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Q3. (a). Prove $||AB||_p \leq ||A||_p ||B||_p$.

(b). Prove that $||A||_2 = \sigma_{max}$. the largest singular value of A.

(a). from the $||A||_p = \max\limits_{x \neq 0} \frac{||Ax||_p}{||x||_p}$ $p = 1, 2, 3 \cdots$

We can get $||A||_p ||x||_p \geq ||Ax||_p$.

$\therefore ||ABx||_p \leq ||A||_p ||Bx||_p$.

$\because ||Bx||_p \leq ||B||_p \cdot ||x||_p$.

$\therefore ||ABx||_p \leq ||A||_p ||B||_p ||x||_p$.

take $||x||_p$ to the left.

$$\frac{||ABx||_p}{||x||_p} \leq ||A||_p ||B||_p.$$

when $x \neq 0$. $\therefore ||AB||_p \leq \max\limits_{x \neq 0.} \frac{||ABx||_p}{||x||_p} \leq ||A||_p ||B||_p$

which equals to : $||AB||_p \leq ||A||_p \cdot ||B||_p$.

(b).

$A = U \Sigma V^T$ < define the $||x||_2 = 1$.

$\therefore AV = U\Sigma$

~~$\therefore ||A||$~~ $\therefore ||AV||_2 = ||U\Sigma||_2$

$\Rightarrow ||A||_2 ||V||_2 = ||U||_2 ||\Sigma||_2$.

since V and U are ~~orgo~~ orthogonal.

$||V||_2 = ||U||_2 = 1$.

$\therefore ||A||_2 = ||\Sigma||_2 = \max ||\Sigma||_2$.

Σ is consist of singular values ~~and~~ on it's diagonal

$\therefore \max ||\Sigma||_2 = \sigma_{max}$

$\therefore ||A||_2 = \sigma_{max}$