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Q2. Let $A=\begin{bmatrix}1 & 1 & 4\\ -1 & 2 & 2\end{bmatrix}$. Determine the 4 foundamental subspaces

Col(A). Col(A^T). Null(A). Null(A^T).

$\Rightarrow$. $A \overset{c}{\sim} \begin{bmatrix}1 & -1 & 2\\ 0 & 1 & 1\end{bmatrix}$. $\therefore$ col(A) $= \lambda_1\begin{bmatrix}1\\0\end{bmatrix}+\lambda_2\begin{bmatrix}-1\\1\end{bmatrix}+\lambda_3\begin{bmatrix}2\\1\end{bmatrix}$

$=\begin{bmatrix}\lambda_1-\lambda_2+2\lambda_3\\ \lambda_2+\lambda_3\end{bmatrix}$ = xy plane.

$(\lambda_1, \lambda_2, \lambda_3)\in R$.

$A\cdot x=0$. $\begin{bmatrix}1 & -1 & 2\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$.

$\begin{cases}x-y+2z=0\\ y+z=0\end{cases}$ $\begin{cases}x=-3z\\ y=-z\end{cases}$. $\therefore$ Null(A) $= k\begin{bmatrix}-3\\-1\\1\end{bmatrix}$. $(k\in R)$

$A^T=\begin{bmatrix}1 & -1\\ 1 & 2\\ 4 & 2\end{bmatrix}\sim\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}$. $\therefore$ col(A^T) $=\lambda_1\begin{bmatrix}1\\0\\0\end{bmatrix}+\lambda_2\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}\lambda_1\\ \lambda_2\\ 0\end{bmatrix}$ = xy plane.

$(\lambda_1, \lambda_2)\in R$.

$A^T x=0$. $\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=0$. $\begin{cases}x=0\\ y=0\end{cases}$ $\therefore$ Null(A^T) $=\begin{bmatrix}0\\0\end{bmatrix}$.