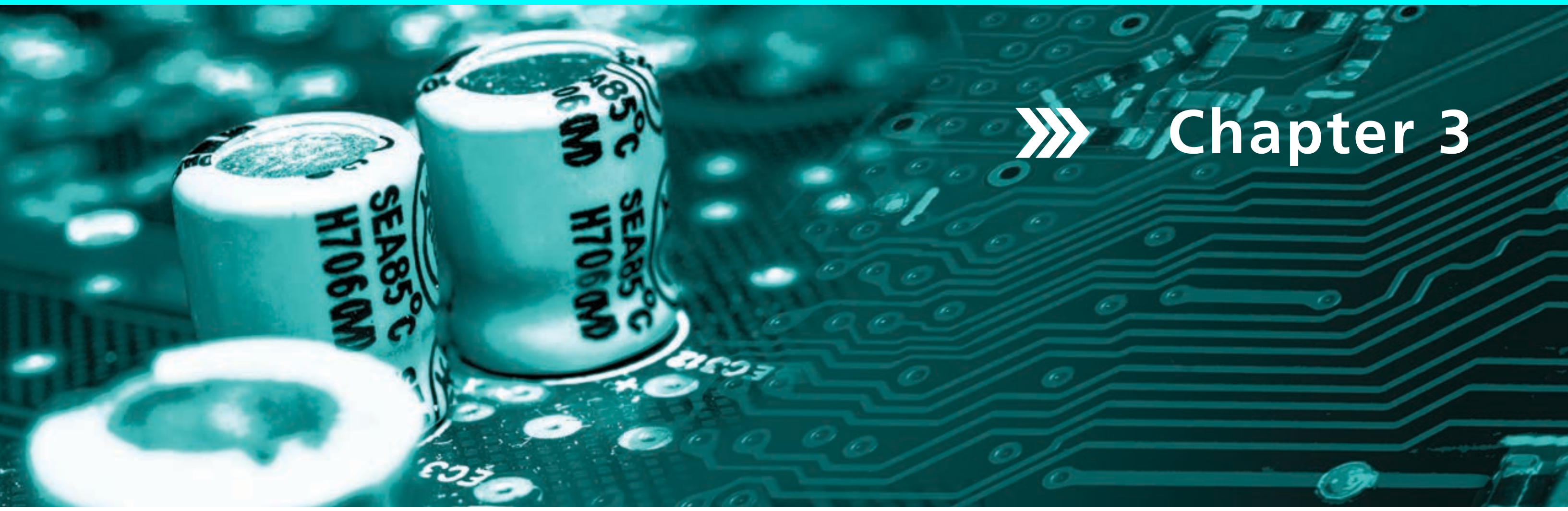


Chapter 3

Circuit Analysis Methods

3.1 Introduction

In this chapter, systematic approaches to analyze electric circuits are presented. The approaches refer to nodal analysis and mesh analysis. These two analysis methods can be universally applied to solve circuit problems.

Nodal analysis is a method of finding all the unknown node voltages of a circuit. The method is based on Kirchhoff's current law (KCL). Nodes can be labeled 1, 2, 3, . . . , or a, b, c, . . . (0 can be used for the reference node), and voltages on these nodes can be labeled V_1, V_2, V_3, . . . , or V_a, V_b, V_c, ... The node voltage of a reference node (0 V) and nodes with specified voltage sources to a reference node are known. As such, for each node whose voltage is unknown, we can write a node-voltage equation by summing the currents leaving (entering, or some entering and the rest leaving) the node. This is tantamount to writing KCL at each node. The currents leaving the node through resistors can be found by applying Ohm's law. A solution to the node voltages is obtained by solving the set of node-voltage equations. Once all the node voltages are computed, the current in each branch can be found using Ohm's law. In a special case, if there is a voltage source connecting two nodes, we can form a "supernode" by first excluding the voltage source and then writing the sum of currents that are leaving its two node-voltage terminals. In writing such an equation instead of two linearly independent equations, the result is one linear equation with two variables. One additional equation, known as the **constraint equation**, therefore, is needed; it is obtained by relating the potential difference across the two node-voltage terminals.

A *mesh* is a fundamental loop formed by circuit elements in which no other closed path exists. Mesh analysis is a method of finding all unknown mesh currents of a circuit, and it is based on Kirchhoff's voltage law (KVL). Like nodes, meshes can be labeled 1, 2, 3, . . . , or a, b, c, . . . ; and mesh currents can be labeled I_1, I_2, I_3, . . . , or I_a, I_b, I_c, ... If there is a current source along the path of a labeled mesh current that is not shared by other meshes, then that mesh current is known. For each mesh whose mesh current is unknown, we can write an equation by summing the voltage drops (or rises) around the mesh. The voltage drops on resistors can be found by applying Ohm's law on each branch that is part of the mesh.

The mesh currents are found by solving these equations. Once all the mesh currents are found, the current through each branch can be found by taking the difference of the two mesh currents. Once all the currents through branches are known, the voltages across all branches and nodes can be found. If there is a current source between two meshes, we can form a supermesh consisting of two meshes. One additional equation is obtained by relating the current from the current source to the two mesh currents.

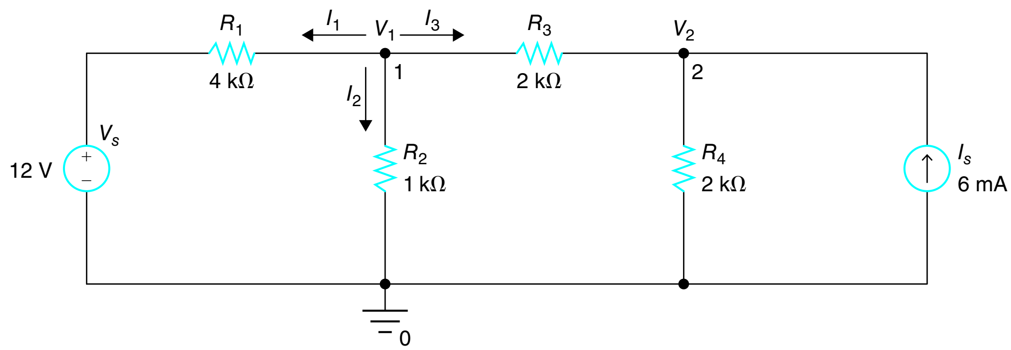
3.2 Nodal Analysis

If the voltages are known for all the nodes in a circuit, the voltage across each branch can be evaluated by subtracting the node voltage on one side of the branch from the node voltage on the other side. Once the voltage across the branch is found, the current through the branch can be evaluated by applying Ohm's law. Once the voltage and current are known for each branch, the power absorbed or supplied by the branch can be found by multiplying the voltage and the current. Thus, a circuit can be analyzed completely by finding the voltage on every node in the given circuit. The nodal analysis provides us voltages at all the nodes of the given circuit. If a voltage source is connected between a node and the ground, the node voltage is already known, and we do not need to find the node voltage on this node.

Excluding the nodes whose voltages are known from the voltage sources, we assign variables such as $V_1, V_2, \ldots, V_n$ to each unknown node voltage. For each node with an assigned variable, write a node equation by applying KCL. Any of the three interpretations of KCL given in Chapter 2 can be used. For example, the sum of the currents leaving a node must be zero. If there are n unknown node voltages, we get n equations in n unknowns. Thus, we can solve these n systems of linear equations with constant coefficients to find the unique solution for the unknown node voltages $V_1, V_2, \ldots, V_n$. If n is a small number, substitution method can be used to solve the system of n linear equations with constant coefficients. Cramer's rule can also be used to solve the system of n linear equations with constant coefficients. MATLAB is useful in finding the solution of an n system of linear equations with constant coefficients.

Labeling a circuit for nodal analysis requires identifying essential nodes where KCL equations can be written. The term *essential node* refers to a node where three or more elements are connected. As an example, consider the circuit shown in Figure 3.1.

FIGURE 3.1 A circuit with two unknown node voltages.



There are only three essential nodes in Figure 3.1. The node formed by resistors R_1, R_2, and R_3 is labeled node 1, and the node formed by resistors R_3, R_4, and the current source is labeled node 2. The node formed by the voltage source, resistors R_2 and R_3, and the current source is labeled node 0; this is the *ground node.* Recognize that with n essential nodes, there can be only $(n - 1)$ KCL equations, and one of the n essential nodes is selected as the ground node.

According to KCL, the sum of the currents leaving a node equals zero. Three branches are connected to node 1. The voltage across R_1 is $(V_1 - V_s)$. The current I_1 through R_1, from right to left, is given by $(V_1 - V_s)/R_1$ (Ohm's law). Similarly, the current I_2 through R_2, from top to bottom, is given by $(V_1 - 0)/R_2$, and the current I_3 through R_3, from left to right, is given by $(V_1 - V_2)/R_3$. The three currents I_1, I_2, and I_3 are shown in Figure 3.1. When these three currents leaving node 1 are added, it should be zero; that is,

$$I_1 + I_2 + I_3 = 0$$

or

$$\frac{V_1 - V_s}{R_1} + \frac{V_1 - 0}{R_2} + \frac{V_1 - V_2}{R_3} = 0 \qquad (3.1)$$

Substituting the values of V_s, R_1, R_2, and R_3, we obtain

$$\frac{V_1 - 12}{4000} + \frac{V_1 - 0}{1000} + \frac{V_1 - V_2}{2000} = 0$$

Multiplication of this equation by 4000 yields

$$V_1 - 12 + 4V_1 + 2V_1 - 2V_2 = 0$$

which can be simplified to

$$7V_1 - 2V_2 = 12 \qquad (3.2)$$

Instead of summing the currents leaving (away from, out of) the node, we can sum the currents entering (into) the node. The direction of current on each branch is arbitrary. If the actual current flows in the opposite direction, it will be negative.

Three branches are connected to node 2. In Figure 3.2, the current I_5 through R_3, from right to left, is given by $(V_2 - V_1)/R_3$ (Ohm's law). Notice that I_5 is identical to $-I_3$ in Figure 3.1. Similarly, the current I_4 through R_4, from top to bottom, is given by $(V_2 - 0)/R_4$, and the current I_6 is given by $I_6 = -I_s = -6$ mA. The current through the current source, from top to bottom, is given by $-I_s = -6$ mA. When these three currents leaving node 2 are added, it should be zero; that is,

$$I_5 + I_4 + I_6 = 0$$

or

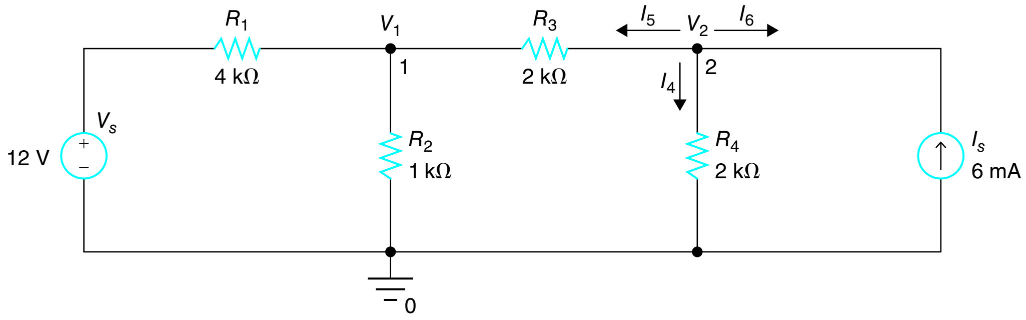
$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - 0}{R_4} - I_s = 0 \qquad (3.3)$$

Substituting the values of I_s, R_3, and R_4, we obtain

$$\frac{V_2 - V_1}{2000} + \frac{V_2 - 0}{2000} - 6 \times 10^{-3} = 0$$

FIGURE 3.2

Currents leaving node 2.



Multiplication of this equation by 2000 yields

$$V_2 - V_1 + V_2 = 12$$

which can be rewritten as

$$-V_1 + 2V_2 = 12 \tag{3.4}$$

The unknown node voltages are found by solving Equations (3.2) and (3.4) or Equations (3.1) and (3.3). Several methods can be used to solve a system of linear equations. For two or three unknowns, we can solve the equations by substitution method. If the number of unknowns is greater than or equal to 2, Cramer's rule can be used to find the solution. MATLAB can be used to find the solution to a system of linear equations. We start with the substitution method. From Equation (3.4), we have

$$V_1 = 2V_2 - 12 \tag{3.5}$$

Substitution of Equation (3.5) into Equation (3.2) yields

$$7(2V_2 - 12) - 2V_2 = 12$$

which can be simplified to

$$12V_2 = 96$$

Thus, we have $V_2 = 8$ V. Substituting this into Equation (3.5), we obtain

$$V_1 = 2V_2 - 12 = 16 - 12 = 4 \text{ V}$$

Equations (3.2) and (3.4) can be put into matrix form as

$$\begin{bmatrix} 7 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \end{bmatrix} \tag{3.6}$$

Let

$$A = \begin{bmatrix} 7 & -2 \\ -1 & 2 \end{bmatrix}, \quad V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad b = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$$

Then, Equation (3.6) becomes

$$AV = b \tag{3.7}$$

Using Cramer's rule on Equations (3.2) and (3.4) or Equation (3.6), we have

$$V_1 = \frac{\begin{vmatrix} 12 & -2 \\ 12 & 2 \end{vmatrix}}{\begin{vmatrix} 7 & -2 \\ -1 & 2 \end{vmatrix}} = \frac{12 \times 2 - (-2) \times 12}{7 \times 2 - (-2) \times (-1)} = \frac{24 + 24}{14 - 2} = \frac{48}{12} = 4 \text{ V}$$

Notice that the denominator is the determinant of matrix A. Let the determinant of A be Δ. Then, we have

$$\Delta = \begin{vmatrix} 7 & -2 \\ -1 & 2 \end{vmatrix} = 7 \times 2 - (-2) \times (-1) = 14 - 2 = 12$$

When the first column of matrix A is replaced by b, we obtain

$$A_1 = \begin{bmatrix} 12 & -2 \\ 12 & 2 \end{bmatrix}$$

The numerator is the determinant of A_1. Let the determinant of A_1 be Δ_1. Then, we have

$$\Delta_1 = \begin{vmatrix} 12 & -2 \\ 12 & 2 \end{vmatrix} = 12 \times 2 - (-2) \times (12) = 24 + 24 = 48$$

The voltage V_1 is the ratio of Δ_1 to Δ; that is,

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{48}{12} = 4 \text{ V}$$

The voltage V_2 is computed as

$$V_2 = \frac{\begin{vmatrix} 7 & 12 \\ -1 & 12 \end{vmatrix}}{\Delta} = \frac{7 \times 12 - 12 \times (-1)}{12} = \frac{84 + 12}{12} = \frac{96}{12} = 8 \text{ V}$$

Notice that the denominator is the determinant of matrix A (that is, Δ). When the second column of matrix A is replaced by b, we obtain

$$A_2 = \begin{bmatrix} 7 & 12 \\ -1 & 12 \end{bmatrix}$$

The numerator is the determinant of A_2. Let the determinant of A_2 be Δ_2. Then, we have

$$\Delta_2 = \begin{vmatrix} 7 & 12 \\ -1 & 12 \end{vmatrix} = 7 \times 12 - (12) \times (-1) = 84 + 12 = 96$$

The voltage V_2 is the ratio of Δ_2 to Δ; that is,

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{96}{12} = 8 \text{ V}$$

Equations (3.2) and (3.4) or Equation (3.6) can be solved using MATLAB, as shown here:

```
clear all;
A=[7 -2;-1 2];b=[12;12];
V=A\b

Answer:
V =
    4.0000
    8.0000
```

Equations (3.1) and (3.3) can be rearranged, respectively, as

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_1 - \frac{1}{R_3} V_2 = \frac{V_s}{R_1}$$

and

$$-\frac{1}{R_3}V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_2 = I_s$$

These equations can be put into matrix form as

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ I_s \end{bmatrix}$$

This equation can be solved using MATLAB, as shown here:

```
clear all;
Vs=12;Is=6e-3;R1=4000;R2=1000;R3=2000;R4=2000;
A=[1/R1+1/R2+1/R3,-1/R3;-1/R3,1/R3+1/R4]
b=[Vs/R1;Is]
V=A\b

Answer:
A =
    0.001750000000000   -0.000500000000000
   -0.000500000000000    0.001000000000000
b =
    0.003000000000000
    0.006000000000000
V =
       4
       8
```

This approach does not require numerical values for the elements of A and b to find V_1 and V_2.

One other method of solving Equations (3.1) and (3.3) is to use the MATLAB function **solve**, as shown here:

```
clear all;
syms V1 V2
Vs=12;Is=6e-3;R1=4000;R2=1000;R3=2000;R4=2000;
[V1,V2]=solve((V1-Vs)/R1+V1/R2+(V1-V2)/R3==0,(V2-V1)/R3+V2/R4-Is==0)
V1=vpa(V1,6)
V2=vpa(V2,6)

Answer:
V1 =
4
V2 =
8
V1 =
4.0
V2 =
8.0
```

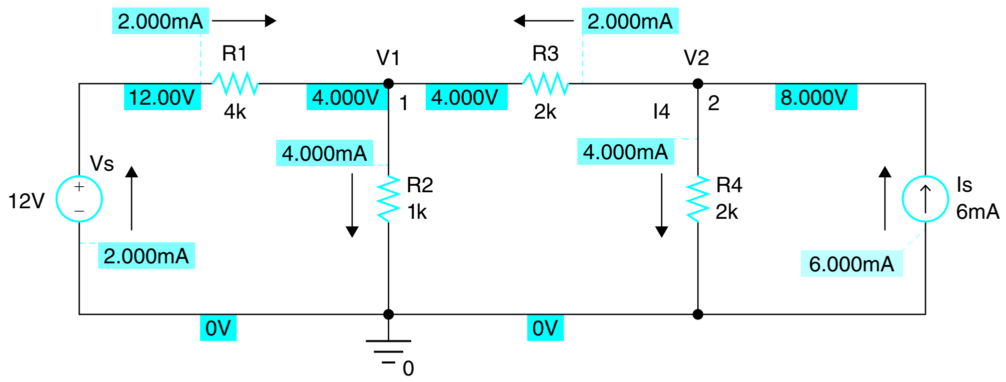
This method provides a solution by entering equations. The equations can be entered without ==0; that is, the line with **solve** can be replaced by

```
[V1,V2]=solve((V1-Vs)/R1+V1/R2+(V1-V2)/R3,(V2-V1)/R3+V2/R4-Is)
```

Once V_1 and V_2 are known, we can find the current through each element. The current I_1 through R_1, from right to left, is given by $I_1 = (V_1 - V_s)/R_1 = (4 - 12)/4 = -2$ mA. The current I_2 through R_2, from top to bottom, is $I_2 = V_1/R_2 = 4/1 = 4$ mA. The current I_3 through R_3, from left to right, is $I_3 = (V_1 - V_2)/R_3 = (4 - 8)/2 = -2$ mA. The current I_4 through R_4, from top to bottom, is $I_4 = V_2/R_4 = 8/2 = 4$ mA. The current I_5 through R_3, from right to left, is $I_5 = (V_2 - V_1)/R_3 = (8 - 4)/2 = 2$ mA. Notice that $I_5 = -I_3$. Figure 3.3 shows the circuit shown in Figure 3.1 with voltages and currents specified. This figure is obtained using PSpice®, a circuit simulation package. The direction of the current through each element is shown in Figure 3.3 by an arrow. In PSpice, the label of current is connected to the terminal of the part where current enters the part. For example, for R_1, label 2.000 mA is connected to the left side of R_1. This implies that current of 2 mA enters R_1 from the left and flows through R_1 from left to right.

FIGURE 3.3

Voltages and currents for the circuit shown in Figure 3.1.

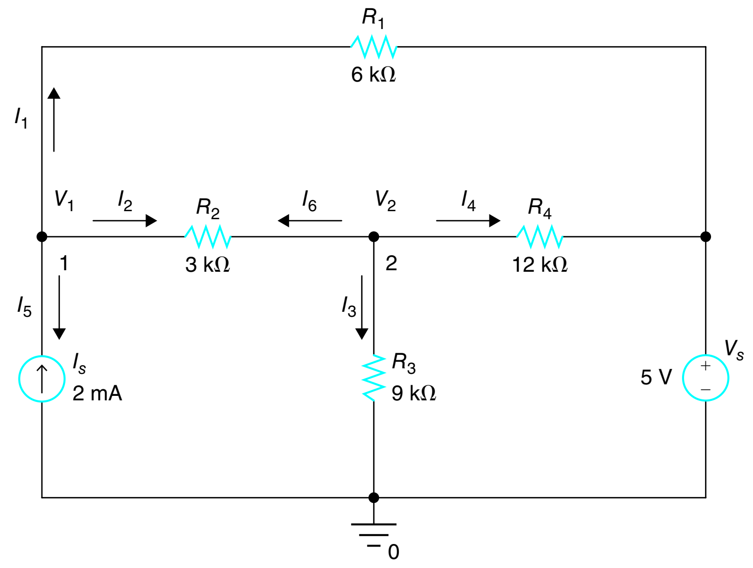


EXAMPLE 3.1

Find the node voltages V_1 and V_2 in the circuit shown in Figure 3.4.

FIGURE 3.4

The circuit for EXAMPLE 3.1.



continued

Example 3.1 continued

Two essential nodes where KCL equations can be written are labeled as node 1 and node 2 in Figure 3.4. According to KCL, the sum of the currents leaving node 1 equals zero; that is, $I_1 + I_2 + I_5 = 0$. The voltage across R_1 is $V_1 - V_s$. From Ohm's law, we have $I_1 = (V_1 - V_s)/R_1$. Similarly, the current I_2 through R_2 is given by $I_2 = (V_1 - V_2)/R_2$. The current I_5 is $-I_s$. Thus, the node equation $I_1 + I_2 + I_5 = 0$ becomes

$$\frac{V_1 - V_s}{R_1} + \frac{V_1 - V_2}{R_2} - I_s = 0$$

Substituting the values of V_s, I_s, R_1, and R_2, we obtain

$$\frac{V_1 - 5}{6000} + \frac{V_1 - V_2}{3000} - 2 \times 10^3 = 0$$

Multiplication by 6000 yields

$$V_1 - 5 + 2V_1 - 2V_2 - 12 = 0$$

which can be simplified to

$$3V_1 - 2V_2 = 17 \tag{3.8}$$

According to KCL, the sum of currents leaving node 2 equals zero; that is, $I_6 + I_4 + I_3 = 0$. The voltage across R_2 is $V_2 - V_1$. From Ohm's law, we have $I_6 = (V_2 - V_1)/R_2$. Notice that $I_6 = -I_2$. The voltage across R_4 is $V_2 - V_s$. From Ohm's law, we have $I_4 = (V_2 - V_s)/R_4$. The voltage across R_3 is V_2. From Ohm's law, we have $I_3 = V_2/R_3$. Thus, the node equation $I_6 + I_4 + I_3 = 0$ becomes

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - V_s}{R_4} + \frac{V_2}{R_3} = 0$$

Substituting the values of V_s, R_2, R_3, and R_4, we obtain

$$\frac{V_2 - V_1}{3000} + \frac{V_2 - 5}{12{,}000} + \frac{V_2}{9000} = 0$$

Multiplication by 36,000 yields

$$12V_2 - 12V_1 + 3V_2 - 15 + 4V_2 = 0$$

which can be simplified to

$$-12V_1 + 19V_2 = 15 \tag{3.9}$$

Equations (3.8) and (3.9) can be solved using the substitution method:

Solving Equation (3.8) for V_2, we obtain

$$V_2 = 1.5V_1 - 8.5$$

Substituting V_2 into Equation (3.9), we get

$$-12V_1 + 19(1.5V_1 - 8.5) = 15$$

continued

Example 3.1 continued

or

$$16.5V_1 = 176.5$$

Thus, we have

$$V_1 = 10.697 \text{ V}$$

and

$$V_2 = 1.5V_1 - 8.5 = 7.5455 \text{ V}$$

Alternatively, Equations (3.8) and (3.9) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 3 & -2 \\ -12 & 19 \end{vmatrix} = 3 \times 19 - (-2) \times (-12) = 57 - 24 = 33$$

$$V_1 = \frac{\begin{vmatrix} 17 & -2 \\ 15 & 19 \end{vmatrix}}{\Delta} = \frac{17 \times 19 - (-2) \times 15}{33} = \frac{353}{33} = 10.6970 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 3 & 17 \\ -12 & 15 \end{vmatrix}}{\Delta} = \frac{3 \times 15 - 17 \times (-12)}{33} = \frac{249}{33} = 7.5455 \text{ V}$$

MATLAB

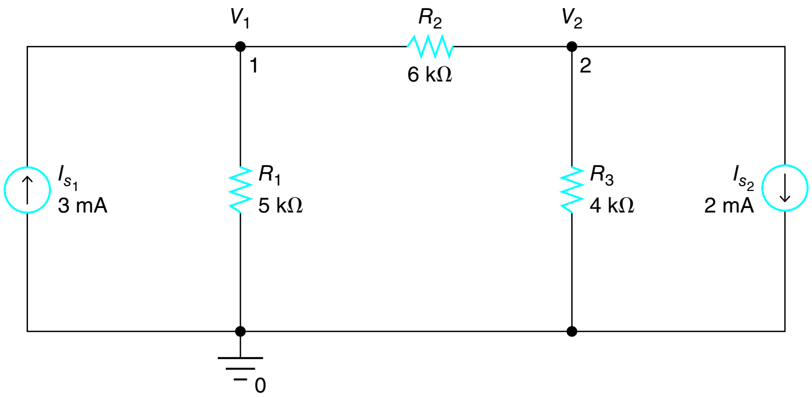
```
>> A=[3 -2;-12 19];b=[17;15];
>> V=A\b
V =
   10.6970
    7.5455
```

Exercise 3.1

Find V_1 and V_2 in the circuit shown in Figure 3.5.

FIGURE 3.5

The circuit for EXERCISE 3.1.



Answer:

$V_1 = 7.3333 \text{ V}, V_2 = -1.8667 \text{ V}.$

EXAMPLE 3.2

For the circuit shown in Figure 3.6, use nodal analysis to find the node voltages at nodes 1 and 2.

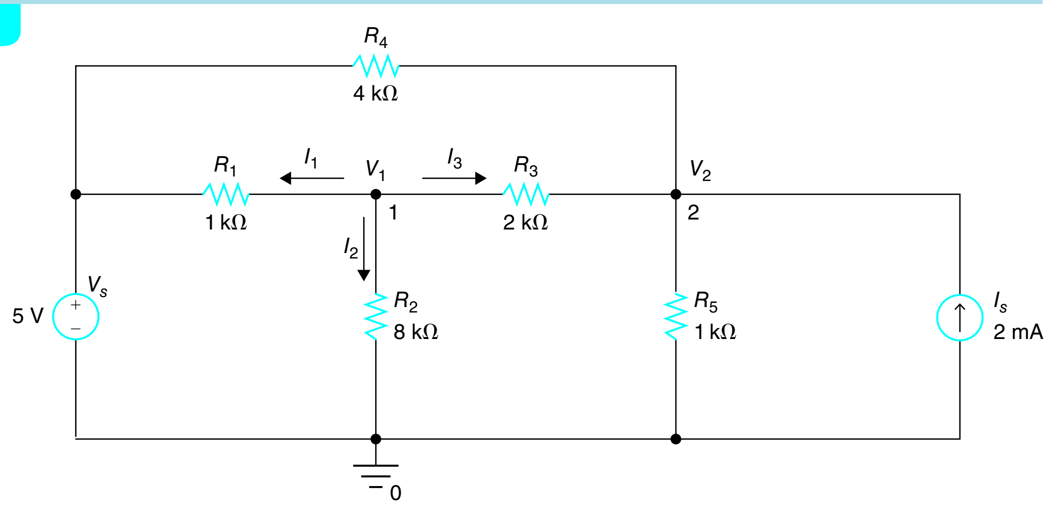


FIGURE 3.6

The circuit for EXAMPLE 3.2.

There are four nodes in the circuit shown in Figure 3.6. The voltage at the ground node (node 0) is zero. The voltage at the node connecting the positive terminal of the voltage source V_s to resistors R_1 and R_4 is 5 V. The voltage V_1 at the node (node 1) connecting R_1, R_2, and R_3 and the voltage V_2 at the node (node 2) connecting the current source I_s and resistors R_3, R_4, and R_5 are unknown. Application of KCL leads us to find these unknown voltages.

According to KCL, the sum of currents leaving a node equals zero. Three branches are connected to node 1. The current I_1 through R_1, from right to left, is given by $(V_1 - V_s)/R_1$ (Ohm's law). Similarly, the current I_2 through R_2, from top to bottom, is given by $(V_1 - 0)/R_2$, and the current I_3 through R_3, from left to right, is given by $(V_1 - V_2)/R_3$. The three currents I_1, I_2, and I_3 are shown in Figure 3.6. When these three currents leaving node 1 are added, it should be zero; that is,

$$I_1 + I_2 + I_3 = 0$$

or

$$\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = 0$$

Substituting the values of V_s, R_1, R_2, and R_3, we obtain

$$\frac{V_1 - 5}{1000} + \frac{V_1}{8000} + \frac{V_1 - V_2}{2000} = 0$$

Multiplication by 8000 yields

$$8V_1 - 40 + V_1 + 4V_1 - 4V_2 = 0$$

which can be simplified to

$$13V_1 - 4V_2 = 40 \tag{3.10}$$

continued

Example 3.2 continued

Four branches are connected to node 2. The current I_6 through R_3, from right to left, is given by $(V_2 - V_1)/R_3$ (Ohm's law). Notice that I_6 is identical to $-I_3$. Similarly, the current I_4 through R_4, from right to left, is given by $(V_2 - V_s)/R_4$, and the current I_5 through R_5, from top to bottom, is given by $(V_2 - 0)/R_5$. The current I_7 is in the opposite direction to I_s. Thus, we have $I_7 = -I_s = -2$ mA. The direction of currents I_4, I_5, I_6, and I_7 are shown in Figure 3.7. When these four currents leaving node 2 are added, it should be zero; that is,

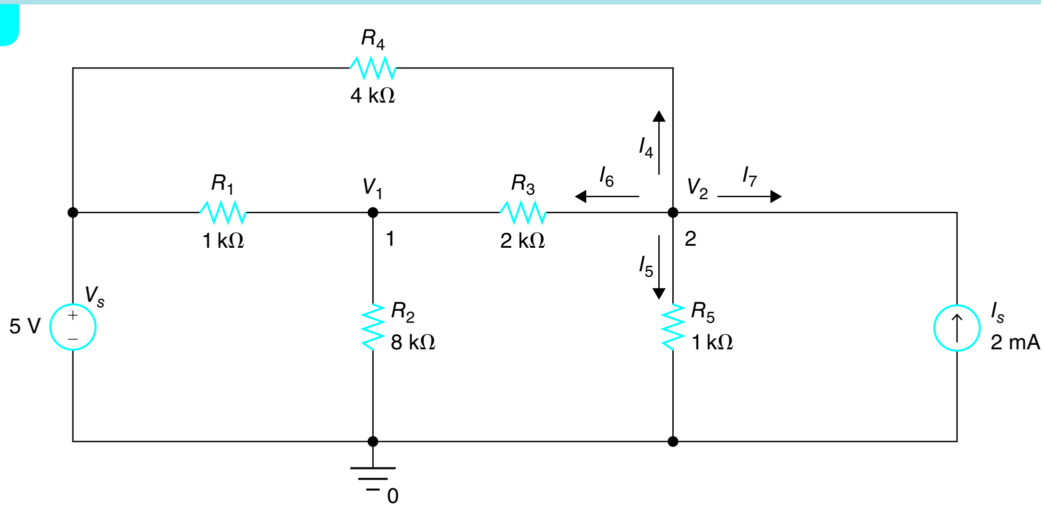
$$I_6 + I_4 + I_5 + I_7 = 0$$

or

$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - V_s}{R_4} + \frac{V_2}{R_5} - I_s = 0$$

FIGURE 3.7

Currents leaving node 2.



Substituting the values of V_s, I_s, R_3, R_4, and R_5, we obtain

$$\frac{V_2 - V_1}{2000} + \frac{V_2 - 5}{4000} + \frac{V_2}{1000} - 2 \times 10^{-3} = 0$$

Multiplication by 4000 yields

$$2V_2 - 2V_1 + V_2 - 5 + 4V_2 - 8 = 0$$

which can be simplified to

$$-2V_1 + 7V_2 = 13 \tag{3.11}$$

Equations (3.10) and (3.11) can be solved using the substitution method:

Solving Equation (3.11) for V_1, we obtain

$$V_1 = 3.5V_2 - 6.5$$

Substituting V_1 into Equation (3.10), we get

$$13(3.5V_2 - 6.5) - 4V_2 = 40$$

or

$$41.5V_2 = 124.5$$

continued

Example 3.2 continued

Thus, we have

$$V_2 = 3\text{ V}$$

and

$$V_1 = 3.5V_2 - 6.5 = 4\text{ V}$$

Alternatively, Equations (3.10) and (3.11) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 13 & -4 \\ -2 & 7 \end{vmatrix} = 13 \times 7 - (-4) \times (-2) = 83$$

$$V_1 = \frac{\begin{vmatrix} 40 & -4 \\ 13 & 7 \end{vmatrix}}{\Delta} = \frac{40 \times 7 - (-4) \times 13}{83} = \frac{332}{83} = 4\text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 13 & 40 \\ -2 & 13 \end{vmatrix}}{\Delta} = \frac{13 \times 13 - 40 \times (-2)}{83} = \frac{249}{83} = 3\text{ V}$$

MATLAB

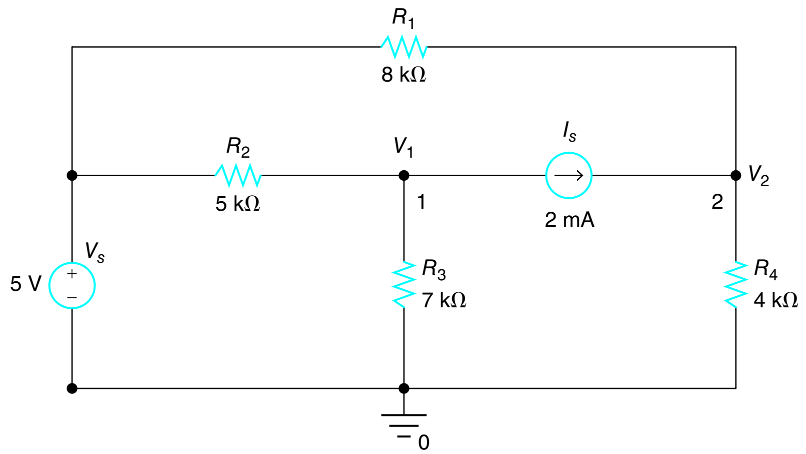
```
clear all;
A=[13 -4;-2 7];b=[40;13];
V=A\b
Answer:
V =
    4.0000
    3.0000
```

Exercise 3.2

Find V_1 and V_2 in the circuit shown in Figure 3.8.

FIGURE 3.8

The circuit for EXERCISE 3.2.



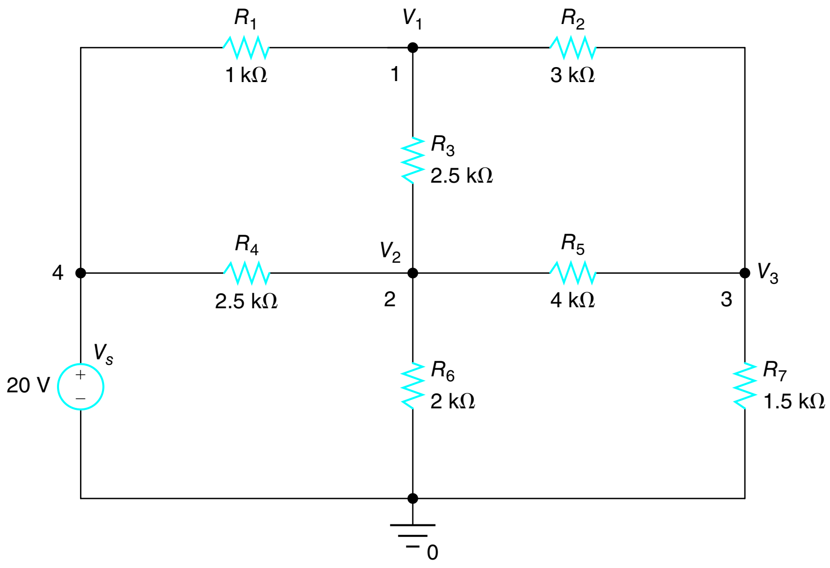
Answer:
$V_1 = -2.9167$ V, $V_2 = 7$ V.

EXAMPLE 3.3

Find V_1, V_2, and V_3 in the circuit shown in Figure 3.9.

FIGURE 3.9

The circuit for EXAMPLE 3.3.



There are five nodes (0, 1, 2, 3, 4) in the circuit shown in Figure 3.9. The voltage at node 0 is zero (reference or ground). The voltage at node 4 is $V_s = 20$ V. The voltages at nodes 1, 2, and 3 are unknown at this point. Let the voltage at node 1 be V_1; at node 2, it is V_2, and at node 3, it is V_3. Summing the currents away from node 1 using Ohm's law, we obtain

$$\frac{V_1 - V_s}{R_1} + \frac{V_1 - V_2}{R_3} + \frac{V_1 - V_3}{R_2} = 0 \tag{3.12}$$

Substituting the resistance and voltage values, we get

$$\frac{V_1 - 20}{1000} + \frac{V_1 - V_2}{2500} + \frac{V_1 - V_3}{3000} = 0$$

Multiplication by 15,000 results in

$$15(V_1 - 20) + 6(V_1 - V_2) + 5(V_1 - V_3) = 0$$

which can be simplified to

$$26V_1 - 6V_2 - 5V_3 = 300 \tag{3.13}$$

Summing the currents away from node 2, using Ohm's law, we obtain

$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - V_s}{R_4} + \frac{V_2}{R_6} + \frac{V_2 - V_3}{R_5} = 0 \tag{3.14}$$

continued

Example 3.3 continued

Substituting the resistance and voltage values, we get

$$\frac{V_2 - V_1}{2500} + \frac{V_2 - 20}{2500} + \frac{V_2}{2000} + \frac{V_2 - V_3}{4000} = 0$$

Multiplication by 20,000 results in

$$8(V_2 - V_1) + 8(V_2 - 20) + 10V_2 + 5(V_2 - V_3) = 0$$

which can be simplified to

$$-8V_1 + 31V_2 - 5V_3 = 160 \qquad (3.15)$$

Summing the currents away from node 3 using Ohm's law, we obtain

$$\frac{V_3 - V_1}{R_2} + \frac{V_3 - V_2}{R_5} + \frac{V_3}{R_7} = 0 \qquad (3.16)$$

Substituting the resistance values, we get

$$\frac{V_3 - V_1}{3000} + \frac{V_3 - V_2}{4000} + \frac{V_3}{1500} = 0$$

Multiplication by 12,000 results in

$$4(V_3 - V_1) + 3(V_3 - V_2) + 8V_3 = 0$$

which can be simplified to

$$-4V_1 - 3V_2 + 15V_3 = 0 \qquad (3.17)$$

Equations (3.13), (3.15), and (3.17) can be solved using the substitution method: Multiplying Equation (3.15) by 3, we obtain

$$-24V_1 + 93V_2 - 15V_3 = 480$$

Adding this equation and Equation (3.17), we get

$$-28V_1 + 90V_2 = 480$$

Solving for V_2, we obtain

$$V_2 = \frac{28}{90}V_1 + \frac{48}{9}$$

Subtracting Equation (3.15) from Equation (3.13), we get

$$34V_1 - 37V_2 = 140$$

Substituting $V_2 = \frac{28}{90}V_1 + \frac{48}{9}$, we obtain

$$34V_1 - 37\left(\frac{28}{90}V_1 + \frac{48}{9}\right) = 140$$

continued

Example 3.3 continued

Thus, we have

$$V_1 = \frac{140 + 37\dfrac{48}{9}}{34 - 37\dfrac{28}{90}} = 15 \text{ V}$$

$$V_2 = \frac{28}{90}V_1 + \frac{48}{9} = 10 \text{ V}$$

From Equation (3.17), we get

$$V_3 = \frac{4}{15}V_1 + \frac{3}{15}V_2 = 6 \text{ V}$$

Alternatively, Equations (3.13), (3.15), and (3.17) can be solved using Cramer's rule:

$$\begin{bmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 300 \\ 160 \\ 0 \end{bmatrix} \tag{3.18}$$

Let

$$A = \begin{bmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{bmatrix}, \; V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}, \; b = \begin{bmatrix} 300 \\ 160 \\ 0 \end{bmatrix}$$

Then, Equation (3.18) can be written as

$$AV = b.$$

Let Δ be the determinant of matrix A. Then, Δ can be written as

$$\Delta = |A| = \begin{vmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{vmatrix}$$

One method of computing the determinant of the 3×3 matrix A is to copy the first two columns and place them at the end as the fourth and fifth columns, and then multiply the numbers in the diagonals. The determinant is the difference of the sum of the products to the right and the sum of the products to the left:

$$\begin{bmatrix} 26 & -6 & -5 & 26 & -6 \\ -8 & 31 & -5 & -8 & 31 \\ -4 & -3 & 15 & -4 & -3 \end{bmatrix}$$

$$- \qquad\qquad\qquad +$$

continued

Example 3.3 continued

Thus, we have

$$\begin{aligned}\Delta &= [(26)\times(31)\times(15)+(-6)\times(-5)\times(-4)+(-5)\times(-8)\times(-3)]\\ &\quad -[(-5)\times(31)\times(-4)+(26)\times(-5)\times(-3)+(-6)\times(-8)\times(15)]\\ &= [12{,}090+(-120)+(-120)]-[620+390+720] = 11{,}850-1730 = 10{,}120\end{aligned}$$

In MATLAB, the determinant of A is found by det(A):

```
>> A=[26 -6 -5;-8 31 -5;-4 -3 15]
A =
    26    -6    -5
    -8    31    -5
    -4    -3    15
>> det(A)
ans =
       10,120
```

The determinant of an $n \times n$ matrix can also be found as a linear combination of determinants of matrices with dimension $(n-1)\times(n-1)$. Choose a row or a column of a matrix. For the 3×3 matrix A given previously, we select the first row. A minor M_{ij} is a determinant of a matrix with the ith row and jth column removed. For matrix A, we have

$$M_{11} = \begin{vmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{vmatrix} = \begin{vmatrix} 31 & -5 \\ -3 & 15 \end{vmatrix} = (31)\times(15)-(-5)\times(-3)$$
$$= 465 - 15 = 450$$

$$M_{12} = \begin{vmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{vmatrix} = \begin{vmatrix} -8 & -5 \\ -4 & 15 \end{vmatrix} = (-8)\times(15)-(-5)\times(-4)$$
$$= -120 - 20 = -140$$

$$M_{13} = \begin{vmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{vmatrix} = \begin{vmatrix} -8 & 31 \\ -4 & -3 \end{vmatrix} = (-8)\times(-3)-(31)\times(-4)$$
$$= 24 - (-124) = 148$$

A cofactor C_{ij} is the product of the minor M_{ij} and $(-1)^{i+j}$; that is,

$$C_{ij} = (-1)^{i+j} M_{ij}$$

If the sum of row index i and column index j is even, $(-1)^{i+j} = 1$, and if $i + j$ is odd, $(-1)^{i+j} = -1$. For the 3×3 matrix A given previously, the cofactors are given by

$$C_{11} = (-1)^{1+1} M_{11} = M_{11} = 450, \quad C_{12} = (-1)^{1+2} M_{12} = -M_{12} = 140,$$
$$C_{13} = (-1)^{1+3} M_{13} = M_{13} = 148$$

continued

Example 3.3 continued

The determinant of a matrix can be expanded as a sum of cofactors. The coefficients of this expansion are the elements of a chosen row or column. For the 3×3 matrix A given previously, with the first row chosen, we have

$$\begin{aligned}\Delta = |A| &= 26C_{11} - 6C_{12} - 5C_{13} = 26 \times 450 - 6 \times 140 - 5 \times 148\\ &= 11{,}700 - 840 - 740 = 10{,}120\end{aligned}$$

The voltages V_1, V_2, and V_3 are given, respectively, as

$$V_1 = \frac{\begin{vmatrix} 300 & -6 & -5 \\ 160 & 31 & -5 \\ 0 & -3 & 15 \end{vmatrix}}{\begin{vmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{vmatrix}}, \quad V_2 = \frac{\begin{vmatrix} 26 & 300 & -5 \\ -8 & 160 & -5 \\ -4 & 0 & 15 \end{vmatrix}}{\begin{vmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{vmatrix}},$$

$$V_3 = \frac{\begin{vmatrix} 26 & -6 & 300 \\ -8 & 31 & 160 \\ -4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 26 & -6 & -5 \\ -8 & 31 & -5 \\ -4 & -3 & 15 \end{vmatrix}}$$

The determinants in the numerators are given, respectively, by

$$\begin{bmatrix} 300 & -6 & -5 & 300 & -6 \\ 160 & 31 & -5 & 160 & 31 \\ 0 & -3 & 15 & 0 & -3 \end{bmatrix}$$

$$-\qquad\qquad\qquad +$$

$$\begin{aligned}\Delta_1 &= [(300) \times (31) \times (15) + (-6) \times (-5) \times (0) + (-5) \times (160) \times (-3)]\\ &\quad - [(-5) \times (31) \times (0) + (300) \times (-5) \times (-3) + (-6) \times (160) \times (15)]\\ &= [139{,}500 + 0 + 2400] - [0 + 4500 - 14{,}400] = 141{,}900 + 9900 = 151{,}800\end{aligned}$$

$$\begin{bmatrix} 26 & 300 & -5 & 26 & 300 \\ -8 & 160 & -5 & -8 & 160 \\ -4 & 0 & 15 & -4 & 0 \end{bmatrix}$$

$$-\qquad\qquad\qquad +$$

$$\begin{aligned}\Delta_2 &= [(26) \times (160) \times (15) + (300) \times (-5) \times (-4) + (-5) \times (-8) \times (0)]\\ &\quad - [(-5) \times (160) \times (-4) + (26) \times (-5) \times (0) + (300) \times (-8) \times (15)]\\ &= [62{,}400 + 6000 + 0] - [3{,}200 + 0 - 36{,}000] = 68{,}400 + 32{,}800 = 101{,}200\end{aligned}$$

$$\begin{bmatrix} 26 & -6 & 300 & 26 & -6 \\ -8 & 31 & 160 & -8 & 31 \\ -4 & -3 & 0 & -4 & -3 \end{bmatrix}$$

$$-\qquad\qquad\qquad +$$

continued

Example 3.3 continued

$$\begin{aligned}\Delta_3 &= [(26)\times(31)\times(0)+(-6)\times(160)\times(-4)+(300)\times(-8)\times(-3)]\\ &\quad -[(300)\times(31)\times(-4)+(26)\times(160)\times(-3)+(-6)\times(-8)\times(0)]\\ &= [0+3{,}840+7{,}200]-[-37{,}200-12{,}480+0]=11{,}040+49{,}680=60{,}720\end{aligned}$$

The voltages V_1, V_2, and V_3 are given, respectively, as

$$V_1=\frac{\Delta_1}{\Delta}=\frac{151{,}800}{10{,}120}=15\text{ V},\quad V_2=\frac{\Delta_2}{\Delta}=\frac{101{,}200}{10{,}120}=10\text{ V},$$

$$V_3=\frac{\Delta_3}{\Delta}=\frac{60{,}720}{10{,}120}=6\text{ V}$$

MATLAB

```
A=[26 -6 -5;-8 31 -5;-4 -3 15];b=[300;160;0];
V=A\b

Answer:
V =
    15
    10
     6
```

The MATLAB function **solve** can be used to solve Equations (3.12), (3.14), and (3.16). The ellipsis (...) is used to continue long statements in multiple lines:

```
%EXAMPLE 3.3
clear all;
Vs=20;R1=1000;R2=3000;R3=2500;R4=2500;R5=4000;R6=2000;R7=1500;
syms V1 V2 V3
[V1,V2,V3]=solve((V1-Vs)/R1+(V1-V2)/R3+(V1-V3)/R2,...
(V2-V1)/R3+(V2-Vs)/R4+V2/R6+(V2-V3)/R5,...
(V3-V1)/R2+(V3-V2)/R5+V3/R7,V1,V2,V3);
V1=vpa(V1,6)
V2=vpa(V2,6)
V3=vpa(V3,6)

Answers:
V1 =
15.0
V2 =
10.0
V3 =
6.0
```

Exercise 3.3

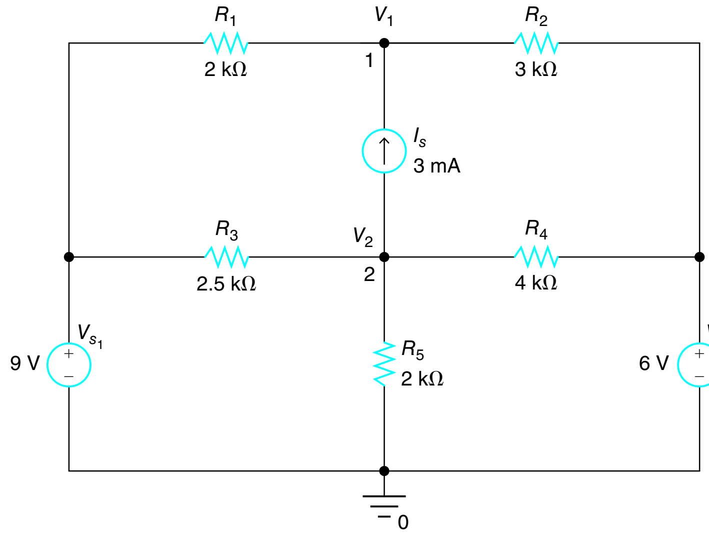
Find V_1 and V_2 in the circuit shown in Figure 3.10.

continued

Exercise 3.3 continued

FIGURE 3.10

The circuit for EXERCISE 3.3.



Answer:

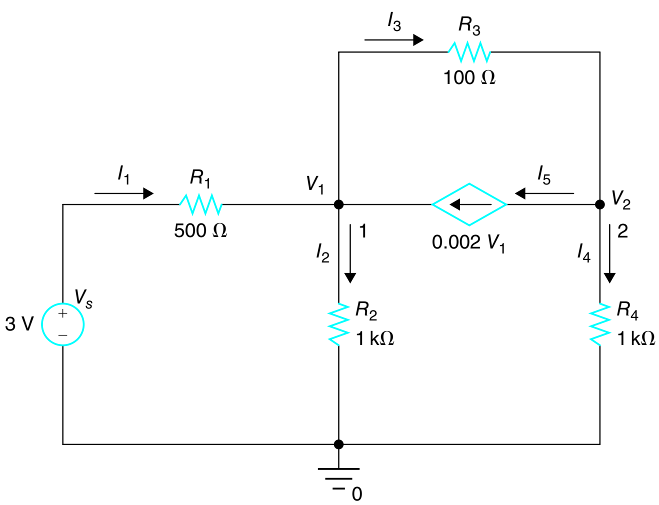
$V_1 = 11.4$ V, $V_2 = 1.8261$ V.

EXAMPLE 3.4

Find V_1 and V_2 in the circuit shown in Figure 3.11. Find currents I_1, I_2, I_3, and I_4, and the current through the voltage-controlled current source.

FIGURE 3.11

The circuit for EXAMPLE 3.4.



Summing the currents leaving node 1, we have

$$\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} - 0.002V_1 = 0$$

Substituting the resistance and voltage values, we get

$$\frac{V_1 - 3}{500} + \frac{V_1}{1000} + \frac{V_1 - V_2}{100} - 0.002V_1 = 0$$

Multiplication by 1000 yields

$$2V_1 - 6 + V_1 + 10V_1 - 10V_2 - 2V_1 = 0$$

which can be simplified to

$$11V_1 - 10V_2 = 6 \tag{3.19}$$

Summing the currents leaving node 2, we have

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_4} + 0.002V_1 = 0$$

continued

Example 3.4 continued

Substituting the resistance values, we obtain

$$\frac{V_2 - V_1}{100} + \frac{V_2}{1000} + 0.002V_1 = 0$$

Multiplication by 1000 yields

$$10V_2 - 10V_1 + V_2 + 2V_1 = 0$$

which can be simplified to

$$-8V_1 + 11V_2 = 0 \tag{3.20}$$

Equations (3.19) and (3.20) can be solved using the substitution method:

Solving Equation (3.19) for V_2, we obtain

$$V_2 = 1.1V_1 - 0.6$$

Substituting V_2 into Equation (3.20), we get

$$-8V_1 + 11(1.1V_1 - 0.6) = 0$$

or

$$4.1V_1 = 6.6$$

Thus, we obtain

$$V_1 = \frac{66}{41} = 1.6098 \text{ V}$$

$$V_2 = 1.1V_1 - 0.6 = \frac{48}{41} = 1.1707 \text{ V}$$

Alternatively, Equations (3.19) and (3.20) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 11 & -10 \\ -8 & 11 \end{vmatrix} = 11 \times 11 - (-10) \times (-8) = 121 - 80 = 41$$

$$V_1 = \frac{\begin{vmatrix} 6 & -10 \\ 0 & 11 \end{vmatrix}}{\Delta} = \frac{6 \times 11 - (-10) \times 0}{41} = \frac{66}{41} = 1.6098 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 11 & 6 \\ -8 & 0 \end{vmatrix}}{\Delta} = \frac{11 \times 0 - 6 \times (-8)}{41} = \frac{48}{41} = 1.1707 \text{ V}$$

The currents are given as follows:

$$I_1 = \frac{V_s - V_1}{R_1} = 2.7805 \text{ mA}, \quad I_2 = \frac{V_1}{R_2} = 1.6098 \text{ mA}, \quad I_3 = \frac{V_1 - V_2}{R_3} = 4.3902 \text{ mA},$$

continued

Example 3.4 continued

$$I_4 = \frac{V_2}{R_4} = 1.1707 \text{ mA}, \quad I_5 = 0.002V_1 = 3.2195 \text{ mA}$$

Notice that the sum of currents leaving node 1 and node 2 are zero; that is,

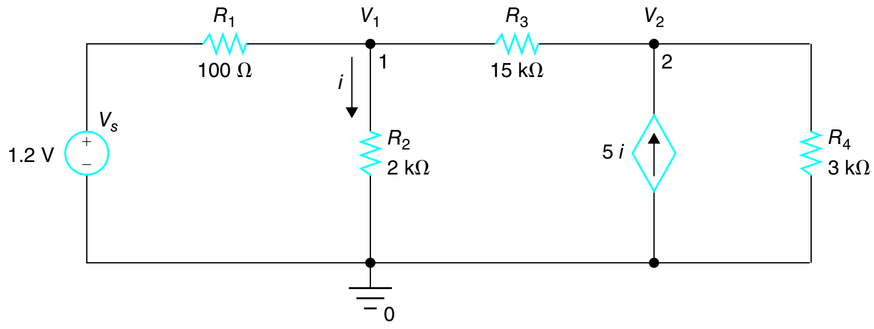
$$-I_1 + I_2 + I_3 - I_5 = 0, \quad -I_3 + I_4 + I_5 = 0$$

Exercise 3.4

Find V_1 and V_2 in the circuit shown in Figure 3.12.

FIGURE 3.12

The circuit for EXERCISE 3.4.



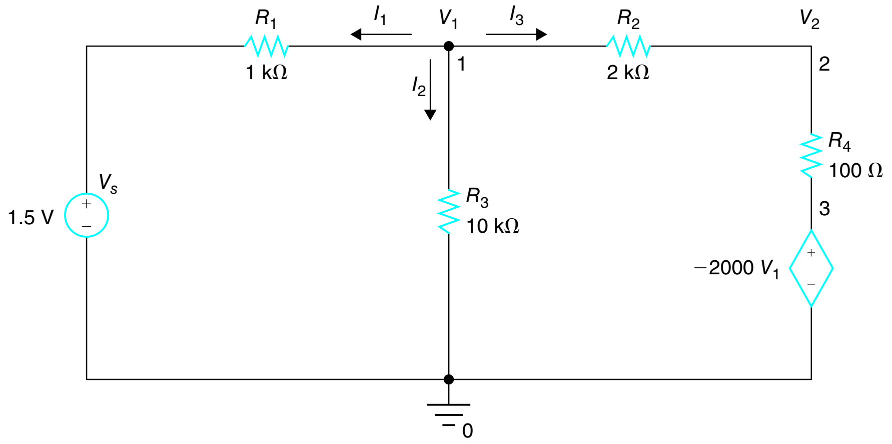
Answer:
$V_1 = 1.1836$ V, $V_2 = 7.5945$ V.

EXAMPLE 3.5

Find V_1 and V_2 in the circuit shown in Figure 3.13.

FIGURE 3.13

The circuit for EXAMPLE 3.5.



This circuit has only one node-voltage, labeled node 1. Summing the currents leaving the node gives

$$I_1 + I_2 + I_3 = 0$$

continued

Example 3.5 continued

or

$$\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_3} + \frac{V_1 - (-2000V_1)}{R_2 + R_4} = 0$$

Substituting the resistance and voltage values, we get

$$\frac{V_1 - 1.5}{1000} + \frac{V_1}{10{,}000} + \frac{V_1 - (-2000V_1)}{2100} = 0 \tag{3.21}$$

Equation (3.21) can be rearranged as

$$\left(\frac{1}{1000} + \frac{1}{10{,}000} + \frac{2001}{2100}\right) V_1 = \frac{1.5}{1000} \tag{3.22}$$

Solving Equation (3.22) for V_1, we obtain

$$V_1 = \frac{\frac{1.5}{1000}}{\frac{1}{1000} + \frac{1}{10{,}000} + \frac{2001}{2100}} = 0.001572397681837 \text{ V} = 1.5724 \text{ mV}$$

The current I_3 is given by

$$I_3 = \frac{V_1 - (-2000V_1)}{R_2 + R_4} = \frac{2001V_1}{2100} = 0.00149827036255 \text{ A}$$

The voltage V_2 is given by

$$\begin{aligned} V_2 &= V_1 - R_2 I_3 \\ &= 1.572397681837 \times 10^{-3} - 2000 \times 0.00149827036255 = -2.994968 \text{ V} \end{aligned}$$

MATLAB

```
%EXAMPLE 3.5
clear all;format long;
Vs=1.5;R1=1000;R2=2000;R3=10000;R4=100;
syms V1 V2
[V1,V2]=solve((V1-Vs)/R1+V1/R3+(V1-V2)/R2, ...
(V2-V1)/R2+(V2+2000*V1)/R4,V1,V2);
I3=(V1-(-2000*V1))/(R2+R4);
V2b=V1-R2*I3;
V1=vpa(V1,15)
V2=vpa(V2,15)
I3=vpa(I3,15)
V2b=vpa(V2,15)

Answers:
V1 =
0.00157239768183656
V2 =
-2.99496832741812
```

continued

Example 3.5 continued

MATLAB continued

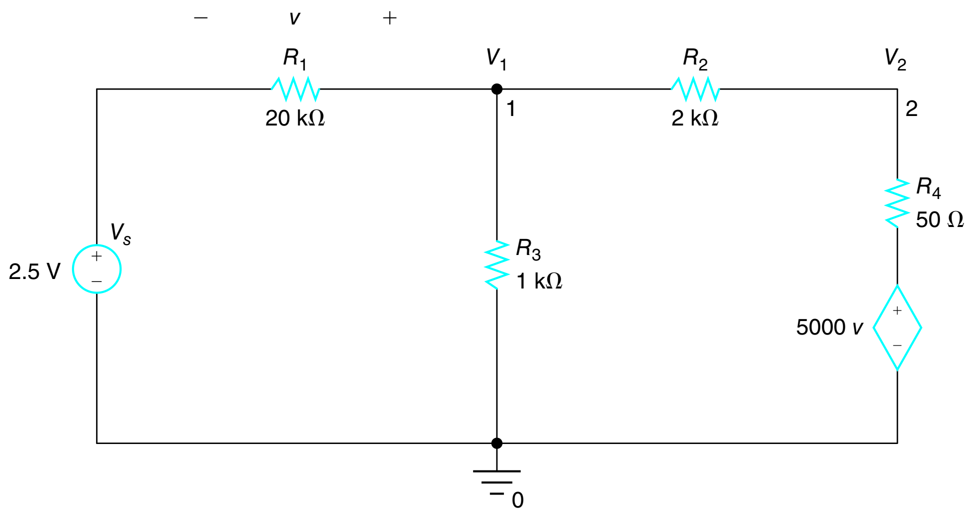
```
I3 =
0.00149827036254998
V2b =
-2.99496832741812
```

Exercise 3.5

Find V_1 and V_2 in the circuit shown in Figure 3.14.

FIGURE 3.14

The circuit for EXERCISE 3.5.



Answer:

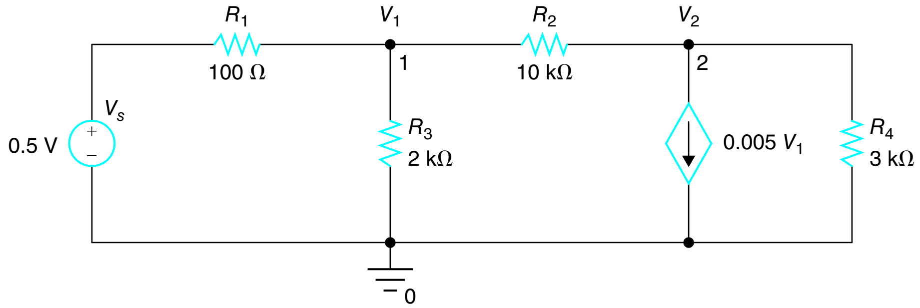
$V_1 = 2.501526$ V, $V_2 = 7.504730$ V.

EXAMPLE 3.6

Find V_1 and V_2 in the circuit shown in Figure 3.15.

FIGURE 3.15

The circuit for EXAMPLE 3.6.



continued

Example 3.6 continued

Summing the currents leaving node 1, we obtain

$$\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_3} + \frac{V_1 - V_2}{R_2} = 0$$

Substituting the resistance and voltage values, we get

$$\frac{V_1 - 0.5}{100} + \frac{V_1}{2000} + \frac{V_1 - V_2}{10{,}000} = 0$$

Multiplication by 10,000 yields

$$100(V_1 - 0.5) + 5V_1 + V_1 - V_2 = 0$$

which can be simplified to

$$106V_1 - V_2 = 50 \tag{3.23}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_2 - V_1}{R_2} + 0.005V_1 + \frac{V_2}{R_4} = 0$$

Substituting the resistance values, we get

$$\frac{V_2 - V_1}{10{,}000} + 0.005V_1 + \frac{V_2}{3000} = 0$$

Multiplication by 30,000 yields

$$3V_2 - 3V_1 + 150V_1 + 10V_2 = 0$$

which can be simplified to

$$147V_1 + 13V_2 = 0 \tag{3.24}$$

Equations (3.23) and (3.24) can be solved using the substitution method: Solving Equation (3.23) for V_2, we obtain

$$V_2 = 106V_1 - 50$$

Substituting V_2 into Equation (3.24), we get

$$147V_1 + 13(106V_1 - 50) = 0$$

or

$$1525V_1 = 650$$

Thus, we obtain

$$V_1 = \frac{650}{1525} = \frac{26}{61} = 0.42622951 \text{ V}$$

$$V_2 = 106V_1 - 50 = -4.8197 \text{ V}$$

continued

Example 3.6 continued

Alternatively, Equations (3.23) and (3.24) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 106 & -1 \\ 147 & 13 \end{vmatrix} = 106 \times 13 - (-1) \times 147 = 1378 - (-147) = 1525$$

$$V_1 = \frac{\begin{vmatrix} 50 & -1 \\ 0 & 13 \end{vmatrix}}{\Delta} = \frac{50 \times 13 - (-1) \times 0}{1525} = \frac{650}{1525} = 0.4262 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 106 & 50 \\ 147 & 0 \end{vmatrix}}{\Delta} = \frac{106 \times 0 - 50 \times 147}{1525} = \frac{-7350}{1525} = -4.8197 \text{ V}$$

MATLAB

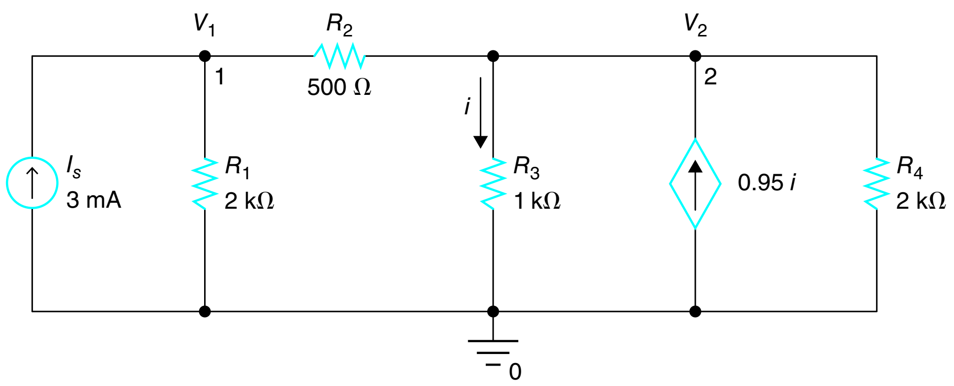
```
%EXAMPLE 3.6
clear all;format long;
Vs=0.5;R1=100;R2=10000;R3=2000;R4=3000;
syms V1 V2
[V1,V2]=solve((V1-Vs)/R1+V1/R3+(V1-V2)/R2, ...
(V2-V1)/R2+0.005*V1+V2/R4,V1,V2);
V1=vpa(V1,10)
V2=vpa(V2,10)

Answers:
V1 =
0.4262295082
V2 =
-4.819672131
```

Exercise 3.6

Find V_1 and V_2 in the circuit shown in Figure 3.16.

FIGURE 3.16

The circuit for EXERCISE 3.6.



Answer:

$V_1 = 3.2211$ V, $V_2 = 2.5263$ V.

3.3 Supernode

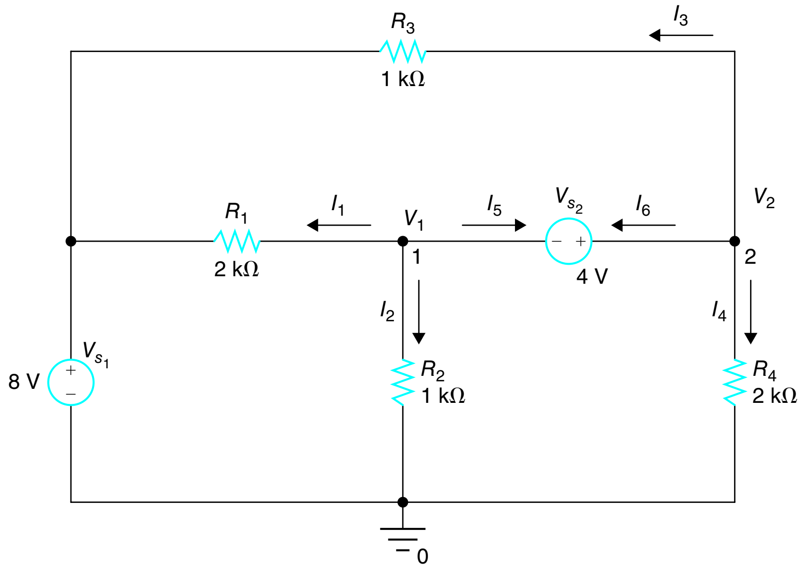
If there is a voltage source in a circuit between two nodes whose voltages are unknown, we do not know the current through the voltage source, and it is not possible to write the node equations for the two nodes that include the voltage source. In this case, combine the two nodes to form a **supernode**. Then, we can write the node equation for this supernode. One additional equation, commonly referred to as a *constraint equation* relating the two node voltages, can be obtained by representing the voltage source as a potential drop or as a potential rise between the two nodes.

As an example, consider the circuit shown in Figure 3.17. Let I_5 be the current through the voltage source V_{s_2} from the negative terminal to the positive terminal (→), and let I_6 be the current through the voltage source V_{s_2} from the positive terminal to the negative terminal (←). Then, $I_6 = -I_5$. Application of KCL to node 1 yields

$$I_1 + I_2 + I_5 = 0 \tag{3.25}$$

FIGURE 3.17

A circuit with a supernode.



Similarly, application of KCL to node 2 yields

$$I_3 + I_4 + I_6 = 0 \tag{3.26}$$

When an ideal voltage source is present between two essential nodes, it is impossible to determine the current through the source because the internal resistance of an ideal voltage source is zero. We cannot apply Ohm's law. However, to proceed with applying KCL, we arbitrarily define a current out of the node. By writing both equations and summing, the arbitrary current vanishes, giving rise to an equation that represents the sum of all currents flowing out of both nodes 1 and 2. Adding Equations (3.25) and (3.26), we obtain

$$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 = 0 \tag{3.27}$$

Since $I_6 = -I_5$, Equation (3.27) becomes

$$I_1 + I_2 + I_3 + I_4 = 0 \tag{3.28}$$

Equation (3.28) suggests that, ignoring the currents I_5 and I_6, the sum of currents leaving nodes 1 and 2 is zero. Nodes 1 and 2 can be treated as a supernode. The supernode for the circuit shown in Figure 3.17 is shown in Figure 3.18. Notice that Equation (3.28) is the sum of currents leaving the supernode. Applying Ohm's law, we can rewrite Equation (3.28) as

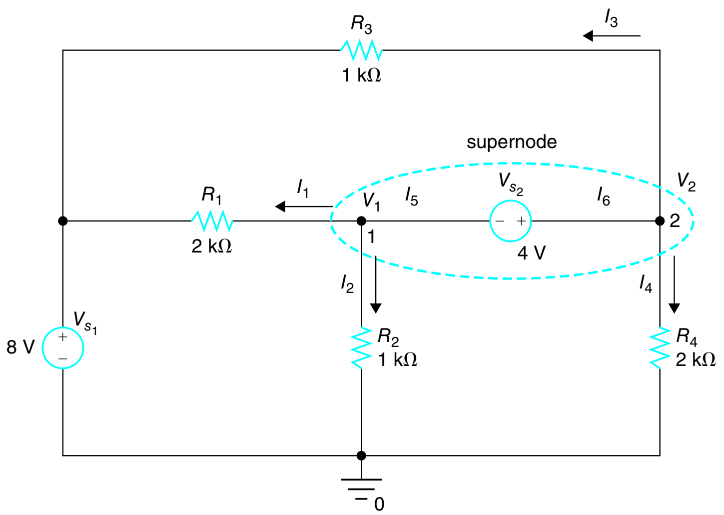
$$\frac{V_1 - V_{s_1}}{R_1} + \frac{V_1}{R_2} + \frac{V_2 - V_{s_1}}{R_3} + \frac{V_2}{R_4} = 0$$

Substituting the resistance and voltage values, we get

$$\frac{V_1 - 8}{2000} + \frac{V_1}{1000} + \frac{V_2 - 8}{1000} + \frac{V_2}{2000} = 0$$

FIGURE 3.18

The supernode includes node 1 and node 2.



Multiplication by 2000 yields

$$V_1 - 8 + 2V_1 + 2V_2 - 16 + V_2 = 0$$

which can be simplified to $3V_1 + 3V_2 = 24$. Thus, we have

$$V_1 + V_2 = 8 \tag{3.29}$$

The voltage at node 2, V_2, is higher than the voltage at node 1, V_1, by $V_{s_2} = 4$ V; that is,

$$V_2 - V_1 = 4$$

or

$$V_1 - V_2 = -4 \tag{3.30}$$

Summing Equations (3.29) and (3.30) yields

$$2V_1 = 4$$

Thus, we have $V_1 = 2$ V. Since V_2 is 4 V higher than V_1, we have $V_2 = 6$ V.

EXAMPLE 3.7

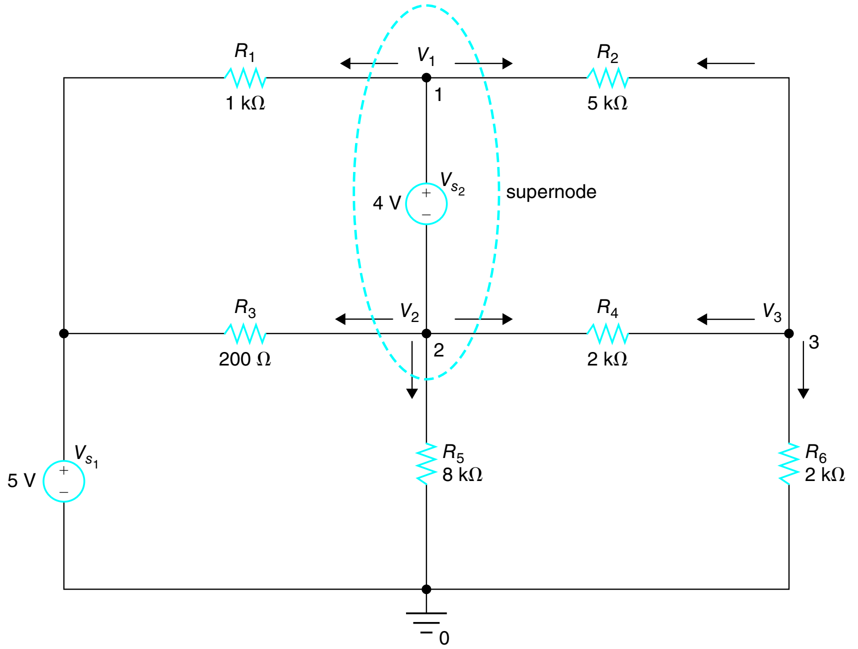
Find V_1, V_2, V_3 in the circuit shown in Figure 3.19.

continued

Example 3.7 continued

FIGURE 3.19

The circuit for EXAMPLE 3.7.



Notice the presence of an ideal voltage source connected between nodes 1 and 2. This makes nodes 1 and 2 part of a supernode. A constraint equation describing this source as a potential drop from node 1 to node 2 is

$$V_1 - V_2 = 4 \tag{3.31}$$

Since the current from node 1 to node 2 is the negative of the current from node 2 to node 1, they cancel each other out. Summing the currents leaving supernode, we obtain

$$\frac{V_1 - V_{s_1}}{R_1} + \frac{V_1 - V_3}{R_2} + \frac{V_2 - V_{s_1}}{R_3} + \frac{V_2}{R_5} + \frac{V_2 - V_3}{R_4} = 0$$

Substituting the resistance and voltage values, we get

$$\frac{V_1 - 5}{1000} + \frac{V_1 - V_3}{5000} + \frac{V_2 - 5}{200} + \frac{V_2}{8000} + \frac{V_2 - V_3}{2000} = 0$$

Multiplication by 40,000 results in

$$40(V_1 - 5) + 8(V_1 - V_3) + 200(V_2 - 5) + 5V_2 + 20(V_2 - V_3) = 0$$

which can be simplified to

$$48V_1 + 225V_2 - 28V_3 = 1200 \tag{3.32}$$

Substitution of Equation (3.31) into Equation (3.32) yields

$$273V_2 - 28V_3 = 1008 \tag{3.33}$$

Summing the currents leaving node 3, we obtain

$$\frac{V_3 - V_1}{R_2} + \frac{V_3 - V_2}{R_4} + \frac{V_3}{R_6} = 0$$

continued

Example 3.7 continued

Substituting the resistance values, we get

$$\frac{V_3 - V_1}{5000} + \frac{V_3 - V_2}{2000} + \frac{V_3}{2000} = 0$$

Multiplication by 10,000 results in

$$2(V_3 - V_1) + 5(V_3 - V_2) + 5V_3 = 0$$

which can be simplified to

$$-2V_1 - 5V_2 + 12V_3 = 0 \tag{3.34}$$

Substitution of Equation (3.31) into Equation (3.34) yields

$$-7V_2 + 12V_3 = 8 \tag{3.35}$$

Equations (3.33) and (3.35) can be solved using the substitution method: Solving Equation (3.35) for V_2, we obtain

$$V_2 = \frac{12}{7}V_3 - \frac{8}{7}$$

Substituting V_2 into Equation (3.33), we get

$$273\left(\frac{12}{7}V_3 - \frac{8}{7}\right) - 28V_3 = 1008$$

or

$$440V_3 = 1320$$

Thus, we obtain

$$V_3 = \frac{132}{44} = \frac{12}{4} = 3\text{ V}$$

$$V_2 = \frac{12}{7}V_3 - \frac{8}{7} = 4\text{ V}$$

$$V_1 = V_2 + 4 = 4 + 4 = 8\text{ V}$$

Alternatively, Equations (3.33) and (3.35) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 273 & -28 \\ -7 & 12 \end{vmatrix} = 273 \times 12 - (-28) \times (-7) = 3276 - 196 = 3080$$

$$V_2 = \frac{\begin{vmatrix} 1008 & -28 \\ 8 & 12 \end{vmatrix}}{\Delta} = \frac{1008 \times 12 - (-28) \times 8}{3080} = \frac{12{,}320}{3080} = 4\text{ V}$$

$$V_3 = \frac{\begin{vmatrix} 273 & 1008 \\ -7 & 8 \end{vmatrix}}{\Delta} = \frac{273 \times 8 - 1008 \times (-7)}{3080} = \frac{9240}{3080} = 3\text{ V}$$

continued

Example 3.7 continued

From Equation (3.31), we have

$$V_1 = V_2 + 4 = 4 + 4 = 8\text{ V}$$

MATLAB

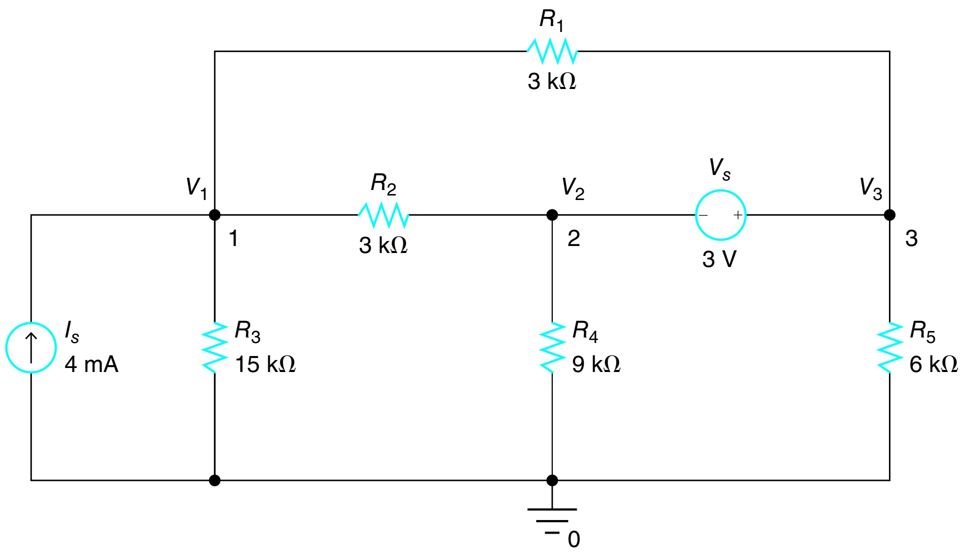
```
%EXAMPLE 3.7
clear all;format long;
Vs1=5;Vs2=4;
R1=1000;R2=5000;R3=200;R4=2000;R5=8000;R6=2000;
syms V1 V2 V3
[V1,V2,V3]=solve(V1-V2==Vs2, ...
(V1-Vs1)/R1+(V1-V3)/R2+(V2-Vs1)/R3+V2/R5+(V2-V3)/R4, ...
(V3-V1)/R2+(V3-V2)/R4+V3/R6,V1,V2,V3);
V1=vpa(V1,15)
V2=vpa(V2,15)
V3=vpa(V3,15)

Answers:
V1 =
8.0
V2 =
4.0
V3 =
3.0
```

Exercise 3.7

Find V_1, V_2, and V_3 in the circuit shown in Figure 3.20.

FIGURE 3.20

The circuit for EXERCISE 3.7.



Answer:

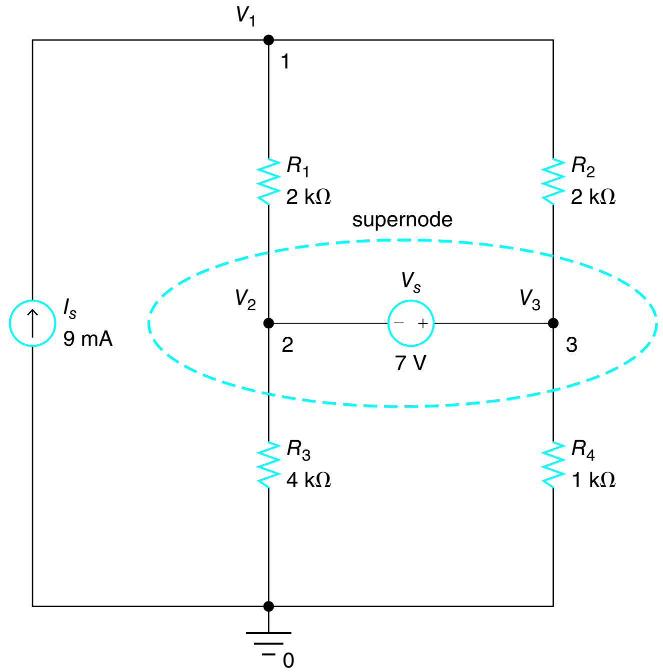
$V_1 = 15\text{ V}$, $V_2 = 9\text{ V}$, $V_3 = 12\text{ V}$.

EXAMPLE 3.8

Find V_1, V_2, and V_3 in the circuit shown in Figure 3.21.

FIGURE 3.21

The circuit for EXAMPLE 3.8.



The source voltage V_s between nodes 2 and 3 represents either a potential rise from node 2 to node 3 or a potential drop from node 3 to node 2. It is good practice to write out equations with the variables listed in the same order. This makes it easy for collecting coefficients to form a set of matrix-vector equations and eliminates the possibilities for errors in algebraic signs. Considering this effect as a potential rise gives the constraint equation

$$V_2 - V_3 = -7 \tag{3.36}$$

Summing the currents leaving the supernode, circled in Figure 3.21, we obtain

$$\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_3} + \frac{V_3 - V_1}{R_2} + \frac{V_3}{R_4} = 0$$

Substituting the resistance values, we get

$$\frac{V_2 - V_1}{2000} + \frac{V_2}{4000} + \frac{V_3 - V_1}{2000} + \frac{V_3}{1000} = 0$$

Multiplication by 4000 yields

$$2(V_2 - V_1) + V_2 + 2(V_3 - V_1) + 4V_3 = 0$$

continued

Example 3.8 continued

which can be simplified to

$$-4V_1 + 3V_2 + 6V_3 = 0 \tag{3.37}$$

Substitution of Equation (3.36) into Equation (3.37) yields

$$-4V_1 + 9V_2 = -42 \tag{3.38}$$

Summing the currents leaving node 1, we have

$$-I_s + \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_2} = 0$$

Substituting the resistance and current values, we get

$$-9 \times 10^{-3} + \frac{V_1 - V_2}{2000} + \frac{V_1 - V_3}{2000} = 0$$

Multiplication by 2000 yields

$$-18 + V_1 - V_2 + V_1 - V_3 = 0$$

which can be simplified to

$$2V_1 - V_2 - V_3 = 18 \tag{3.39}$$

Substitution of Equation (3.36) into Equation (3.39) yields

$$2V_1 - 2V_2 = 25 \tag{3.40}$$

Equations (3.38) and (3.40) can be solved using the substitution method:

Solving Equation (3.40) for V_2, we obtain

$$V_2 = V_1 - 12.5$$

Substituting V_2 into Equation (3.38), we get

$$-4V_1 + 9(V_1 - 12.5) = -42$$

or

$$5V_1 = 70.5$$

Thus, we obtain

$$V_1 = \frac{70.5}{5} = 14.1 \text{ V}$$

$$V_2 = V_1 - 12.5 = 1.6 \text{ V}$$

$$V_3 = V_2 + 7 = 1.6 + 7 = 8.6 \text{ V}$$

continued

Example 3.8 continued

Alternatively, Equations (3.38) and (3.40) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} -4 & 9 \\ 2 & -2 \end{vmatrix} = (-4) \times (-2) - 9 \times 2 = 8 - 18 = -10$$

$$V_1 = \frac{\begin{vmatrix} -42 & 9 \\ 25 & -2 \end{vmatrix}}{\Delta} = \frac{(-42) \times (-2) - 9 \times 25}{-10} = \frac{-141}{-10} = 14.1 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} -4 & -42 \\ 2 & 25 \end{vmatrix}}{\Delta} = \frac{(-4) \times 25 - (-42) \times 2}{-10} = \frac{-16}{-10} = 1.6 \text{ V}$$

From Equation (3.36), we have

$$V_3 = V_2 + 7 = 1.6 + 7 = 8.6 \text{ V}$$

MATLAB

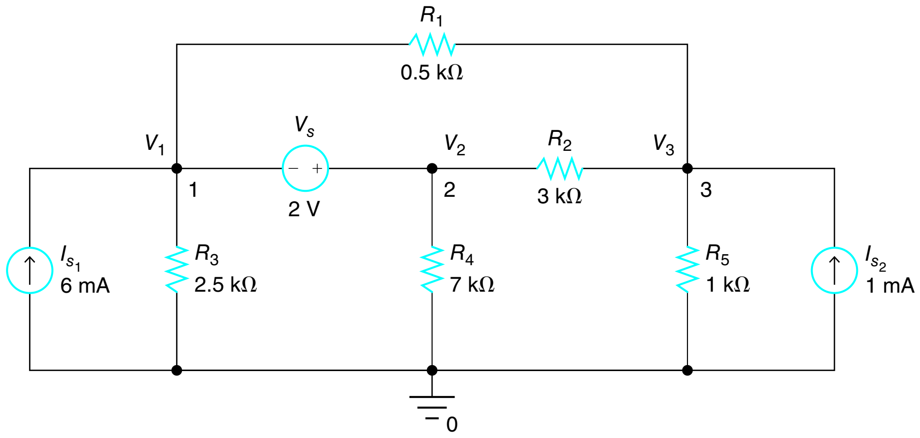
```
A=[-4 9;2 -2];b=[-42;25];
V=A\b
V =
   14.1000
    1.6000
```

Exercise 3.8

Find the voltages V_1, V_2, and V_3 for the circuit shown in Figure 3.22.

FIGURE 3.22

The circuit for EXERCISE 3.8.



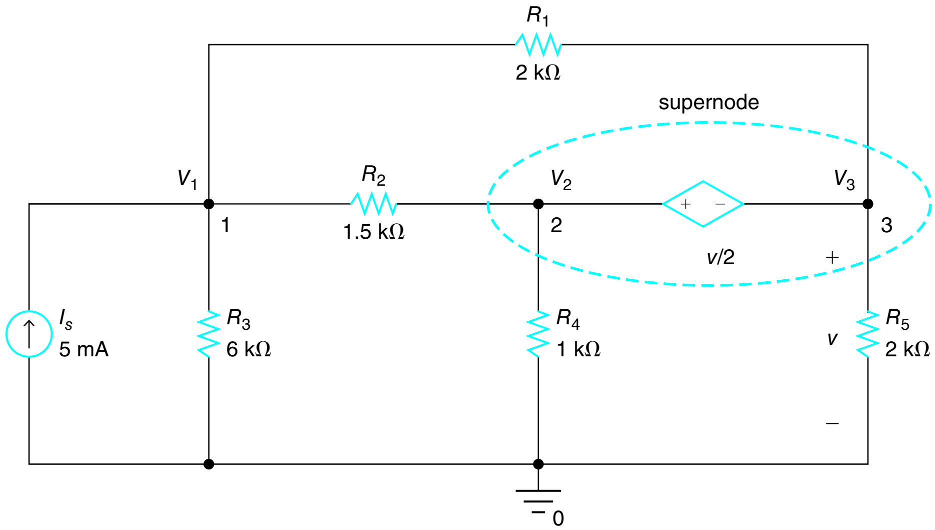
Answer:
$V_1 = 5$ V, $V_2 = 7$ V, $V_3 = 4$ V.

EXAMPLE 3.9

Find V_1, V_2, and V_3 for the circuit shown in Figure 3.23.

FIGURE 3.23

The circuit with VCVS.



The presence of a dependent source across two essential nodes is no different than an ideal voltage source across two essential nodes for application of nodal analysis. The key is to use the dependent variable in terms of the appropriate node voltage variable. Notice that $v = V_3$. Also notice that

$$V_2 = V_3 + v/2 = V_3 + V_3/2 = 1.5V_3 \tag{3.41}$$

Summing the currents leaving the supernode consisting of nodes 2 and 3, we obtain

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_4} + \frac{V_3 - V_1}{R_1} + \frac{V_3}{R_5} = 0$$

Substituting the resistance values, we get

$$\frac{V_2 - V_1}{1500} + \frac{V_2}{1000} + \frac{V_3 - V_1}{2000} + \frac{V_3}{2000} = 0$$

Multiplication by 6000 yields

$$4V_2 - 4V_1 + 6V_2 + 3V_3 - 3V_1 + 3V_3 = 0$$

which can be simplified to

$$-7V_1 + 10V_2 + 6V_3 = 0 \tag{3.42}$$

Substitution of Equation (3.41) into Equation (3.42) yields

$$-7V_1 + 21V_3 = 0 \tag{3.43}$$

continued

Example 3.9 continued

Summing the currents leaving node 1, we obtain

$$-I_s + \frac{V_1 - V_3}{R_1} + \frac{V_1}{R_3} + \frac{V_1 - V_2}{R_2} = 0$$

Substituting the resistance and current values, we get

$$-5 \times 10^{-3} + \frac{V_1 - V_3}{2000} + \frac{V_1}{6000} + \frac{V_1 - V_2}{1500} = 0$$

Multiplication by 6000 yields

$$-30 + 3V_1 - 3V_3 + V_1 + 4V_1 - 4V_2 = 0$$

which can be simplified to

$$8V_1 - 4V_2 - 3V_3 = 30 \qquad (3.44)$$

Substitution of Equation (3.41) into Equation (3.44) yields

$$8V_1 - 9V_3 = 30 \qquad (3.45)$$

Equations (3.43) and (3.45) can be solved using the substitution method:

Solving Equation (3.43) for V_1, we obtain

$$V_1 = 3V_3$$

Substituting V_1 into Equation (3.45), we get

$$8(3V_3) - 9V_3 = 30$$

or

$$15V_3 = 30$$

Thus, we obtain

$$V_3 = 2 \text{ V}$$

$$V_1 = 3V_3 = 6 \text{ V}$$

$$V_2 = 1.5V_3 = 3 \text{ V}$$

Alternatively, Equations (3.43) and (3.45) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} -7 & 21 \\ 8 & -9 \end{vmatrix} = (-7) \times (-9) - 21 \times 8 = 63 - 168 = -105$$

$$V_1 = \frac{\begin{vmatrix} 0 & 21 \\ 30 & -9 \end{vmatrix}}{\Delta} = \frac{0 \times (-9) - 21 \times 30}{-105} = \frac{-630}{-105} = 6 \text{ V}$$

continued

Example 3.9 continued

$$V_3 = \frac{\begin{vmatrix} -7 & 0 \\ 8 & 30 \end{vmatrix}}{\Delta} = \frac{(-7) \times 30 - 0 \times 8}{-105} = \frac{-210}{-105} = 2\text{ V}$$

From Equation (3.41), we obtain

$$V_2 = 1.5V_3 = 3\text{ V}$$

MATLAB

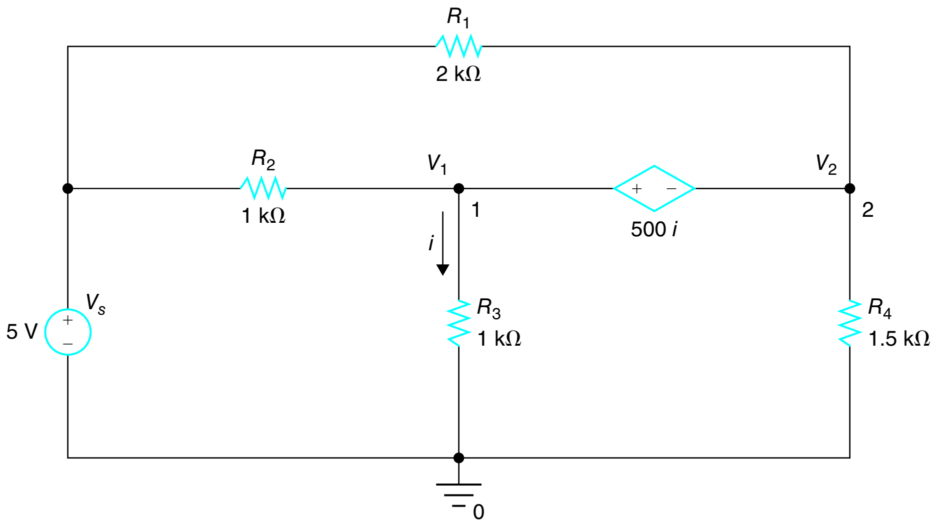
```
%EXAMPLE 3.9
clear all;
Is1=5e-3;R1=2000;R2=1500;R3=6000;R4=1000;R5=2000;R6=2000;R7=1500;
syms V1 V2 V3
[V1,V2,V3]=solve(-Is1+(V1-V3)/R1+V1/R3+(V1-V2)/R2, ...
V2==V3+V3/2, ...
(V2-V1)/R2+V2/R4+(V3-V1)/R1+V3/R5,V1,V2,V3);
V1=vpa(V1,6)
V2=vpa(V2,6)
V3=vpa(V3,6)

Answers:
V1 =
6.0
V2 =
3.0
V3 =
2.0
```

Exercise 3.9

Find V_1 and V_2 for the circuit shown in Figure 3.24.

FIGURE 3.24

The circuit for EXERCISE 3.9.



Answer:
$V_1 = 2.9032$ V, $V_2 = 1.4516$ V.

3.4 Mesh Analysis

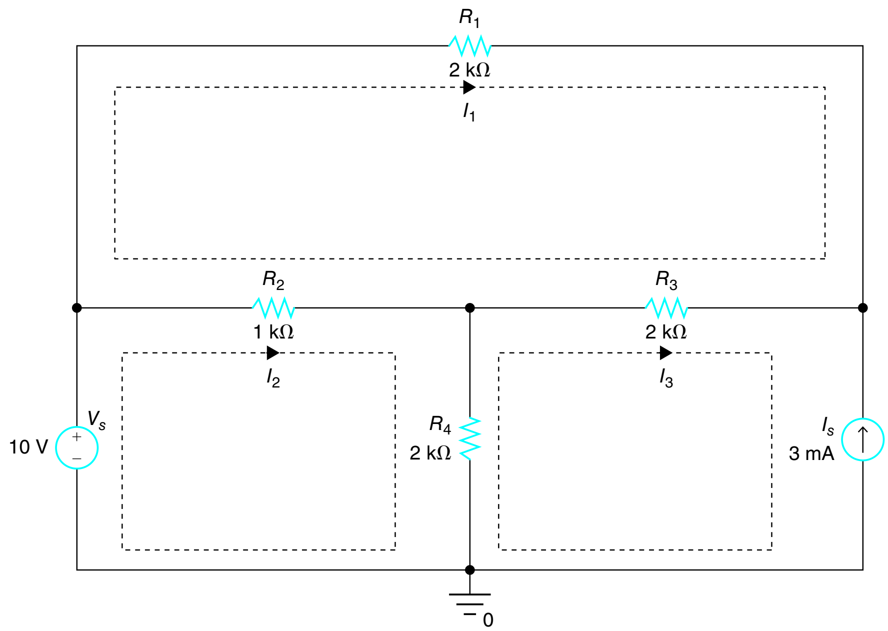
A **loop** in a circuit is a closed path in which no node is encountered more than once. A **mesh** is a loop that does not contain any other loops. A circuit is **planar** if it can be drawn in a two-dimensional space without wires crossing over other wires. We are interested in finding mesh currents in a planar circuit. From the mesh currents, we can find the current through every branch of the circuit. If the current through every branch of a circuit is known, the voltage across every branch can be found by applying Ohm's law. The power on every branch can be found by applying $P = IV$. Thus, if we can find the current through every branch, we can find the voltages and powers everywhere. Mesh analysis is based on KVL. We assign mesh current variables such as $I_1, I_2, I_3, \ldots, I_n$ in the meshes whose currents are unknown. Then, for each mesh with unknown mesh current, we apply KVL. Specifically, we sum the voltage drops around the mesh and let that equal zero. If a mesh contains a current source, the mesh current is the same as the current from the current source if they point in the same direction. If the direction is opposite, the mesh current is the negative of the current from the current source.

If there are n unknown mesh currents, we get n equations in n unknowns. Thus, we can solve these n system of linear equations with constant coefficients to find the unique solution for the unknown mesh currents $I_1, I_2, I_3, \ldots, I_n$. A simple algorithm to find the solution of the n system of linear equations with constant coefficients is Cramer's rule. MATLAB is useful in finding the solution of an n system of linear equations with constant coefficients. The presence of dependent sources in the circuit requires extra equations describing the controlling voltages or currents, and controlled voltages and currents in terms of mesh currents.

As an example, consider the circuit shown in Figure 3.25.

FIGURE 3.25

Circuit with three meshes.



The circuit shown in Figure 3.25 has three meshes. We assign mesh currents for these three meshes. At the outset, it is good practice to assign all mesh currents in the same direction. It is common practice to choose all three mesh currents in the clockwise direction. Figure 3.25 shows the circuit with mesh currents I_1, I_2, and I_3. Notice that the mesh current I_3 is in the opposite direction of the current from current source I_s. Thus, $I_3 = -I_s = -3$ mA. This leaves two unknown mesh currents, I_1 and I_2.

Three resistors, R_1, R_3, and R_2, form mesh 1 with mesh current I_1. Starting from the left terminal of R_1, we add voltage drops across each resistor using Ohm's law. The voltage drops are measured in the clockwise direction following the direction specified by the arrow. According to KVL, the sum of voltage drops in any closed loop must be zero. The voltage drop across R_1, from left to right, is R_1I_1. The directions of I_1 and I_3 are opposite on R_3. The net current through R_3, from right to left, is $I_1 - I_3$. Thus, the voltage drop across R_3, from right to left, is $R_3(I_1 - I_3)$. Similarly, the voltage drop across R_2, from right to left, is $R_2(I_1 - I_2)$. Summing all three voltage drops in the clockwise direction, we have the mesh equation for mesh 1:

$$R_1I_1 + R_3(I_1 - I_3) + R_2(I_1 - I_2) = 0 \tag{3.46}$$

Substituting the component values, we have

$$2000I_1 + 2000(I_1 - I_3) + 1000(I_1 - I_2) = 0 \tag{3.47}$$

Since $I_3 = -3$ mA $= -0.003$ A, Equation (3.47) becomes

$$5000I_1 - 1000I_2 = 2000 \times (-0.003) = -6 \tag{3.48}$$

Summing voltage drops around mesh 2 with mesh current I_2 starting from the negative terminal of the voltage source V_s, we obtain the mesh equation for mesh 2:

$$-V_s + R_2(I_2 - I_1) + R_4(I_2 - I_3) = 0 \tag{3.49}$$

Substituting the component values, we have

$$-10 + 1000(I_2 - I_1) + 2000(I_2 - I_3) = 0 \tag{3.50}$$

Since $I_3 = -3$ mA $= -0.003$ A, Equation (3.50) becomes

$$-1000I_1 + 3000I_2 = 10 + 2000 \times (-0.003) = 4 \tag{3.51}$$

Multiplying Equation (3.48) by 3, we obtain

$$15{,}000I_1 - 3000I_2 = -18$$

Adding this equation and Equation (3.51), we get

$$14{,}000I_1 = -14$$

Thus, we have

$$I_1 = -0.001\ \text{A} = -1\ \text{mA}$$

Substituting I_1 into Equation (3.48), we obtain

$$I_2 = 0.001\ \text{A} = 1\ \text{mA}$$

Alternatively, the two mesh currents I_1 and I_2 can be found by applying Cramer's rule to Equations (3.48) and (3.51):

$$\Delta = \begin{vmatrix} 5000 & -1000 \\ -1000 & 3000 \end{vmatrix} = 1.5 \times 10^7 - 0.1 \times 10^7 = 1.4 \times 10^7$$

$$I_1 = \frac{\begin{vmatrix} -6 & -1000 \\ 4 & 3000 \end{vmatrix}}{\Delta} A = \frac{-1.8 \times 10^4 + 0.4 \times 10^4}{1.4 \times 10^7} A = \frac{-1.4 \times 10^4}{1.4 \times 10^7} A = -1\ \text{mA}$$

$$I_2 = \frac{\begin{vmatrix} 5000 & -6 \\ -1000 & 4 \end{vmatrix}}{\Delta} A = \frac{2.0 \times 10^4 - 0.6 \times 10^4}{1.4 \times 10^7} A = \frac{1.4 \times 10^4}{1.4 \times 10^7} A = 1\ \text{mA}$$

MATLAB

```
A=[5000 -1000;-1000 3000];b=[-6;4];
I=A\b
I =
   1.0e-03 *
   -1.0000
    1.0000
```

Equations (3.46) and (3.49), respectively, can be rearranged as

$$(R_1 + R_3 + R_2)I_1 - R_2I_2 - R_3I_3 = 0$$

and

$$-R_2I_1 + (R_2 + R_4)I_2 - R_4I_3 = V_s$$

Also, we have

$$I_3 = -I_s$$

These equations can be put into matrix form as

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_2 & -R_3 \\ -R_2 & R_2 + R_4 & -R_4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ -I_s \end{bmatrix}$$

This equation can be solved using MATLAB, as shown here:

```
clear all;
Vs=10;Is=3e-3;R1=2000;R2=1000;R3=2000;R4=2000;
A=[R1+R2+R3,-R2,-R3;-R2,R2+R4,-R4;0 0 1]
b=[0;Vs;-Is]
I=A\b

Answer:
A =
        5000        -1000        -2000
       -1000         3000        -2000
           0            0            1
b =
         0
   10.0000
   -0.0030
I =
   -0.0010
    0.0010
   -0.0030
```

This approach does not require numerical values for the elements of A and b to find I_1 and I_2.

One other method of solving Equations (3.46) and (3.49) is to use the MATLAB function **solve**, as shown here:

```
clear all;
syms I1 I2 I3
Vs=10;Is=3e-3;R1=2000;R2=1000;R3=2000;R4=2000;
[I1,I2,I3]=solve(R1*I1+R3*(I1-I3)+R2*(I1-I2),...
-Vs+R2*(I2-I1)+R4*(I2-I3),I3==-Is)
I1=vpa(I1,7)
I2=vpa(I2,7)
I3=vpa(I3,7)
```

```
Answers:
I1 =
-1/1000
I2 =
1/1000
I3 =
-3/1000
I1 =
-0.001
I2 =
0.001
I3 =
-0.003
```

This method provides the solution by entering equations.

Let I_{R_1} be the current through R_1, from right to left, as shown in Figure 3.26. Then, $I_{R_1} = -I_1$; that is, $I_{R_1} = -I_1 = 1$ mA. The voltage across R_1, from right to left, is $V_{R_1} = R_1 \times I_{R_1} = 2\ k\Omega \times 1$ mA $= 2$ V. Let I_{R_2} be the current through R_2, from left to right, as shown in Figure 3.26. Then, I_{R_2} can be represented as $I_{R_2} = I_2 - I_1 = 1$ mA $- (-1$ mA$) =$ 2 mA. The voltage across R_2, from left to right, is $V_{R_2} = R_2 \times I_{R_2} = 1\ k\Omega \times 2$ mA $= 2$ V. Let I_{R_3} be the current through R_3, from right to left, as shown in Figure 3.26. Then, $I_{R_3} = I_1 - I_3 = -1$ mA $- (-3$ mA$) = 2$ mA. The voltage across R_3, from right to left, is $V_{R_3} = R_3 \times I_{R_3} = 2\ k\Omega \times 2$ mA$= 4$ V. Let I_{R_4} be the current through R_4, from top to bottom, as shown in Figure 3.26. Then, $I_{R_4} = I_2 - I_3 = 1$ mA $- (-3$ mA$) = 4$ mA. The voltage across R_4, from top to bottom, is $V_{R_4} = R_4 \times I_{R_4} = 2\ k\Omega \times 4$ mA $= 8$ V. Notice that the voltage at node 1 (V_1 in Figure 3.26) is given by $V_1 = V_{R_4} = 8$ V, and the voltage at node 2 is given by $V_2 = V_1 + V_{R_3} = 8$ V $+ 4$ V $= 12$ V. The current through the voltage source, from bottom to top, is given by $I_{V_s} = I_{R_4} - I_s = 1$ mA.

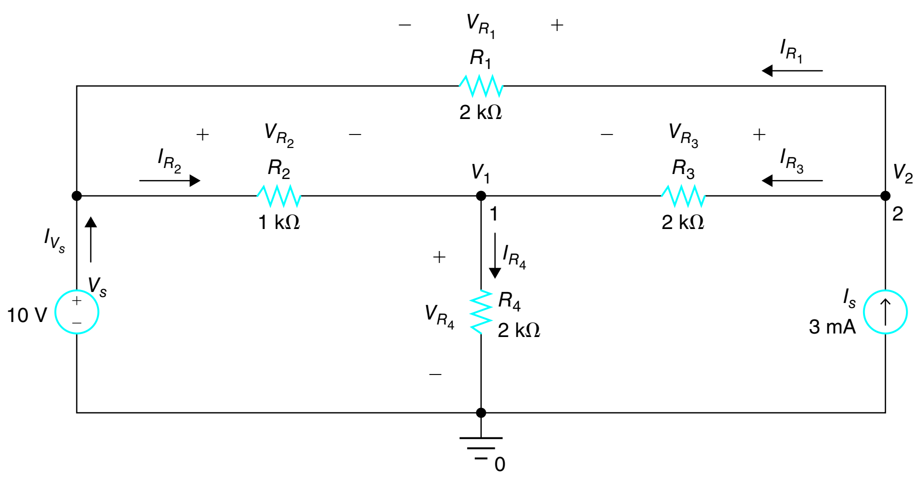


FIGURE 3.26

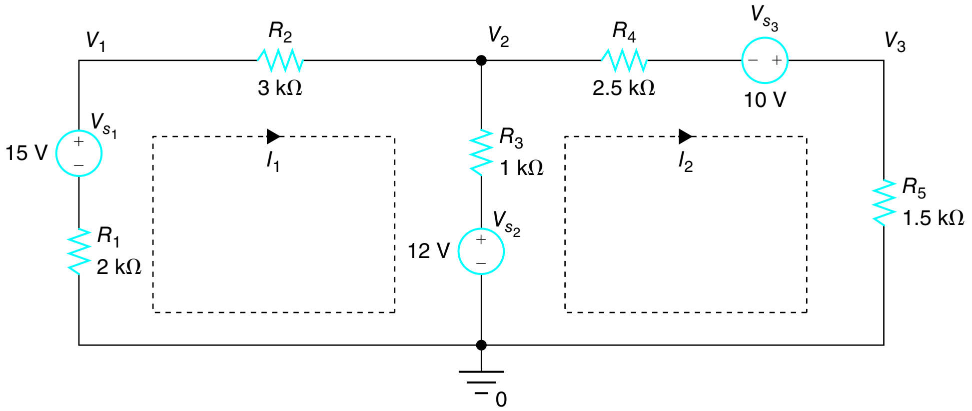
The circuit shown in Figure 3.25 with current and voltage directions on the branches.

EXAMPLE 3.10

Find the mesh currents I_1 and I_2 in the circuit shown in Figure 3.27. Also, find V_1, V_2, V_3 and the power absorbed or supplied by all elements.

FIGURE 3.27

The circuit for EXAMPLE 3.10.



Summing the voltage drops around the mesh 1 (left side), we obtain

$$R_1I_1 - V_{s_1} + R_2I_1 + R_3(I_1 - I_2) + V_{s_2} = 0$$

Substituting the resistance and voltage values, we have

$$2000I_1 - 15 + 3000I_1 + 1000(I_1 - I_2) + 12 = 0$$

which can be simplified to

$$6000I_1 - 1000I_2 = 3 \quad (3.52)$$

Summing the voltage drops around mesh 2 (on the right side), we obtain

$$-V_{s_2} + R_3(I_2 - I_1) + R_4I_2 - V_{s_3} + R_5I_2 = 0$$

Substituting the resistance and voltage values, we have

$$-12 + 1000(I_2 - I_1) + 2500I_2 - 10 + 1500I_2 = 0$$

which can be simplified to

$$-1000I_1 + 5000I_2 = 22 \quad (3.53)$$

Equations (3.52) and (3.53) can be solved using the substitution method:

Multiplying Equation (3.52) by 5, we obtain

$$30{,}000I_1 - 5000I_2 = 15$$

Adding this equation and Equation (3.53), we get

$$29{,}000I_1 = 37$$

continued

Example 3.10 continued

Thus, we obtain

$$I_1 = \frac{37}{29{,}000} = 1.2759 \text{ mA}$$

Solving Equation (3.52) for I_2, we get

$$I_2 = 6I_1 - \frac{3}{1000} = \frac{135}{29{,}000} = 4.6552 \text{ mA}$$

Alternatively, Equations (3.52) and (3.53) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 6000 & -1000 \\ -1000 & 5000 \end{vmatrix} = 6000 \times 5000 - (-1000) \times (-1000) = 2.9 \times 10^7$$

$$I_1 = \frac{\begin{vmatrix} 3 & -1000 \\ 22 & 5000 \end{vmatrix}}{\Delta} A = \frac{3 \times 5000 - (-1000) \times 22}{2.9 \times 10^7} A = \frac{37{,}000}{2.9 \times 10^7} A = 1.2759 \text{ mA}$$

$$I_2 = \frac{\begin{vmatrix} 6000 & 3 \\ -1000 & 22 \end{vmatrix}}{\Delta} A = \frac{6000 \times 22 - 3 \times (-1000)}{2.9 \times 10^7} A = \frac{135{,}000}{2.9 \times 10^7} A = 4.6552 \text{ mA}$$

The voltages are given by

$$V_1 = V_{s_1} - R_1 \times I_1 = (15 - 2000 \times 1.2759 \times 10^{-3})\text{V} = 12.4483 \text{ V}$$

$$V_2 = R_3 \times (I_1 - I_2) + V_{s_2} = [1000 \times (1.2759 \times 10^{-3} - 4.6552 \times 10^{-3}) + 12] \text{ V}$$
$$= 8.6207 \text{ V}$$

$$V_3 = R_5 \times I_2 = 1500 \times 4.6552 \times 10^{-3} \text{ V} = 6.9828 \text{ V}$$

The powers absorbed or supplied by all elements are given by

$$P_{R_1} = I_1^2 R_1 = 3.2556 \text{ mW}$$

$$P_{R_2} = I_1^2 R_2 = 4.8835 \text{ mW}$$

$$P_{R_3} = (I_1 - I_2)^2 R_3 = 11.4197 \text{ mW}$$

$$P_{R_4} = I_2^2 R_4 = 54.1766 \text{ mW}$$

$$P_{R_5} = I_2^2 R_5 = 32.5059 \text{ mW}$$

$$P_{Vs_1} = (-I_1)V_{s_1} = -19.1379 \text{ mW}$$

$$P_{Vs_2} = (I_1 - I_2)V_{s_2} = -40.5517 \text{ mW}$$

$$P_{Vs_3} = (-I_2)V_{s_3} = -46.5517 \text{ mW}$$

Notice that $P_{R_1} + P_{R_2} + P_{R_3} + P_{R_4} + P_{R_5} + P_{Vs_1} + P_{Vs_2} + P_{Vs_3} = 0$.

continued

Example 3.10 continued

MATLAB

```
%EXAMPLE 3.10
clear all;format long;
Vs1=15;Vs2=12;Vs3=10;
R1=2000;R2=3000;R3=1000;R4=2500;R5=1500;
syms I1 I2
[I1,I2]=solve(R1*I1-Vs1+R2*I1+R3*(I1-I2)+Vs2, ...
-Vs2+R3*(I2-I1)+R4*I2-Vs3+R5*I2,I1,I2);
V1=Vs1-R1*I1;
V2=R3*(I1-I2)+Vs2;
V3=R5*I2;
PR1=I1^2*R1;
PR2=I1^2*R2;
PR3=(I1-I2)^2*R3;
PR4=I2^2*R4;
PR5=I2^2*R5;
Ps1=-I1*Vs1;
Ps2=(I1-I2)*Vs2;
Ps3=-I2*Vs3;
SumP=PR1+PR2+PR3+PR4+PR5+Ps1+Ps2+Ps3;
I1=vpa(I1,7)
I2=vpa(I2,7)
V1=vpa(V1,7)
V2=vpa(V2,7)
V3=vpa(V3,7)
PR1=vpa(PR1,7)
PR2=vpa(PR2,7)
PR3=vpa(PR3,7)
PR4=vpa(PR4,7)
PR5=vpa(PR5,7)
Ps1=vpa(Ps1,7)
Ps2=vpa(Ps2,7)
Ps3=vpa(Ps3,7)
SumP=vpa(SumP,7)

Answers:
I1 =
0.001275862
I2 =
0.004655172
V1 =
12.44828
V2 =
8.62069
V3 =
6.982759
PR1 =
0.003255648
PR2 =
0.004883472
PR3 =
0.01141974
PR4 =
0.05417658
```

continued

Example 3.10 continued

MATLAB continued

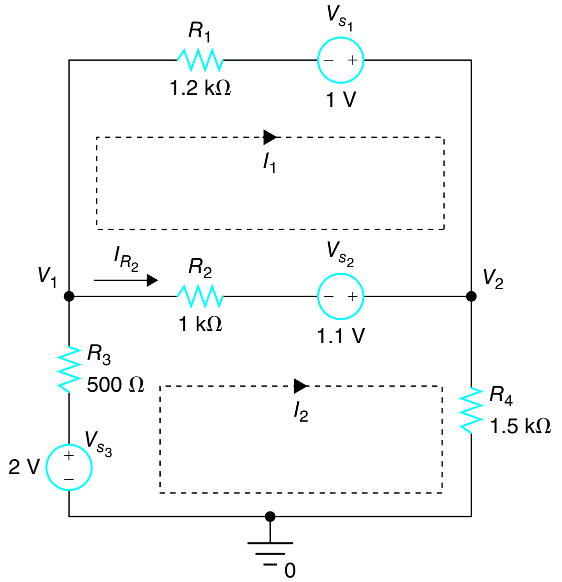
```
PR5 =
0.03250595
Ps1 =
-0.01913793
Ps2 =
-0.04055172
Ps3 =
-0.04655172
SumP =
0.0
```

Exercise 3.10

Find the mesh currents I_1, I_2, voltages V_1, V_2, and current I_{R_2} in the circuit shown in Figure 3.28.

FIGURE 3.28

The circuit for EXERCISE 3.10.



Answer:

$I_1 = 0.5$ mA, $I_2 = 1.2$ mA, $V_1 = 1.4$ V, $V_2 = 1.8$ V, $I_{R_2} = 0.7$ mA.

EXAMPLE 3.11

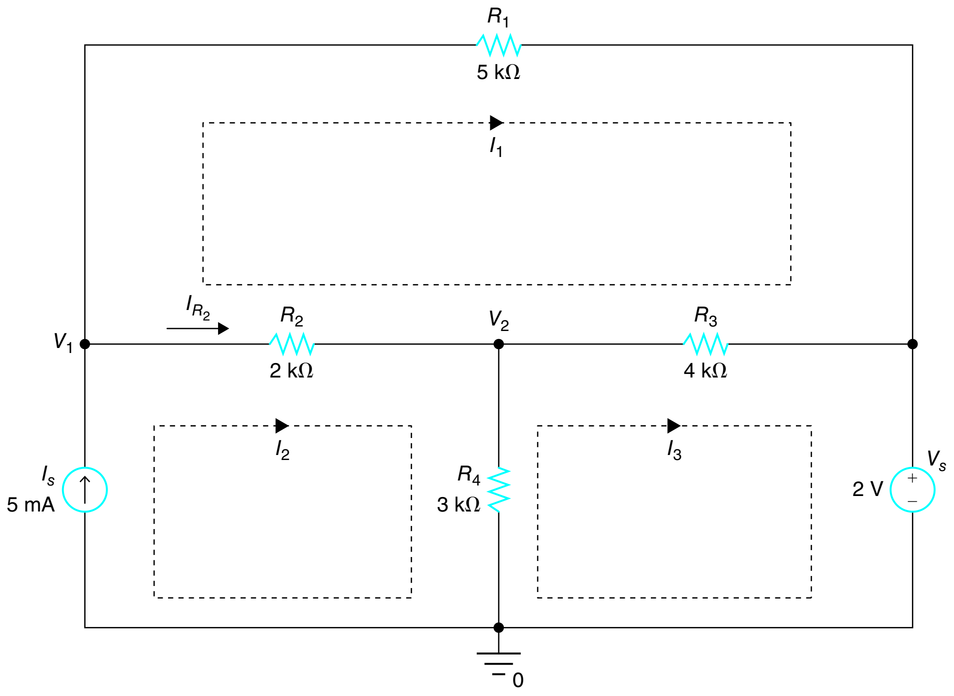
Find the mesh currents I_1, I_2, and I_3, voltages V_1 and V_2, and current I_{R_2} in the circuit shown in Figure 3.29.

continued

Example 3.11 continued

FIGURE 3.29

The circuit for EXAMPLE 3.11.



The mesh current I_2 is I_s. Thus, we have

$$I_2 = 5 \text{ mA}$$

Summing the voltage drops around mesh 1 (on the upper side), we obtain

$$R_1 I_1 + R_3(I_1 - I_3) + R_2(I_1 - I_2) = 0$$

Substituting the resistance values, we have

$$5000I_1 + 4000(I_1 - I_3) + 2000(I_1 - I_2) = 0$$

which can be simplified to

$$11{,}000I_1 - 2000I_2 - 4000I_3 = 0$$

Substitution of $I_2 = 5 \text{ mA} = 5 \times 10^{-3}$ A and dividing by 1000 yields

$$11I_1 - 4I_3 = 0.01 \tag{3.54}$$

Summing the voltage drops around mesh 3 (on the lower right side), we obtain

$$R_4(I_3 - I_2) + R_3(I_3 - I_1) + V_s = 0$$

Substituting the resistance and voltage values, we have

$$3000(I_3 - I_2) + 4000(I_3 - I_1) + 2 = 0$$

which can be simplified to

$$-4000I_1 - 3000I_2 + 7000I_3 = -2$$

continued

Example 3.11 continued

Substitution of $I_2 = 5\text{ mA} = 5 \times 10^{-3}\text{ A}$ yields

$$-4000I_1 + 7000I_3 = 13 \tag{3.55}$$

Equations (3.54) and (3.55) can be solved using the substitution method:

Solving Equation (3.54) for I_3, we obtain

$$I_3 = \frac{11}{4}I_1 - \frac{1}{400}$$

Substituting I_3 into Equation (3.55), we get

$$-4000I_1 + 7000\left(\frac{11}{4}I_1 - \frac{1}{400}\right) = 13$$

which can be rearranged as

$$\frac{61{,}000}{4}I_1 = \frac{12{,}200}{400}$$

Thus, we obtain

$$I_1 = \frac{122}{61{,}000} = 2\text{ mA}$$

$$I_3 = \frac{11}{4}I_1 - \frac{1}{400} = \frac{11}{4} \times 0.002 - \frac{1}{400} = 3\text{ mA}$$

Alternatively, Equations (3.54) and (3.55) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 11{,}000 & -4000 \\ -4000 & 7000 \end{vmatrix} = 11{,}000 \times 7000 - (-4000) \times (-4000) = 6.1 \times 10^7$$

$$I_1 = \frac{\begin{vmatrix} 10 & -4000 \\ 13 & 7000 \end{vmatrix}}{\Delta}A = \frac{10 \times 7000 - (-4000) \times 13}{6.1 \times 10^7}A = \frac{122{,}000}{6.1 \times 10^7}A = 2\text{ mA}$$

$$I_3 = \frac{\begin{vmatrix} 11{,}000 & 10 \\ -4000 & 13 \end{vmatrix}}{\Delta}A = \frac{11{,}000 \times 13 - 10 \times (-4000)}{6.1 \times 10^7}A = \frac{183{,}000}{6.1 \times 10^7}A = 3\text{ mA}$$

The voltages are given by

$$V_1 = V_s + R_1 \times I_1 = (2 + 5000 \times 2 \times 10^{-3})\text{V} = 12\text{ V}$$

$$V_2 = R_4 \times (I_2 - I_3) = 3000 \times (5 \times 10^{-3} - 3 \times 10^{-3})\text{V} = 6\text{ V}$$

The current through R_2 is given by

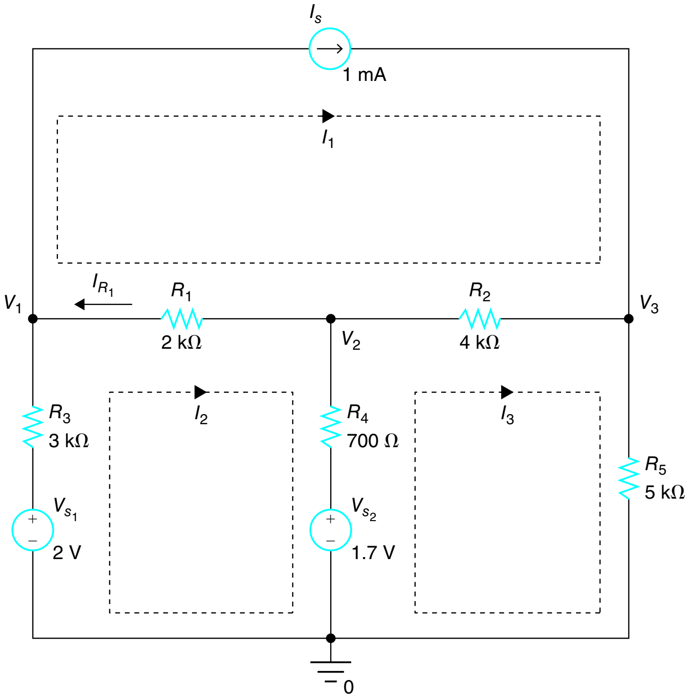
$$I_{R_2} = I_2 - I_1 = 5\text{ mA} - 2\text{ mA} = 3\text{ mA}$$

Exercise 3.11

Find the mesh currents I_1, I_2, I_3, voltages V_1, V_2, and current I_{R_1} in the circuit shown in Figure 3.30.

FIGURE 3.30

The circuit for EXERCISE 3.11.



Answer:

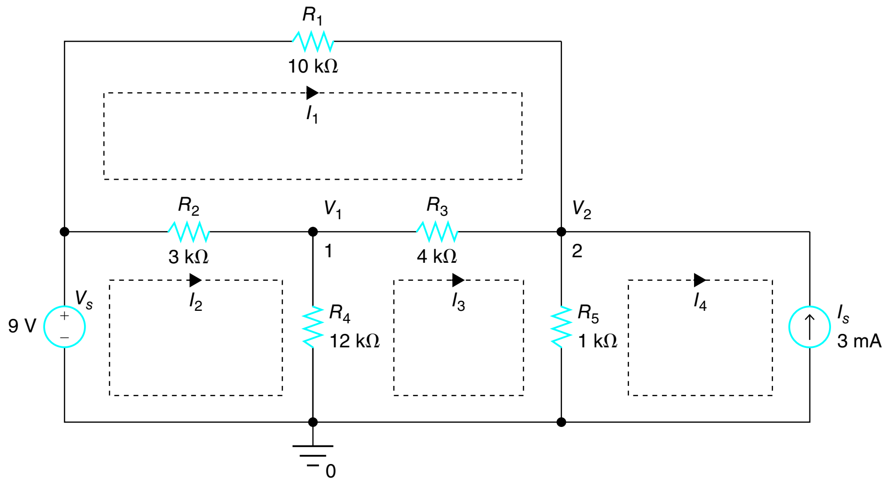
$I_1 = 1$ mA, $I_2 = 0.4799$ mA, $I_3 = 0.6223$ mA, $V_1 = 0.5602$ V, $V_2 = 1.6004$ V, $V_3 = 3.1113$ V, $I_{R_1} = 0.5201$ mA.

EXAMPLE 3.12

Use mesh analysis to find the voltages and currents everywhere for the circuit shown in Figure 3.31.

FIGURE 3.31

The circuit for EXAMPLE 3.12.



continued

Example 3.12 continued

The circuit shown in Figure 3.31 has four meshes. We assign mesh currents for these four meshes. Figure 3.31 shows the circuit with mesh currents.

The mesh with mesh current I_4 includes the current source I_s. Since the mesh current I_4 is in the opposite direction of the current from the current source, we have $I_4 = -I_s = -3$ mA. Since the mesh current I_4 is already known, we only need to write mesh equations for the other three meshes. Three resistors (R_1, R_3, and R_2) form mesh 1 with mesh current I_1. Starting from the left terminal of R_1, we add voltage drops on each resistor using Ohm's law. The voltage drops are measured in the clockwise direction following the direction specified by the arrow. According to KVL, the sum of voltage drops in any closed mesh must be zero. The voltage drop across R_1, from left to right, is R_1I_1. The directions of I_1 and I_3 are opposite on R_3. The net current through R_3, from right to left, is $I_1 - I_3$. Thus, the voltage drop across R_3, from right to left, is $R_3(I_1 - I_3)$. Similarly, the voltage drop across R_2, from right to left, is $R_2(I_1 - I_2)$. Summing all three voltage drops in the clockwise direction, we have a mesh equation for mesh 1:

$$R_1I_1 + R_3(I_1 - I_3) + R_2(I_1 - I_2) = 0 \tag{3.56}$$

Substituting resistance values, we get

$$10{,}000I_1 + 4000(I_1 - I_3) + 3000(I_1 - I_2) = 0 \tag{3.57}$$

Equation (3.57) can be simplified to

$$17{,}000I_1 - 3000I_2 - 4000I_3 = 0$$

Dividing by 1000, we get

$$17I_1 - 3I_2 - 4I_3 = 0 \tag{3.58}$$

Summing voltage drops around mesh 2 with mesh current I_2 starting from the negative terminal of the voltage source V_s, we obtain the mesh equation for mesh 2:

$$-V_s + R_2(I_2 - I_1) + R_4(I_2 - I_3) = 0 \tag{3.59}$$

Substitution of the component values yields

$$-9 + 3000(I_2 - I_1) + 12{,}000(I_2 - I_3) = 0 \tag{3.60}$$

Equation (3.60) can be simplified to

$$-3000I_1 + 15{,}000I_2 - 12{,}000I_3 = 9$$

Dividing by 3000, we get

$$-I_1 + 5I_2 - 4I_3 = 0.003 \tag{3.61}$$

Summing voltage drops around mesh 3 with mesh current I_3 starting from the bottom terminal of the resistor R_4, we obtain the mesh equation for mesh 3:

$$R_4(I_3 - I_2) + R_3(I_3 - I_1) + R_5(I_3 - I_4) = 0 \tag{3.62}$$

Substitution of the component values yields

$$12{,}000(I_3 - I_2) + 4000(I_3 - I_1) + 1000(I_3 + 0.003) = 0 \tag{3.63}$$

Equation (3.63) can be simplified to

$$-4000I_1 - 12{,}000I_2 + 17{,}000I_3 = -3$$

continued

Example 3.12 continued

Dividing by 1000, we get

$$-4I_1 - 12I_2 + 17I_3 = -0.003 \tag{3.64}$$

Equations (3.58), (3.61), and (3.64) can be solved using the substitution method: Solving Equation (3.61) for I_1, we obtain

$$I_1 = 5I_2 - 4I_3 - 0.003$$

Substituting I_1 into Equations (3.58) and (3.64) respectively, we get

$$17(5I_2 - 4I_3 - 0.003) - 3I_2 - 4I_3 = 0$$

$$-4(5I_2 - 4I_3 - 0.003) - 12I_2 + 17I_3 = -0.003$$

These two equations can be simplified to

$$82I_2 - 72I_3 = 0.051$$

$$-32I_2 + 33I_3 = -0.015$$

Solving $-32I_2 + 33I_3 = -0.015$ for I_2, we obtain

$$I_2 = \frac{33}{32}I_3 + \frac{0.015}{32}$$

Substituting I_2 into $82I_2 - 72I_3 = 0.051$, we get

$$82\left(\frac{33}{32}I_3 + \frac{0.015}{32}\right) - 72I_3 = 0.051$$

Thus, we have

$$I_3 = \frac{0.051 - \frac{82}{32} \times 0.015}{82 \times \frac{33}{32} - 72} = 1 \text{ mA}$$

$$I_2 = \frac{33}{32}I_3 + \frac{0.015}{32} = 1.5 \text{ mA}$$

$$I_1 = 5I_2 - 4I_3 - 0.003 = 0.5 \text{ mA}$$

Alternatively, the three mesh currents I_1, I_2, and I_3 can be found by applying Cramer's rule to Equations (3.58), (3.61), and (3.64):

$$\Delta = \begin{vmatrix} 17 & -3 & -4 \\ -1 & 5 & -4 \\ -4 & -12 & 17 \end{vmatrix} = 17\begin{vmatrix} 5 & -4 \\ -12 & 17 \end{vmatrix} + 3\begin{vmatrix} -1 & -4 \\ -4 & 17 \end{vmatrix} - 4\begin{vmatrix} -1 & 5 \\ -4 & -12 \end{vmatrix}$$

$$= 17[5 \times 17 - (-4)(-12)] + 3[(-1)(17) - (-4)(-4)] - 4[(-1)(-12) - (5)(-4)]$$

$$= 17[85 - 48] + 3[-17 - 16] - 4[12 + 20] = 17 \times 37 - 3 \times 33 - 4 \times 32 = 402$$

continued

Example 3.12 continued

$$I_1 = \frac{\begin{vmatrix} 0 & -3 & -4 \\ 0.003 & 5 & -4 \\ -0.003 & -12 & 17 \end{vmatrix}}{\Delta} = \frac{0\begin{vmatrix} 5 & -4 \\ -17 & 17 \end{vmatrix} + 3\begin{vmatrix} 0.003 & -4 \\ -0.003 & 17 \end{vmatrix} - 4\begin{vmatrix} 0.003 & 5 \\ -0.003 & -12 \end{vmatrix}}{\Delta}$$

$$= \frac{0.201}{402} = 0.0005\,\text{A} = 0.5\,\text{mA}$$

$$I_2 = \frac{\begin{vmatrix} 17 & 0 & -4 \\ -1 & 0.003 & -4 \\ -4 & -0.003 & 17 \end{vmatrix}}{\Delta} = \frac{17\begin{vmatrix} 0.003 & -4 \\ -0.003 & 17 \end{vmatrix} + 0\begin{vmatrix} -1 & -4 \\ -4 & 17 \end{vmatrix} - 4\begin{vmatrix} -1 & 0.003 \\ -4 & -0.003 \end{vmatrix}}{\Delta}$$

$$= \frac{0.603}{402} = 0.0015\,\text{A} = 1.5\,\text{mA}$$

$$I_3 = \frac{\begin{vmatrix} 17 & -3 & 0 \\ -1 & 5 & 0.003 \\ -4 & -12 & -0.003 \end{vmatrix}}{\Delta} = \frac{17\begin{vmatrix} 5 & 0.003 \\ -12 & -0.003 \end{vmatrix} + 3\begin{vmatrix} -1 & 0.003 \\ -4 & -0.003 \end{vmatrix} + 0\begin{vmatrix} -1 & 5 \\ -4 & -12 \end{vmatrix}}{\Delta}$$

$$= \frac{0.402}{402} = 0.001\,\text{A} = 1\,\text{mA}$$

MATLAB

```
%EXAMPLE 3.12
clear all;
syms I1 I2 I3 I4
Vs=9;Is=3e-3;
R1=10000;R2=3000;R3=4000;R4=12000;R5=1000;
[I1,I2,I3,I4]=solve(R1*I1+R3*(I1-I3)+R2*(I1-I2),...
I4==-Is,...
-Vs+R2*(I2-I1)+R4*(I2-I3),...
R4*(I3-I2)+R3*(I3-I1)+R5*(I3-I4),I1,I2,I3,I4);
IR1=I1;
IR2=I2-I1;
IR3=I3-I1;
IR4=I2-I3;
IR5=I3-I4;
V1=R4*IR4;
V2=R5*IR5;
I1=vpa(I1,7)
I2=vpa(I2,7)
I3=vpa(I3,7)
I4=vpa(I4,7)
IR1=vpa(IR1,7)
IR2=vpa(IR2,7)
IR3=vpa(IR3,7)
IR4=vpa(IR4,7)
IR5=vpa(IR5,7)
V1=vpa(V1,7)
V2=vpa(V2,7)
```

continued

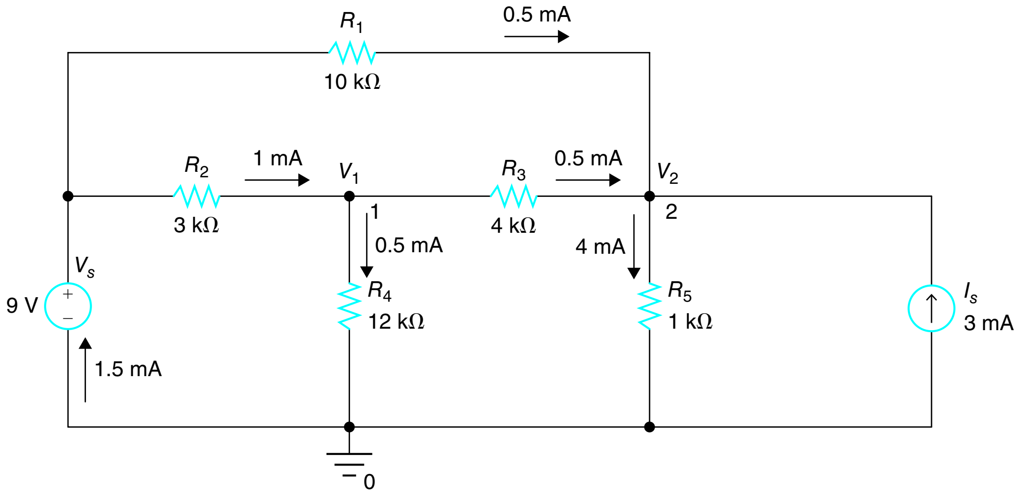
Example 3.12 continued
MATLAB continued

```
Answers:
I1 =
0.0005
I2 =
0.0015
I3 =
0.001
I4 =
-0.003
IR1 =
0.0005
IR2 =
0.001
IR3 =
0.0005
IR4 =
0.0005
IR5 =
0.004
V1 =
6.0
V2 =
4.0
```

Let I_{R_4} be the current through R_4, from top to bottom, as shown in Figure 3.32. Then, I_{R_4} can be represented as $I_{R_4} = I_2 - I_3 = 1.5 \text{ mA} - 1 \text{ mA} = 0.5 \text{ mA}$. The voltage across R_4 is given by $V_{R_4} = R_4 I_{R_4} = 12 \text{ k}\Omega \times 0.5 \text{ mA} = 6 \text{ V}$. Notice that $V_1 = V_{R_4} = 6$ V. Let I_{R_5} be the current through R_5, from top to bottom, as shown in Figure 3.32. Then, I_{R_5} can be represented as $I_{R_5} = I_3 - I_4 = 1 \text{ mA} - (-3 \text{ mA}) = 4 \text{ mA}$. The voltage across R_5 is given by $V_{R_5} = R_5 I_{R_5} = 1 \text{ k}\Omega \times 4 \text{ mA} = 4 \text{ V}$. Notice that $V_2 = V_{R_5} = 4$ V. Figure 3.32 shows currents through all branches. Also, we have $V_{R_1} = V_s - V_2 = 5 \text{ V}$, $V_{R_2} = V_s - V_1 = 3 \text{ V}$, and $V_{R_3} = V_1 - V_2 = 2 \text{ V}$.

FIGURE 3.32

A circuit with currents.

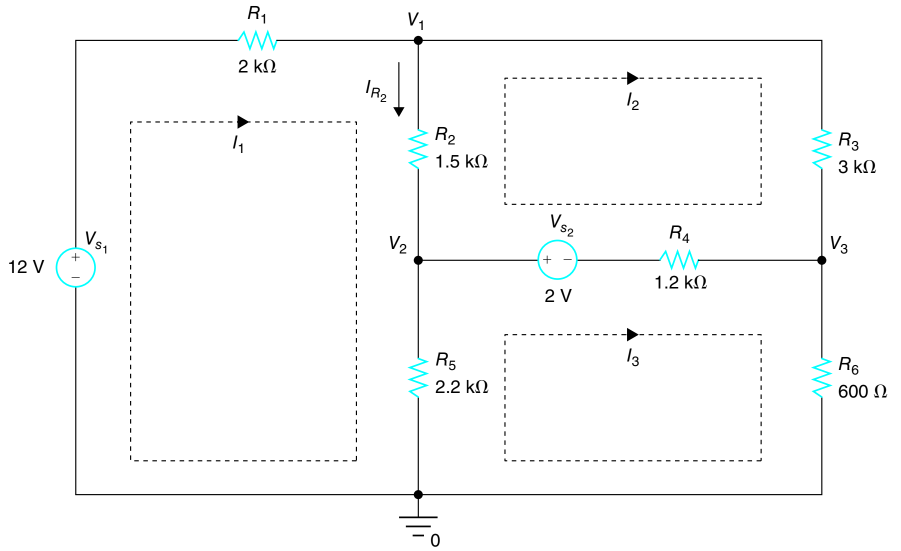


Exercise 3.12

Find the mesh currents I_1, I_2, and I_3, voltages V_1 and V_2, and current I_{R_2} in the circuit shown in Figure 3.33.

FIGURE 3.33

The circuit for EXERCISE 3.12.



Answer:

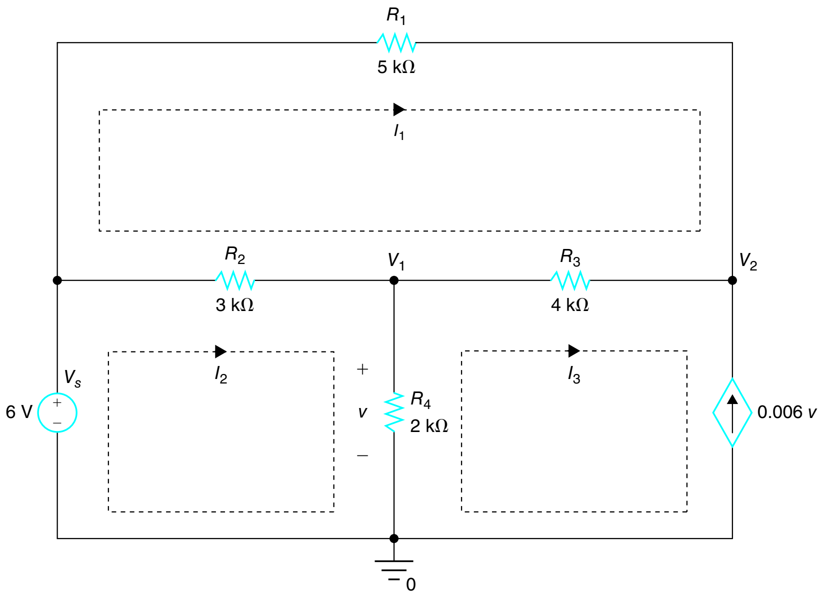
$I_1 = 3.1707$ mA, $I_2 = 1.5447$ mA, $I_3 = 1.7073$ mA, $V_1 = 5.6585$ V, $V_2 = 3.2195$ V, $V_3 = 1.02439$ V, $I_{R_2} = 1.6260$ mA.

EXAMPLE 3.13

Find the mesh currents I_1, I_2, and I_3, voltages V_1 and V_2, and powers on all elements in the circuit shown in Figure 3.34.

FIGURE 3.34

The circuit for EXAMPLE 3.13.



continued

Example 3.13 continued

Here, the voltage-dependent current source produces current that is in the opposite direction of the mesh current I_3; that is, $I_3 = -0.006v$, where $v = R_4(I_2 - I_3)$. Hence,

$$I_3 = -0.006v = -0.006R_4(I_2 - I_3) = -0.006 \times 2000(I_2 - I_3) = -12(I_2 - I_3)$$

Thus,

$$I_3 = \frac{12}{11} I_2 = 1.0909I_2 \tag{3.65}$$

Summing the voltage drops around mesh 1 (on the upper side), we obtain

$$R_1I_1 + R_3(I_1 - I_3) + R_2(I_1 - I_2) = 0$$

Substituting the resistance values, we get

$$5000I_1 + 4000(I_1 - I_3) + 3000(I_1 - I_2) = 0$$

which can be simplified to

$$12{,}000I_1 - 3000I_2 - 4000I_3 = 0 \tag{3.66}$$

Substitution of Equation (3.65) into Equation (3.66) yields

$$12{,}000I_1 - 7363.6364I_2 = 0 \tag{3.67}$$

Summing the voltage drops around mesh 2 (on the lower left side), we obtain

$$-V_s + R_2(I_2 - I_1) + R_4(I_2 - I_3) = 0$$

Substituting the resistance and voltage values, we get

$$-6 + 3000(I_2 - I_1) + 2000(I_2 - I_3) = 0$$

which can be simplified to

$$-3000I_1 + 5000I_2 - 2000I_3 = 6 \tag{3.68}$$

Substitution of Equation (3.65) into Equation (3.68) yields

$$-3000I_1 + 2818.1818I_2 = 6 \tag{3.69}$$

Equations (3.67) and (3.69) can be solved using the substitution method:

Multiplying Equation (3.69) by 4, we obtain

$$-12{,}000I_1 + 11{,}272.7272I_2 = 24$$

Adding this equation and Equation (3.67), we get

$$3909.0908I_2 = 24$$

Thus, we obtain

$$I_2 = \frac{24}{3909.0908} = 6.139535 \text{ mA}$$

continued

Example 3.13 continued

$$I_1 = \frac{7363.6364 I_2}{12{,}000} = 3.767442 \text{ mA}$$

$$I_3 = \frac{12}{11} I_2 = 1.0909 I_2 = 6.69767 \text{ mA}$$

Alternatively, Equations (3.67) and (3.69) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 12{,}000 & -7363.6364 \\ -3000 & 2818.1818 \end{vmatrix} = 1.172727 \times 10^7$$

$$I_1 = \frac{\begin{vmatrix} 0 & -7363.6364 \\ 6 & 2818.1818 \end{vmatrix}}{\Delta} A = \frac{44{,}181.8182}{1.172727 \times 10^7} A = 3.767442 \text{ mA}$$

$$I_2 = \frac{\begin{vmatrix} 12{,}000 & 0 \\ -3000 & 6 \end{vmatrix}}{\Delta} A = \frac{72{,}000}{1.172727 \times 10^7} A = 6.139535 \text{ mA}$$

$$I_3 = \frac{12}{11} I_2 = 1.0909 I_2 = 1.0909 \times 6.139535 \text{ mA} = 6.69767 \text{ mA}$$

The voltages are given by

$$V_1 = R_4 \times (I_2 - I_3) = 2000 \times (6.139535 \times 10^{-3} - 6.69767442 \times 10^{-3}) \text{V}$$
$$= -1.11628 \text{ V}$$

$$V_2 = V_s - R_1 \times I_1 = (6 - 5000 \times 3.767442 \times 10^{-3}) \text{V} = -12.83721 \text{ V}$$

The power absorbed or supplied by all elements are computed as:

$$P_{R_1} = I_1^2 R_1 = 70.9681 \text{ mW}$$

$$P_{R_2} = (I_1 - I_2)^2 R_2 = 16.8805 \text{ mW}$$

$$P_{R_3} = (I_1 - I_3)^2 R_3 = 34.3451 \text{ mW}$$

$$P_{R_4} = (I_2 - I_3)^2 R_4 = 0.6230 \text{ mW}$$

$$P_{Vs} = (-I_2) V_s = -36.83721 \text{ mW}$$

$$P_{VCCS} = I_3 V_2 = -85.97945 \text{ mW}$$

Notice that $P_{R_1} + P_{R_2} + P_{R_3} + P_{R_4} + P_{Vs} + P_{VCCS} = 0$.

MATLAB

```
%EXAMPLE 3.13
clear all;format long;
Vs=6;
R1=5000;R2=3000;R3=4000;R4=2000;
syms I1 I2 I3
[I1,I2,I3]=solve(R1*I1+R3*(I1-I3)+R2*(I1-I2), ...
-Vs+R2*(I2-I1)+R4*(I2-I3), ...
I3==-0.006*R4*(I2-I3),I1,I2,I3);
V1=R4*(I2-I3);
V2=R3*(I1-I3)+V1;
PR1=R1*I1^2;
PR2=R2*(I1-I2)^2;
```

continued

Example 3.13 continued

MATLAB continued

```
PR3=R3*(I1-I3)^2;
PR4=R4*(I2-I3)^2;
PVCCS=I3*V2;
PVs=-I2*Vs;
PSum=PR1+PR2+PR3+PR4+PVCCS+PVs;
I1=vpa(I1,7)
I2=vpa(I2,7)
I3=vpa(I3,7)
V1=vpa(V1,7)
V2=vpa(V2,7)
PR1=vpa(PR1,7)
PR2=vpa(PR2,7)
PR3=vpa(PR3,7)
PR4=vpa(PR4,7)
PVCCS=vpa(PVCCS,7)
PVs=vpa(PVs,7)
PSum=vpa(PSum,7)

Answers:
I1 =
0.003767442
I2 =
0.006139535
I3 =
0.006697674
V1 =
-1.116279
V2 =
-12.83721
PR1 =
0.07096809
PR2 =
0.01688048
PR3 =
0.03434505
PR4 =
0.0006230395
PVCCS =
-0.08597945
PVs =
-0.03683721
PSum =
0.0
```

Exercise 3.13

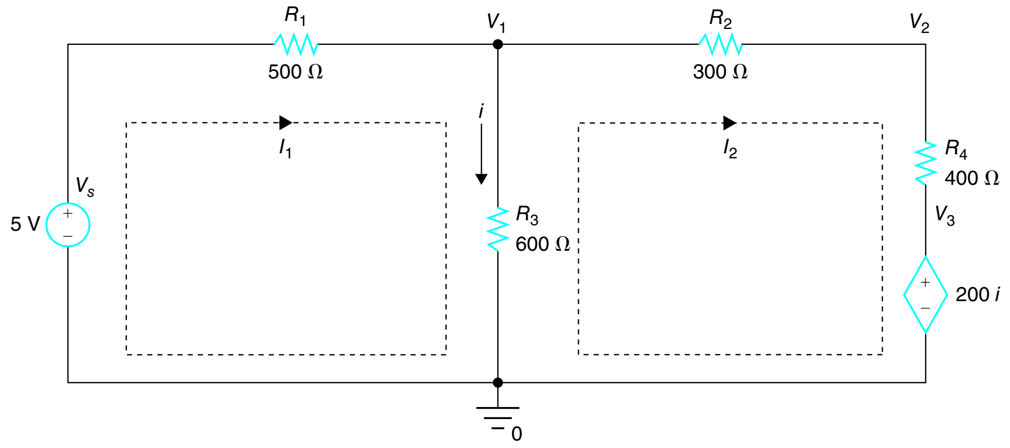
Find the mesh currents I_1, I_2 and voltages V_1, V_2, and V_3 in the circuit shown in Figure 3.35.

continued

Exercise 3.13 continued

FIGURE 3.35

The circuit for EXERCISE 3.13.



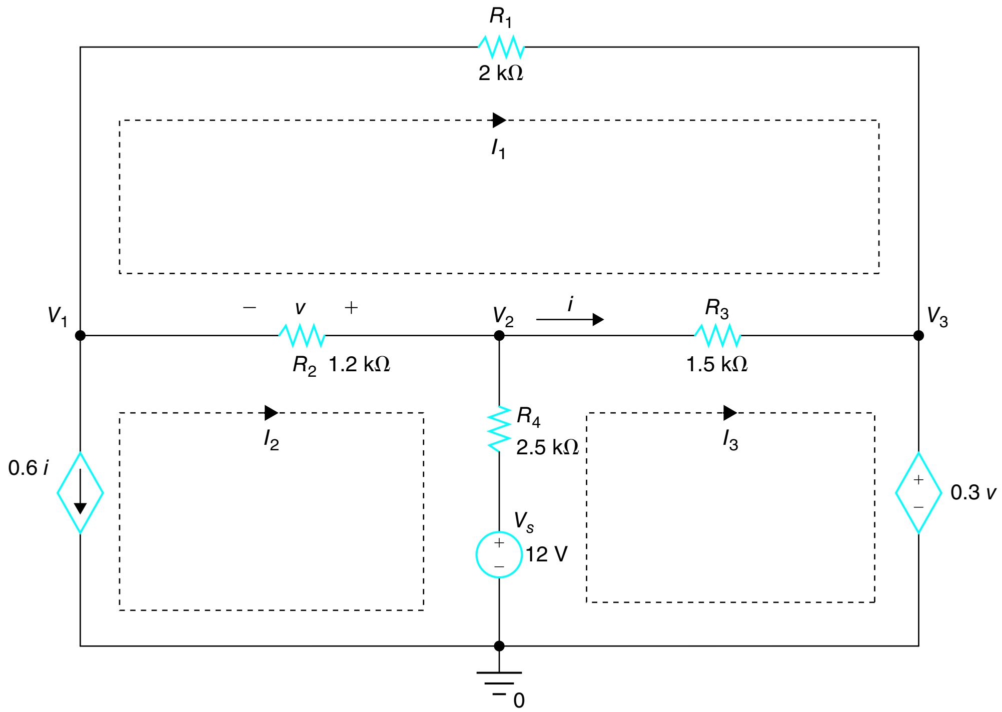
Answer:
$I_1 = 5.6701$ mA, $I_2 = 2.0619$ mA, $V_1 = 2.16495$ V, $V_2 = 1.5464$ V, $V_3 = 0.72165$ V.

EXAMPLE 3.14

Find the mesh currents I_1, I_2, and I_3 and voltages V_1, V_2, and V_3 in the circuit shown in Figure 3.36.

FIGURE 3.36

A circuit with dependent sources.



continued

Example 3.14 continued

In this circuit, the current-dependent current source has a direction opposite that of the direction chosen for mesh current I_2; that is,

$$I_2 = -0.6i = -0.6(I_3 - I_1) \quad (3.70)$$

Summing the voltage drops around mesh 1 (on the upper side), we obtain

$$R_1 I_1 + R_3(I_1 - I_3) + R_2(I_1 - I_2) = 0$$

Substituting the resistance values, we get

$$2000I_1 + 1500(I_1 - I_3) + 1200(I_1 - I_2) = 0$$

which can be simplified to

$$4700I_1 - 1200I_2 - 1500I_3 = 0 \quad (3.71)$$

Substitution of Equation (3.70) into Equation (3.71) yields

$$3980I_1 - 780I_3 = 0 \quad (3.72)$$

Summing the voltage drops around mesh 3 (on the lower right side), we obtain

$$-V_s + R_4(I_3 - I_2) + R_3(I_3 - I_1) + 0.3R_2(I_1 - I_2) = 0$$

Substituting the resistance and voltage values, we get

$$-12 + 2500(I_3 - I_2) + 1500(I_3 - I_1) + 0.3 \times 1200 \times (I_1 - I_2) = 0$$

which can be simplified to

$$-1140I_1 - 2860I_2 + 4000I_3 = 12 \quad (3.73)$$

Substitution of Equation (3.70) into Equation (3.73) yields

$$-2856I_1 + 5716I_3 = 12 \quad (3.74)$$

Equations (3.72) and (3.74) can be solved using the substitution method:

Solving Equation (3.72) for I_3, we obtain

$$I_3 = \frac{398}{78} I_1$$

Substituting I_3 into Equation (3.74), we get

$$-2856I_1 + 5716 \times \frac{398}{78} I_1 = 12$$

Thus, we obtain

$$I_1 = \frac{12}{-2856 + 5716 \times \frac{398}{78}} = 456.0959\ \mu\text{A}$$

continued

Example 3.14 continued

$$I_3 = \frac{398}{78} I_1 = 2.3273 \text{ mA}$$

$$I_2 = -0.6(I_3 - I_1) = -1.1227 \text{ mA}$$

Alternatively, Equations (3.72) and (3.74) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 3980 & -780 \\ -2856 & 5716 \end{vmatrix} = 3980 \times 5716 - (-780) \times (-2856) = 2.0522 \times 10^7$$

$$I_1 = \frac{\begin{vmatrix} 0 & -780 \\ 12 & 5715 \end{vmatrix}}{\Delta} A = \frac{9360}{2.0522 \times 10^7} A = 456.0959 \ \mu\text{A}$$

$$I_3 = \frac{\begin{vmatrix} 3980 & 0 \\ -2856 & 12 \end{vmatrix}}{\Delta} A = \frac{47{,}760}{2.0522 \times 10^7} A = 2.3273 \text{ mA}$$

$$I_2 = -0.6(I_3 - I_1) = -1.1227 \text{ mA}$$

The voltages are given by

$$V_2 = V_s - R_4 \times (I_3 - I_2) = 3.3751 \text{ V}$$

$$V_1 = V_2 - R_2 \times (I_1 - I_2) = 1.4806 \text{ V}$$

$$V_3 = V_1 - R_1 \times I_1 = 0.5684 \text{ V}$$

MATLAB

```
%EXAMPLE 3.14
clear all;format long;
Vs=12;R1=2000;R2=1200;R3=1500;R4=2500;
syms I1 I2 I3
[I1,I2,I3]=solve(R1*I1+R3*(I1-I3)+R2*(I1-I2), ...
I2==-0.6*(I3-I1), ...
-Vs+R4*(I3-I2)+R3*(I3-I1)+0.3*R2*(I1-I2),I1,I2,I3);
V2=Vs-R4*(I3-I2);
V1=V2-R2*(I1-I2);
V3=V1-R1*I1;
I1=vpa(I1,10)
I2=vpa(I2,10)
I3=vpa(I3,10)
V1=vpa(V1,10)
V2=vpa(V2,10)
V3=vpa(V3,10)

Answers:
I1 =
0.0004560958971
I2 =
-0.001122697593
I3 =
0.002327258552
```

continued

Example 3.14 continued

MATLAB continued

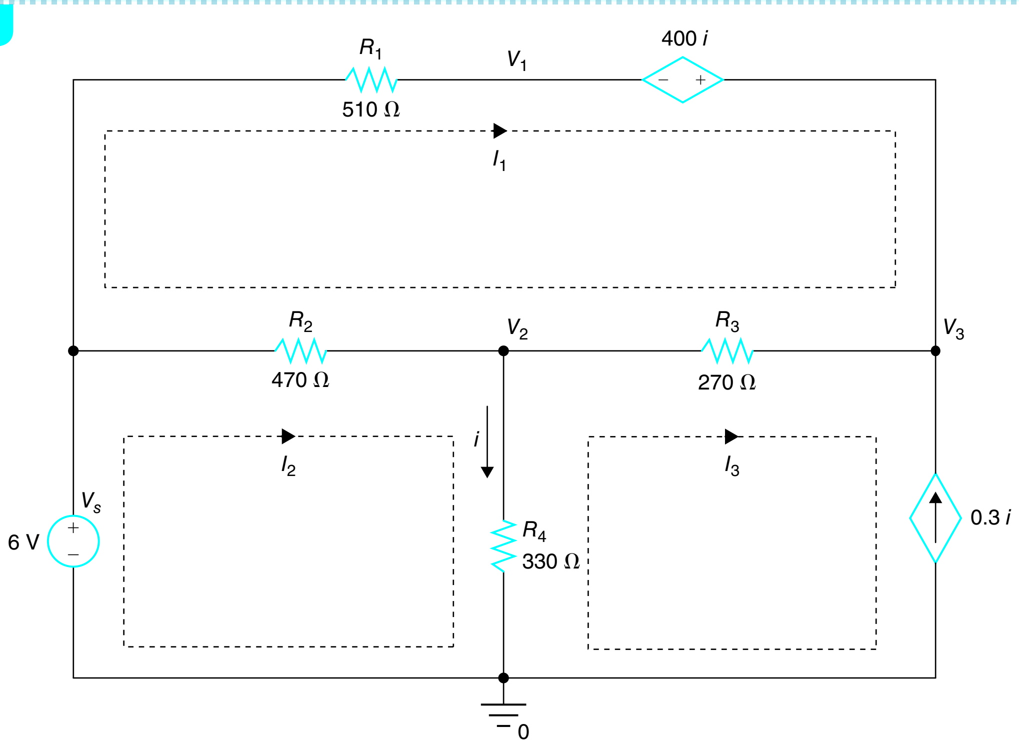
```
V1 =
1.480557451
V2 =
3.375109638
V3 =
0.5683656564
```

Exercise 3.14

Find the mesh currents I_1, I_2, and I_3 and voltages V_1, V_2, and V_3 in the circuit shown in Figure 3.37.

FIGURE 3.37

The circuit for EXERCISE 3.14.



Answer:

$I_1 = 7.4886$ mA, $I_2 = 10.1119$ mA, $I_3 = -4.3337$ mA, $V_1 = 2.1808$ V, $V_2 = 4.7670$ V, $V_3 = 7.9591$ V.

3.5 Supermesh

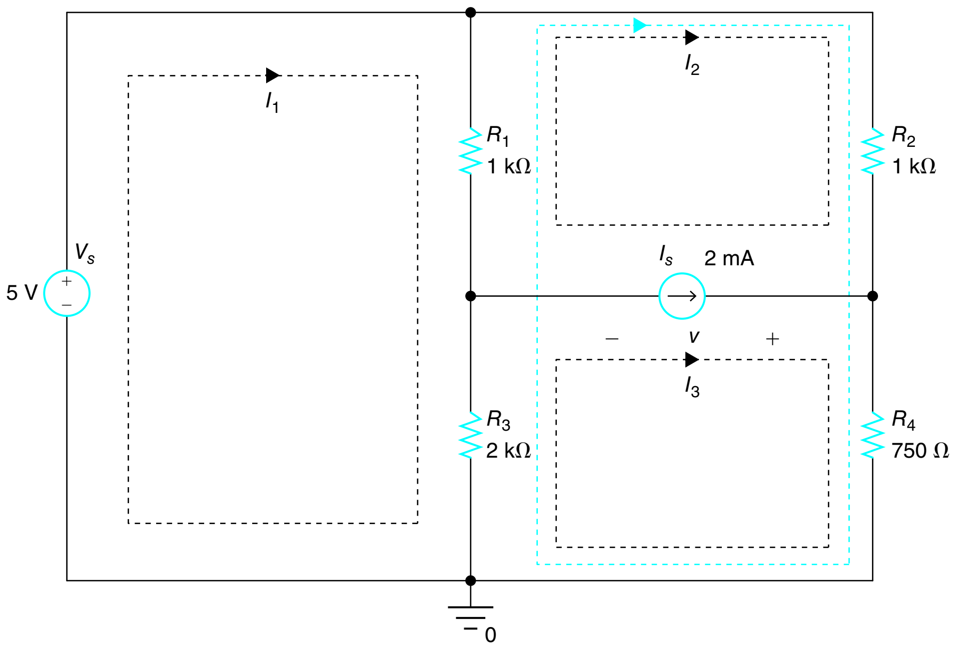
Recognizing that we use KVL in mesh analysis, if there is a current source that is a common branch between two different meshes, we do not know the voltage drop across the current source. Let the unknown voltage across the current source be included in the loop equation. Ohm's law cannot be applied because of the unknown internal resistance of the current source. However, to overcome this, and to complete the loop equation, let the unknown voltage across the current source be *v*. Then, write the mesh equation for each mesh and add the two equations to remove the unknown voltage *v*. The voltage *v* is removed when added because in one equation, the voltage drop across the current source is v and in the other equation, the voltage drop across the current source is $-v$. Alternatively, the sum of the two equations can be obtained directly by defining a **supermesh** consisting of the two meshes, excluding the current source. When the voltage drops around the supermesh are added, we get the same equation that we obtain by adding the two equations. The extra equation needed to find the mesh currents is obtained by representing the current of the current source by the difference of the two mesh currents. The current of the current source is obtained by subtracting the mesh current pointing in the opposite direction from the mesh current pointing in the same direction as the current source.

EXAMPLE 3.15

Use mesh analysis to find the voltage v across the current source in the circuit shown in Figure 3.38.

FIGURE 3.38

The circuit for EXAMPLE 3.15.



Summing the voltage drops around mesh 1 in the clockwise direction, we obtain

$$-V_s + R_1(I_1 - I_2) + R_3(I_1 - I_3) = 0$$

Substituting the resistance and voltage values, we get

$$-5 + 1000(I_1 - I_2) + 2000(I_1 - I_3) = 0$$

which can be rearranged as

$$3000I_1 - 1000I_2 - 2000I_3 = 5 \qquad (3.75)$$

Summing the voltage drops around mesh 2 in the clockwise direction, we obtain

$$R_1(I_2 - I_1) + R_2I_2 + v = 0$$

continued

Example 3.15 continued

Substituting the resistance values, we get

$$1000(I_2 - I_1) + 1000I_2 + v = 0 \tag{3.76}$$

Summing the voltage drops around mesh 3 in the clockwise direction, we obtain

$$R_3(I_3 - I_1) - v + R_4I_3 = 0$$

Substituting the resistance values, we get

$$2000(I_3 - I_1) - v + 750I_3 = 0 \tag{3.77}$$

The unknown voltage v across the current source can be removed by adding Equations (3.76) and (3.77):

$$1000(I_2 - I_1) + 1000I_2 + 750I_3 + 2000(I_3 - I_1) = 0 \tag{3.78}$$

which can be rearranged as

$$-3000I_1 + 2000I_2 + 2750I_3 = 0 \tag{3.79}$$

Notice that Equation (3.78) can be directly obtained by summing the voltage drops around the supermesh consisting of mesh 2 and mesh 3, shown in blue in Figure 3.38. The current through the current source $I_s = 2$ mA is given by

$$I_3 - I_2 = I_s = 2 \text{ mA}$$

which can be rearranged as

$$-I_2 + I_3 = 0.002 \tag{3.80}$$

Equations (3.75), (3.79), and (3.80) can be solved using the substitution method: Solving Equation (3.80) for I_3, we obtain

$$I_3 = I_2 + 0.002$$

Substituting I_3 into Equations (3.75) and (3.79), we obtain

$$3000\,I_1 - 3000\,I_2 = 9$$

$$-3000\,I_1 + 4750\,I_2 = -5.5$$

Adding these two equations, we obtain

$$1750\,I_2 = 3.5$$

Thus, we get

$$I_2 = \frac{3.5}{1750} = 0.002 \text{ A} = 2 \text{ mA}$$

$$I_3 = I_2 + 0.002 = 0.004 \text{ A} = 4 \text{ mA}$$

continued

Example 3.15 continued

From Equation (3.79), we get

$$I_1 = \frac{2000}{3000} I_2 + \frac{2750}{3000} I_3 = 0.005 \text{ A} = 5 \text{ mA}$$

Alternatively, Equations (3.75), (3.79), and (3.80) can be solved using Cramer's rule:

Let

$$\Delta = \begin{vmatrix} 3000 & -1000 & -2000 \\ -3000 & 2000 & 2750 \\ 0 & -1 & 1 \end{vmatrix} = 5{,}250{,}000$$

$$\Delta_1 = \begin{vmatrix} 5 & -1000 & -2000 \\ 0 & 2000 & 2750 \\ 0.002 & -1 & 1 \end{vmatrix} = 26{,}250$$

$$\Delta_2 = \begin{vmatrix} 3000 & 5 & -2000 \\ -3000 & 0 & 2750 \\ 0 & 0.002 & 1 \end{vmatrix} = 10{,}500$$

$$\Delta_3 = \begin{vmatrix} 3000 & -1000 & 5 \\ -3000 & 2000 & 0 \\ 0 & -1 & 0.002 \end{vmatrix} = 21{,}000$$

Then, we have

$$I_1 = \frac{\Delta_1}{\Delta} = 5 \text{ mA}, \quad I_2 = \frac{\Delta_2}{\Delta} = 2 \text{ mA}, \quad I_3 = \frac{\Delta_3}{\Delta} = 4 \text{ mA}$$

MATLAB

```
A=[3000 -1000 -2000;-3000 2000 2750;0 -1 1];
b=[5;0;0.002];
I=A\b

Answer:
I =
    0.005000000000000
    0.002000000000000
    0.004000000000000
```

The voltage across R_3 is given by

$$V_{R_3} = R_3(I_1 - I_3) = 2000(0.005 - 0.004) = 2 \text{ V}$$

The voltage across R_4 is given by

$$V_{R_4} = R_4 I_3 = 750 \times 0.004 = 3 \text{ V}$$

Thus, the voltage v across the current source is given by

$$v = V_{R_4} - V_{R_3} = 3 \text{ V} - 2 \text{ V} = 1 \text{ V}$$

Exercise 3.15

Use mesh analysis to find V_1 in the circuit shown in Figure 3.39.

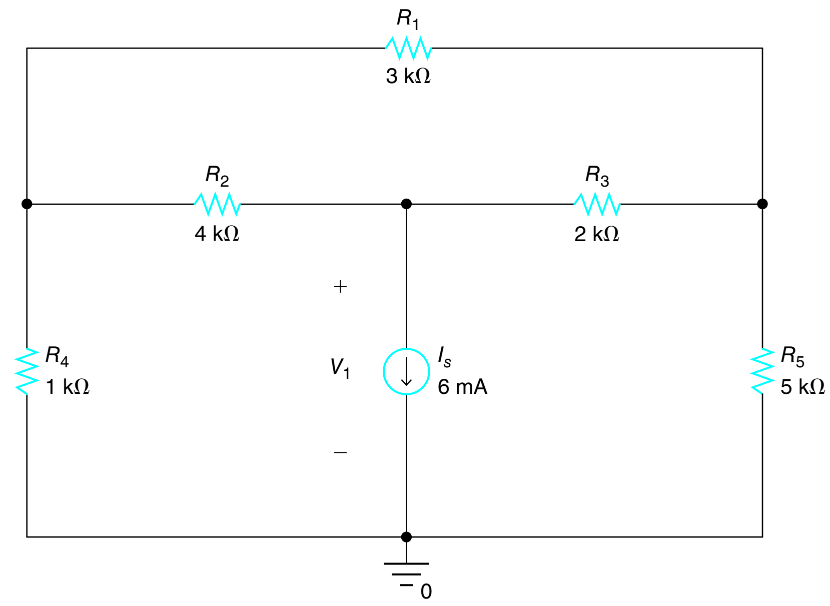


FIGURE 3.39
The circuit for EXERCISE 3.15.

Answer:
$V_1 = -15.25$ V.

EXAMPLE 3.16

Find the mesh currents I_1, I_2, I_3, and I_4 for the circuit shown in Figure 3.40.

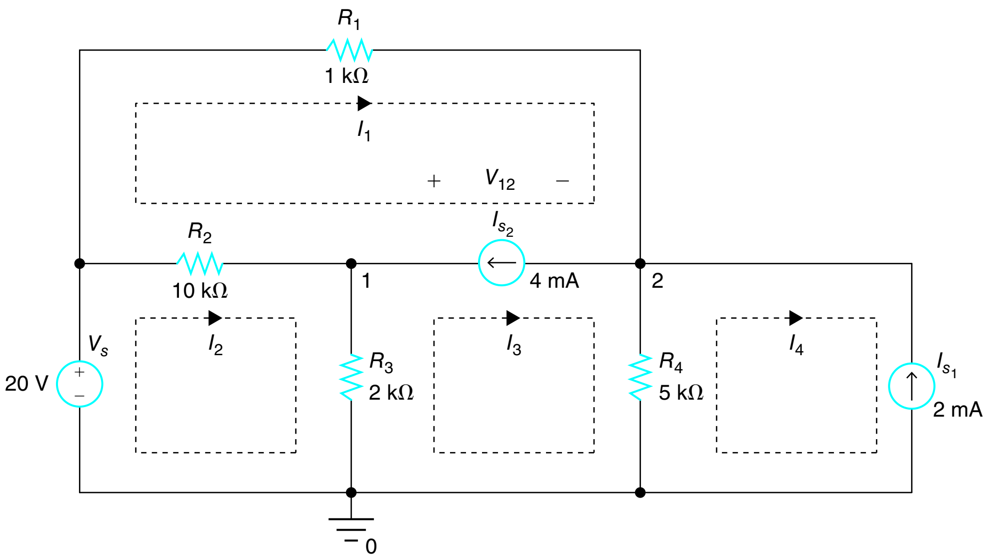


FIGURE 3.40
The circuit for EXAMPLE 3.16.

continued

Example 3.16 continued

Mesh current I_4 is in the opposite direction of current source I_{s_1}. Thus, $I_4 = -I_{s_1} = -2$ mA. Mesh 1 with mesh current I_1 and mesh 3 with mesh current I_3 share a current source I_{s_2}. Since the direction of I_1 coincides with I_{s_2}, we have

$$I_1 - I_3 = I_{s_2} \tag{3.81}$$

Let the voltage across the current source I_{s_2} from node 1 to node 2 be V_{12}, as shown in Figure 3.40. The mesh equation on mesh 1 with mesh current I_1 is given by, starting from the left side of R_1 in the clockwise direction,

$$R_1 I_1 - V_{12} + R_2(I_1 - I_2) = 0 \tag{3.82}$$

The mesh equation on mesh 3 with mesh current I_3 is given by, starting from node 1 in the clockwise direction,

$$V_{12} + R_4(I_3 - I_4) + R_3(I_3 - I_2) = 0 \tag{3.83}$$

The unknown voltage V_{12} across the current source is eliminated by adding Equations (3.82) and (3.83):

$$R_1 I_1 + R_4(I_3 - I_4) + R_3(I_3 - I_2) + R_2(I_1 - I_2) = 0 \tag{3.84}$$

Equation (3.84) suggests that if a supermesh consisting of meshes 1 and 3 without the current source I_{s_2} is formed as shown in Figure 3.41 (blue dashed line), the sum of voltage drops around this supermesh will be zero. Notice that the original mesh currents I_1 and I_3 are not changed.

For mesh 2 with mesh current I_2, the mesh equation is given by

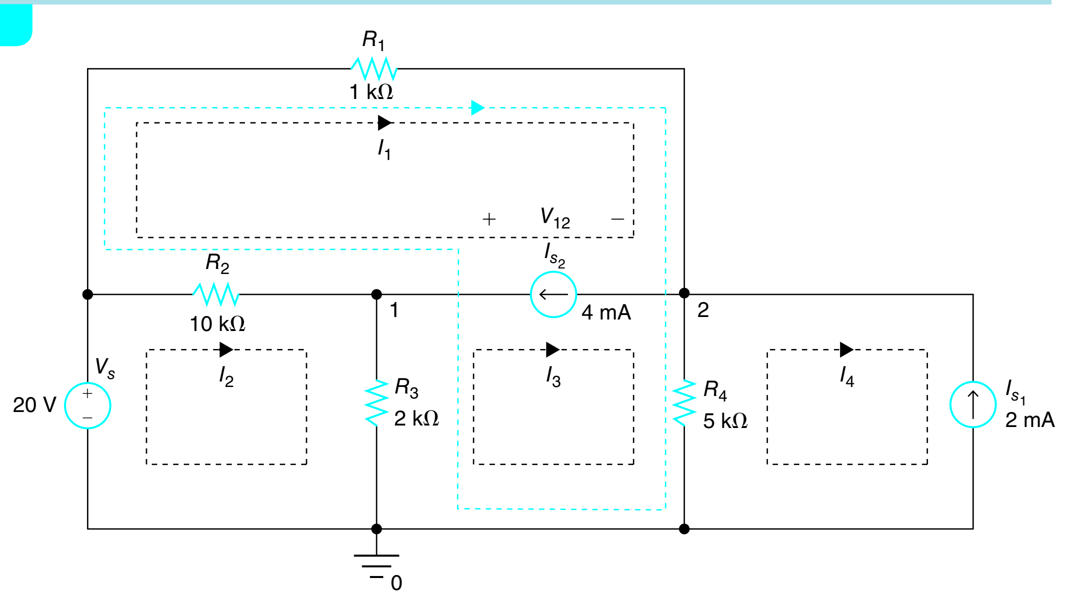
$$-V_s + R_2(I_2 - I_1) + R_3(I_2 - I_3) = 0 \tag{3.85}$$

Substitution of $V_s = 20$ V, $I_4 = -I_{s_1} = -2$ mA, $I_{s_2} = 4$ mA, $R_1 = 1\ k\Omega$, $R_2 = 10\ k\Omega$, $R_3 = 2\ k\Omega$, $R_4 = 5\ k\Omega$ into Equations (3.81), (3.84), and (3.85) results in

$$I_1 - I_3 = 0.004 \tag{3.86}$$

FIGURE 3.41

The supermesh is outlined in the blue dashed line.



continued

Example 3.16 continued

$$1000I_1 + 5000[I_3 - (-0.002)] + 2000(I_3 - I_2) + 10{,}000(I_1 - I_2) = 0 \quad (3.87)$$

$$-20 + 10{,}000(I_2 - I_1) + 2000(I_2 - I_3) = 0 \quad (3.88)$$

Simplifying Equations (3.86)–(3.88), we obtain

$$I_1 - I_3 = 0.004 \quad (3.89)$$

$$11I_1 - 12I_2 + 7I_3 = -0.01 \quad (3.90)$$

$$-10I_1 + 12I_2 - 2I_3 = 0.02 \quad (3.91)$$

Equations (3.89), (3.90), and (3.91) can be solved using the substitution method:

Solving Equation (3.89) for I_3, we obtain

$$I_3 = I_1 - 0.004$$

Substituting I_3 into Equations (3.90) and (3.91), we obtain

$$18I_1 - 12I_2 = 0.018$$

$$-12I_1 + 12I_2 = 0.012$$

Adding these two equations, we obtain

$$6I_1 = 0.03$$

Thus, we obtain

$$I_1 = \frac{0.03}{6} = 0.005 \text{ A} = 5 \text{ mA}$$

$$I_3 = I_1 - 0.004 = 0.001 \text{ A} = 1 \text{ mA}$$

From Equation (3.90), we get

$$I_2 = \frac{11}{12}I_1 + \frac{7}{12}I_3 + \frac{0.01}{12} = 0.006 \text{ A} = 6 \text{ mA}$$

Alternatively, Equations (3.89), (3.90), and (3.91) can be solved using Cramer's rule:

$$\Delta = \begin{vmatrix} 1 & 0 & -1 \\ 11 & -12 & 7 \\ -10 & 12 & -2 \end{vmatrix} = -(0)\begin{vmatrix} 11 & 7 \\ -10 & -2 \end{vmatrix} + (-12)\begin{vmatrix} 1 & -1 \\ -10 & -2 \end{vmatrix} - 12\begin{vmatrix} 1 & -1 \\ 11 & 7 \end{vmatrix}$$
$$= (-12)[(1)(-2) - (-1)(-10)] - 12[(1)(7) - (-1)(11)]$$
$$= (-12)[-2 - 10] - 12[7 + 11] = 144 - 216 = -72$$

continued

Example 3.16 continued

$$I_1 = \frac{\begin{vmatrix} 0.004 & 0 & -1 \\ -0.01 & -12 & 7 \\ 0.02 & 12 & -2 \end{vmatrix}}{\Delta} = \frac{-(0)\begin{vmatrix} -0.01 & 7 \\ 0.02 & -2 \end{vmatrix} + (-12)\begin{vmatrix} 0.004 & -1 \\ 0.02 & -2 \end{vmatrix} - 12\begin{vmatrix} 0.004 & -1 \\ -0.01 & 7 \end{vmatrix}}{\Delta}$$

$$= \frac{(-12)[(0.004)(-2) - (-1)(0.02)] - 12[(0.004)(7) - (-1)(-0.01)]}{\Delta}$$

$$= \frac{(-12)[-0.008 + 0.02] - 12[0.028 - 0.01]}{\Delta} = \frac{-0.144 - 0.216}{-72} = \frac{0.36}{72} = 0.005 \text{ A} = 5 \text{ mA}$$

$$I_2 = \frac{\begin{vmatrix} 1 & 0.004 & -1 \\ 11 & -0.01 & 7 \\ -10 & 0.02 & -2 \end{vmatrix}}{\Delta} = \frac{(1)\begin{vmatrix} -0.01 & 7 \\ 0.02 & -2 \end{vmatrix} - (0.004)\begin{vmatrix} 11 & 7 \\ -10 & -2 \end{vmatrix} + (-1)\begin{vmatrix} 11 & -0.01 \\ -10 & 0.02 \end{vmatrix}}{\Delta}$$

$$= \frac{(1)[(-0.01)(-2) - (7)(0.02)] - 0.004[(11)(-2) - (7)(-10)] - [(11)(0.02) - (-0.01)(-10)]}{\Delta}$$

$$= \frac{[0.02 - 0.14] - 0.004[-22 + 70] - [0.22 - 0.1]}{\Delta} = \frac{-0.12 - 0.192 - 0.12}{-72}$$

$$= \frac{-0.432}{-72} = 0.006 \text{ A} = 6 \text{ mA}$$

$$I_3 = \frac{\begin{vmatrix} 1 & 0 & 0.004 \\ 11 & -12 & -0.01 \\ -10 & 12 & 0.02 \end{vmatrix}}{\Delta} = \frac{-(0)\begin{vmatrix} 11 & -0.01 \\ -10 & 0.02 \end{vmatrix} + (-12)\begin{vmatrix} 1 & 0.004 \\ -10 & 0.02 \end{vmatrix} - (12)\begin{vmatrix} 1 & 0.004 \\ 11 & -0.01 \end{vmatrix}}{\Delta}$$

$$= \frac{(-12)[(1)(0.02) - (0.004)(-10)] - (12)[(1)(-0.01) - (0.004)(11)]}{\Delta}$$

$$= \frac{(-12)[0.02 + 0.04] - (12)[-0.01 - 0.044]}{\Delta} = \frac{-0.72 + 0.648}{-72} = \frac{-0.072}{-72} = 0.001 \text{ A} = 1 \text{ mA}$$

MATLAB

```
A=[1 0 -1;11 -12 7;-10 12 -2]
b=[0.004;-0.01;0.02]
I=A\b

Answer:
I =
    0.005000000000000
    0.006000000000000
    0.001000000000000
```

Alternative solution:

```
%EXAMPLE 3.16
clear all;
Vs=20;Is1=2e-3;Is2=4e-3;R1=1000;R2=10000;R3=2000;R4=5000;
syms I1 I2 I3 I4
[I1,I2,I3,I4]=solve(R1*I1+R4*(I3-I4)+R3*(I3-I2)+R2*(I1-I2),...
-Vs+R2*(I2-I1)+R3*(I2-I3),...
I1-I3==Is2,...
```

continued

Example 3.16 continued

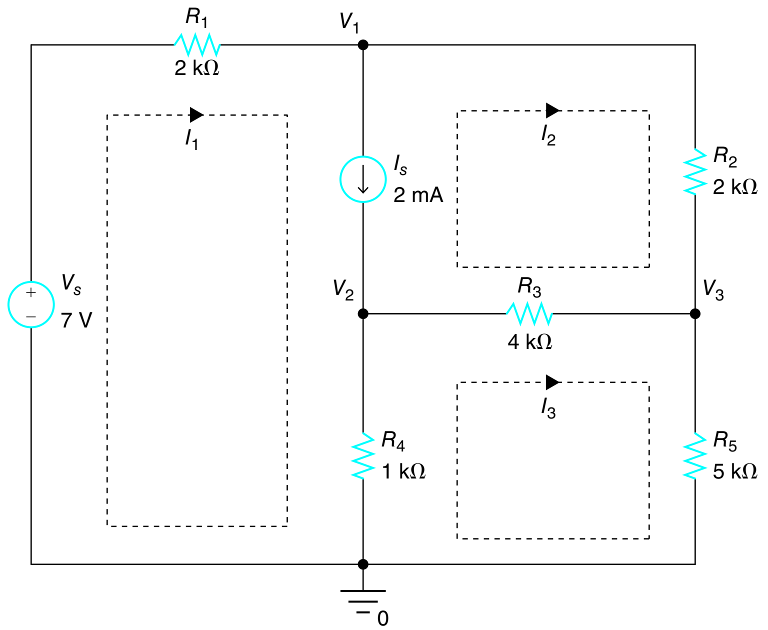
```
I4==-Is1,I1,I2,I3,I4);
I1=vpa(I1,7)
I2=vpa(I2,7)
I3=vpa(I3,7)
I4=vpa(I4,7)

Answers:
I1 =
0.005
I2 =
0.006
I3 =
0.001
I4 =
-0.002
```

Exercise 3.16

Find mesh currents I_1, I_2, and I_3 and voltages V_1, V_2, and V_3 in the circuit shown in Figure 3.42.

FIGURE 3.42

The circuit for EXERCISE 3.16.



Answer:
$I_1 = 2.3077$ mA, $I_2 = 307.6923$ μA, $I_3 = 353.8462$ μA, $V_1 = 2.3846$ V, $V_2 = 1.9538$ V, $V_3 = 1.7692$ V.

EXAMPLE 3.17

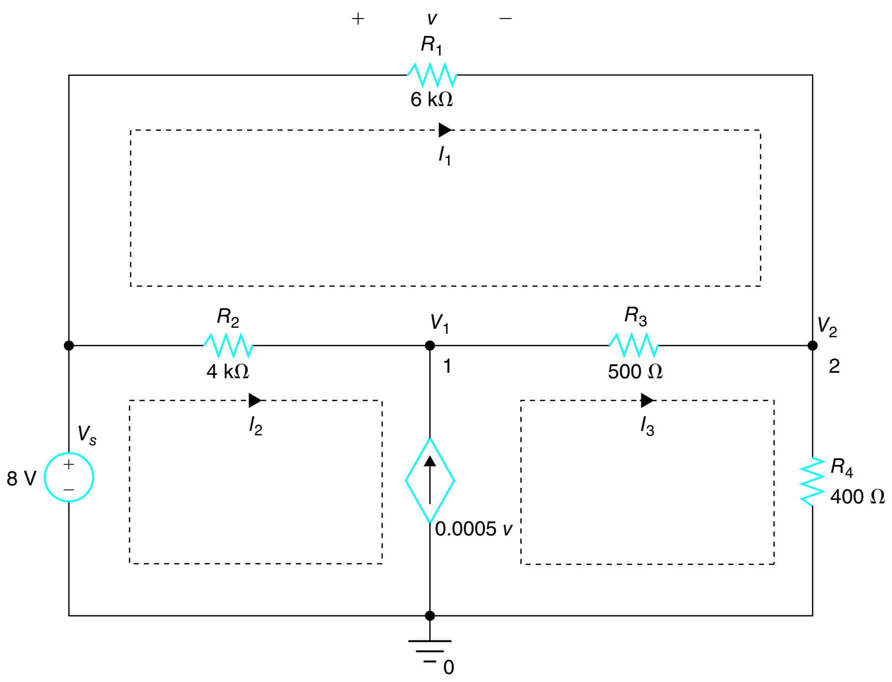
Use mesh analysis to find voltages V_1 and V_2 in the circuit shown in Figure 3.43.

continued

Example 3.17 continued

FIGURE 3.43

The circuit for EXAMPLE 3.17.



Mesh currents I_1, I_2, and I_3 are shown in Figure 3.43. Due to the presence of a voltage-controlled current source (VCCS) between meshes 2 and 3, meshes 2 and 3 form a supermesh. The current through the VCCS equals $I_3 - I_2$. Thus, we have

$$I_3 - I_2 = 0.0005v = 0.0005 \times 6000I_1$$

or

$$-3I_1 - I_2 + I_3 = 0 \tag{3.92}$$

Summing the voltage drops around mesh 1 in the clockwise direction, we obtain

$$R_1I_1 + R_3(I_1 - I_3) + R_2(I_1 - I_2) = 0$$

Substituting the resistance values, we get

$$6000I_1 + 500(I_1 - I_3) + 4000(I_1 - I_2) = 0$$

or

$$10.5I_1 - 4I_2 - 0.5I_3 = 0 \tag{3.93}$$

Summing the voltage drops around a supermesh consisting of meshes 2 and 3 in the clockwise direction, we obtain

$$-V_s + R_2(I_2 - I_1) + R_3(I_3 - I_1) + R_4I_3 = 0$$

Substituting the resistance and voltage values, we get

$$-8 + 4000(I_2 - I_1) + 500(I_3 - I_1) + 400I_3 = 0$$

continued

Example 3.17 continued

or

$$-4.5I_1 + 4I_2 + 0.9I_3 = 0.008 \tag{3.94}$$

Equations (3.92), (3.93), and (3.94) can be solved using the substitution method: Multiplying Equation (3.92) by 4, we obtain

$$-12I_1 - 4I_2 + 4I_3 = 0$$

Adding this equation and Equation (3.94), we obtain

$$-16.5I_1 + 4.9I_3 = 0.008$$

Adding Equations (3.93) and (3.94), we obtain

$$6I_1 + 0.4I_3 = 0.008$$

Solving this equation for I_3, we get

$$I_3 = 0.02 - 15I_1$$

Substituting I_3 into $-16.5I_1 + 4.9I_3 = 0.008$, we obtain

$$-16.5I_1 + 4.9(0.02 - 15I_1) = 0.008$$

or

$$-90I_1 = -0.09$$

Thus, we get

$$I_1 = \frac{-0.09}{-90} = 0.001 \text{ A} = 1 \text{ mA}$$

$$I_3 = 0.02 - 15I_1 = 5 \text{ mA}$$

From Equation (3.92), we get

$$I_2 = -3I_1 + I_3 = 2 \text{ mA}$$

Alternatively, with the application of Cramer's rule to Equations (3.92)–(3.94), we get

$$\begin{aligned}\Delta &= \begin{vmatrix} -3 & -1 & 1 \\ 10.5 & -4 & -0.5 \\ -4.5 & 4 & 0.9 \end{vmatrix} = (-3)\begin{vmatrix} -4 & -0.5 \\ 4 & 0.9 \end{vmatrix} - (-1)\begin{vmatrix} 10.5 & -0.5 \\ -4.5 & 0.9 \end{vmatrix} + (1)\begin{vmatrix} 10.5 & -4 \\ -4.5 & 4 \end{vmatrix} \\ &= (-3)[(-4)(0.9) - (-0.5)(4)] + [(10.5)(0.9) - (-0.5)(-4.5)] \\ &\quad + [(10.5)(4) - (-4)(-4.5)] \\ &= (-3)[-3.6 + 2] + [9.45 - 2.25] + [42 - 18] = 4.8 + 7.2 + 24 = 36\end{aligned}$$

$$\begin{aligned}I_1 &= \frac{\begin{vmatrix} 0 & -1 & 1 \\ 0 & -4 & -0.5 \\ 0.008 & 4 & 0.9 \end{vmatrix}}{\Delta} = \frac{0.008\begin{vmatrix} -1 & 1 \\ -4 & -0.5 \end{vmatrix}}{36} = \frac{0.008[(-1)(-0.5) - (1)(-4)]}{36} \\ &= \frac{0.008[0.5 + 4]}{36} = \frac{0.036}{36} = 0.001 \text{ A} = 1 \text{ mA}\end{aligned}$$

continued

Example 3.17 continued

$$I_2 = \frac{\begin{vmatrix} -3 & 0 & 1 \\ 10.5 & 0 & -0.5 \\ -4.5 & 0.008 & 0.9 \end{vmatrix}}{\Delta} = \frac{-0.008\begin{vmatrix} -3 & 1 \\ 10.5 & -0.5 \end{vmatrix}}{36}$$

$$= \frac{-0.008[(-3)(-0.5) - (1)(10.5)]}{36}$$

$$= \frac{-0.008[1.5 - 10.5]}{36} = \frac{0.072}{36} = 0.002 \text{ A} = 2 \text{ mA}$$

$$I_3 = \frac{\begin{vmatrix} -3 & -1 & 0 \\ 10.5 & -4 & 0 \\ -4.5 & 4 & 0.008 \end{vmatrix}}{\begin{vmatrix} -3 & -1 & 1 \\ 10.5 & -4 & -0.5 \\ -4.5 & 4 & 0.9 \end{vmatrix}} = \frac{0.008\begin{vmatrix} -3 & 1 \\ 10.5 & -4 \end{vmatrix}}{36} = \frac{0.008[(-3)(-4) - (-1)(10.5)]}{36}$$

$$= \frac{0.008[12 + 10.5]}{36} = \frac{0.18}{36} = 0.005 \text{ A} = 5 \text{ mA}$$

$$V_1 = V_s - R_2(I_2 - I_1) = 8 \text{ V} - 4000\ \Omega \times (0.002 - 0.001) \text{ A} = 4 \text{ V}$$

$$V_2 = R_4 I_3 = 400\ \Omega \times 0.005 \text{ A} = 2 \text{ V}$$

MATLAB

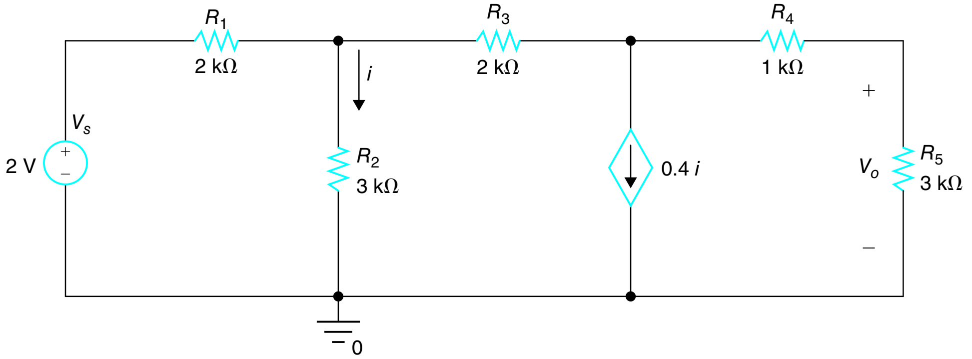
```
A=[-3 -1 1;10.5 -4 -0.5;-4.5 4 0.9]
b=[0;0;0.008]
I=A\b

I =
   0.001000000000000
   0.002000000000000
   0.005000000000000
```

Exercise 3.17

Use mesh analysis to find V_0 in the circuit shown in Figure 3.44.

FIGURE 3.44

The circuit for EXERCISE 3.17.



Answer:

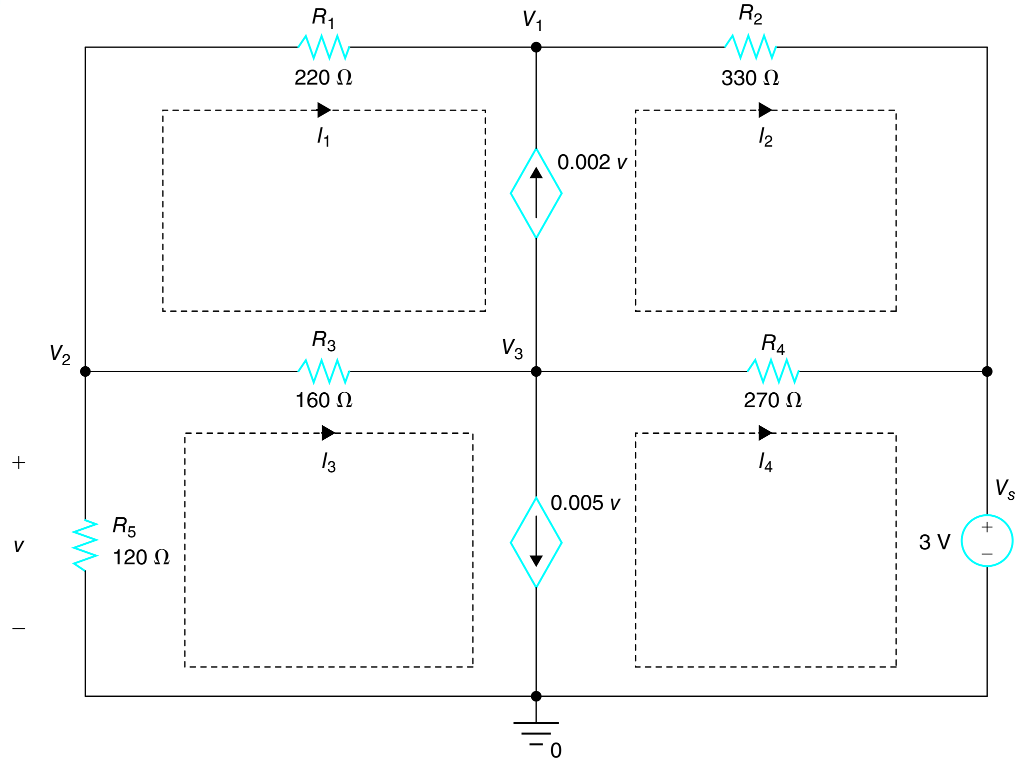
$V_0 = 0.3367$ V.

EXAMPLE 3.18

Find the mesh currents I_1, I_2, I_3, and I_4 and voltages V_1, V_2, and V_3 in the circuit shown in Figure 3.45.

FIGURE 3.45

The circuit for EXAMPLE 3.18.



In this circuit, mesh 1 and mesh 2 form supermesh 1, while mesh 3 and mesh 4 form supermesh 2. From supermesh 1, we have

$$I_2 - I_1 = 0.002v = 0.002 \times 120 \times (-I_3) = -0.24I_3$$

Solving for I_2, we have

$$I_2 = I_1 - 0.24I_3 \tag{3.95}$$

From supermesh 2, we have

$$I_3 - I_4 = 0.005v = 0.005 \times 120 \times (-I_3) = -0.6I_3$$

Thus, we get

$$I_4 = 1.6I_3 \tag{3.96}$$

Summing the voltage drops around supermesh 1, we obtain

$$R_1I_1 + R_2I_2 + R_4(I_2 - I_4) + R_3(I_1 - I_3) = 0$$

Substituting the resistance values, we get

$$220I_1 + 330I_2 + 270(I_2 - I_4) + 160(I_1 - I_3) = 0$$

continued

Example 3.18 continued

which can be simplified to

$$380I_1 + 600I_2 - 160I_3 - 270I_4 = 0 \quad (3.97)$$

Substitution of Equations (3.95) and (3.96) into Equation (3.97) yields

$$980I_1 - 736I_3 = 0 \quad (3.98)$$

Summing the voltage drops around supermesh 2, we obtain

$$R_5I_3 + R_3(I_3 - I_1) + R_4(I_4 - I_2) + V_s = 0$$

Substituting the resistance and voltage values, we get

$$120I_3 + 160(I_3 - I_1) + 270(I_4 - I_2) + 3 = 0$$

which can be simplified to

$$-160I_1 - 270I_2 + 280I_3 + 270I_4 = -3 \quad (3.99)$$

Substitution of Equations (3.95) and (3.96) into Equation (3. 99) yields

$$-430I_1 + 776.8I_3 = -3 \quad (3.100)$$

Equations (3.98) and (3.100) can be solved using the substitution method:

Solving Equation (3.98) for I_3, we obtain

$$I_3 = \frac{980}{736}I_1$$

Substituting I_3 into Equation (3.100), we get

$$-430I_1 + 776.8\left(\frac{980}{736}I_1\right) = -3$$

Thus, we have

$$I_1 = \frac{-3}{-430 + 776.8 \times \dfrac{980}{736}} = -4.9642 \text{ mA}$$

$$I_3 = \frac{980}{736}I_1 = -6.60995 \text{ mA}$$

Alternatively, the application of Cramer's rule to Equations (3.98) and (3.100) yields

$$\Delta = \begin{vmatrix} 980 & -736 \\ -430 & 776.8 \end{vmatrix} = 980 \times 776.8 - (-736) \times (-430) = 4.44784 \times 10^5$$

$$I_1 = \frac{\begin{vmatrix} 0 & -736 \\ -3 & 776.8 \end{vmatrix}}{\Delta} A = \frac{-2208}{4.44784 \times 10^5} A = -4.9642 \text{ mA}$$

continued

Example 3.18 continued

$$I_3 = \frac{\begin{vmatrix} 980 & 0 \\ -430 & -3 \end{vmatrix}}{\Delta} A = \frac{-2940}{4.44784 \times 10^5} A = -6.60995 \text{ mA}$$

$$I_2 = I_1 - 0.24 I_3 = -3.3778 \text{ mA}$$

$$I_4 = 1.6 I_3 = -10.5759 \text{ mA}$$

The voltages are given by

$$V_1 = V_s + R_2 \times I_2 = 1.8853 \text{ V}$$

$$V_2 = V_1 + R_1 \times I_1 = 0.7932 \text{ V}$$

$$V_3 = V_s - R_4 \times (I_2 - I_4) = 1.0565 \text{ V}$$

MATLAB

```
%EXAMPLE 3.18
clear all;
Vs=3;R1=220;R2=330;R3=160;R4=270;R5=120;
syms I1 I2 I3 I4 v
[I1,I2,I3,I4,v]=solve(v==-R5*I3,I2-I1==0.002*v,I3-I4==0.005*v,...
R1*I1+R2*I2+R4*(I2-I4)+R3*(I1-I3),...
R5*I3+R3*(I3-I1)+R4*(I4-I2)+Vs,I1,I2,I3,I4,v);
V1=R2*I2+Vs;
V2=v;
V3=R4*(I4-I2)+Vs;
I1=vpa(I1,7)
I2=vpa(I2,7)
I3=vpa(I3,7)
I4=vpa(I4,7)
V1=vpa(V1,7)
V2=vpa(V2,7)
V3=vpa(V3,7)

Answers:
I1 =
-0.004964207
I2 =
-0.003377819
I3 =
-0.00660995
I4 =
-0.01057592
V1 =
1.88532
V2 =
0.793194
V3 =
1.056513
```

Exercise 3.18

Find the mesh currents I_1, I_2, and I_3 and voltages V_1, V_2, V_3, and V_4 in the circuit shown in Figure 3.46.

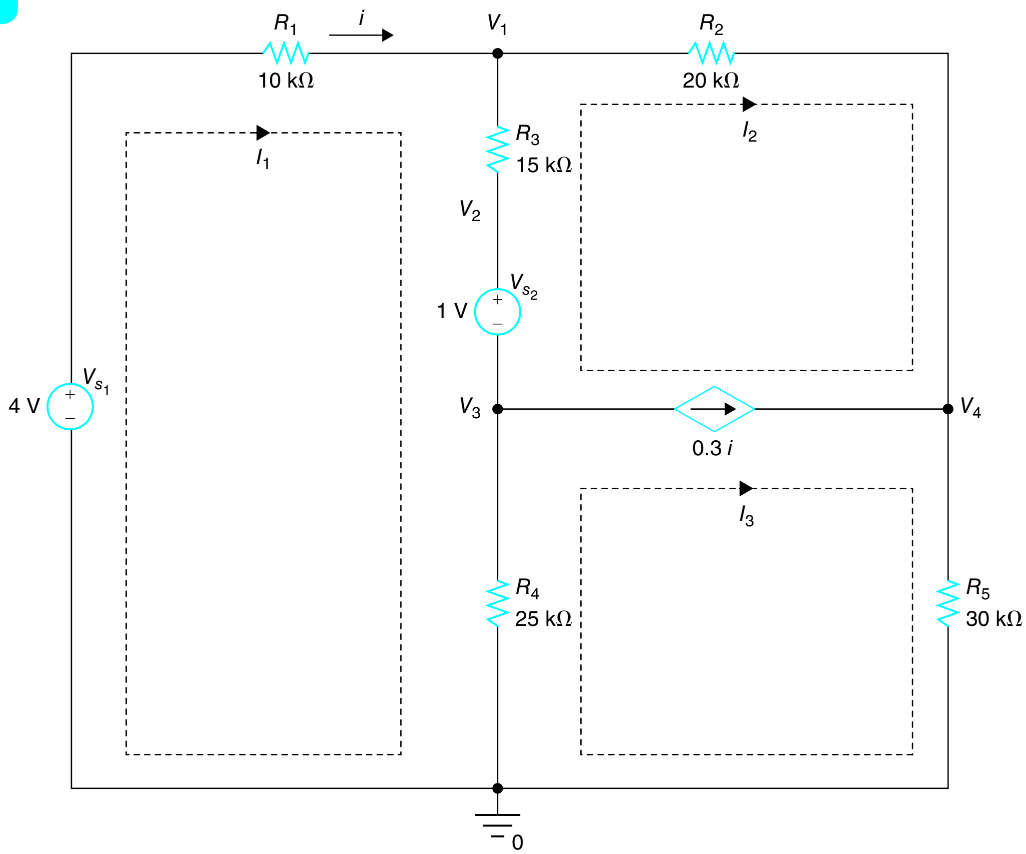


FIGURE 3.46 The circuit for EXERCISE 3.18.

Answer:
$I_1 = 107.4523\ \mu A$, $I_2 = 39.1681\ \mu A$, $I_3 = 71.4038\ \mu A$, $V_1 = 2.9255$ V, $V_2 = 1.9012$ V, $V_3 = 0.9012$ V, $V_4 = 2.1421$ V.

3.6 PSpice and Simulink

3.6.1 PSpice

We present PSpice simulation for circuits with dependent sources.

3.6.2 VCVS

Consider the circuit shown in Figure 3.47. It can be shown that $V_1 = 1.99938$ V and $V_2 = 5.998017$ V. The PSpice schematic for this circuit is shown in Figure 3.48. The part name of VCVS starts with E. Double-click on E_1 and enter 10,000 (the voltage gain). The controlling voltage v is the voltage across R_1, and the controlled voltage source is placed between node 3 and the ground. From the PSpice simulation, we get $V_1 = 1.999$ V and $V_2 = 5.998$ V, as shown in Figure 3.48.

FIGURE 3.47

A circuit with VCVS.

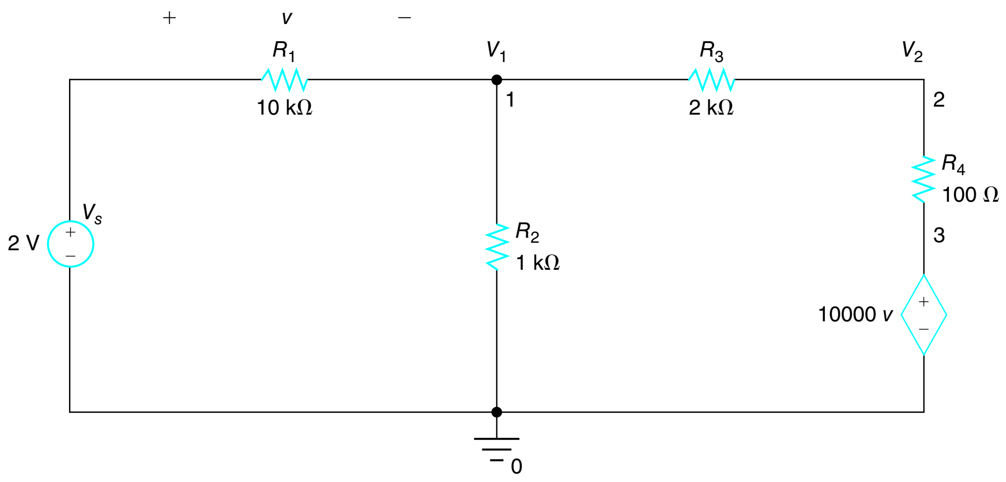
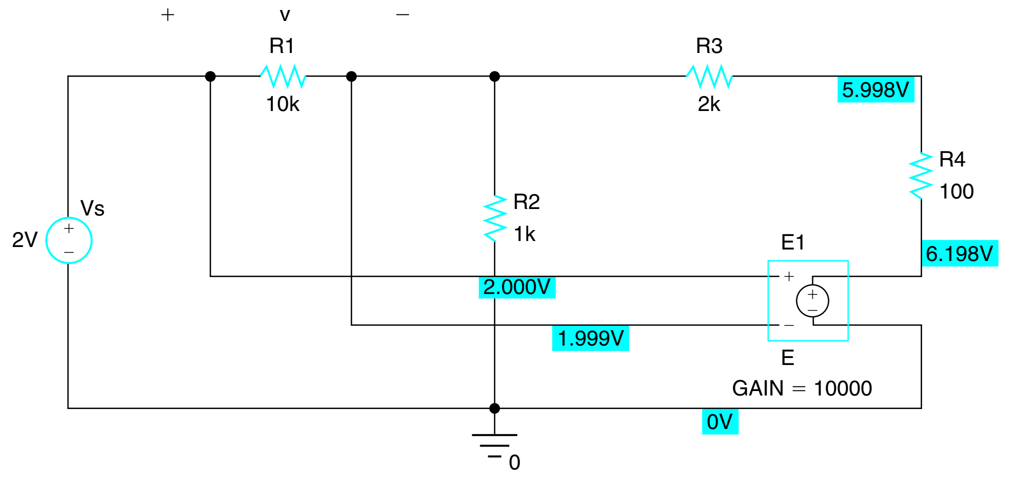


FIGURE 3.48

The PSpice schematic of the circuit shown in Figure 3.47.



3.6.3 VCCS

Consider the circuit shown in Figure 3.49. It can be shown that $V_1 = 472.0930$ mV and $V_2 = -4.8023$ V. The PSpice schematic for this circuit is shown in Figure 3.50. The part name of VCCS starts with G. Double-click on G_1 and enter 0.005 (the conductance). The controlling voltage v is the voltage across R_2, and the controlled current source is placed between node 2 and ground. From the PSpice simulation, we get $V_1 = 472.1$ mV and $V_2 = -4.802$ V, as shown in Figure 3.50.

FIGURE 3.49

A circuit with VCCS.

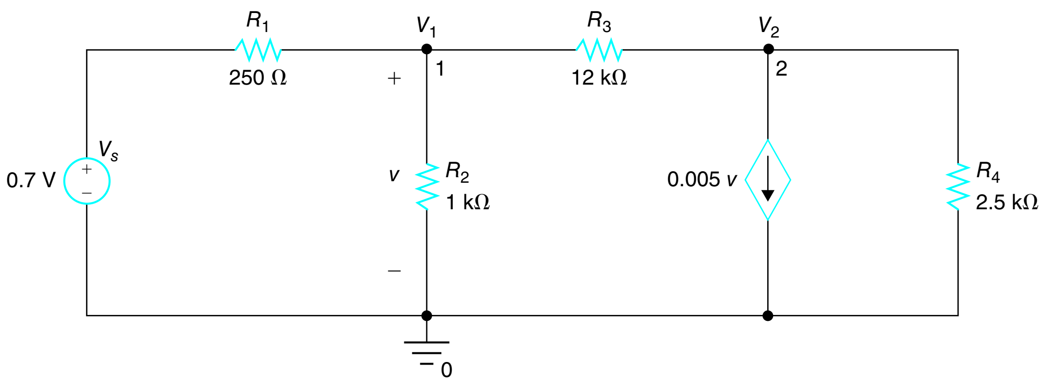
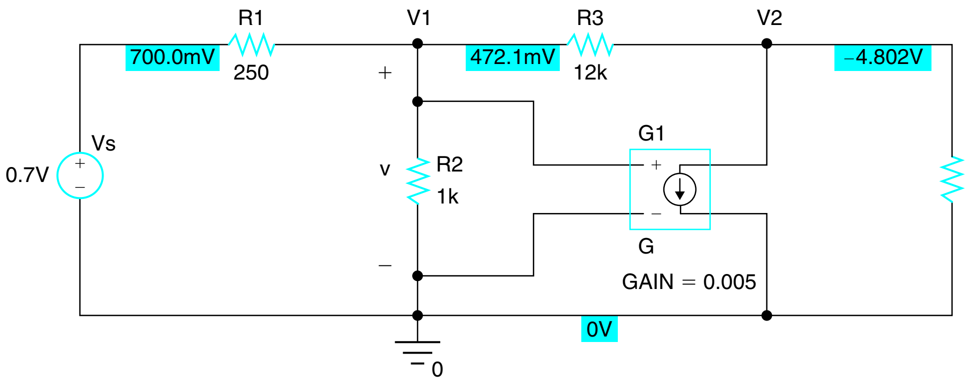


FIGURE 3.50

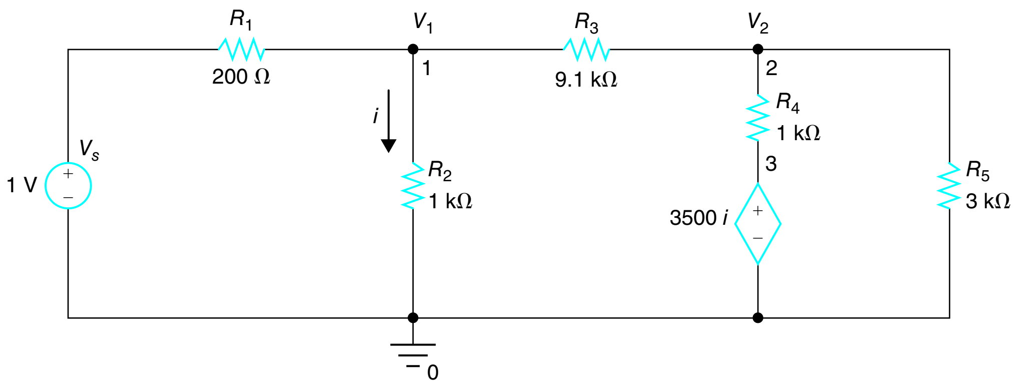


The PSpice schematic of the circuit shown in Figure 3.49.

3.6.4 CCVS

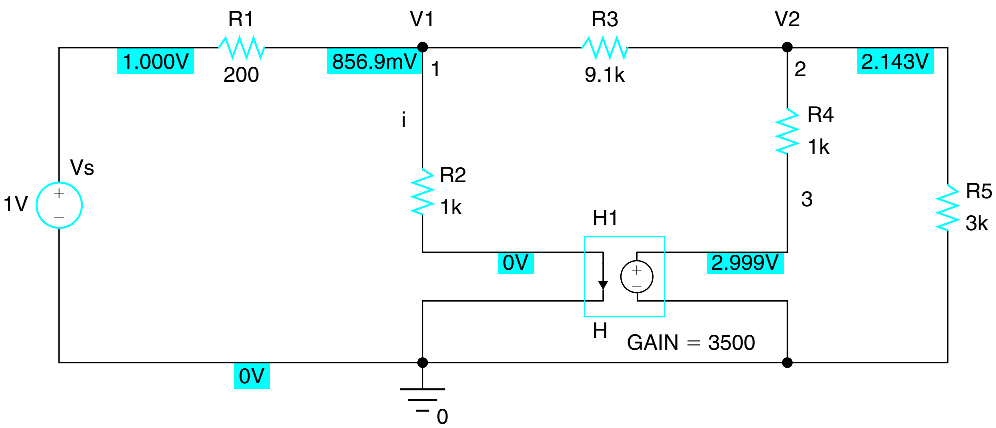
Consider the circuit shown in Figure 3.51. It can be shown that $V_1 = 856.88943$ mV and $V_2 = 2.1433$ V. The PSpice schematic for this circuit is shown in Figure 3.52. The part name of CCVS starts with H. Double-click on H_1 and enter 3500 (the resistance). The controlling current i is the current through R_2 and the controlled voltage source is placed between node 3 and ground. From the PSpice simulation, we get $V_1 = 856.9$ mV and $V_2 = 2.143$ V, as shown in Figure 3.52.

FIGURE 3.51



A circuit with CCVS.

FIGURE 3.52



The PSpice schematic of the circuit shown in Figure 3.51.

3.6.5 CCCS

Consider the circuit shown in Figure 3.53. It can be shown that $V_1 = 2.7068$ V and $V_2 = -6.3158$ V. The PSpice schematic for this circuit is shown in Figure 3.54. The part name of CCCS starts with F. Double-click on F_1 and enter 2 (the current gain). The controlling current i is the current through R_2, and the controlled current source is placed between node 2 and the ground. From the PSpice simulation, we get $V_1 = 2.707$ V and $V_2 = -6.316$ V, as shown in Figure 3.54.

FIGURE 3.53

A circuit with CCCS.

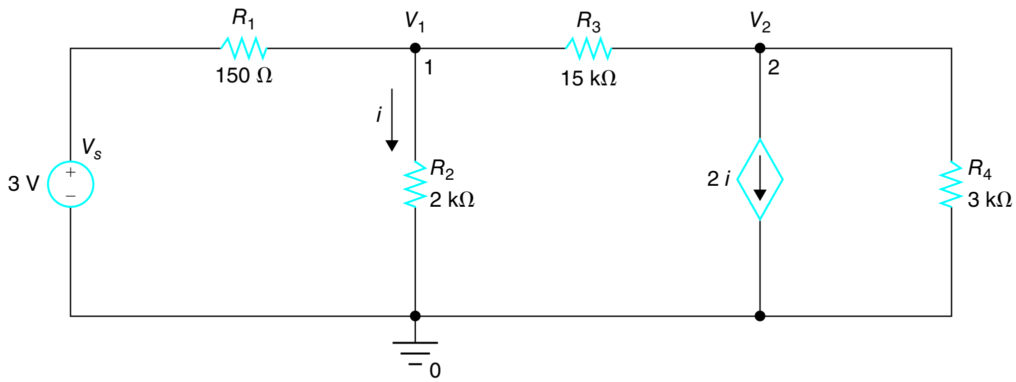
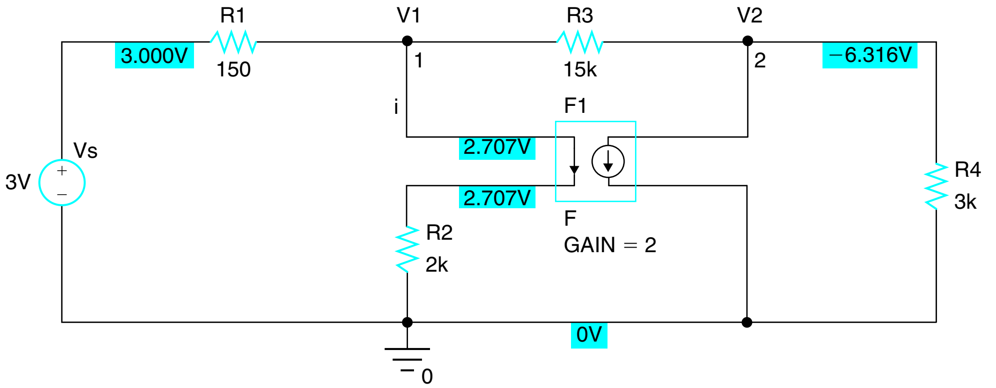


FIGURE 3.54

The PSpice schematic of the circuit shown in Figure 3.53.

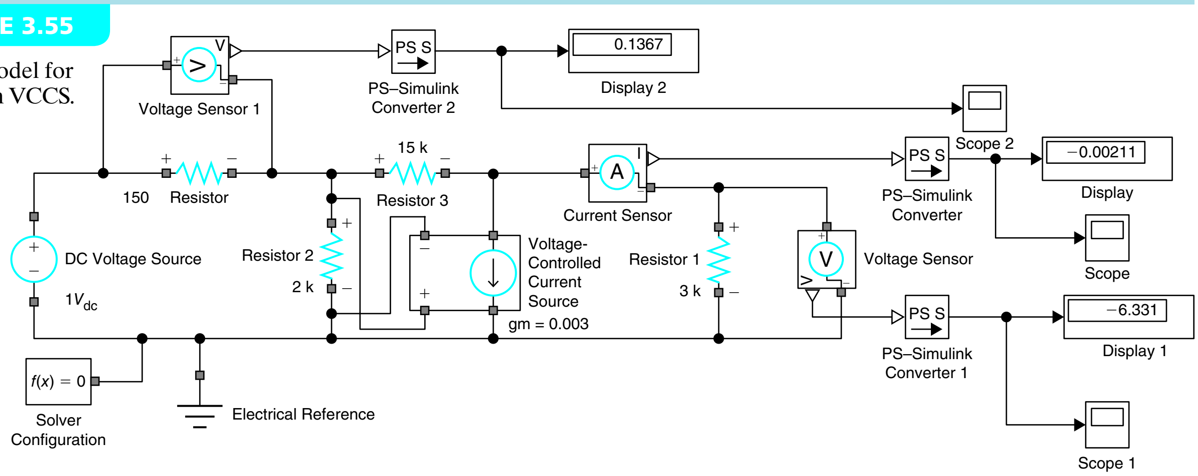


3.6.6 SIMULINK

A simscape of Simulink® makes it possible to simulate a circuit with VCCS, as shown in Figure 3.55.

FIGURE 3.55

A Simulink model for a circuit with VCCS.



SUMMARY

Two analysis methods, nodal analysis and mesh analysis, are widely used in analyzing electrical and electronic circuits.

The nodal analysis is a circuit analysis method that finds all unknown node voltages. For each node whose voltage is unknown, we sum the currents leaving (or entering) the node. If there are n unknown node voltages, we obtain n equations in n unknowns. The node voltages are found by solving these n equations using Cramer's rule or MATLAB. Once all node voltages are found, we can find the branch currents and the powers everywhere.

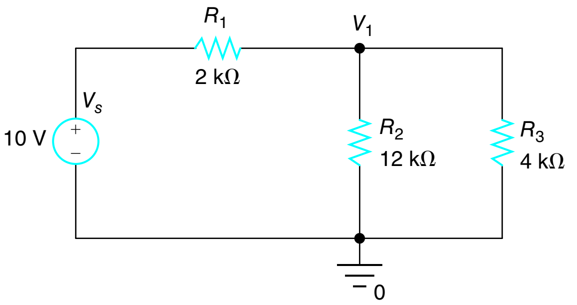
The mesh analysis is a circuit analysis method that finds all unknown mesh currents. For each mesh whose mesh current is unknown, we sum the voltage drops around the mesh. If there are n unknown mesh currents, we obtain n equations in n unknowns. The mesh currents are found by solving these n equations using Cramer's rule or MATLAB. Once all mesh currents are found, we can find the branch currents, the node voltages, and the powers everywhere.

PROBLEMS

Nodal Analysis

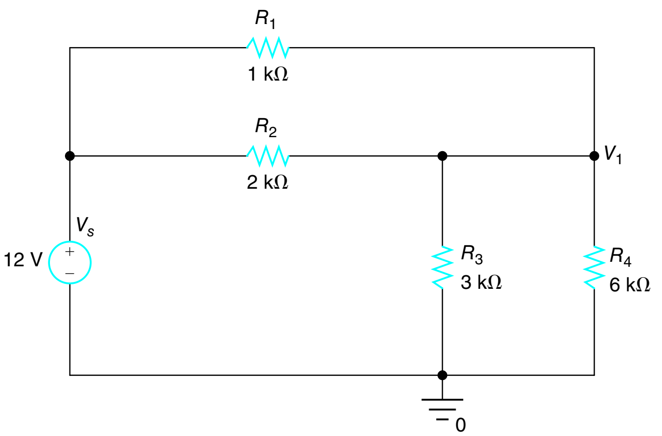
3.1 Find the voltage V_1 in the circuit shown in Figure P3.1.

FIGURE P3.1



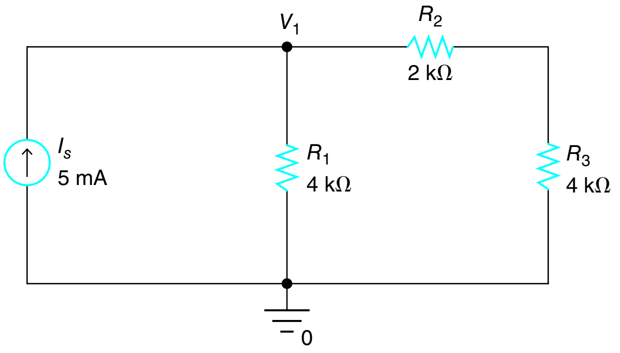
3.2 Find the voltage V_1 in the circuit shown in Figure P3.2.

FIGURE P3.2



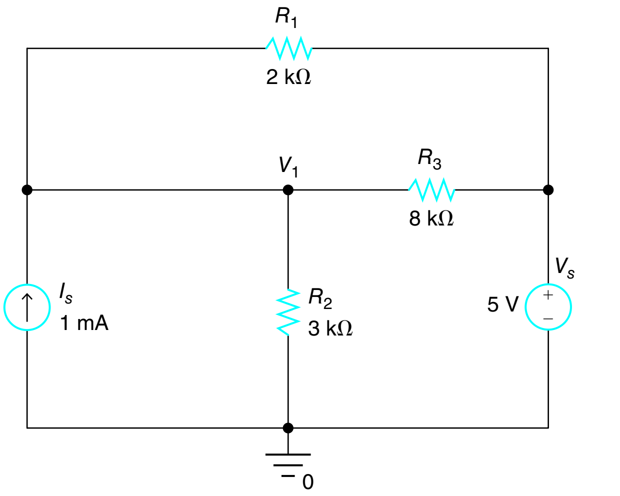
3.3 Find the voltage V_1 in the circuit shown in Figure P3.3.

FIGURE P3.3



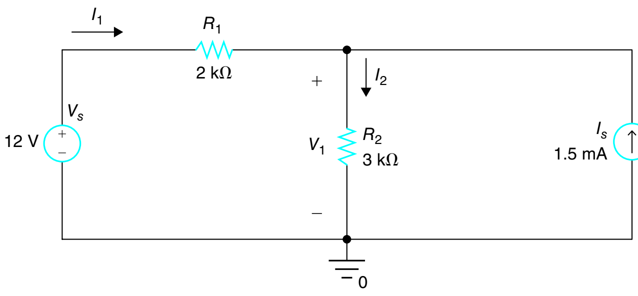
3.4 Find the voltage V_1 in the circuit shown in Figure P3.4.

FIGURE P3.4



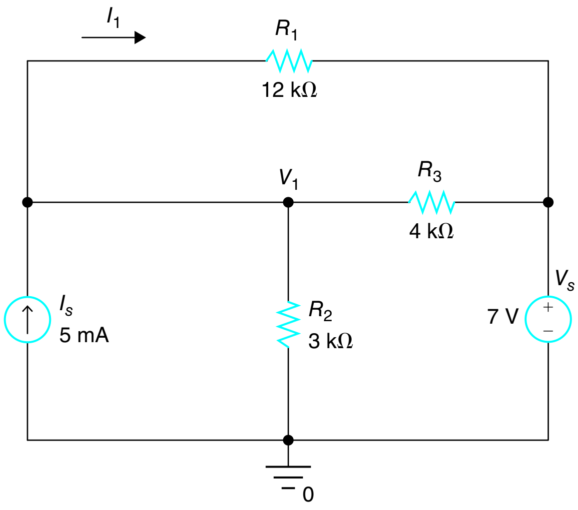
3.5 Find V_1, I_1, and I_2 in the circuit shown in Figure P3.5.

FIGURE P3.5



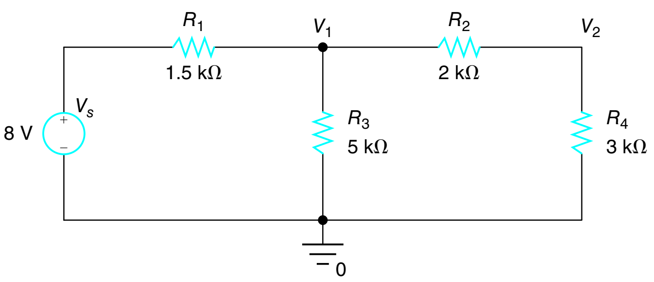
3.6 Find V_1 and I_1 in the circuit shown in Figure P3.6.

FIGURE P3.6



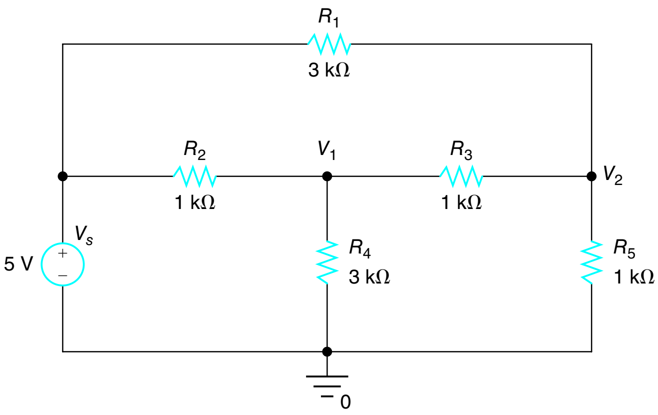
3.7 Find the voltages V_1 and V_2 in the circuit shown in Figure P3.7.

FIGURE P3.7



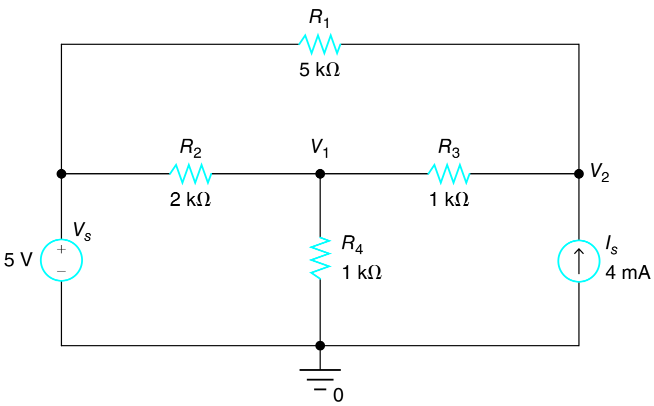
3.8 Find the voltages V_1 and V_2 in the circuit shown in Figure P3.8.

FIGURE P3.8



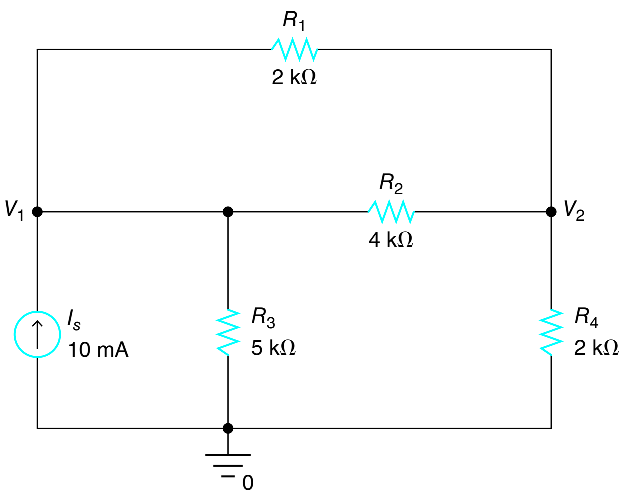
3.9 Find the voltages V_1 and V_2 in the circuit shown in Figure P3.9.

FIGURE P3.9



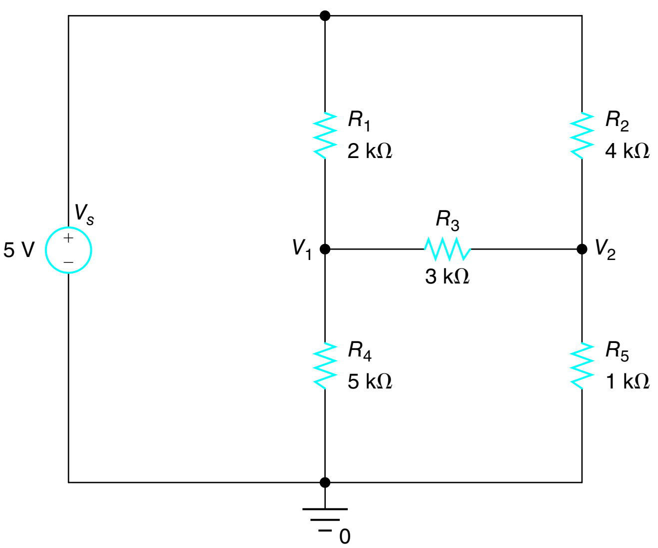
3.10 Find the voltages V_1 and V_2 in the circuit shown in Figure P3.10.

FIGURE P3.10



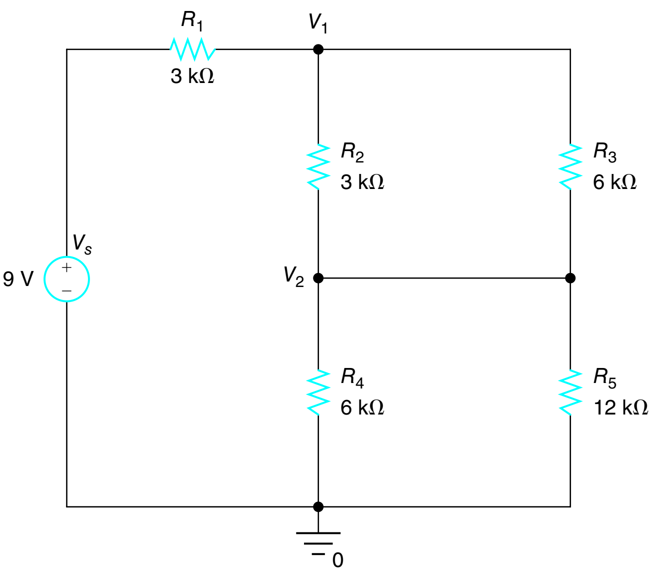
3.11 Find the voltages V_1 and V_2 in the circuit shown in Figure P3.11.

FIGURE P3.11



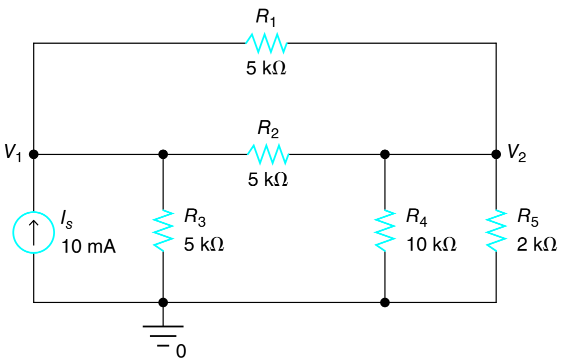
3.12 Find the voltages V_1 and V_2 in the circuit shown in Figure P3.12.

FIGURE P3.12



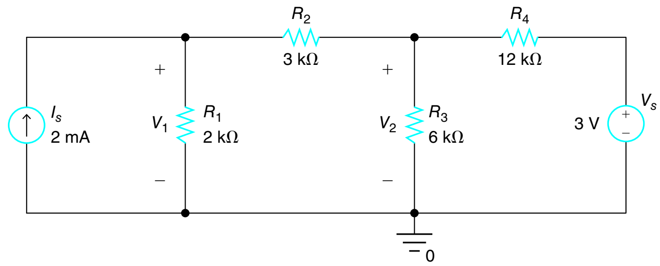
3.13 Find the voltages V_1 and V_2 in the circuit shown in Figure P3.13.

FIGURE P3.13



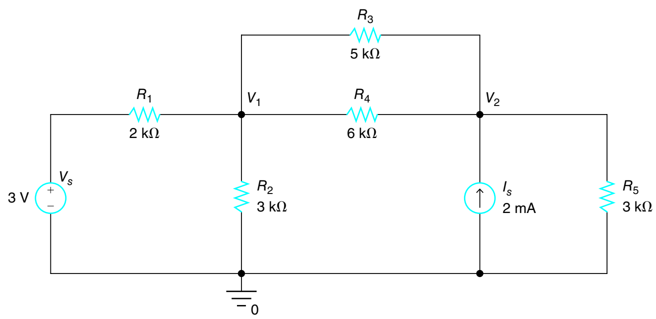
3.14 Determine V_1 and V_2 for the circuit shown in Figure P3.14.

FIGURE P3.14



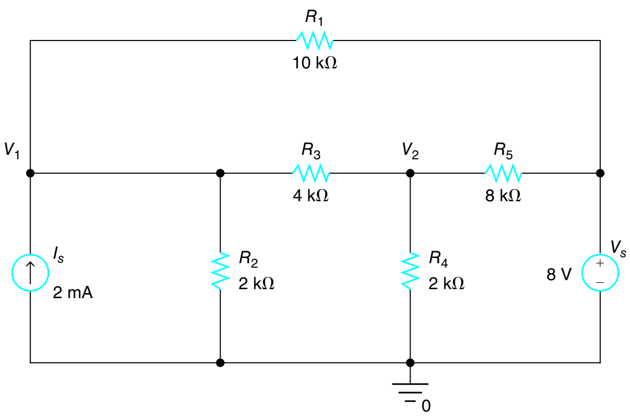
3.15 Find V_1 and V_2 in the circuit shown in Figure P3.15.

FIGURE P3.15



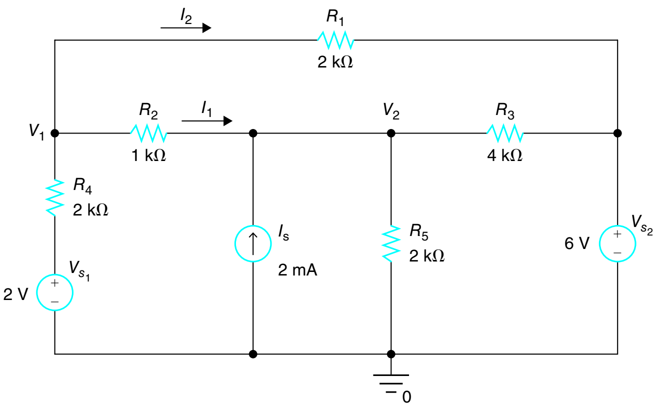
3.16 Find V_1 and V_2 in the circuit shown in Figure P3.16.

FIGURE P3.16



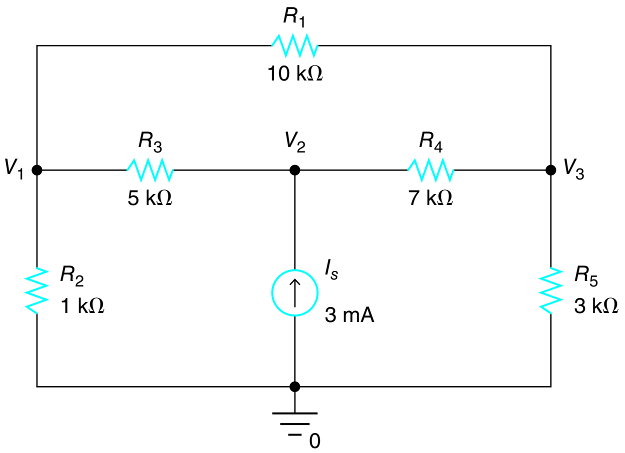
3.17 Find V_1, V_2, I_1, and I_2 in the circuit shown in Figure P3.17.

FIGURE P3.17



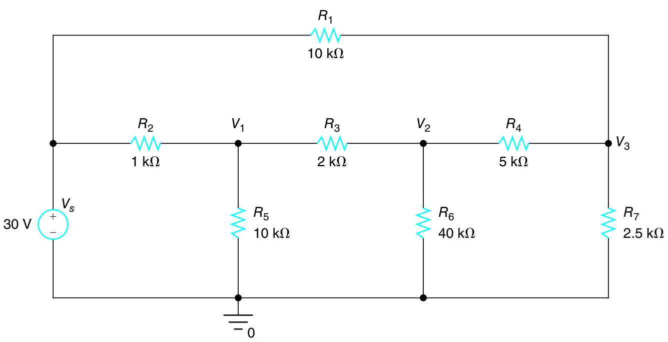
3.18 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.18.

FIGURE P3.18



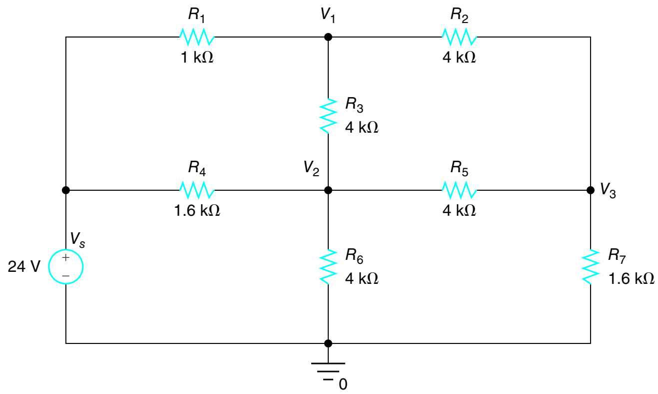
3.19 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.19.

FIGURE P3.19



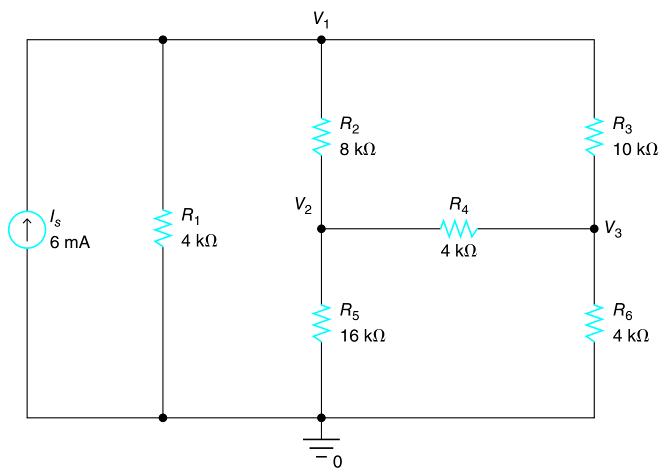
3.20 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.20.

FIGURE P3.20



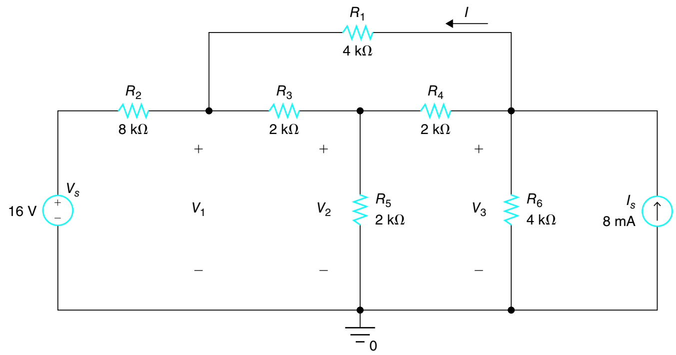
3.21 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.21.

FIGURE P3.21



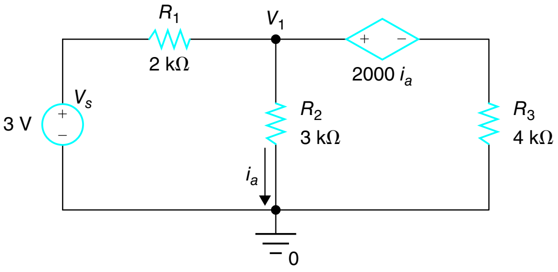
3.22 Find V_1, V_2, V_3, and I in the circuit shown in Figure P3.22.

FIGURE P3.22



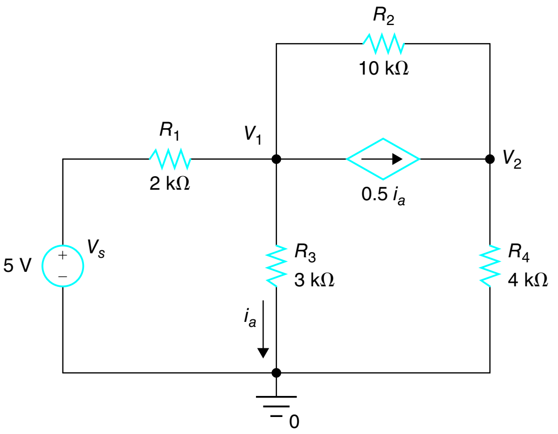
3.23 Find V_1 in the circuit shown in Figure P3.23.

FIGURE P3.23



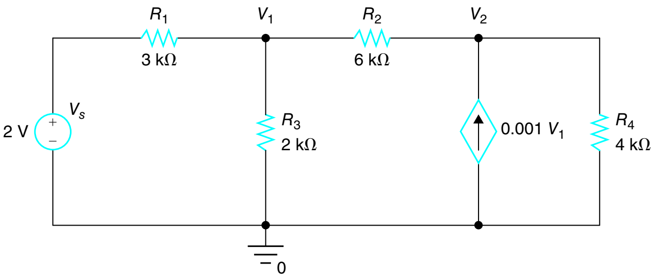
3.24 Find V_1 and V_2 in the circuit shown in Figure P3.24.

FIGURE P3.24



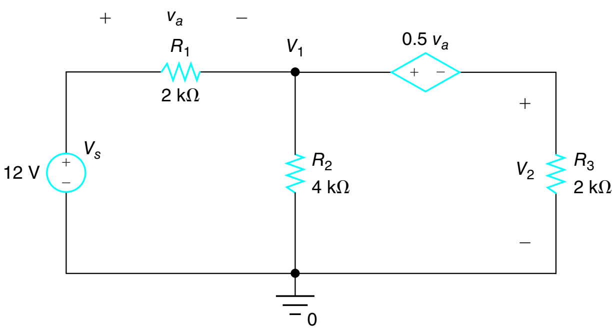
3.25 Find V_1 and V_2 in the circuit shown in Figure P3.25.

FIGURE P3.25



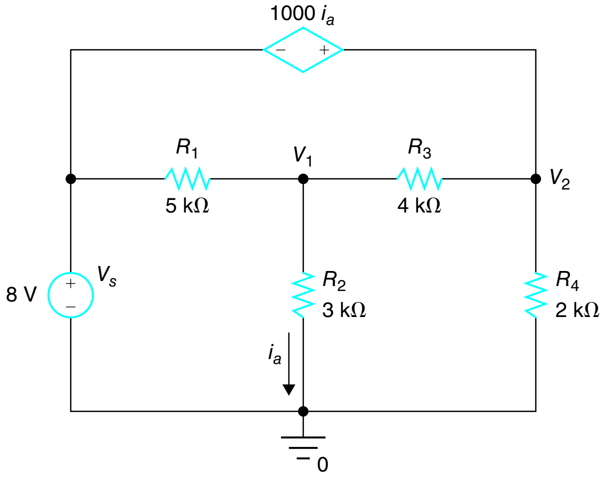
3.26 Find V_1 and V_2 in the circuit shown in Figure P3.26.

FIGURE P3.26



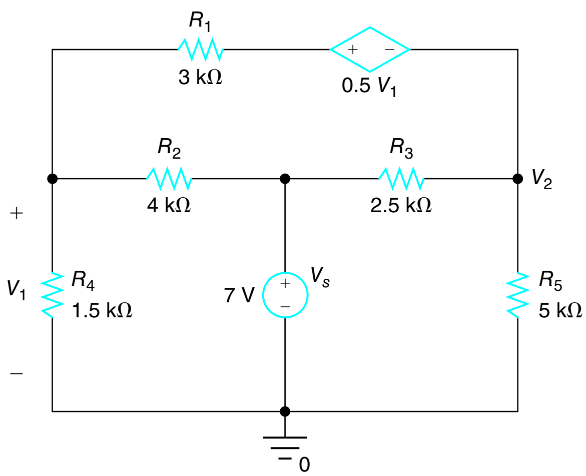
3.27 Find V_1 and V_2 in the circuit shown in Figure P3.27.

FIGURE P3.27



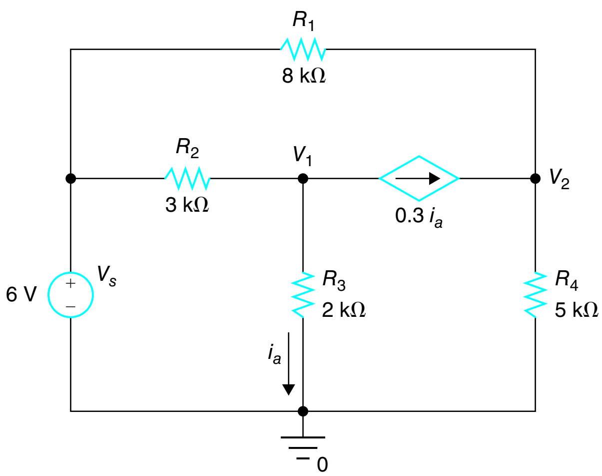
3.28 Find V_1 and V_2 in the circuit shown in Figure P3.28.

FIGURE P3.28



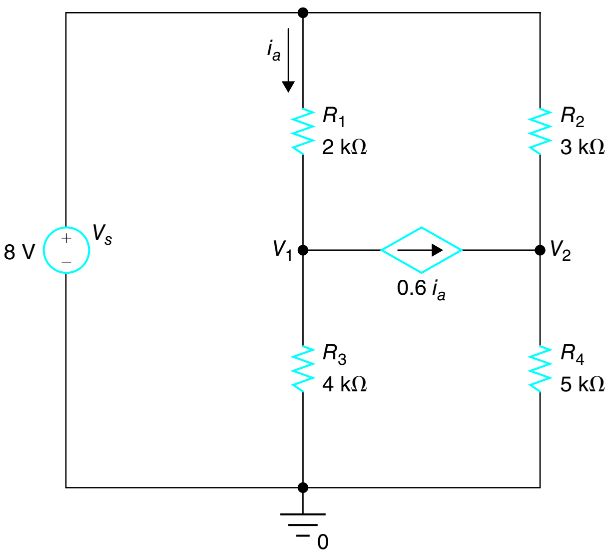
3.29 Find V_1 and V_2 in the circuit shown in Figure P3.29.

FIGURE P3.29



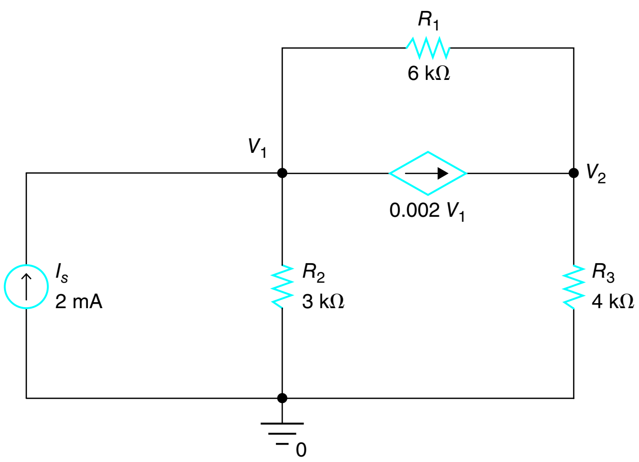
3.30 Find V_1 and V_2 in the circuit shown in Figure P3.30.

FIGURE P3.30



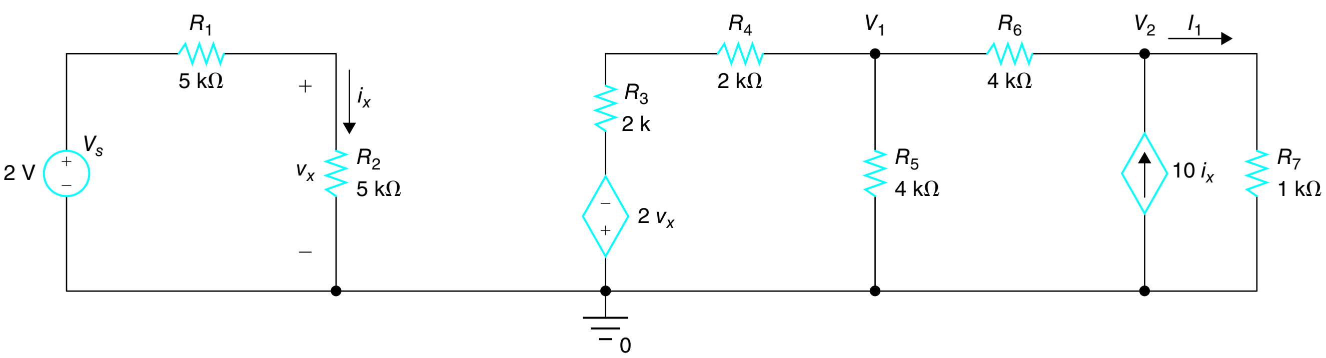
3.31 Find V_1 and V_2 in the circuit shown in Figure P3.31.

FIGURE P3.31



3.32 Find V_1, V_2, and I_1 in the circuit shown in Figure P3.32.

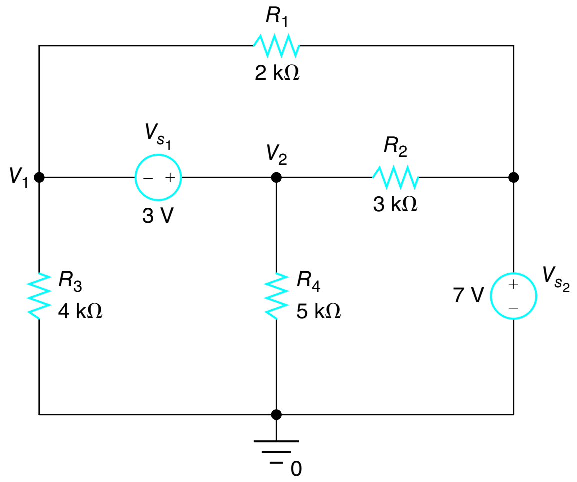
FIGURE P3.32



Supernode

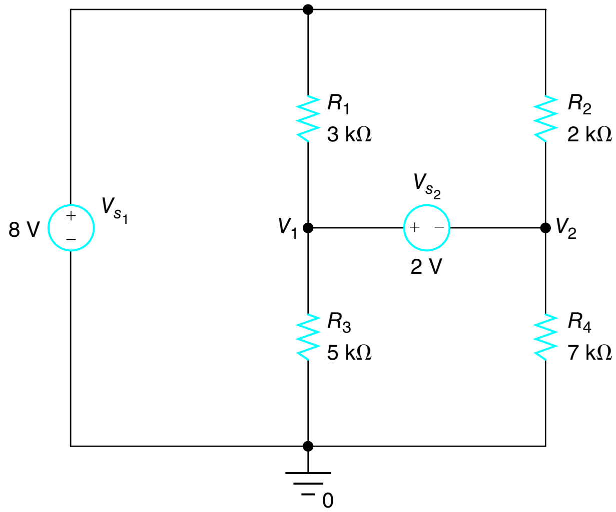
3.33 Find V_1 and V_2 in the circuit shown in Figure P3.33.

FIGURE P3.33



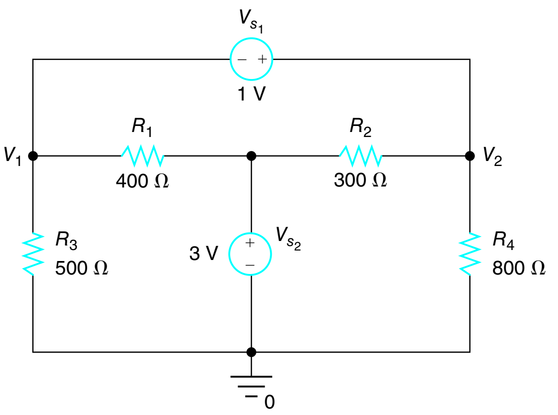
3.34 Find V_1 and V_2 in the circuit shown in Figure P3.34.

FIGURE P3.34



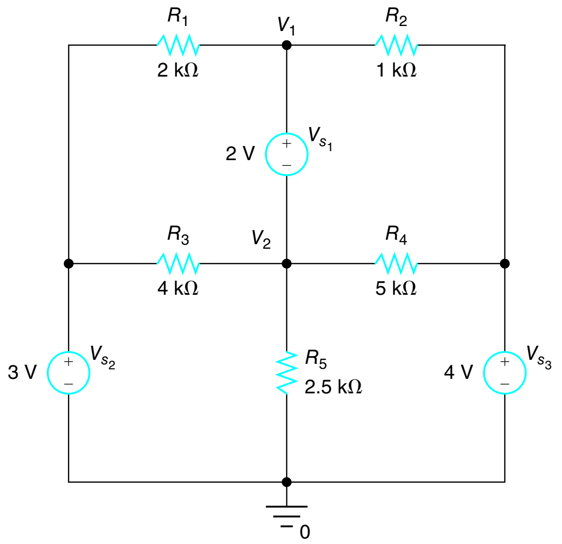
3.35 Find V_1 and V_2 in the circuit shown in Figure P3.35.

FIGURE P3.35



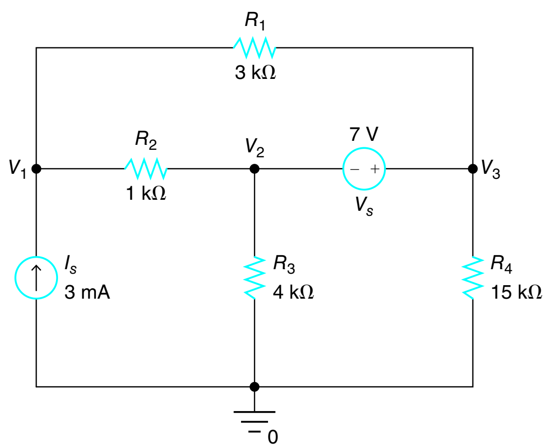
3.36 Find V_1 and V_2 in the circuit shown in Figure P3.36.

FIGURE P3.36



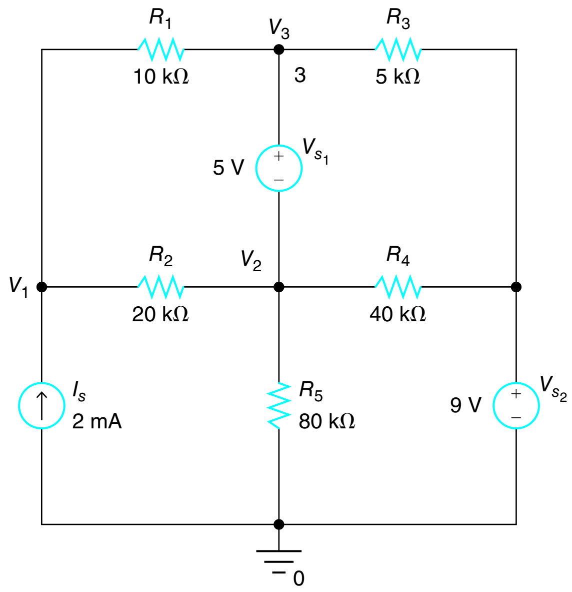
3.37 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.37.

FIGURE P3.37



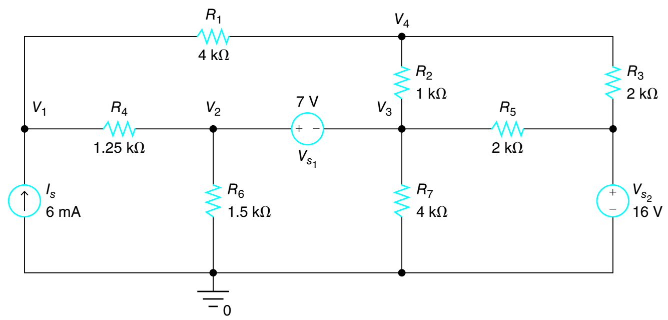
3.38 Find V_1, V_2, and V_3 in the circuit shown in Figure P3.38.

FIGURE P3.38



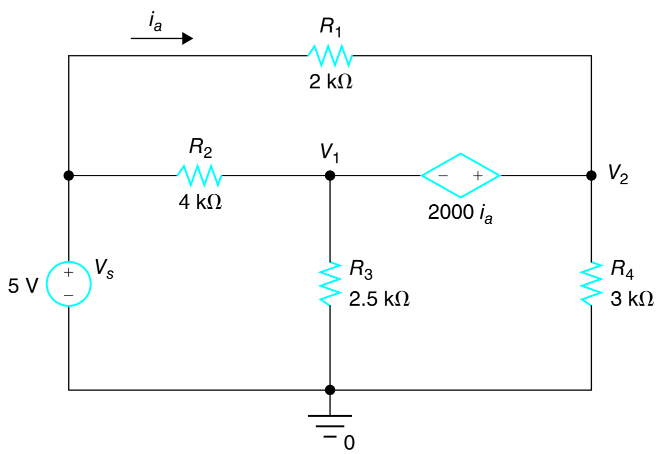
3.39 Find the voltages V_1, V_2, V_3, and V_4 in the circuit shown in Figure P3.39.

FIGURE P3.39



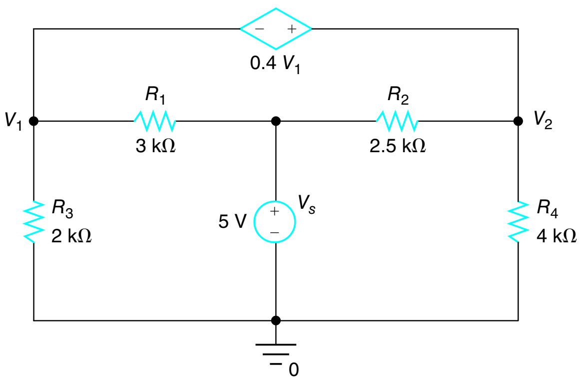
3.40 Find V_1 and V_2 in the circuit shown in Figure P3.40.

FIGURE P3.40



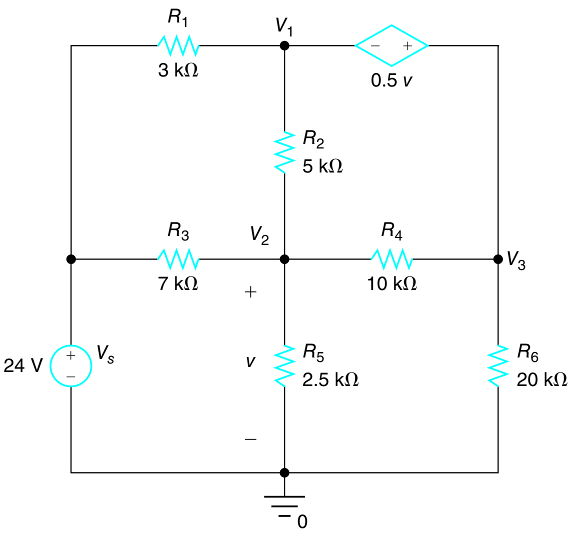
3.41 Find V_1 and V_2 in the circuit shown in Figure P3.41.

FIGURE P3.41



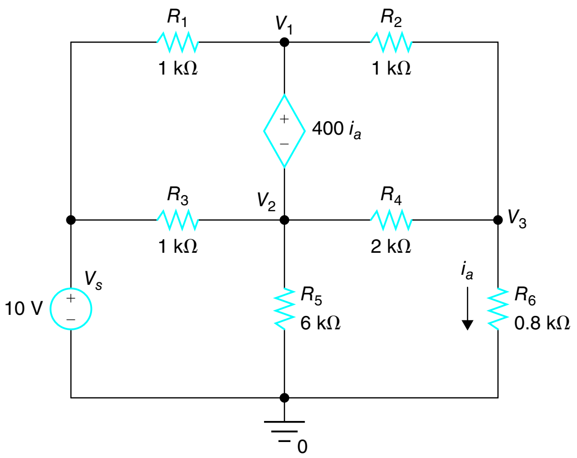
3.42 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.42.

FIGURE P3.42



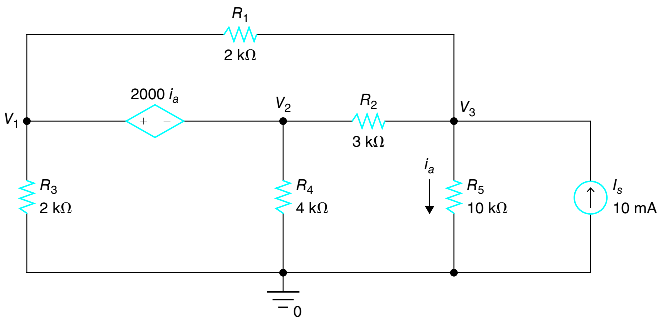
3.43 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.43.

FIGURE P3.43



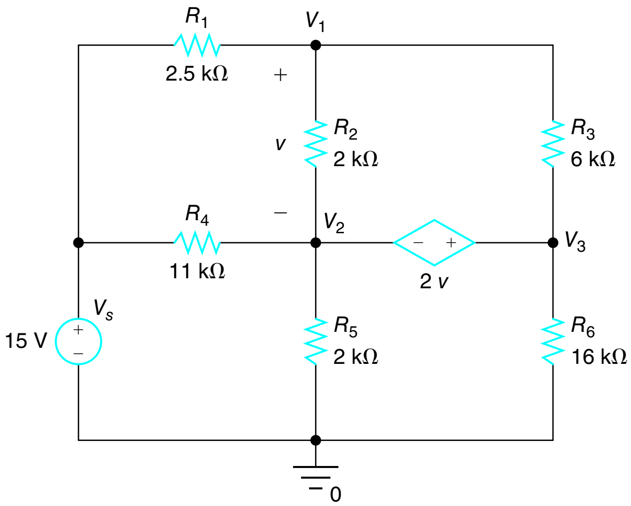
3.44 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.44.

FIGURE P3.44



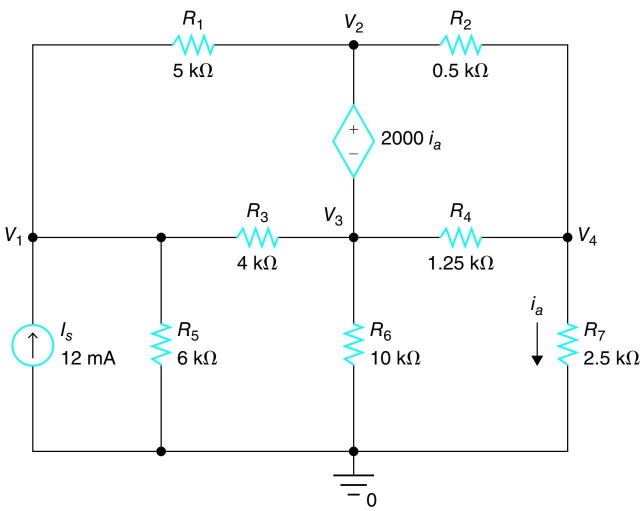
3.45 Find the voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.45.

FIGURE P3.45



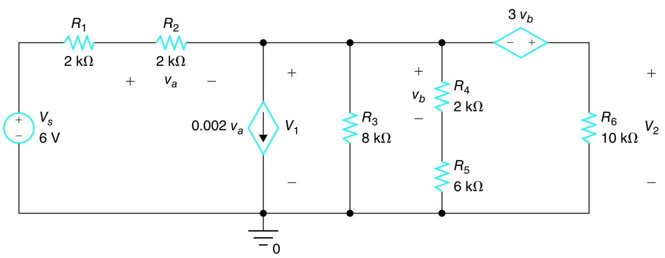
3.46 Find the voltages V_1, V_2, V_3, and V_4 in the circuit shown in Figure P3.46.

FIGURE P3.46



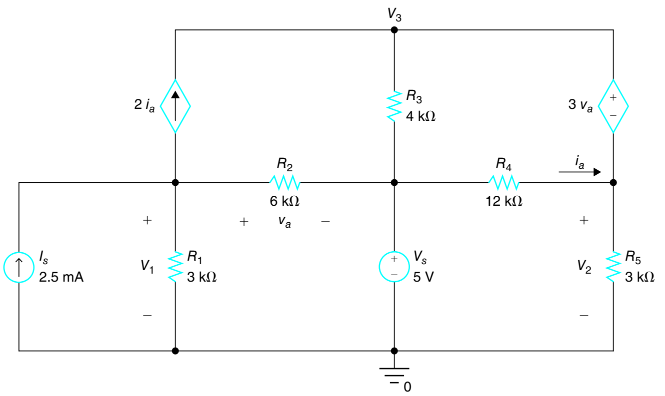
3.47 Find V_1 and V_2 in the circuit shown in Figure P3.47.

FIGURE P3.47



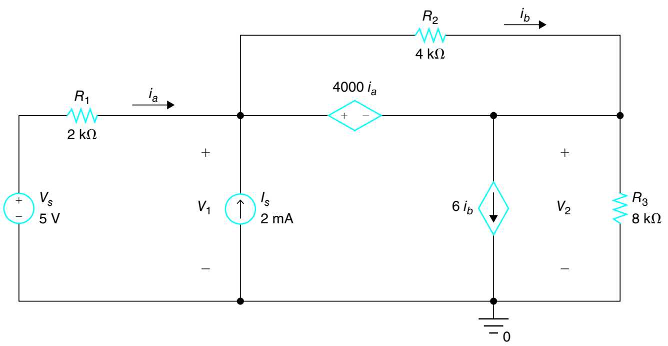
3.48 Find V_1, V_2, and V_3 in the circuit shown in Figure P3.48.

FIGURE P3.48



3.49 Find V_1 and V_2 in the circuit shown in Figure P3.49.

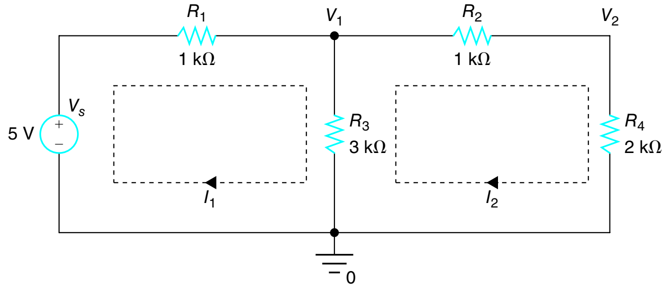
FIGURE P3.49



Mesh Analysis

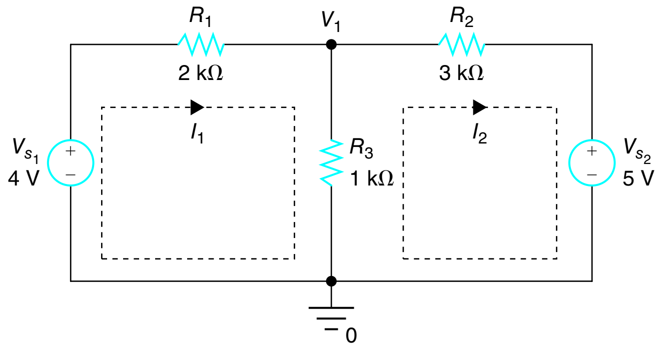
3.50 Find mesh currents I_1 and I_2 and node voltages V_1 and V_2 in the circuit shown in Figure P3.50.

FIGURE P3.50



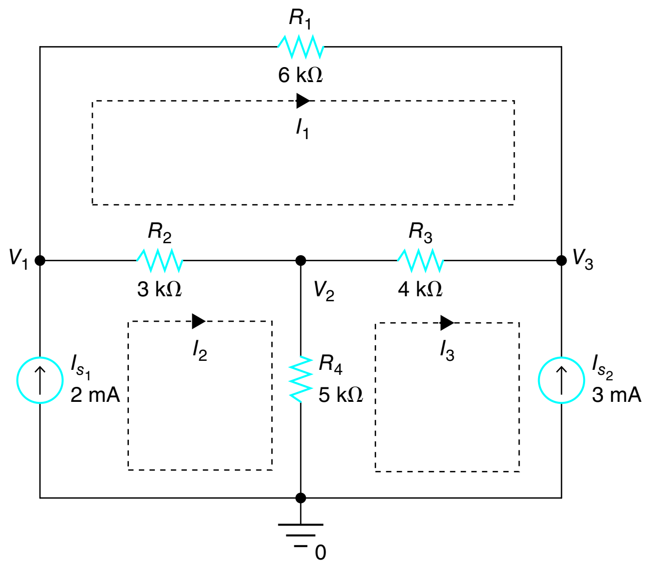
3.51 Find mesh currents I_1 and I_2 and node voltage V_1 in the circuit shown in Figure P3.51.

FIGURE P3.51



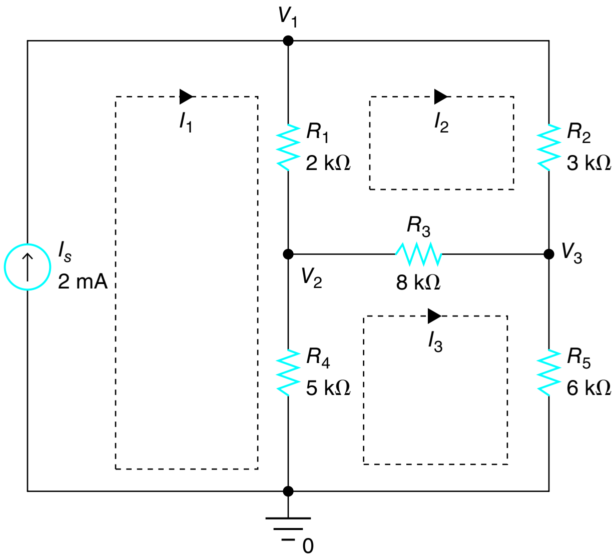
3.52 Find mesh currents I_1, I_2, and I_3 and node voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.52.

FIGURE P3.52



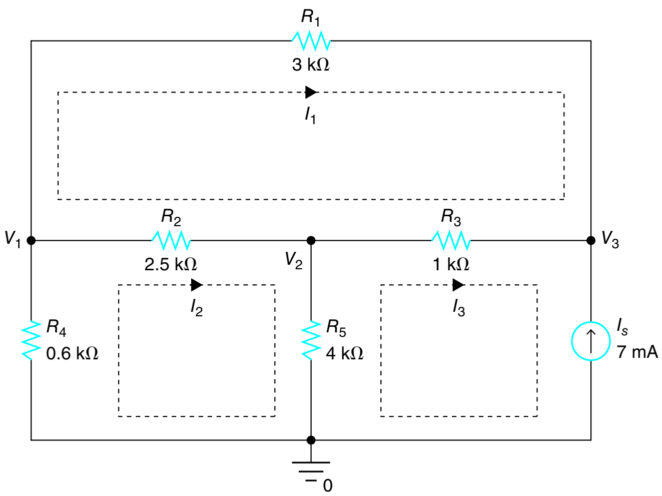
3.53 Find mesh currents I_1, I_2, and I_3 and node voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.53.

FIGURE P3.53



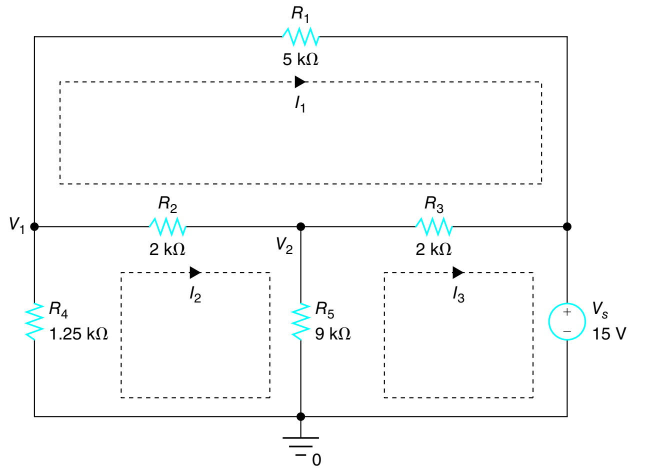
3.54 Find mesh currents I_1, I_2, and I_3 and node voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.54.

FIGURE P3.54



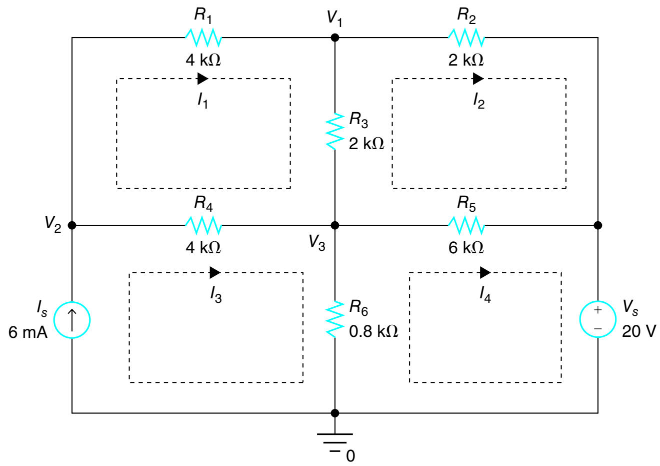
3.55 Find mesh currents I_1, I_2, and I_3 and node voltages V_1 and V_2 in the circuit shown in Figure P3.55.

FIGURE P3.55



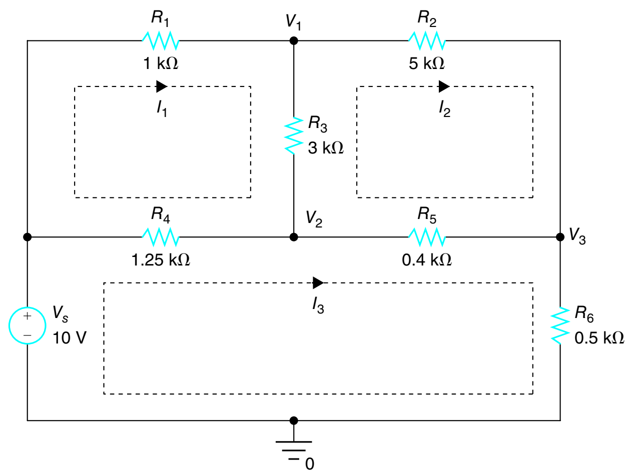
3.56 Find mesh currents I_1, I_2, I_3, and I_4 and node voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.56.

FIGURE P3.56



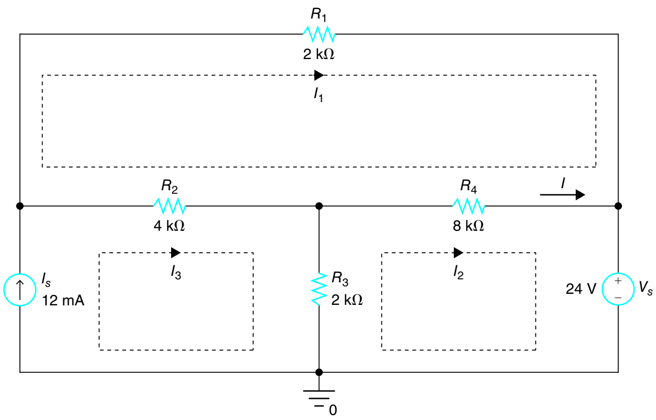
3.57 Find mesh currents I_1, I_2, and I_3 and node voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.57.

FIGURE P3.57



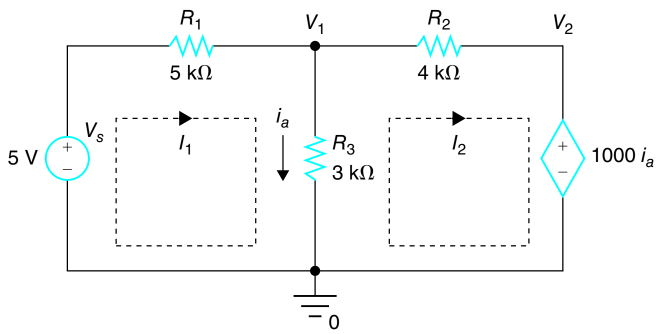
3.58 Use mesh analysis to find the current I in the circuit shown in Figure P3.58.

FIGURE P3.58



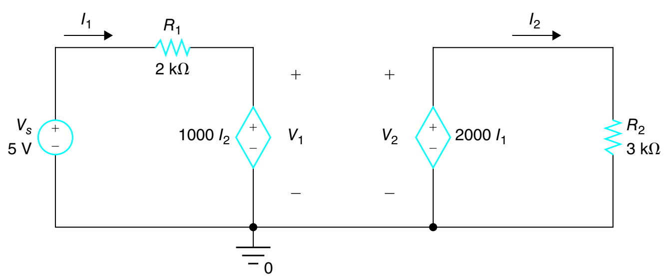
3.59 Find mesh currents I_1 and I_2 and node voltages V_1 and V_2 in the circuit shown in Figure P3.59.

FIGURE P3.59



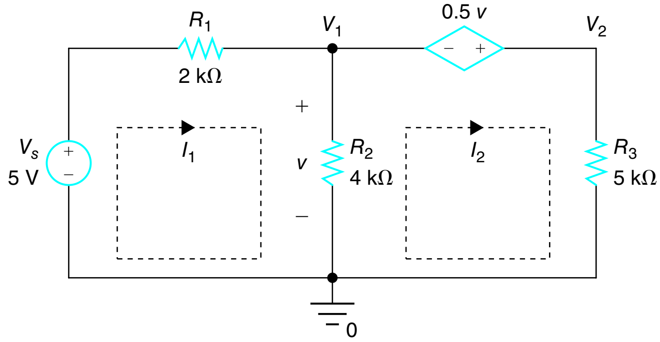
3.60 Find mesh currents I_1 and I_2 and node voltages V_1 and V_2 in the circuit shown in Figure P3.60.

FIGURE P3.60



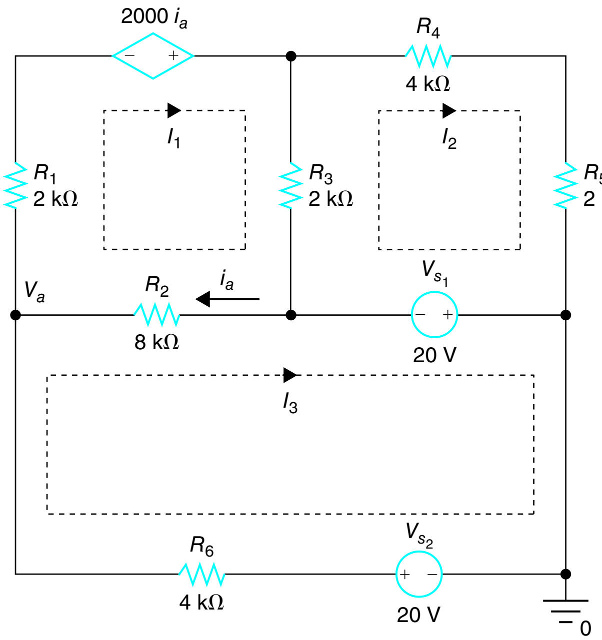
3.61 Find mesh currents I_1 and I_2 and node voltages V_1 and V_2 in the circuit shown in Figure P3.61.

FIGURE P3.61



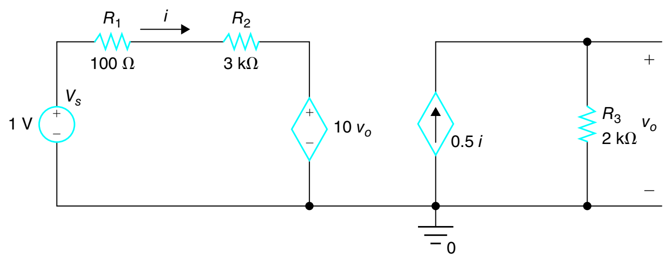
3.62 Use mesh analysis to find V_a in the circuit shown in Figure P3.62.

FIGURE P3.62



3.63 Use mesh analysis to find v_o in the circuit shown in Figure P3.63.

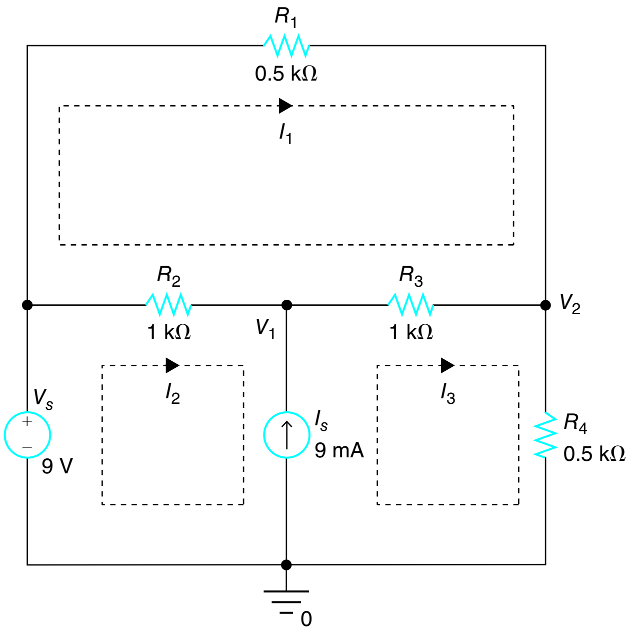
FIGURE P3.63



Supermesh

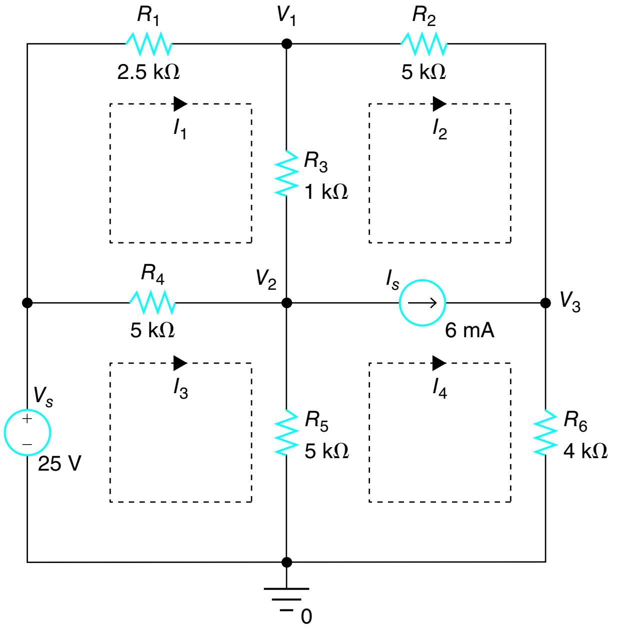
3.64 Find mesh currents I_1, I_2, and I_3, and node voltages V_1 and V_2 in the circuit shown in Figure P3.64.

FIGURE P3.64



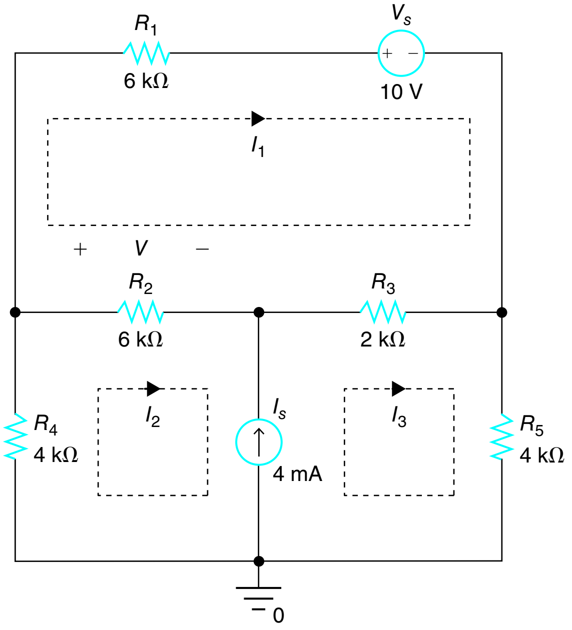
3.65 Find the mesh currents I_1, I_2, I_3, and I_4, and node voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.65.

FIGURE P3.65



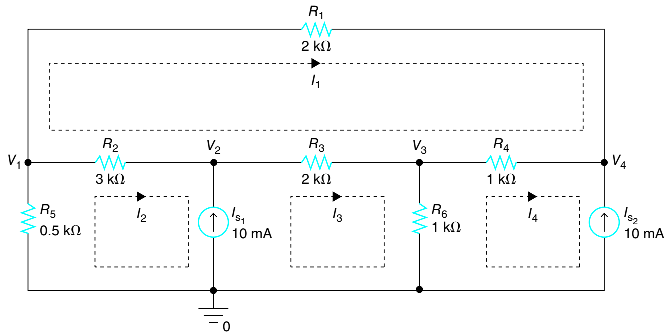
3.66 Use mesh analysis to find the voltage V in the circuit shown in Figure P3.66.

FIGURE P3.66



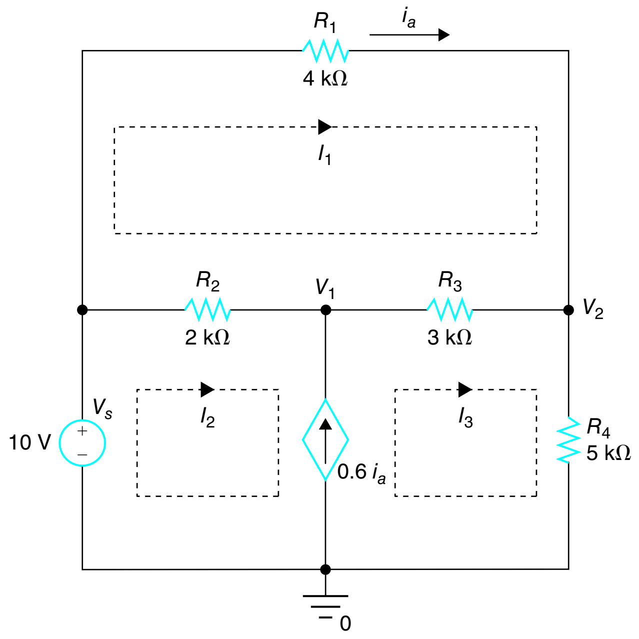
3.67 Find mesh currents I_1, I_2, I_3, and I_4, and node voltages V_1, V_2, V_3, and V_4 in the circuit shown in Figure P3.67.

FIGURE P3.67



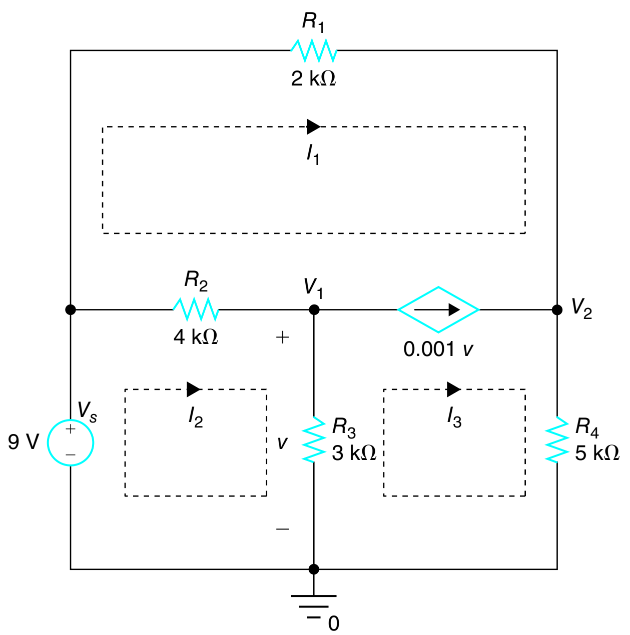
3.68 Find mesh currents I_1, I_2, and I_3, and node voltages V_1 and V_2 in the circuit shown in Figure P3.68.

FIGURE P3.68



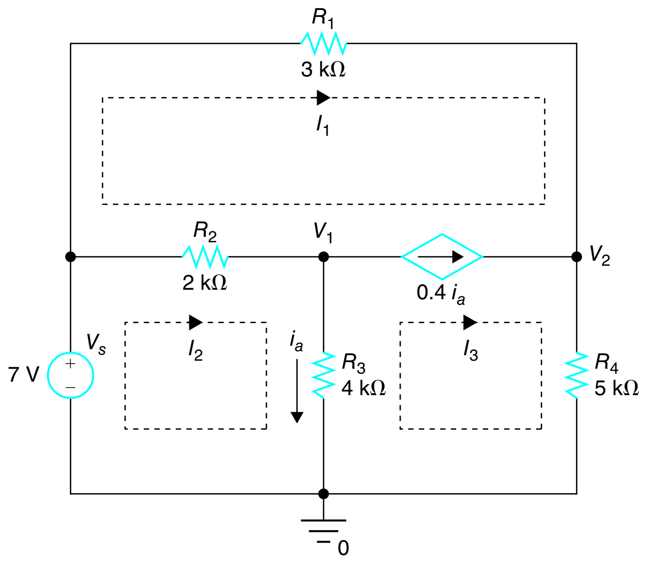
3.69 Find mesh currents I_1, I_2, and I_3, and node voltages V_1 and V_2 in the circuit shown in Figure P3.69.

FIGURE P3.69



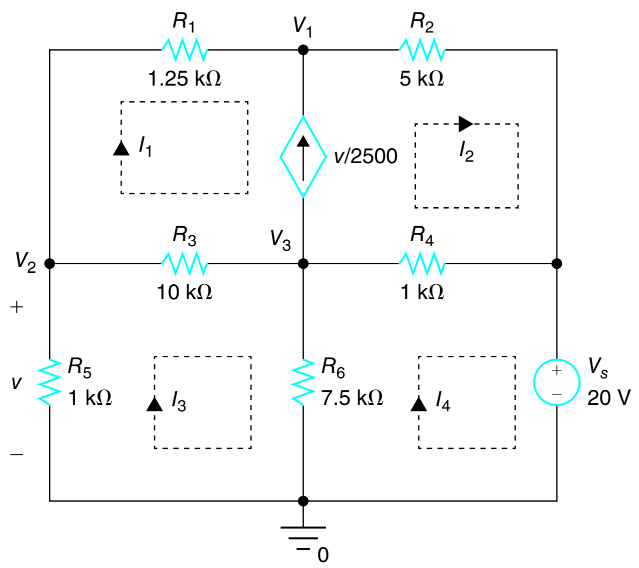
3.70 Find mesh currents I_1, I_2, and I_3, and node voltages V_1 and V_2 in the circuit shown in Figure P3.70.

FIGURE P3.70



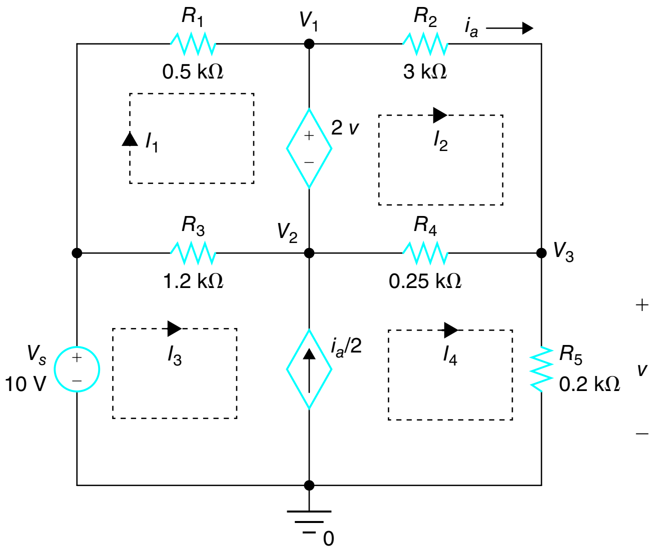
3.71 Find mesh currents I_1, I_2, I_3, and I_4, and node voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.71.

FIGURE P3.71



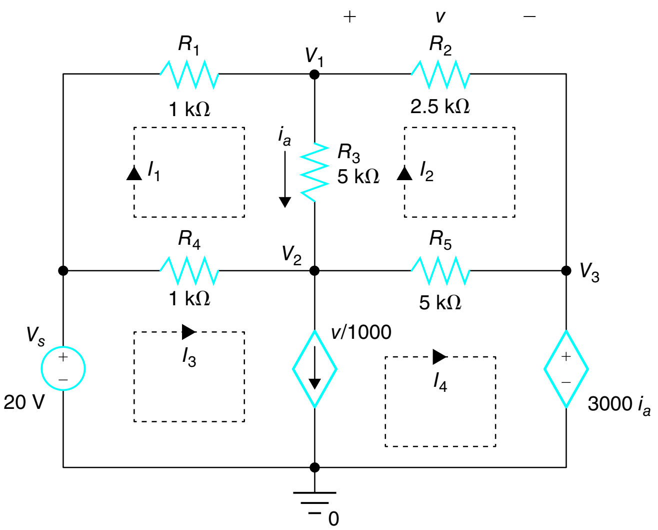
3.72 Find mesh currents I_1, I_2, I_3, and I_4, and node voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.72.

FIGURE P3.72



3.73 Find mesh currents I_1, I_2, I_3, and I_4, and voltages V_1, V_2, and V_3 in the circuit shown in Figure P3.73.

FIGURE P3.73



3.74 Use mesh analysis to find voltage V_3 across R_3 for the circuit shown in Figure P3.74.

FIGURE P3.74