Summary

Introduction

Materials and Methods

Basic explanation of the models. We modeled a stage-structured population in two stages: immatures and matures. The demography is given by a transition matrix, with...

From Engen et al. (2011), we derived equations for mean variation of phenotype on our model.

We have for variations of phenotype, under weak selection:

$$\Delta\overline{z} = (\theta_f - \overline{z})\left[\frac{v_I u_I G_I s_0 m \overline{f_1}}{\lambda(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \overline{f_2}}{\lambda(P_M + \omega_f)}\right] + (\theta_s - \overline{z})\left[\frac{v_I u_I G_I \overline{s_I}(1-m)}{\lambda(P_I + \omega_s)}\right] \tag{1}$$

Within the square brackets, we see weighting average of fecundity and survival. Thus, we define them as γ_f and γ_s such as:

$$\gamma_f = \frac{v_I u_I G_I s_0 m \overline{f_1}}{\lambda(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \overline{f_2}}{\lambda(P_M + \omega_f)} \tag{2a}$$

and

$$\gamma_s = \frac{v_I u_I G_I \overline{s_I}(1-m)}{\lambda(P_I + \omega_s)} \tag{2b}$$

We supposed an auto-correlated fluctuating environment influencing optimum such as $\theta_f = \overline{\theta}_f + \alpha_f \epsilon_t$. With $\epsilon_{t+1} = (1-\rho)\overline{\epsilon} + \rho\epsilon_t + \xi$ with ξ a gaussian noise vector with variance σ_ξ^2 and mean 0.

Using Lande (2009), under weak selection we have:

$$\Delta\overline{z} = \frac{d\ln\overline{\lambda}(\overline{z})}{d\overline{z}} = \frac{1}{\overline{\lambda}(\overline{z})}\frac{d\overline{\lambda}(\overline{z})}{d\overline{z}} \tag{3}$$

And we have:

$$\begin{aligned}\overline{\lambda}(\overline{z}) &= \sum_{i,j} v_i u_j \overline{a_{ij}} \\ &= v_I u_I \overline{a_{II}} + v_I u_M \overline{a_{IM}} + v_M u_I \overline{a_{MI}} + v_M u_M \overline{a_{MM}}\end{aligned}$$

With $\overline{a_{ij}}$ the expected values of the coefficent of the transition matrix. Thus,

$$\begin{aligned}\overline{\lambda}(\overline{z}) = v_I u_I &\left[\overline{f_1}(\overline{z}) m s_0 + (1-m)\overline{s_I}(\overline{z})\right] + v_I u_M s_0 \overline{f_2}(\overline{z}) \\ &+ v_M u_I m s_M + v_M u_M s_M\end{aligned} \tag{4}$$

$$\frac{d\overline{\lambda}(\overline{z})}{d\overline{z}} = v_I u_I\left[\frac{d\overline{f_1}(\overline{z})}{d\overline{z}} m s_0 + (1-m)\frac{d\overline{s_I}(\overline{z})}{d\overline{z}}\right] + v_I u_M s_0 \frac{d\overline{f_2}(\overline{z})}{d\overline{z}} \tag{5}$$

Because f_i and s_I are gaussians we can write the population means $\overline{f_i}$ and $\overline{s_I}$ easily.

$$\overline{f_1}(\overline{z}) = f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\exp\left(-\frac{(\overline{z_I} - \theta_f)^2}{2(\omega_f + P_I)}\right) \tag{6a}$$

$$\overline{f_2}(\overline{z}) = f_2(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_M}}\exp\left(-\frac{(\overline{z_M} - \theta_f)^2}{2(\omega_f + P_M)}\right) \tag{6b}$$

$$\overline{s_I}(\overline{z}) = s_I(\theta_s)\sqrt{\frac{\omega_s}{\omega_s + P_I}}\exp\left(-\frac{(\overline{z_I} - \theta_s)^2}{2(\omega_s + P_I)}\right) \tag{6c}$$

Thus we can derive these expression with respect to $\overline{z}$:

$$\begin{aligned}
\frac{\partial \overline{f_1}(\overline{z})}{\partial \overline{z}} &= f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\frac{\partial \exp\left(-\frac{(\overline{z_I}-\theta_f)^2}{2(\omega_f+P_I)}\right)}{\partial \overline{z}} \\
&= f_1(\theta_f)\sqrt{\frac{\omega_f}{\omega_f + P_I}}\exp\left(-\frac{(\overline{z_I}-\theta_f)^2}{2(\omega_f+P_I)}\right)\frac{\theta_f - \overline{z_I}}{\omega_f + P_I} \\
&= \overline{f_1}(\overline{z})\frac{\theta_f - \overline{z_I}}{\omega_f + P_I}
\end{aligned} \tag{7}$$

We obtain similar formulas for $\overline{f_2}$ and $\overline{s_I}$. Plugging (7) into (5)

Results

Subheading1

Subheading2

Discussion

Authors Contributions and Acknowledgments

References

(Barfield et al., 2011)

References

Barfield, M., Holt, R. D. and Gomulkiewicz, R. (2011). Evolution in Stage-Structured Populations (2 versions). The American Naturalist *177*, 397--409.

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