Chapter 5

Operational Amplifier Circuits

5.1 Introduction

An operational amplifier (commonly called *op amp* or *opamp*) is a device that can be used to perform mathematical operations such as addition, subtraction, amplification, attenuation, integration, and differentiation. It is a versatile integrated circuit (IC) chip that is widely used in amplifiers, filters, signal conditioning, and instrumentation circuits. The circuit symbol for an op amp is shown in Figure 5.1.

As can be seen from Figure 5.1, there are two input terminals and one output terminal for an op amp. The voltage at the positive input terminal (i.e., the noninverting input terminal) is v_p, the voltage at the negative input terminal (i.e., the inverting input terminal) is v_n, and the voltage at the output terminal is v_o. Figure 5.2 shows pin configuration for a typical 8-pin package. Pin number 2 is the inverting input terminal, and pin number 3 is the noninverting input terminal. The output signal is available at pin number 6. Pin number 7 is the positive power supply, V_{cc^+}, and pin number 4 is the negative power supply, V_{cc^-}.

FIGURE 5.1

Circuit symbol for an op amp.

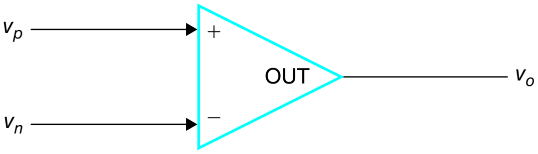
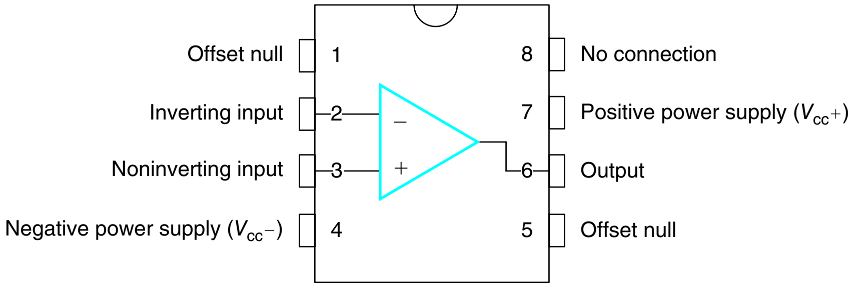


FIGURE 5.2

Pin configuration of typical 8-pin package.

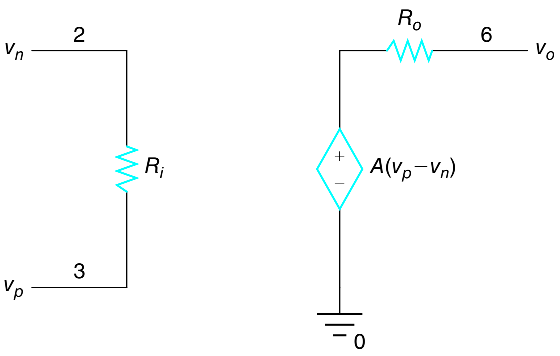


5.2 Ideal Op Amp

Op amps can be modeled as a voltage-controlled voltage source (VCVS), as shown in Figure 5.3.

FIGURE 5.3

A model for op amp.

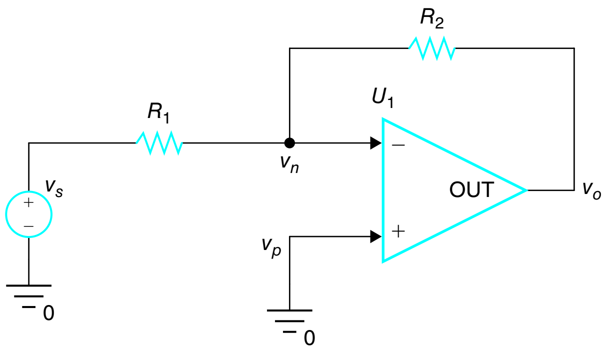


In the model, v_n is the voltage on the inverting input (pin 2) of the op amp, v_p is the voltage on the noninverting input (pin 3) of the op amp, v_o is the voltage on the output (pin 6) of the op amp, R_i is the input resistance, R_o is the output resistance, and A is the unloaded voltage gain. In general, the input resistance R_i is large, the output resistance R_o is small, the gain A is large. For $\mu A741$ (or $uA741$) general-purpose operational amplifier from Texas Instruments, typical value of the input resistance is 2 MΩ, output resistance is 75 Ω, and large-signal differential voltage amplification is 200,000. If $R_i = \infty$, $R_o = 0$, and $A = \infty$, the op amp is called ideal. If an op amp is assumed to be ideal, the analysis of circuits with op amps is greatly simplified. The controlled voltage is proportional to the difference of the two inputs, $v_d = v_p - v_n$. The voltage v_d is called *differential input*.

Figure 5.4 shows the inverting configuration of an op amp.

FIGURE 5.4

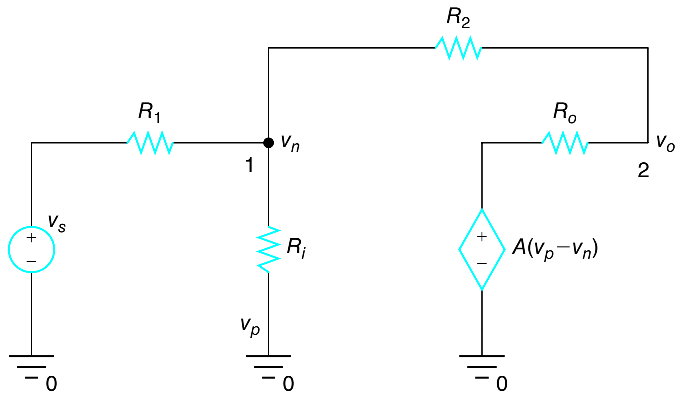
Inverting configuration.



The noninverting input terminal is grounded ($v_p = 0$). The input voltage v_s is applied to the inverting input through a resistor R_1. Resistor R_2 provides a feedback path between the output terminal and the inverting input terminal. When the op amp is replaced by the model shown in Figure 5.3, we obtain the circuit shown in Figure 5.5.

FIGURE 5.5

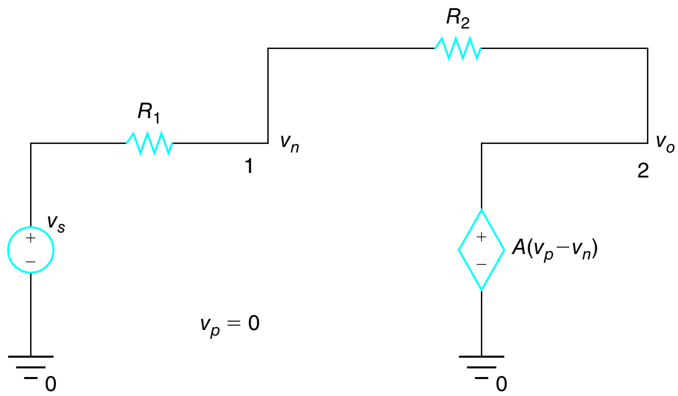
A model for inverting configuration.



More detailed analysis of the circuit shown in Figure 5.5 is presented in Section 5.6 of this chapter. For now, in this section, we assume $R_o = 0$, $R_i = \infty$ and A is large. Under these conditions, the circuit shown in Figure 5.5 reduces to the one shown in Figure 5.6.

FIGURE 5.6

A simplified model for inverting configuration.



We can apply nodal analysis to the circuit shown in Figure 5.6. Summing the currents leaving node 1, we obtain

$$\frac{v_n - v_s}{R_1} + \frac{v_n - v_o}{R_2} = 0$$

which can be revised as

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_n = \frac{1}{R_1}v_s + \frac{1}{R_2}v_o$$

Solving for v_n, we obtain

$$v_n = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}}v_s + \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}v_o = \frac{R_2}{R_1 + R_2}v_s + \frac{R_1}{R_1 + R_2}v_o \tag{5.1}$$

The voltage at the inverting input is the sum of input component, $v_sR_2/(R_1 + R_2)$, and the feedback component from the output, $v_oR_1/(R_1 + R_2)$. Let the voltage difference between noninverting input v_p and the inverting input v_n be v_d. Then, we have

$$v_d = v_p - v_n$$

The output v_o is given by

$$v_o = Av_d = A(v_p - v_n) = A(0 - v_n) = -Av_n \tag{5.2}$$

Substitution of Equation (5.1) into Equation (5.2) yields

$$v_o = -\frac{AR_2}{R_1 + R_2}v_s - \frac{AR_1}{R_1 + R_2}v_o \tag{5.3}$$

Equation (5.3) can be rearranged as

$$v_o = \frac{-\frac{AR_2}{R_1 + R_2}}{1 + \frac{AR_1}{R_1 + R_2}}v_s = \frac{-\frac{R_2}{R_1 + R_2}}{\frac{1}{A} + \frac{R_1}{R_1 + R_2}}v_s \tag{5.4}$$

Without the negative feedback component from the output, which is the second term of Equation (5.3), the output v_o consists of the first term of Equation (5.3) only, which is due to input v_s. The output v_o will be large due to large gain A. Equation (5.4) shows that the negative feedback component provides comparable gain in the denominator $[AR_1/(R_1 + R_2)]$ to offset the effects of large gain in the numerator $[AR_2/(R_1 + R_2)]$. It is called *negative feedback* because the feedback from the output is subtracted from the positive input v_p.

Equation (5.4) can be written as

$$v_o = -\frac{AR_2}{R_1 + R_2 + AR_1}v_s = -\frac{\dfrac{R_2}{R_1 + R_2}A}{1 + \dfrac{R_1}{R_1 + R_2}A}v_s = -\frac{R_2}{\dfrac{R_1 + R_2}{A} + R_1}v_s \tag{5.5}$$

Substitution of Equation (5.5) into Equation (5.1) results in

$$\begin{aligned} v_n &= \frac{R_2}{R_1 + R_2}v_s - \frac{R_1}{R_1 + R_2}\frac{AR_2}{R_1 + R_2 + AR_1}v_s = \frac{R_2(R_1 + R_2 + AR_1) - R_1R_2A}{(R_1 + R_2)(R_1 + R_2 + AR_1)}v_s \\ &= \frac{R_2}{R_1 + R_2 + AR_1}v_s = \frac{\dfrac{R_2}{R_1 + R_2}}{1 + \dfrac{R_1}{R_1 + R_2}A}v_s = \frac{\dfrac{R_2}{A}}{\dfrac{R_1 + R_2}{A} + R_1}v_s \end{aligned} \tag{5.6}$$

Since $A \gg (R_1 + R_2)$, $(R_1 + R_2)/A \approx 0$ and Equation (5.5) reduces to

$$v_o \cong -\frac{R_2}{R_1}v_s \tag{5.7}$$

Since $A \gg (R_1 + R_2)$, Equation (5.6) reduces to

$$v_n \cong \frac{R_2}{R_1A}v_s \approx 0 \tag{5.8}$$

The voltage difference is given by

$$v_d = v_p - v_n = 0 - v_n = -\frac{\dfrac{R_2}{A}}{\dfrac{R_1 + R_2}{A} + R_1}v_s \cong -\frac{R_2}{R_1A}v_s \approx 0 \tag{5.9}$$

The current through R_i can be approximated by

$$i_{R_i} = \frac{v_d}{R_i} = \frac{-v_n}{R_i} \cong -\frac{R_2}{R_1AR_i}v_s \approx 0 \tag{5.10}$$

Notice that the denominator of Equation (5.10) includes AR_i, which is the product of two large numbers, making the current through R_i very small. From Equations (5.9) and (5.10), we conclude that the current through R_i is close to zero, and the voltage difference, $v_d = v_p - v_n$, is close to zero (i.e., $v_p \approx v_n$). The voltage drop across R_i is close to zero.

Figure 5.7 shows a noninverting configuration of an op amp.

The input voltage v_s is applied to the noninverting input terminal of the op amp ($v_p = v_s$). There is a resistor R_1 between inverting input and the ground, and another resistor R_2 between the inverting input and the output v_o provides feedback from output v_o to v_n. When the op amp is replaced by the model shown in Figure 5.3, we obtain the circuit shown in Figure 5.8.

FIGURE 5.7

Noninverting configuration of an op amp.

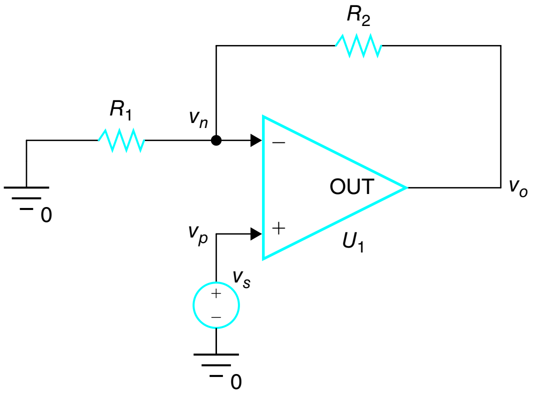


FIGURE 5.8

A model of a noninverting configuration.

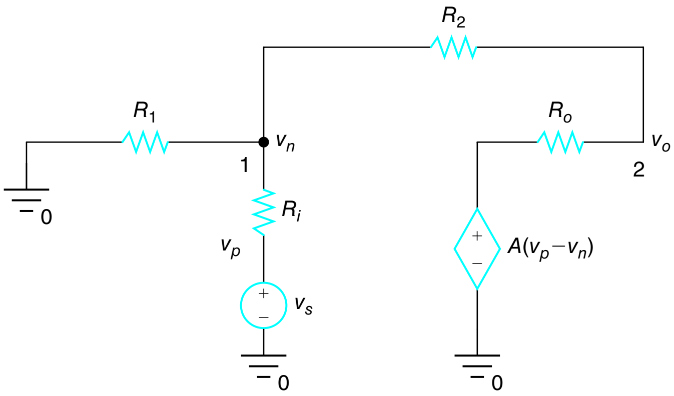
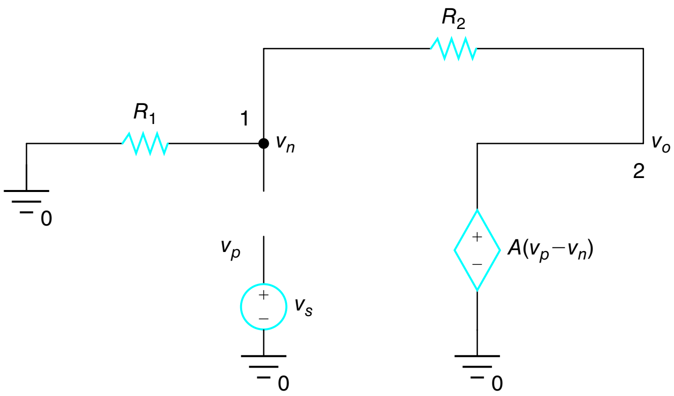


FIGURE 5.9

A simplified model for noninverting configuration.



More detailed analysis of the circuit shown in Figure 5.8 is presented in Section 5.7 of this chapter. In this section, we assume that $R_o = 0$, $R_i = \infty$ and A is large. Under these conditions, the circuit shown in Figure 5.8 reduces to the one shown in Figure 5.9.

We can apply nodal analysis to the circuit shown in Figure 5.9. Summing the currents leaving node 1, we obtain

$$\frac{v_n}{R_1} + \frac{v_n - v_o}{R_2} = 0$$

which can be revised as

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_n = \frac{1}{R_2}v_o$$

Solving for v_n, we obtain

$$v_n = \frac{\dfrac{1}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}v_o = \frac{R_1}{R_1 + R_2}v_o \tag{5.11}$$

The output signal is given by

$$v_o = A(v_p - v_n) = A(v_s - v_n) \tag{5.12}$$

Substitution of Equation (5.11) into Equation (5.12) yields

$$v_o = Av_s - \frac{AR_1}{R_1 + R_2}v_o \tag{5.13}$$

Equation (5.13) can be revised as

$$v_o = \frac{A}{1 + \dfrac{R_1}{R_1 + R_2}A}v_s = \frac{1}{\dfrac{1}{A} + \dfrac{R_1}{R_1 + R_2}}v_s \tag{5.14}$$

Without the negative feedback component, which is the second term of Equation (5.13), the output v_o consists of the first term of Equation (5.13) only, due to input v_s. The output v_o will be large due to large gain A. Equation (5.14) shows that the negative feedback component provides comparable gain $[AR_1/(R_1 + R_2)]$ in the denominator to offset the effects of large gain (A) in the numerator.

Equation (5.14) can be written as

$$v_o = \frac{A(R_1 + R_2)}{R_1 + R_2 + AR_1}v_s = \frac{R_1 + R_2}{\dfrac{R_1 + R_2}{A} + R_1}v_s \tag{5.15}$$

Substitution of Equation (5.15) into Equation (5.11) results in

$$v_n = \frac{R_1}{R_1 + R_2}\frac{A(R_1 + R_2)}{R_1 + R_2 + AR_1}v_s = \frac{AR_1}{R_1 + R_2 + AR_1}v_s = \frac{R_1}{\dfrac{R_1 + R_2}{A} + R_1}v_s \tag{5.16}$$

Since $A \gg (R_1 + R_2)$, $(R_1 + R_2)/A \approx 0$ and Equation (5.15) reduces to

$$v_o \cong \frac{R_1 + R_2}{R_1} v_s = \left(1 + \frac{R_2}{R_1}\right) v_s \tag{5.17}$$

Since $A \gg (R_1 + R_2)$, Equation (5.16) reduces to

$$v_n = \frac{R_1}{\dfrac{R_1 + R_2}{A} + R_1} v_s \cong v_s \tag{5.18}$$

The voltage difference v_d is given by

$$\begin{aligned} v_d &= v_p - v_n = v_s - \frac{AR_1}{R_1 + R_2 + AR_1} v_s = \frac{R_1 + R_2}{R_1 + R_2 + AR_1} v_s \\ &\cong \frac{R_1 + R_2}{AR_1} v_s \approx 0 \end{aligned} \tag{5.19}$$

The current through R_i can be approximated by

$$i_{R_i} = \frac{v_d}{R_i} \cong \frac{R_1 + R_2}{AR_1R_i} v_s \approx 0 \tag{5.20}$$

Notice that the denominator of Equation (5.20) includes AR_i, which is the product of two large numbers, making the current through R_i very small. From Equations (5.19) and (5.20), we conclude that the current through R_i is close to zero, and the voltage difference $v_d = v_p - v_n$ is close to zero (i.e., $v_p \approx v_n$). The voltage drop across R_i is close to zero.

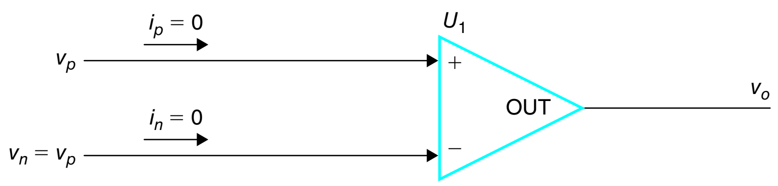
In summary, in the ideal op amp model for both inverting and noninverting configuration,

a. The current flowing into the op amp from the positive input terminal is zero ($i_p = 0$).
b. The current flowing into the op amp from the negative input terminal is zero ($i_n = 0$).
c. The voltage difference between v_p and v_n is zero; that is, $v_p = v_n$.

The fact that $v_p = v_n$ is called **virtual short**. Figure 5.10 shows an op amp with these three properties for both inverting and noninverting configuration.

FIGURE 5.10

An op amp showing zero input currents and a virtual short.



These properties make it simpler to analyze circuits with op amps. Since the error between the exact solution based on practical op amp model and the ideal op amp model is so small, we can use the ideal op amp model for many applications. We can use the ideal op amp model to find the output of the inverting amplifier shown in Figure 5.11.

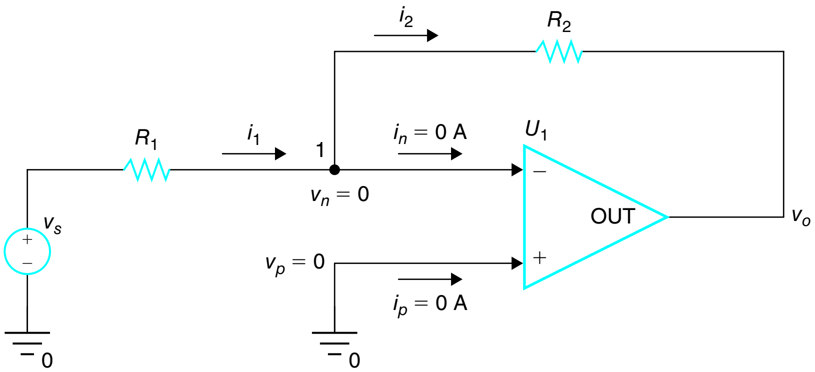
Since $v_p = v_n$ and $v_p = 0$, we have $v_n = 0$. From Ohm's law, the current i_1 through R_1 is given by

$$i_1 = \frac{v_s - v_n}{R_1} = \frac{v_s - 0}{R_1} = \frac{v_s}{R_1}$$

Since the current i_n flowing into the op amp at the inverting input terminal (node 1) is zero, according to Kirchhoff's current law (KCL), the current through R_2 is the same as i_1;

FIGURE 5.11

Inverting amplifier.



that is, $i_2 = i_1 = v_s/R_1$. The voltage across R_2 is $-i_2R_2$ (positive on the right side); that is, $v_{R_2} = -i_2R_2 = (-R_2/R_1)v_s$. Since $v_o = v_{R_2} + v_n = v_{R_2} + 0$, we have $v_o = (-R_2/R_1)v_s$. Instead of this intuitive approach, we can simply write a node equation at the inverting input of the op amp by summing the currents leaving node 1:

$$\frac{v_n - v_s}{R_1} + \frac{v_n - v_o}{R_2} = 0$$

Setting $v_n = 0$, we have

$$\frac{0 - v_s}{R_1} + \frac{0 - v_o}{R_2} = 0$$

Solving for v_o, we obtain

$$v_o = -\frac{R_2}{R_1}v_s$$

The output voltage is the input voltage multiplied by $-R_2/R_1$. The voltage gain of the amplifier, defined by $G = v_o/v_s$, is

$$G = -\frac{R_2}{R_1} \tag{5.21}$$

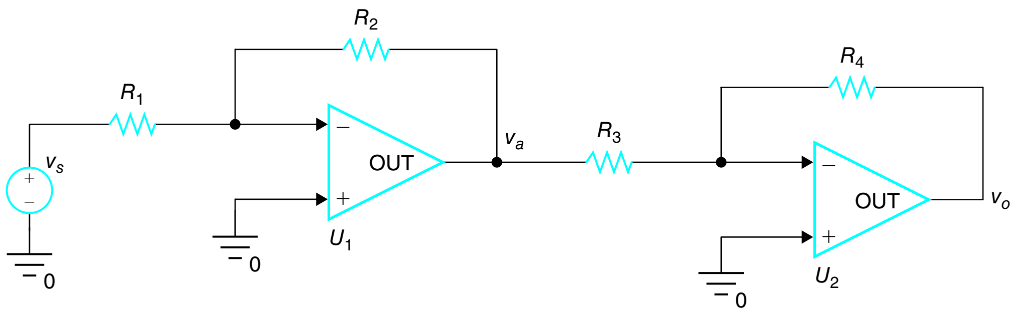
with sign inversion.

Figure 5.12 shows two inverting amplifiers that are cascaded. The voltage gain of the first amplifier is $v_a/v_s = -R_2/R_1$ and the voltage gain of the second amplifier is $v_o/v_a = -R_4/R_3$. The overall gain of the two cascaded amplifiers is given by

$$G = \frac{v_o}{v_s} = \frac{v_a}{v_s} \times \frac{v_o}{v_a} = \left(-\frac{R_2}{R_1}\right) \times \left(-\frac{R_4}{R_3}\right) = \frac{R_2R_4}{R_1R_3}$$

FIGURE 5.12

Two inverting amplifiers are cascaded.

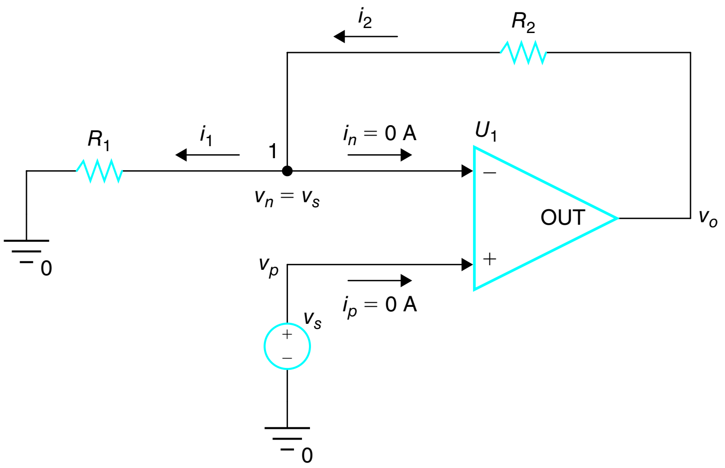


The output signal has the same sign as the input signal, and the voltage gain is the product of the voltage gain of each amplifier.

We can use the ideal op amp model to find the output of the noninverting amplifier shown in Figure 5.13.

FIGURE 5.13

Noninverting amplifier.



Since $v_p = v_n$ and $v_p = v_s$, we have $v_n = v_s$. The current i_1 through R_1 is given by

$$i_1 = \frac{v_n - 0}{R_1} = \frac{v_s - 0}{R_1} = \frac{v_s}{R_1}$$

Since the current i_n flowing into the op amp at the inverting input terminal (node 1) is zero, according to KCL, the current through R_2 is the same as i_1; that is, $i_2 = i_1 = v_s/R_1$. The voltage across R_2 is i_2R_2 (positive on the right side); that is, $v_{R_2} = i_2R_2 = (R_2/R_1)v_s$. Since $v_o = v_n + v_{R_2}$, we have

$$v_o = v_n + v_{R_2} = v_s + \frac{R_2}{R_1}v_s = \left(1 + \frac{R_2}{R_1}\right)v_s = \frac{R_1 + R_2}{R_1}v_s$$

Instead of this intuitive approach, we can simply write a node equation at the inverting input of the op amp. Summing the currents leaving node 1, we have

$$\frac{v_n - 0}{R_1} + \frac{v_n - v_o}{R_2} = 0$$

Setting $v_n = v_s$, we get

$$\frac{v_s}{R_1} + \frac{v_s - v_o}{R_2} = 0$$

Solving for v_o, we obtain

$$v_o = \left(1 + \frac{R_2}{R_1}\right)v_s = \frac{R_1 + R_2}{R_1}v_s$$

The noninverting configuration can be viewed as a feedback circuit from the output v_o to the input. Since the current into the op amp from the inverting input is zero ($i_n = 0$), we can apply the voltage divider rule: the output voltage v_o is divided between

R_1 and R_2 in proportion to the resistance values. Thus, the voltage at the inverting input is given by

$$v_s = \frac{R_1}{R_1 + R_2} v_o$$

Solving for v_o, we have

$$v_o = \frac{R_1 + R_2}{R_1} v_s = \left(1 + \frac{R_2}{R_1}\right) v_s$$

The output voltage is the input voltage multiplied by $(1 + R_2/R_1)$. The voltage gain of the amplifier, $G = v_o/v_s$, is

$$G = 1 + \frac{R_2}{R_1} \tag{5.22}$$

without sign inversion. The voltage gain is always greater than or equal to 1.

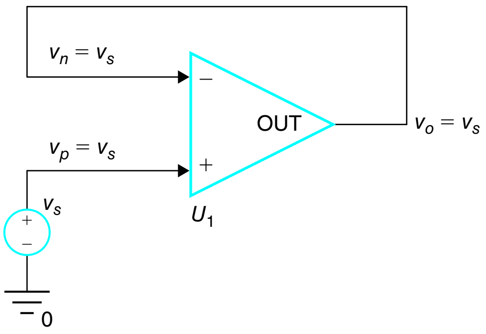
5.2.1 VOLTAGE FOLLOWER

The circuit shown in Figure 5.14 is called a *voltage follower*, or a *buffer amplifier*.

The input signal is applied to the noninverting input terminal of the op amp, and the inverting input terminal is directly connected to the output terminal. Since $v_n = v_p$ and $v_p = v_s$, we have $v_n = v_s$. Since $v_o = v_n$, we have $v_o = v_s$. Thus, the output voltage follows the input voltage exactly. That is why this circuit is called a *voltage follower*. When the op amp in the voltage follower circuit shown in Figure 5.14 is replaced by a VCVS model, we obtain the circuit shown in Figure 5.15.

FIGURE 5.14

A voltage follower circuit.



Notice that $v_n = v_o$. Summing the currents leaving node 2, we obtain

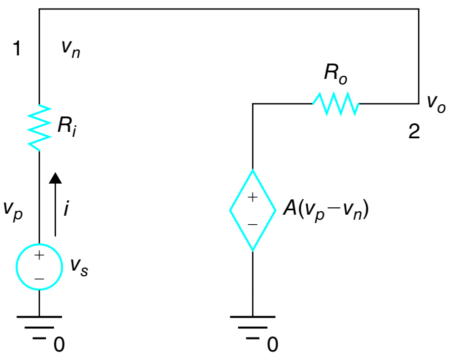
$$\frac{v_o - v_s}{R_i} + \frac{v_o - A(v_s - v_o)}{R_o} = 0$$

Solving for v_o, we obtain

$$v_o = v_n = \frac{\dfrac{1}{R_i} + \dfrac{A}{R_o}}{\dfrac{1}{R_i} + \dfrac{1}{R_o} + \dfrac{A}{R_o}} v_s$$

FIGURE 5.15

A voltage follower circuit with the op amp model.



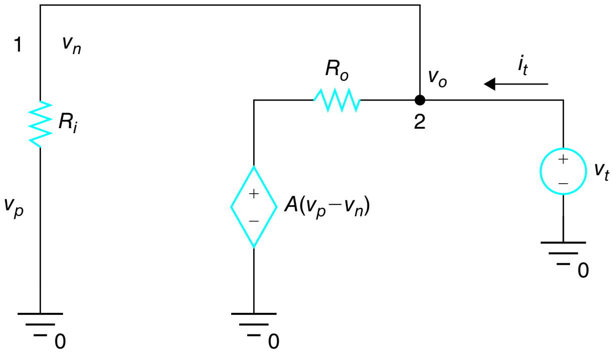
The current flowing out of the positive terminal of v_s is given by

$$i = \frac{v_s - v_n}{R_i} = \frac{1 - \dfrac{\dfrac{1}{R_i} + \dfrac{A}{R_o}}{\dfrac{1}{R_i} + \dfrac{1}{R_o} + \dfrac{A}{R_o}}}{R_i} v_s = \frac{\dfrac{\dfrac{1}{R_o}}{\dfrac{1}{R_i} + \dfrac{1}{R_o} + \dfrac{A}{R_o}}}{R_i} v_s$$

$$= \frac{\dfrac{1}{R_o}}{1 + \dfrac{R_i}{R_o} + \dfrac{AR_i}{R_o}} v_s = \frac{1}{R_o + R_i + AR_i} v_s$$

FIGURE 5.16

A voltage follower circuit with test voltage at the output.



The input resistance R_{in} is defined as the ratio of v_s to i. For the voltage follower model shown in Figure 5.15, the input resistance is given by

$$R_{in} = \frac{v_s}{i} = (A + 1)R_i + R_o \approx AR_i$$

Since both A and R_i are large, the input resistance of the voltage follower is large. To find the output resistance of the voltage follower model shown in Figure 5.15, we deactivate the voltage source and apply a test voltage at the output, as shown in Figure 5.16. Notice that $v_p = 0$ and $v_n = v_o = v_t$.

The current flowing out of the test voltage source is given by

$$i_t = \frac{v_t}{R_i} + \frac{v_t - A(0 - v_t)}{R_o} = \left(\frac{1}{R_i} + \frac{1 + A}{R_o}\right)v_t$$

The output resistance R_{out} is defined as the ratio of v_t to i_t. For the voltage follower model shown in Figure 5.16, the output resistance is given by

$$R_{out} = \frac{v_t}{i_t} = \frac{1}{\dfrac{1}{R_i} + \dfrac{1 + A}{R_o}} \approx \frac{R_o}{A + 1} \approx \frac{R_o}{A}$$

Since R_o is small and A is large, the output resistance of voltage follower is small.

The voltage gain of the voltage follower is 1. What is the purpose of using a unity gain voltage follower? The op amp provides large input resistance and small output resistance, as shown previously. The purpose of the voltage follower is to provide a buffer between the source and load to reduce the loading effect. Consider a voltage source v_s with source resistance $R_S = 200\ \Omega$. If this source is directly applied to a load with resistance $R_L = 1\ k\Omega$, as shown in Figure 5.17, the voltage across the load is $v_s \times 1000/(1000 + 200) = 0.833v_s$. Direct connection of the source and load results in a 16.7% reduction in the load voltage.

FIGURE 5.17

A source connected directly to the load.

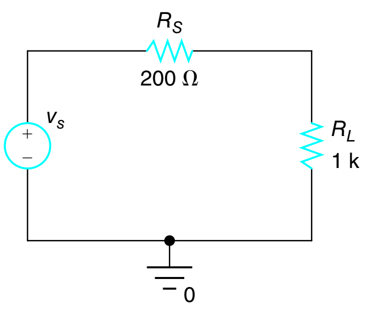
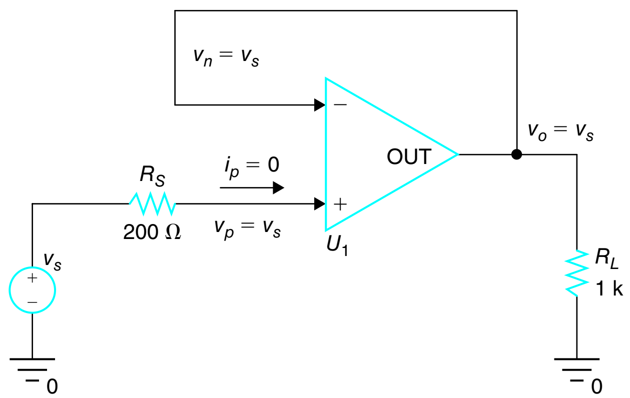


Figure 5.18 shows a circuit that connects the voltage source with source resistance R_S to the positive input terminal of the op amp, and the load resistor is connected to the output of the op amp. For the ideal op amp, since the current flowing into the op amp is zero ($i_p = 0$), the voltage drop across R_S is zero. Thus, the voltage at the noninverting input terminal is v_s ($v_p = v_s$). Due to a virtual short, the voltage

FIGURE 5.18

A source connected to the load through voltage follower.



at the negative input terminal is also v_s ($v_n = v_p = v_s$). Since the output terminal is directly connected to negative input terminal, we have $v_o = v_s$. The voltage across the load impedance is v_s, eliminating the effect of R_S. The voltage follower can be used to isolate different sections of circuits.

EXAMPLE 5.1

Express V_o in the circuit shown in Figure 5.19 as a function of V_s, R_1, R_2, R_3, and R_4. Find the numerical value of V_o if $V_s = 0.5$ V, $R_1 = 2$ kΩ, $R_2 = 5$ kΩ, $R_3 = 1.5$ kΩ, $R_4 = 9$ kΩ.

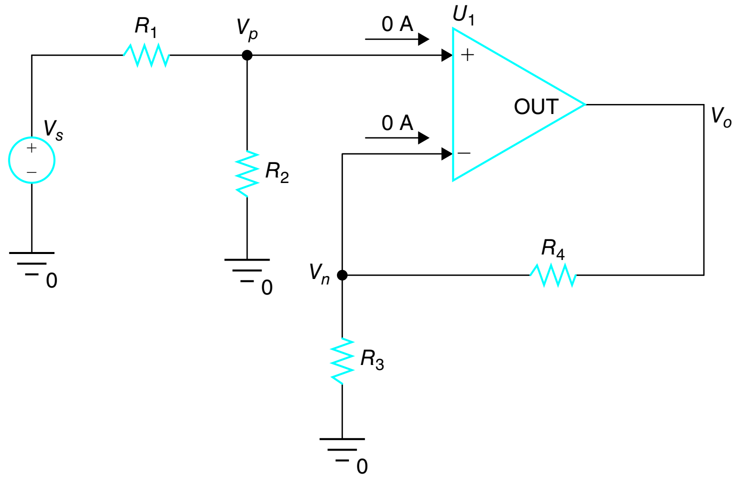


FIGURE 5.19 Circuit for EXAMPLE 5.1.

Since the current entering the positive input of the op amp is zero, according to the voltage divider rule, the voltage at the positive input of the op amp, which is the voltage across R_2, is given by

$$V_p = \frac{R_2}{R_1 + R_2} V_s$$

Similarly, since the current entering the negative input of the op amp is zero, according to the voltage divider rule, the voltage at the negative input of the op amp, which is the voltage across R_3, is given by

$$V_n = \frac{R_3}{R_3 + R_4} V_o$$

Due to a virtual short, we have $V_n = V_p$. Thus, we have

$$\frac{R_3}{R_3 + R_4} V_o = \frac{R_2}{R_1 + R_2} V_s$$

Solving for V_o, we obtain

$$V_o = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{R_3 + R_4}{R_3}\right) V_s = \left(\frac{R_2}{R_1 + R_2}\right)\left(1 + \frac{R_4}{R_3}\right) V_s$$

continued

Example 5.1 continued

If $V_s = 0.5$ V, $R_1 = 2\ k\Omega$, $R_2 = 5\ k\Omega$, $R_3 = 1.5\ k\Omega$, $R_4 = 9\ k\Omega$, V_o is given by

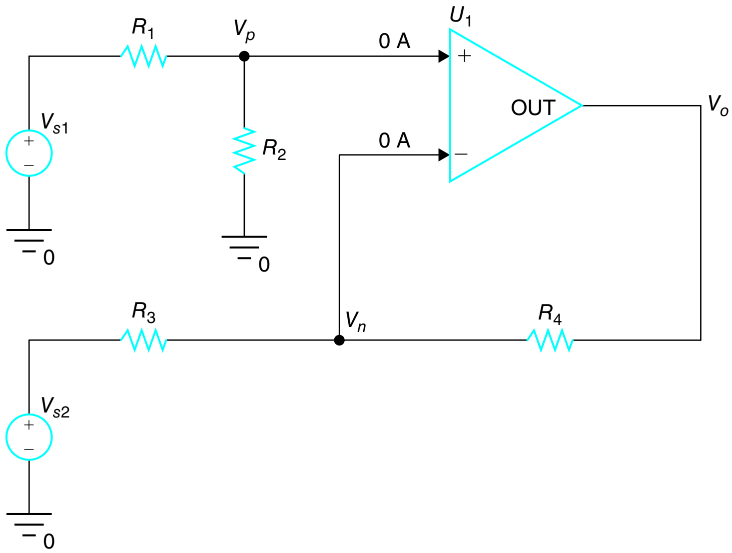
$$V_o = \left(\frac{5\ k\Omega}{2\ k\Omega + 5\ k\Omega}\right)\left(1 + \frac{9\ k\Omega}{1.5\ k\Omega}\right) \times 0.5\ \text{V} = 2.5\ \text{V}$$

Exercise 5.1

Express V_o in the circuit shown in Figure 5.20 as a function of V_{s_1}, V_{s_2}, R_1, R_2, R_3, and R_4. Find V_o as a function of V_{s_1} and V_{s_2} if $R_1 = 2\ k\Omega$, $R_2 = 4\ k\Omega$, $R_3 = 1.5\ k\Omega$, $R_4 = 7.5\ k\Omega$.

FIGURE 5.20

Circuit for EXERCISE 5.1.



Answer:

From the superposition principle, we have

$$V_o = \left(\frac{R_2}{R_1 + R_2}\right)\left(1 + \frac{R_4}{R_3}\right)V_{s_1} - \frac{R_4}{R_3}V_{s_2}$$

$$V_o = \left(\frac{4}{6}\right)(1 + 5)V_{s_1} - \frac{7.5}{1.5}V_{s_2} = 4V_{s_1} - 5V_{s_2}$$

EXAMPLE 5.2

Find v_o in the circuit shown in Figure 5.21.

The voltage at the inverting input of the op amp is equal to zero due to a virtual short. Thus, we have $v_1 = 0$. Summing the currents leaving node 1 (inverting input of the op amp), we get

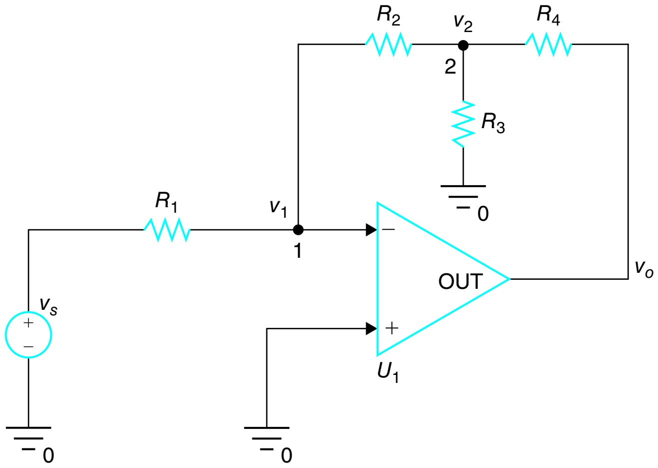
$$\frac{0 - v_s}{R_1} + \frac{0 - v_2}{R_2} = 0$$

continued

Example 5.2 continued

FIGURE 5.21

Circuit for EXAMPLE 5.2.



Solving for v_2, we obtain

$$v_2 = -\frac{R_2}{R_1} v_s$$

Summing the currents leaving node 2, we get

$$\frac{v_2 - 0}{R_2} + \frac{v_2 - 0}{R_3} + \frac{v_2 - v_o}{R_4} = 0$$

This equation can be simplified as

$$v_o = R_4\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)v_2$$

Substituting $v_2 = -\frac{R_2}{R_1} v_s$ into $v_o = R_4\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)v_2$, we get

$$\begin{aligned} v_o &= -\frac{R_2}{R_1} R_4\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)v_s = -\frac{1}{R_1}\left(R_4 + R_2 + \frac{R_2 R_4}{R_3}\right)v_s \\ &= -\frac{1}{R_1}\left[R_4 + R_2\left(1 + \frac{R_4}{R_3}\right)\right]v_s \end{aligned}$$

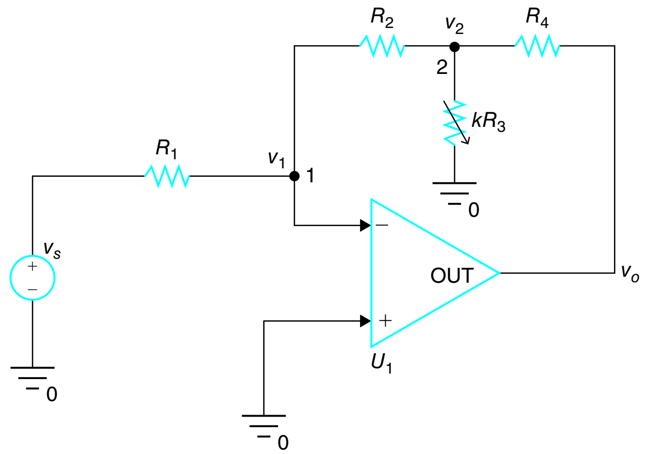
The single feedback resistor is replaced by a T circuit consisting of R_2, R_3, and R_4. The combined resistance of the T circuit is given by $R_4 + R_2\left(1 + \frac{R_4}{R_3}\right)$. If $R_4 > R_3$, the value of the combined resistance will be very large due to the product of R_2 and $1 + \frac{R_4}{R_3}$.

Exercise 5.2

Find v_o in the circuit shown in Figure 5.22 as a function of v_s, R_1, R_2, R_3, k, and R_4. The resistance of the variable resistor is kR_3, where $0 < k < \infty$. Let $v_s = 1$ V, $R_1 = R_2 = R_3 = R_4 = 1$ kΩ. Find v_o for $k = 0.1, 0.5, 1, 10$, and 1000.

FIGURE 5.22

Circuit for EXERCISE 5.2.



Answer:

$$v_o = -\frac{1}{R_1}\left(R_4 + R_2\left(1 + \frac{R_4}{kR_3}\right)\right)v_s$$

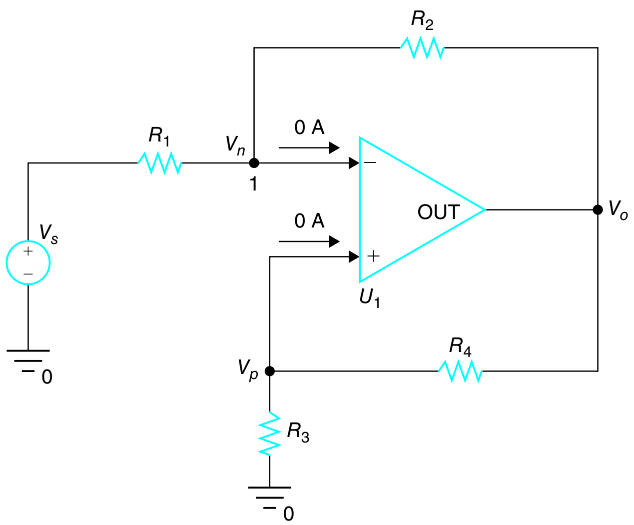
$$v_o = -12 \text{ V}, -4 \text{ V}, -3 \text{ V}, -2.1 \text{ V}, -2.001 \text{ V}.$$

EXAMPLE 5.3

Express V_o in the circuit shown in Figure 5.23 as a function of V_s.

FIGURE 5.23

Circuit for EXAMPLE 5.3.



Since the current entering the positive input of the op amp is zero, according to the voltage divider rule, the voltage at the positive input of the op amp, which is the voltage across R_3, is given by

$$V_p = \frac{R_3}{R_3 + R_4}V_o \quad \text{(5.23)}$$

Summing the currents leaving node 1, we obtain

$$\frac{V_n - V_s}{R_1} + \frac{V_n - V_o}{R_2} = 0$$

Solving for V_o, we get

$$V_o = \left(1 + \frac{R_2}{R_1}\right)V_n - \frac{R_2}{R_1}V_s \quad \text{(5.24)}$$

continued

Example 5.3 continued

Due to a virtual short, we have $V_n = V_p$. Substitution of Equation (5.23) into Equation (5.24) yields

$$V_o = \left(1 + \frac{R_2}{R_1}\right)\frac{R_3}{R_3 + R_4}V_o - \frac{R_2}{R_1}V_s$$

which can be rewritten as

$$V_o = \frac{-\dfrac{R_2}{R_1}V_s}{1 - \left(\dfrac{R_1 + R_2}{R_1}\right)\dfrac{R_3}{R_3 + R_4}} = \frac{-R_2(R_3 + R_4)}{R_1R_4 - R_2R_3}V_s \tag{5.25}$$

Notice that if $R_1R_4 > R_2R_3$, the voltage gain V_o/V_s is negative (i.e., inverting amplifier), and if $R_1R_4 < R_2R_3$, the voltage gain V_o/V_s is positive (i.e., noninverting amplifier).

Exercise 5.3

(a) Let $R_1 = 1\ k\Omega$, $R_2 = 2\ k\Omega$, $R_3 = 2\ k\Omega$, and $R_4 = 2.5\ k\Omega$. Find the voltage gain V_o/V_s for the circuit shown in Figure 5.23.

(b) Let $R_1 = 3.25\ k\Omega$, $R_2 = 2\ k\Omega$, $R_3 = 2.5\ k\Omega$, and $R_4 = 2\ k\Omega$. Find the voltage gain V_o/V_s for the circuit shown in Figure 5.23.

Answer:

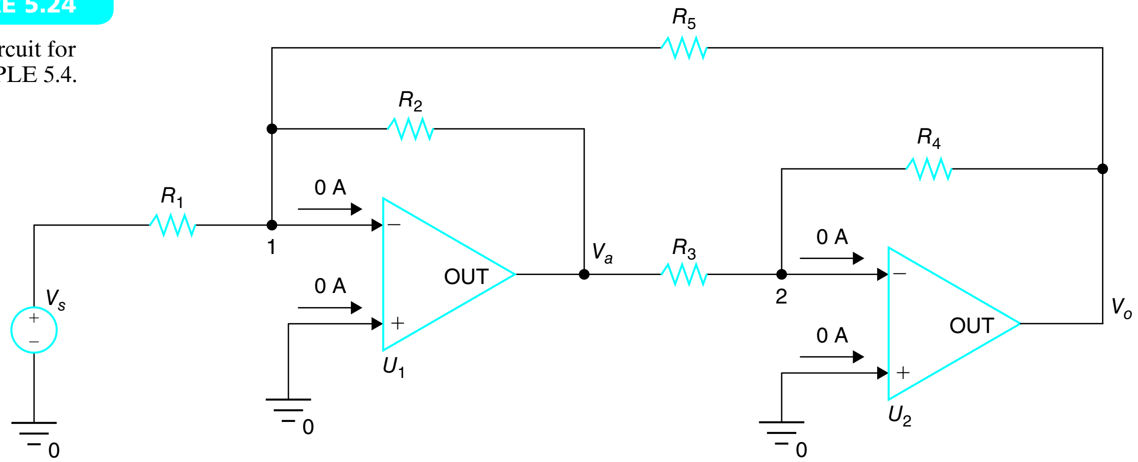
(a) $V_o/V_s = 6$ V/V (a) $V_o/V_s = -6$ V/V

EXAMPLE 5.4

Express V_o in the circuit shown in Figure 5.24 as a function of V_s.

FIGURE 5.24

Circuit for EXAMPLE 5.4.



continued

Example 5.4 continued

Summing the currents leaving node 2, we obtain

$$\frac{0 - V_a}{R_3} + \frac{0 - V_o}{R_4} = 0$$

Solving for V_a, we obtain

$$V_a = -\frac{R_3}{R_4}V_o \tag{5.26}$$

Summing the currents leaving node 1, we obtain

$$\frac{0 - V_s}{R_1} + \frac{0 - V_o}{R_5} + \frac{0 - V_a}{R_2} = 0$$

Solving for V_o and substituting Equation (5.26), we obtain

$$V_o = -\frac{R_5}{R_1}V_s - \frac{R_5}{R_2}V_a = -\frac{R_5}{R_1}V_s - \frac{R_5}{R_2}\left(-\frac{R_3}{R_4}V_o\right) = -\frac{R_5}{R_1}V_s + \frac{R_3R_5}{R_2R_4}V_o \tag{5.27}$$

Solving for V_o, we get

$$V_o = \frac{-\dfrac{R_5}{R_1}}{1 - \dfrac{R_3R_5}{R_2R_4}}V_s = -\frac{R_2R_4R_5}{R_1(R_2R_4 - R_3R_5)}V_s \tag{5.28}$$

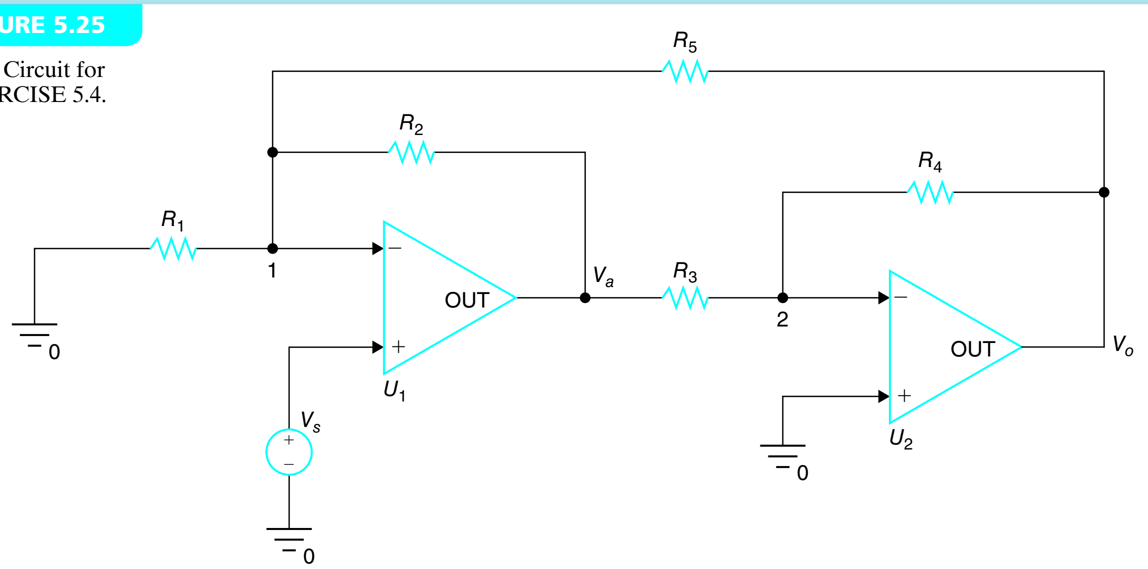
Notice that $R_3 - R_4$ provides a feedback from V_o to V_a, as shown by Equation (5.26). In turn, this voltage is amplified by $R_2 - R_5$, as shown by the second term of Equation (5.27). The first term of Equation (5.27) is the direct path from V_s to V_o with voltage gain $-R_5/R_1$.

Exercise 5.4

Express V_o in the circuit shown in Figure 5.25 as a function of V_s.

FIGURE 5.25

Circuit for EXERCISE 5.4.



continued

Exercise 5.4 continued

Answer:

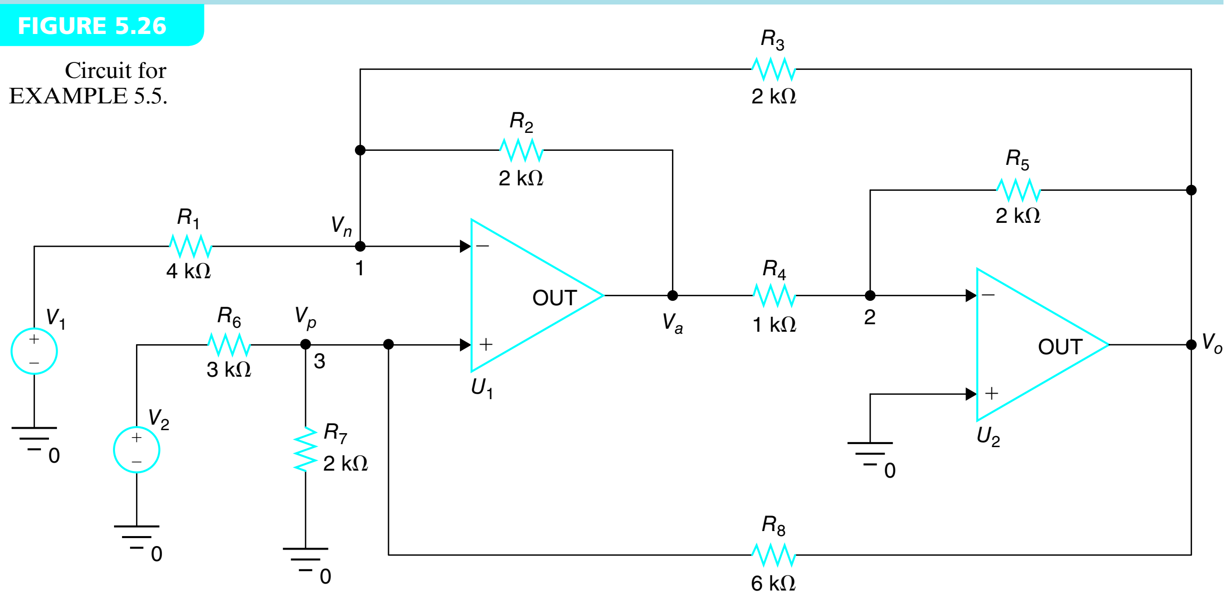
$$V_o = \frac{R_4(R_1R_2 + R_1R_5 + R_2R_5)}{R_1(R_2R_4 - R_3R_5)} V_s$$

EXAMPLE 5.5

Express V_o in the circuit shown in Figure 5.26 as a function of V_1 and V_2.

FIGURE 5.26

Circuit for EXAMPLE 5.5.



Summing the currents leaving node 2, we obtain

$$\frac{0 - V_a}{1000} + \frac{0 - V_o}{2000} = 0$$

Multiplication by 2000 yields

$$-2V_a - V_o = 0,$$

which can be simplified to

$$V_a = -0.5V_o \tag{5.29}$$

Summing the currents leaving node 1, we obtain

$$\frac{V_n - V_1}{4000} + \frac{V_n - V_o}{2000} + \frac{V_n - V_a}{2000} = 0$$

continued

Example 5.5 continued

Multiplication by 4000 yields

$$V_n - V_1 + 2(V_n - V_o) + 2(V_n - V_a) = 0$$

which can be simplified to

$$5V_n - V_1 - 2V_o - 2V_a = 0 \tag{5.30}$$

Substitution of Equation (5.29) into Equation (5.30) yields

$$5V_n - V_1 - 2V_o + V_o = 0$$

Solving for V_n, we obtain

$$V_n = 0.2V_1 + 0.2V_o \tag{5.31}$$

Summing the currents leaving node 3, we obtain

$$\frac{V_p - V_2}{3000} + \frac{V_p}{2000} + \frac{V_p - V_o}{6000} = 0$$

Multiplication by 6000 yields

$$2(V_p - V_2) + 3V_p + V_p - V_o = 0$$

which can be simplified to

$$6V_p - 2V_2 - V_o = 0$$

Solving for V_p, we obtain

$$V_p = \frac{1}{3}V_2 + \frac{1}{6}V_o \tag{5.32}$$

Since $V_n = V_p$ due to a virtual short, from Equations (5.31) and (5.32), we obtain

$$\frac{1}{5}V_1 + \frac{1}{5}V_o = \frac{1}{3}V_2 + \frac{1}{6}V_o$$

which can be simplified to

$$\frac{1}{30}V_o = -\frac{1}{5}V_1 + \frac{1}{3}V_2$$

Multiplying by 30, we obtain

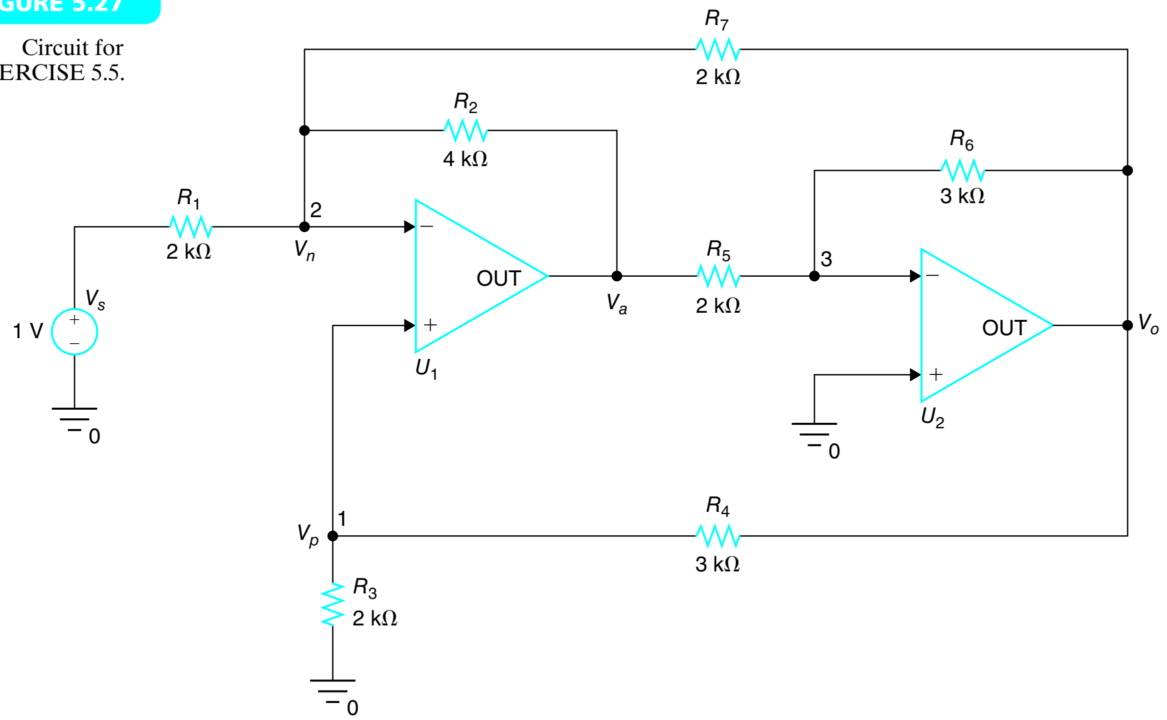
$$V_o = -6V_1 + 10V_2$$

Exercise 5.5

Find V_o in the circuit shown in Figure 5.27.

FIGURE 5.27

Circuit for EXERCISE 5.5.



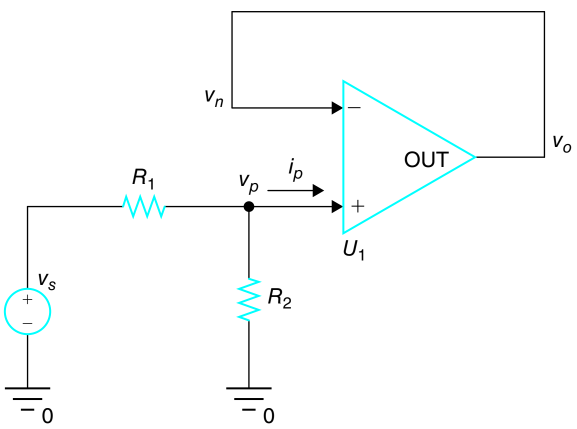
Answer:
$V_o = 3$ V.

EXAMPLE 5.6

Design an op amp circuit that will provide a voltage gain of 0.8.

FIGURE 5.28

Circuit for EXAMPLE 5.6.



A circuit shown in Figure 5.28 can be used to provide a positive voltage gain of less than 1. Since the current flowing into the positive input terminal of the op amp is zero, according to the voltage divider rule, the voltage v_p is given by

$$v_p = \frac{R_2}{R_1 + R_2} v_s$$

Due to a virtual short, we have $v_n = v_p$. Since the output v_o is connected to v_n, we have

$$v_o = \frac{R_2}{R_1 + R_2} v_s$$

continued

Example 5.6 continued

The voltage gain is

$$G = \frac{R_2}{R_1 + R_2}$$

Taking the inverses on both sides, we obtain

$$\frac{R_1 + R_2}{R_2} = \frac{1}{G}$$

For $G = 0.8, 1/G = 1.25$. Let us choose $R_2 = 10\ k\Omega$. Then, we need $R_1 = 2.5\ k\Omega$.

Exercise 5.6

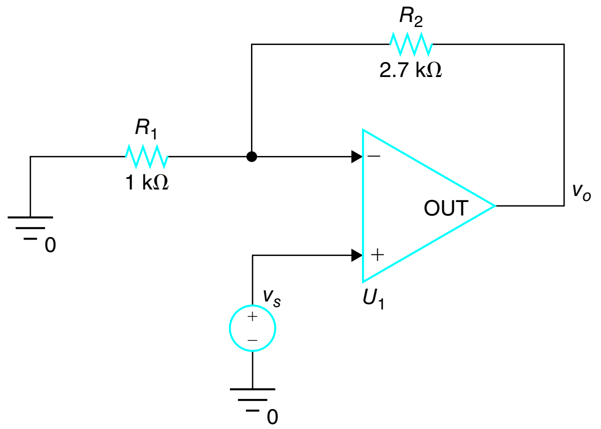
Design an op amp circuit that will provide a voltage gain of 3.7.

Answer:

Since the gain is positive, we can use the noninverting amplifier shown in Figure 5.13 earlier in this chapter. Since the voltage gain is $G = 1 + \frac{R_2}{R_1} = 3.7$, we can choose $R_1 = 1\ k\Omega$ and $R_2 = 2.7\ k\Omega$. The circuit with these values is shown in Figure 5.29.

FIGURE 5.29

Circuit for a voltage gain of 3.7.



5.3 Sum and Difference

In this section, we discuss op amp circuits that can be used to add input signals or subtract one signal from another. The summing amplifiers can be implemented in the inverting configuration or noninverting configuration.

5.3.1 SUMMING AMPLIFIER (INVERTING CONFIGURATION)

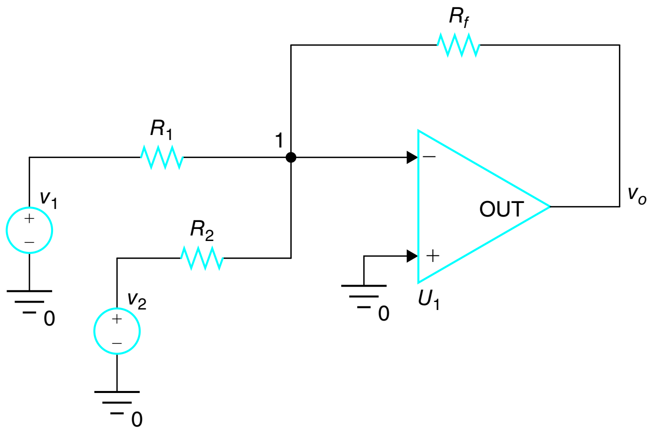
Figure 5.30 shows an inverting amplifier with two inputs, v_1 and v_2.

Due to a virtual short, the voltage at the inverting input of the op amp is zero ($v_n = v_p = 0$). Summing the currents leaving node 1, we obtain

$$\frac{0 - v_1}{R_1} + \frac{0 - v_2}{R_2} + \frac{0 - v_o}{R_f} = 0$$

FIGURE 5.30

A summing amplifier with an inverting configuration.



Solving for v_o, we have

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right) \tag{5.33}$$

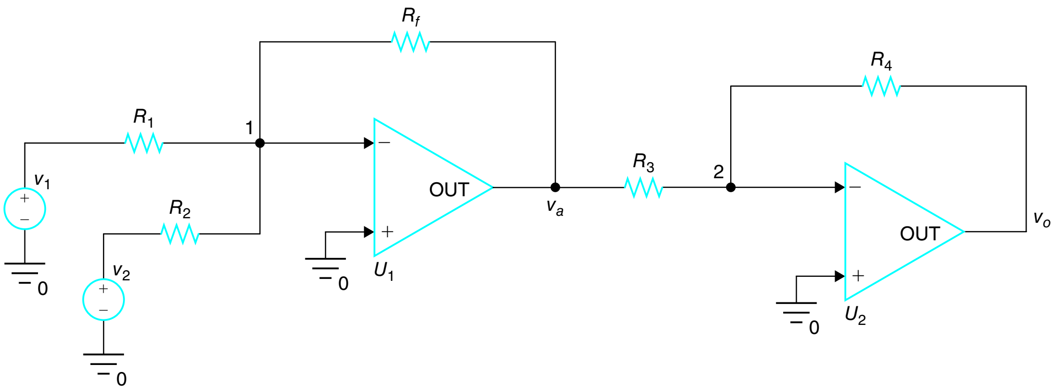
If $R_1 = R_2 = R_f$, we have

$$v_o = -(v_1 + v_2) \tag{5.34}$$

If we want a positive sum instead of a negative sum, we can reverse the sign by applying the output to another inverting amplifier with $R_3 = R_4$, as shown in Figure 5.31.

FIGURE 5.31

Having a sign inverter added to the summing amplifier.



Let v_a be the voltage at the output of the first op amp. Summing the currents leaving node 2, we obtain

$$\frac{0 - v_a}{R_3} + \frac{0 - v_o}{R_4} = 0$$

Solving for v_o, we get

$$v_o = -\frac{R_4}{R_3}v_a$$

If $R_4 = R_3$, we have $v_o = -v_a$. Summing the currents leaving node 1, we obtain

$$\frac{0 - v_1}{R_1} + \frac{0 - v_2}{R_2} + \frac{0 - v_a}{R_f} = 0$$

Solving for v_a, we obtain

$$v_a = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right)$$

From $v_o = -\frac{R_4}{R_3}v_a$, the output voltage is given by

$$v_o = \frac{R_4}{R_3}\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right) \tag{5.35}$$

If $R = R_1 = R_2 = R_3 = R_4 = R_f$, the output voltage becomes

$$v_o = v_1 + v_2 \tag{5.36}$$

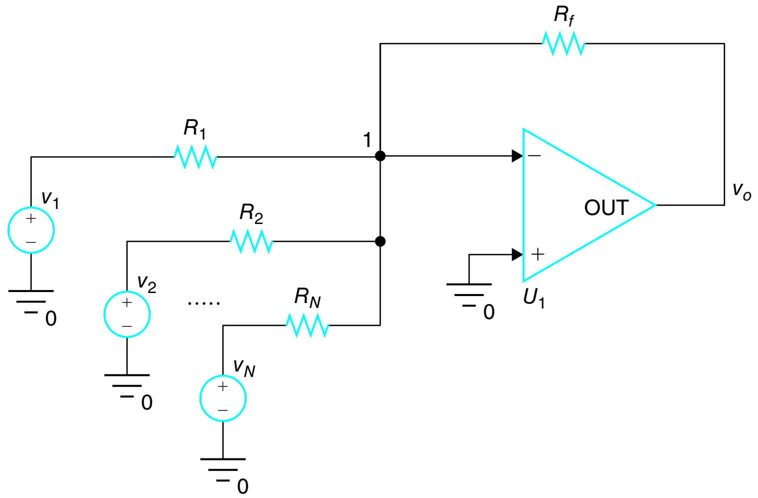
If $R = R_3 = R_4 = R_f$, $R_1 = R/k_1$, and $R_2 = R/k_2$, the output voltage becomes

$$v_o = k_1 v_1 + k_2 v_2 \tag{5.37}$$

Figure 5.32 shows an inverting amplifier with N inputs, $v_1, v_2, \ldots, v_N$.

FIGURE 5.32

Summing amplifier with n inputs.



Due to a virtual short, the voltage at the inverting input of the op amp is zero ($v_n = v_p = 0$). Summing the currents leaving node 1, we obtain

$$\frac{0 - v_1}{R_1} + \frac{0 - v_2}{R_2} + \frac{0 - v_3}{R_3} + \cdots + \frac{0 - v_N}{R_N} + \frac{0 - v_o}{R_f} = 0$$

Solving for v_o, we have

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \cdots + \frac{R_f}{R_N}v_N\right) \tag{5.38}$$

If $R_1 = R_2 = R_3 = \cdots = R_N = R_f$, we have

$$v_o = -(v_1 + v_2 + v_3 + \cdots + v_N) \tag{5.39}$$

If we want a positive sum instead of a negative sum, we can reverse the sign by applying the output to an inverting amplifier with the same resistance values.

If $R_1 = R/k_1, R_2 = R/k_2, R_3 = R/k_3, \ldots, R_N = R/k_N$, and $R_f = R$, we have

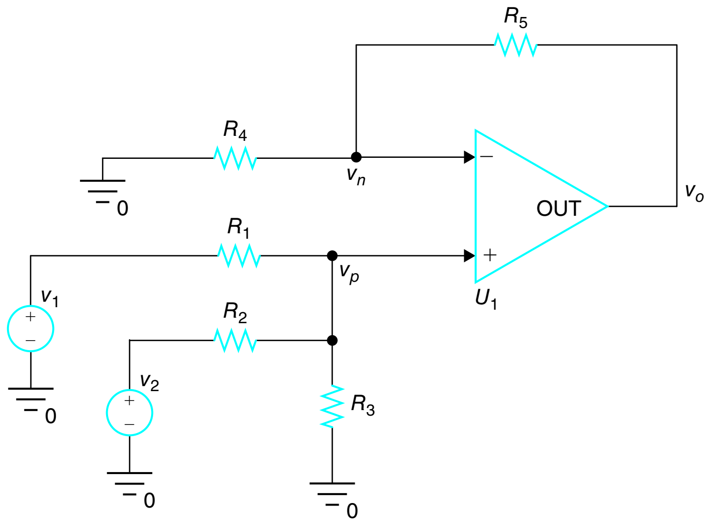
$$v_o = -(k_1 v_1 + k_2 v_2 + k_3 v_3 + \cdots + k_N v_N) \tag{5.40}$$

5.3.2 SUMMING AMPLIFIER (NONINVERTING CONFIGURATION)

The circuit shown in Figure 5.33 can be used to add two signals, v_1 and v_2. There is no sign change in this configuration.

FIGURE 5.33

A summing amplifier with a noninverting configuration.



Summing the currents leaving the negative input of the op amp, we have

$$\frac{v_n - 0}{R_4} + \frac{v_n - v_o}{R_5} = 0$$

Solving for v_n, we obtain

$$v_n = \frac{\dfrac{1}{R_5}}{\dfrac{1}{R_4} + \dfrac{1}{R_5}} v_o \tag{5.41}$$

Summing the currents leaving the positive input of the op amp, we have

$$\frac{v_p - v_1}{R_1} + \frac{v_p - v_2}{R_2} + \frac{v_p - 0}{R_3} = 0$$

which can be rewritten as

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_p = \frac{v_1}{R_1} + \frac{v_2}{R_2} \tag{5.42}$$

Substituting Equation (5.41) into Equation (5.42), we obtain

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \frac{1}{\dfrac{1}{R_4} + \dfrac{1}{R_5}} \frac{v_o}{R_5} = \frac{v_1}{R_1} + \frac{v_2}{R_2}$$

Solving for v_o, we have

$$v_o = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}\left(1 + \frac{R_5}{R_4}\right)\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right) \tag{5.43}$$

Let $R = R_1 = R_2 = R_3$. Then, Equation (5.43) becomes

$$v_o = \frac{R}{3}\left(\frac{R_4 + R_5}{R_4}\right)\frac{1}{R}(v_1 + v_2) \tag{5.44}$$

Let $R_4 = R$ and $R_5 = 2R$. Then, we have

$$v_o = v_1 + v_2 \tag{5.45}$$

Let $R_3 = R_4 = R$, $R_1 = R/k_1$, $R_2 = R/k_2$, and $R_5 = R(k_1 + k_2)$. Then, Equation (5.43) becomes

$$v_o = \frac{1}{\frac{k_1}{R} + \frac{k_2}{R} + \frac{1}{R}}(1 + k_1 + k_2)\frac{1}{R}(k_1 v_1 + k_2 v_2) = k_1 v_1 + k_2 v_2 \tag{5.46}$$

EXAMPLE 5.7

Design an op amp circuit for $v_o = 0.5v_1 + 3v_2$.

Notice that $k_1 = 0.5$ and $k_2 = 3$ in Equation (5.46). Let $R = 3\ k\Omega$. Then, we have

$$R_1 = R/k_1 = 3\ k\Omega/0.5 = 6\ k\Omega,\ R_2 = R/k_2 = 3\ k\Omega/3 = 1\ k\Omega,\ R_3 = R_4 = R = 3\ k\Omega,$$
$$R_5 = R(k_1 + k_2) = 3\ k\Omega \times (0.5 + 3) = 10.5\ k\Omega$$

The circuit with these resistance values is shown in Figure 5.34. There are many other choices for the resistance values.

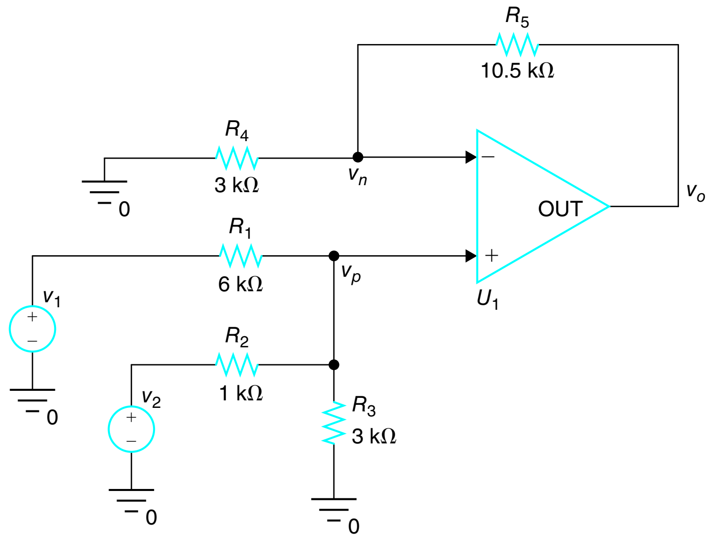


FIGURE 5.34

Circuit for $v_o = 0.5v_1 + 3v_2$.

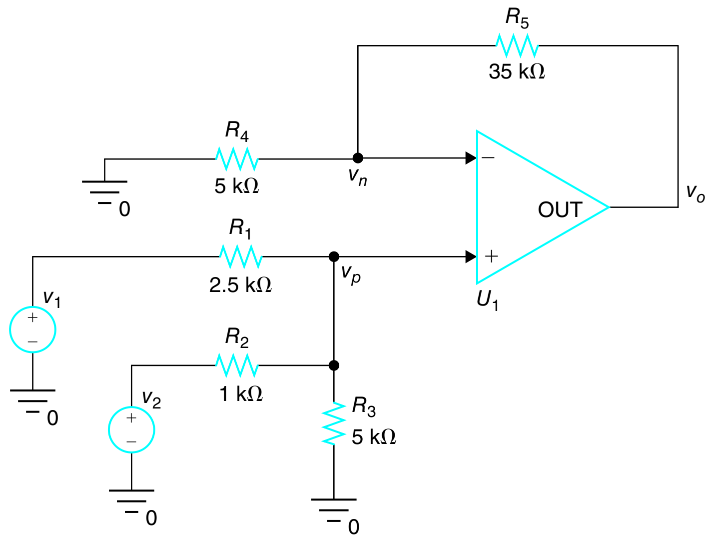
Exercise 5.7

Design an op amp circuit for $v_o = 2v_1 + 5v_2$.

Answer:
$k_1 = 2, k_2 = 5, R = 5\ k\Omega, R_1 = R/k_1 = 2.5\ k\Omega, R_2 = R/k_2 = 1\ k\Omega, R_3 = R_4 = R = 5\ k\Omega,$
$R_5 = R(k_1 + k_2) = 5\ k\Omega \times (2 + 5) = 35\ k\Omega$. The op amp circuit is shown in Figure 5.35.

FIGURE 5.35

Circuit for $v_o = 2v_1 + 5v_2$.



EXAMPLE 5.8

Design a circuit for converting a polar binary signal (1 is represented by a pulse with 5 V, and 0 is represented by a pulse with −5 V) to a unipolar binary signal (1 is represented by a pulse with 5 V, and 0 is represented by a pulse with 0 V).

Let v_1 be the polar signal and v_o be the unipolar signal. Then, adding 5 V to v_1 results in a signal with 10 V (for 1) or 0 V (for 0). We can obtain the unipolar signal by dividing $(v_1 + 5)$ V by 2. Thus, we have

$$v_o = \frac{1}{2}(v_1 + 5)$$

Let v_2 be 5 V. Then choosing $R_4 = 2R$ and $R_5 = R$ in Equation (5.44) results in

$$v_o = \frac{R}{3}\left(\frac{R_4 + R_5}{R_4}\right)\frac{1}{R}(v_1 + v_2) = \frac{R}{3}\left(\frac{2R + R}{2R}\right)\frac{1}{R}(v_1 + v_2) = \frac{1}{2}(v_1 + v_2)$$

In conclusion, if we choose $R_1 = R_2 = R_3 = R_5 = R, R_4 = 2R, V_2 = 5$ V, we can convert a polar signal to a unipolar signal. Figure 5.36 shows the circuit with $R = 10\ k\Omega$, and Figure 5.37 shows sample waveforms for v_1 and v_o. In PSpice®, VPULSE is the voltage source v_1. If the amplitude is other than 5 V, change v_2 to the amplitude of the input signal.

continued

Example 5.8 continued

FIGURE 5.36

Circuit for $v_o = 0.5(v_1 + v_2)$.

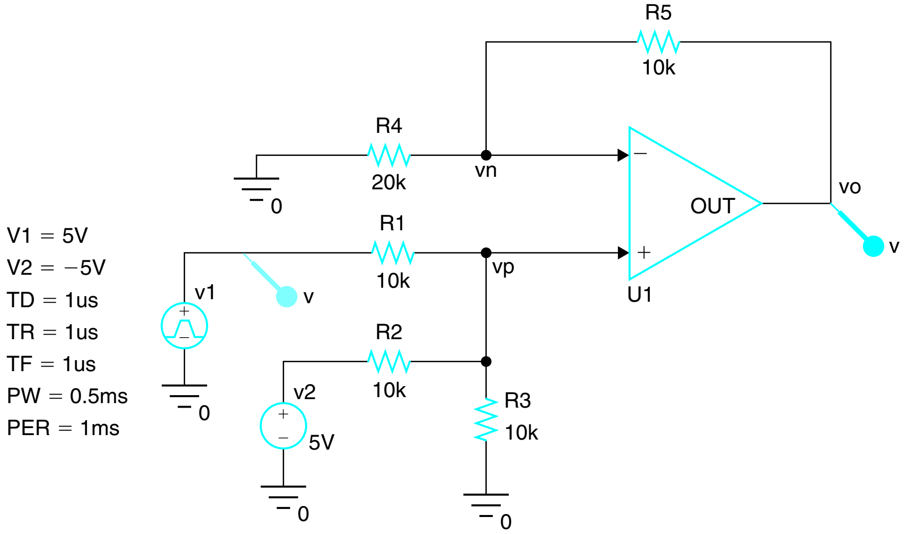


FIGURE 5.37

Sample waveforms.

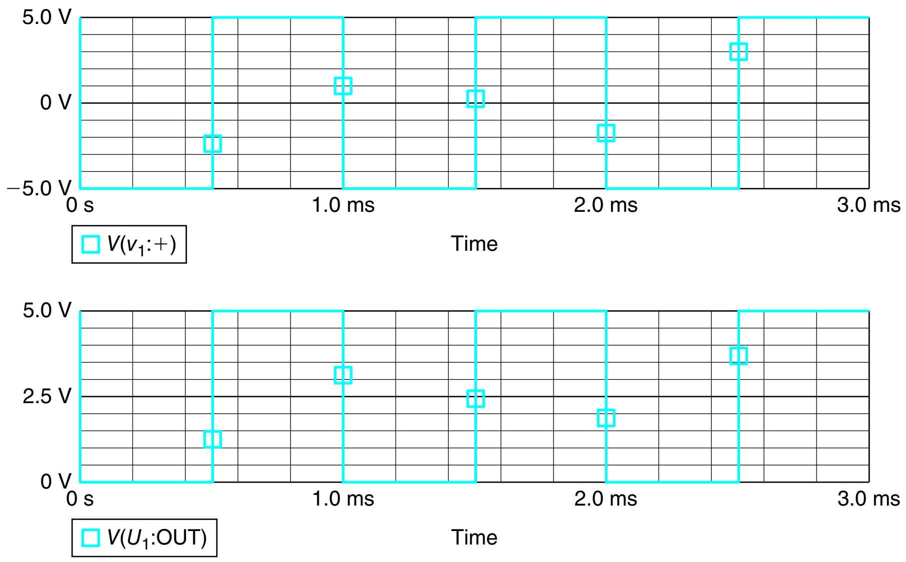


Figure 5.38 shows a noninverting summing amplifier with N inputs. Summing the currents leaving the negative input of the op amp, we have

$$\frac{v_n - 0}{R_{N+2}} + \frac{v_n - v_o}{R_{N+3}} = 0$$

Solving for v_n, we obtain

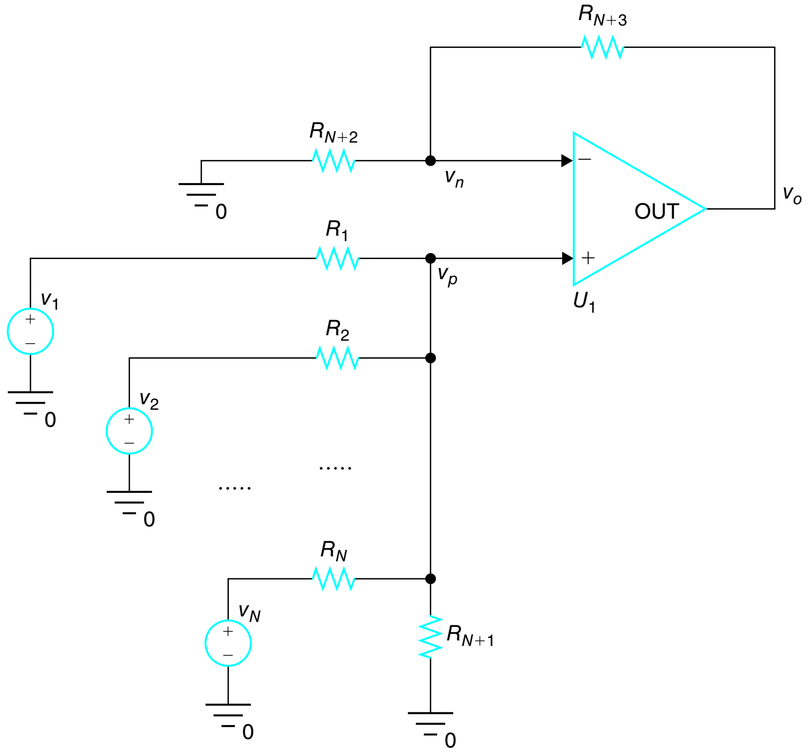
$$v_n = \frac{\dfrac{1}{R_{N+3}}}{\dfrac{1}{R_{N+2}} + \dfrac{1}{R_{N+3}}} v_o \tag{5.47}$$

continued

Example 5.8 continued

FIGURE 5.38

A summing amplifier with a noninverting configuration with n inputs.



Summing the currents leaving the positive input of the op amp, we have

$$\frac{v_p - v_1}{R_1} + \cdots + \frac{v_p - v_N}{R_N} + \frac{v_p - 0}{R_{N+1}} = 0$$

which can be rewritten as

$$\left(\frac{1}{R_1} + \cdots + \frac{1}{R_N} + \frac{1}{R_{N+1}}\right) v_p = \frac{v_1}{R_1} + \cdots + \frac{v_N}{R_N} \tag{5.48}$$

Substitution of Equation (5.47) into Equation (5.48) yields

$$\left(\frac{1}{R_1} + \cdots + \frac{1}{R_N} + \frac{1}{R_{N+1}}\right) \frac{1}{\dfrac{1}{R_{N+2}} + \dfrac{1}{R_{N+3}}} \frac{v_o}{R_{N+3}} = \frac{v_1}{R_1} + \cdots + \frac{v_N}{R_N}$$

Solving for v_o, we have

$$v_o = \frac{1}{\dfrac{1}{R_1} + \cdots + \dfrac{1}{R_N} + \dfrac{1}{R_{N+1}}} \left(1 + \frac{R_{N+3}}{R_{N+2}}\right) \left(\frac{v_1}{R_1} + \cdots + \frac{v_N}{R_N}\right) \tag{5.49}$$

Let $R = R_1 = R_2 = \cdots = R_N = R_{N+1}$. Then, Equation (5.49) becomes

$$v_o = \frac{R}{N+1} \left(\frac{R_{N+2} + R_{N+3}}{R_{N+2}}\right) \frac{1}{R} (v_1 + \cdots + v_N) \tag{5.50}$$

continued

Example 5.8 continued

Let $R_{N+2} = R$ and $R_{N+3} = NR$. Then, Equation (5.50) becomes

$$v_o = v_1 + \cdots + v_N \tag{5.51}$$

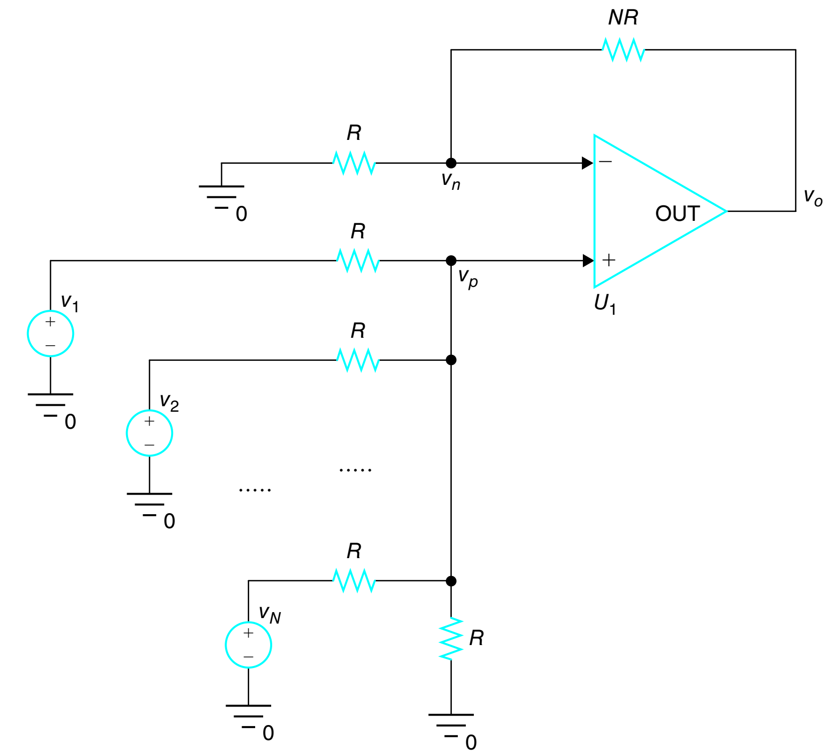
Let $R_{N+1} = R_{N+2} = R$, $R_1 = R/k_1$, $R_2 = R/k_2, \ldots, R_N = R/k_N$, $R_{N+3} = R(k_1 + k_2 + \cdots + k_N)$. Then, Equation (5.49) becomes

$$v_o = \frac{1}{\frac{k_1}{R} + \frac{k_2}{R} + \cdots + \frac{k_N}{R} + \frac{1}{R}}(1 + k_1 + k_2 + \cdots + k_N) \times \frac{1}{R}(k_1v_1 + k_2v_2 + \cdots + k_Nv_N) = k_1v_1 + k_2v_2 + \cdots + k_Nv_N \tag{5.52}$$

Figure 5.39 shows the circuit that produces the output $v_o = v_1 + \cdots + v_N$.

FIGURE 5.39

A summing amplifier with a noninverting configuration with n inputs and gain of 1.



5.3.3 ALTERNATIVE SUMMING AMPLIFIER (NONINVERTING CONFIGURATION)

Another summing amplifier with a noninverting configuration is shown in Figure 5.40.

Application of the voltage divider rule to $R_4 - R_3$ yields

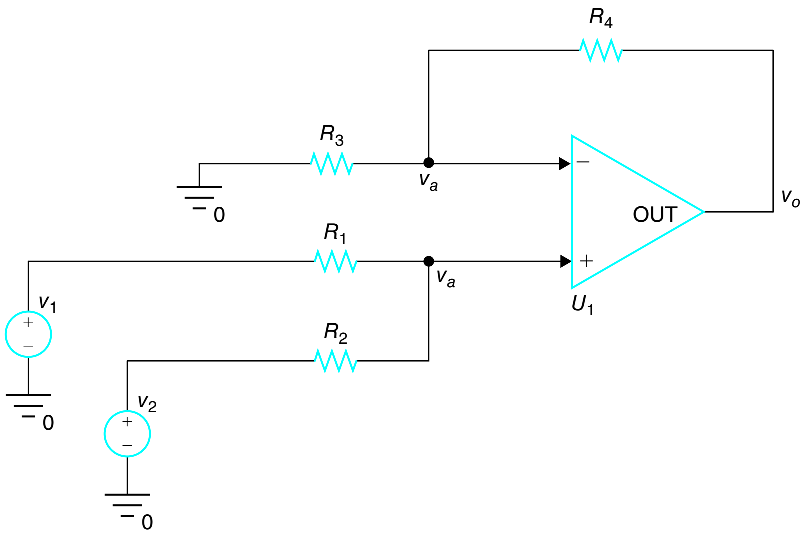
$$v_a = \frac{R_3}{R_3 + R_4} v_o \tag{5.53}$$

Summing the currents leaving the noninverting input of the op amp, we obtain

$$\frac{v_a - v_1}{R_1} + \frac{v_a - v_2}{R_2} = 0$$

FIGURE 5.40

A summing amplifier with a noninverting configuration.



which can be revised as

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_a=\frac{v_1}{R_1}+\frac{v_2}{R_2} \tag{5.54}$$

Substitution of Equation (5.53) into Equation (5.54) yields

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{R_3}{R_3+R_4}v_o=\frac{v_1}{R_1}+\frac{v_2}{R_2}$$

Solving for v_o, we obtain

$$v_o=\frac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}}\,\frac{R_3+R_4}{R_3}\left(\frac{v_1}{R_1}+\frac{v_2}{R_2}\right) \tag{5.55}$$

Let $R = R_1 = R_2$. Then, Equation (5.55) becomes

$$v_o=\frac{R}{2}\,\frac{R_3+R_4}{R_3}\,\frac{1}{R}(v_1+v_2) \tag{5.56}$$

Let $R_3 = R_4$. Then, we have

$$v_o=\frac{R}{2}\,\frac{2R_3}{R_3}\,\frac{1}{R}(v_1+v_2)=v_1+v_2 \tag{5.57}$$

If $R_1 = R/k_1, R_2 = R/k_2, R_3 = R, R_4 = (k_1 + k_2 - 1)R$, Equation (5.55) becomes

$$v_o=\frac{1}{\dfrac{k_1}{R}+\dfrac{k_2}{R}}\,\frac{R+(k_1+k_2-1)R}{R}\left(\frac{k_1v_1}{R}+\frac{k_2v_2}{R}\right)=k_1v_1+k_2v_2 \tag{5.58}$$

The constants k_1 and k_2 should be selected such that $k_1 + k_2 > 1$.

In general, to design an adder that adds N inputs, $v_1, v_2, v_3, \ldots, v_N$, that is,

$$v_o = v_1 + v_2 + v_3 + \cdots + v_N$$

choose $R = R_1 = R_2 = \cdots = R_N$ and choose $R_{N+2} = (N-1)R_{N+1}$, where R_{N+2} is the feedback resistor connecting the inverting input and output and R_{N+1} is the resistor connecting the inverting input and ground.

5.3.4 DIFFERENCE AMPLIFIER

Figure 5.41 shows a difference amplifier. For an ideal op amp model, the current flowing into the op amp from the noninverting input terminal is zero. Summing the currents leaving the noninverting input, we have

$$\frac{v_p - v_2}{R_3} + \frac{v_p - 0}{R_4} = 0$$

Solving this equation for v_p, we get

$$v_p = \frac{R_4}{R_3 + R_4} v_2$$

Alternatively, we can apply the voltage divider rule to find v_p.

Since $v_n = v_p$ (a virtual short), we have

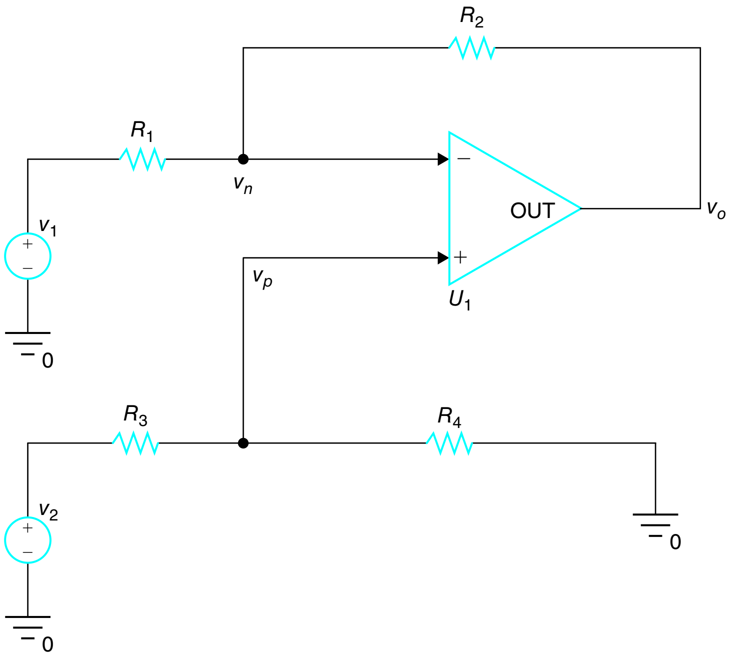
$$v_n = \frac{R_4}{R_3 + R_4} v_2 \tag{5.59}$$

Summing the currents leaving the inverting input, we obtain

$$\frac{v_n - v_1}{R_1} + \frac{v_n - v_o}{R_2} = 0$$

FIGURE 5.41

Difference amplifier.



Solving for v_o, we get

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_n - \frac{R_2}{R_1} v_1 \tag{5.60}$$

Substitution of Equation (5.59) into Equation (5.60) yields

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1 \tag{5.61}$$

If $R = R_1 = R_2 = R_3 = R_4$, the output voltage becomes

$$v_o = v_2 - v_1 \tag{5.62}$$

which is the difference between v_2 and v_1.

EXAMPLE 5.9

Design a circuit for converting a unipolar binary signal (1 is represented by a pulse with 5 V, and 0 is represented by a pulse with 0 V) to a polar binary signal (1 is represented by a pulse with 5 V, and 0 is represented by a pulse with −5V).

continued

Example 5.9 continued

Let v_2 be the unipolar signal and v_o be the polar signal. Then, subtracting 2.5 V from v_2 results in a signal with 2.5 V (for 1) or −2.5 V (for 0). We can obtain the polar signal by multiplying (v_2 − 2.5 V) by 2. Thus, we have

$$v_o = 2(v_2 - 2.5)$$

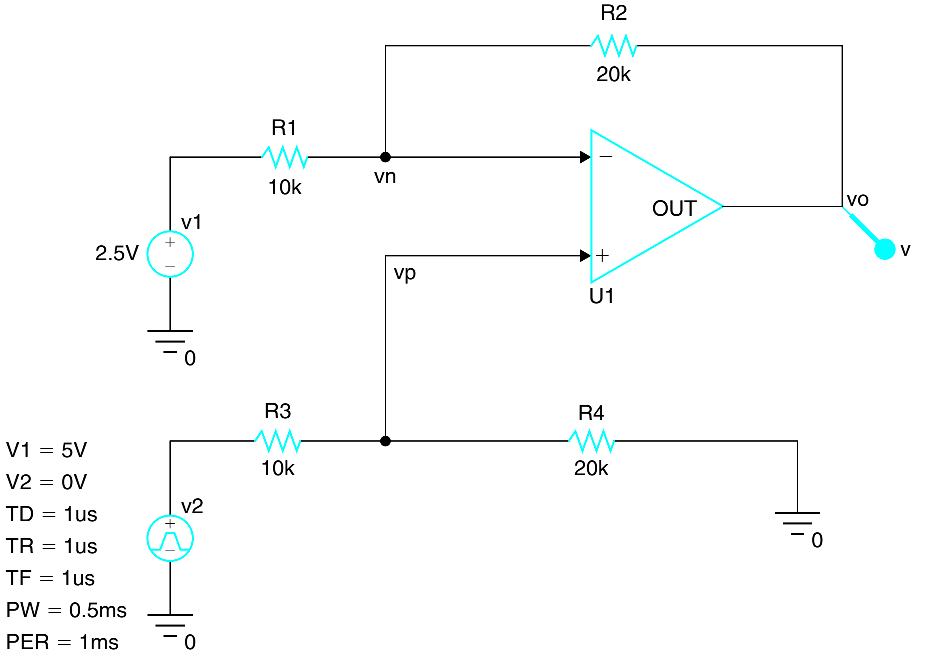
Choosing $R_3 = R_1$, $R_4 = R_2$, $R_2 = 2R_1$, and $v_1 = 2.5$ V in Equation (5.61) results in

$$\begin{aligned} v_o &= \left(\frac{R_1 + R_2}{R_1}\right)\frac{R_2}{R_1 + R_2}v_2 - \frac{R_2}{R_1}v_1 = \frac{R_2}{R_1}(v_2 - v_1) \\ &= \frac{2R_1}{R_1}(v_2 - v_1) = 2(v_2 - v_1) = 2(v_2 - 2.5) \end{aligned} \tag{5.63}$$

In conclusion, if we choose $R_3 = R_1$, $R_4 = R_2$, $R_2 = 2R_1$, $v_1 = 2.5$ V, we can convert a unipolar signal to a polar signal. Figure 5.42 shows the circuit with $R_1 = 10\ k\Omega$, and Figure 5.43 shows sample waveforms for v_2 and v_o. If the amplitude is other than 5 V, change v_1 to the amplitude of the input signal divided by 2.

FIGURE 5.42

Circuit for $v_o = 2(v_2 - v_1)$.



An op amp circuit can be used to generate a signal such as

$$v_o = 7v_1 - 2v_2$$

Signal v_o is the linear combination of the two input signals v_1 and v_2. We can use the superposition principle to show that the circuit shown in Figure 5.44 provides an output voltage given by $7v_1 - 2v_2$. We first deactivate voltage source v_2 by short-circuiting it. When v_2 is short-circuited, the voltage at node 1 is zero. Due to a virtual short, the voltage at the negative input of op amp 1 is zero. The voltage v_a at the output of op amp 1 is zero as well. The circuit reduces to a noninverting amplifier (Figure 5.13). The output due to v_1 is given by

$$v_{o_1} = \left(1 + \frac{R_4}{R_3}\right)v_1 = \left(1 + \frac{6\ k\Omega}{1\ k\Omega}\right)v_1 = 7v_1$$

continued

Example 5.9 continued

FIGURE 5.43

Sample waveforms.

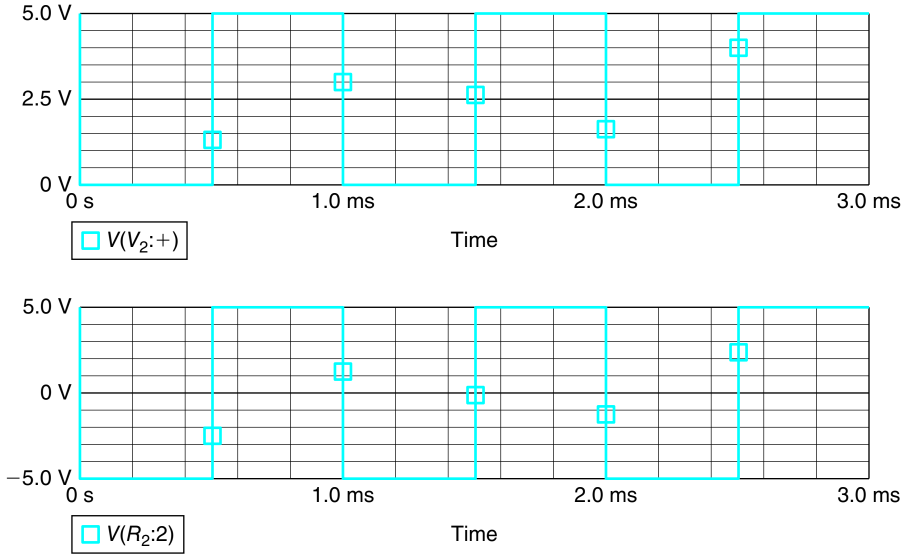
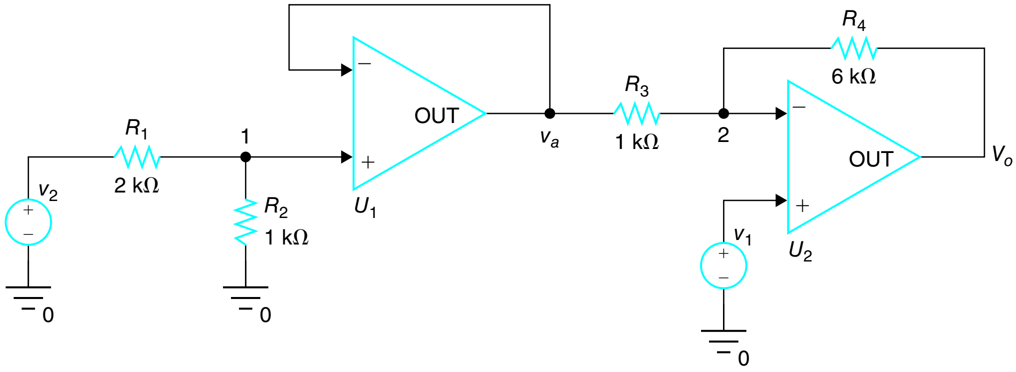


FIGURE 5.44

Circuit to implement $v_o = 7v_1 - 2v_2$.



Now, we deactivate voltage source v_1 by short-circuiting it. Op amp 2 becomes an inverting amplifier (Figure 5.11). The input to the inverting amplifier is provided by the output v_a of the voltage follower. Due to a virtual short, v_a is the voltage at node 1. From the voltage divider rule, we obtain

$$v_a = \frac{R_2}{R_1 + R_2} v_2 = \frac{1\ k\Omega}{1\ k\Omega + 2\ k\Omega} v_2 = \frac{1}{3} v_2$$

The output due to v_2 is given by

$$v_{o_2} = -\frac{R_4}{R_3} \times \frac{1}{3} v_2 = -\frac{6\ k\Omega}{1\ k\Omega} \times \frac{1}{3} v_2 = -2v_2$$

Adding v_{o_1} and v_{o_2}, we get

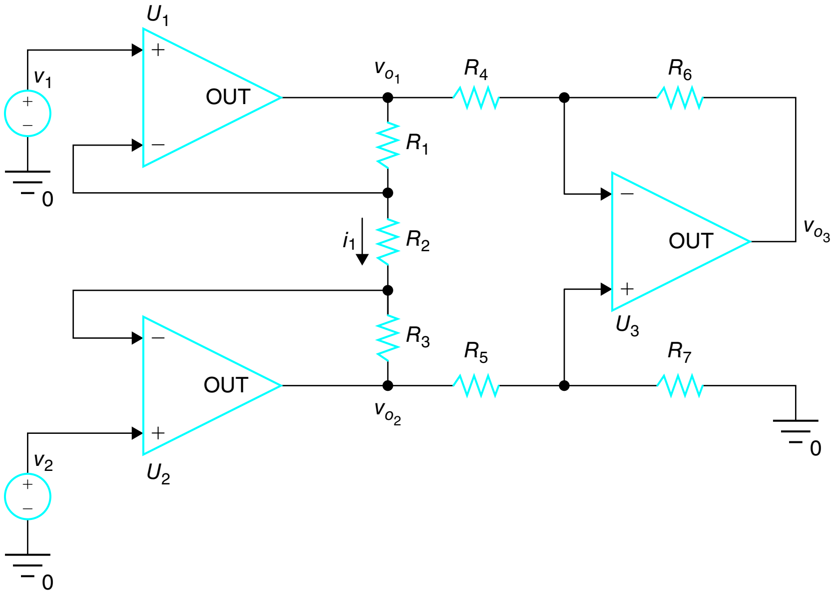
$$v_o = v_{o_1} + v_{o_2} = 7v_1 - 2v_2$$

5.4 Instrumentation Amplifier

Figure 5.45 shows an instrumentation amplifier.

FIGURE 5.45

Instrumentation amplifier.



The voltage at the inverting (−) input of the op amp U_1 is v_1 due to a virtual short. Likewise, the voltage at the inverting input of the op amp U_2 is v_2. Thus, the current through R_2 (↓) is given by

$$i_1 = \frac{v_1 - v_2}{R_2} \tag{5.64}$$

Since the current into the inverting (−) input terminal of the op amp U_1 and the current into the inverting input terminal of the op amp U_2 are both zero, the current through R_1 and the current through R_3 are also i_1. The voltage at the output of the op amp U_1 is given by

$$v_{o_1} = i_1 R_1 + v_1 = \frac{R_1}{R_2}(v_1 - v_2) + v_1 = \left(\frac{R_1 + R_2}{R_2}\right)v_1 - \frac{R_1}{R_2}v_2 \tag{5.65}$$

The voltage at the output of the op amp U_2 is given by

$$v_{o_2} = -i_1 R_3 + v_2 = -\frac{R_3}{R_2}(v_1 - v_2) + v_2 = -\frac{R_3}{R_2}v_1 + \left(\frac{R_3 + R_2}{R_2}\right)v_2 \tag{5.66}$$

If $R_3 = R_1$, Equation (5.66) becomes

$$v_{o_2} = -\frac{R_1}{R_2}v_1 + \left(\frac{R_1 + R_2}{R_2}\right)v_2 \tag{5.67}$$

The difference in voltage between the two outputs is given by

$$v_{o_2} - v_{o_1} = \frac{2R_1 + R_2}{R_2}(v_2 - v_1) = \left(1 + \frac{2R_1}{R_2}\right)(v_2 - v_1) \tag{5.68}$$

The voltage drop across $R_1 - R_2 - R_3$ is given by

$$v_{o_1} - v_{o_2} = (R_1 + R_2 + R_3)i_1$$

If $R_3 = R_1$, we have

$$v_{o_1} - v_{o_2} = (2R_1 + R_2)i_1 = \frac{2R_1 + R_2}{R_2}(v_1 - v_2) = \left(1 + \frac{2R_1}{R_2}\right)(v_1 - v_2) \tag{5.69}$$

Since the current into the positive terminal of the op amp U_3 is zero, the voltage at the positive terminal of the op amp U_3 is given by, based on the voltage divider rule,

$$v_{p_3} = \frac{R_7}{R_5 + R_7}v_{o_2} \tag{5.70}$$

Since the current into the negative terminal of the op amp U_3 is zero, the voltage at the negative terminal of the op amp U_3 is given by, based on the voltage divider rule,

$$v_{n_3} = v_{o_1} + \frac{R_4}{R_4 + R_6}(v_{o_3} - v_{o_1}) = \frac{R_6}{R_4 + R_6}v_{o_1} + \frac{R_4}{R_4 + R_6}v_{o_3} \tag{5.71}$$

Since $v_{p_3} = v_{n_3}$, from Equations (5.70) and (5.71), we have

$$\frac{R_7}{R_5 + R_7}v_{o_2} = \frac{R_6}{R_4 + R_6}v_{o_1} + \frac{R_4}{R_4 + R_6}v_{o_3}$$

If $R_4 = R_5$ and $R_6 = R_7$, the output of the op amp U_3 is given by

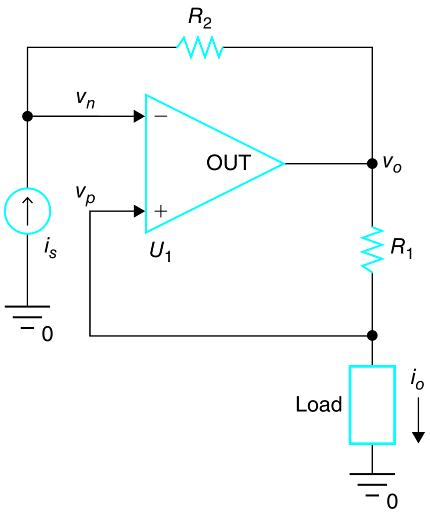
$$v_{o_3} = \frac{R_6}{R_4}(v_{o_2} - v_{o_1}) = \left(1 + \frac{2R_1}{R_2}\right)\frac{R_6}{R_4}(v_2 - v_1) \tag{5.72}$$

Thus, the output voltage of the op amp U_3 is proportional to the difference between v_2 and v_1. The instrumentation amplifier provides large input resistances for inputs v_1 and v_2. This results in no load down by finite input resistance like the difference circuit shown in Figure 5.41.

5.5 Current Amplifier

FIGURE 5.46

Current amplifier.



A current amplifier is shown in Figure 5.46.

The current through R_1, which is also the current through the load, is given by

$$i_o = \frac{v_o - v_p}{R_1} \tag{5.73}$$

The voltage at the output of the op amp is given by

$$v_o = v_n - R_2 i_s \tag{5.74}$$

Substitution of Equation (5.74) into Equation (5.73) yields

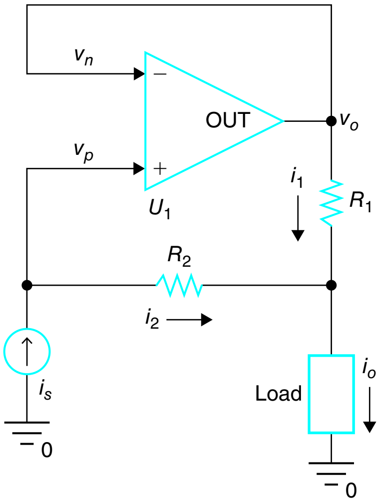
$$i_o = \frac{v_o - v_p}{R_1} = \frac{v_n - R_2 i_s - v_p}{R_1} \tag{5.75}$$

Since $v_n = v_p$ (i.e., a virtual short), Equation (5.75) reduces to

$$i_o = \frac{-R_2}{R_1}i_s \tag{5.76}$$

FIGURE 5.47

Another current amplifier.



The output current is the input current multiplied by $-R_2/R_1$, independent of the value of the load resistance. The current gain is R_2/R_1 with sign inversion. If $R_2 = R_1$, $i_o = -i_s$. The circuit is called a current reverser or a current mirror.

Figure 5.47 shows another current amplifier.

Since the current flowing into the op amp from the positive input terminal is zero, the current through R_2 is i_s; that is,

$$i_2 = i_s$$

Due to a virtual short, we have $v_n = v_p = v_o$. Thus, the voltage drop across R_1 and R_2 are the same; that is,

$$R_1 i_1 = R_2 i_2$$

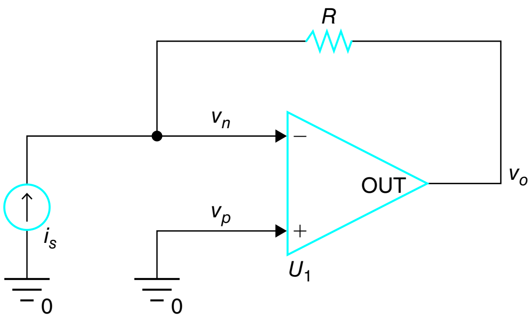
The current into the load is the sum of i_1 and i_2. Thus, we have

$$i_o = i_1 + i_2 = \frac{R_2}{R_1} i_2 + i_2 = \left(1 + \frac{R_2}{R_1}\right) i_2 = \left(1 + \frac{R_2}{R_1}\right) i_s \tag{5.77}$$

The output current is the input current multiplied by $(1 + R_2/R_1)$, independent of the value of the load resistance. The current gain is $(1 + R_2/R_1)$ without sign inversion. The current gain is always greater than or equal to 1.

FIGURE 5.48

Current-to-voltage converter.



5.5.1 CURRENT TO VOLTAGE CONVERTER (TRANSRESISTANCE AMPLIFIER)

A current-to-voltage converter is shown in Figure 5.48.

Since the current flowing into the op amp from the negative input terminal is zero, the current from the current source flows through R. Thus, we have

$$v_o = -R i_s \tag{5.78}$$

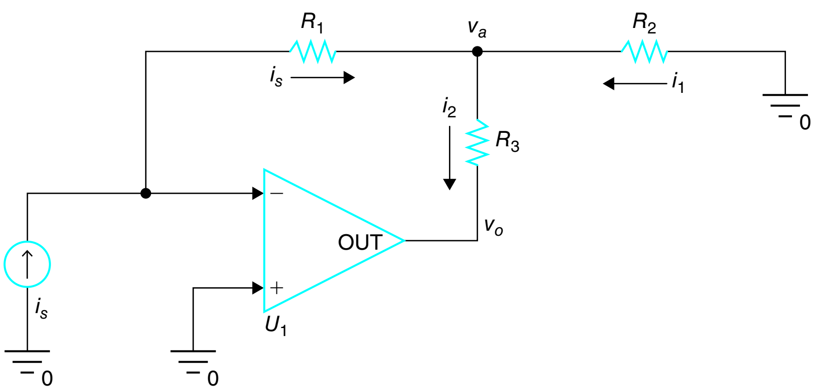
Another implementation of a current-to-voltage converter is shown in Figure 5.49.

Due to a virtual short, the voltage at the negative input terminal of the op amp is zero. Since the current flowing into the op amp from the negative input terminal is zero, the current from the current source flows through R_1. Thus, we have

$$v_a = -R_1 i_s$$

FIGURE 5.49

Another example of a current-to-voltage converter.



From Ohm's law, the current through R_2 is given by

$$i_1 = \frac{0 - v_a}{R_2} = \frac{R_1}{R_2} i_s$$

The current i_2 is the sum of i_s and i_1. Thus, we get

$$i_2 = i_s + i_1 = \left(1 + \frac{R_1}{R_2}\right) i_s$$

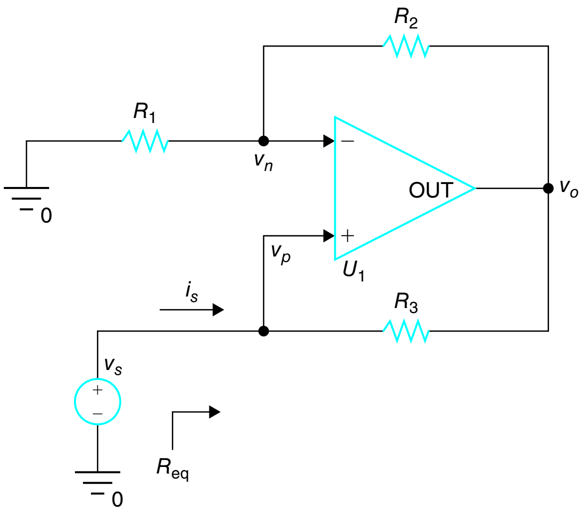
The output voltage of the op amp is given by

$$v_o = -i_2 R_3 + v_a = -R_3\left(1 + \frac{R_1}{R_2}\right) i_s - R_1 i_s = -R_1\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right) i_s \quad (5.79)$$

Compared to the circuit shown in Figure 5.48, assuming that $R = R_1$, the gain of the transresistance amplifier is increased by a factor of $\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right)$.

FIGURE 5.50

Negative resistance circuit.



5.5.2 NEGATIVE RESISTANCE CIRCUIT

A negative resistance circuit is shown in Figure 5.50.

Summing the currents leaving the negative input of the op amp, we obtain

$$\frac{v_n - 0}{R_1} + \frac{v_n - v_o}{R_2} = 0$$

Solving for v_o, we have

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_n$$

Due to a virtual short, the voltage at the inverting input and at the noninverting input is the same as the input voltage v_s; that is,

$$v_n = v_p = v_s$$

Thus, the output voltage is given by

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_s$$

Notice that this is the output voltage of the noninverting amplifier. The output voltage v_o is greater than the input voltage v_s by $(R_2/R_1)v_s$. The current flows through R_3 from right to left. Thus, the current flowing out of the positive terminal of the voltage source is negative. The current i_s is given by

$$i_s = \frac{v_s - v_o}{R_3} = \frac{v_s - \left(1 + \frac{R_2}{R_1}\right) v_s}{R_3} = \frac{v_s - v_s - \frac{R_2}{R_1} v_s}{R_3} = -\frac{R_2}{R_1 R_3} v_s$$

The equivalent resistance of the circuit from the voltage source v_s is given by

$$R_{\text{eq}} = \frac{v_s}{i_s} = -\frac{R_1 R_3}{R_2} \tag{5.80}$$

which proves that the circuit shown in Figure 5.50 provides negative resistance.

5.5.3 VOLTAGE-TO-CURRENT CONVERTER (TRANSCONDUCTANCE AMPLIFIER)

The circuit shown in Figure 5.51 is called a Howland current pump circuit.

Due to a virtual short, we have

$$v_L = v_p = v_n$$

Summing the currents leaving node 1, we have

$$\frac{v_n}{R_1} + \frac{v_n - v_o}{R_2} = 0,$$

which can be rewritten as

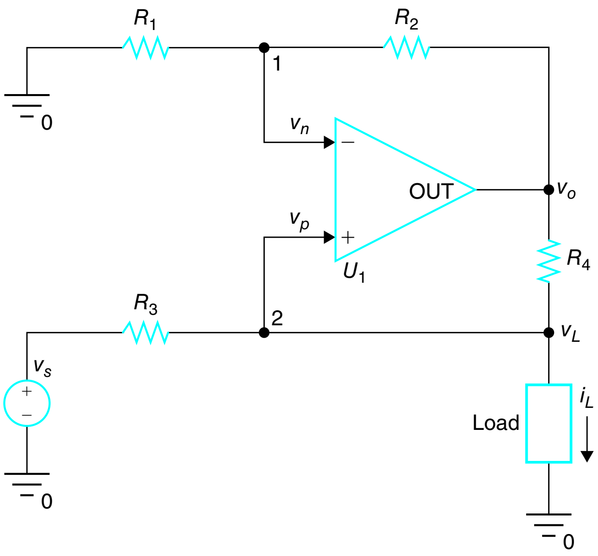
$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_n = \frac{v_o}{R_2}$$

Solving for v_n, we obtain

$$v_n = v_p = v_L = \frac{\dfrac{1}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} v_o = \frac{R_1}{R_1 + R_2} v_o \tag{5.81}$$

FIGURE 5.51

Howland current pump circuit.



Solving Equation (5.81) for v_o, we obtain

$$v_o = \frac{R_1 + R_2}{R_1} v_n = \left(1 + \frac{R_2}{R_1}\right) v_n \tag{5.82}$$

The current i_L through the load is given by

$$\begin{aligned}
i_L &= \frac{v_s - v_L}{R_3} + \frac{v_o - v_L}{R_4} = \frac{v_s - v_L}{R_3} + \frac{\dfrac{R_1 + R_2}{R_1} v_L - v_L}{R_4} = \frac{v_s}{R_3} - \left(\frac{1}{R_3} - \frac{R_2}{R_1 R_4}\right) v_L \\
&= \frac{v_s}{R_3} - \frac{R_1 R_4 - R_2 R_3}{R_1 R_3 R_4} v_L = \frac{v_s}{R_3} - \frac{1}{\dfrac{R_1 R_3 R_4}{R_1 R_4 - R_2 R_3}} v_L \\
&= \frac{v_s}{R_3} - \frac{1}{\dfrac{R_4}{\dfrac{R_1 R_4}{R_1 R_3} - \dfrac{R_2 R_3}{R_1 R_3}}} v_L = \frac{v_s}{R_3} - \frac{\dfrac{v_L}{R_4}}{\dfrac{R_4}{R_3} - \dfrac{R_2}{R_1}}
\end{aligned} \tag{5.83}$$

Let

$$R_a = \frac{R_4}{\dfrac{R_4}{R_3} - \dfrac{R_2}{R_1}} \quad (5.84)$$

Then, Equation (5.83) becomes

$$i_L = \frac{v_s}{R_3} - \frac{v_L}{R_a} \quad (5.85)$$

If

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} \quad (5.86)$$

the denominator of R_a becomes zero, and R_a is infinity ($R_a = \infty$). Thus, the current through the load is given by

$$i_L = \frac{v_s}{R_3} \quad (5.87)$$

The current through the load does not depend on the voltage across the load. The current depends only on the input voltage v_s. For a discussion of an improved Howland current pump circuit, refer to Sergio Franco.[1] One application of constant current source is in the light-emitting diode (LED) driver design. The current through LED is related to voltage across it exponentially. Due to variability of the current-voltage characteristics of LEDs, for the given constant voltage, the current can vary significantly from LED to LED. If a constant voltage driver is used, there are LEDs that will violate the absolute maximum current rating and compromise reliability. If a constant current driver is used, the current through LED will be constant, regardless of the current-voltage characteristics of LED.

5.6 Analysis of Inverting Configuration

Figure 5.4, earlier in this chapter, showed the inverting configuration of an op amp. When the op amp is replaced by the model shown in Figure 5.3, we obtain the circuit shown in Figure 5.5, which is repeated here in Figure 5.52.

The circuit shown in Figure 5.52 is analyzed in Section 5.2, earlier in this chapter, assuming that $R_i = \infty$ and $R_o = 0$. In this section, we analyze the circuit shown in Figure 5.52, including R_i and R_o.

Summing the currents leaving node 2, we obtain

$$\frac{v_o - v_n}{R_2} + \frac{v_o - A(v_p - v_n)}{R_o} = 0 \quad (5.88)$$

Since $v_p = 0$, Equation (5.88) becomes

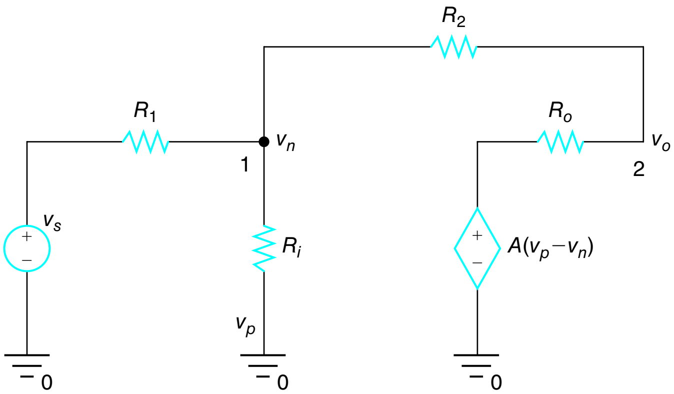
$$\frac{v_o - v_n}{R_2} + \frac{v_o + Av_n}{R_o} = 0 \quad (5.89)$$

Solving Equation (5.89) for v_n, we obtain

$$v_n = \frac{R_o + R_2}{R_o - R_2 A} v_o \quad (5.90)$$

FIGURE 5.52

A model for an inverting configuration.



[1] Sergio Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, McGraw-Hill, New York, 1988.

Solving Equation (5.90) for v_o, we obtain

$$v_o = \frac{R_o - R_2A}{R_o + R_2}v_n \tag{5.91}$$

Summing the currents leaving node 1, we obtain

$$\frac{v_n - v_s}{R_1} + \frac{v_n}{R_i} + \frac{v_n - v_o}{R_2} = 0 \tag{5.92}$$

which can be revised as

$$\left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right)v_n - \frac{v_o}{R_2} = \frac{v_s}{R_1} \tag{5.93}$$

Substitution of Equation (5.91) into Equation (5.93) yields

$$\left(\frac{R_2R_i + R_1R_2 + R_1R_i}{R_1R_2R_i} - \frac{1}{R_2}\frac{R_o - R_2A}{R_o + R_2}\right)v_n = \frac{v_s}{R_1} \tag{5.94}$$

Solving Equation (5.94) for v_n, we obtain

$$\begin{aligned} v_n &= \frac{\dfrac{v_s}{R_1}}{\dfrac{R_2R_i + R_1R_2 + R_1R_i}{R_1R_2R_i} - \dfrac{1}{R_2}\dfrac{R_o - R_2A}{R_o + R_2}} \\ &= \frac{R_2R_i(R_o + R_2)v_s}{(R_2R_i + R_1R_2 + R_1R_i)(R_o + R_2) - R_1R_i(R_o - R_2A)} \\ &= \frac{R_i(R_o + R_2)v_s}{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i} \end{aligned} \tag{5.95}$$

Substitution of Equation (5.95) into Equation (5.91) yields

$$v_o = \frac{-R_i(-R_o + R_2A)v_s}{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i} \tag{5.96}$$

Dividing every term in Equations (5.95) and (5.96) by R_i, we obtain

$$v_n = \frac{(R_o + R_2)v_s}{R_o + \dfrac{R_oR_1}{R_i} + R_2 + \dfrac{R_1R_2}{R_i} + R_1 + AR_1} \tag{5.97}$$

$$v_o = \frac{-(-R_o + R_2A)v_s}{R_o + \dfrac{R_oR_1}{R_i} + R_2 + \dfrac{R_1R_2}{R_i} + R_1 + AR_1} \tag{5.98}$$

If $R_o = 0$ and $R_i = \infty$, Equations (5.97) and (5.98) become, respectively,

$$v_n = \frac{R_2v_s}{R_1 + R_2 + AR_1} = \frac{\dfrac{R_2}{A}v_s}{\dfrac{R_1 + R_2}{A} + R_1} \tag{5.99}$$

$$v_o = \frac{-R_2 A v_s}{R_1 + R_2 + AR_1} = \frac{-R_2 v_s}{\dfrac{R_1 + R_2}{A} + R_1} \tag{5.100}$$

These equations are identical to Equations (5.6) and (5.5), respectively. Since A $\gg$ $(R_1 + R_2)$, Equation (5.99) reduces to

$$v_n \cong \frac{R_2}{R_1 A} v_s \approx 0 \tag{5.101}$$

Equation (5.101) is identical to Equation (5.8). Since A $\gg (R_1 + R_2)$, Equation (5.100) reduces to

$$v_o \cong -\frac{R_2}{R_1} v_s \tag{5.102}$$

Equation (5.102) is identical to Equation (5.7).

Alternatively, if every term in Equations (5.95) and (5.96) is divided by $R_i A$, we obtain

$$v_n = \frac{\dfrac{R_o + R_2}{A} v_s}{\dfrac{R_o}{A} + \dfrac{R_o R_1}{R_i A} + \dfrac{R_2}{A} + \dfrac{R_1 R_2}{R_i A} + \dfrac{R_1}{A} + R_1} \tag{5.103}$$

$$v_o = \frac{-\left(-\dfrac{R_o}{A} + R_2\right) v_s}{\dfrac{R_o}{A} + \dfrac{R_o R_1}{R_i A} + \dfrac{R_2}{A} + \dfrac{R_1 R_2}{R_i A} + \dfrac{R_1}{A} + R_1} \tag{5.104}$$

In typical op amps, the input resistance is large ($R_i \gg 1, R_i \gg R_1, R_i \gg R_2$), the output resistance is small ($R_o \ll R_i$), and the unloaded voltage gain is large ($A \gg 1$, $A \gg R_1$, $A \gg R_2$). If we ignore the terms that are divided by A or $R_i A$, Equations (5.103) and (5.104) become, respectively,

$$v_n \cong \frac{R_2}{R_1 A} v_s \approx 0 \tag{5.105}$$

$$v_o = -\frac{R_2}{R_1} v_s \tag{5.106}$$

The current flowing into the op amp from the negative input terminal (node 1) is obtained by dividing v_n by R_i:

$$i_n = \frac{v_n}{R_i} = \frac{(R_o + R_2) v_s}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + A R_1 R_i} \tag{5.107}$$

If we divide each term of Equation (5.107) by AR_i, we have

$$i_n = \frac{v_n}{R_i} = \frac{\dfrac{R_o + R_2}{AR_i} v_s}{\dfrac{R_o R_i}{AR_i} + \dfrac{R_o R_1}{AR_i} + \dfrac{R_i R_2}{AR_i} + \dfrac{R_1 R_2}{AR_i} + \dfrac{R_1 R_i}{AR_i} + R_1} = \frac{\dfrac{R_o + R_2}{AR_i} v_s}{\dfrac{R_o}{A} + \dfrac{R_o R_1}{AR_i} + \dfrac{R_2}{A} + \dfrac{R_1 R_2}{AR_i} + \dfrac{R_1}{A} + R_1} \quad (5.108)$$

From Equation (5.108), we get

$$i_n = \frac{v_n}{R_i} \cong \frac{(R_o + R_2)v_s}{R_1 A R_i} \cong \frac{R_2 v_s}{R_1 A R_i} \approx 0 \quad (5.109)$$

5.6.1 INPUT RESISTANCE

The current through R_1, from left to right, is given by

$$i_1 = \frac{v_s - v_n}{R_1} = \frac{v_s - \dfrac{R_i(R_o + R_2)v_s}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i}}{R_1}$$

$$= \frac{\dfrac{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i - R_i R_o - R_i R_2}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i} v_s}{R_1}$$

$$= \frac{\dfrac{R_o R_1 + R_1 R_2 + R_1 R_i + AR_1 R_i}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i} v_s}{R_1}$$

$$= \frac{R_o + R_2 + R_i + AR_i}{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i} v_s$$

The input resistance R_{in} is given by v_s/i_1:

$$R_{in} = \frac{v_s}{i_1} = \frac{R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + AR_1 R_i}{R_o + R_2 + R_i + AR_i} \quad (5.110)$$

Dividing every term by $R_i A$, we get

$$R_{in} = \frac{v_S}{i_1} = \frac{\dfrac{R_o}{A} + \dfrac{R_o R_1}{AR_i} + \dfrac{R_2}{A} + \dfrac{R_1 R_2}{AR_i} + \dfrac{R_1}{A} + R_1}{\dfrac{R_o}{AR_i} + \dfrac{R_2}{AR_i} + \dfrac{1}{A} + 1} \approx R_1 \quad (5.111)$$

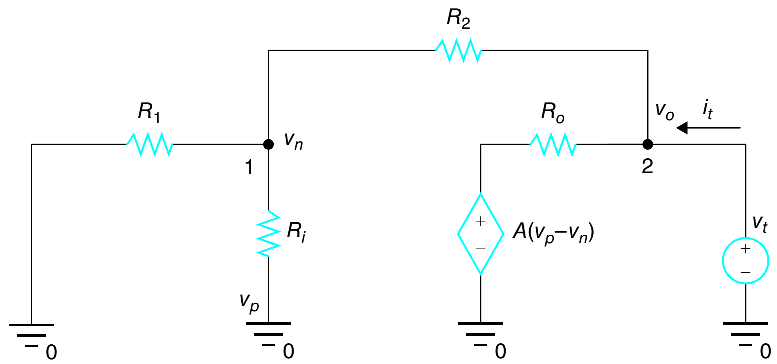
The input resistance of the inverting configuration is R_1. The noninverting configuration has a higher value of input resistance.

5.6.2 OUTPUT RESISTANCE

Deactivate v_s (i.e., short-circuit it) and apply test voltage v_t to the output of the op amp. The circuit is shown in Figure 5.53.

FIGURE 5.53

Circuit for finding the output resistance.



From the voltage divider rule, we obtain

$$v_n = \frac{\dfrac{R_1R_i}{R_1 + R_i}}{R_2 + \dfrac{R_1R_i}{R_1 + R_i}} v_t = \frac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i} v_t \tag{5.112}$$

The current flowing out of the test voltage source is given by

$$i_t = \frac{v_t - v_n}{R_2} + \frac{v_t - A(0 - v_n)}{R_o} = \frac{v_t - \dfrac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i} v_t}{R_2} + \frac{v_t + A\dfrac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i} v_t}{R_o} \tag{5.113}$$

The output resistance is the ratio of v_t to i_t:

$$R_{out} = \frac{v_t}{i_t} = \frac{1}{\dfrac{1 - \dfrac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i}}{R_2} + \dfrac{1 + A\dfrac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i}}{R_o}} \tag{5.114}$$

Equation (5.114) can be simplified to

$$R_{out} = \frac{R_o(R_1R_2 + R_2R_i + R_1R_i)}{R_oR_1 + R_oR_i + AR_1R_i + R_1R_2 + R_2R_i + R_1R_i} \tag{5.115}$$

If we keep only the AR_1R_i term from the denominator and ignore the R_1R_2 term from the numerator of Equation (5.115), we obtain

$$R_{out} \cong \frac{R_o(R_2 + R_1)}{AR_1} \approx 0 \tag{5.116}$$

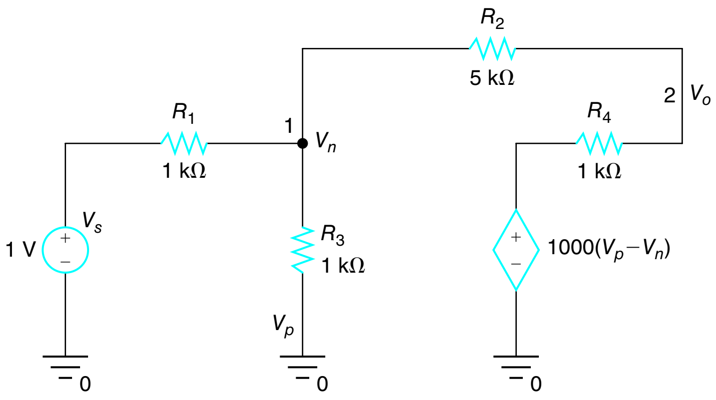
The output resistance is close to zero for the inverting configuration.

EXAMPLE 5.10

Find V_o in the circuit shown in Figure 5.54.

FIGURE 5.54

Circuit for EXAMPLE 5.10.



Summing the currents leaving node 1, we obtain

$$\frac{V_n - 1}{1000} + \frac{V_n}{1000} + \frac{V_n - V_o}{5000} = 0$$

Multiplication by 5000 yields

$$5V_n - 5 + 5V_n + V_n - V_o = 0$$

which can be simplified to

$$11V_n - V_o = 5 \tag{5.117}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_o - V_n}{5000} + \frac{V_o - 1000(0 - V_n)}{1000} = 0$$

Multiplication by 5000 yields

$$V_o - V_n + 5V_o + 5000V_n = 0$$

which can be simplified to

$$4999V_n + 6V_o = 0 \tag{5.118}$$

Multiplication of Equation (5.117) by 6 yields

$$66V_n - 6V_o = 30 \tag{5.119}$$

Adding Equations (5.118) and (5.119), we get

$$5065V_n = 30$$

continued

Example 5.10 continued

Thus, we have

$$V_n = 30/5065 = 0.005923 \text{ V}$$

From Equation (5.117), we obtain

$$V_o = 11V_n - 5 = 330/5065 - 5 = -4.934847 \text{ V}$$

For an ideal op amp, $V_o = -5$ V. Since the circuit shown in Figure 5.54 is not ideal (R_i $(= R_3)$ is small, A is small, and R_o $(= R_4)$ is large), the output voltage is not -5 V.

MATLAB

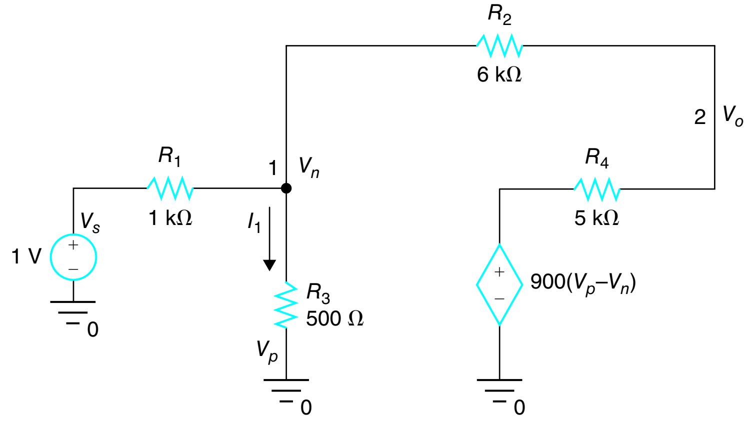
```
% EXAMPLE 5.10
clear all;
R1=1000;R2=5000;R3=1000;R4=1000;Vs=1;A=1000;
syms Vn Vo
[Vn,Vo]=solve((Vn-Vs)/R1+Vn/R3+(Vn-Vo)/R2,...
(Vo-Vn)/R2+(Vo-A*(0-Vn))/R4,Vn,Vo);
Vn=vpa(Vn,11)
Vo=vpa(Vo,11)

Answers:
Vn =
0.0059230009872
Vo =
-4.9348469891
```

Exercise 5.8

Find V_n and I_1 in the circuit shown in Figure 5.55.

FIGURE 5.55

Circuit for EXERCISE 5.8.



Answer:

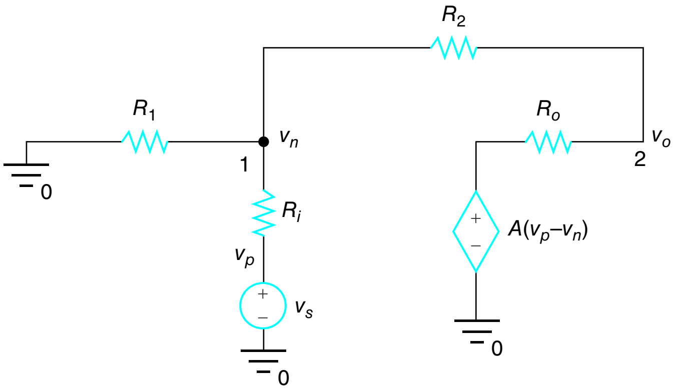
$V_n = 11.7773$ mV, $I_1 = 23.5546\ \mu$A.

5.7 Analysis of Noninverting Configuration

Figure 5.7, earlier in this chapter, showed a noninverting configuration of an op amp. When the op amp is replaced by the model shown in Figure 5.3, we obtain the circuit shown in Figure 5.8, which is repeated in Figure 5.56.

FIGURE 5.56

Model of a noninverting configuration.



Summing the currents leaving node 2, we obtain

$$\frac{v_o - v_n}{R_2} + \frac{v_o - A(v_p - v_n)}{R_o} = 0 \tag{5.120}$$

Since $v_p = v_s$, Equation (5.120) becomes

$$\frac{v_o - v_n}{R_2} + \frac{v_o - A(v_s - v_n)}{R_o} = 0 \tag{5.121}$$

Summing the currents leaving node 1, we obtain

$$\frac{v_n}{R_1} + \frac{v_n - v_s}{R_i} + \frac{v_n - v_o}{R_2} = 0 \tag{5.122}$$

which can be revised as

$$\left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right)v_n = \frac{v_s}{R_i} + \frac{v_o}{R_2} \tag{5.123}$$

Solving Equations (5.121) and (5.123), we have

$$v_n = \frac{R_1(R_o + R_2 + AR_i)v_s}{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i} \tag{5.124}$$

$$v_o = \frac{(R_oR_1 + R_1R_iA + R_2R_iA)v_s}{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i} \tag{5.125}$$

If each term of Equations (5.124) and (5.125) is divided by R_i, we get

$$v_n = \frac{R_1\left(\dfrac{R_o}{R_i} + \dfrac{R_2}{R_i} + A\right)v_s}{R_o + \dfrac{R_oR_1}{R_i} + R_2 + \dfrac{R_1R_2}{R_i} + R_1 + AR_1} \tag{5.126}$$

$$v_o = \frac{\left(\dfrac{R_oR_1}{R_i} + R_1A + R_2A\right)v_s}{R_o + \dfrac{R_oR_1}{R_i} + R_2 + \dfrac{R_1R_2}{R_i} + R_1 + AR_1} \tag{5.127}$$

If $R_o = 0$ and $R_i = \infty$, Equations (5.126) and (5.127) reduce, respectively, to

$$v_n = \frac{AR_1v_s}{R_1 + R_2 + AR_1} = \frac{R_1v_s}{\dfrac{R_1 + R_2}{A} + R_1} \tag{5.128}$$

$$v_o = \frac{A(R_1 + R_2)v_s}{R_1 + R_2 + AR_1} = \frac{(R_1 + R_2)v_s}{\dfrac{R_1 + R_2}{A} + R_1} \tag{5.129}$$

These equations are identical to Equations (5.16) and (5.15), respectively. Since $A \gg (R_1 + R_2)$, Equation (5.129) reduces to

$$v_o \cong \frac{R_1 + R_2}{R_1} v_s = \left(1 + \frac{R_2}{R_1}\right) v_s \tag{5.130}$$

which is identical to Equation (5.17). Since $A \gg (R_1 + R_2)$, Equation (5.128) reduces to

$$v_n = \frac{R_1}{\dfrac{R_1 + R_2}{A} + R_1} v_s \cong v_s \tag{5.131}$$

which is identical to Equation (5.18).

Alternatively, if every term in Equations (5.124) and (5.125) is divided by $R_i A$, we obtain

$$v_n = \frac{R_1\left(\dfrac{R_o + R_2}{R_i A} + 1\right) v_s}{\dfrac{R_o}{A} + \dfrac{R_o R_1}{R_i A} + \dfrac{R_2}{A} + \dfrac{R_1 R_2}{R_i A} + \dfrac{R_1}{A} + R_1} \tag{5.132}$$

$$v_o = \frac{\left(\dfrac{R_o R_1}{R_i A} + R_1 + R_2\right) v_s}{\dfrac{R_o}{A} + \dfrac{R_o R_1}{R_i A} + \dfrac{R_2}{A} + \dfrac{R_1 R_2}{R_i A} + \dfrac{R_1}{A} + R_1} \tag{5.133}$$

The current flowing into the op amp from the negative input terminal (node 1) is obtained by dividing $v_n - v_s$ by R_i:

$$i_n = \frac{v_n - v_s}{R_i} = \frac{(R_o R_1 + R_1 R_2 + A R_i R_1) v_s}{R_i (R_o R_i + R_o R_1 + R_i R_2 + R_1 R_2 + R_1 R_i + A R_1 R_i)} - \frac{v_s}{R_i} \tag{5.134}$$

If we divide each term by AR_i, we have

$$i_n = \frac{v_n - v_s}{R_i} = \frac{\left(\dfrac{R_o R_1}{A R_i} + \dfrac{R_1 R_2}{A R_i} + R_1\right) v_s}{R_i\left(\dfrac{R_o}{A} + \dfrac{R_o R_1}{A R_i} + \dfrac{R_2}{A} + \dfrac{R_1 R_2}{A R_i} + \dfrac{R_1}{A} + R_1\right)} - \frac{v_s}{R_i} \tag{5.135}$$

In typical op amps, the input resistance is large ($R_i \gg 1$), the output resistance is small ($R_o \ll R_i$), and the unloaded voltage gain is large ($A \gg 1, A \gg R_1, A \gg R_2$). If we ignore the terms that are divided by A or $R_i A$, Equations (5.132), (5.133), and (5.135) become, respectively,

$$v_n = v_s \tag{5.136}$$

$$v_o = \frac{R_1 + R_2}{R_1} v_s = \left(1 + \frac{R_2}{R_1}\right) v_s \tag{5.137}$$

$$i_n = \frac{v_s}{R_i} - \frac{v_s}{R_i} = 0 \tag{5.138}$$

5.7.1 INPUT RESISTANCE

The current through R_i, from bottom to top, is given by

$$i_i = \frac{v_s - v_n}{R_i} = \frac{v_s - \dfrac{R_1R_o + R_1R_2 + R_1AR_i}{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i}v_s}{R_i}$$

$$= \frac{\dfrac{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i - R_1R_o - R_1R_2 - R_1AR_i}{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i}}{R_i}v_s$$

$$= \frac{R_o + R_2 + R_1}{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i}v_s$$

The input resistance R_{in} is given by v_s/i_i:

$$R_{in} = \frac{v_s}{i_i} = \frac{R_oR_i + R_oR_1 + R_iR_2 + R_1R_2 + R_1R_i + AR_1R_i}{R_o + R_2 + R_1} \tag{5.139}$$

Approximate input resistance is given by

$$R_{in} \simeq \frac{AR_iR_1}{R_2 + R_1} \tag{5.140}$$

Due to the multiplication of A and R_i in the numerator, the input resistance is large compared to R_1 for the inverting configuration.

5.7.2 OUTPUT RESISTANCE

Deactivate v_s (i.e., short-circuit it) and apply test voltage v_t to the output of the op amp. The circuit is shown in Figure 5.57.

From the voltage divider rule, we can find voltage v_n at node 1:

$$v_n = \frac{\dfrac{R_1R_i}{R_1 + R_i}}{R_2 + \dfrac{R_1R_i}{R_1 + R_i}}v_t = \frac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i}v_t$$

The current flowing out of the test voltage source is given by

$$i_t = \frac{v_t - v_n}{R_2} + \frac{v_t - A(0 - v_n)}{R_o} = \frac{v_t - \dfrac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i}v_t}{R_2}$$

$$+ \frac{v_t + A\dfrac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i}v_t}{R_o}$$

The output resistance is the ratio of v_t to i_t:

$$R_{out} = \frac{v_t}{i_t} = \frac{1}{\dfrac{1 - \dfrac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i}}{R_2} + \dfrac{1 + A\dfrac{R_1R_i}{R_1R_2 + R_2R_i + R_1R_i}}{R_o}}$$

which can be simplified to

$$R_{out} = \frac{R_o(R_1R_2 + R_2R_i + R_1R_i)}{R_oR_1 + R_oR_i + AR_1R_i + R_1R_2 + R_2R_i + R_1R_i} \tag{5.141}$$

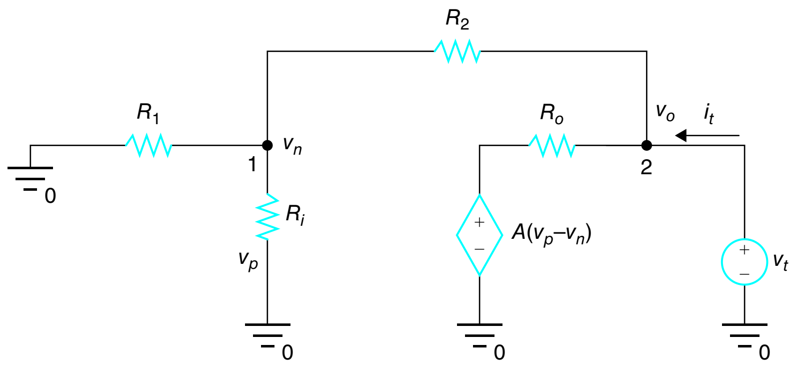
If the largest terms are kept, Equation (5.141) becomes

$$R_{out} \cong \frac{R_o(R_2 + R_1)R_i}{AR_1R_i} = \frac{R_o(R_2 + R_1)}{AR_1} \approx 0 \tag{5.142}$$

The output resistance is close to zero.

FIGURE 5.57

A circuit with test voltage.

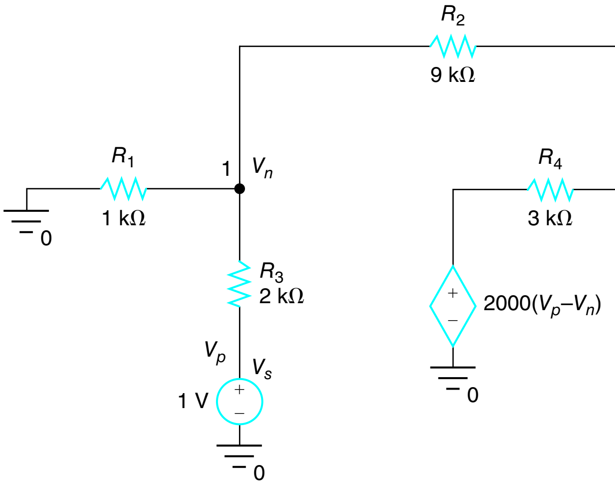


EXAMPLE 5.11

Find V_o in the circuit shown in Figure 5.58.

FIGURE 5.58

Circuit for EXAMPLE 5.11.



Summing the currents leaving node 1, we obtain

$$\frac{V_n - 1}{2000} + \frac{V_n}{1000} + \frac{V_n - V_o}{9000} = 0$$

Multiplication by 18,000 yields

$$9V_n - 9 + 18V_n + 2V_n - 2V_o = 0$$

which can be simplified to

$$29V_n - 2V_o = 9 \tag{5.143}$$

Summing the currents leaving node 2, we obtain

$$\frac{V_o - V_n}{9000} + \frac{V_o - 2000(1 - V_n)}{3000} = 0$$

Multiplication by 9000 yields

$$V_o - V_n + 3V_o - 6000 + 6000V_n = 0$$

continued

Example 5.11 continued

which can be simplified to

$$5999V_n + 4V_o = 6000 \quad (5.144)$$

Multiplying Equation (5.143) by 2, we obtain

$$58V_n - 4V_o = 18$$

Adding this equation to Equation (5.144), we obtain

$$6057V_n = 6018$$

Thus,

$$V_n = \frac{6018}{6057} = 0.99356117 \text{ V}$$

From Equation (5.143), we get

$$V_o = \frac{29}{2}V_n - \frac{9}{2} = 9.90664 \text{ V}$$

Alternatively, application of Cramer's rule to Equations (5.143) and (5.144) yields

$$V_o = \frac{\begin{vmatrix} 29 & 9 \\ 5999 & 6000 \end{vmatrix}}{\begin{vmatrix} 29 & -2 \\ 5999 & 4 \end{vmatrix}} = 9.90664 \text{ V}$$

For an ideal op amp, $V_o = 10$ V. Since the circuit shown in Figure 5.58 is not ideal (R_i ($= R_3$) is small, A is small, and R_o ($= R_4$) is large), the output voltage is not 10 V.

MATLAB

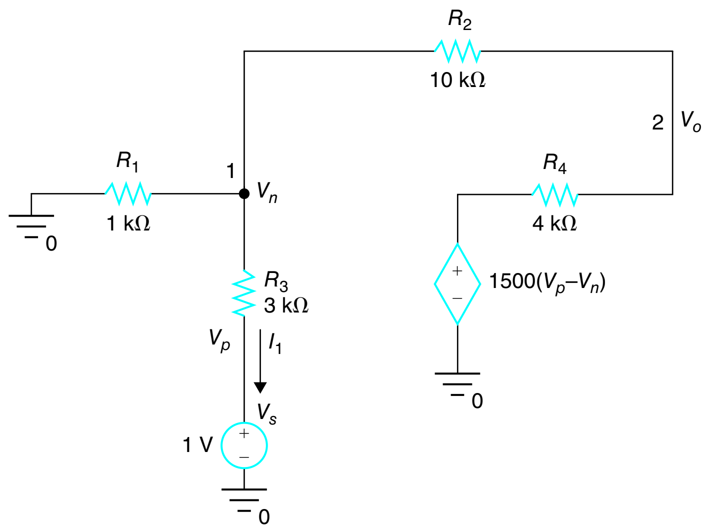
```
% EXAMPLE 5.11
clear all;
R1=1000;R2=9000;R3=2000;R4=3000;Vs=1;A=2000;
Vp=Vs;
syms Vn Vo
[Vn,Vo]=solve(Vn/R1+(Vn-Vs)/R3+(Vn-Vo)/R2,...
(Vo-Vn)/R2+(Vo-A*(Vp-Vn))/R4,Vn,Vo);
Vn=vpa(Vn,11)
Vo=vpa(Vo,11)

Answers:
Vn =
0.9935611689
Vo =
9.906636949
```

Exercise 5.9

Find V_n and I_1 in the circuit shown in Figure 5.59.

FIGURE 5.59

Circuit for EXERCISE 5.9.



Answer:
$V_n = 0.99013$ V, $I_1 = -3.2902\ \mu$A.

5.8 PSpice and Simulink

The PSpice schematic for the inverting configuration of μA741 op amp is shown in Figure 5.60. Figure 5.61 shows the simulation settings that enable voltage gain, input resistance, and output resistance, as well as the results of the simulation.

FIGURE 5.60

The PSpice schematic for the μA741 op amp in an inverting configuration.

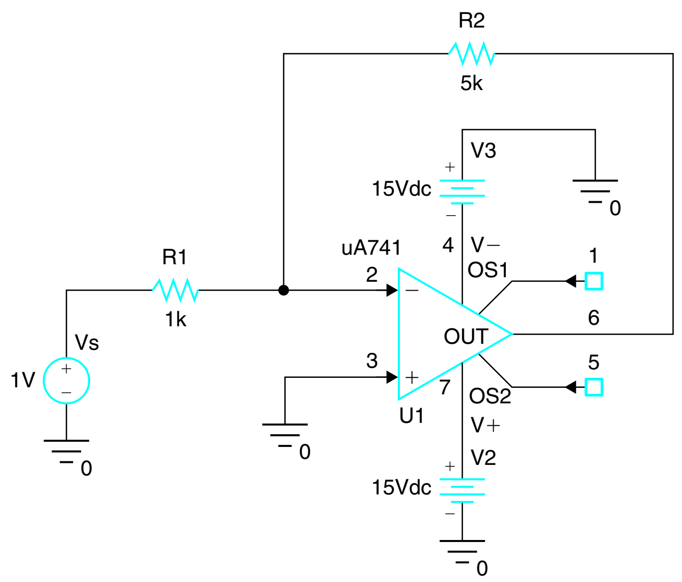
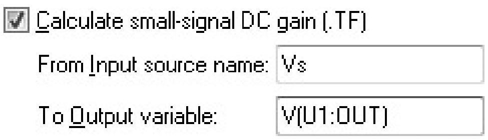


FIGURE 5.61

Simulation settings for the transfer function, and the result of the simulation. (*Source: OrCAD PSpice by Cadence*)




```
****     SMALL-SIGNAL CHARACTERISTICS
      V(N00212)/V_Vs = -5.000E+00
      INPUT RESISTANCE AT V_Vs =  1.000E+03
      OUTPUT RESISTANCE AT V(N00212) = 5.000E+03
```

The PSpice schematic for the inverting configuration of the μA741 op amp is shown in Figure 5.62. Simulation settings for DC sweep analysis are shown in Figure 5.63. The input voltage is swept from -5 V to 5 V at an interval of 0.1 V. Since the gain of the op amp is -5, the output changes from 25 V to -25 V. However, the output cannot exceed the power supply voltages. The output is limited between -15 V and 15 V. Beyond this range, the voltage stays at -15 V or 15 V. The saturation of the output voltage is shown in Figure 5.64.

FIGURE 5.62

Schematic for the μA741 op amp.

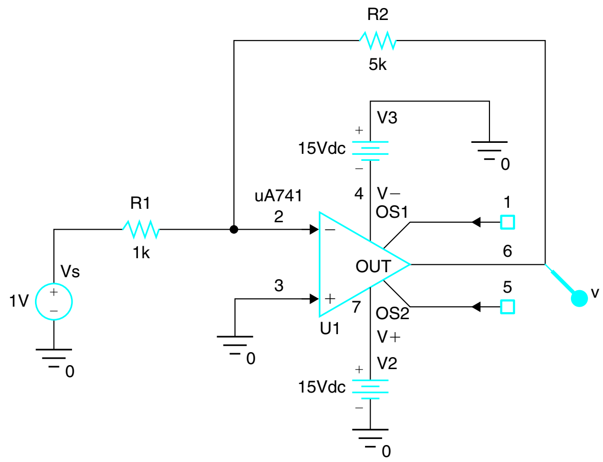


FIGURE 5.63

Simulation settings for DC sweep. (*Source: OrCAD PSpice by Cadence*)

FIGURE 5.64

The voltage at the output of the op amp.

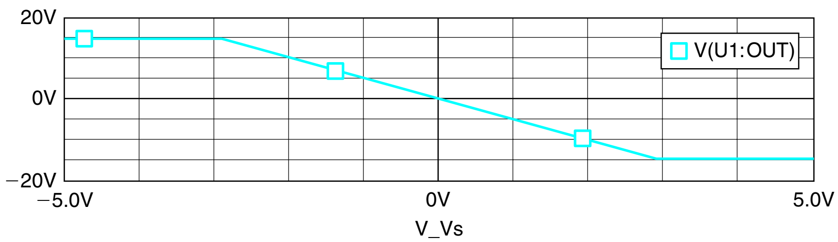
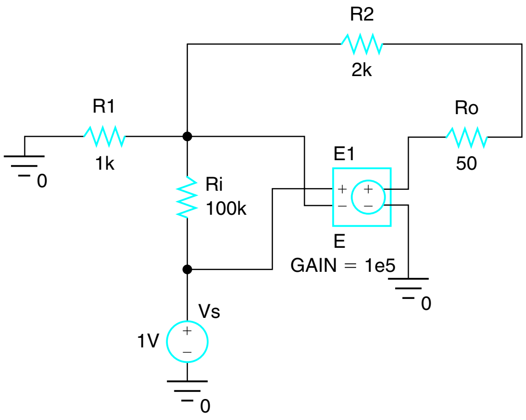


FIGURE 5.65

The PSpice model for a noninverting amplifier using VCVS.



The PSpice model for a noninverting configuration using VCVS is shown in Figure 5.65. The results from the small-signal DC gain are as follows:

```
****       SMALL-SIGNAL CHARACTERISTICS
    V(N00248)/V_Vs = 3.000E+00
    INPUT RESISTANCE AT V_Vs = 3.279E+09
    OUTPUT RESISTANCE AT V(N00248) = 1.510E-03
```

In the Wheatstone bridge circuit shown in Figure 5.66, R_4 represents a sensor. When the bridge is balanced, the resistance values are: $R_1 = 1\ k\Omega$, $R_2 = 1\ k\Omega$, $R_3 = 1\ k\Omega$, $R_4 = 1\ k\Omega$. Under certain physical condition, the resistance value of R_4 increases by $\Delta R = 10\ \Omega$ from $1\ k\Omega$ to $1.010\ k\Omega$. According to Equation (2.54), the voltage difference $v_1 - v_2$ is given by

$$v_o = v_1 - v_2 = \left(\frac{R_2}{R_1 + R_2} - \frac{R_4 + \Delta R}{R_3 + R_4 + \Delta R}\right) \times V_s$$

$$= \left(\frac{1000}{1000 + 1000} - \frac{1010}{1000 + 1010}\right) \times 5\,\text{V} = -0.012437810945274\ \text{V}$$

Thus, the voltage difference $v_2 - v_1$ is given by

$$-v_o = v_2 - v_1 = 0.012437810945274\ \text{V}$$

According to Equation (2.56), the voltage difference $v_2 - v_1$ can be approximated by

$$-v_o = v_2 - v_1 \approx \frac{+\Delta R \times V_s}{4R} = \frac{+10 \times 5}{4000}\,\text{V} = 0.0125\ \text{V}$$

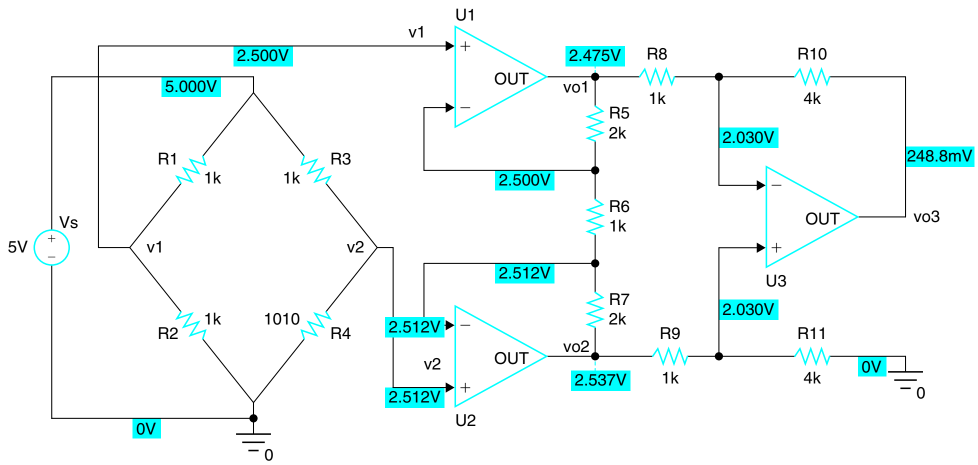
The voltage difference $v_2 - v_1$ is amplified by an instrumentation amplifier as shown in Figure 5.66. According to Equation (5.72), the output of the instrumentation amplifier is given by

$$v_{o_3} = \frac{R_{10}}{R_8}(v_{o_2} - v_{o_1}) = \left(1 + \frac{2R_5}{R_6}\right)\frac{R_{10}}{R_8}(v_2 - v_1)$$

$$= \left(1 + \frac{2 \times 2\ k\Omega}{1\ k\Omega}\right)\frac{4\ k\Omega}{1\ k\Omega}(0.012437811) = 0.24875622\ \text{V}$$

FIGURE 5.66

Wheatstone bridge circuit connected to an instrumentation amplifier.



When the approximation 0.0125 V is amplified by the same instrumentation amplifier, we get

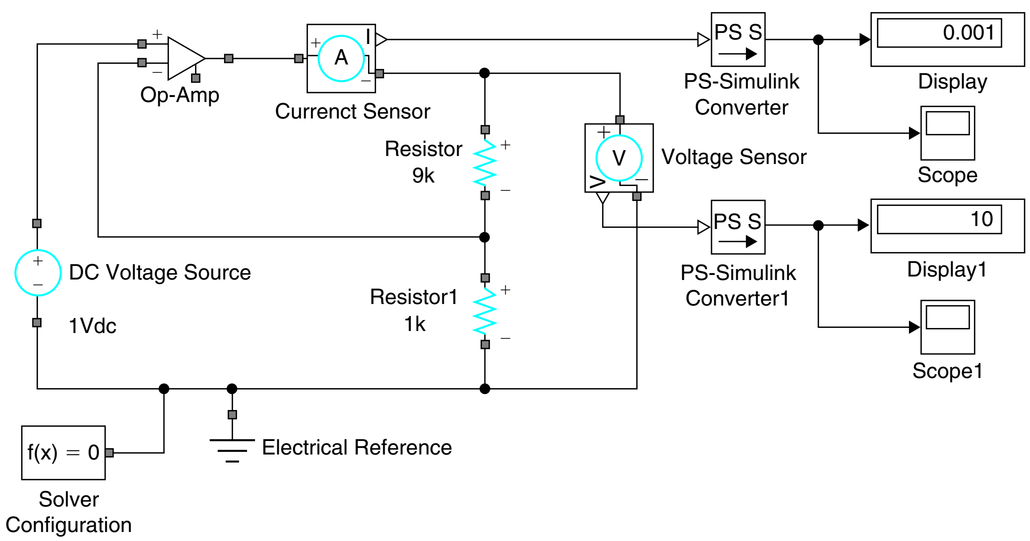
$$v_{o_{3b}} = \left(1 + \frac{2 \times 2\,k\Omega}{1\,k\Omega}\right)\frac{4\,k\Omega}{1\,k\Omega}(0.0125) = 0.25\,\text{V}$$

The percent error between approximation and exact output is

$$\%\text{Error} = \frac{v_{o_{3b}} - v_{o_3}}{v_{o_3}} \times 100\% = \frac{0.25 - 0.24875622}{0.24875622} \times 100\% = 0.5\%$$

Figure 5.66 shows the voltage values from PSpice® simulation.
The Simulink model for the noninverting configuration is shown in Figure 5.67.

FIGURE 5.67
The Simulink model for a noninverting amplifier.



The model for inverting configuration shown in Figure 5.52 can be simulated in Simulink. From Equation (5.91), we have

$$v_o = \frac{R_2A - R_o}{R_o + R_2}(0 - v_n) = Bv_d \tag{5.145}$$

where

$$B = \frac{R_2A - R_o}{R_o + R_2} \tag{5.146}$$

Since $v_p = 0$, we have $v_d = v_p - v_n = -v_n$, and Equation (5.145) becomes

$$v_o = -Bv_n$$

From Equation (5.93), we get

$$v_n = \frac{\dfrac{1}{R_1}}{\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}}v_s + \frac{\dfrac{1}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}}v_o = \alpha v_s + \beta v_o \tag{5.147}$$

where

$$\alpha = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}} \quad (5.148)$$

and

$$\beta = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}} \quad (5.149)$$

Since $v_p = 0$, we have $v_o = -Bv_n$, and Equation (5.147) becomes

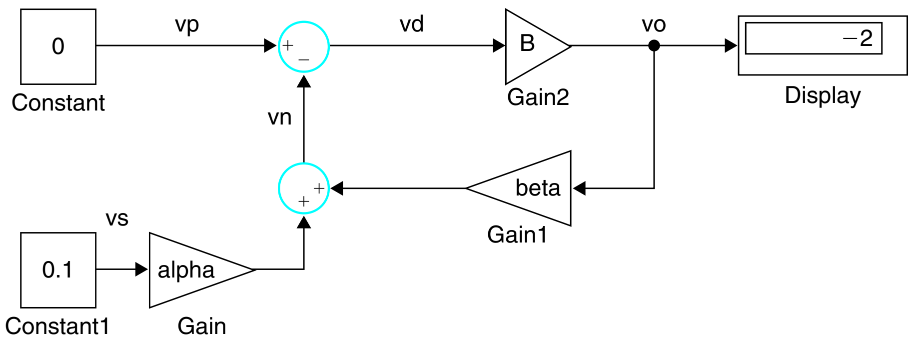
$$v_o = \frac{-B\alpha}{1 + \beta B} v_s = \frac{-\alpha}{\frac{1}{B} + \beta} v_s \approx \frac{-\alpha}{\beta} v_s$$

The Simulink model for Equations (5.145) and (5.147) is shown in Figure 5.68. The values of coefficients can be calculated in the Command window:

```
>> R1=1e3;R2=20e3;Ri=100e3;Ro=100;A=100e3;
>> B=(R2*A-Ro)/(Ro+R2)
B =
   9.9502e+04
>> alpha=1/R1/(1/R1+1/Ri+1/R2)
alpha =
     0.9434
>> beta=1/R2/(1/R1+1/Ri+1/R2)
beta =
     0.0472
```

FIGURE 5.68

The Simulink model for the inverting configuration.



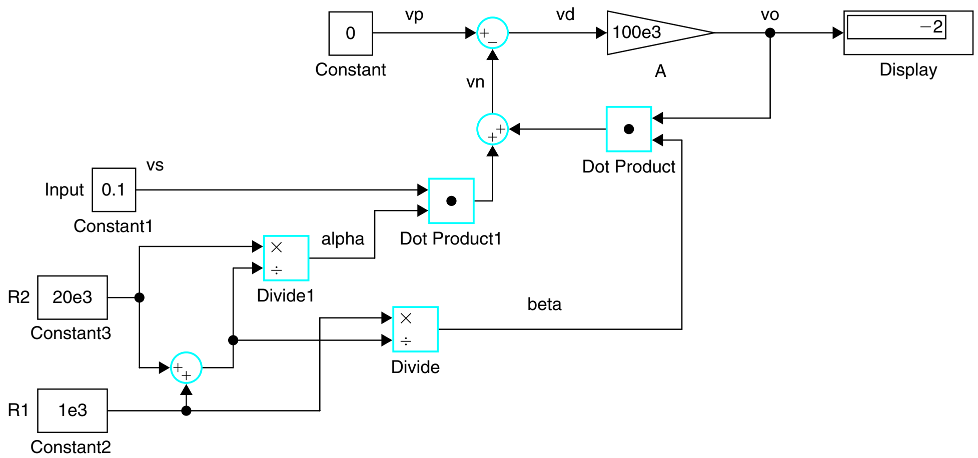
If $R_o = 0$ and $R_i = \infty$, we have

$$B = A, \quad \alpha = R_2/(R_1 + R_2), \quad \beta = R_1/(R_1 + R_2),$$
$$v_o = Av_d, \quad v_n = \alpha v_s + \beta v_o \quad (5.150)$$

The model shown in Figure 5.68 can be redrawn as the one shown in Figure 5.69.

FIGURE 5.69

The Simulink model for the inverting configuration with $R_o = 0$ and $R_i = \infty$.



EXAMPLE 5.12

Build a Simulink model similar to the one shown in Figure 5.68 for the noninverting configuration.

From Equation (5.120), we get

$$v_o = \frac{R_o}{R_o + R_2} v_n + \frac{R_2 A}{R_o + R_2}(v_p - v_n) = C v_n + B(v_p - v_n) \tag{5.151}$$

where

$$C = \frac{R_o}{R_o + R_2} \tag{5.152}$$

$$B = \frac{R_2 A}{R_o + R_2} \tag{5.153}$$

From Equation (5.123), we get

$$v_n = \frac{\dfrac{1}{R_i}}{\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}} v_s + \frac{\dfrac{1}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}} v_o = \alpha v_s + \beta v_o \tag{5.154}$$

where

$$\alpha = \frac{\dfrac{1}{R_i}}{\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}} \tag{5.155}$$

$$\beta = \frac{\dfrac{1}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}} \tag{5.156}$$

continued

Example 5.12 continued

From Equations (5.151) and (5.154), we obtain

$$v_o = \frac{C\alpha + (1-\alpha)B}{1 - C\beta + \beta B} v_s = \frac{\frac{C\alpha}{B} + 1 - \alpha}{\frac{1 - C\beta}{B} + \beta} v_s \approx \frac{1-\alpha}{\beta} v_s$$

The Simulink model is shown in Figure 5.70.

```
>> R1=1e3;R2=20e3;Ri=100e3;Ro=100;A=100e3;
>> B=R2*A/(Ro+R2)
B =
   9.9502e+04
>> C=Ro/(Ro+R2)
C =
     0.0050
>> alpha=1/Ri/(1/R1+1/Ri+1/R2)
alpha =
     0.0094
>> beta=1/R2/(1/R1+1/Ri+1/R2)
beta =
     0.0472
```

If $R_o = 0$ and $R_i = \infty$, we have

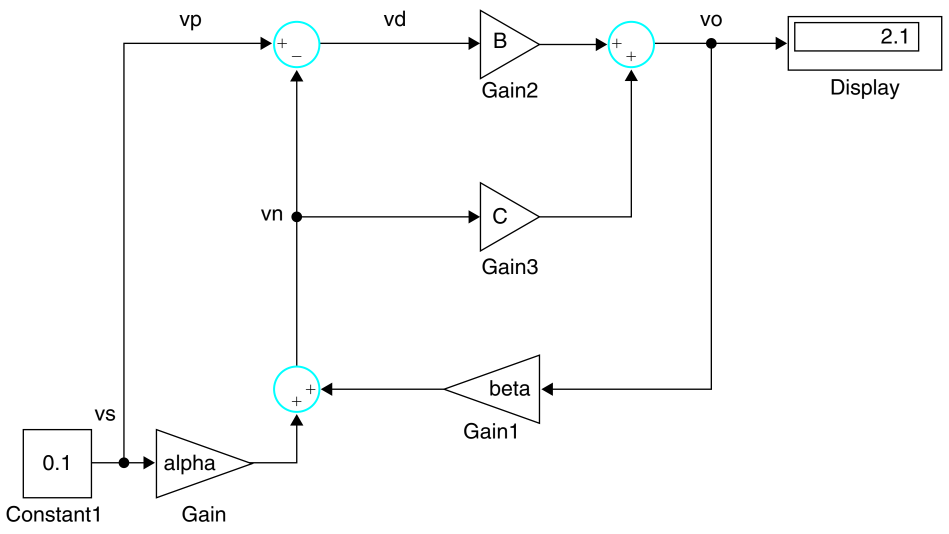
$$C = 0, \quad B = A, \quad \alpha = 0, \quad \beta = \frac{R_1}{R_1 + R_2}, \quad v_n = \beta v_o, \quad v_o = A(v_p - v_n) \qquad (5.157)$$

The model shown in Figure 5.70 reduces to the one shown in Figure 5.71.

```
>> R1=1e3;R2=20e3;A=100e3;
>> beta=R1/(R1+R2)
beta =
     0.0476
```

FIGURE 5.70

The Simulink model for the noninverting configuration.

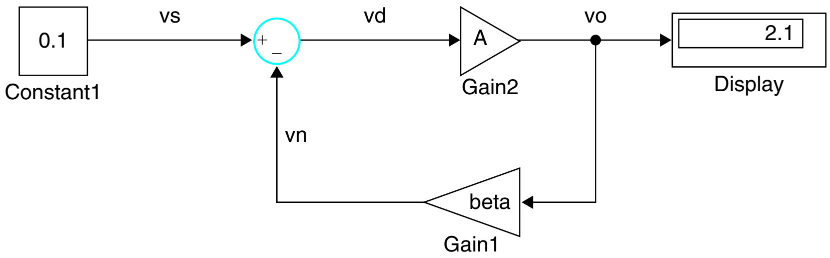


continued

Example 5.12 continued

FIGURE 5.71

The Simulink model for the noninverting configuration for $R_o = 0$ and $R_i = \infty$.



Exercise 5.10

Build a Simulink model for the summing amplifier shown in Figure 5.34 by modifying the model shown in Figure 5.71. Assume that $v_1 = 5$ V and $v_2 = 2$ V.

Answer:

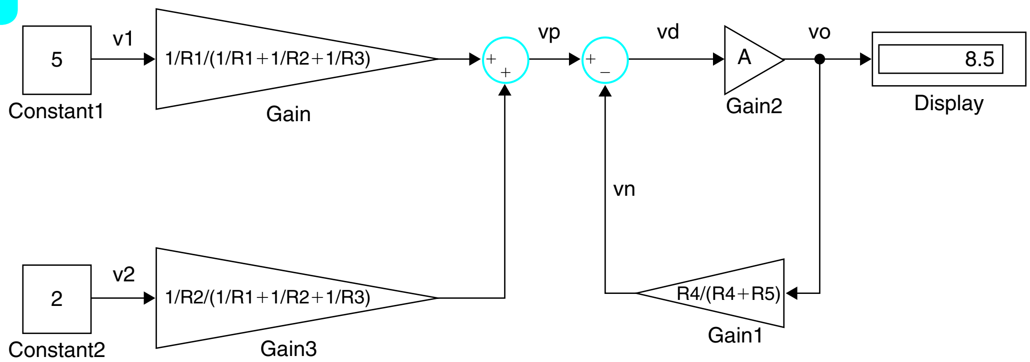
$$v_p = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} v_1 + \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} v_2, \beta = \frac{R_4}{R_4 + R_5}.$$

The model is shown in Figure 5.72.

```
>> R1=6e3;R2=1e3;R3=3e3;R4=3e3;R5=10.5e3;A=1e6;
```

FIGURE 5.72

The Simulink model for $v_o = 0.5v_1 + 3v_2$.



SUMMARY

In this chapter, the analysis and design of circuits that include op amp have been presented. Both the ideal and practical models of op amp are analyzed. In the ideal model, it is found that the values of the current entering the op amp from the positive and the negative input terminals are both zero, and the voltage on the positive input terminal is identical to the voltage on the negative input terminal for both the noninverting configuration and the inverting configuration. Both the noninverting configuration and the inverting configuration provide negative feedback from the output to the input. In the practical model, the effect of output resistance R_o, the input resistance R_i, and the open loop gain A are included.

In the inverting configuration, the output voltage v_o is given by

$$v_o = -\frac{R_2}{R_1} v_s$$

where v_s is the input voltage. In the noninverting configuration, the output voltage v_o is given by

$$v_o = \left(1 + \frac{R_2}{R_1}\right)v_s$$

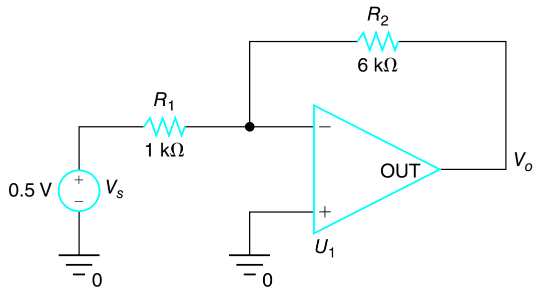
Applications of the op amp in amplifying signals, adding and subtracting signals, converting current to voltage, and converting voltage to current have been discussed. Circuits with op amps can be modeled in PSpice and Simulink. Several examples of simulating circuits using PSpice and Simulink are presented.

PROBLEMS

Ideal Op Amp

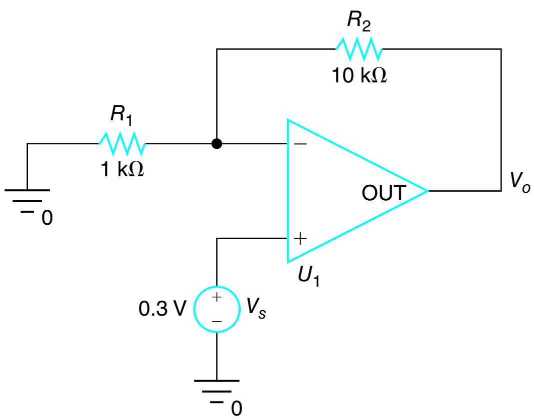
5.1 Find V_o in the circuit shown in Figure P5.1.

FIGURE P5.1



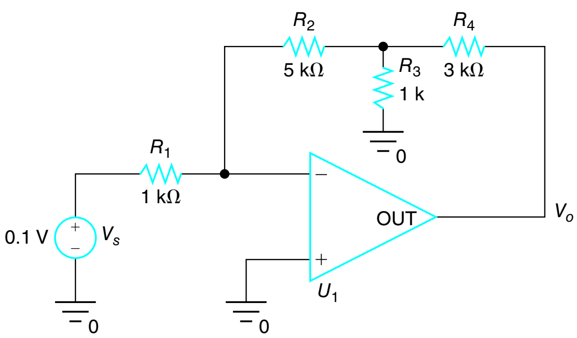
5.2 Find V_o in the circuit shown in Figure P5.2.

FIGURE P5.2



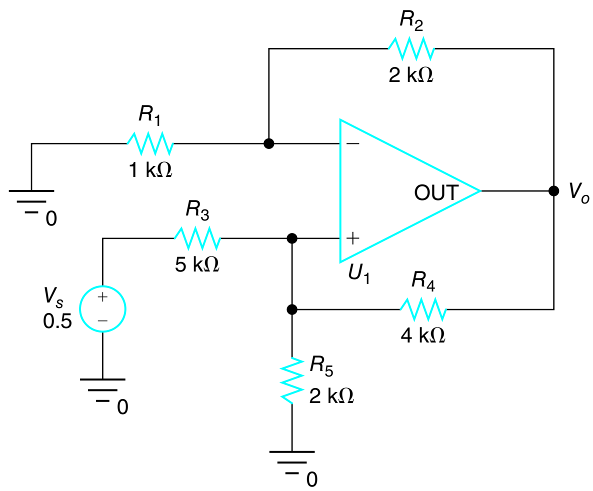
5.3 Find V_o in the circuit shown in Figure P5.3.

FIGURE P5.3



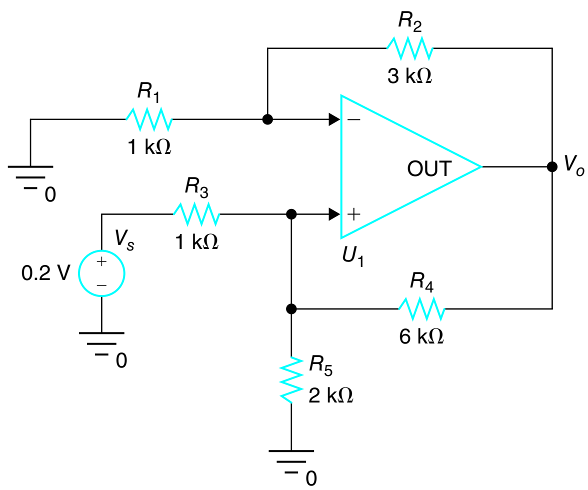
5.4 Find V_o in the circuit shown in Figure P5.4.

FIGURE P5.4



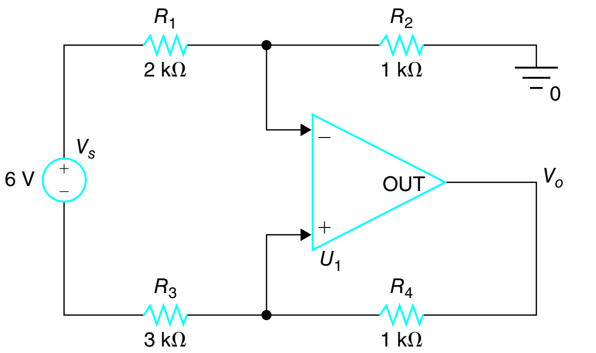
5.5 Find V_o in the circuit shown in Figure P5.5.

FIGURE P5.5



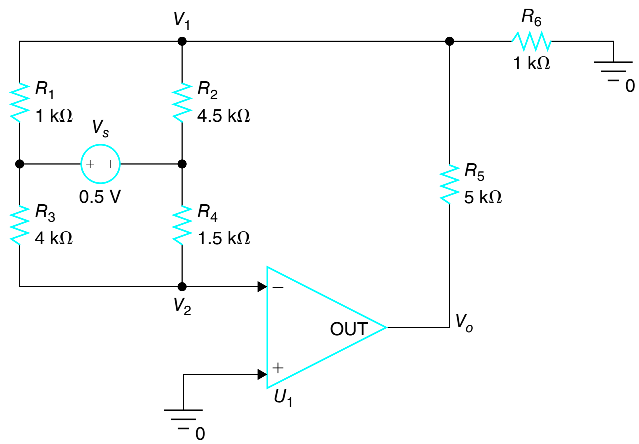
5.6 Find V_o in the circuit shown in Figure P5.6.

FIGURE P5.6



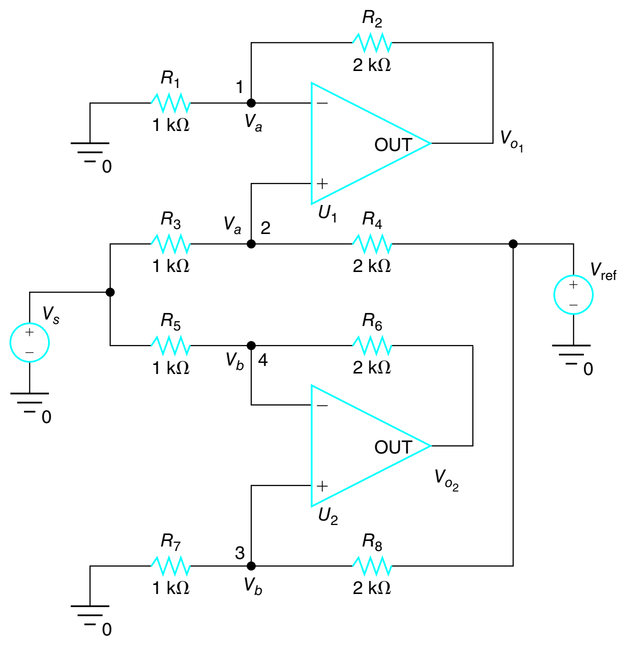
5.7 Find V_o in the circuit shown in Figure P5.7.

FIGURE P5.7



5.8 An op amp circuit is shown in Figure P5.8.

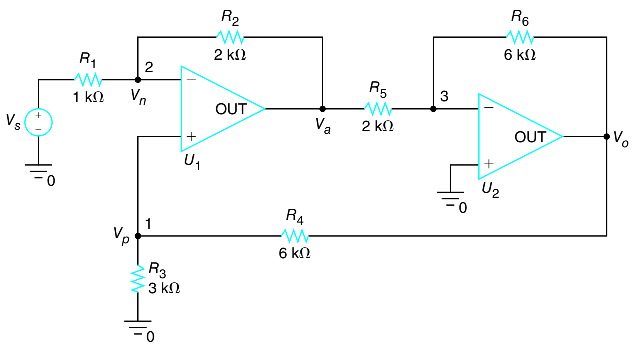
FIGURE P5.8



a. Write a node equation at node 1 by summing the currents away from node 1. The voltage at node 1 is V_a.
b. Write a node equation at node 2 by summing the currents away from node 2. The voltage at node 2 is V_a.
c. Solve the two node equations from (a) and (b) and express V_{o_1} as a function of V_s and V_{ref}.
d. Write a node equation at node 3 by summing the currents away from node 3. The voltage at node 3 is V_b.
e. Write a node equation at node 4 by summing the currents away from node 4. The voltage at node 4 is V_b.
f. Solve the two node equations from (d) and (e) and express V_{o_2} as a function of V_s and V_{ref}.
g. If $V_s = 1$ V and $V_{ref} = 0$ V, what are the numerical values of V_{o_1} and V_{o_2}?
h. If $V_s = 1$ V and $V_{ref} = 5$ V, what are the numerical values of V_{o_1} and V_{o_2}?

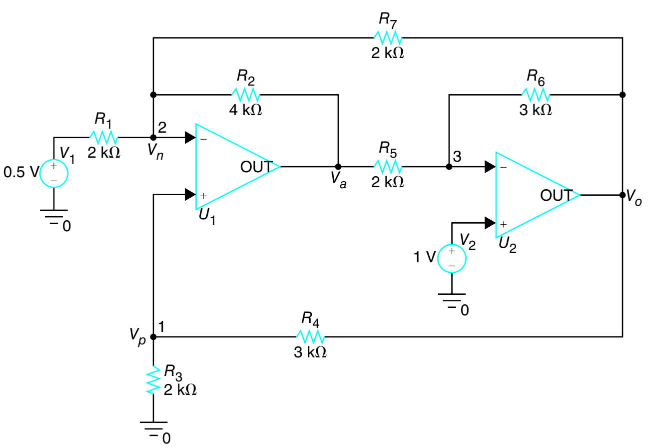
5.9 In the circuit shown in Figure P5.9, if $V_s = 0.5$ V, what is the value of V_o?

FIGURE P5.9



5.10 In the circuit shown in Figure P5.10,

FIGURE P5.10



a. write a node equation at node 1 by summing the currents leaving node 1. Express V_p as a function of V_o.
b. write a node equation at node 3 by summing the currents leaving node 3. Express V_a as a function of V_o.
c. write a node equation at node 2 by summing the currents leaving node 2.
d. express V_o as a function of V_1 and V_2
e. find the numerical values of V_o, V_p, and V_a.

5.11 **Design an op amp circuit that amplifies the input signal by −4.5 times (i.e., $v_o = -4.5\, v_{in}$). You can use one op amp and two resistors. The resistance value of the resistor connecting the inverting input and the voltage source is 1 $k\Omega$. Find the value of the other resistor.**

5.12 **Design an op amp circuit that amplifies the input signal by −0.9 times (i.e., $v_o = -0.9\, v_{in}$). You can use one op amp and two resistors. The resistance value of the resistor connecting the inverting input and the voltage source is 10 $k\Omega$. Find the value of the other resistor.**

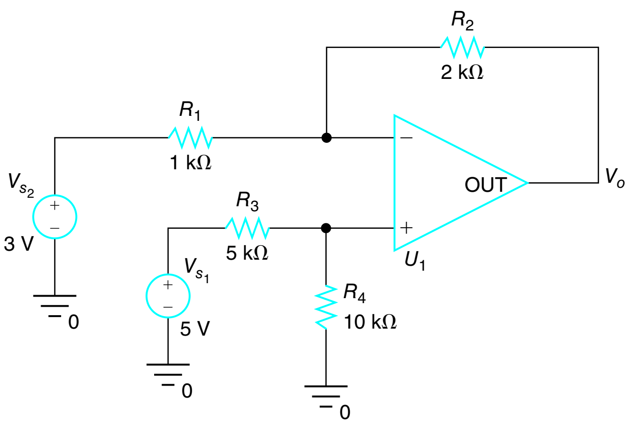
5.13 **Design an op amp circuit that amplifies the input signal by 0.75 times (i.e., $v_o = 0.75\, v_{in}$). You can use one op amp and two resistors. The resistance value of the resistor connecting the noninverting input and the voltage source is 1 $k\Omega$. Find the value of the other resistor.**

5.14 **Design an op amp circuit that amplifies the input signal by 2.5 times (i.e., $v_o = 2.5\, v_{in}$). You can use one op amp and two resistors. The resistance value of the resistor connecting the inverting input and ground is 1 $k\Omega$. Find the value of the other resistor.**

Sum and Difference

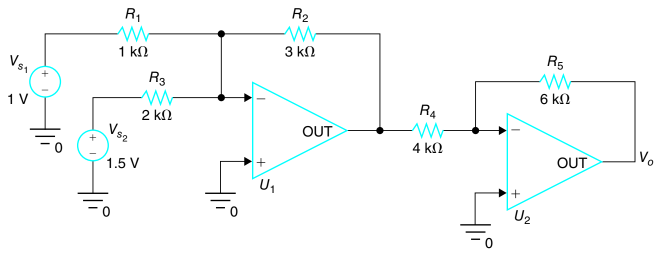
5.15 **Find V_o in the circuit shown in Figure P5.15.**

FIGURE P5.15



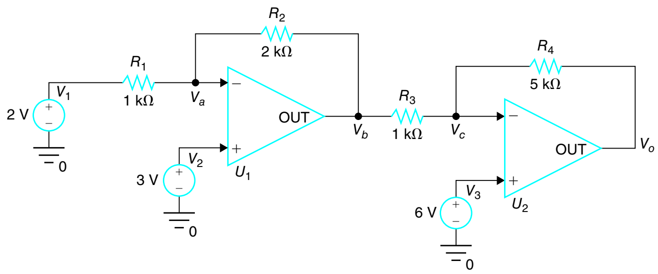
5.16 **Find V_o in the circuit shown in Figure P5.16.**

FIGURE P5.16



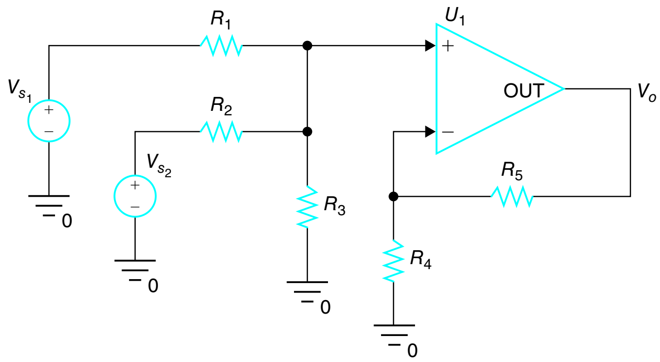
5.17 **Find V_a, V_b, V_c, and V_o in the circuit shown in Figure P5.17.**

FIGURE P5.17



5.18 **In the circuit shown in Figure P5.18,**

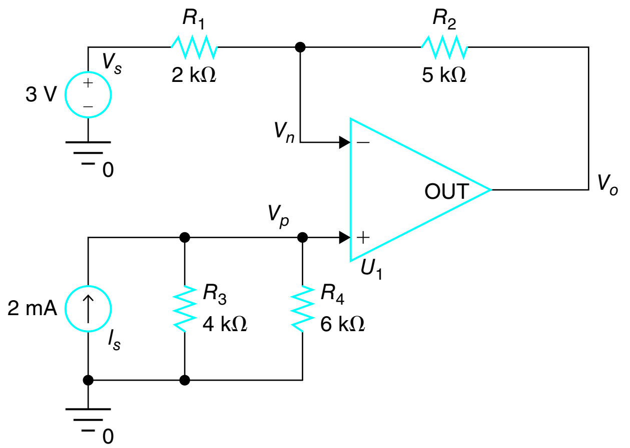
FIGURE P5.18



a. find the expression of the output voltage V_o as a function of V_{s_1} and V_{s_2}.
b. find the expression of the output voltage V_o as a function of V_{s_1} and V_{s_2} when $R_3 = R_4 = 12\ k\Omega$, $R_1 = 4\ k\Omega$, $R_2 = 3\ k\Omega$, and $R_5 = 84\ k\Omega$.
c. find the expression of the output voltage V_o as a function of V_{s_1} and V_{s_2} when $R_3 = R_4 = 6\ k\Omega$, $R_1 = 20\ k\Omega$, $R_2 = 1.5\ k\Omega$, and $R_5 = 25.8\ k\Omega$.

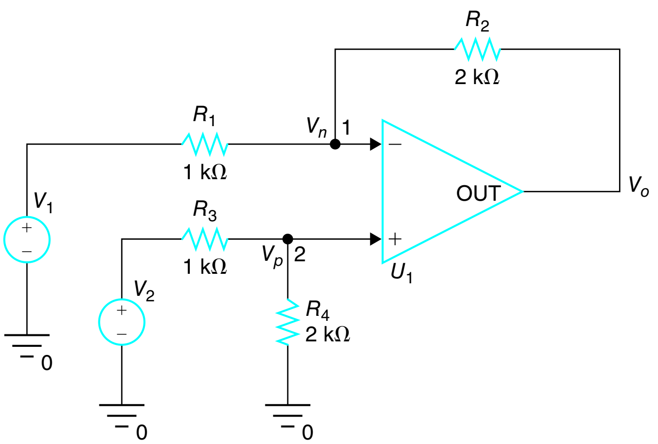
5.19 Find V_o in the circuit shown in Figure P5.19.

FIGURE P5.19



5.20 The op amp in the circuit shown in Figure P5.20 is ideal.

FIGURE P5.20



a. Write a node equation at node 1 by summing the currents leaving node 1. Represent V_o as a function of V_1 and V_n.
b. Write a node equation at node 2 by summing the currents leaving node 2. Represent V_p as a function of V_2.
c. Represent V_o as a function of V_1 and V_2.
d. Find the numerical value of V_o when $V_1 = 3.5$ V and $V_2 = 5$ V.
e. Find the numerical value of V_o when $V_1 = 3.5$ V and $V_2 = 0$ V.

5.21 Design an op amp circuit with two inputs and one output. The output of the op amp is given by $V_o = -2V_{s_1} - 3V_{s_2}$. There is one op amp and three resistors in this circuit. Find the values of the two resistors connected to the input signals when the feedback resistor between the inverting input and the output of the op amp is given by 12 $k\Omega$.

5.22 Design an op amp circuit with three inputs and one output. The output of the op amp is given by $V_o = 2V_1 + 3V_2 + 6V_3$. There is one op amp and six resistors in this circuit. Find the values of the resistors.

5.23 Design an op amp circuit with three inputs and one output. The output of the op amp is given by $V_o = V_1/2 + V_2/4 + V_3/8$.

5.24 Design an op amp circuit with two inputs and one output. The output of the op amp is given by $V_o = 5(V_{s_1} - V_{s_2})$. There is one op amp and four resistors in this circuit. Find the values of the two remaining resistors when the resistors connected to two inputs are 2 $k\Omega$.

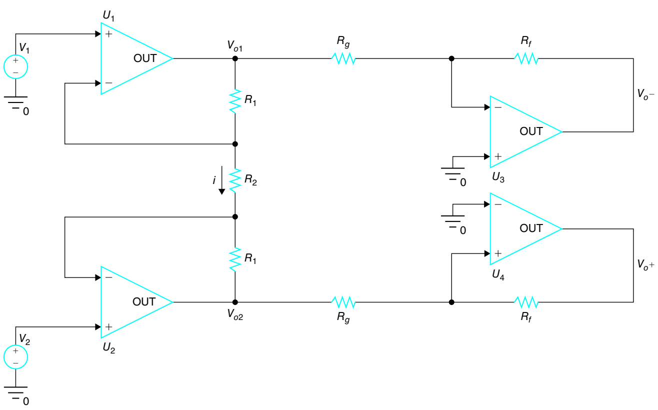
5.25 Use op amps to design a circuit that provides an output given by

$$V_o = 0.7V_1 + 1.2V_2 - 2V_3$$

Instrumentation Amplifier

5.26 In the fully differential amplifier shown in Figure P5.26, there are two outputs, V_{o+} and V_{o-}.

FIGURE P5.26

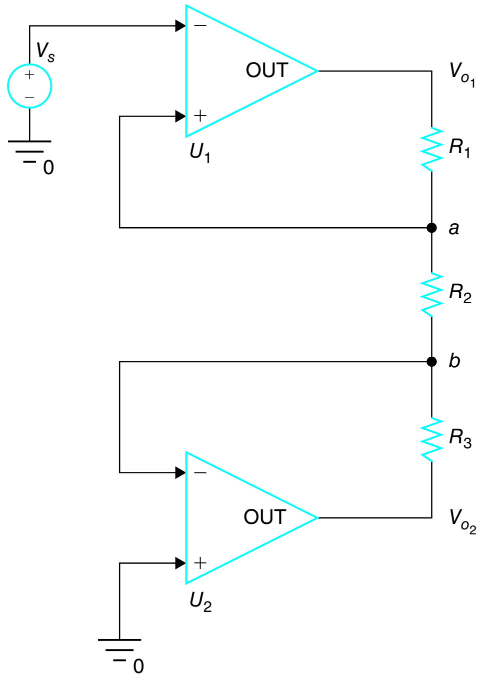


a. Find the current i through R_2 as a function of V_1, V_2, and R_2.
b. Find V_{o_1} as a function of V_1, V_2, R_1, and R_2.
c. Find V_{o_2} as a function of V_1, V_2, R_1, and R_2.
d. Find $V_{o_1} - V_{o_2}$ as a function of V_1, V_2, R_1, and R_2.
e. Find $V_{o+} - V_{o-}$ as a function of V_1, V_2, R_1, R_2, R_g, and R_f.
f. Find V_{o+} and V_{o-} when $V_2 = -V_1$.

5.27 Consider the circuit shown in Figure P5.27.

a. Find V_{o_1} as a function of V_s, R_1, and R_2.
b. Find V_{o_2} as a function of V_s, R_2, and R_3.

FIGURE P5.27

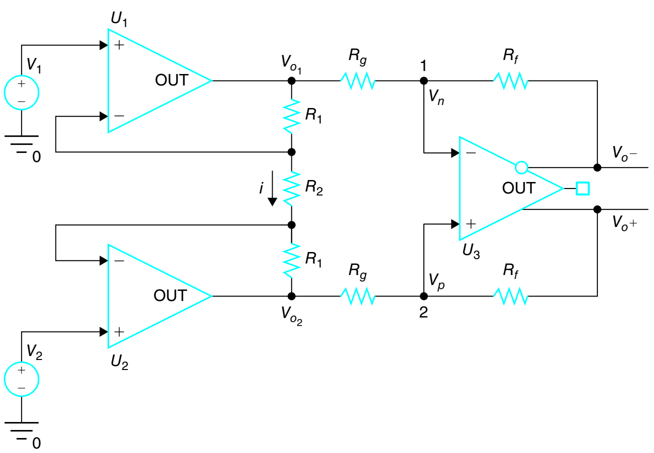


c. Find the numerical values of V_{o_1} and V_{o_2} when $V_s = 1$ V, $R_1 = 4\ k\Omega$, $R_2 = 1\ k\Omega$, and $R_3 = 5\ k\Omega$.

d. It is desired to obtain $V_{o_1} = 10$ V and $V_{o_2} = -5$ V when $V_s = 1$ V. Find one solution for R_1, R_2, and R_3.

5.28 In the fully differential amplifier shown in Figure P5.28, there are two outputs, V_{o+} and V_{o-}.

FIGURE P5.28



a. Find the current i through R_2 as a function of V_1, V_2, and R_2.

b. Find V_{o_1} as a function of V_1, V_2, R_1, and R_2.

c. Find V_{o_2} as a function of V_1, V_2, R_1, and R_2.

d. Find $V_{o_1} - V_{o_2}$ as a function of V_1, V_2, R_1, and R_2.

e. Write a node equation at node 1 by summing the currents away from node 1.

f. Write a node equation at node 2 by summing the currents away from node 2.

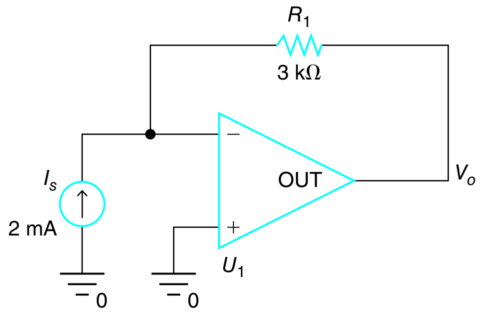
g. Find $V_{o+} - V_{o-}$ as a function of V_1, V_2, R_1, R_2, R_g, and R_f.

h. Find $V_{o+} - V_{o-}$ if $V_1 = 1$ V, $V_2 = 0.5$ V, $R_1 = 1\ k\Omega$, $R_2 = 1\ k\Omega$, $R_g = 1\ k\Omega$, and $R_f = 2\ k\Omega$.

Current Amplifier

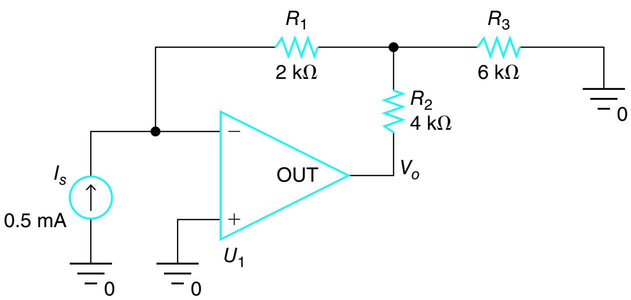
5.29 Find V_o in the circuit shown in Figure P5.29.

FIGURE P5.29



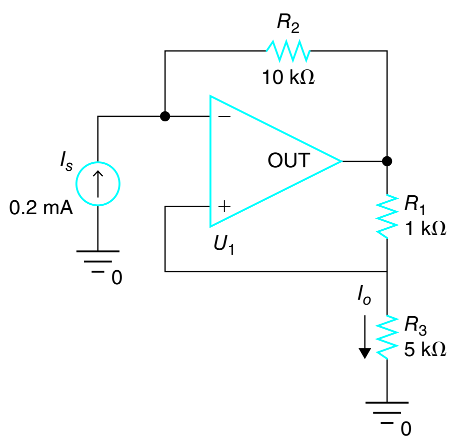
5.30 Find V_o in the circuit shown in Figure P5.30.

FIGURE P5.30



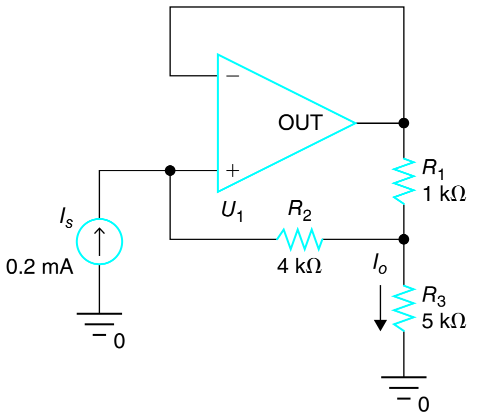
5.31 Find I_o in the circuit shown in Figure P5.31.

FIGURE P5.31



5.32 Find I_o in the circuit shown in Figure P5.32.

FIGURE P5.32

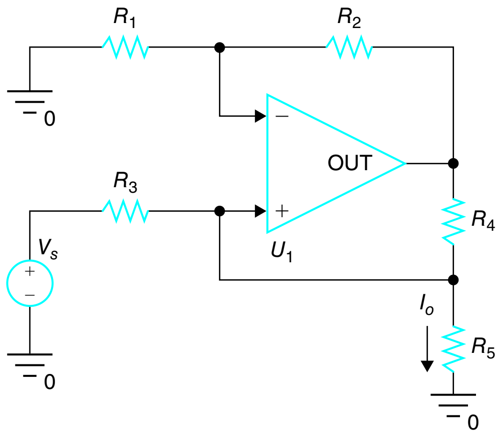


5.33 a. Represent I_o as a function of V_s in the circuit shown in Figure P5.33.

b. If $V_s = 0.5$ V, $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $R_3 = 1$ kΩ, $R_4 = 2$ kΩ, and $R_5 = 5$ kΩ, what is I_o?

c. If $V_s = 0.5$ V, $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $R_3 = 1$ kΩ, $R_4 = 2.1$ kΩ, and $R_5 = 5$ kΩ, what is I_o?

FIGURE P5.33



5.34 Design an op amp current amplifier that amplifies input current by a factor of 3; that is, $i_o = 3i_s$.

5.35 Design an op amp current amplifier that amplifies input current by a factor of −2; that is, $i_o = -2i_s$.

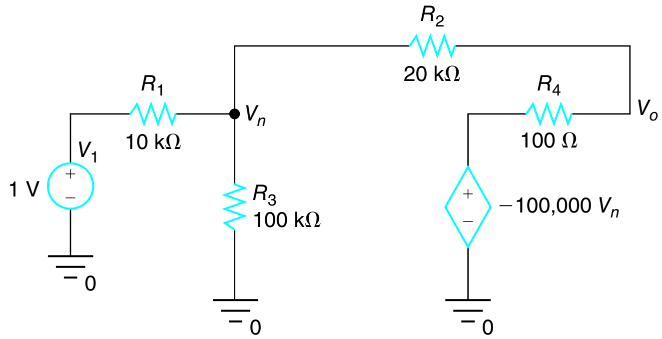
5.36 Design an op amp circuit that converts input current 1 mA to output voltage of 5 V.

5.37 Design an op amp circuit that provides a constant current of 2 mA to a load resistor from a 5-V voltage source.

Inverting Configuration

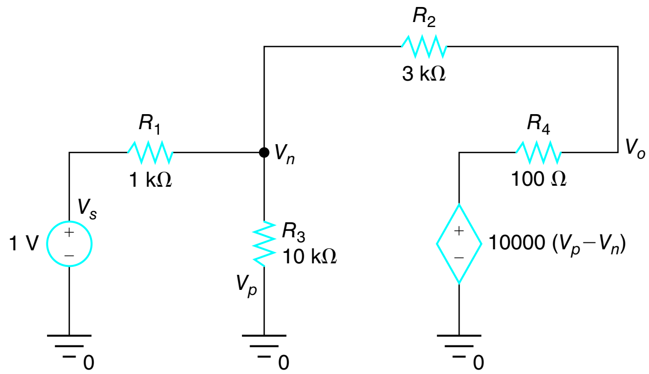
5.38 Find V_o in the circuit shown in Figure P5.38.

FIGURE P5.38



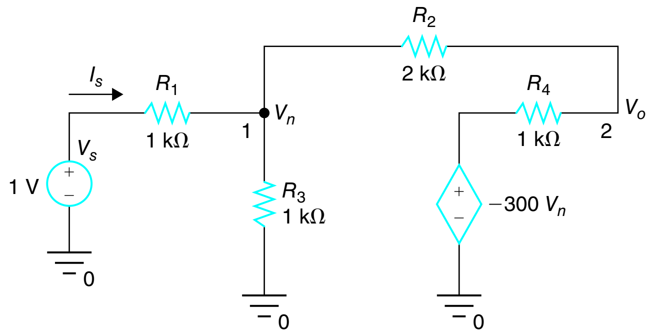
5.39 Find V_o in the circuit shown in Figure P5.39.

FIGURE P5.39



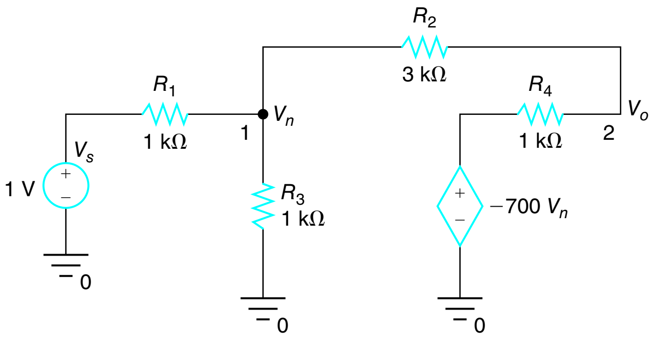
5.40 In the circuit shown in Figure P5.40, find input resistance R_{in}.

FIGURE P5.40



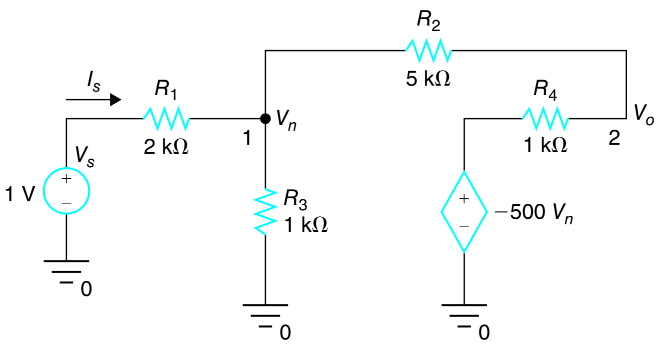
5.41 In the circuit shown in Figure P5.41, find output resistance R_{out}.

FIGURE P5.41



5.42 In the circuit shown in Figure P5.42,

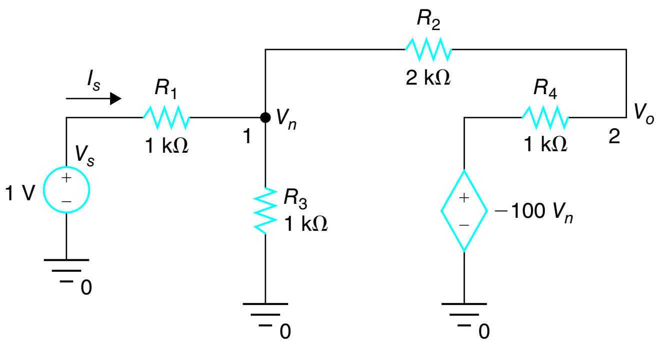
FIGURE P5.42



a. write a node equation at node 1 by summing the currents leaving node 1.
b. write a node equation at node 2 by summing the currents leaving node 2.
c. solve the two node equations to find numerical values of V_n and V_o.
d. find the value of I_s through R_1. Find the input resistance $R_{in} = V_s/I_s$.

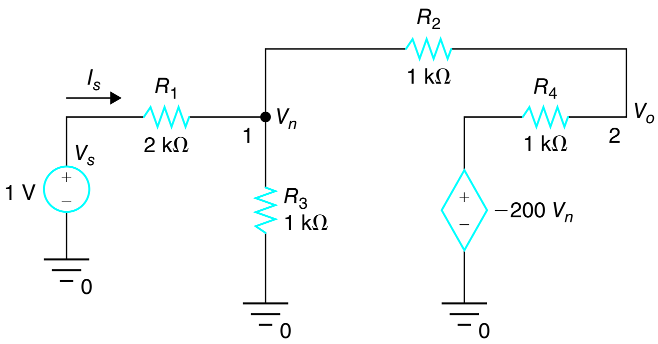
5.43 In the circuit shown in Figure P5.43, find input resistance $R_{in} = V_s/I_s$.

FIGURE P5.43



5.44 In the circuit shown in Figure P5.44,

FIGURE P5.44

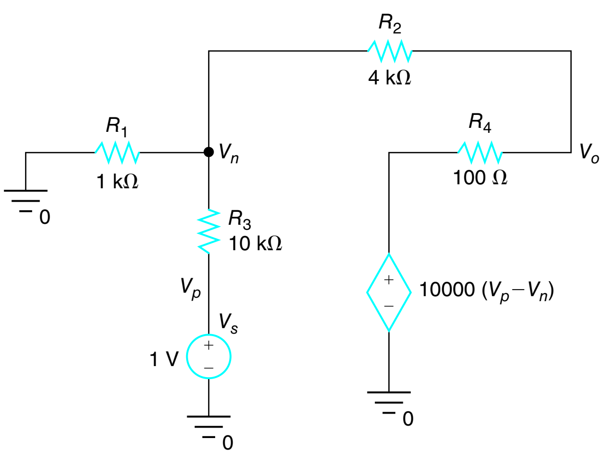


a. write a node equation at node 1 by summing the currents leaving node 1.
b. write a node equation at node 2 by summing the currents leaving node 2.
c. solve the two node equations to find numerical values of V_n and V_o.
d. find the value of I_s through R_1. Find the input resistance $R_{in} = V_s/I_s$.

Noninverting Configuration

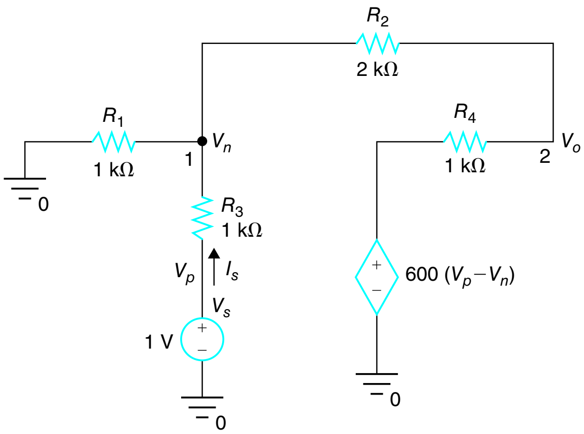
5.45 Find V_o in the circuit shown in Figure P5.45.

FIGURE P5.45



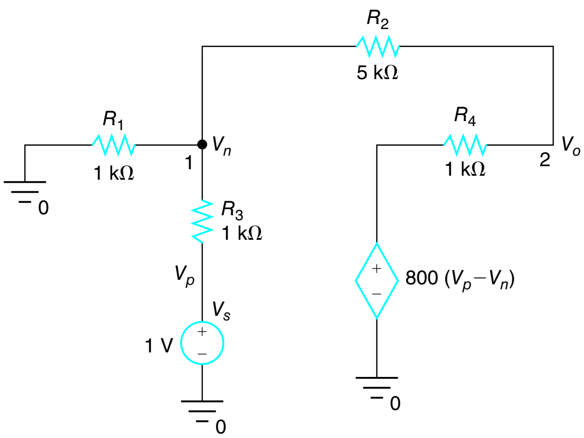
5.46 In the circuit shown in Figure P5.46, find input resistance R_{in}.

FIGURE P5.46



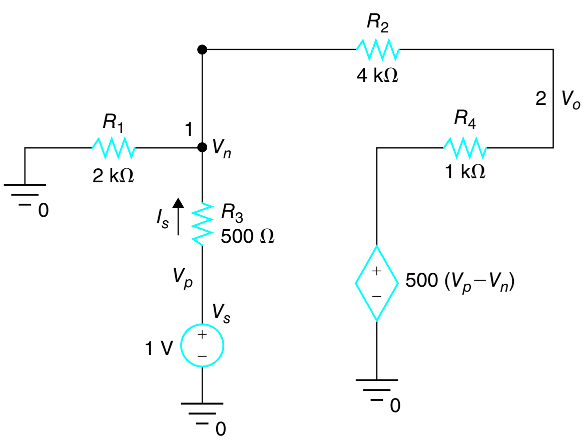
5.47 In the circuit shown in Figure P5.47, find output resistance R_{out}.

FIGURE P5.47



5.48 In the circuit shown in Figure P5.48,

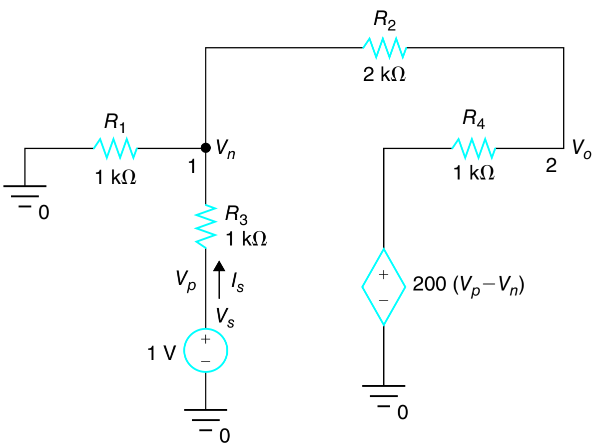
FIGURE P5.48



a. write a node equation at node 1 by summing the currents away from node 1.
b. write a node equation at node 2 by summing the currents away from node 2. ($V_p = V_s = 1$ V)
c. solve the two node equations to find numerical values of V_n and V_o.
d. find the value of I_s through R_3. Find the input resistance $R_{in} = V_s/I_s$.

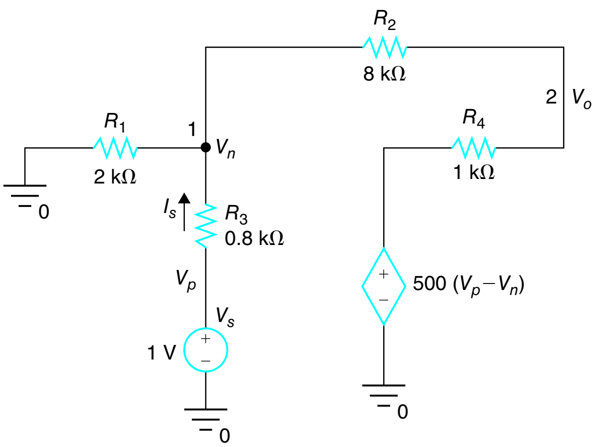
5.49 In the circuit shown in Figure P5.49, find input resistance $R_{in} = V_s/I_s$.

FIGURE P5.49



5.50 In the circuit shown in Figure P5.50,

FIGURE P5.50



a. write a node equation at node 1 by summing the currents away from node 1.
b. write a node equation at node 2 by summing the currents away from node 2. ($V_p = V_s = 1$ V)
c. solve the two node equations to find numerical values of V_n and V_o.
d. find the value of I_s through R_3. Find the input resistance $R_{in} = V_s/I_s$.