# fdcoexist equation only

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This document presents the relationships between functional traits of species and an environmental gradient.

Our coexistence model is developed following this equation:

$$N_{t+1,i,x} = \frac{R_{i,x} \times N_{t,i,x}}{1 + A \times \alpha_i} \tag{1}$$

with

$$\alpha_i = \sum_{j=1, j\neq i}^{S} N_{t,j,x} \times (1 - \delta_{ij}) \tag{2}$$

$$R_{i,x} = k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \tag{3}$$

If we replace $\alpha_i$ and $R_{i,x}$ in the first equation it gives:

$$N_{t+1,i,x} = \frac{k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \times N_{t,i,x}}{1 + A \times \sum_{j=1, j\neq i}^{S} N_{t,j,x} \times (1 - \delta_{ij})} \tag{4}$$

The equation above only considers inter-specific competition when $j \neq i$ in the sum. We can however add intra-specific competition when $j = i$. Each site has a species-specific carrying capacity $K$ as the number of individuals approaches this carrying capacity the intra-specific competition increases:

$$\alpha_{ii} = B \times N_{t,i,x} \tag{5}$$

Thus the equation becomes:

$$N_{t+1,i,x} = \frac{k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \times N_{t,i,x}}{1 + A \left(\sum_{j=1,\ j\neq i}^{S} N_{t,j,x}(1 - \delta_{ij}) + \frac{B}{A} \times N_{t,i,x}\right)} \tag{6}$$

with $A$ the coefficient scaling inter-specific competition and $B$ the one for intra-specific competition.

Because several traits participate to the growth term depending on their contribution we can rewrite the growth term as:

$$R_{i,x} = \sum_{g=1}^{T} w_g \times k \times \exp\left(-\frac{(\text{trait}_{g,i} - \text{env}_x)^2}{2 \times \text{width}^2}\right) \tag{7}$$

with $g$ the trait number, $0 \leq w_g \leq 1$ the contribution of this trait to growth (and $\sum_{g=1}^{T} w_g = 1$), $\text{trait}_{g,i}$ the trait number $g$ of species $i$.

If we add hierarchical competition, the species with the largest trait has an increased growth. We can include this as a "bonus" term in the computation of the growth term $R_{i,x}$ such as:

$$R_{i,x} = R_{i,x,\text{env}} + R_{i,x,\text{hierarch.}} \tag{8}$$

$$R_{i,x,\text{hierarch.}} = \sum_{c=1}^{T_c} w_c \times H \times \frac{t_i}{\max(t_i)} \tag{9}$$

$$R_{i,x} = k \left[ \sum_{g=1}^{T} w_g \times \exp\left( -\frac{(\text{trait}_{g,i} - \text{env}_x)^2}{2 \times \text{width}^2} \right) + \frac{H}{k} \sum_{c=1}^{T_c} w_c \frac{\text{trait}_i}{\max(\text{trait}_i)} \right], \tag{10}$$

with$w_g$ the weight of traits contributing to growth, $w_c$ the weight of traits contributing to competition (as many as $T_c$).

So the final equation looks like the following:

$$N_{t+1,i,x} = \frac{k \left[ \sum_{g=1}^{T} w_g \times \exp\left( -\frac{(\text{trait}_{g,i} - \text{env}_x)^2}{2 \times \text{width}^2} \right) + \frac{H}{k} \sum_{c=1}^{T_c} w_c \frac{\text{trait}_i}{\max(\text{trait}_i)} \right] \times N_{t,i,x}}{1 + A \left( \sum_{j=1,\ j \neq i}^{S} N_{t,j,x}(1 - \delta_{ij}) + \frac{B}{A} \times N_{t,i,x} \right)} \tag{11}$$