fdcoexist equation only

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Basic Equation

This document resumes the equation used in our model and details the different processes. Our coexistence model is developed following this equation (Beverton-Holt equation):

$$N_{t+1,i,x} = \frac{R_{i,x} \times N_{t,i,x}}{1 + A \times \alpha_i} \tag{1}$$

with

$$\alpha_i = \sum_{j=1, j \neq i}^{S} N_{t,j,x} \times (1 - \delta_{ij}) \tag{2}$$

$$R_{i,x} = k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \tag{3}$$

If we replace α_i and $R_{i,x}$ in the first equation it gives:

$$N_{t+1,i,x} = \frac{k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \times N_{t,i,x}}{1 + A \times \sum_{j=1, j \neq i}^{S} N_{t,j,x} \times (1 - \delta_{ij})} \tag{4}$$

Detailing the computation of δ_{ij}

δ_{ij} is the functional dissimilarity coefficient between species i and species j. It reflects the fact that the more two species are dissimilar the less competition they experience from each other.

First we can compute δ_{ij} as the euclidean distance between species using multiple traits:

$$\delta_{ij} = \sqrt{\sum_{c=1}^{T_c} (\text{trait}_{i,c} - \text{trait}_{j,c})^2}, \tag{5}$$

with T_c the number of traits contributing to competition, $\text{trait}_{i,c}$ trait c of species i and $\text{trait}_{j,c}$ trait c of species j. If want to take the contribution of traits to competition we have to modify the trait values as follow:

$$\delta_{ij} = \sqrt{\sum_{c=1}^{T_c} [\sqrt{w_c}(\text{trait}_{i,c} - \text{trait}_{j,c})]^2} \tag{6}$$

$$\delta_{ij} = \sqrt{\sum_{c=1}^{T_c} w_c \times (\text{trait}_{i,c} - \text{trait}_{j,c})^2}, \tag{7}$$

with w_c the contribution of trait c in limiting similarity.

Adding intra-specific competition

The equation above only considers inter-specific competition when $j \neq i$ in the sum. We can however add intra-specific competition when $j = i$. Each site has a species-specific carrying capacity K as the number of individuals approaches this carrying capacity the intra-specific competition increases:

$$\alpha_{ii} = B \times N_{t,i,x} \tag{8}$$

Thus the equation becomes:

$$N_{t+1,i,x} = \frac{k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \times N_{t,i,x}}{1 + A\left(\sum_{j=1,\ j\neq i}^{S} N_{t,j,x}(1-\delta_{ij}) + \frac{B}{A} \times N_{t,i,x}\right)} \tag{9}$$

with A the coefficient scaling inter-specific competition and B the one for intra-specific competition.

Multi-trait growth term

Because several traits participate to the growth term depending on their contribution we can rewrite the growth term as:

$$R_{i,x} = \sum_{g=1}^{T} w_g \times k \times \exp\left(-\frac{(\text{trait}_{g,i} - \text{env}_x)^2}{2 \times \text{width}^2}\right) \tag{10}$$

with g the trait number, $0 \leq w_g \leq 1$ the contribution of this trait to growth (and $\sum_{g=1}^{T} w_g = 1$), $\text{trait}_{g,i}$ the trait number g of species i.

Adding Hierarchical competition

If we add hierarchical competition, the species with the largest trait has an increased growth. We can include this as a "bonus" term in the computation of the growth term $R_{i,x}$ such as:

$$R_{i,x} = R_{i,x,\text{env}} + R_{i,x,\text{hierarch.}} \tag{11}$$

$$R_{i,x,\text{hierarch.}} = \sum_{c=1}^{T_c} w_c \times H \times \frac{t_i}{\max(t_i)} \tag{12}$$

$$R_{i,x} = k\left[\sum_{g=1}^{T} w_g \times \exp\left(-\frac{(\text{trait}_{g,i} - \text{env}_x)^2}{2 \times \text{width}^2}\right) + \frac{H}{k}\sum_{c=1}^{T_c} w_c \frac{\text{trait}_i}{\max(\text{trait}_i)}\right], \tag{13}$$

withw_g the weight of traits contributing to growth, w_c the weight of traits contributing to competition (as many as T_c).

So the final equation looks like the following:

$$N_{t+1,i,x} = \frac{k\left[\sum_{g=1}^{T} w_g \times \exp\left(-\frac{(\text{trait}_{i,g} - \text{env}_x)^2}{2 \times \text{width}^2}\right) + \frac{H}{k}\sum_{c=1}^{T_c} w_c \frac{\text{trait}_{i,c}}{\max(\text{trait}_{i,c})}\right] \times N_{t,i,x}}{1 + A\left(\sum_{j=1,\ j\neq i}^{S} N_{t,j,x}(1-\delta_{ij}) + \frac{B}{A} \times N_{t,i,x}\right)} \tag{14}$$

If we consider that traits contributing to limiting similarity may not contribute to hierarchical competition we can develop the total equation as follow:

$$N_{t+1,i,x} = \frac{k\left[\sum_{g=1}^{T_g} w_g \times \exp\left(-\frac{(\text{trait}_{i,g} - \text{env}_x)^2}{2 \times \text{width}^2}\right) + \frac{H}{k}\sum_{h=1}^{T_h} w_h \frac{\text{trait}_{i,h}}{\max(\text{trait}_{i,h})}\right] \times N_{t,i,x}}{1 + A\left[\sum_{j=1,\ j\neq i}^{S} N_{t,j,x}\left(1 - \sqrt{\sum_{c=1}^{T_c} w_c \times (\text{trait}_{i,c} - \text{trait}_{j,c})^2}\right) + \frac{B}{A} \times N_{t,i,x}\right]}, \tag{15}$$

with w_g, w_h and w_c, the contribution respectively of trait to growth, hierarchical competition and limiting similarity with T_g, T_h and T_c the number of traits respectively contributing to growth, hierarchical competition and limiting similarity.