

fdcoexist

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This document presents the relationships between functional traits of species and an environmental gradient. Our coexistence model is developed following this equation:

$$N_{t+1,i,x} = \frac{R_{i,x} \times N_{t,i,x}}{1 + A \times \alpha_i} \quad (1)$$

with

$$\alpha_i = \sum_{j=1, j \neq i}^S N_{t,j,x} \times (1 - \delta_{ij}) \quad (2)$$

$$R_{i,x} = k \times \exp \left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2} \right) \quad (3)$$

If we replace α_i and $R_{i,x}$ in the first equation it gives:

$$N_{t+1,i,x} = \frac{k \times \exp \left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2} \right) \times N_{t,i,x}}{1 + A \times \sum_{j=1, j \neq i}^S N_{t,j,x} \times (1 - \delta_{ij})} \quad (4)$$

The equation above only considers inter-specific competition when $j \neq i$ in the sum. We can however add intra-specific competition when $j = i$. Each site has a species-specific carrying capacity K as the number of individuals approaches this carrying capacity the intra-specific competition increases:

$$\alpha_{ii} = B \times N_{t,i,x} \quad (5)$$

Thus the equation becomes:

$$N_{t+1,i,x} = \frac{k \times \exp \left(-\frac{(\text{trait}_i - \text{env}_x)^2}{2 \times \text{width}^2} \right) \times N_{t,i,x}}{1 + A \left(\sum_{j=1, j \neq i}^S N_{t,j,x} (1 - \delta_{ij}) + \frac{B}{A} \times N_{t,i,x} \right)} \quad (6)$$

with A the coefficient scaling inter-specific competition and B the one for intra-specific competition.

Because several traits participate to the growth term depending on their contribution we can rewrite the growth term as:

$$R_{i,x} = \sum_{g=1}^T w_g \times k \times \exp \left(-\frac{(\text{trait}_{g,i} - \text{env}_x)^2}{2 \times \text{width}^2} \right) \quad (7)$$

with g the trait number, $0 \leq w_g \leq 1$ the contribution of this trait to growth (and $\sum_{g=1}^T w_g = 1$), $\text{trait}_{g,i}$ the trait number g of species i .

If we add hierarchical competition, the species with the largest trait has an increased growth. We can include this as a “bonus” term in the computation of the growth term $R_{i,x}$ such as:

$$R_{i,x} = R_{i,x,\text{env}} + R_{i,x,\text{hierarch.}} \quad (8)$$

$$R_{i,x,\text{hierarch.}} = \sum_{g=1}^T w_g \times H \times \frac{t_i}{\max(t_i)} \quad (9)$$

$$R_{i,x} = \sum_{g=1}^T w_g \times k \left[\exp \left(-\frac{(\text{trait}_{g,i} - \text{env}_x)^2}{2 \times \text{width}^2} \right) + \frac{H}{k} \times \frac{t_i}{\max(t_i)} \right] \quad (10)$$

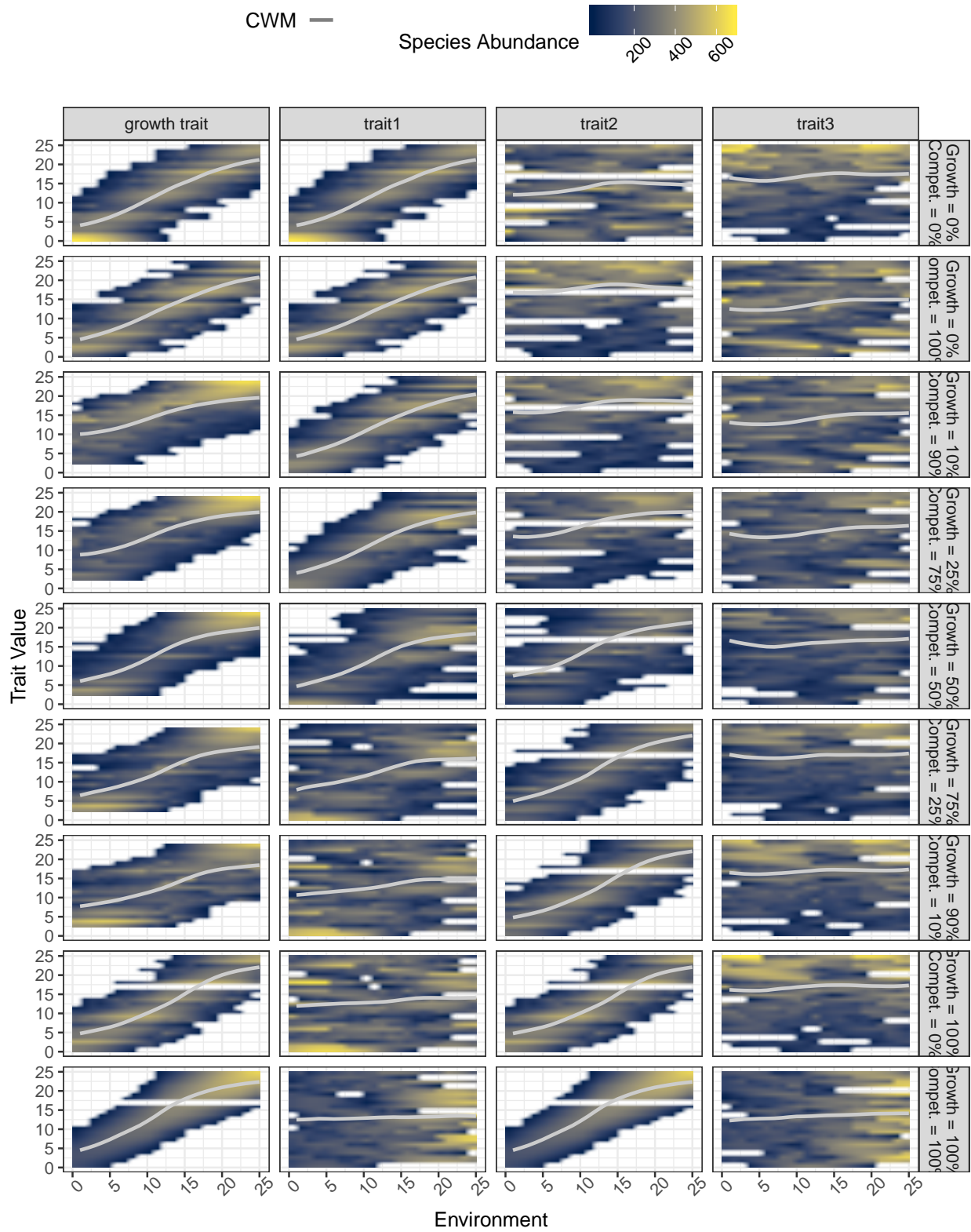
Constant environmental filtering strength

Without Competition (only intra-specific competition)

We can run the simulations without any competition $\mathbf{A} = 0$ to see if we see the theoretical patterns.

No correlations among traits

Only intra comp.; 0% dispersal; 3 uncorrelated traits



Low correlations among traits

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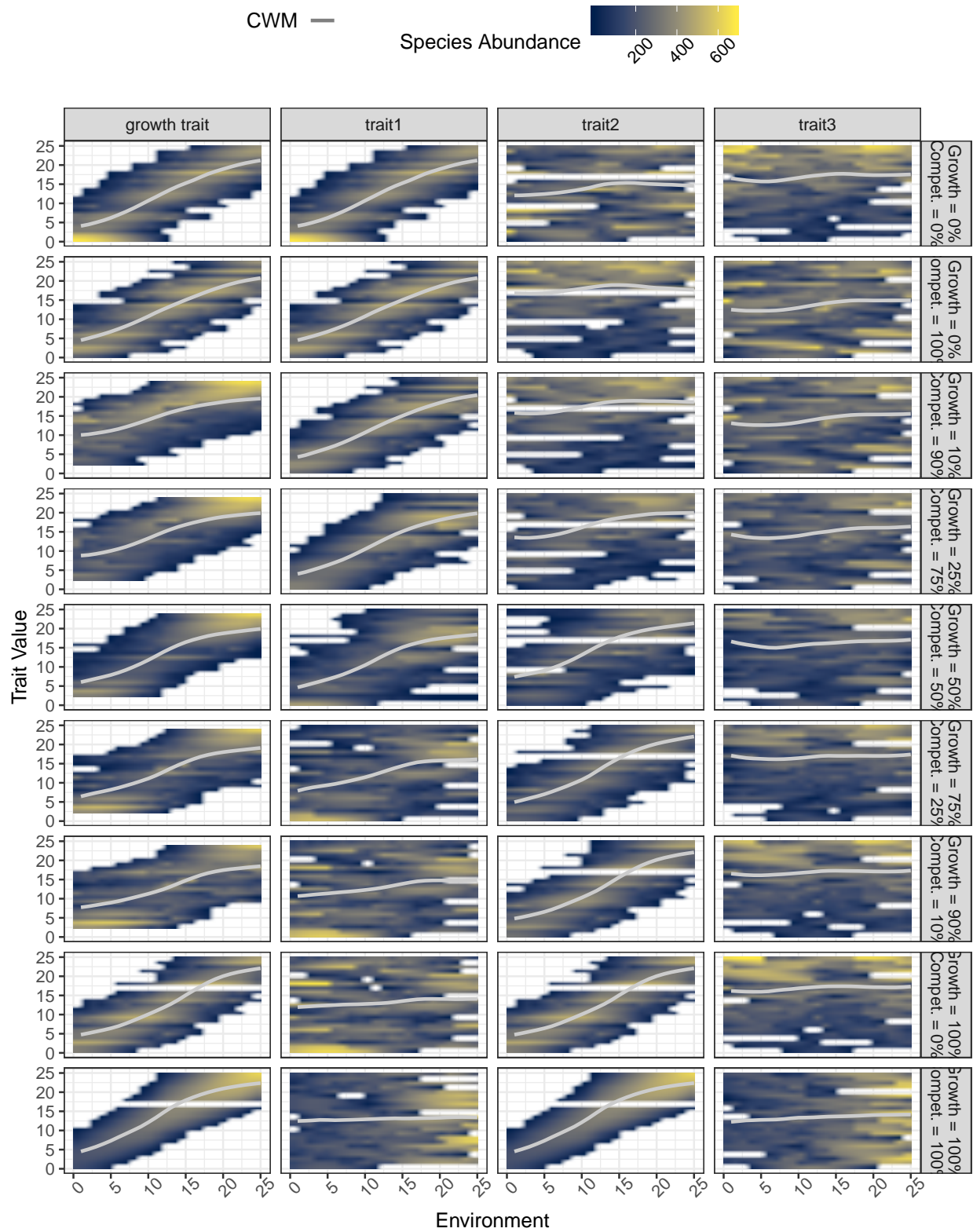

High correlations among traits

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With competition

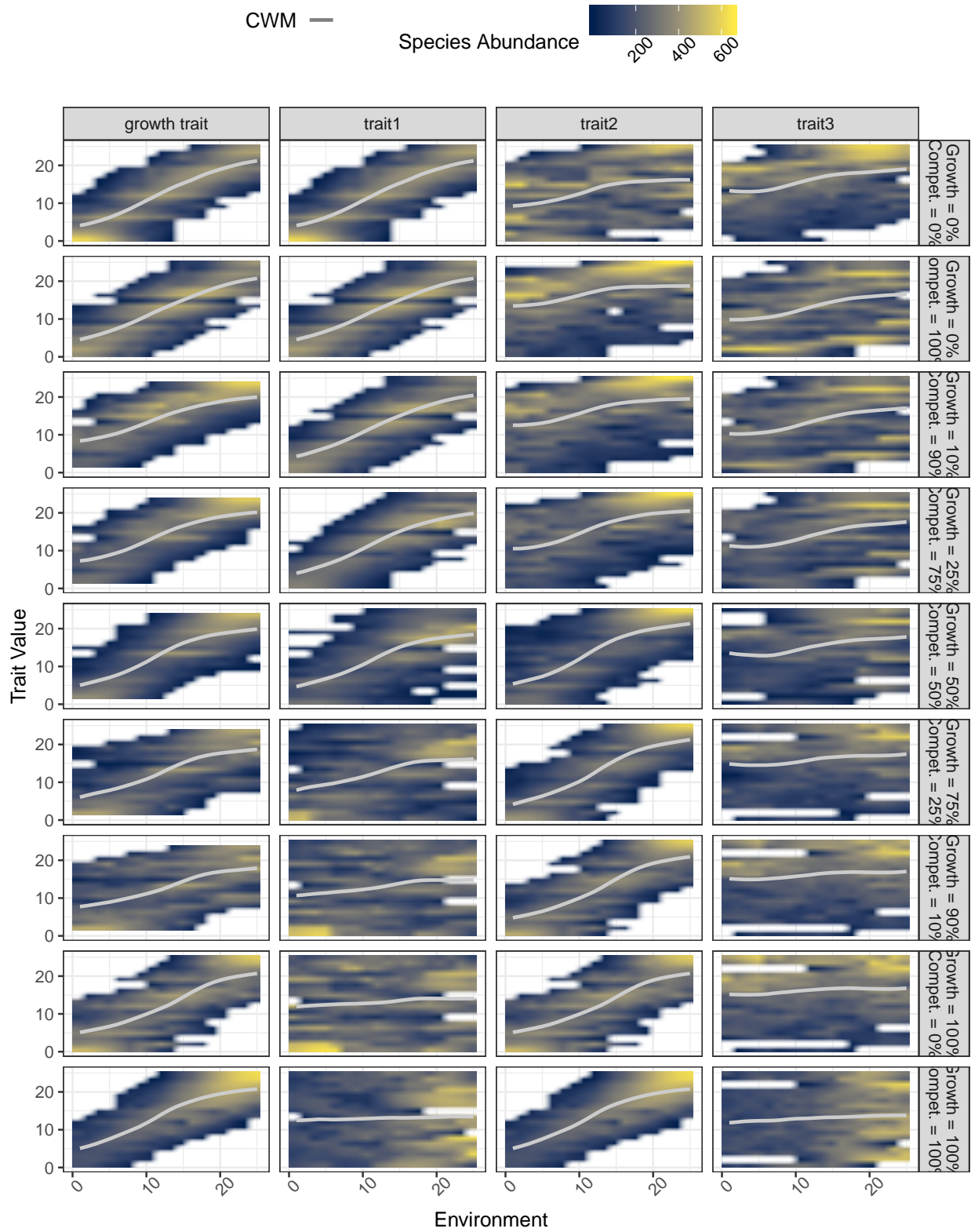
No correlations among traits

Competition ($A = 1e-05$); 0% dispersal; 3 uncorrelated traits



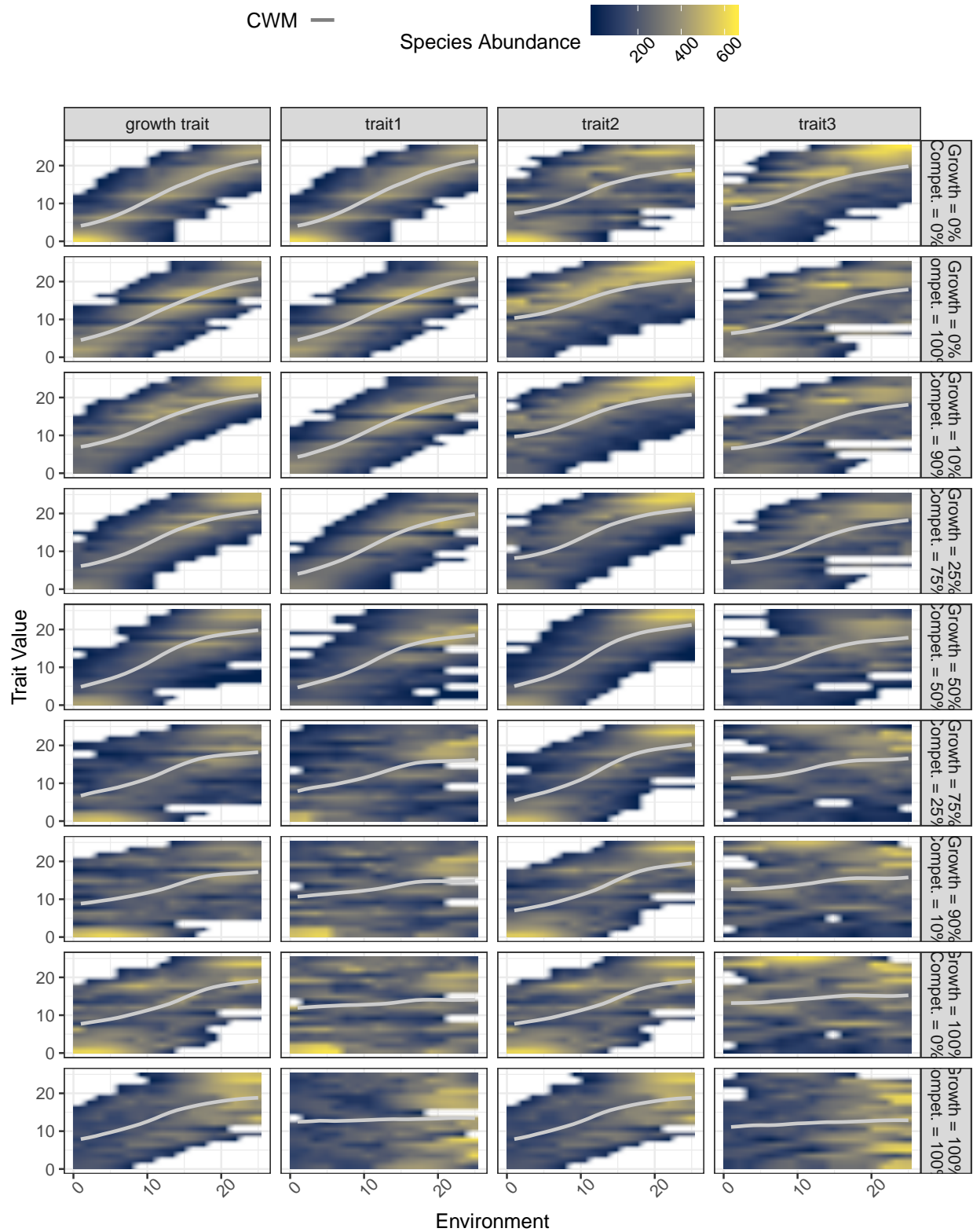
Low correlations among traits

Competition ($A = 1e-05$); 0% dispersal; 3 correlated traits ($r = 0.3$)



High correlations among traits

Competition ($A = 1e-05$); 0% dispersal; 3 correlated traits ($r = 0.7$)



With varying environmental filtering strength

Without Competition (only intra-specific competition)

In this section, the environmental filtering selects for a narrower trait range towards the end of the environmental gradient.

No correlations among traits

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Low correlations among traits

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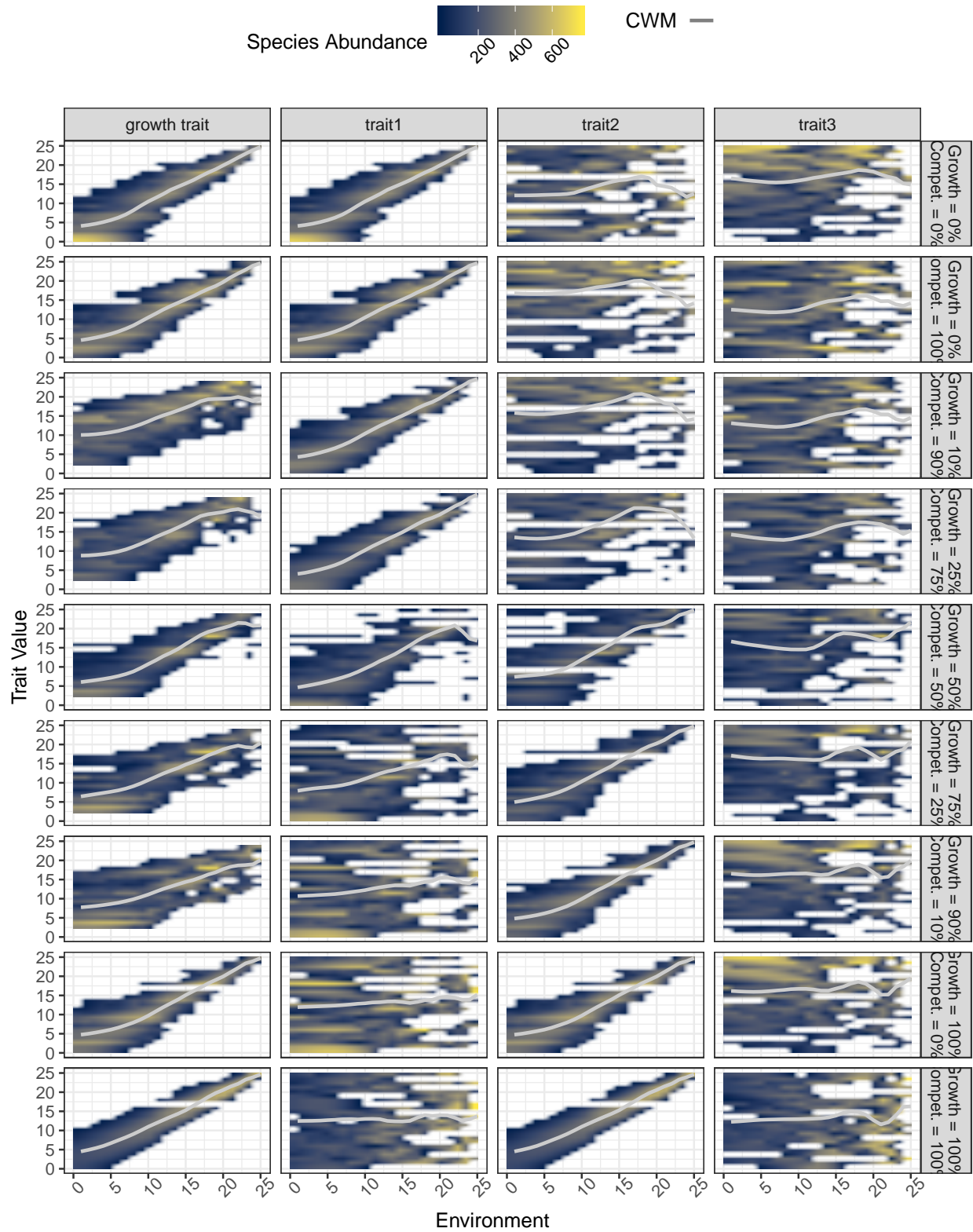

High correlations among traits

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With competition

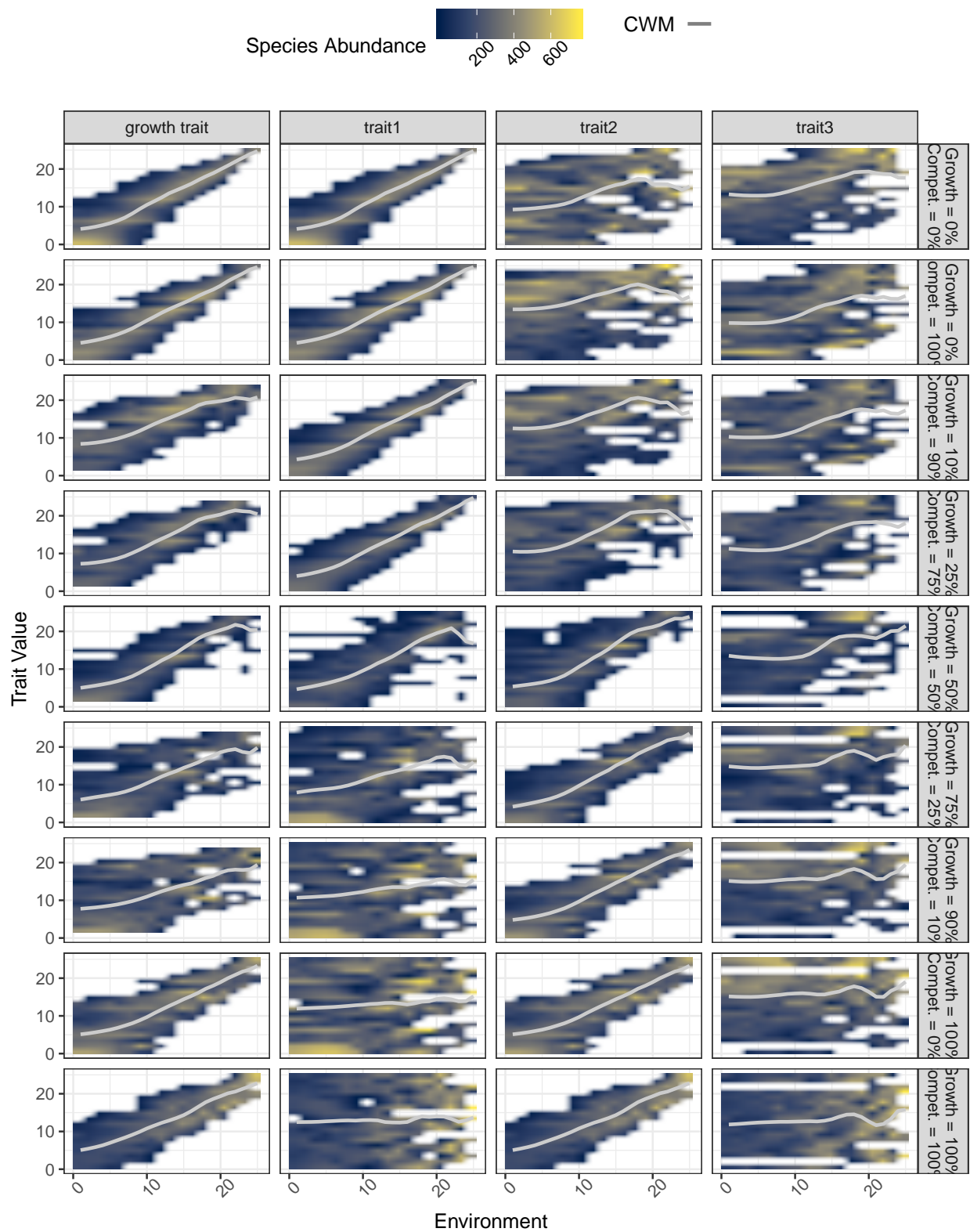
No correlations among traits

Competition ($A = 1e-05$); 0% dispersal; 3 uncorrelated traits



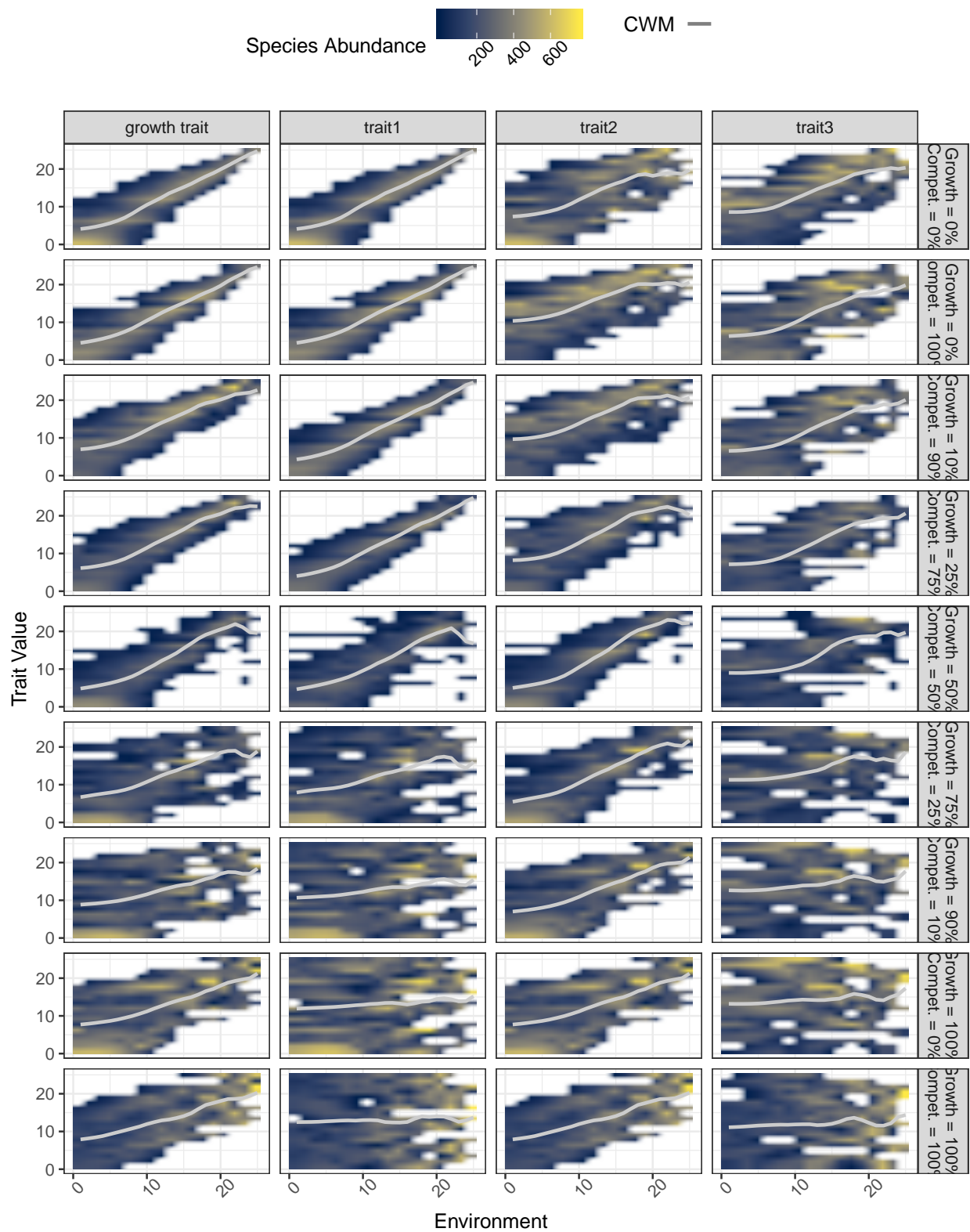
Low correlations among traits

Competition ($A = 1e-05$); 0% dispersal; 3 correlated traits ($r = 0.3$)



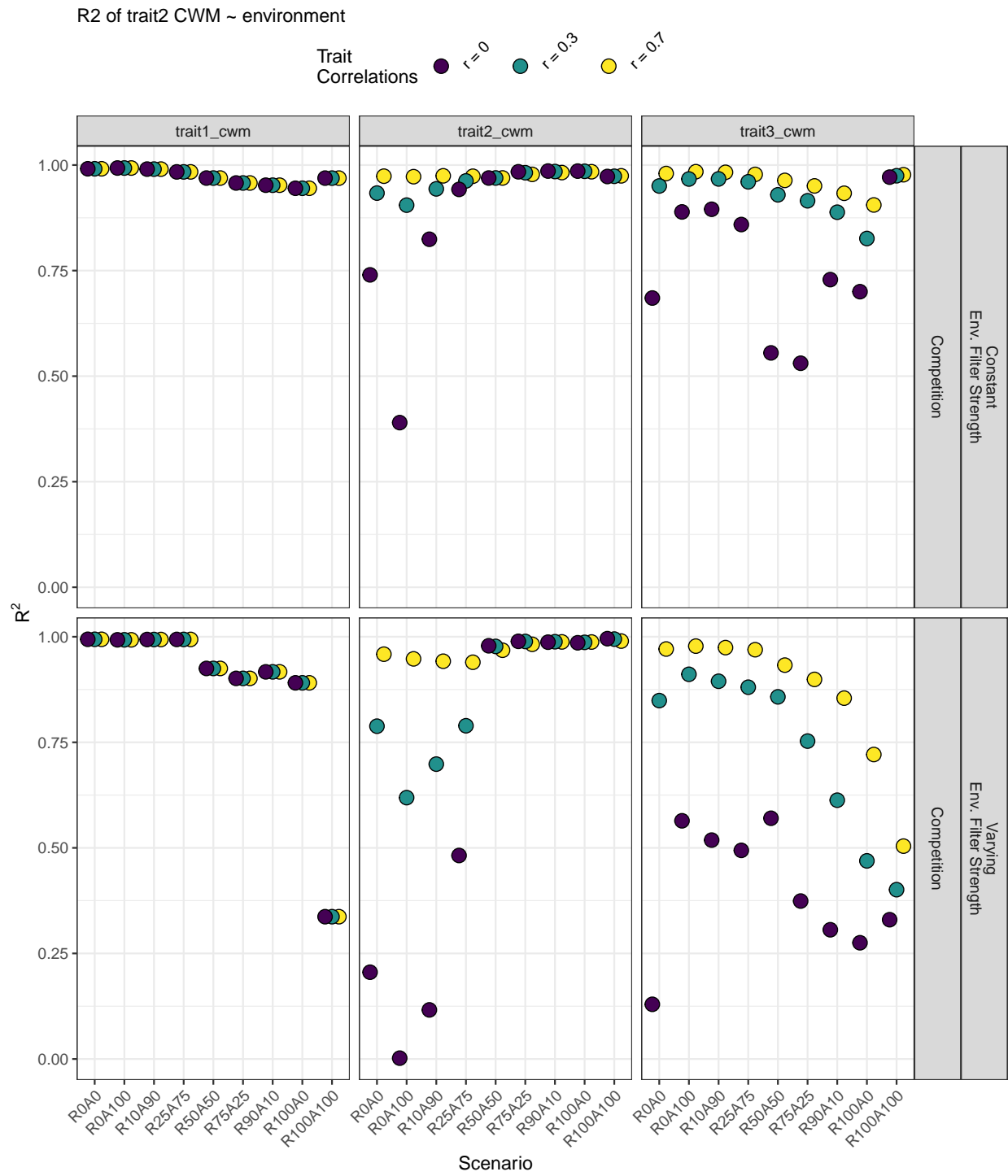
High correlations among traits

Competition ($A = 1e-05$); 0% dispersal; 3 correlated traits ($r = 0.7$)

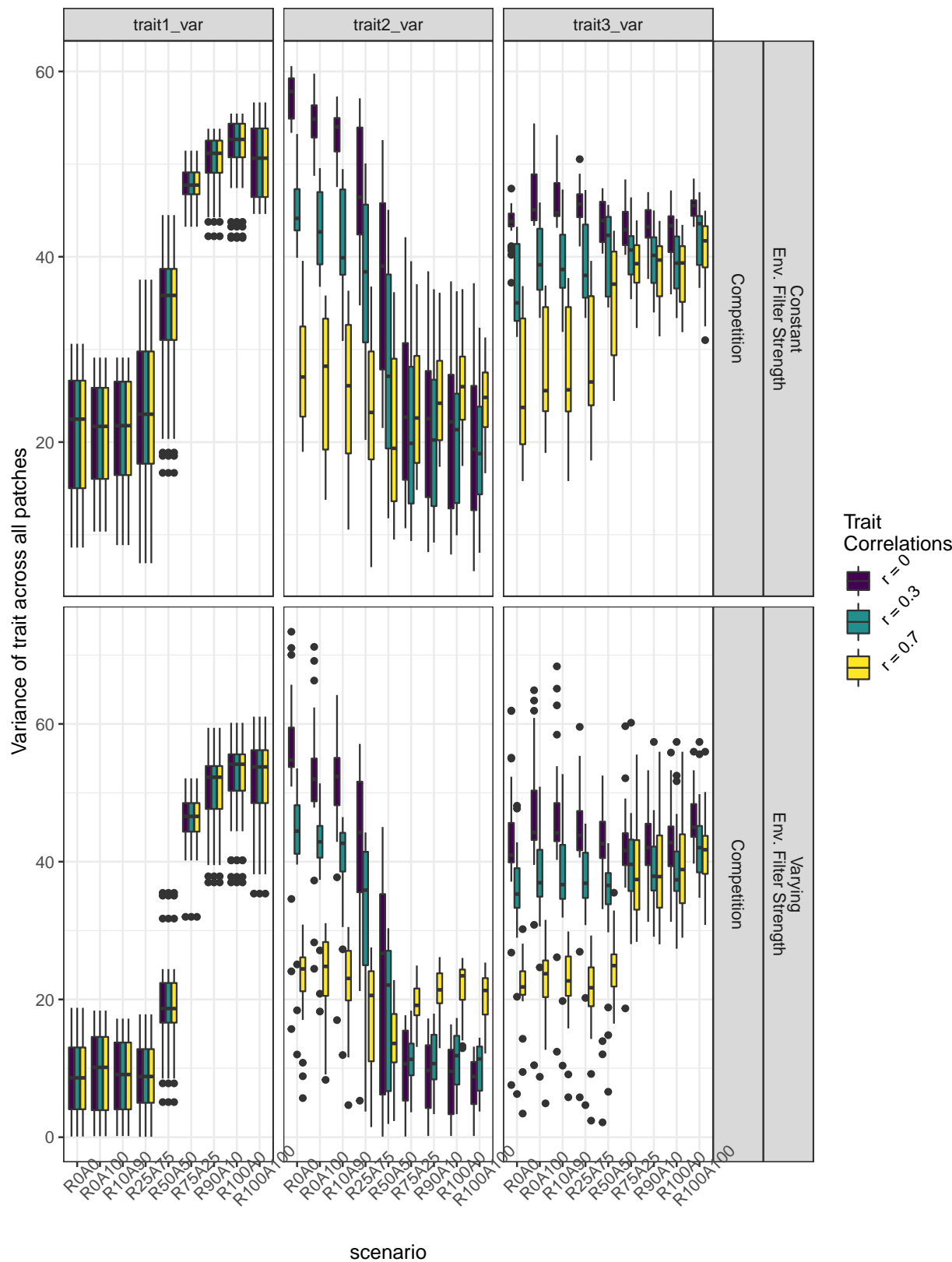


Synthetic plots

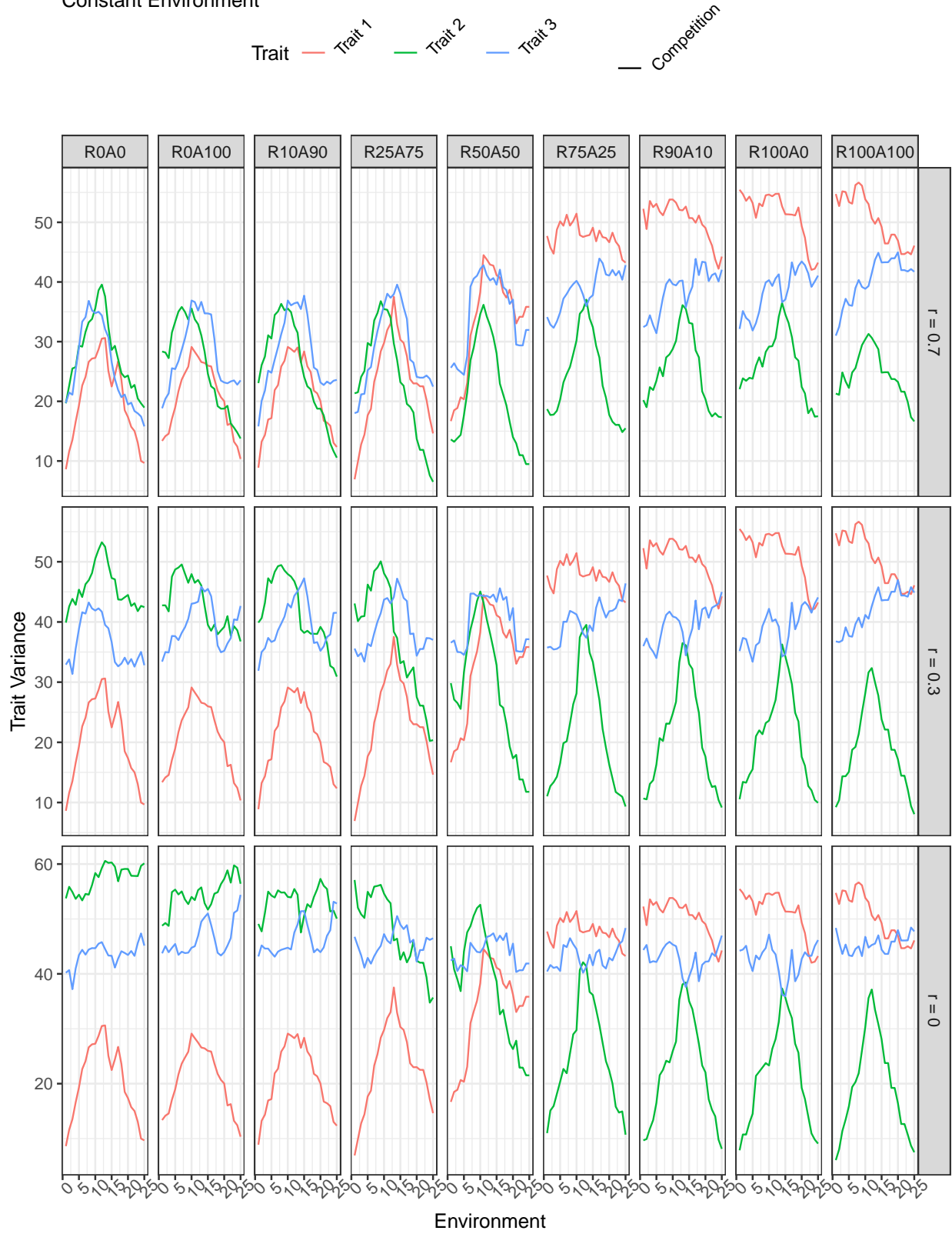
R² CWM against environment



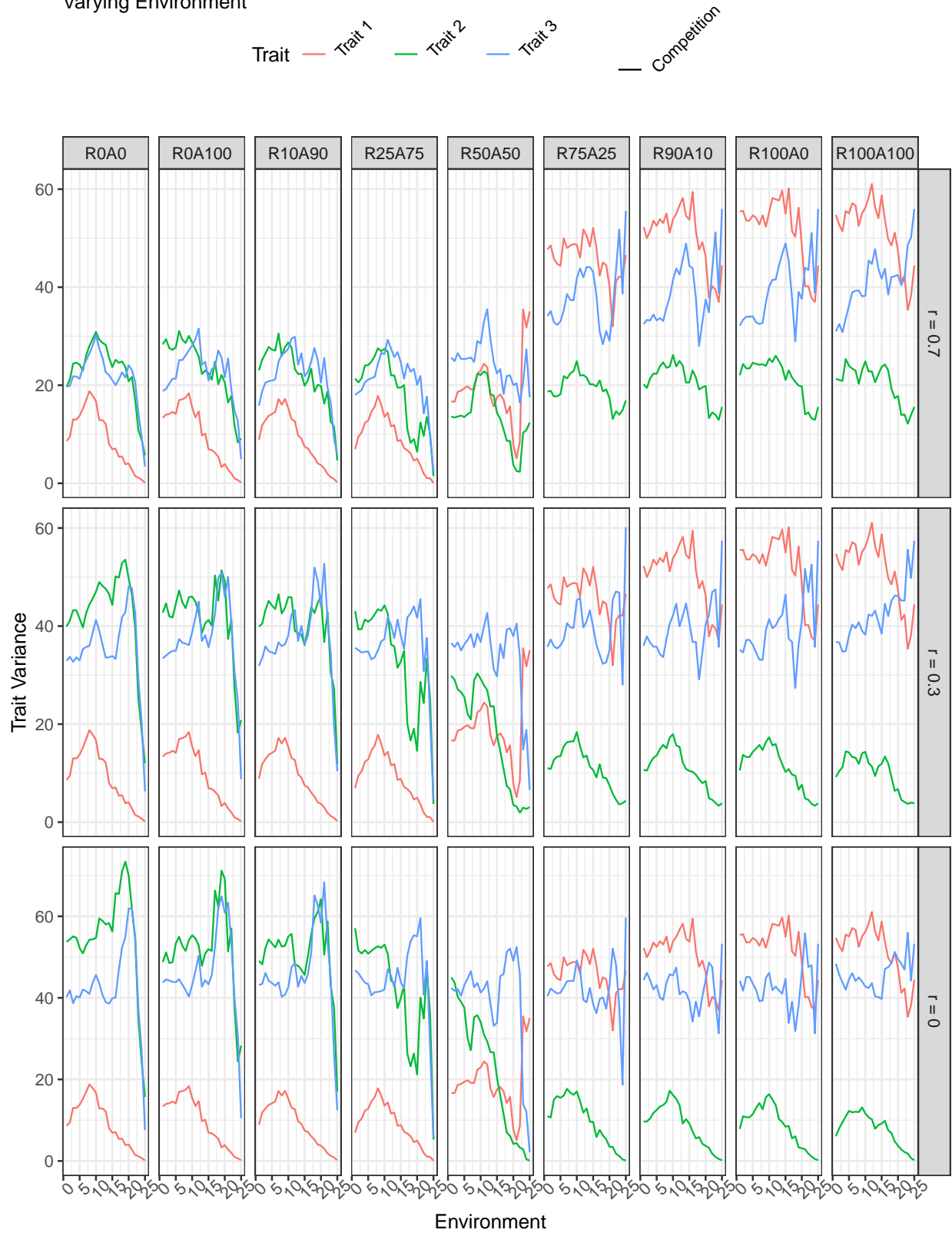
Trait variance against environment



Constant Environment



Varying Environment



CWV against environment

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Varying environment; 0% dispersal; Competition

