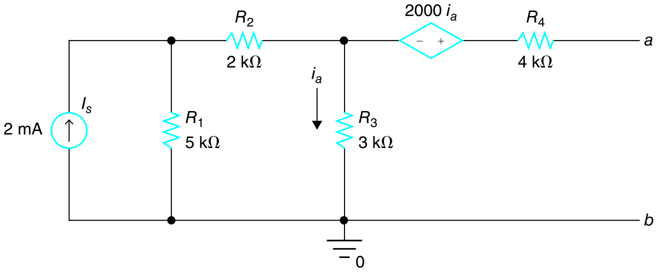
4.72 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.72.

FIGURE P4.72



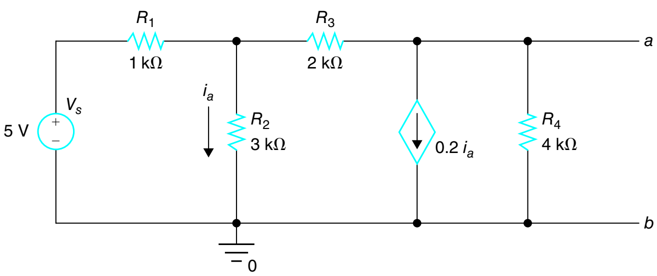
4.73 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.73.

FIGURE P4.73



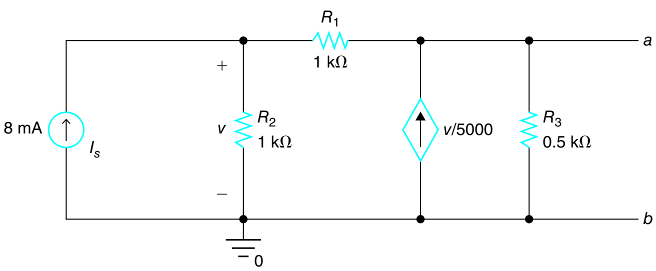
4.74 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.74.

FIGURE P4.74



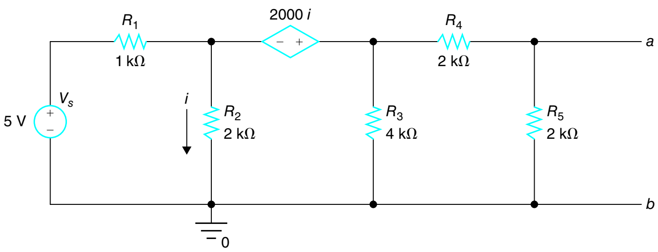
4.75 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.75.

FIGURE P4.75



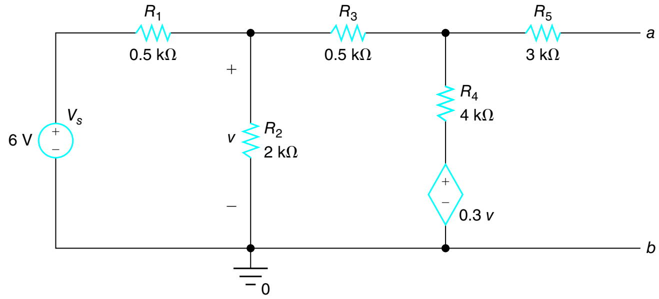
4.76 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.76.

FIGURE P4.76



4.77 Find the Norton equivalent circuit between *a* and *b* for the circuit shown in Figure P4.77.

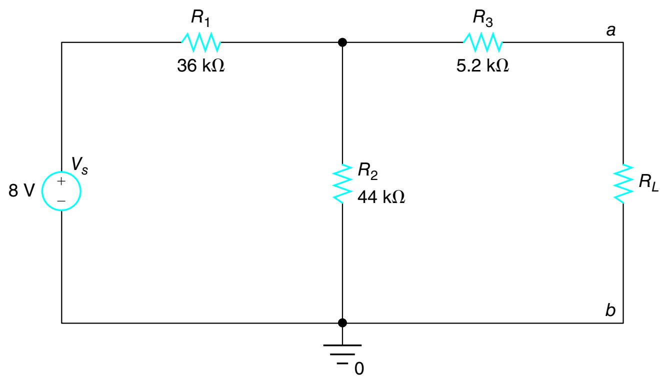
FIGURE P4.77



Maximum Power Transfer

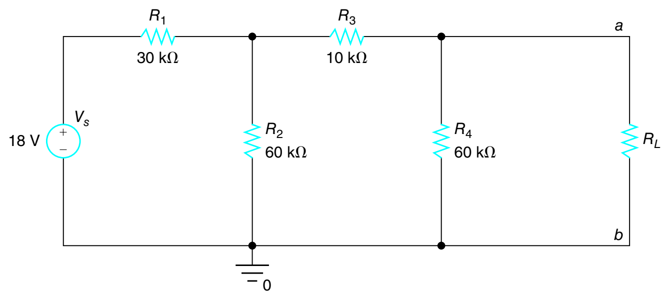
4.78 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.78.

FIGURE P4.78



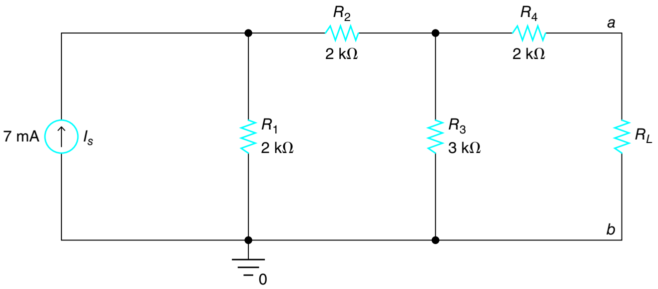
4.79 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.79.

FIGURE P4.79



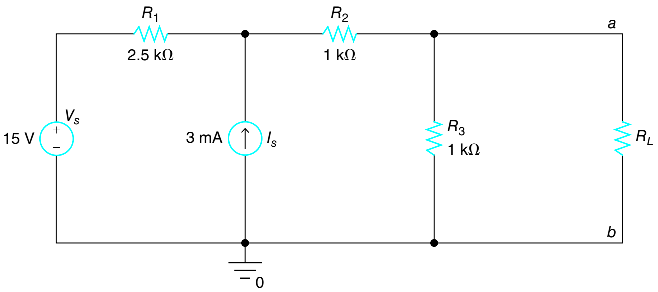
4.80 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.80.

FIGURE P4.80



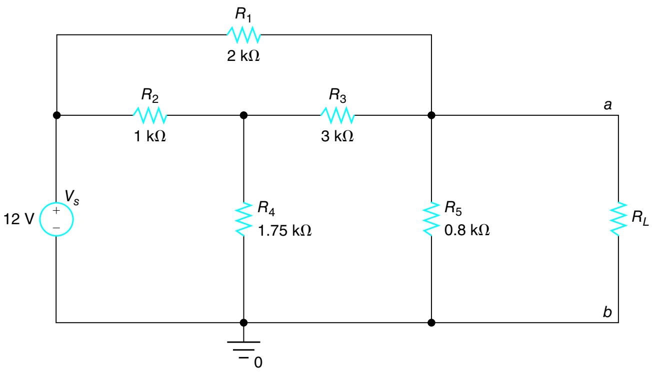
4.81 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.81.

FIGURE P4.81



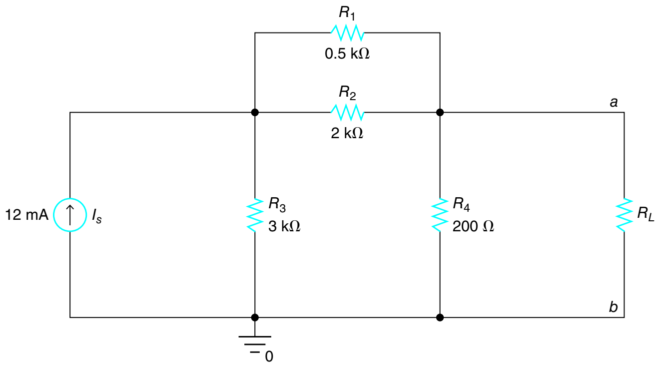
4.82 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.82.

FIGURE P4.82



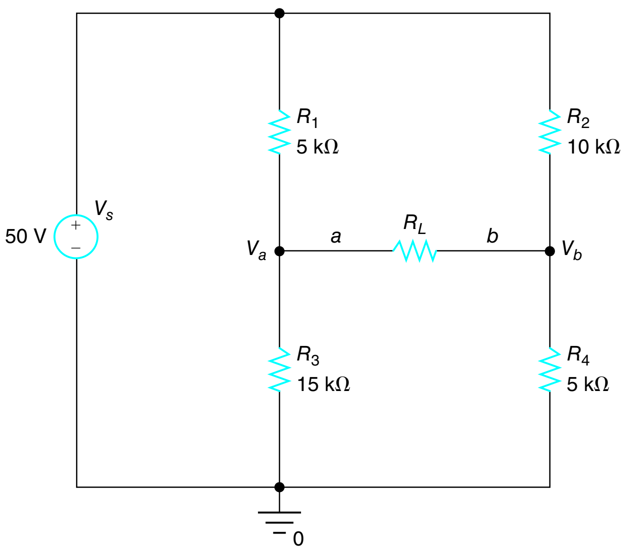
4.83 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.83.

FIGURE P4.83



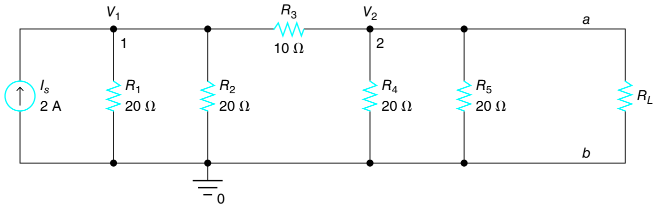
4.84 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.84.

FIGURE P4.84



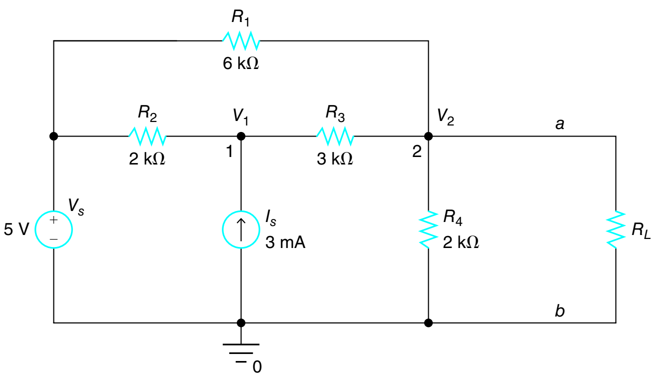
4.85 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.85.

FIGURE P4.85



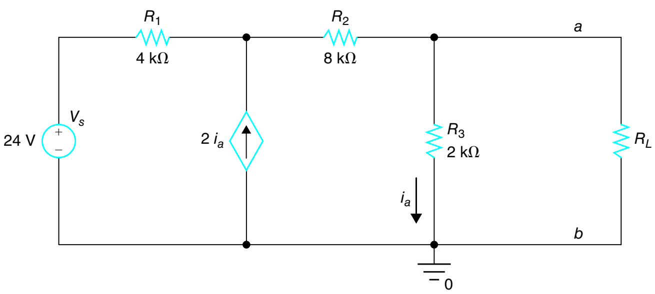
4.86 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.86.

FIGURE P4.86



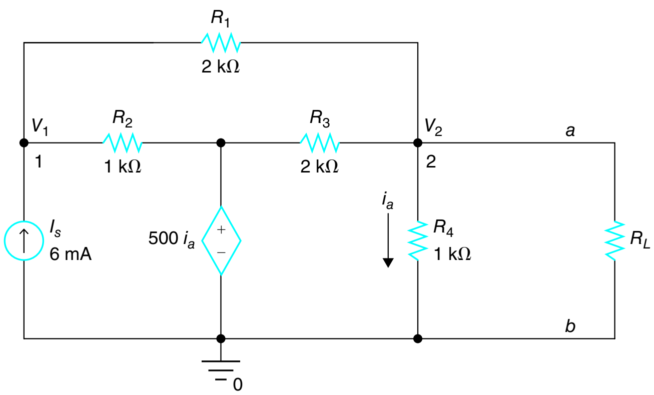
4.87 Find the value of R_L for the maximum power transfer for the circuit shown in Figure P4.87. Also, find the maximum power dissipated at R_L.

FIGURE P4.87



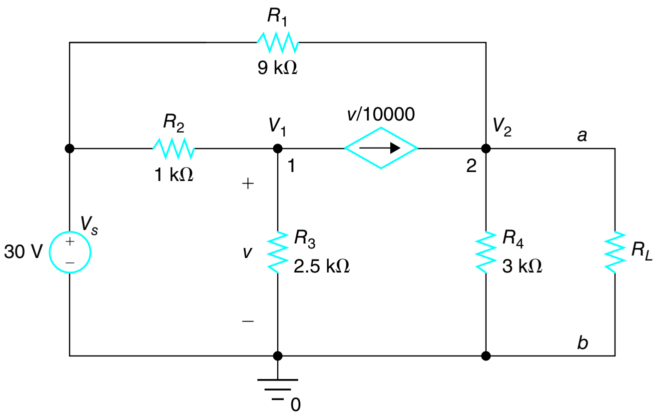
4.88 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.88.

FIGURE P4.88



4.89 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.89.

FIGURE P4.89



4.90 Find the load resistance value R_L for maximum power transfer, and find the maximum power transferred to the load for the circuit shown in Figure P4.90.

FIGURE P4.90