1.5. CONCEPT OF IMPEDANCE AND ADMITTANCE

Impedance (Z) – represents the total opposition to the flow of alternating current, expressed in ohms (Ω) . - represents the passive elements R, L, C and their combination in the frequency domain Admittance (Y) – the reciprocal of impedance, expressed in siemens (S) If $I = I_m \angle \theta_1$ is the resulting current drawn by a passive, linear RLC circuit from source a voltage $V = V_m \angle \theta_V$, then

$$Z = \frac{V}{I} = \frac{V_m \angle \theta_V}{I_m \angle \theta_I} = Z \angle \theta = Z \cos \theta + jZ \sin \theta = R + jX = \sqrt{R^2 + X^2} \angle \tan^{-1} \frac{X}{R}$$

$$Z = \frac{V_m}{I_m} = \sqrt{R^2 + X^2} = magnitude of the impedance$$

where
$$Z = \frac{V_m}{I_m} = \sqrt{R^2 + X^2} = magnitude$$
 of the impedance $\theta = \theta_V - \theta_I = \tan^{-1} \frac{X}{R} = phase$ angle of the impedance

$$R = Z \cos \theta$$
 = active or real component of the impedance

$$X = Z \sin \theta = reactive or quadrature component of impedance$$





Similarly,

$$Y = \frac{I_m}{V_m} \angle \theta_I - \theta_V = Y \angle \theta_y = Y \cos \theta_y + jY \sin \theta_y = G + jB = \sqrt{G^2 + B^2} \angle \tan^{-1} \frac{B}{G}$$
where

$$Y = \frac{I_m}{V_m} = \sqrt{G^2 + B^2} = \frac{1}{Z} = magnitude of the admittance$$

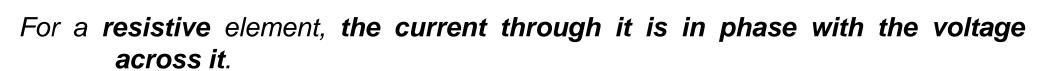
$$\theta_y = \theta_I - \theta_V = -\theta = \tan^{-1} \frac{B}{G} = phase angle of the$$

$$G = Y \cos \theta_v = conductive$$
 (or conductance) component

$$B = Y \sin \theta_y = susceptive (or susceptance) component$$







$$Z = R$$
 and $Y = 1/R = G$

For a purely inductive element, the current through it lags the voltage across it by 90°.

$$Z = jX_L$$
 and $Y = -jB_L$
where $X_L = \omega L = 2\pi f L$ and $B_L = 1/X_L$

For a purely capacitive element, the current through it leads the voltage across it by 90°.

$$Z = -jX_C$$
 and $Y = jB_C$
where $X_C = 1/\omega C = 1/2\pi fC$ and $B_c = 1/X_C$

For the **inductive** circuit (series RL or parallel RL), the current **lags** the voltage by an angle less than 90° (equal to the angle of the equivalent impedance). Z = R + jX

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$$



For the **capacitive** circuit (series RC or parallel RC), the current **leads** the voltage by an angle less than 90° (equal to the angle of the equivalent impedance).

$$Z = R - jX$$
 and $Y = \frac{1}{Z} = \frac{1}{R - jX} = \frac{R + jX}{R^2 + X^2}$

1.6. FREQUENCY DOMAIN ANALYSIS OF SIMPLE RLC SYSTEMS

Series circuits

$$I = I_1 = I_2 = I_3 = \dots$$

 $V = V_1 + V_2 + V_3 + \dots$
 $Z = Z_1 + Z_2 + Z_3 + \dots$

Voltage Division Rule:

In a series circuit, the ratio of any two voltages is also the ratio of the corresponding impedances.





Parallel circuits

$$V = V_1 = V_2 = V_3 = \dots$$

$$I = I_1 + I_2 + I_3 + \dots$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + K = Y = Y_1 + Y_2 + Y_3 + \dots$$

Current Division Rule:

In a parallel circuit, the ratio of any two currents is also the ratio of the corresponding admittances or the inverse ratio of the corresponding impedances.





1.7. DELTA-WYE (OR WYE-DELTA) TRANSFORMATION

- involves the conversion of a network with the □ (or delta) configuration into a network with the T (or wye) configuration, and vice versa, in order to facilitate network analysis.

Delta to wye conversion:

 $Z_{wye} = \frac{\text{product of adjacent impedances in delta}}{\text{sum of impedances in delta}}$

Wye to delta conversion:

 $Z_{delta} = \frac{\text{sum of product of adjacent impedances in wye}}{\text{opposite impedance in wye}}$

Note: For balanced delta or wye, $Z_{wye} = Z_{delta}/3$ and $Z_{delta} = 3 Z_{wye}$



