Got Tensor Contractions?

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1 DEFINITIONS

- 1. The values of consecutive indices which are not separated by commas are assumed to form an ascending sequence (partially symmetrical storage).
- 2. Index groups separated by commas run independently.
- 3. Index ranges:

m,n,l: full occupied orbital range (e.g., [0..99]);

e,f,d: full virtual orbital range (e.g., [100..999]);

I,J,K: a small (active) subrange of the occupied orbital range (e.g., [90..99]);

A,B,C: a small (active) subrange of the virtual orbital range (e.g., [990..999]);

i,j,k: the occupied orbital range excluding the active subrange (e.g., [0..89]);

a,b,c: the virtual orbital range excluding the active subrange (e.g., [100..989]).

4. Despite multiple forms in which S/L/C tensors may appear in contractions, the only unique elements of these tensors are those which have both upper and lower indices ordered in an ascending order (separately).

2 TENSOR EXPRESSIONS

(1)
$$3: Z += H_{d_1}^{l_1} S_{l_1}^{d_1}$$

(2)
$$9: Z += H_{d_1 d_2}^{l_1 l_2} S_{l_1 l_2}^{d_1 d_2}$$

(3)
$$18: Z += H_{d_1, d_2}^{l_1, l_2} S_{l_1}^{d_1} S_{l_2}^{d_2} \cdot 1/2$$

$$(4) 1: Z_{m_1}^{e_1} + = H_{m_1}^{e_1}$$

(5)
$$4: Z_{m_1}^{e_1} + = H_{d_1}^{e_1} S_{m_1}^{d_1}$$

(6)
$$5: Z_{m_1}^{e_1} + = H_{m_1}^{l_1} S_{l_1}^{e_1} \cdot -1/1$$

(7)
$$6: Z_{m_1}^{e_1} + = H_{m_1, d_1}^{e_1, l_1} S_{l_1}^{d_1}$$

(8)
$$10: Z_{m_1}^{e_1} + = H_{d_1}^{l_1} S_{m_1, l_1}^{e_1, d_1}$$

(9)
$$11: Z_{m_1}^{e_1} += H_{d_1 d_2}^{e_1, l_1} S_{m_1, l_1}^{d_1 d_2}$$

$$(10) 12: Z_{m_1}^{e_1} + = H_{m_1,d_1}^{l_1 l_2} S_{l_1 l_2}^{e_1,d_1} \cdot -1/1$$

$$(11) \qquad 19: Z_{m_1}^{e_1} + = H_{d_1}^{l_1} S_{l_1}^{e_1} S_{m_1}^{d_1} \cdot -1/1$$

$$(12) 20: Z_{m_1}^{e_1} + = H_{d_1, d_2}^{e_1, l_1} S_{m_1}^{d_1} S_{l_1}^{d_2}$$

$$(13) 21: Z_{m_1}^{e_1} + = H_{m_1,d_1}^{l_1,l_2} S_{l_1}^{e_1} S_{l_2}^{d_1} \cdot -1/1$$

(14)
$$25: Z_{m_1}^{e_1} + = H_{d_1 d_2}^{l_1, l_2} S_{l_1}^{e_1} S_{m_1, l_2}^{d_1 d_2} \cdot -1/1$$

$$(15) \hspace{1cm} 26: Z_{m_1}^{e_1} + = H_{d_1,d_2}^{l_1 l_2} S_{m_1}^{d_1} S_{l_1 l_2}^{e_1,d_2} \cdot -1/1$$

(16)
$$27: Z_{m_1}^{e_1} + = H_{d_1, d_2}^{l_1, l_2} S_{l_1}^{d_1} S_{m_1, l_2}^{e_1, d_2}$$

(17)
$$40: Z_{m_1}^{e_1} + = H_{d_1, d_2}^{l_1, l_2} S_{l_1}^{e_1} S_{m_1}^{d_1} S_{l_2}^{d_2} \cdot -1/1$$

(18)
$$2: Z_{m_1 m_2}^{e_1 e_2} + = H_{m_1 m_2}^{e_1 e_2}$$

(19)
$$7: Z_{m_1 m_2}^{e_1 e_2} += H_{m_1, d_1}^{e_1 e_2} S_{m_2}^{d_1}$$

(20)
$$8: Z_{m_1m_2}^{e_1e_2} + = H_{m_1m_2}^{e_1,l_1} S_{l_1}^{e_2} \cdot -1/1$$

(21)
$$13: Z_{m_1m_2}^{e_1e_2} += H_{d_1}^{e_1} S_{m_1m_2}^{e_2,d_1} \cdot -1/1$$

(22)
$$14: Z_{m_1m_2}^{e_1e_2} + = H_{m_1}^{l_1} S_{m_2,l_1}^{e_1e_2}$$

(23)
$$15: Z_{m_1m_2}^{e_1e_2} += H_{d_1d_2}^{e_1e_2} S_{m_1m_2}^{d_1d_2}$$

(24)
$$16: Z_{m_1 m_2}^{e_1 e_2} += H_{m_1, d_1}^{e_1, l_1} S_{m_2, l_1}^{e_2, d_1}$$

(25)
$$17: Z_{m_1m_2}^{e_1e_2} + = H_{m_1m_2}^{l_1l_2} S_{l_1l_2}^{e_1e_2}$$

$$(26) 22: Z_{m_1 m_2}^{e_1 e_2} += H_{d_1, d_2}^{e_1 e_2} S_{m_1}^{d_1} S_{m_2}^{d_2} \cdot 1/2$$

$$(27) \hspace{1cm} 23: Z_{m_1m_2}^{e_1e_2} + = H_{m_1,d_1}^{e_1,l_1} S_{l_1}^{e_2} S_{m_2}^{d_1} \cdot -1/1$$

(28)
$$24: Z_{m_1m_2}^{e_1e_2} + = H_{m_1m_2}^{l_1, l_2} S_{l_1}^{e_1} S_{l_2}^{e_2} \cdot 1/2$$

(29)
$$28: Z_{m_1m_2}^{e_1e_2} + = H_{d_1}^{l_1} S_{l_1}^{e_1} S_{m_1m_2}^{e_2, d_1}$$

$$(30) \hspace{1cm} 29: Z_{m_1m_2}^{e_1e_2} + = H_{d_1}^{l_1} S_{m_1}^{d_1} S_{m_2,l_1}^{e_1e_2}$$

$$(31) \qquad \ \ 30: Z_{m_1m_2}^{e_1e_2} + = H_{d_1d_2}^{e_1,l_1} S_{l_1}^{e_2} S_{m_1m_2}^{d_1d_2} \cdot -1/1$$

(32)
$$31: Z_{m_1m_2}^{e_1e_2} + = H_{d_1,d_2}^{e_1,l_1} S_{m_1}^{d_1} S_{m_2,l_1}^{e_2,d_2}$$

$$(33) 32: Z_{m_1m_2}^{e_1e_2} + = H_{d_1,d_2}^{e_1,l_1} S_{l_1}^{d_1} S_{m_1m_2}^{e_2,d_2}$$

$$(34) \hspace{1cm} 33: Z_{m_1m_2}^{e_1e_2} + = H_{m_1,d_1}^{l_1,l_2} S_{l_1}^{e_1} S_{m_2,l_2}^{e_2,d_1} \cdot -1/1$$

$$(35) 34: Z_{m_1m_2}^{e_1e_2} + = H_{m_1,d_1}^{l_1l_2} S_{m_2}^{d_1} S_{l_1l_2}^{e_1e_2}$$

$$(36) \hspace{1cm} 35: Z_{m_1m_2}^{e_1e_2} + = H_{m_1,d_1}^{l_1,l_2} S_{l_1}^{d_1} S_{m_2,l_2}^{e_1e_2} \cdot -1/1$$

$$(37) \qquad 36: Z_{m_1m_2}^{e_1e_2} + = H_{d_1d_2}^{l_1,l_2} S_{m_1,l_1}^{e_1e_2} S_{m_2,l_2}^{d_1d_2} \cdot -1/1$$

$$(38) \hspace{1cm} 37: Z_{m_1m_2}^{e_1e_2} + = H_{d_1d_2}^{l_1l_2} S_{l_1l_2}^{e_1e_2} S_{m_1m_2}^{d_1d_2}$$

$$(39) \qquad 38: Z_{m_1m_2}^{e_1e_2} + = H_{d_1,d_2}^{l_1l_2} S_{m_1m_2}^{e_1,d_1} S_{l_1l_2}^{e_2,d_2} \cdot -1/1$$

$$(40) \qquad 39: Z_{m_1m_2}^{e_1e_2} += H_{d_1,d_2}^{l_1,l_2} S_{m_1,l_1}^{e_1,d_1} S_{m_2,l_2}^{e_2,d_2} \cdot 1/2$$

$$(41) 41: Z_{m_1 m_2}^{e_1 e_2} + = H_{d_1, d_2}^{e_1, l_1} S_{l_1}^{e_2} S_{m_1}^{d_1} S_{m_2}^{d_2} \cdot -1/2$$

$$(42) \qquad 42: Z_{m_1m_2}^{e_1e_2} + = H_{m_1,d_1}^{l_1,l_2} S_{l_1}^{e_1} S_{l_2}^{e_2} S_{m_2}^{d_1} \cdot 1/2$$

$$(43) \qquad 43: Z_{m_1m_2}^{e_1e_2} + = H_{d_1d_2}^{l_1,l_2} S_{l_1}^{e_1} S_{l_2}^{e_2} S_{m_1m_2}^{d_1d_2} \cdot 1/2$$

$$(44) \qquad 44: Z_{m_1m_2}^{e_1e_2} + = H_{d_1,d_2}^{l_1,l_2} S_{l_1}^{e_1} S_{m_1}^{d_1} S_{m_2,l_2}^{e_2,d_2} \cdot -1/1$$

$$(45) 45: Z_{m_1m_2}^{e_1e_2} + = H_{d_1,d_2}^{l_1,l_2} S_{l_1}^{e_1} S_{l_2}^{d_1} S_{m_1m_2}^{e_2,d_2} \cdot -1/1$$

$$(46) 46: Z_{m_1 m_2}^{e_1 e_2} + = H_{d_1, d_2}^{l_1 l_2} S_{m_1}^{d_1} S_{m_2}^{d_2} S_{l_1 l_2}^{e_1 e_2} \cdot 1/2$$

$$(47) 47: Z_{m_1 m_2}^{e_1 e_2} + = H_{d_1, d_2}^{l_1, l_2} S_{m_1}^{d_1} S_{l_1}^{d_2} S_{m_2, l_2}^{e_1 e_2} \cdot -1/1$$

(48)
$$48: Z_{m_1m_2}^{e_1e_2} + = H_{d_1,d_2}^{l_1,l_2} S_{l_1}^{e_1} S_{l_2}^{e_2} S_{m_1}^{d_1} S_{m_2}^{d_2} \cdot 1/4$$