

# Summary

## Introduction

## Materials and Methods

Basic explanation of the models. We modeled a stage-structured population in two stages: immatures and matures. The demography is given by a transition matrix, with...

### Under constant environment, no plasticity

Using Lande (2009), under weak selection we have:

$$\Delta \bar{z} = \frac{d \ln \bar{\lambda}(\bar{z})}{d \bar{z}} = \frac{1}{\bar{\lambda}(\bar{z})} \frac{d \bar{\lambda}(\bar{z})}{d \bar{z}} \quad (1)$$

And we have:

$$\begin{aligned} \bar{\lambda}(\bar{z}) &= \sum_{i,j} v_i u_j \bar{a}_{ij} \\ &= v_I u_I \bar{a}_{II} + v_I u_M \bar{a}_{IM} + v_M u_I \bar{a}_{MI} + v_M u_M \bar{a}_{MM} \end{aligned}$$

With  $\bar{a}_{ij}$  the expected values of the coefficient of the transition matrix. Thus,

$$\begin{aligned} \bar{\lambda}(\bar{z}) &= v_I u_I [\bar{f}_1(\bar{z}) m s_0 + (1 - m) \bar{s}_I(\bar{z})] + v_I u_M s_0 \bar{f}_2(\bar{z}) \\ &\quad + v_M u_I m s_M + v_M u_M s_M \end{aligned} \quad (2)$$

$$\frac{d \bar{\lambda}(\bar{z})}{d \bar{z}} = v_I u_I \left[ \frac{d \bar{f}_1(\bar{z})}{d \bar{z}} m s_0 + (1 - m) \frac{d \bar{s}_I(\bar{z})}{d \bar{z}} \right] + v_I u_M s_0 \frac{d \bar{f}_2(\bar{z})}{d \bar{z}} \quad (3)$$

Because  $f_i$  and  $s_I$  are gaussians we can write the population means  $\bar{f}_i$  and  $\bar{s}_I$  easily.

$$\bar{f}_1(\bar{z}) = f_1(\theta_f) \sqrt{\frac{\omega_f}{\omega_f + P_I}} \exp \left( -\frac{(\bar{z} - \theta_f)^2}{2(\omega_f + P_I)} \right) \quad (4a)$$

$$\bar{f}_2(\bar{z}) = f_2(\theta_f) \sqrt{\frac{\omega_f}{\omega_f + P_M}} \exp \left( -\frac{(\bar{z} - \theta_f)^2}{2(\omega_f + P_M)} \right) \quad (4b)$$

$$\bar{s}_I(\bar{z}) = s_I(\theta_s) \sqrt{\frac{\omega_s}{\omega_s + P_I}} \exp \left( -\frac{(\bar{z} - \theta_s)^2}{2(\omega_s + P_I)} \right) \quad (4c)$$

Thus we can derive these expression with respect to  $\bar{z}$ :

$$\begin{aligned} \frac{\partial \bar{f}_1(\bar{z})}{\partial \bar{z}} &= f_1(\theta_f) \sqrt{\frac{\omega_f}{\omega_f + P_I}} \frac{\partial \exp \left( -\frac{(\bar{z} - \theta_f)^2}{2(\omega_f + P_I)} \right)}{\partial \bar{z}} \\ &= f_1(\theta_f) \sqrt{\frac{\omega_f}{\omega_f + P_I}} \exp \left( -\frac{(\bar{z} - \theta_f)^2}{2(\omega_f + P_I)} \right) \frac{\theta_f - \bar{z}}{\omega_f + P_I} \\ &= \bar{f}_1(\bar{z}) \frac{\theta_f - \bar{z}}{\omega_f + P_I} \end{aligned} \quad (5)$$

We obtain similar formulas for  $\bar{f}_2$  and  $\bar{s}_I$ . Plugging (5) into (3) we have:

$$\frac{d\bar{\lambda}(\bar{z})}{d\bar{z}} = v_I u_I \left[ \frac{\theta_f - \bar{z}}{\omega_f + P_I} m s_0 + (1 - m) \frac{\theta_s - \bar{z}}{\omega_s + P_I} \right] + v_I u_M s_0 \frac{\theta_f - \bar{z}}{\omega_f + P_M} \quad (6)$$

Using (6) into (1) gives us after rearranging: We have for variations of phenotype, under weak selection:

$$\Delta \bar{z} = (\theta_f - \bar{z}) \left[ \frac{v_I u_I G_I s_0 m \bar{f}_1}{\lambda(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \bar{f}_2}{\lambda(P_M + \omega_f)} \right] + (\theta_s - \bar{z}) \left[ \frac{v_I u_I G_I \bar{s}_I (1 - m)}{\lambda(P_I + \omega_s)} \right] \quad (7)$$

Within the square brackets, we see weighting average of fecundity and survival. Thus, we define them as  $\gamma_f$  and  $\gamma_s$  such as:

$$\gamma_f = \frac{v_I u_I s_0 m \bar{f}_1}{\lambda(P_I + \omega_f)} + \frac{v_I u_M \frac{G_M}{G_I} s_0 \bar{f}_2}{\lambda(P_M + \omega_f)} \quad (8a)$$

and

$$\gamma_s = \frac{v_I u_I \bar{s}_I (1 - m)}{\lambda(P_I + \omega_s)} \quad (8b)$$

We end up having a simpler expression for  $\Delta \bar{z}$  under constant environment:

$$\begin{aligned} \Delta \bar{z} &= -G_I [\gamma_f (\bar{z} - \theta_f) + \gamma_s (\bar{z} - \theta_s)] \\ \Delta \bar{z} &= -G_I \gamma (\bar{z} - \theta_v) \end{aligned} \quad (9)$$

with

$$\gamma = \gamma_f + \gamma_s \quad (10)$$

$$\theta_v = \frac{\frac{\gamma_f}{\gamma_s} \theta_f + \theta_s}{\frac{\gamma_f}{\gamma_s} + 1} \quad (11)$$

## Under varying environment, without plasticity

From Engen et al. (2011), we derived equations for mean variation of phenotype on our model.

We supposed an auto-correlated fluctuating environment  $\epsilon_t$  influencing optimums  $\theta_i$  such as:

$$\begin{cases} \theta_i(t) = \bar{\theta}_i + \alpha_i \epsilon_t \\ \epsilon_{t+1} = (1 - \rho) \bar{\epsilon} + \rho \epsilon_t + \xi \end{cases} \quad (12)$$

with  $\alpha_i$  the dependence factor of the optimum on the environment,  $\rho$  the auto-correlation coefficient of the environment,  $\bar{\epsilon}$  the expected environment and  $\xi$  a gaussian noise vector with variance  $\sigma_\xi^2$  and mean 0. We chose  $\bar{\epsilon} = 0$  to simplify the calculations so that  $\epsilon_{t+1} = \rho \epsilon_t + \xi$ , then:

$$\begin{aligned} \theta_i(t+1) &= \bar{\theta}_i + \alpha_i \epsilon_{t+1} \\ &= \bar{\theta}_i + \alpha_i (\rho \epsilon_t + \xi) \\ &= \bar{\theta}_i + \alpha_i \rho \left( \frac{\theta_i(t) - \bar{\theta}_i}{\alpha_i} \right) + \alpha_i \xi \\ \theta_i(t+1) &= \bar{\theta}_i (1 - \rho) + \rho \theta_i(t) + \alpha_i \xi \end{aligned} \quad (13)$$

Using the same approach as in a constant environment, under weak selection, we end up having a similar equation than (7) but with optimum depending on environment:

$$\Delta \bar{z}_t = (\theta_f(t) - \bar{z}) \left[ \frac{v_I u_I G_I s_0 m \bar{f}_1}{\lambda_t(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \bar{f}_2}{\lambda_t(P_M + \omega_f)} \right] + (\theta_s(t) - \bar{z}) \left[ \frac{v_I u_I G_I \bar{s}_I (1 - m)}{\lambda_t(P_I + \omega_s)} \right] \quad (14)$$

Plugging (12) in (14) we obtain

$$\Delta \bar{z}_t = (\bar{\theta}_f + \alpha_f \epsilon_t - \bar{z}) \left[ \frac{v_I u_I G_I s_0 m \bar{f}_1}{\lambda_t(P_I + \omega_f)} + \frac{v_I u_M G_M s_0 \bar{f}_2}{\lambda_t(P_M + \omega_f)} \right] + (\bar{\theta}_s + \alpha_s \epsilon_t - \bar{z}) \left[ \frac{v_I u_I G_I \bar{s}_I (1 - m)}{\lambda_t(P_I + \omega_s)} \right] \quad (15a)$$

Defining the same  $\gamma_f$  and  $\gamma_s$  as in (8a) and (8b):

$$\begin{aligned} \Delta \bar{z}_t &= G_I [(\bar{\theta}_f + \alpha_f \epsilon_t - \bar{z}) \gamma_f + (\bar{\theta}_s + \alpha_s \epsilon_t - \bar{z}) \gamma_s] \\ &= -G_I [\gamma_f (\bar{z} - \theta_f) + \gamma_s (\bar{z} - \theta_s)] - G_I \epsilon_t (\gamma_f \alpha_f + \gamma_s \alpha_s) \\ \Delta \bar{z}_t &= \Delta \bar{z} + G_I \epsilon_t (\gamma_f \alpha_f + \gamma_s \alpha_s) \end{aligned} \quad (15b)$$

$$\begin{aligned} &= -G_I \gamma (\bar{z} - \theta_v) + G_I \epsilon_t (\gamma_f \alpha_f + \gamma_s \alpha_s) \\ &= -G_I \gamma \left( \bar{z} - \theta_v - \epsilon_t \frac{\gamma_f \alpha_f + \gamma_s \alpha_s}{\gamma} \right) \\ \Delta \bar{z}_t &= -G_I \gamma (\bar{z} - \theta_v - \alpha_v \epsilon_t) \end{aligned} \quad (15c)$$

With  $\alpha_v$  a weighted component between  $\alpha_f$  and  $\alpha_s$ , defined in similar fashion as  $\theta_v$  in (11):

$$\begin{aligned} \alpha_v &= \frac{\gamma_f \alpha_f + \gamma_s \alpha_s}{\gamma} \\ &= \frac{\gamma_f \alpha_f + \gamma_s \alpha_s}{\gamma_f + \gamma_s} \\ \alpha_v &= \frac{\frac{\gamma_f}{\gamma_s} \alpha_f + \alpha_s}{\frac{\gamma_f}{\gamma_s} + 1} \end{aligned} \quad (16)$$

### Estimating variance of $\bar{z}_t$

Taking (15c) we can estimate variance of  $\bar{z}_t$ . Using the same process as Engen et al. (2011). Indeed, (15c) has the form:

$$\Delta A_t = -D A_t + e_t \quad (17)$$

with  $A_t = \bar{z} - \theta_v$ ,  $D = G_I \gamma$  and  $e_t = D \alpha_v \epsilon_t$ . (17) has a stationary solution:

$$A_{t+1} = (1 - D)^{t+1} A_0 + \sum_{r=0}^t (1 - D)^r e_{t-r} \quad (18a)$$

If we consider the evolution of  $A_t$  over a long time, (18a) becomes, because  $(1 - D) \inf 1$ :

$$A_t = \sum_{r=0}^{\infty} e_{t-r} (1 - D)^r \quad (18b)$$

We want to estimate how will  $\bar{z}_t$  move away from the mean because of environmental fluctuations, that is why we compute its variance. From (18a):

$$\begin{aligned}
\text{Var}(A_t) &= \text{Var} \left[ \sum_{r=0}^{\infty} e_{t-r} (1-D)^r \right] \\
&= \sum_{r=0}^{\infty} \text{Var} [e_{t-r} (1-D)^r] \\
&= \sum_{r=0}^{\infty} \text{Var} [\epsilon_{t-r} D \alpha_v (1-D)^r] \\
&\stackrel{\text{def}}{=} \sigma_{\epsilon}^2 D^2 \alpha_v^2 \sum_{r=0}^{\infty} (1-D)^{2r} \\
\text{Var}(A_t) &= \sigma_{\epsilon}^2 D^2 \alpha_v^2 \frac{1}{1 - (1-D)^2} \\
\text{Var}(\bar{z}) &\stackrel{\text{def}}{=} \sigma_{\epsilon}^2 G_I^2 \gamma^2 \alpha_v^2 \frac{1}{G_I \gamma (2 - G_I \gamma)} \\
\text{Var}(\bar{z}) &\stackrel{\gamma \rightarrow 0}{=} \frac{1}{2} G_I \gamma \sigma_{\epsilon}^2 \alpha_v^2
\end{aligned} \tag{19}$$

## Results

### Subheading1

### Subheading2

## Discussion

## Authors Contributions and Acknowledgments

## References

(Barfield et al., 2011)

## References

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