Chapter 2 Notes and Exercises

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This document are notes taken when reading *Statistical Rethinking* from Richard McElreath

Practice

answer questions

Easy

2E1

(2) Pr(rain|Monday)

2E2

(3) The probability that it is Monday, given that it is raining

2E3

Probability that it is Monday given that it is raining:

(1) Pr(Monday|rain) and (4) (from Bayes' theorem)

$$\frac{\Pr(\text{rain}|\text{Monday})\Pr(\text{Monday})}{\Pr(\text{rain})} \tag{1}$$

Medium

2M1

```r
globe_water = function(n, number_W, size) {
  p_grid = seq(0, 1, length.out = n)

  prior = rep(1, n)

  likelihood = dbinom(number_W, size = size, prob = p_grid)

  non_std_post = likelihood * prior

  posterior = non_std_post / sum(non_std_post)

  cat("Most probable percentage of water is: ", p_grid[which.max(posterior)])

  plot(p_grid, posterior, type = "b", xlab = "Percentage of water",
```

```r
       ylab = "Density")
}

par(mfrow = c(3, 1), mai = rep(0.3,4))

globe_water(50, 3, 3)
```

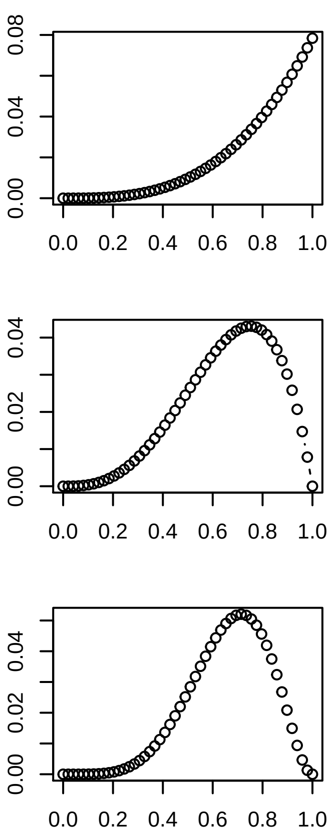
```
## Most probable percentage of water is:  1
```

```r
globe_water(50, 3, 4)
```

```
## Most probable percentage of water is:  0.755102
```

```r
globe_water(50, 5, 7)
```

```
## Most probable percentage of water is:  0.7142857
```



2M2

Same question but changed prior

```r
globe_water2 = function(n, number_W, size) {
  p_grid = seq(0, 1, length.out = n)

  prior = c(rep(0, n/2), rep(1, n/2))

  likelihood = dbinom(number_W, size = size, prob = p_grid)

  non_std_post = likelihood * prior

  posterior = non_std_post / sum(non_std_post)

  cat("Most probable percentage of water is: ", p_grid[which.max(posterior)])

  plot(p_grid, posterior, type = "b", xlab = "Percentage of water",
       ylab = "Probability Density")
}

par(mfrow = c(3, 1), mai = rep(0.3,4))

globe_water2(50, 3, 3)
```

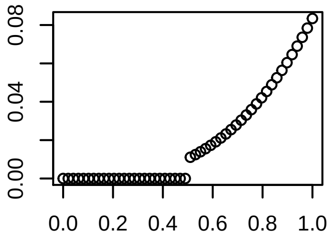
```
## Most probable percentage of water is:  1
```

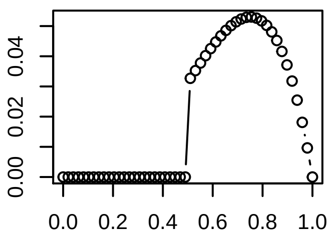
```r
globe_water2(50, 3, 4)
```

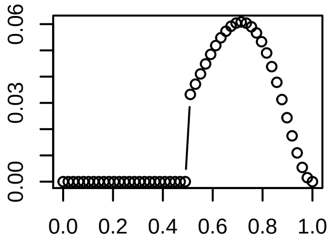
```
## Most probable percentage of water is:  0.755102
```

```r
globe_water2(50, 5, 7)
```

```
## Most probable percentage of water is:  0.7142857
```







2M3

From Bayes' Theorem:

$$\begin{aligned} \Pr(\text{Earth}|\text{land}) &= \frac{\Pr(\text{land}|\text{Earth})\Pr(\text{Earth})}{\Pr(\text{land})} && (2) \\ &= \frac{0.3 \times 0.5}{\frac{1.3}{2}} && (3) \\ &= 0.23 && (4) \end{aligned}$$

2M4

Three cards B/B, B/W, B/B 3 ways of having a black side up (B/B two sides and one side of B/W) then only 2 ways of having the other side black (B1/B2 and B2/B1) so

$$P = \frac{2}{3} \quad (5)$$

2M5

Same as above but 5 ways of having a black side up and only 4 ways then to have the other side black so:

$$P = \frac{4}{5} \quad (6)$$

2M6

As above but this time the deck could be as follow: one B/B card, two B/W cards and three W/W cards. 4 ways of a black side up (the two B/W cards and the two sides of the B/B card) and only then 2 ways of having the other side black

2M7

By counting everything there is 8 ways to have black side up first then white side up, of which only 6 have first a card with two black sides so $P = 6/8$

Hard

2H1

$$\begin{aligned} \mathrm{P}(\text{twins}) &= \mathrm{P}(\text{twins}|A)\mathrm{P}(A) + \mathrm{P}(\text{twins}|B)\mathrm{P}(B) && (7) \\ &= 0.1 \times 0.5 + 0.2 \times 0.5 && (8) \\ &= 0.15 && (9) \end{aligned}$$

2H2

From Bayes' theorem:

$$\begin{aligned} \mathrm{P}(A|\text{twins}) &= \frac{\mathrm{P}(\text{twins}|A)\mathrm{P}(A)}{\mathrm{P}(\text{twins})} && (10) \\ &= \frac{0.1 \times 0.5}{0.15} && (11) \\ &= \frac{1}{3} && (12) \end{aligned}$$

2H3

From Bayes' theorem:

$$\begin{aligned} \mathrm{P}(A|\text{single}, \text{twins}) &= \frac{\mathrm{P}(\text{single}, \text{twins}|A)\mathrm{P}(A)}{\mathrm{P}(\text{single}, \text{twins})} && (13) \\ &= \frac{\mathrm{P}(\text{single}|A)\mathrm{P}(\text{twins}|A)\mathrm{P}(A)}{\mathrm{P}(\text{single}|A)\mathrm{P}(\text{twins}|A)\mathrm{P}(A) + \mathrm{P}(\text{single}|B)\mathrm{P}(\text{twins}|B)\mathrm{P}(B)} && (14) \\ &= \frac{0.9 \times 0.1 \times 0.5}{0.9 \times 0.1 \times 0.5 + 0.8 \times 0.2 \times 0.5} && (15) \\ &= 0.36 && (16) \end{aligned}$$

2H4

From Bayes' theorem: (with idA being the event "panda indetified as species A")

$$\begin{aligned}
\mathrm{P}(A|idA) &= \frac{\mathrm{P}(idA|A)\mathrm{P}(A)}{\mathrm{P}(idA)} && (17)\\
&= \frac{\mathrm{P}(idA|A)\mathrm{P}(A)}{\mathrm{P}(idA|A)\mathrm{P}(A) + \mathrm{P}(idA|B)\mathrm{P}(B)} && (18)\\
&= \frac{0.8 \times 0.5}{0.8 \times 0.5 + 0.35 \times 0.5} && (19)\\
&= 0.6956522 && (20)
\end{aligned}$$