Summary

Introduction

Materials and Methods

Basic explanation of the models. We modeled a stage-structured population in two stages: immatures and matures. The demography is given by a transition matrix, with...

Under constant environment, no plasticity

Using Lande (2009), under weak selection we have:

$$\Delta\overline{z} = \frac{d\ln\overline{\lambda}(\overline{z})}{d\overline{z}} = \frac{1}{\overline{\lambda}(\overline{z})}\frac{d\overline{\lambda}(\overline{z})}{d\overline{z}} \tag{1}$$

And we have:

$$\begin{aligned}\overline{\lambda}(\overline{z}) &= \sum_{i,j} v_i u_j \overline{a_{ij}} \\ &= v_I u_I \overline{a_{II}} + v_I u_M \overline{a_{IM}} + v_M u_I \overline{a_{MI}} + v_M u_M \overline{a_{MM}}\end{aligned}$$

With $\overline{a_{ij}}$ the expected values of the coefficent of the transition matrix. Thus,

$$\begin{aligned}\overline{\lambda}(\overline{z}) &= v_I u_I \left[\overline{f_1}(\overline{z}) m s_0 + (1-m)\overline{s_I}(\overline{z})\right] + v_I u_M s_0 \overline{f_2}(\overline{z}) \\ &\quad + v_M u_I m s_M + v_M u_M s_M \end{aligned} \tag{2}$$

$$\frac{d\overline{\lambda}(\overline{z})}{d\overline{z}} = v_I u_I \left[\frac{d\overline{f_1}(\overline{z})}{d\overline{z}} m s_0 + (1-m)\frac{d\overline{s_I}(\overline{z})}{d\overline{z}}\right] + v_I u_M s_0 \frac{d\overline{f_2}(\overline{z})}{d\overline{z}} \tag{3}$$

Because f_i and s_I are gaussians we can write the population means $\overline{f_i}$ and $\overline{s_I}$ easily.

$$\overline{f_1}(\overline{z}) = f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\exp\left(-\frac{(\overline{z}-\theta_f)^2}{2(\omega_f+P_I)}\right) \tag{4a}$$

$$\overline{f_2}(\overline{z}) = f_2(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_M}}\exp\left(-\frac{(\overline{z}-\theta_f)^2}{2(\omega_f+P_M)}\right) \tag{4b}$$

$$\overline{s_I}(\overline{z}) = s_I(\theta_s)\sqrt{\frac{\omega_s}{\omega_s + P_I}}\exp\left(-\frac{(\overline{z}-\theta_s)^2}{2(\omega_s+P_I)}\right) \tag{4c}$$

Thus we can derive these expression with respect to $\overline{z}$:

$$\begin{aligned}\frac{\partial\overline{f_1}(\overline{z})}{\partial\overline{z}} &= f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\frac{\partial\exp\left(-\frac{(\overline{z}-\theta_f)^2}{2(\omega_f+P_I)}\right)}{\partial\overline{z}} \\ &= f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\exp\left(-\frac{(\overline{z}-\theta_f)^2}{2(\omega_f+P_I)}\right)\frac{\theta_f-\overline{z}}{\omega_f+P_I} \\ &= \overline{f_1}(\overline{z})\frac{\theta_f-\overline{z}}{\omega_f+P_I}\end{aligned} \tag{5}$$

We obtain similar formulas for $\overline{f_2}$ and $\overline{s_I}$. Plugging (5) into (3) we have:

$$\frac{d\overline{\lambda}(\overline{z})}{d\overline{z}} = v_I u_I \left[\frac{\theta_f - \overline{z}}{\omega_f + P_I} m s_0 + (1-m)\frac{\theta_s - \overline{z}}{\omega_s + P_I}\right] + v_I u_M s_0 \frac{\theta_f - \overline{z}}{\omega_f + P_M} \tag{6}$$

Using (6) into (1) gives us after rearranging: We have for variations of phenotype, under weak selection:

$$\Delta\overline{z} = (\theta_f - \overline{z})\left[\frac{v_I u_I G_I s_0 m \overline{f_1}}{\lambda(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \overline{f_2}}{\lambda(P_M + \omega_f)}\right] + (\theta_s - \overline{z})\left[\frac{v_I u_I G_I \overline{s_I}(1-m)}{\lambda(P_I + \omega_s)}\right] \tag{7}$$

Within the square brackets, we see weighting average of fecundity and survival. Thus, we define them as γ_f and γ_s such as:

$$\gamma_f = \frac{v_I u_I s_0 m \overline{f_1}}{\lambda(P_I + \omega_f)} + \frac{v_I u_M \frac{G_M}{G_I} s_0 \overline{f_2}}{\lambda(P_M + \omega_f)} \tag{8a}$$

and

$$\gamma_s = \frac{v_I u_I \overline{s_I}(1-m)}{\lambda(P_I + \omega_s)} \tag{8b}$$

We end up having a simpler expression for $\Delta\overline{z}$ under constant environment:

$$\begin{aligned}\Delta\overline{z} &= -G_I\left[\gamma_f(\overline{z} - \theta_f) + \gamma_s(\overline{z} - \theta_s)\right] \\ \Delta\overline{z} &= -G_I \gamma(\overline{z} - \theta_v)\end{aligned} \tag{9}$$

with

$$\gamma = \gamma_f + \gamma_s \tag{10}$$

$$\theta_v = \frac{\frac{\gamma_f}{\gamma_s}\theta_f + \theta_s}{\frac{\gamma_f}{\gamma_s} + 1} \tag{11}$$

Under varying environment, without plasticity

From Engen et al. (2011), we derived equations for mean variation of phenotype on our model.

We supposed an auto-correlated fluctuating environment ϵ_t influencing optimums θ_i such as:

$$\begin{cases}\theta_i(t) = \overline{\theta}_i + \alpha_i \epsilon_t \\ \epsilon_{t+1} = (1-\rho)\overline{\epsilon} + \rho\epsilon_t + \xi\end{cases} \tag{12}$$

with α_i the dependence factor of the optimum on the environment, ρ the auto-correlation coefficient of the environment, $\overline{\epsilon}$ the expected environment and ξ a gaussian noise vector with variance σ_ξ^2 and mean 0. We chose $\overline{\epsilon} = 0$ to simplify the calculations so that $\epsilon_{t+1} = \rho\epsilon_t + \xi$, we can see that:

$$\begin{aligned}\theta_i(t+1) &= \overline{\theta}_i + \alpha_i \epsilon_{t+1} \\ &= \overline{\theta}_i + \alpha_i(\rho\epsilon_t + \xi) \\ &= \overline{\theta}_i + \alpha_i \rho\left(\frac{\theta_i(t) - \overline{\theta}_i}{\alpha_i}\right) + \alpha_i \xi \\ \theta_i(t+1) &= \overline{\theta}_i(1-\rho) + \rho\theta_i(t) + \alpha_i \xi\end{aligned} \tag{13}$$

The auto-correlation in the environment ϵ_t causes θ_i to be auto-correlated with the same correlation coefficient ρ.

Using the same approach as in a constant environment, under weak selection, we end up having a similar equation than (7) but with optimum depending on environment:

$$\Delta\overline{z}_t = (\theta_f(t) - \overline{z_t})\left[\frac{v_I u_I G_I s_0 m \overline{f_1}}{\lambda_t(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \overline{f_2}}{\lambda_t(P_M + \omega_f)}\right] + (\theta_s(t) - \overline{z_t})\left[\frac{v_I u_I G_I \overline{s_I}(1-m)}{\lambda_t(P_I + \omega_s)}\right] \quad (14)$$

Plugging (12) in (14) we obtain

$$\Delta\overline{z}_t = (\overline{\theta}_f + \alpha_f\epsilon_t - \overline{z_t})\left[\frac{v_I u_I G_I s_0 m \overline{f_1}}{\lambda_t(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \overline{f_2}}{\lambda_t(P_M + \omega_f)}\right] + (\overline{\theta}_s + \alpha_s\epsilon_t - \overline{z_t})\left[\frac{v_I u_I G_I \overline{s_I}(1-m)}{\lambda_t(P_I + \omega_s)}\right] \quad (15a)$$

Defining the same γ_f, γ_s and θ_v as in (8a), (8b) and (11), respectively:

$$\begin{aligned}
\Delta\overline{z}_t &= G_I\left[(\overline{\theta}_f + \alpha_f\epsilon_t - \overline{z_t})\gamma_f + (\overline{\theta}_s + \alpha_s\epsilon_t - \overline{z_t})\gamma_s\right] \\
&= -G_I\left[\gamma_f(\overline{z_t} - \theta_f) + \gamma_s(\overline{z_t} - \theta_s)\right] - G_I\epsilon_t\left(\gamma_f\alpha_f + \gamma_s\alpha_s\right) \\
\Delta\overline{z}_t &= \Delta\overline{z} + G_I\epsilon_t\left(\gamma_f\alpha_f + \gamma_s\alpha_s\right) && (15b) \\
&= -G_I\gamma(\overline{z_t} - \theta_v) + G_I\epsilon_t\left(\gamma_f\alpha_f + \gamma_s\alpha_s\right) \\
&= -G_I\gamma\left(\overline{z_t} - \theta_v - \epsilon_t\frac{\gamma_f\alpha_f + \gamma_s\alpha_s}{\gamma}\right) \\
\Delta\overline{z}_t &= -G_I\gamma(\overline{z_t} - \theta_v - \alpha_v\epsilon_t) && (15c)
\end{aligned}$$

With α_v a weighted component between α_f and α_s, defined in similar fashion as θ_v in (11):

$$\begin{aligned}
\alpha_v &= \frac{\gamma_f\alpha_f + \gamma_s\alpha_s}{\gamma} \\
&= \frac{\gamma_f\alpha_f + \gamma_s\alpha_s}{\gamma_f + \gamma_s} \\
\alpha_v &= \frac{\frac{\gamma_f}{\gamma_s}\alpha_f + \alpha_s}{\frac{\gamma_f}{\gamma_s} + 1} && (16)
\end{aligned}$$

Estimating variance of $\overline{z}_t$

Taking (15c) we can estimate variance of $\overline{z}_t$. Using the same process as Engen et al. (2011). Indeed, (15c) has the form:

$$\Delta A_t = -DA_t + e_t \quad (17)$$

with $A_t = \overline{z_t} - \theta_v$, $D = G_I\gamma$ and $e_t = D\alpha_v\epsilon_t$. (17) has a stationary solution:

$$A_{t+1} = (1-D)^{t+1}A_0 + \sum_{r=0}^{t}(1-D)^r e_{t-r} \quad (18a)$$

If we consider the evolution of A_t over a long time, (18a) becomes, because $(1-D) < 1$:

$$A_t = \sum_{r=0}^{\infty} e_{t-r}(1-D)^r \quad (18b)$$

We want to estimate how will $\overline{z}_t$ move away from the mean because of environmental fluctuations, that is why we compute its variance. From (18a):

$$\begin{aligned}
\mathrm{Var}(A_t) &= \mathrm{Var}\left[\sum_{r=0}^{\infty} e_{t-r}(1-D)^r\right] \\
&= \sum_{r=0}^{\infty} \mathrm{Var}\left[e_{t-r}(1-D)^r\right] \\
&= \sum_{r=0}^{\infty} \mathrm{Var}\left[\epsilon_{t-r} D \alpha_v (1-D)^r\right] \\
&\stackrel{\text{def}}{=} \sigma_\epsilon^2 D^2 \alpha_v^2 \sum_{r=0}^{\infty} (1-D)^{2r} \\
\mathrm{Var}(A_t) &= \sigma_\epsilon^2 D^2 \alpha_v^2 \frac{1}{1-(1-D)^2} \\
\mathrm{Var}(\overline{z_t}) &\stackrel{\text{def}}{=} \sigma_\epsilon^2 G_I^2 \gamma^2 \alpha_v^2 \frac{1}{G_I\gamma(2-G_I\gamma)} \\
\mathrm{Var}(\overline{z_t}) &\stackrel{\gamma \to 0}{=} \frac{1}{2} G_I \gamma \sigma_\epsilon^2 \alpha_v^2
\end{aligned} \tag{19}$$

Results

Subheading1

Subheading2

Discussion

Authors Contributions and Acknowledgments

References

(Barfield et al., 2011)

References

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