**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676 #**Option B** is correct answer
4. 0.5
5. 0.6987

**ANS) B 0.2676**

mu =45+10

mu

SD = 8

from scipy.stats import norm

z=norm(mu,SD)

z

#probability that the car will be ready in 1 hour (60 minutes)

z.cdf(60)

#probability that the car will not be ready in 1 hour (60 minutes)

(1-z.cdf(60)).round(5)

#0.2659855

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

Ans: Both the statements are True

A) More employees at the processing center are older than 44 than between 38 and 44.

Ans) We have a normal distribution with μ= 38 and σ = 6.

Let X be the number of employees. So according to question.

#Probabilty of employees greater than age of 44

= P(X>44)P(X > 44)

= 1 -P(X ≤ 44).

Z = (X -μ)/ σ

= (X -38)/6.

Thus the question can be answered by using the normal table to find P(X ≤ 44)

= P(Z ≤ (44 -38)/6)

= P(Z ≤ 1)=84.1345%

Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age

= P(X<44)-0.5

=84.1345-0.5

= 34.1345%

Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

B) Probabilty of employees less than age of 30 = P(X<30).

Z = (X -μ)/ σ

= (30 -38)/6

Thus the question can be answered by using the normal table to find P(X ≤ 30)

= P(Z ≤ (30 -38)/6)

= P(Z ≤ -1.333)

=9.12%

So the number of employees with probability 0.912 of them being under age 30 =0.0912\*400

=36.48( or 36 employees).

Therefore the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

The difference between the random variables 2X1 and X1 + X2, where X1 and X2 are independent and identically distributed (i.i.d.) normal random variables with mean μ and variance σ².

**Distribution of 2X1:**

If X1 ~ N(μ, σ²), then 2X1 ~ N(2μ, 4σ²).

The mean of 2X1 is 2μ.

The variance of 2X1 is 4σ².

**Distribution of X1 + X2:**

Since X1 and X2 are i.i.d., X1 + X2 follows a normal distribution with mean μ + μ = 2μ and variance σ² + σ² = 2σ².

Therefore, X1 + X2 ~ N(2μ, 2σ²).

The mean of X1 + X2 is 2μ.

The variance of X1 + X2 is 2σ².

In summary, both 2X1 and X1 + X2 have the same mean (2μ) but different variances. 2X1 has a variance of 4σ², while X1 + X2 has a variance of 2σ².

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5 **Option D is correct**
6. 90.1, 109.9

**ANS) D)** 48.5, 151.5

Convert the problem to the standard normal distribution:

We know that the standard normal distribution has a mean (μ) of 0 and a standard deviation (σ) of 1.

To transform X ~ N(100, 20^2) into the standard normal distribution, we use the formula:

Z = (X - μ) / σ

In this case, μ = 100 and σ = 20. So, we have:

Z = (X - 100) / 20.

We can use a standard normal distribution table or calculator to find the Z-scores corresponding to the cumulative probability of 0.005 and 0.995 (since we want the middle 99% of the distribution).

Z\_a = Z-score for P(Z < 0.005)

Z\_b = Z-score for P(Z < 0.995)

Transform Z\_a and Z\_b back to the original distribution:

Once you have Z\_a and Z\_b, you can transform them back to the original distribution using the formula:

X = Z \* σ + μ

In this case, σ = 20 and μ = 100. So, you have:

a = Z\_a \* 20 + 100

b = Z\_b \* 20 + 100

Now, calculate a and b using the Z-scores obtained in step 2:

Using python

import scipy.stats as stats

# Find Z-scores for P(Z < 0.005) and P(Z < 0.995)

Z\_a = stats.norm.ppf(0.005)

Z\_b = stats.norm.ppf(0.995)

# Transform Z\_a and Z\_b back to the original distribution

sigma = 20

mu = 100

a = Z\_a \* sigma + mu

b = Z\_b \* sigma + mu

print("a =", a) # a = 48.483413929021985

print("b =", b) # b = 151.516586070978

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

**Ans)**

import scipy.stats as stats

# Constants

USD\_to\_Rupees = 45

mean\_profit\_1\_USD = 5

variance\_profit\_1\_USD = 32

mean\_profit\_2\_USD = 7

variance\_profit\_2\_USD = 42

# Calculate the mean and variance in Rupees

mean\_profit\_1\_Rupees = mean\_profit\_1\_USD \* USD\_to\_Rupees

variance\_profit\_1\_Rupees = variance\_profit\_1\_USD \* (USD\_to\_Rupees\*\*2)

mean\_profit\_2\_Rupees = mean\_profit\_2\_USD \* USD\_to\_Rupees

variance\_profit\_2\_Rupees = variance\_profit\_2\_USD \* (USD\_to\_Rupees\*\*2)

# A. Calculate the Rupee range containing 95% probability

total\_mean\_Rupees = mean\_profit\_1\_Rupees + mean\_profit\_2\_Rupees

total\_variance\_Rupees = variance\_profit\_1\_Rupees + variance\_profit\_2\_Rupees

# Calculate the standard deviation

std\_dev\_Rupees = (total\_variance\_Rupees) \*\* 0.5

# Calculate the Z-score for a 95% confidence interval

confidence\_interval\_Z = stats.norm.ppf(0.975)

# Calculate the lower and upper limits in Rupees

lower\_limit = total\_mean\_Rupees - confidence\_interval\_Z \* std\_dev\_Rupees

upper\_limit = total\_mean\_Rupees + confidence\_interval\_Z \* std\_dev\_Rupees

print("A. Rupee range with 95% probability:", (lower\_limit, upper\_limit))

# B. Calculate the 5th percentile of profit in Rupees

percentile\_5\_Z = stats.norm.ppf(0.05)

percentile\_5\_Rupees = total\_mean\_Rupees + percentile\_5\_Z \* std\_dev\_Rupees

print("B. 5th percentile of profit in Rupees:", percentile\_5\_Rupees)

# C. Calculate the probability of making a loss for each division

# Define the standard normal distribution for calculations

normal\_dist = stats.norm(0, 1)

# Calculate the Z-scores for each division's loss

z\_score\_profit\_1 = (0 - mean\_profit\_1\_USD) / (variance\_profit\_1\_USD \*\* 0.5)

z\_score\_profit\_2 = (0 - mean\_profit\_2\_USD) / (variance\_profit\_2\_USD \*\* 0.5)

# Calculate the probabilities of making a loss for each division

probability\_loss\_profit\_1 = normal\_dist.cdf(z\_score\_profit\_1)

probability\_loss\_profit\_2 = normal\_dist.cdf(z\_score\_profit\_2)

print("C. Probability of making a loss for Profit1:", probability\_loss\_profit\_1)

print(" Probability of making a loss for Profit2:", probability\_loss\_profit\_2)

A. Rupee range with 95% probability: (-218.7111468016103, 1298.7111468016103)

B. 5th percentile of profit in Rupees: -96.73046620701132

C. Probability of making a loss for Profit1: 0.188379558905791

Probability of making a loss for Profit2: 0.14004360540574878