Solving Equation to get A value

Matthias Grenié

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Analytical derivation of A value

We start from this equation

$$N_{t+1,i,x} = \frac{k \times \exp\left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2}\right) \times N_{t,i,x}}{1 + A \times \sum\limits_{j=1,\ j\neq i}^{S} N_{t,j,x}(1-\delta_{ij}) + B \times N_{t,i,x}}$$

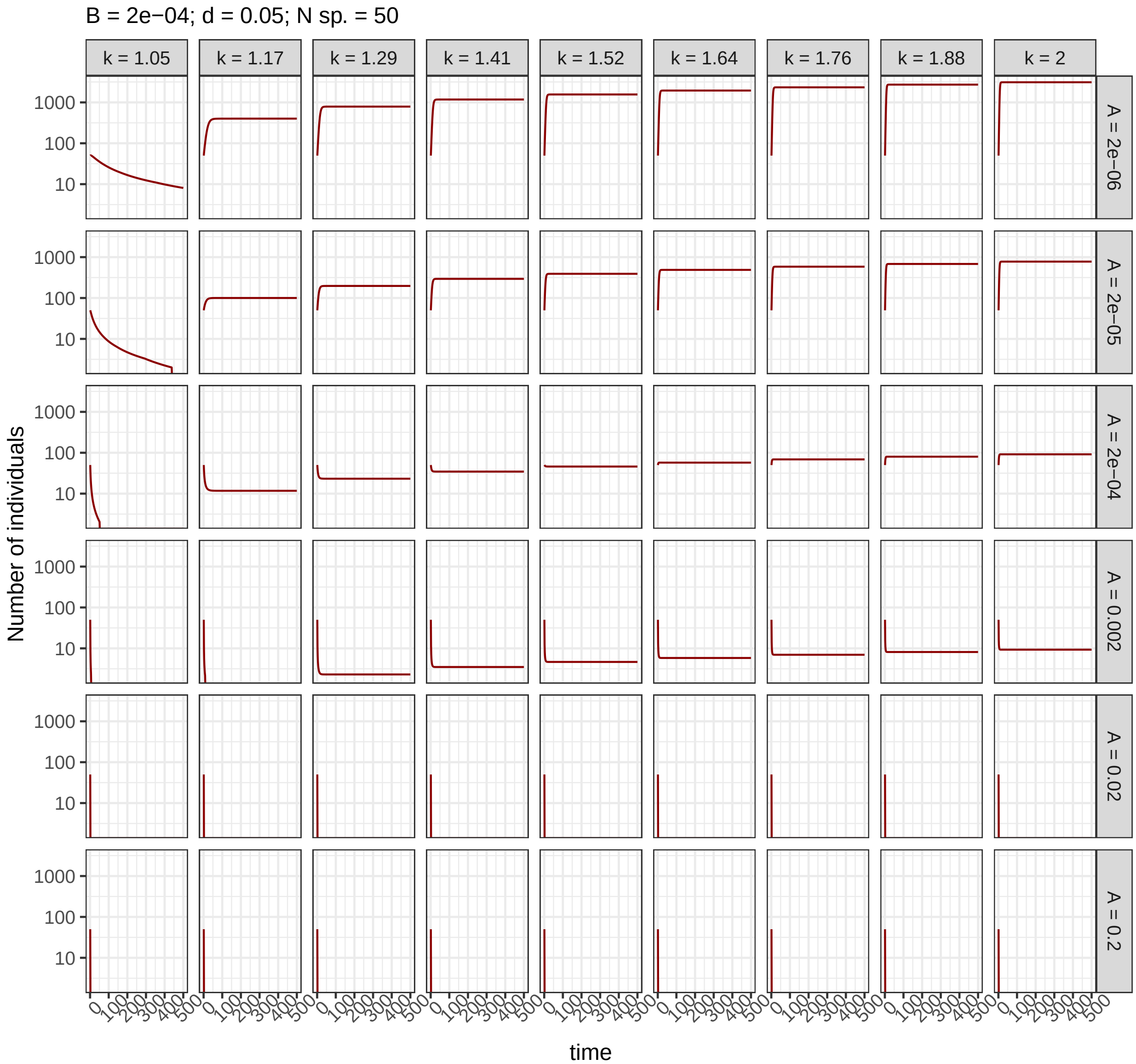
to get realistic value for A and B. We solve $N_{t+1,i,x} = N_{t,i,x}$ with $A = B$ and $1 - \delta_{ij} = 1$ for all i and j and for $\text{trait}_i - \text{env}_x = 0$. We thus get:

$$\begin{aligned}
N_{t+1,i,x} =& N_{t,i,x} \\
\Leftrightarrow N_{t,i,x} =& \frac{k \times N_{t,i,x}}{1 + A \times \sum\limits_{j=1,\ j\neq i}^{S} N_{t,j,x} + A \times N_{t,i,x}} \\
\Leftrightarrow N_{t,i,x} =& \frac{k \times N_{t,i,x}}{1 + A \times \sum\limits_{j=1}^{S} N_{t,j,x}} \\
\Leftrightarrow \cancel{N_{t,i,x}} =& \frac{k \times \cancel{N_{t,i,x}}}{1 + A \times \sum\limits_{j=1}^{S} N_{t,j,x}} \\
\Leftrightarrow 1 =& \frac{k}{1 + A \times \sum\limits_{j=1}^{S} N_{t,j,x}} \\
\Leftrightarrow 1 + A \times \sum_{j=1}^{S} N_{t,j,x} = k& \\
\Leftrightarrow & \boxed{A = \frac{k-1}{\sum\limits_{j=1}^{S} N_{t,j,x}}}
\end{aligned} \tag{1}$$

We can find a value for A heuristically computing it at the last generation for a number of sites.

Heuristical value of A

Another way of choosing the A-value is to simulate the growth of 100 species with exactly the same traits and to see when the growth is canceled, with a number of maximal growth parameters.


B = 2e−04; d = 0.05; N sp. = 50
k = 1.05
k = 1.17
k = 1.29
k = 1.41
k = 1.52
k = 1.64
k = 1.76
k = 1.88
k = 2
A = 2e−06
A = 2e−05
A = 2e−04
A = 0.002
A = 0.02
A = 0.2
Number of individuals
1000
100
10
0
100
200
300
400
500
time