Solving Equation to get A value

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Analytical derivation of A value

We start from this equation

$$N_{t+1,i,x} = \frac{k \times \exp\left(-\dfrac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \times N_{t,i,x}}{1 + A \times \displaystyle\sum_{j=1,\ j\neq i}^{S} N_{t,j,x}(1 - \delta_{ij}) + B \times N_{t,i,x}}$$

to get realistic value for A and B. We solve $N_{t+1,i,x} = N_{t,i,x}$ with $A = B$ and $1 - \delta_{ij} = 1$ for all i and j and for $\text{trait}_i - \text{env}_x = 0$. We thus get:

$$
\begin{aligned}
N_{t+1,i,x} &= N_{t,i,x} \\
\Leftrightarrow N_{t,i,x} &= \frac{k \times N_{t,i,x}}{1 + A \times \displaystyle\sum_{j=1,\ j\neq i}^{S} N_{t,j,x} + A \times N_{t,i,x}} \\
\Leftrightarrow N_{t,i,x} &= \frac{k \times N_{t,i,x}}{1 + A \times \displaystyle\sum_{j=1}^{S} N_{t,j,x}} \\
\Leftrightarrow \cancel{N_{t,i,x}} &= \frac{k \times \cancel{N_{t,i,x}}}{1 + A \times \displaystyle\sum_{j=1}^{S} N_{t,j,x}} \\
\Leftrightarrow 1 &= \frac{k}{1 + A \times \displaystyle\sum_{j=1}^{S} N_{t,j,x}} \\
\Leftrightarrow 1 + A \times \sum_{j=1}^{S} N_{t,j,x} &= k \\
\Leftrightarrow \boxed{A = \frac{k - 1}{\displaystyle\sum_{j=1}^{S} N_{t,j,x}}}
\end{aligned}
\tag{1}
$$

We can find a value for A heuristically computing it at the last generation for a number of sites.

Heuristical value of A

Another way of choosing the A-value is to simulate the growth of 100 species with exactly the same traits and to see when the growth is canceled, with a number of maximal growth parameters.

B = 2e−04; d = 0.05; N sp. = 100