# fdcoexist

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This document presents the relationships between functional traits of species and an environmental gradient.

Our coexistence model is developed following this equation:

$$N_{t+1,i,x} = \frac{R_{i,x} \times N_{t,i,x}}{1 + A \times \alpha_i} \tag{1}$$

with

$$\alpha_i = \sum_{j=1, j \neq i}^{S} N_{t,j,x} \times (1 - \delta_{ij}) \tag{2}$$

$$R_{i,x} = k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \tag{3}$$

If we replace $\alpha_i$ and $R_{i,x}$ in the first equation it gives:

$$N_{t+1,i,x} = \frac{k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \times N_{t,i,x}}{1 + A \times \sum_{j=1, j \neq i}^{S} N_{t,j,x} \times (1 - \delta_{ij})} \tag{4}$$

The equation above only considers inter-specific competition when $j \neq i$ in the sum. We can however add intra-specific competition when $j = i$. Each site has a species-specific carrying capacity $K$ as the number of individuals approaches this carrying capacity the intra-specific competition increases:

$$\alpha_{ii} = B \times N_{t,i,x} \tag{5}$$

Thus the equation becomes:

$$N_{t+1,i,x} = \frac{k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \times N_{t,i,x}}{1 + A \left(\sum_{j=1,\ j \neq i}^{S} N_{t,j,x}(1 - \delta_{ij}) + \frac{B}{A} \times N_{t,i,x}\right)} \tag{6}$$

with $A$ the coefficient scaling inter-specific competition and $B$ the one for intra-specific competition.

Because several traits participate to the growth term depending on their contribution we can rewrite the growth term as:

$$R_{i,x} = \sum_{g=1}^{T} w_g \times k \times \exp\left(-\frac{(\text{trait}_{g,i} - \text{env}_x)^2}{2 \times \text{width}^2}\right) \tag{7}$$

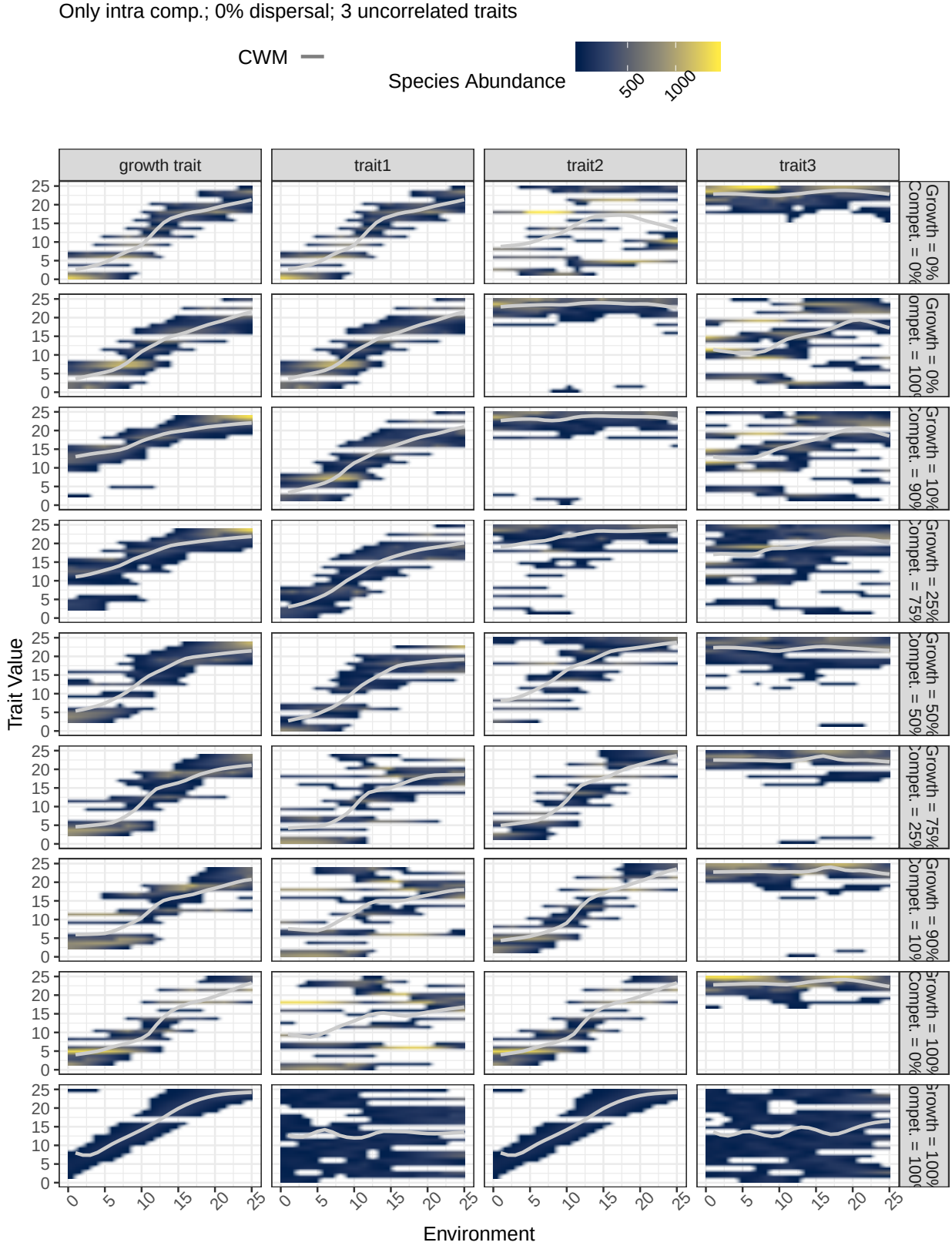
with $g$ the trait number, $0 \leq w_g \leq 1$ the contribution of this trait to growth (and $\sum_{g=1}^{T} w_g = 1$), $\text{trait}_{g,i}$ the trait number $g$ of species $i$.

# Constant environmental filtering strength

## Without Competition (only intra-specific competition)

We can run the simulations without any competition `A = 0` to see if we see the theoretical patterns.

### No correlations among traits



**Low correlations among traits**

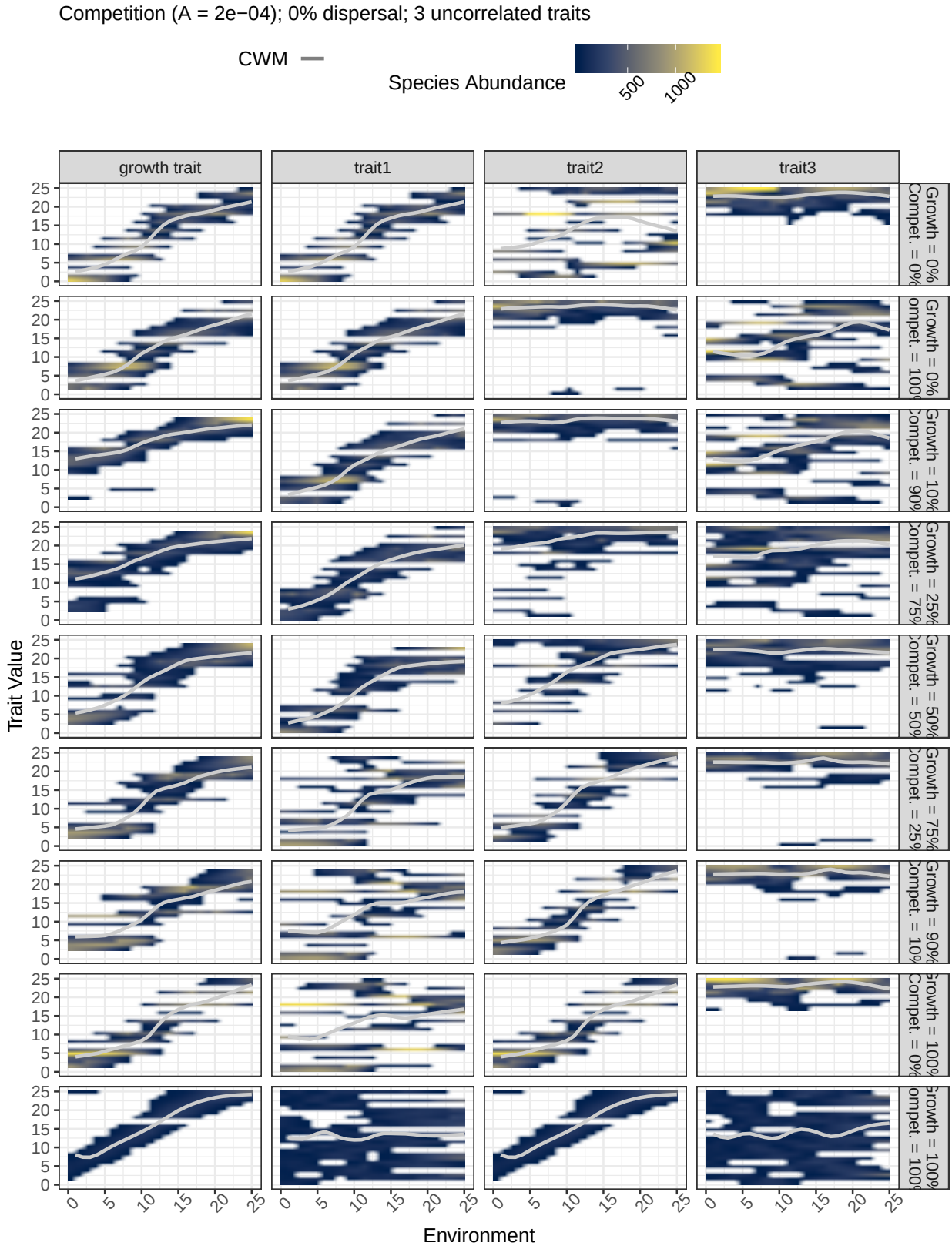
```
## Error: Faceting variables must have at least one value
```

**High correlations among traits**

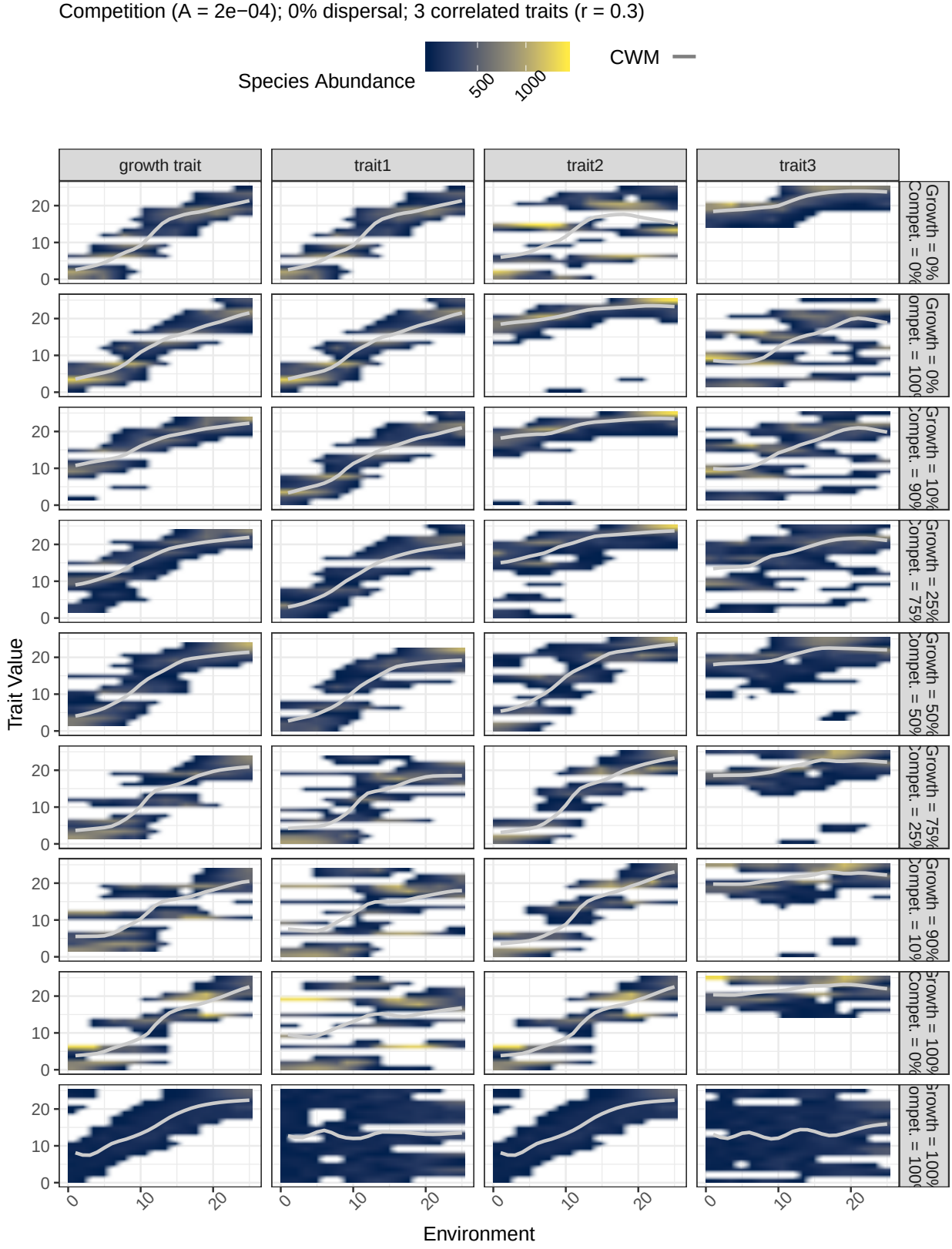
```
## Error: Faceting variables must have at least one value
```

## With competition

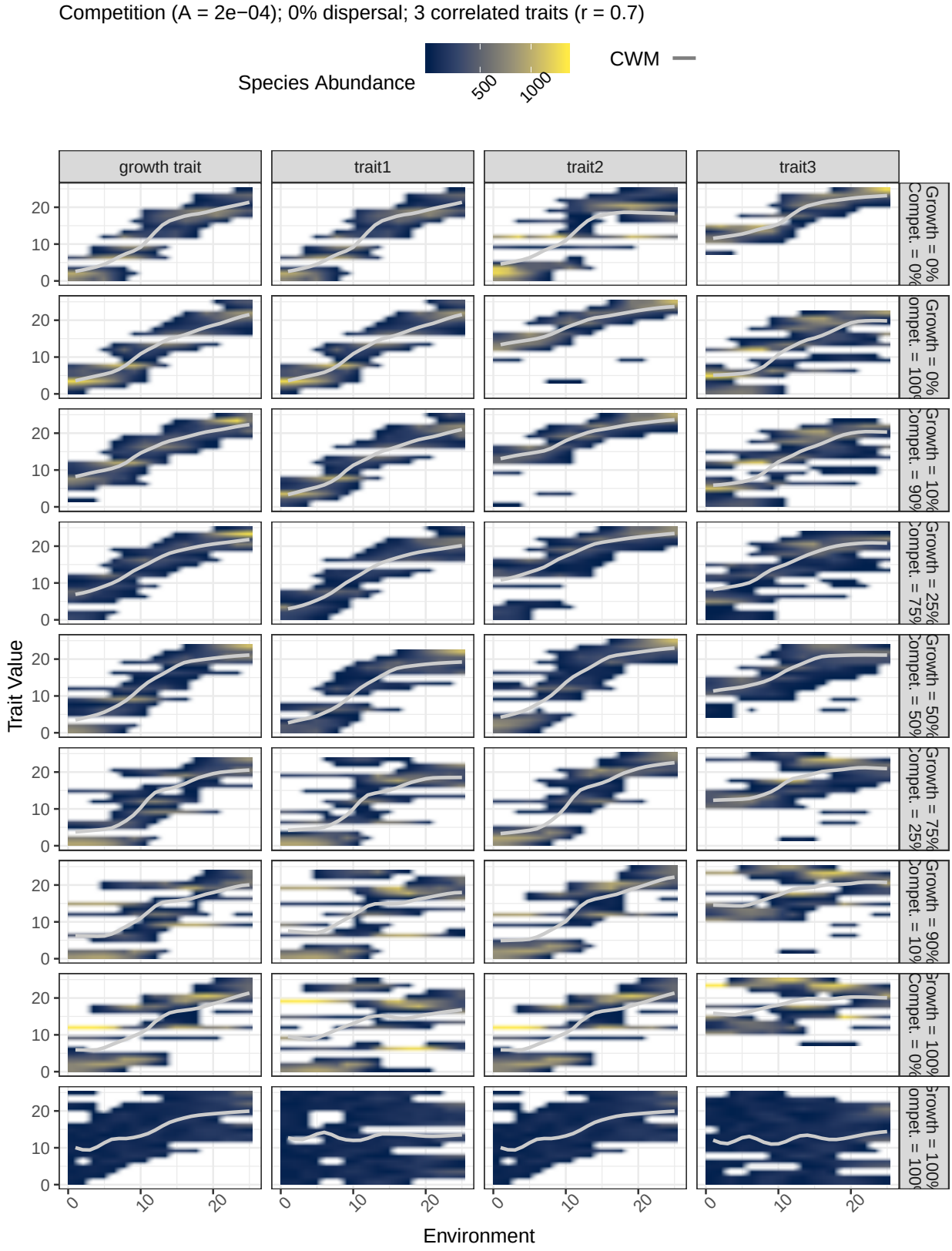
### No correlations among traits



**Low correlations among traits**



### High correlations among traits



# With varying environmental filtering strength

## Without Competition (only intra-specific competition)

In this section, the environmental filtering selects for a narrower trait range towards the end of the environmental gradient.

### No correlations among traits

```
## Error: Faceting variables must have at least one value
```

**Low correlations among traits**

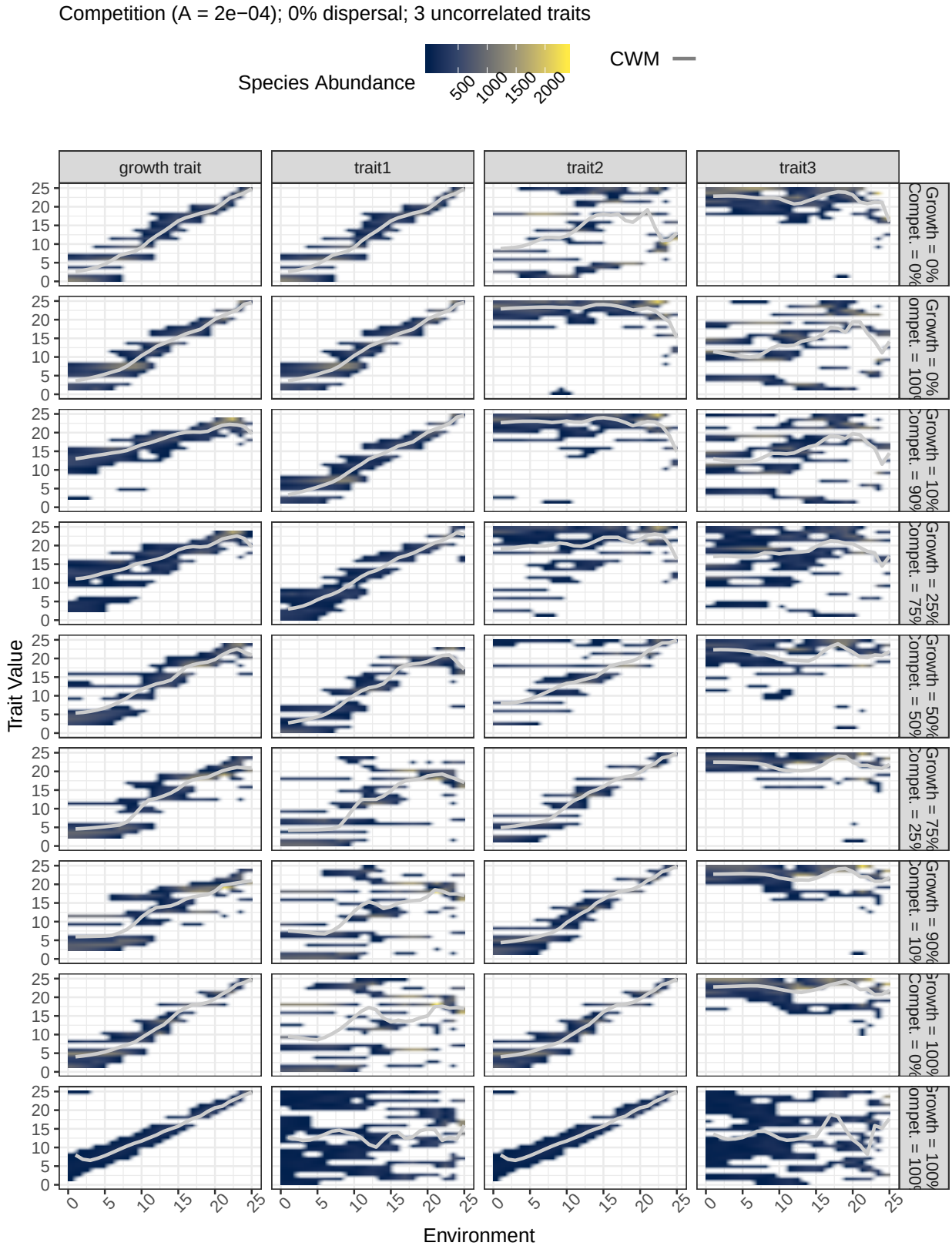
```
## Error: Faceting variables must have at least one value
```

**High correlations among traits**

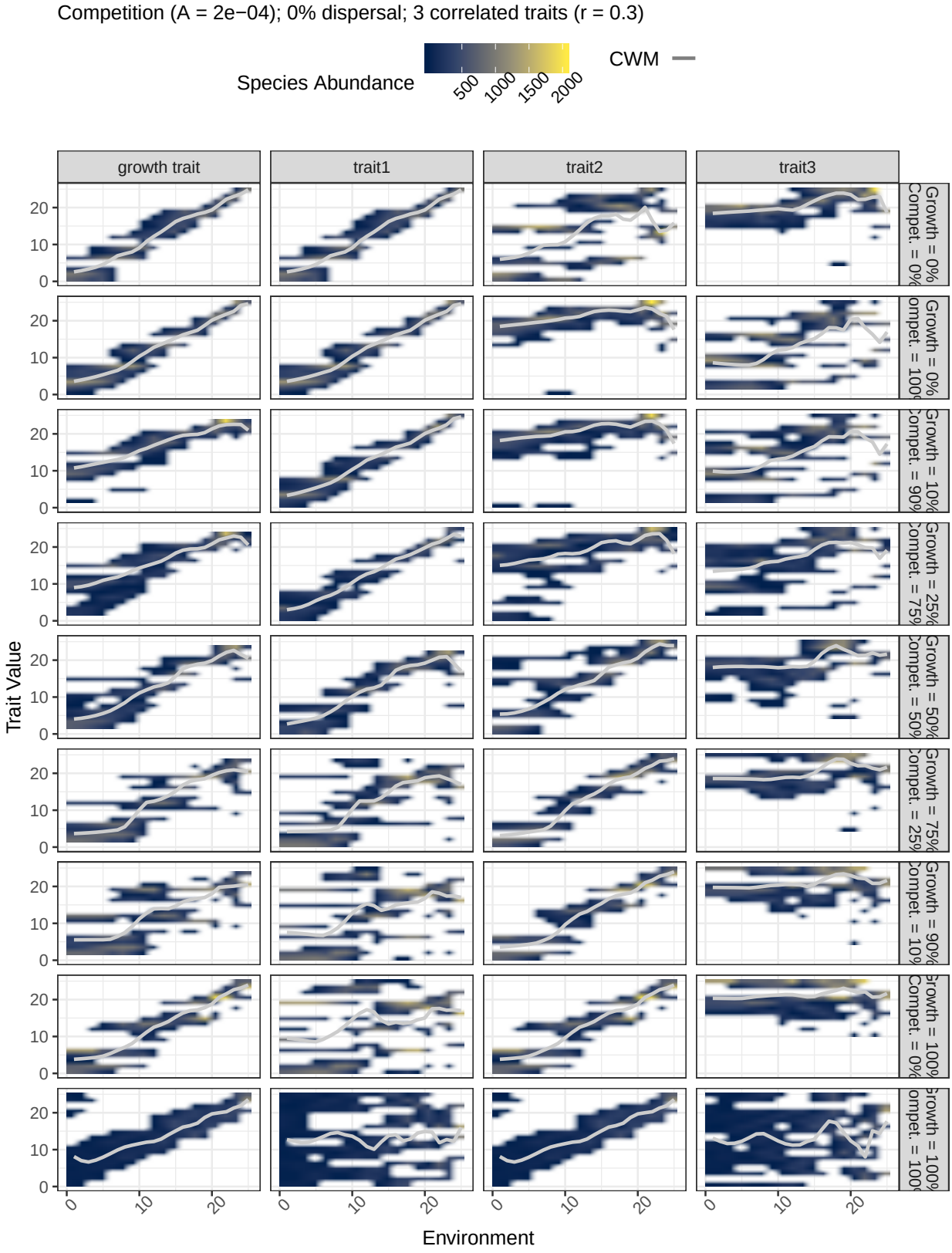
```
## Error: Faceting variables must have at least one value
```

## With competition

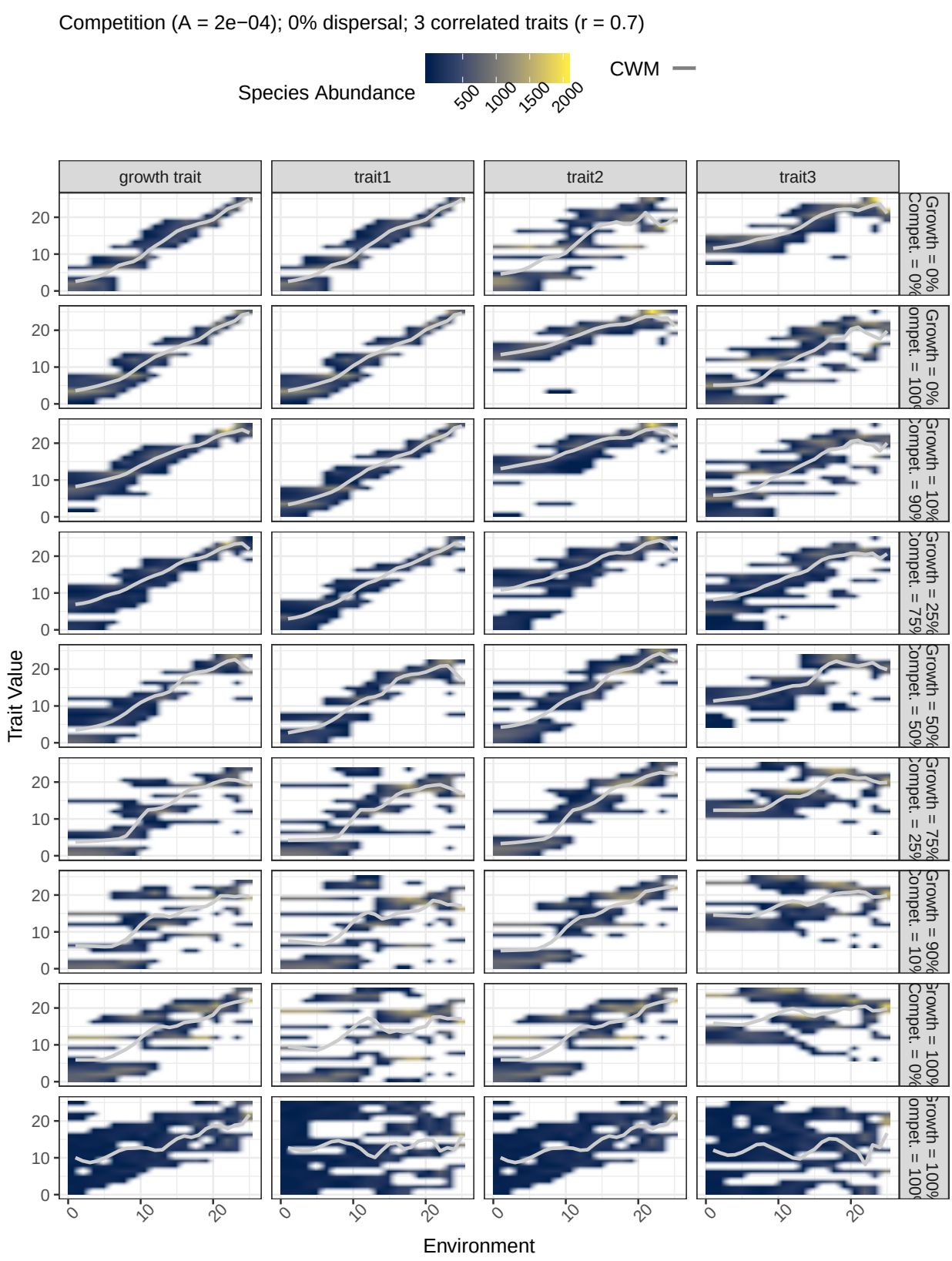
### No correlations among traits



**Low correlations among traits**



### High correlations among traits



# Synthetic plots

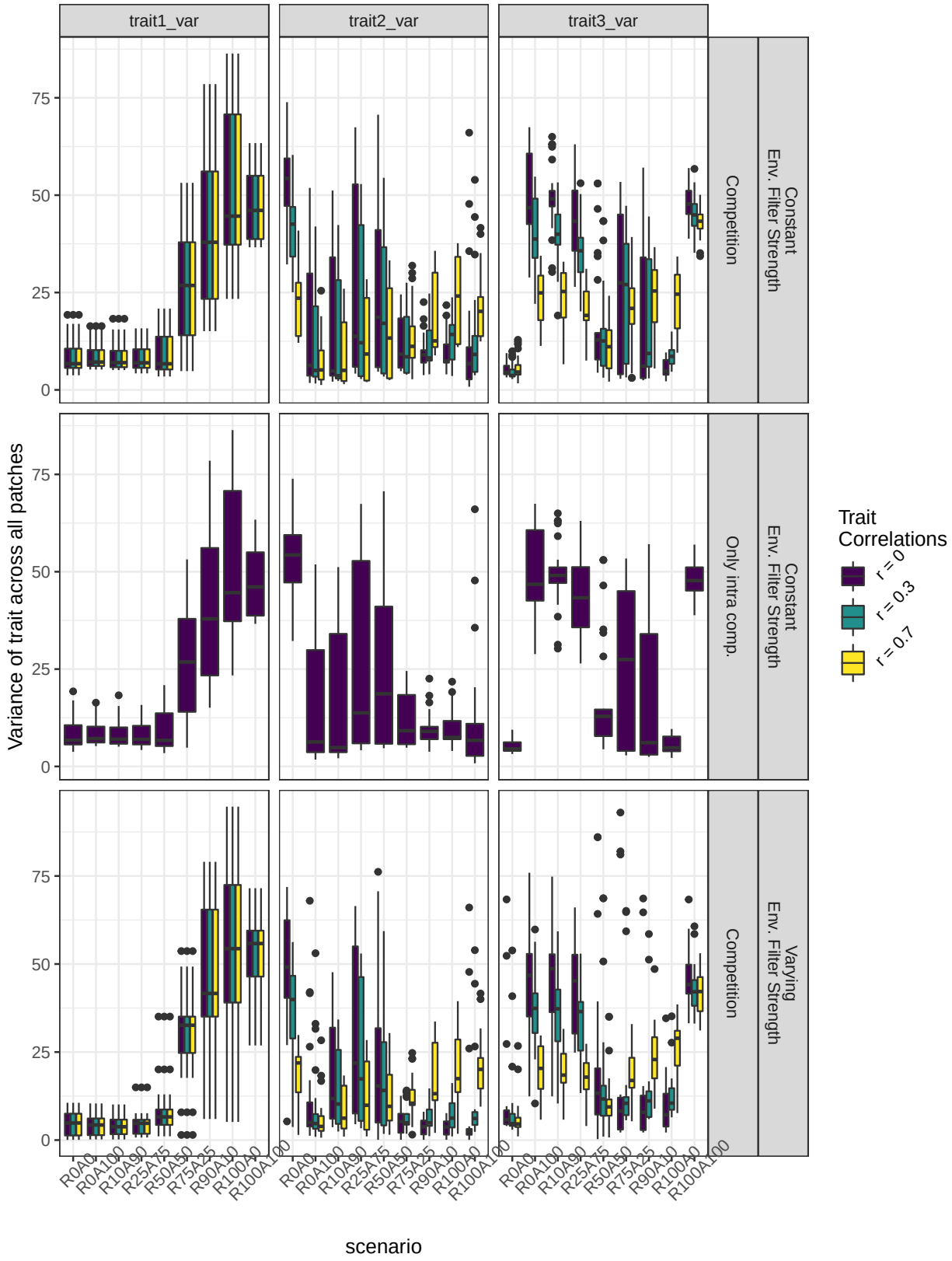
## $R^2$ CWM against environment

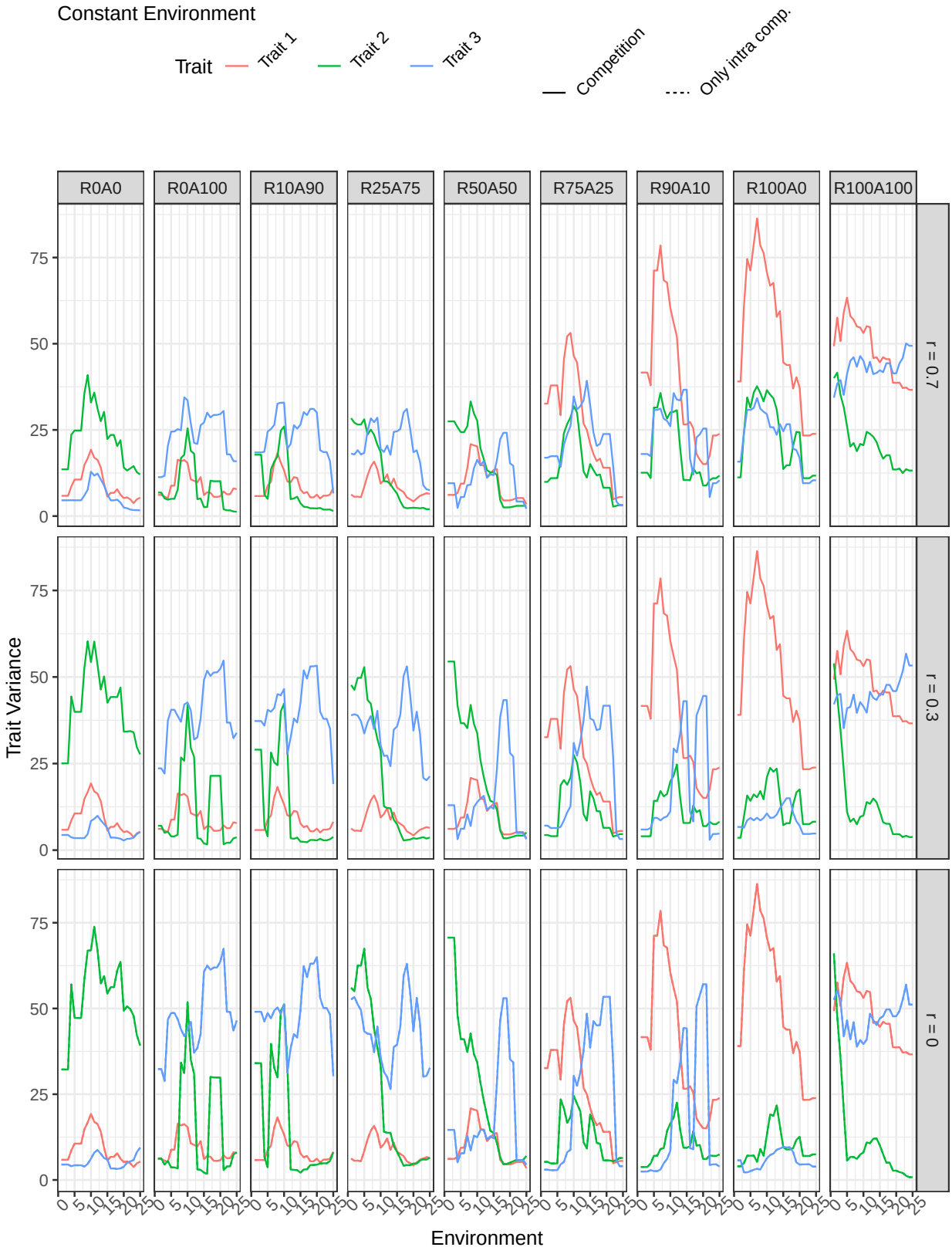
```
## Error in lm.fit(x, y, offset = offset, singular.ok = singular.ok, ...): 0 (non-NA) cases
```

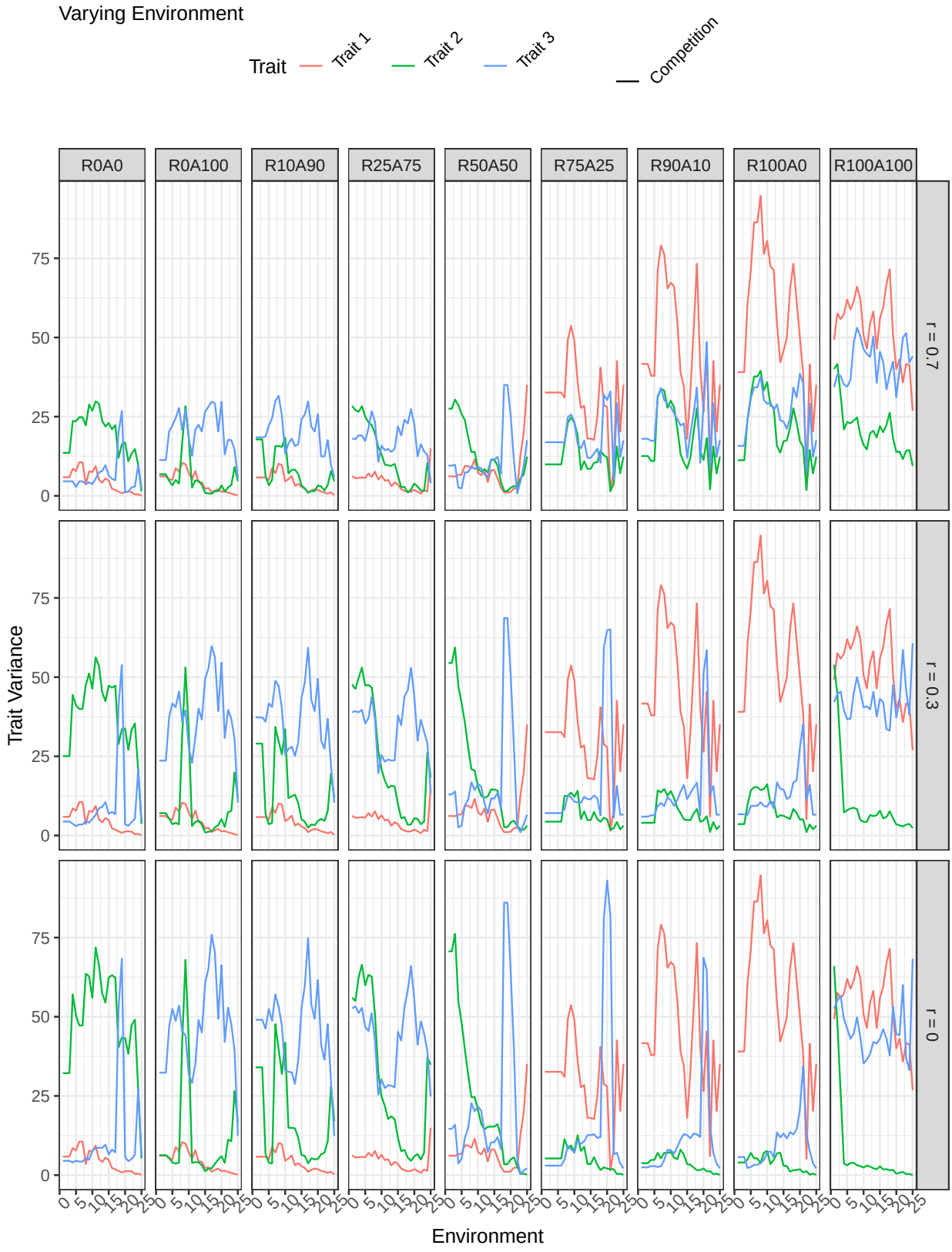
```
## Error in eval(lhs, parent, parent): objet 'mod_df' introuvable
```

```
## Error in eval(expr, envir, enclos): objet 'plot_r2_cwm_env' introuvable
```

## Trait variance against environment




Constant Environment
Trait
Trait 1
Trait 2
Trait 3
Competition
Only intra comp.
R0A0
R0A100
R10A90
R25A75
R50A50
R75A25
R90A10
R100A0
R100A100
r = 0.7
r = 0.3
r = 0
Trait Variance
Environment


Varying Environment
Trait
Trait 1
Trait 2
Trait 3
Competition
R0A0
R0A100
R10A90
R25A75
R50A50
R75A25
R90A10
R100A0
R100A100
r = 0.7
r = 0.3
r = 0
Trait Variance
Environment

## CWV against environment