Types of Numbers

Natural Numbers:

Whole Numbers: The numbers 0,1,2,3.... are called whole numbers. Whole numbers include “0”.

Integers: The numbers .... -3, -2, -1, 0, 1, 2, 3,....are called integers.

Negative Integers: The numbers -1, -2, -3, .. are called negative integers.

Positive Fractions: The numbers $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{8}$... are called positive fractions.

Negative Fractions: The numbers $-\frac{6}{8}$, $-\frac{7}{19}$, $-\frac{12}{47}$ ... are called negative fractions.

Rational Numbers: Any number which is a positive or negative integer or fraction, or zero is called a rational number. A rational number is one which can be expressed in the following format $\Rightarrow \frac{a}{b}$, where b $\neq$ 0 and a & b are positive or negative integers.

Irrational Numbers: An infinite non recurring decimal number is known as an irrational number. These numbers cannot be expressed in the form of a proper fraction a/b where b $\neq$ 0. e.g. $\sqrt{2}$, $\sqrt{5}$, π, etc.

Surds: Any root of a number, which cannot be exactly found is called a surd. Essentially, all surds are irrational numbers. e.g. $\sqrt{2}$, $\sqrt{5}$, etc.
Surds of the form x + $\sqrt{y}$, x - $\sqrt{y}$ are called binomial quadratic surds, where x + $\sqrt{y}$ and x - $\sqrt{y}$ are called conjugate surds, each being the conjugate of the other.

Even Numbers: The numbers which are divisible by 2 are called even numbers e.g. -4, 0, 2, 16 etc.

Odd Numbers: The numbers which are not divisible by 2 are odd numbers e.g. -7, -15, 5, 9 etc.

Prime Numbers: Those numbers, which are divisible only by themselves and 1, are called prime numbers. In other words, a number, which has only two factors - 1 and itself, is called a prime number. e.g. 2, 3, 5, 7, etc.

2 is the only even prime number.

There are 25 prime numbers upto 100. These are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 & 97. These should be learnt by heart.

Two numbers are considered to be relatively prime to each other when their HCF is 1. e.g. 5 and 21 are prime to each other. In other words, 5 and 21 are **co-prime.**

Tests of Divisibility

1. **By 2** - A number is divisible by 2 when its units place is 0 or divisible by 2. e.g. 120, 138.
2. **By 3** - 19272 is divisible by 3 when the sum of the digits of 19272 - *21* is divisible by 3. Note that if n is odd, then $2^n + 1$ is divisible by 3 and if n is even, then $2^n - 1$ is divisible by 3.
3. **By 4** - A number is divisible by 4 when the last two digits of the number are 0s or are divisible by 4. As 100 is divisible by 4, it is sufficient if the divisibility test is restricted to the last two digits. e.g. 145896, 128, 18400
4. **By 5** - A number is divisible by 5, if its unit's digit is 5 or 0. e.g. 895, 100
5. **By 6** - A number is divisible by 6, if it is divisible by both 2 and by 3. i.e. the number should be an even number and the sum of its digits should be divisible by 3.
6. **By 8** - A number is divisible by 8, if the last three digits of the number are 0s or are divisible by 8. As 1000 is divisible by 8, it is sufficient if the divisibility test is restricted to the last three digits e.g. 135128, 45000
7. **By 9** - A number is divisible by 9, if the sum of its digits is divisible by 9. e.g. 810, 92754
8. **By 11** - A number is divisible by 11, if the difference between the sum of the odd digits and the even digits of the number is either 0 or a multiple of 11.

   e.g. 121, 65967. In the first case 1+1 - 2 = 0. In the second case 6+9+7 = 22 and 5+6 = 11 and the difference is 11. Therefore, both these numbers are divisible by 11.
9. **By 12** - A number is divisible by 12, if it both divisible by 3 and by 4. i.e., the sum of the digits should be divisible by 3 and the last two digits should be divisible by 4. e.g. 144, 8136.
10. **By 15 –** A number is divisible by 15, if it is divisible by both 5 and 3.
11. **By 25** – 2358975 is divisible by 25 if the last two digits of 2358975 are divisible by 25 or the last two digits are 0.
12. **By 75** - A number is divisible by 75, if it is both divisible by 3 and by 25. i.e. the sum of the digits should be divisible by 3 and the last two digits should be divisible by 25.

Decimals and Fractions to be remembered

You get many questions in the exams based on Percentage, Profit, Interest etc. in which you have to calculate, say 87.5 % of 800, 58.33 % of 2400 etc. Calculating these values with the help of traditional methods is time-consuming. IF you have the fraction approach, you can crack these easily i.e., if you know that 87.5 % is just $\frac{7}{8}^{th}$ of the number and 58.33 % is $\frac{7}{12}^{th}$ of the number, then it becomes easy to calculate.

% age	**Fraction**		**% age**	**Fraction**
50 %	$\frac{1}{2}$		$55\frac{5}{9}$ %	$\frac{5}{9}$
$33\frac{1}{3}$ %	$\frac{1}{3}$		$77\frac{7}{9}$ %	$\frac{7}{9}$
$66\frac{2}{3}$ %	$\frac{2}{3}$		$88\frac{8}{9}$ %	$\frac{8}{9}$
25 %	$\frac{1}{4}$			
75 %	$\frac{3}{4}$		$9\frac{1}{11}$ %	$\frac{1}{11}$
20 %	$\frac{1}{5}$		$18\frac{2}{11}$ %	$\frac{2}{11}$
40 %	$\frac{2}{5}$		$27\frac{3}{11}$ %	$\frac{3}{11}$
60 %	$\frac{3}{5}$		$36\frac{4}{11}$ %	$\frac{4}{11}$
80 %	$\frac{4}{5}$		$45\frac{5}{11}$ %	$\frac{5}{11}$
			$54\frac{6}{11}$ %	$\frac{6}{11}$
$16\frac{2}{3}$ %	$\frac{1}{6}$		$63\frac{7}{11}$ %	$\frac{7}{11}$
$83\frac{1}{3}$ %	$\frac{5}{6}$		$72\frac{8}{11}$ %	$\frac{8}{11}$
$14\frac{2}{7}$ %	$\frac{1}{7}$		$81\frac{9}{11}$ %	$\frac{9}{11}$
			$90\frac{10}{11}$ %	$\frac{10}{11}$
$12\frac{1}{2}$ %	$\frac{1}{8}$			
$37\frac{1}{2}$ %	$\frac{3}{8}$		$8\frac{1}{3}$ %	$\frac{1}{12}$
$62\frac{1}{2}$ %	$\frac{5}{8}$		$41\frac{2}{3}$ %	$\frac{5}{12}$
$87\frac{1}{2}$ %	$\frac{7}{8}$		$58\frac{1}{3}$ %	$\frac{7}{12}$
			$91\frac{2}{3}$ %	$\frac{11}{12}$
$11\frac{1}{9}$ %	$\frac{1}{9}$		$6\frac{2}{3}$ %	$\frac{1}{15}$
$22\frac{2}{9}$ %	$\frac{2}{9}$		$6\frac{1}{4}$ %	$\frac{1}{16}$
$44\frac{4}{9}$ %	$\frac{4}{9}$		5 %	$\frac{1}{20}$

Anything doubles to increase by 100 % and becomes 200 %.
Anything trebles to increase by 200 % and becomes 300 %

LCM & HCF

1. ***Least Common Multiple - LCM***

The least common multiple (LCM) of two or more numbers is the smallest of the numbers, which is exactly divisible by each of them.
e.g Consider two number 12 and 15
Multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, ...
Multiples of 15 are: 15, 30, 45, 60, 75, 90, 105, 120, 135, ...
The common multiples of both 12 and 15 are 60, 120, 180, ...
The least common multiple is 60.

How to find the LCM of two or more numbers?
The LCM of two numbers can be found by the product of the factors of the two numbers after eliminating repetition of the common factors.
In the above example, the common factor for 12 and 15 are 3. Therefore, the LCM will be 3*4*5 = 60.
Alternatively, LCM is the product of all prime factors of the given numbers, the common factors among them being in their highest degree. e.g, The LCM of $5x^2y^3z^5$ and $3xy^2z^7$ will be $5*3*x^2y^3z^7 = 15x^2y^3z^7$, where x, y and z are the prime factors.

2. ***Greatest Common Divisor (GCD)/ Highest Common Factor (HCF):***

The highest common factor of two or more numbers is the greatest number, which divides each of those numbers exact number of times. e.g HCF of 24 and 36 is 12.

How to find the HCF of two or more numbers?
a) Express the two numbers as product of prime numbers separately.
b) Take the product of prime numbers common to both numbers.

3. ***LCM and HCF of Fractions***

LCM of fractions = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$;

e.g. LCM of $\frac{3}{4}$ and $\frac{1}{2}$ = $\frac{3\text{ (LCM of numerators)}}{2\text{ (HCF of denominators)}}$

HCF of fractions = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

e.g. HCF of $\frac{3}{4}$ and $\frac{1}{2}$ = $\frac{1\text{(HCF of numerators)}}{4\text{(LCM of denominators)}}$

Note that the product of the two fractions is always equal to the product of LCM and HCF of the two fractions.

The product of the two fractions = $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

The product of the LCM and HCF = $\frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$.

Important Points

- In case of HCF, if some remainders are given, then first those remainders are subtracted from the numbers given and then their HCF is calculated.
- In case of LCM, if a single remainder is given, then firstly the LCM is calculated and then that single reminder is added in that.
- In case of LCM, if for different numbers different remainders are given, then the difference between the number and its respective remainder will be equal. In that case, firstly the LCM is calculated, then that common difference between the number and its respective remainder is subtracted from that.
- Sometimes in case of HCF questions, the same remainder is required is given and the remainder is not given.

Fractions

1. ***Types of fractions***

<u>Common Fractions:</u> Fractions such 3/4, 32/43 etc are called common or vulgar fractions.

<u>Decimal Fractions:</u> Fractions whose denominators are 10, 100, 1000, ... are called decimal fractions.

<u>Proper Fraction:</u> A fraction whose numerator is less than its denominator is known as a proper fraction e.g. 3/4

<u>Improper Fraction:</u> A fraction whose numerator is greater than its denominator is known as an improper fraction. e.g. 4/3

Mixed Fractions: Fractions which consists of an integral part and a fractional part are called mixed fractions. All improper fractions can be expressed as mixed fractions and vice versa. e.g. $1\frac{3}{4}$.

Recurring Decimals: A decimal in which a set of figures is repeated continually is called a recurring or periodic or a circulating decimal.
e.g. $1/7$ = 0.142857…….. the dots indicate that the figure between 1 and 7 will repeat continuously.

Addition of Fractions

Mixed fractions can be added by adding the integral and fractional part separately or by converting them into improper fractions.

e.g $3\frac{4}{5} + \frac{11}{12}$ can be added as either

$3 + \frac{4}{5} + \frac{11}{12} \Rightarrow 3 + \frac{4\times12+11\times5}{60}$

$\Rightarrow 3 + \frac{103}{60} \Rightarrow 3 + 1\frac{43}{60} = 4\frac{46}{60} = \frac{283}{60}$

or by converting the mixed fractions into improper fraction, $\frac{19}{5} + \frac{11}{12} = \frac{(19\times12+11\times5)}{60} = \frac{283}{60}$.

Averages

Average means Arithmetic mean of the items and it is = $\frac{\text{Sum of Items}}{\text{Number of Items}}$

When the difference between all the items is same, then average is equal to $\frac{n+1}{2}$ item,

where *n* is the total number of items.

Average speed : If a man covers some journey from A to B at *u* km/hr and returns back to A at B uniform speed of *v* km/hr.,

then the average speed during the whole journey is $\frac{2uv}{u+v}$ km/hr

➨ If *x* is added in all the items, then average increases by *x*.

- ➡ If x is subtracted from all the items, then average decreases by x.
- ➡ If every item is multiplied by x, then average also gets multiplied by x.
- ➡ If every item is divided by x, then average also gets divided by x.
- ➡ This means if average increases by Y, it can be assumed that Y is added in all the items.

Basic Formulae

I. $(a + b)^2 = a^2 + b^2 + 2ab$.

II. $(a - b)^2 = a^2 + b^2 - 2ab$.

III. $(a^2 - b^2) = (a + b)(a - b)$.

IV. $(a + b)^2 - (a - b)^2 = 4ab$.

V. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$.

VI. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.

VII. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.

VIII. $(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$.

IX. $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$.

X. $(a + b + c)^2 = [a^2 + b^2 + c^2 + 2(ab + bc + ca)]$.

XI. $(a + b + c + \mathrm{d})^2 =$
$[a^2 + b^2 + c^2 + \mathrm{d}^2 + 2a(b + c + \mathrm{d}) + 2b(c + \mathrm{d}) + 2c\mathrm{d}]$.

XII. $(a^3 + b^3 + c^3 - 3abc) =$
$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$.
If $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$.

XIII. $(x + a)(x + b) = x^2 + (a + b)x + ab$.

XIV. $(x + a)(x + b)(x + c) =$
$x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$.

Basics of Algebra

- General Algebra is based on the operations of arithmetic and on the concept of an unknown quantity, or variable. Letters such as x or n are used to represent unknown quantities. For example, suppose Pam has 5 more pencils than Fred. If F represents the number of pencils that Fred has, then the number of pencils that Pam has is F + 5. As another example, if Jim's present salary S is increased by 7 %, then his new salary is 1.07S. A combination of letters and arithmetic operations, such as $F + 5$, $\frac{3x^2}{2x-5}$ and $19x^2 - 6x + 3$, is called an algebraic expression.

- The expression $19x^2 - 6x + 3$ consists of the terms $19x^2$, $-6x$, and 3, where 19 is the coefficient of x^2, - 6 is the coefficient of x, and 3 is a constant term (or coefficient of $x^0 = 1$). Such an expression is called a second degree (or quadratic) polynomial in x since the highest power of x is 2. The expression F + 5 is a first degree (or linear) polynomial in F since the highest power of F is 1.

- Often when working with algebraic expressions, it is necessary to simplify them by factoring or combining like terms. For example, the expression $6x + 5x$ is equivalent to $(6 + 5)x$, or $11x$. In the expression $9x - 3y$, 3 is a factor common to both terms: $9x - 3y = 3(3x - y)$. In the expression $5x^2 + 6y$, there are no like terms and no common factors. If there are common factors in the numerator and denominator of an expression they can be divided out, provided that they are not equal to zero. For example, if $x \neq 3$, then $\frac{x-3}{x-3}$ is equal to 1; therefore, $\frac{3xy-9y}{x-3} = \frac{3y(x-3)}{x-3} = 3y$
- To multiply two algebraic expressions, each term of one expression is multiplied by each term of the other expression. For example:
  $(3x - 4)(9y + x) = 3x(9y + x) - 4(9y + x)$
  $= (3x)(9y) + (3x)(x) + (-4)(9y) + (-4)(x)$
  $= 27xy + 3x^2 - 36y - 4x$.

- An algebraic expression can be evaluated by substituting values of the unknowns in the expression. For example, if $x = 3$ and $y = -2$, then $3xy - x^2 + y$ can be evaluated as $3(3)(-2) - (3)^2 + (-2) = -18 - 9 - 2 = -29$.

- A major focus of algebra is to solve equations involving algebraic expressions. Some examples of such equations are
  $5x - 2 = 9 - x$ (a linear equation with one unknown)
  $3x + 1 = y - 2$ (a linear equation with two unknowns)
  $5x^2 + 3x - 2 = 7x$ (a quadratic equation with one unknown)
  $\frac{x(x-3)(x^2+5)}{x-4} = 0$ (an equation that is factored on one side with 0 on the other).

  The solutions of an equation with one or more unknowns are those values that make the equation true, or "satisfy the equation," when they are substituted for the unknowns of the equation. An equation may have no solution or one or more solutions. If two or more equations are to be solved together, the solutions must satisfy all of the equations simultaneously.

There are two systems of equations.

Consistent System: A system, which could have two or more simultaneous linear equations is known as consistent if it has at least one solution.

Inconsistent System: A system of two simultaneous linear equations is said to be inconsistent, if it has no solution at all.

To get the number of solutions a set of two equations has the following rules can be applied. The equations are of the form of $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

- The equations have a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. lines are intersecting

- The equations have infinitely many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ i.e. lines are parallel
- The equations have no solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ i.e. lines are coinciding

Linear Algebra

Linear Equations with one unknown

To solve a linear equation with one unknown (that is, to find the value of the unknown that satisfies the equation), the unknown should be isolated on one side of the equation. This can be done by performing the same mathematical operations on both sides of the equation. Remember that if the same number is added to or subtracted from both sides of the equation, this does not change the equality; likewise, multiplying or dividing both sides by the same nonzero number does not change the equality. For example, to solve the equation

$\frac{5x-6}{3}$ = 4 for *x*, x can be isolated using the following steps:

$\Rightarrow$ 5*x* – 6 = 12

(multiplying by 3)

5*x* = 12 + 6 = 18 (adding 6)

x = $\frac{18}{5}$ (dividing by 5)

Linear Equations with two unknowns

There are several methods of solving two linear equations in two unknowns. With any method, if a contradiction is reached, then the equations have no solution; if a trivial equation such as 0 = 0 is reached, then the equations are equivalent and have infinitely many solutions. Otherwise, a unique solution can be found.

One way to solve for the two unknowns is to express one of the unknowns in terms of the other using one of the equations, and then substitute the expression into the remaining equation to obtain an equation with one unknown. This equation can solved and the value of the unknown substituted into either of the original equations to find the value of the other unknown. For example, the following two equations can be solved for x and y.

(1) 3*x* + 2*y* = 11

(2) *x* – *y* = 2

In equation (2), *x* = 2 + *y*. Substitute (2 + *y*) in equation (1) for *x*:

3(2 + *y*) + 2*y* = 11

$\Rightarrow$ 6 + 3*y* + 2*y* = 11 $\Rightarrow$ 6 + 5*y* = 11 $\Rightarrow$ 5*y* = 5 $\Rightarrow$ *y* = 1.

If *y* = 1, then *x* = 2 + 1 = 3.

There is another way to solve for x and y by eliminating one of the unknowns. This can be done by making the coefficients of one of the unknowns the same (disregarding the sign) in both equations and either adding the equations or subtracting one equation from the other. For example, to solve the equations

(1) $6x + 5y = 29$

(2) $4x - 3y = -6$

by this method, multiply equation (1) by 3 and equation (2) by 5 to get

$18x + 15y = 87$

$20x - 15y = -30$.

Adding the two equations eliminates y,
yielding $38x = 57$, or $x = \frac{57}{38} = \frac{3}{2}$.
Finally, substituting $\frac{3}{2}$ for x in one of the equations gives $y = 4$.

Quadratic Equations

Solving Quadratic Equations by Factoring

The standard form for a quadratic equation is $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$; for example:

$x^2 + 6x + 5 = 0$, $3x^2 - 2x = 0$, and $x^2 + 4 = 0$.

Some equations can be solved by factoring. To do this, first add or subtract expressions to bring all the expressions to one side of the equation, with 0 on the other side. Then try to factor the nonzero side into a product of expressions. Each of the factors can be set equal to 0, yielding several simpler equations that possibly can be solved. The solutions of the simpler equations will be solutions of the factored equation. As an example, consider the equation $x^2 - 7x = -12$:

$x^2 - 7x + 12 = 0$ (taking all terms on one side and putting the expression equal to zero)

Now try to break b into two parts, such that the sum of those two parts = 'b' and the product is equal to the product of 'a' and 'c'.

$x^2 - 4x - 3x + 12 = 0 \Rightarrow x(x-4) - 3(x-4) = 0 \Rightarrow (x-3)(x-4) = 0$.

Putting these separately equal to 0 $\Rightarrow x - 3 = 0$, $x = 3$ and $x - 4 = 0$, $x = 4$.

Thus the solutions of the equation are 3 and 4.

The solutions of an equation are also called the roots of the equation.

A quadratic equation has at most two real roots and may have just one or even no real root. For example, the equation $x^2 - 6x + 9 = 0$ can be expressed as $(x-3)^2 = 0$, or $(x-3)(x-3) = 0$; thus the only root is 3. The equation $x^2 + 4 = 0$ has no real root; since the square of any real number is greater than or equal to zero, $x^2 + 4$ must be greater than zero.

An expression of the form $a^2 - b^2$ can be factored as $(a-b)(a+b)$.

For example, the quadratic equation $9x^2 - 25 = 0$ can be solved as follows.

$(3x - 5)(3x + 5) = 0$

$3x - 5 = 0$ or $3x + 5 = 0$

$x = 5/3$ or $x = -5/3$

Solving Equations by using the Quadratic Formula

If a quadratic expression is not easily factored, then its roots can always be found using the quadratic formula: If $ax^2 + bx + c = 0$ ($a \neq 0$), then the roots are

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

These are two distinct real numbers unless $b^2 - 4ac < 0$.

If $b^2 - 4ac = 0$; then these two expressions for x are equal to $-b/2a$ and the equation has only one root.

If $(b^2 - 4ac) < 0$, then $\sqrt{b^2-4ac}$ is not a real number and the equation has no real roots.

To solve the quadratic equation $x^2 - 7x + 8 = 0$ using the above formula, note that $a = 1$, $b = -7$, and $c = 8$, and hence the roots are

$$x = \frac{-7+\sqrt{7^2-4\times1\times8}}{2\times1} = \frac{7+\sqrt{17}}{2} = 5.6 \text{ approx. and } x = \frac{7-\sqrt{17}}{2} = 1.4 \text{ approx.}$$

$b^2 - 4ac$ is called the discriminant and is denoted by the symbol Δ or is represented by the letter D. Following are some of the important points relating to the discriminant and its relation with the nature of the roots.

- If $\Delta > 0$, then both the roots will be real and unequal and the value of roots will be $\frac{-b\pm\sqrt{\Delta}}{2a}$. If Δ is a perfect square, then roots are rational otherwise they are irrational.
- If $\Delta = 0$, then roots are real, equal and rational. In this case the value of roots will be $-b/2a$.
- If $\Delta < 0$, then roots will be imaginary, unequal and conjugates of each other.
- If α and β are the roots of the equation $ax^2 + bx + c = 0$, then sum of the roots i.e. $\alpha + \beta = -b/a$.
- If α and β are the roots of the equation $ax^2 + bx + c = 0$, then product of the roots i.e. $\alpha\beta = c/a$.
- If α and β, the two roots of a quadratic equation is given, then the equation will be $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

  The equation is x^2 – (sum of roots)x + product of roots = 0

These were some very important points relating to the quadratic equations. The following are some properties regarding the roots of the equation.

- If in the equation $b = 0$, then roots are equal in magnitude, but opposite in sign.
- If $a = c$, then roots are reciprocal of each other.
- If $c = 0$, then one of the roots will be zero.
- If one root of a quadratic equation be a complex number, the other root must be its conjugate complex number i.e. $\alpha = j + \sqrt{-k}$, then $\beta = j - \sqrt{-k} \Rightarrow \alpha = j + ik$ and $\beta = j - ik$

Progressions

Arithmetic Progression An arithmetic progression is a sequence of numbers in which each term is derived from the preceding term by adding or subtracting a fixed number called the common difference. Example, the sequence 9,6,3,0,-3,.... is an arithmetic progression with –3 as the common difference. The progression –3, 0, 3, 6, 9 is an Arithmetic Progression (AP) with 3 as the common difference.

- The general form of an Arithmetic Progression is $a, a + d, a + 2d, a + 3d$ and so on. Thus nth term of an AP series is $T_n = a + (n - 1)d$. Where $T_n = n^{th}$ term and a = first term.

  d = common difference = $T_m - T_{m-1}$.
- Sometimes the last term is given and either 'd' is asked or 'a' is asked. Then formula becomes $l = a + (n - 1)d$
- The formula, to find the sum of first n terms of an AP ; $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$
- The sum of n terms is also equal to the formula $S_n = \frac{n}{2}(a + l)$ where l is the last term.
- When three quantities are in AP, the middle one is called as the arithmetic mean of the other two. If a, b and c are three terms in AP then $b = \frac{a+c}{2}$.

Geometric Progression A geometric progression is a sequence in which each term is derived by multiplying or dividing the preceding term by a fixed number called the common ratio. The sequence 4, -2 , 1, $-\frac{1}{2}$,... is a Geometric Progression (GP) for which ($-\frac{1}{2}$) is the common ratio.

- The general form of a GP is a, ar, ar^2, ar^3 and so on. Thus nth term of a GP series is $T_n = ar^{n-1}$, where a = first term and r = common ratio = T_m/T_{m-1}.
- The formula applied to calculate sum of first n terms of a GP

  $S_n = \frac{a(r^n - 1)}{r - 1} \text{ where} \rightarrow r > 1$ and $S_n = \frac{a(1 - r^n)}{1 - r} \text{ where} \Rightarrow r < 1$

- When three quantities are in GP, the middle one is called as the geometric mean of the other two. If *a, b* and *c* are three quantities in GP and *b* is the geometric mean of *a* and c i.e. $b = \sqrt{ac}$
- The sum of infinite terms of a GP series $S_\infty = \frac{a}{1-r}$

Harmonic Progression A series of terms is known as a HP series when their reciprocals are in arithmetic progression. E.g. $1/a$, $\frac{1}{a+d}$, $\frac{1}{a+2d}$ and so on are in HP because *a, a + d, a +* 2*d* are in AP.

- The n^{th} term of a HP series is $T_n = \frac{1}{a+(n-1)d}$
- In order to solve a problem on Harmonic Progression, one should make the corresponding AP series and then solve the problem.
- If three terms *a, b, c* are in HP then $b = \frac{2ac}{a+c}$.

Percentages

Definition: A decimal fraction is one in which the denominator of the fraction is a power of 10 i.e. 10, 100, 1000 etc. That decimal fraction which has 100 as its denominator is known as Percentage. The numerator of such a fraction is known as Rate Per Cent.

15 % and $15/100$ mean one and the same quantity.

Any number written in the form of a fraction with 100 as the denominator is a percentage.

e.g. 13 = $1300/100$ = 1300 %,

$3/5$ = $60/100$ = 60 %,

$62.5/100$ = 62.5 %

Alternatively, X % of a number $Y = \frac{X \times Y}{100}$

e.g. 16 $2/3$ % of 300 = $50/3 \times 300/100$ = 50

A. ***Conversion from a Fraction to Percent and vice versa***

1. ***Fraction to Percent:*** Multiply the fraction by 100 to convert it into a percent.
   e.g. 0.2 = 0.2 $\times$ 100 = 20 %
   $3/8 = 3/8 \times 100$ = 37.5 %
2. ***Percent to Fraction:*** Reversing the earlier operation will convert a percent to a fraction - i.e. divide the percent by 100.
   e.g. 40 % = $40/100$ = 0.4,
   55 % = $55/100$ = 0.55 = 55 : 100

B. ***Percentage Increase or Decrease of a Quantity***:

Here, one point is to be noted, that the increase or the decrease is always on the original quantity. If the increase or decrease is given in absolute and the % age increase or decrease is to be calculated, then the following formula is applied to do so.

% increase /decrease = 100 × Quantity increase or decrease / original quantity

The point worth remembering is that the denominator is the *ORIGINAL QUANTITY*

e.g. the salary of a man goes up from Rs 100 to Rs 125. What is the percentage increase in his salary

Increase = 125 – 100 = Rs. 25.

$\therefore$ % increase = $25/100 \times 100\ \% = 25\ \%$

Alternatively, if the salary of the same man had been reduced from Rs. 125 to Rs 100, what is the percentage decrease in his salary?

Decrease = 125 – 100 = Rs. 25

$\therefore$ % decrease = $25/125 \times 100\ \% = 20\ \%$.

Note that for the same quantity of increase or decrease the % increase and % decrease have two different answers. The change in the denominator – which is the original value changes in the above two situations and hence the difference.

C. ***To Increase a Number by x %:***

If a number is increased by 10 %, then it becomes 1.1 times of itself.
If a number is increased by 20 %, then it becomes 1.2 times of itself.
If a number is increased by 30 %, then it becomes 1.3 times of itself.
If a number is increased by 40 %, then it becomes 1.4 times of itself.

D. ***To Decrease a Number by x %:***

If a number is decreased by 10 %, then it becomes 0.90 times of itself.
If a number is decreased by 20 %, then it becomes 0.80 times of itself.
If a number is decreased by 30 %, then it becomes 0.70 times of itself.
If a number is decreased by 40 %, then it becomes 0.60 times of itself.

Simple Interest

"40 % return is assured. You'll surely get back 40 % of your deposit after a year."

If I borrow money from you for a certain time period, then at the end of the time period, I return not only the borrowed money but also some additional money. This additional money that a borrower pays is called interest. The actual borrowed money is called Principal. The interest is usually calculated as a percentage of the principal and this is called the interest rate.
There is a well-accepted norm about the interest rate. It is always assumed to be per annum, i.e. for a period of one year, unless stated otherwise.

Interest can be computed in two basic ways. The first way, with simple annual interest, the interest computed on the principal only and is equal to (principle) × (rate) × (time)/100.
Where rate is taken as percent per annum and time is taken in years. Sometimes the interest is given and time or one of other two items is missing. Then the formula for calculating the time becomes (interest × 100)/(principal × rate). And similarly the rate and principal can be calculated.

Compound Interest

Going back to the case where I had borrowed money from you. Now at the end of the time period, I am not able to make the payment of interest. In the entire duration of the time period, the principal amount, which is used as the basis for calculation of the interest has remained constant. After I default, you add the unpaid interest to the principal (compounding matters for me) and we now have agreed to compound the interest.

We need not wait till the end of the time period but can compound the principal in between also. Assume that in the initial agreement that we had I was supposed to pay you every quarter of a year. I have of course defaulted on that, so you can keep on adding the interest amount every quarter and change the principal. There is a well-accepted norm about compounding. It is always assumed to be annually, i.e. after a period of one year, unless stated otherwise. As is the case in the compounded growth problem stated earlier in the chapter, the formula for the amount A at the end of T years for an initial amount of P and at an interest rate of r% per annum is

$$A = P\left(1 + \frac{R}{100}\right)^{T}$$

Profit & Loss

- ***Cost Price (CP):*** *The sum of money that is paid for the product. All overhead expenses such as transportation, taxes etc. are also included in the cost price.*
- ***Selling Price (SP)***: *The sum of money, which is finally received for the product.*

- ***Marked Price (MP)**: The price, which is listed or marked on the product, this is also known as printed price/quotation price/invoice price/catalogue price.*
- ***Profit:** There is gain in a transaction if the selling price is more than the cost price. The excess of the selling price to the cost price is the profit in the transaction.*

PROFIT = SELLING PRICE – COST PRICE

E.g. Let the cost price of a quintal of rice be Rs 1000 and the shopkeeper sells a kg of rice for Rs 12.5, then

Cost price = Rs 1000 / quintal = Rs 10/kg as 1 quintal = 100 kg

Selling price = Rs 12.5 / kg.

$\therefore$ Profit = Rs 12.5 – Rs. 10 = Rs 2.5 / kg or Rs 250 per quintal

- ***Loss:** When the selling price is less than the cost price there is loss in the transaction. The excess of cost price over the selling price is the loss in the transaction.*

LOSS = COST PRICE – SELLING PRICE

E.g. The cost price of a score of mangoes is Rs. 220. The fruit vendor retails each mango for Rs. 10, then

Cost price = Rs. 220 / score = Rs. 11 / mango

(1 score = 20 nos.)

Selling price = Rs. 10 / mango

$\therefore$ Loss = Rs. 11 – Rs. 10 = Re. 1 per mango

***Note**: Profit and loss percentage is always calculated on cost price, unless otherwise specified.*

- ***% Profit:***

$$\%\,\mathrm{Pr}\,ofit = 100 \times \frac{Profit}{Costprice}$$

- ***% Loss:***

$$\%Loss = 100 \times \frac{Loss}{Costprice}$$

- ***Equal % profit & loss on the same selling price of two articles:***

If two items are sold each at Rs *X*, one at a gain of *p* % and the other at a loss of *p* %, then the two transactions have resulted in an overall loss of $\frac{p^2}{100}$%

The absolute value of the loss = Rs $\frac{2 \cdot p^2 \cdot X}{100^2 - p^2}$

- ***Equal % profit & loss on the same cost price of two articles:***

If the cost price of two items are X, and one is sold at a profit of p % and the other at a loss of p %, then the two transactions have resulted in no gain or no loss.

- ***Trade Discount:*** To attract customers it is a common practice to announce discount on the marked price of an article.

  Note: *The discount is always taken as a % of the marked price only unless otherwise specified.*

  E.g. suppose the list price of an article be Rs. 450. A discount of 5 % on its list price is announced.

  Therefore, the new selling price = $\frac{95}{100} \times 450$ = Rs. 422.5

- ***Cash Discount:***

  In addition to trade discount, the manufacturer may offer an additional discount called the Cash Discount if the buyer makes full payment within a certain specified time.

  Cash Discount is usually offered on the net price (the price after subtracting discount from the marked price).

  Therefore, Cash Price = Net Price - Cash Discount

  Note: *Cash discount is always calculated on net price, unless otherwise specified.*

- ***Wrong Weight****: When a tradesman professes to sell at cost price, but uses a false weight, then the percentage profit earned*

  $$= \frac{100 \times \text{Error}}{\text{True Weight - Error}}$$

- ***Successive Discounts****: When a tradesman offers more than one discount to the customer, then sometimes you need to calculate the single discount, which is equal to the two discounts given. There you can apply the method of decimals learned in the concepts of percentages.*

  e.g. a tradesman offers two successive discounts of 20 % and 10 %, which single discount is equal to these two successive discounts.

  You can apply the principle, that after the first discount of 20 % the remaining price is 0.8 and after the second discount of 10 %, the remaining part is 0.9. Net the remaining part is $0.8 \times 0.9 = 0.72 \Rightarrow$ the discount is $1 - 0.72 = 0.28$ i.e. 28 %.

- *Or a straight method can be applied for two discounts.*

  Single discount, which is equal to two successive discounts of *m* % and *n* % = $(m + n + \frac{mn}{100})\%$

- *When the SP of x articles is equal to CP of y articles. What is the profit percent earned?*

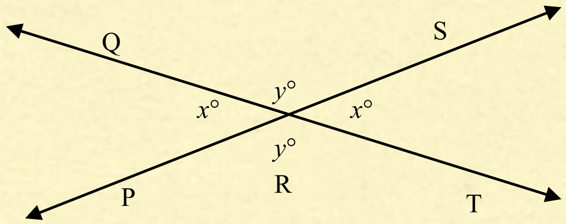
  $$\text{Profit percent} = \frac{100 \times \text{difference in x and y}}{\text{X}}.$$

Geometry (Lines and Angles)

Intersecting Lines And Angles: If two lines intersect, the opposite angles are called vertical angles and have the same measure. In the figure given below

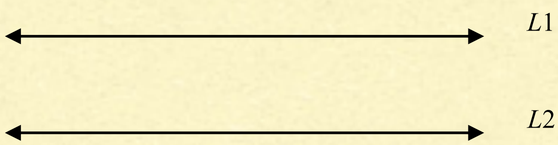
Also, $x + y = 180°$ since PRS is a straight line.

$\angle$PRQ and $\angle$ SRT are vertical angles and $\angle$QRS and $\angle$PRT are vertical angles.



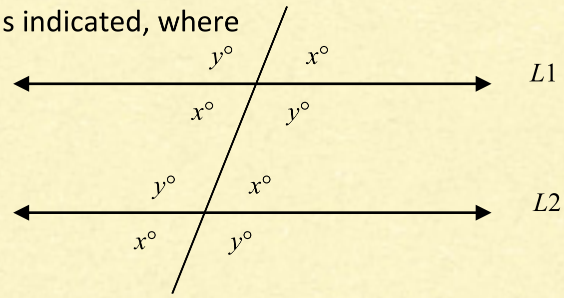
Parallel Lines:

If two lines that are in the same plane do not intersect, the two lines are parallel. In the figure given below,



lines $L1$, and $L2$ are parallel, denoted by $Ll \mid\mid L2$.

If two parallel lines are intersected by a third line, as shown below, then the angle measures are related as indicated, where



$x + y = 180°$.

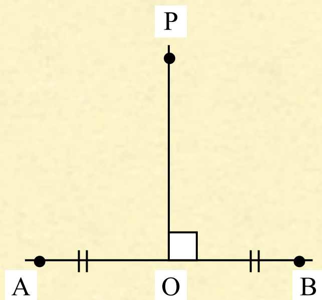
Some important points:

(i) Three or more points are said to be collinear if they lie on a line, otherwise they are said to be non-collinear.

(ii) Two or more lines are said to be coplanar if they lie in the same plane, otherwise they are said to be non-coplanar.

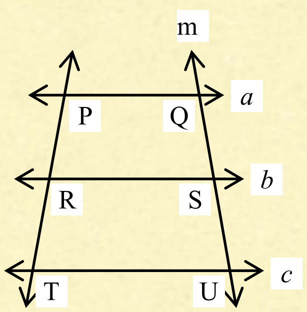
(iii) A line, which is perpendicular to a line segment i.e., intersects at 90° and passes through the midpoint of the segment is called he perpendicular bisector of the segment.

(iv) Every point on the perpendicular bisector of a segment is equidistant from the two endpoints of the segment.
Conversely, if any point is equidistant from the two endpoints of the segment, then it must lie on the perpendicular bisector of the segment.
If PO is the perpendicular bisector of segment AB, then, AP = PB.
Also, if AP = PB, then P lies on the perpendicular bisector of segment AB.



(v) The ratio of intercepts made by three parallel lines on a transversal is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

If line $a \parallel$ line $b \parallel$ line c and line ℓ and line m are two transversals, then $\frac{PR}{RT} = \frac{QS}{SU}$.



Types of Angles:

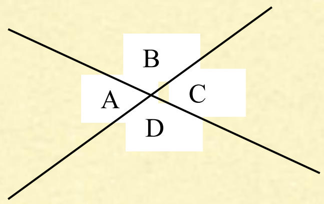
(i) An angle greater than 180°, but less than 360° is called a reflex angle.

(ii) Two angles whose sum is 90° are called complementary angles.

(iii) Two angles having a sum of 180° are called supplementary angles.

(iv) When two lines intersect, two pairs of vertically opposite angles are equal. The sum of 2 adjacent angles is 180°.

As given in the above diagram $\angle A = \angle C$ & $\angle B = \angle D$.



Secondly $\angle A + \angle B = 180°$ & $\angle C + \angle D = 180°$.

Two lines are parallel to each other if

- They are parallel to a 3rd line.
- They are opposite sides of a rectangle/ square/ rhombus/ parallelogram.
- If they are perpendicular to a 3rd line.
- If one of them is a side of the triangle & other joins the midpoints of the remaining two sides.
- If one of them is a side of a triangle & other divides other 2 sides proportionately.

Two lines are perpendicular to each other if

- They are arms of a right-angle triangle.
- If the adjacent angles formed by them are equal and supplementary.
- They are adjacent sides of a rectangle or a square.
- If they are diagonals of a rhombus.
- If one of them is a tangent & other is radius of the circle through the point of contact.
- If the sum of their squares is equal to the square of line joining their ends.

Two angles are said to be equal if

- They are vertically opposite angles.
- Their arms are parallel to each other.
- They are the corresponding angles of two congruent triangles.
- They are the opposite angles of a parallelogram.
- They are the angles of an equilateral triangle.

- They are the angles of a regular polygon.
- They are in same segment of a circle.
- One of them lies between a tangent & a chord thorough the point of contact & other is in the alternate segment, in a circle.

Two sides are equal to each other if

- They are corresponding sides of two congruent triangles.
- They are sides of an equilateral triangle.
- They are opposite sides of a parallelogram.
- They are the sides of a regular polygon.
- They are radii of the same circle.
- They are chords equidistant from centre of circle.
- They are tangents to a circle from an external point.

1. An angle is twice its complement. Find the angle.

Sol: If the complement is $x°$, then angle = $2x°$.

So $2x + x = 90° \therefore x = 30°$

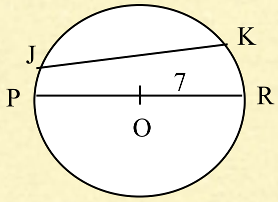
$\therefore$ The angle is $2 \times 30 = 60°$.

Circles

A circle is a set of points in a plane that are all located at the same distance from a fixed point (the center of the circle).

A **chord** of a circle is a line segment that has its endpoints on the circle. A chord that passes through the center of the circle is a diameter of the circle. A radius of a circle is a segment from the center of the circle to a point on the circle. The words "diameter" and "radius" are also used to refer to the lengths of these segments.

The **circumference** of a circle is the distance around the circle. If r is the radius of the circle, then the circumference is equal to $2\pi r$, where π approximately 3.14.The area of a circle of radius r is is equal to πr^2.



In the circle above, O is the center of the circle and JK and PR are chords. PR is a diameter and OR is a radius. If OR = 7, then the circumference of the circle is 2π (7) = 14π, and the area of the circle is $\pi(7)^2 = 49\pi$.

<u>Arc</u> An arc is a part of a circle. A minor arc is an arc less than the semicircle and a major arc is an arc greater than a semicircle.

<u>Central Angle</u> An angle in the plane of the circle with its vertex at the centre is called a central angle.

<u>Measure of an arc</u>

(i) The measure of a semicircle is 180°.
(ii) The measure of a minor arc is equal to the measure of its central angle.
(iii) The measure of a major arc = 360° - (measure of corresponding minor arc).

Some Important Properties

(i) The perpendicular from the centre of a circle to a chord of the circle bisects the chord.

Conversely, the line joining the centre of the circle and the midpoint of a chord is perpendicular to the chord.

(ii) Equal chords of a circle or congruent circles are equidistant from the centre.

Conversely, two chords of a circle or congruent circles that are equidistant from the centre are equal.

(iii) In a circle or congruent circles, equal chords subtend equal angles at the centre.

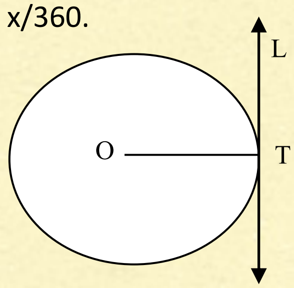
Conversely, chords, which subtend equal angles at the centre of the same or congruent circles, are equal.

(iv) If the two circles touch each other externally, distance between their centres = sum of their radii.

If the two circles touch each other internally, distance between their centres = difference of their radii.

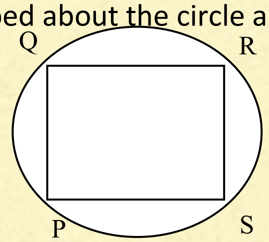
(a) For the two circles with centres A and B, PQ and RS are the direct common tangents, and CD and EF are the transverse common tangents. (Only two of both transverse common tangents and direct common tangents are possible.)

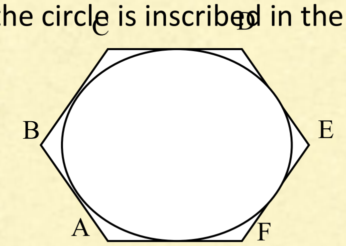
- Length of arc RS = $2\pi r \times x/360$. ∵ the complete circle is having 360 degrees & any part of that shall be equal to x/360.
- Area of Sector ORS = $\pi r^2 \times x/360$. ∵ the complete circle is having 360 degrees & any part of that shall be equal to x/360.



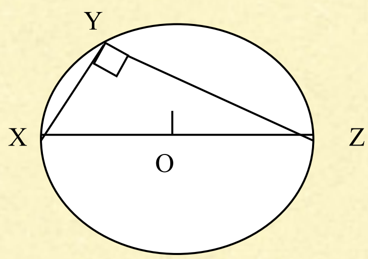
A line that has exactly one point in common with a circle is said to be tangent to the circle, and that common point is called the point of contact. A radius or diameter with an endpoint at the point of contact is perpendicular to the tangent line, and, conversely, a line that is perpendicular to a diameter at one of its endpoints is tangent to the circle at that endpoint.

The line L above is tangent to the circle and radius OT is perpendicular to L. If each vertex of a polygon lies on a circle, then the polygon is inscribed in a circle and the circle is circumscribed about the polygon. If each side of a polygon is tangent to a circle, then the polygon is circumscribed about the circle and the circle is inscribed in the polygon.





In the figure above, quadrilateral PQRS is inscribed in a circle and hexagon ABCDEF is circumscribed about a circle.



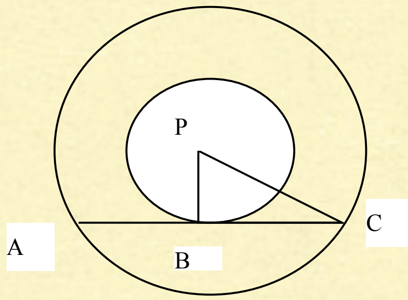
If a triangle is inscribed in a circle so that one of its sides is a diameter of the circle, then the triangle is a right triangle.

In the circle above, XZ is a diameter and the measure of angle XYZ is $90°$.

3. Two concentric circles with centre P have radii 6.5 cm and 3.3 cm. Through a point A of the larger circle, a tangent is drawn to the smaller circle touching it at B. Find AC.

Sol: $\angle PBC = 90°$ (A tangent is perpendicular to the radius at the point of contact)

So $(6.5)^2 = (3.3)^2 + (BC)^2$. So BC = 5.6. Hence AC = $2 \times 5.6 = 11.2$ cm.



Triangles

The plane figure bounded by the union of three lines, which join three non collinear points, is called a triangle.

There are several special types of triangles with important properties. But one property that all triangles share is that the sum of the lengths of any two of the sides is greater than the length of the third side or difference of the lengths of any two of the sides is less than the length of the third side, as illustrated below.

Types of Triangles

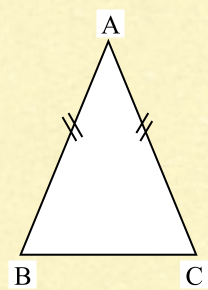
With regard to their sides, triangles are of three types:

(i) Scalene Triangle: A triangle in which none of the three sides is equal is called a scalene triangle.

(ii) Isosceles Triangle: A triangle in which at least two sides are equal is called an isosceles triangle. In an isosceles triangle, the angles opposite to the congruent sides are congruent.

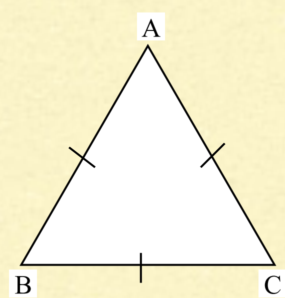
Conversely, if two angles of a triangle are congruent, then the sides opposite to them are congruent.

In ΔABC, $AB = AC$, $\angle ABC = \angle ACB$



(iii) Equilateral Triangle: A triangle in which all the three sides are equal is called an equilateral triangle. In an equilateral triangle, all the angles are congruent and equal to 60°.

In ΔABC, $AB = BC = AC$. $\angle ABC = \angle BCA = \angle CAB = 60^\circ$

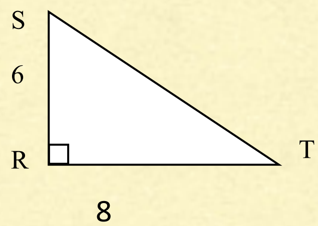


With regard to their angles, triangles are of five types:

(i) Acute triangle: If all the three angles of a triangle are acute i.e., less than 90°, then the triangle is an acute-angled triangle.

(ii) Obtuse triangle: If any one angle of a triangle is obtuse i.e., greater than 90°, then the triangle is an obtuse-angled triangle. The other two angles of the obtuse triangle will be acute.

(iii) Right Triangle: A triangle that has a right angle is a right triangle. In a right triangle, the side opposite the right angle is the hypotenuse, and the other two sides are the legs. An important theorem concerning right triangles is the Pythagorean theorem, which states: In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



In the figure above, ΔRST is a right triangle, so $(RS)^2 + (RT)^2 = (ST)^2$. Here, RS = 6 and RT = 8, so ST = 10, since $6^2 + 8^2 = 36 + 64 = 100 = (ST)^2$ and ST = $\sqrt{100}$. Any triangle in which the lengths of the sides are in the ratio 3:4 is a right triangle. In general, if a, b, and c are the lengths of the sides of a triangle in which $a^2 + b^2 = c^2$, then the triangle is a right triangle. There are some standard Pythagorean triplets, which are repeatedly used in the questions. It is better to remember these triplets by heart.

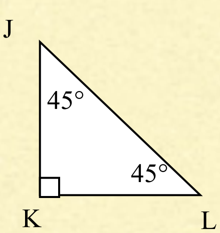
♠ 3, 4, 5 ♠ 5, 12, 13 ♠ 7, 24, 25 ♠ 8, 15, 17

♠ 9, 40, 41 ♠ 11, 60, 61 ♠ 12, 35, 37 ♠ 16, 63, 65

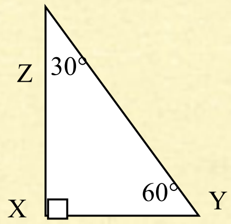
♠ 20, 21, 29 ♠ 28, 45, 53.

Any multiple of these triplets will also be a triplet i.e. when we say 3, 4, 5 is a triplet, if we multiply all the numbers by 2, it will also be a triplet i.e. 6, 8, 10 will also be a triplet.

(iv) 45°- 45° - 90° Triangle: If the angles of a triangle are 45°, 45° and 90°, then the perpendicular sides are $1/\sqrt{2}$ times the hypotenuse. In a 45°- 45°- 90° triangle, the lengths of the sides are in the ratio $1 : 1 : \sqrt{2}$. For example, in ΔJKL, if JL = 2, then JK = $\sqrt{2}$ and KL = $\sqrt{2}$.



(v) 30°- 60° - 90° Triangle: In 30°- 60° - 90° triangle, the lengths of the sides are in the ratio 1: √3 : 2. For example, in ΔXYZ, if XZ = 3, then XY = 3√3 and YZ = 6. In short, the following formulas can be applied to calculate the two sides of a 30°- 60°-90° triangle, when the third side is given.



Side opposite to 30° = ½ of hypotenuse.

Side opposite to 60° = √3/2 of hypotenuse.

(vi) Area of a triangle = $\frac{1}{2} \times base \times height$ = $\sqrt{s(s-a)(s-b)(s-c)} = r \times s = \frac{abc}{4R}$ where, a, b and c are the sides of the triangle,
s = semi perimeter, r = in-radius, R = circum-radius.

Congruency of triangles: If the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle, then the two triangles are said to be congruent.

Two triangles are congruent if

- Two sides & the included angle of a triangle are respectively equal to two sides & included angle of other triangle (SAS).
- Two angles & one side of a triangle are respectively equal to two angles & the corresponding side of the other triangle (AAS).
- Three sides of a triangle are respectively congruent to three sides of the other triangle (SSS).
- One side & hypotenuse of a right-triangle are respectively congruent to one side & hypotenuse of other right triangle (RHS).

Similarity of triangles:

Two triangles are similar if they alike in shape only. The corresponding angles are congruent, but corresponding sides are only proportional. All congruent triangles are similar but all similar triangles are not necessarily congruent.

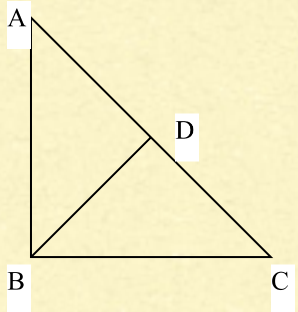
Two similar if triangles are

- Three sides of a triangle are proportional to the three sides of the other triangle (SSS).
- Two angles of a triangle are respectively equal to the two angles of the other triangle (AA).
- Two sides of a triangle are proportional to two sides of the other triangle & the included angles are equal (SAS).

Properties of similar triangles:

- If two triangles are similar, ratios of sides = ratio of heights = ratio of medians = ratio of angle bisectors = ratio of inradii = ratio of circumradii.
- Ratio of areas = $b_1h_1/b_2h_2 = (s_1)^2/(s_2)^2$, where b_1 & h_1 are the base & height of first triangle and b_2 & h_2 are the base & height of second triangle. s_1 & s_2 are the corresponding sides of first and second triangle respectively.
- The triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other.

  Δ DBA is similar to ΔDCB which is further similar to Δ BCA.
- The altitude from the vertex of the right angle to the hypotenuse is the geometric mean of the segments into which the hypotenuse is divided.
- I.e. $(DB)^2 = AD \times DC$

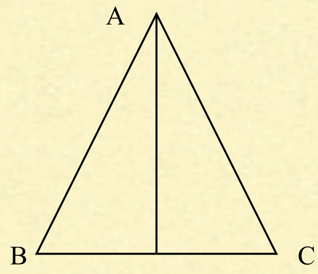


Some important theorems:

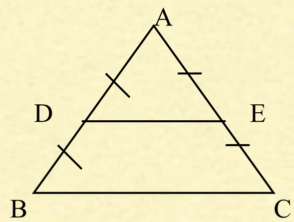
(i) The Angle Bisector Theorem: The angle bisector divides the opposite side in ratio of the lengths of its adjacent arms.

If AD is the angle bisector, then AB/AC = BD/DC.

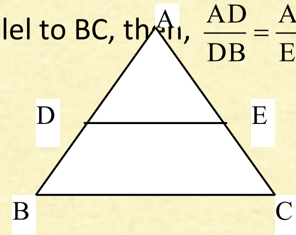
D



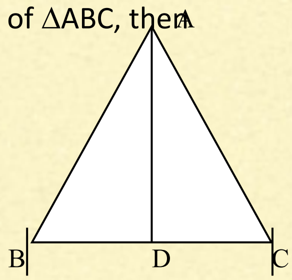
(ii) Midpoint Theorem: The segment joining the midpoints of any two sides of a triangle is parallel to the third side and is half of the third side.
If AD = DB, AE = EC, then DE is parallel to BC and
DE = $\frac{1}{2}$BC.



(iii) Basic Proportionality Theorem: If a line is drawn parallel to one side of a triangle and intersects the other sides in two distinct points, then the other sides are divided in the same ratio by it. If DE is parallel to BC, then, $\frac{AD}{DB} = \frac{AE}{EC}$

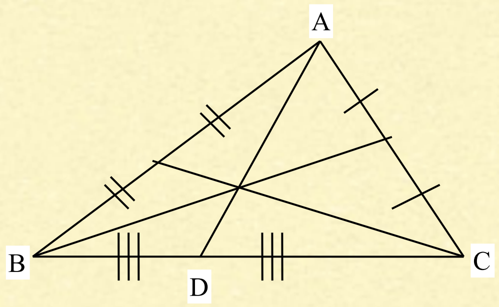


(iv) Appollonius Theorem: The sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and the square of half the third side. If AD is the median of ΔABC, then
$AB^2 + AC^2 = 2(AD^2 + DC^2)$



4. In a triangle ABC, AB = 9cm, BC = 10cm, AC = 13cm. Find the length of median AD. If G is the centroid, find GA and GD.

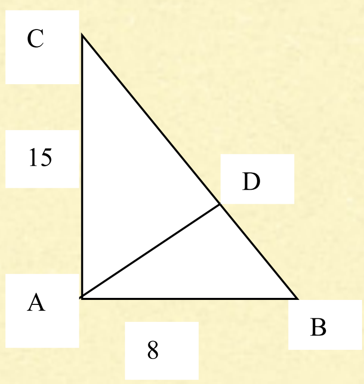
Sol: By Apollonius theorem, $AB^2 + AC^2 = 2 \times (AD)^2 + 2 \times (DC)^2 \therefore 81 + 169 = 2 \times (AD)^2 + 2 \times (5)^2$
$\therefore 250 = 2 \times (AD)^2 + 2 \times (5)^2 \therefore 125 = AD^2 + 25$ .....(Dividing by 2)
$\therefore 100 = AD^2 \therefore 10 = AD$ $\therefore$ median = 10 cm.
Since G divides AD in the ratio 2 : 1



So $GA = \frac{2}{3} \times AD = \frac{2}{3} \times 10 = \frac{20}{3}$ cm, $GD = \frac{1}{3} \times 10 = \frac{10}{3}$ cm.

5. ΔABC is right angled at A and AD is the altitude to BC. If AB = 8 cm and AC = 15 , find BC and altitude AD. If M is the midpoint of BC, find AM.

Sol: By the theorem of Pythagoras, $BC^2 = 8^2 + 15^2 = 64 + 225 = 289 \therefore BC = \sqrt{289} = 17$



Area of the triangle = $\frac{1}{2}\times$ Product of perpendicular sides $=\frac{1}{2}\times 8\times 15=60\,cm^2$.

Also area = $\frac{1}{2}\times BC\times AD=60=\frac{1}{2}\times 17\times AD=60 \;\; \therefore AD=\frac{120}{17}$ cm Again, AM is the median to the hypotenuse.

$\therefore AM=\frac{1}{2}\times \text{hypotenuse}=\frac{1}{2}\times 17=8.5$ cm.

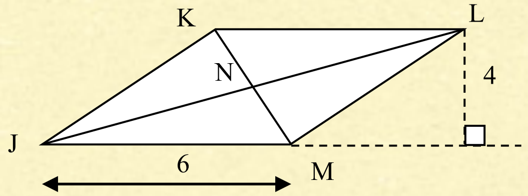
Quadrilaterals

A polygon with four sides is a quadrilateral. In a quadrilateral, sum of all angles is 360°. Area of a quadrilateral = ½ × one of the diagonals × sum of the perpendiculars drawn to that diagonal from the opposite vertices.

The different kinds of quadrilaterals are parallelogram, rectangle, square, rhombus, trapezium and kite.

Parallelogram:

A quadrilateral in which both pairs of opposite sides are parallel is a parallelogram. The opposite sides of a parallelogram also have equal length. In a parallelogram opposite sides are parallel and equal. Opposite angles are equal. Diagonals bisect each other. Sum of any 2 adjacent angles = 180°. A parallelogram inscribed in a circle is always a rectangle. Parallelogram circumscribed about a circle is always a Rhombus.



In parallelogram JKLM, JK || LM and JK = LM, KL || JM and KL = JM, The diagonals of a parallelogram bisect each other that is, KN = NM and JN = NL.

The area of a parallelogram is equal to: Base × height

The area of JKLM is equal to 4 x 6 = 24 cm. Every diagonal of a parallelogram divides it into two triangles of equal area.

Parallelograms that lie on the same base and between the same parallels are equal in area.

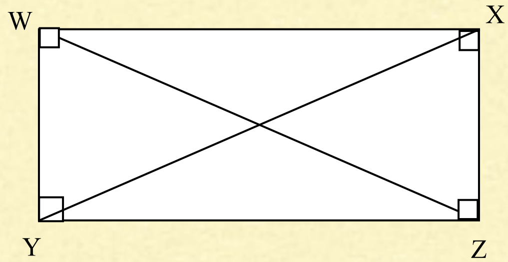
Rectangle:

A parallelogram with right angles is a rectangle. In a rectangle, each pair of opposite sides is parallel and equal. Diagonals are equal and bisect each other, but not at right angles. A parallelogram is a rectangle if its diagonals are equal.

Perimeter of rectangle = 2(L + B), where L = Length, B = Breadth

Area of rectangle = LB. Area is written in the square units of sides. $Diagonal^2 = L^2 + B^2$

The perimeter of WXYZ = 2[3 + 4] = 14 cm and the area of WXYZ is equal to 3 × 4 = 12 cm^2. The diagonals of a rectangle are equal; therefore WY = XZ = $\sqrt{(9 + 16)} = \sqrt{25}$ = 5 cm.



Square:

A rectangle with all sides equal is known as square. In a square, all 4 sides are equal. All the 4 angles are equal & each angle is equal to 90°. Diagonals are equal and bisect each other at right angles. The perimeter of a square is '4a' and the area of the square is 'a^2', where 'a' is the side of the square. Every square is a rhombus, rectangle and parallelogram.

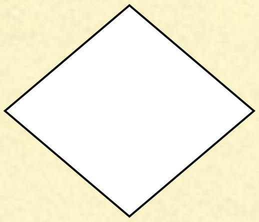
When a square is inscribed in a circle, the diagonal is equal to the diameter of the circle. When a circle is inscribed in a square, side of the square is equal to the diameter of the circle.

Rhombus:

In a rhombus all the sides are equal and all the angles are not equal. In a rhombus, the two pairs of opposite sides are parallel. Diagonals are not equal but they bisect each other at right angles.

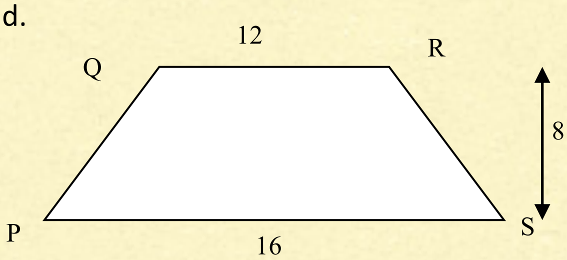
Opposite angles are equal.

Area = ½ d_1d_2, where d_1 & d_2 are two diagonals of a rhombus. $(\text{side})^2 = (\text{½ one diagonal})^2 + (\text{1/2 other diagonal})^2$
Every rectangle, square and rhombus is a parallelogram.



Trapezium:

A quadrilateral with two sides that are parallel but the other two sides are not parallel, as shown below is a trapezoid.



The area of trapezoid PQRS may be calculated as follows:

½ × sum of parallel sides × height = ½ × (QR + PS)(8) = ½ × (28 × 8) = 112 cm.

A trapezium inscribed in a circle is an isosceles trapezium. In an isosceles trapezium, the oblique sides (the sides which are not parallel) are equal. Angles made by each parallel side with the oblique side are equal.

Rectangular Solids and Cylinders

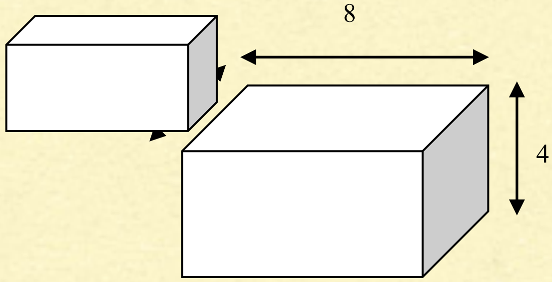
Solids: Solids are three – dimensional objects, bound by one or more surfaces. When plane surfaces bound a solid, they are called its faces. The lines of intersection of adjacent faces are called its edges. For any regular solid,

Number of faces + Number of vertices = Number of edges + 2. This formula is called Euler's formula.

Volume: Volume of a solid figure is the amount of space enclosed by its bounding surfaces. Volume is measured in cubic units.

Cuboid: A cuboid is a three-dimensional figure formed by six rectangular surfaces, as shown below. Each rectangular surface is a face. Each solid line segment is an edge, and each point at which the edges meet is a vertex. A rectangular solid has six faces, twelve edges, and eight vertices. Edges mean sides and vertices mean corners. Opposite faces are parallel rectangles that have the same dimensions.

The surface area of a rectangular solid is equal to the sum of the areas of all the faces. The volume is equal to (length) × (width) × (height); in other words, (area of base) × (height).



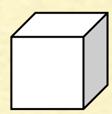
In the rectangular solid above, the dimensions are 3 cm, 4 cm, and 8 cm.

The surface area is equal to 2[(3 × 4) + (3 × 8) + (4 × 8)] = 136. The volume is equal to 3 × 4 × 8 = 96 cm^3.

Body diagonal of a cuboid = Length of the longest rod that can be kept inside a rectangular room is $\sqrt{L^2+B^2+H^2}$.

Cube: A rectangular solid in which all edges are of equal length is a cube. In a cube, just like cuboid, there are six faces, eight vertices & twelve edges.

Volume = a^3.



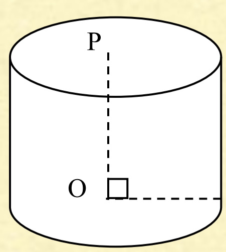
Surface Area = $6a^2$, where ‘a’ is the side of a cube.

Body Diagonal = Length of the longest rod inside a cubical room = $a\sqrt{3}$

Cylinder: The figure given below is a right circular cylinder. The two bases are circles of the same size with centers O and P, respectively, and altitude (height) OP is perpendicular to the bases.

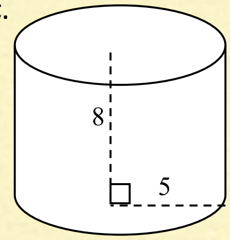
The **surface area** of a right circular cylinder with a base of **radius ‘r’** and **height ‘h’** is equal to $2(\pi r^2) + 2\pi rh$ (the sum of the areas of the two bases plus the area of the curved surface).

The **volume** of a cylinder is equal to $\pi r^2 h$, that is (area of base) x (height).



Volume of material of a hollow cylinder = $\pi (R^2 - r^2) h$, where R is the outer radius and r is the inner radius.

In the cylinder given below, the surface area is equal to $2(25\pi) + 2\pi(5)(8) = 130\pi$, and the volume is equal to $25\pi(8) = 200\pi$.



Cone: A cone is having one circle on one of its ending & rest is the curved circle part with a corner on the other end.
Volume = $1/3\ \pi r^2 h$. Surface Area (curved) = $\pi r l$, where l = slant height.
As per the Pythagoras theorem, $l^2 = r^2 + h^2$.
Surface Area (total) = $\pi r l + \pi r^2$.

Frustum of a cone: A frustum is the lower part of a cone, containing the base, when it is cut by a plane parallel to the base of the cone.

Slant height, L = $\sqrt{h^2 + (R - r)^2}$ Curved Surface area of cone = $\pi (R + r) L$.

Total surface area of frustum = Base area + Area of upper circle + Area of lateral surface = $\pi (R^2 + r^2 + RL + rL)$.

Volume of frustum = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$

Sphere: The set of all points in space, which are at a fixed distance from a fixed point, is called a sphere. The fixed point is the centre of the sphere and the fixed distance is the radius of the sphere.
Volume = $4/3\pi r^3$. Surface Area (curved and total) = $4\pi r^2$.

Hemisphere: A sphere cut by a plane passing through its centre forms two hemispheres. The upper surface of a hemisphere is a circular region.
Volume = $2/3\pi r^3$. Surface Area (curved) =$2\pi r^2$. Surface Area (Total) = $2\pi r^2 + \pi r^2 \Rightarrow 3\pi r^2$.

Spherical shell: If R and r are the outer and inner radius of a hollow sphere, then volume of material in a spherical shell = $4/3\pi (R^3 - r^3)$.

6. A rectangle 7 cm × 5 cm is rotated about its smaller edge as axis. Find the curved surface area and volume of solid generated.

Sol: Curved Surface Area = $2\pi rh = 2 \times 22/7 \times 7 \times 5 = 220$ sq. cm.
Volume of solid = $\pi r^2 h = 22/7 \times 7 \times 7 \times 5 = 770$ cu. cm.

7. The perpendicular sides of the base of a right triangular metallic prism are 6 cm and 8 cm. It weighs 810 g. Find its height if density of metal is 13.5 g/cc.

Sol: Volume = $\frac{Weight}{Density} = \frac{810}{13.5} = 60$ cc. Volume = 1/3 × Base area × height.

60 = 1/3 × ½ × 6 × 8 × height

$\Rightarrow$ height = 7.5 cm.

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