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Assignment 3.

Q1: Let $Q = I - \frac{2V \cdot V^T}{V^T \cdot V}$

(a). show that Q is symmetric

(b). show that Q is orthogonal.

(c). Let y be any orthogonal to V. and x be any vector. show that x - Qx is orthogonal to y.

⇒ (a). As $\left(\frac{2V \cdot V^T}{V^T \cdot V}\right)^T = \frac{(V^T V)^T}{(2VV^T)^T} = \frac{VV^T}{\frac{1}{2} V^T V} = \frac{2V \cdot V^T}{V^T V}$

equals to the origin matrix.

so $\frac{2VV^T}{V^T \cdot V}$ is symmetric.

since I is symmetric

so $I - \frac{2VV^T}{V^T V}$ is symmetric. which means Q is symmetric.

(b). ~~∵~~ since Q is symmetric

∴ $Q \cdot Q^T = Q \cdot Q$

$= I + \frac{4V \cdot V^T \cancel{\cdot V \cdot V^T}}{V^T \cancel{V} \cdot V^T \cdot V} - \frac{\cancel{4}V \cdot V^T}{V^T \cdot V}$

$= I + \frac{4VV^T}{V^T V} - \frac{4V \cdot V^T}{V^T V}$

$= I.$

∴ Q is orthogonal.

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(c). $x - Qx = I \cdot x - Qx = (I - Q)x = \frac{2V \cdot V^T}{V^T \cdot V} x.$

$\therefore$ when $\frac{2V \cdot V^T}{V^T \cdot V} x \cdot y.$

$\because$ x, y are all vectors and $[y, v] = 0$

$\therefore \frac{2V \cdot V^T}{V^T \cdot V} x \cdot y = \frac{2V \cdot V^T}{V^T \cdot V} y \cdot x = 0.$

$\therefore$ $x - Qx$ is orthogonal to y.