Summary

Introduction

Materials and Methods

Basic explanation of the models. We modeled a stage-structured population in two stages: immatures and matures. The demography is given by a transition matrix, with...

Under constant environment

Using Lande (2009), under weak selection we have:

$$\Delta\overline{z} = \frac{d\ln\overline{\lambda}(\overline{z})}{d\overline{z}} = \frac{1}{\overline{\lambda}(\overline{z})}\frac{d\overline{\lambda}(\overline{z})}{d\overline{z}} \tag{1}$$

And we have:

$$\begin{aligned}\overline{\lambda}(\overline{z}) &= \sum_{i,j} v_i u_j \overline{a_{ij}} \\ &= v_I u_I \overline{a_{II}} + v_I u_M \overline{a_{IM}} + v_M u_I \overline{a_{MI}} + v_M u_M \overline{a_{MM}}\end{aligned}$$

With $\overline{a_{ij}}$ the expected values of the coefficent of the transition matrix. Thus,

$$\begin{aligned}\overline{\lambda}(\overline{z}) = v_I u_I \left[\overline{f_1}(\overline{z}) m s_0 + (1-m)\overline{s_I}(\overline{z})\right] + v_I u_M s_0 \overline{f_2}(\overline{z}) \\ + v_M u_I m s_M + v_M u_M s_M \end{aligned} \tag{2}$$

$$\frac{d\overline{\lambda}(\overline{z})}{d\overline{z}} = v_I u_I \left[\frac{d\overline{f_1}(\overline{z})}{d\overline{z}} m s_0 + (1-m)\frac{d\overline{s_I}(\overline{z})}{d\overline{z}}\right] + v_I u_M s_0 \frac{d\overline{f_2}(\overline{z})}{d\overline{z}} \tag{3}$$

Because f_i and s_I are gaussians we can write the population means $\overline{f_i}$ and $\overline{s_I}$ easily.

$$\overline{f_1}(\overline{z}) = f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\exp\left(-\frac{(\overline{z}-\theta_f)^2}{2(\omega_f + P_I)}\right) \tag{4a}$$

$$\overline{f_2}(\overline{z}) = f_2(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_M}}\exp\left(-\frac{(\overline{z}-\theta_f)^2}{2(\omega_f + P_M)}\right) \tag{4b}$$

$$\overline{s_I}(\overline{z}) = s_I(\theta_s)\sqrt{\frac{\omega_s}{\omega_s + P_I}}\exp\left(-\frac{(\overline{z}-\theta_s)^2}{2(\omega_s + P_I)}\right) \tag{4c}$$

Thus we can derive these expression with respect to $\overline{z}$:

$$\begin{aligned}\frac{\partial\overline{f_1}(\overline{z})}{\partial\overline{z}} &= f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\frac{\partial\exp\left(-\frac{(\overline{z}-\theta_f)^2}{2(\omega_f+P_I)}\right)}{\partial\overline{z}} \\ &= f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\exp\left(-\frac{(\overline{z}-\theta_f)^2}{2(\omega_f + P_I)}\right)\frac{\theta_f - \overline{z}}{\omega_f + P_I} \\ &= \overline{f_1}(\overline{z})\frac{\theta_f - \overline{z}}{\omega_f + P_I}\end{aligned} \tag{5}$$

We obtain similar formulas for $\overline{f_2}$ and $\overline{s_I}$. Plugging (5) into (3) we have:

$$\frac{d\overline{\lambda}(\overline{z})}{d\overline{z}} = v_I u_I \left[\frac{\theta_f - \overline{z}}{\omega_f + P_I} m s_0 + (1-m)\frac{\theta_s - \overline{z}}{\omega_s + P_I}\right] + v_I u_M s_0 \frac{\theta_f - \overline{z}}{\omega_f + P_M} \tag{6}$$

Using (6) into (1) gives us after rearranging: We have for variations of phenotype, under weak selection:

$$\Delta\overline{z} = (\theta_f - \overline{z})\left[\frac{v_I u_I G_I s_0 m \overline{f_1}}{\lambda(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \overline{f_2}}{\lambda(P_M + \omega_f)}\right] + (\theta_s - \overline{z})\left[\frac{v_I u_I G_I \overline{s_I}(1-m)}{\lambda(P_I + \omega_s)}\right] \tag{7}$$

Within the square brackets, we see weighting average of fecundity and survival. Thus, we define them as γ_f and γ_s such as:

$$\gamma_f = \frac{v_I u_I s_0 m \overline{f_1}}{\lambda(P_I + \omega_f)} + \frac{v_I u_M \frac{G_M}{G_I} s_0 \overline{f_2}}{\lambda(P_M + \omega_f)} \tag{8a}$$

and

$$\gamma_s = \frac{v_I u_I \overline{s_I}(1-m)}{\lambda(P_I + \omega_s)} \tag{8b}$$

We end up having a simpler expression for $\Delta\overline{z}$ under constant environment:

$$\Delta\overline{z} = -G_I\left[\gamma_f(\overline{z} - \theta_f) + \gamma_s(\overline{z} - \theta_s)\right] \tag{9}$$

Under varying environment

From Engen et al. (2011), we derived equations for mean variation of phenotype on our model.

We supposed an auto-correlated fluctuating environment ϵ_t influencing optimums θ_i such as:

$$\begin{cases} \theta_i(t) = \overline{\theta}_i + \alpha_i \epsilon_t \\ \epsilon_{t+1} = (1-\rho)\overline{\epsilon} + \rho\epsilon_t + \xi \end{cases} \tag{10}$$

with $alpha_i$ the dependence factor of the optimum on the environment, ρ the auto-correlation coefficient of the environment, $\overline{\epsilon}$ the expected environment and ξ a gaussian noise vector with variance σ_ξ^2 and mean 0. We chose $\overline{\epsilon} = 0$ to simplify the calculations so that $\epsilon_{t+1} = \rho\epsilon_t + \xi$, then:

$$\begin{aligned} \theta_i(t+1) &= \overline{\theta}_i + \alpha_i \epsilon_{t+1} \\ &= \overline{\theta}_i + \alpha_i(\rho\epsilon_t + \xi) \\ &= \overline{\theta}_i + \alpha_i\rho\left(\frac{\theta_i(t) - \overline{\theta}_i}{\alpha_i}\right) + \alpha_i\xi \\ \theta_i(t+1) &= \overline{\theta}_i(1-\rho) + \rho\theta_i(t) + \alpha_i\xi \end{aligned} \tag{11}$$

Results

Subheading1

Subheading2

Discussion

Authors Contributions and Acknowledgments

References

(Barfield et al., 2011)

References

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