Observing Brownian Motion and Optical Trapping using Micron-Scale Particles and Optical Tweezers

James Michael Patrick Brady
School of Physics and Astronomy
The University of Manchester

(Dated: March 2023)

The motion of micron-scale beads in distilled water was observed using a microscope to find a mean value for the viscosity of water. Fitting using the observed error on measurement yielded a value of 1.1 ± 0.24 mPas, which agrees with experimental results but is not precise due to analytical mistakes. Using the resolving limit of the lens yielded a value of 0.5 ± 0.1 mPas, which is more precise but unlikely to be correct. Particles were successfully trapped by using a laser, however the force supplied at each power is smaller than the theoretical limit, also partially due to a combination of mistakes in method and analysis. Further consideration of the minutiae of the experiment will be required for accurate results.

I. INTRODUCTION

Brownian motion is a type of thermally distributed rrandom motion observed in fluids, named after Robert Brown, who first described it in 1827 and was initially modelled by Albert Einstein in 1905 [1]. It becomes an important physical effect to consider at the micron scale, as said thermal motion means that micron-scale particles become more difficult to confine and control, However, in 1986, Ashkin and Chu devised a simple, non-contact method to confine particles using a laser, as previously observed by Ashkin in 1970, referred to as optical tweezing and allowing for the precise manipulation of structures such as proteins and containment of individual atoms [2] [3] .The aims of this experiment are to recreate Einstein's experiment to find the viscosity of water and further investigate the use of optical tweezers to trap micron-scale beads.

II. BROWNIAN MOTION

A. Theory of Brownian Motion and Diffusive Fluids

Brownian motion is a type of linear random motion observed in micron-scale particles suspended in a diffusive fluid made of far smaller molecules. Fluid molecules move randomly according to the Maxwell-Boltzmann distribution and collide with the suspended particles. Overall, this leads to a net random motion in the suspended particles. Motion in fluids that have a drag effect on suspended particles can be modelled via the Langevin Equation [4]:

$$m\ddot{x} + \gamma\dot{x} + \kappa x = F_B(t) \tag{1}$$

. There are four parts to the Langevin Equation: $m\ddot{x}$, the inertial motion of a particle (which is ignored as it is a relatively small effect), $\gamma\dot{x}$, the force due to drag, κx, the force due to a spring-like potential and F_B, the force due to thermal fluctuations. Solving the Langevin equation for small timescales, $t < \frac{\gamma}{\kappa}$ produces the following linear solution for the Mean Square (MSD) of suspended particles:

$$\langle (x(t+\tau) - x(t))^2 \rangle = \langle x^2 \rangle \approx \frac{2k_B T}{\kappa} \tag{2}$$

Stokes' Law can be used to find the form of γ as follows:

$$\gamma = 6\pi\eta r \tag{3}$$

where η is the fluid's viscosity and r is the diameter of a particle suspended in the fluid [4] . Using Equation 2 and Equation 3, along with the time step, τ, the viscosity of the fluid can be found:

$$\eta = \frac{k_B T \tau}{6\pi r \langle x^2 \rangle} \tag{4}$$

Water is a diffusive fluid, meaning that there is a linear relation between $\langle x^2 \rangle$ and τ and so linear fitting is appropriate.

B. Equipment and Method

A microscope with a CMOS camera and an objective lens with an immersion oil interface and a resolving limit of 410 nm were used to observe the motion of beads in water. A micrometer slide was used to calibrate the microscope. Slides were prepared by diluting $1\mu m$ radius bead solutions in distilled water. Small drops were put on a slide and were then sealed to prevent evaporation from causing a diffusion gradient during observation.

Under the microscope individual beads were isolated and the region of interest for recording was reduced to allow for more frequent data points. The recorded frames were converted to .avi files (with integer frame rates only) and passed to a motion tracker. Under the microscope, using a micrometer slide and averaging measurements, the length a pixel corresponded to was 93.8 ± 2.5 nm. Errors were directly propagated to the MSD and was passed to NumPy's polyfit method with the timestep and MSD to perform a linear fit, the error on the gradient being extracted from the calculated covariance matrix. However, the errors on the MSD were found to be too large as they were not treated as standard errors, hence the resolving limit $\pm 410nm$ was also used and reduced to a standard error to perform comparison fits.

C. Results

	Using Pixel Error		Using Resolving Limit	
FPS	Gradient ($\mu m^2/s$)	χ^2_{red}	Gradient ($\mu m^2/s$)	χ^2_{red}
59.0	2.35 ± 0.09	4.6	2.86 ± 0.01	0.2
45.7	2.02 ± 0.04	4.1	2.72 ± 0.02	2.4
45.3	1.22 ± 0.05	3.6	0.98 ± 0.01	3.6
31.5	0.54 ± 0.07	3.6	0.572 ± 0.008	0.5
40.9	1.62 ± 0.09	3.6	1.79 ± 0.008	1.1
71.6	2.28 ± 0.1	3.6	3.17 ± 0.02	6.7
59.0	2.05 ± 0.1	3.6	2.42 ± 0.01	0.4
58.4	2.86 ± 0.1	3.6	3.62 ± 0.02	2.7
59.0	2.57 ± 0.1	3.6	2.10 ± 0.01	2.1
37.2	$2.22 \pm 0,1$	3.6	2.68 ± 0.03	1.9

TABLE I: Gradients of MSD against timestep found for polystyrene and silica beads

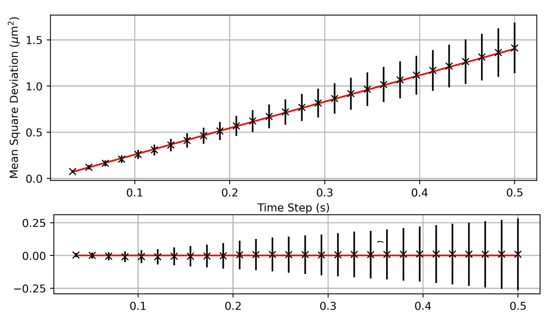

FIG. 1: Linear fit of $\langle x^2 \rangle$ against τ using the pixel error

From Table I and using Equation 4 and a standard temperature of 293.15 K, the mean viscosity was found to be 1.1 ± 0.24 mPas when using the errors measured by pixel size, agreeing with the NIST value [5] and 0.5 ± 0.1 mPas when using the errors found from the diffraction limit. As most of the χ^2_{red} values are close to the when considering the diffraction limit, it is likely that water is diffusive, as expected. The value of η found when fitting with the pixel errors is more likely to be accurate, despite the errors being too large, because they prevent the conversion to an integer frame rate from influencing the fit as seen in Fig. 1. The errors due to the diffraction limit are more likely to be accurate, however the mean from those fits is affected by the residuals from converting to integer frame rates, as seen in the residuals Fig. 2. Accounting for the integer frame rate conversion should allow for a more accurate result.

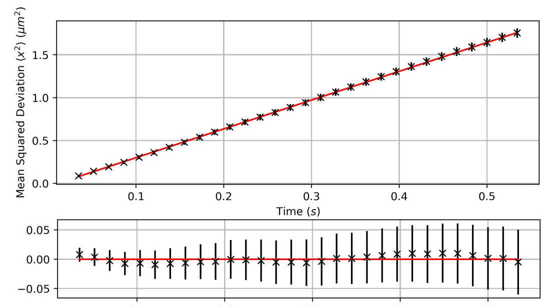

FIG. 2: Subsequent fit of $\langle x^2 \rangle$ against τ using the resolving limit

III. OPTICAL TRAPPING

Photons carry momentum, which can be imparted on larger dielectric particles. The laser used in this experiment has a wavelength of 680 nm, meeting the condition of the Mie approximation ($r \gg \frac{\lambda}{2\pi}$), hence allowing the beads to be treated as dielectrics [3]. A beam splitter is used to prevent passing the beam to the camera, but allowing the sample image to pass. The force provided by the beam is dependent on the intensity gradient of the beam, greater differences will provide greater force, which can be achieved if the beam is focused and calibrated properly. As the beam has a Gaussian distribution along both axes of the plane of observation, the net force exerted on a particle acts towards the centre of the beam, keeping the particle constrained. Given a laser of sufficient power, the force provided by the laser will resist thermal motion centrally, hence a spring potential becomes a good approximation.

A. Method

As before, a trapped particle was recorded using the microscope and its motion was tracked. The laser's power was varied between 1 mW and 6 mW and the positional data was binned and fitted to a Gaussian. The standard deviation of the fits was used as an approximation of $\langle x \rangle$ and $\langle y \rangle$. Using the potential of a spring and the thermal energy of the system, the spring force can be found to be:

$$F_s = \frac{k_B T}{\langle x \rangle} \tag{5}$$

Theoretically, the force provided by a laser of power P through a medium of refractive index n should be equal to:

$$F_{\gamma} = \frac{QnP}{c} \tag{6}$$

, where Q acts as a dimensionless efficiency factor [4]. It should be expected that the force provided by the laser will increase with the current, as the current directly relates to the intensity of the beam, which directly relates to the gradient of the beam. In addition, if the same analysis of the MSD is performed for the positions of a trapped particle, the gradient should approach zero as the timestep increases. Errors on measurements were found by propagating the error on the standard deviation (as it is used as $\langle x \rangle$) found from the Gaussian fits.

B. Results

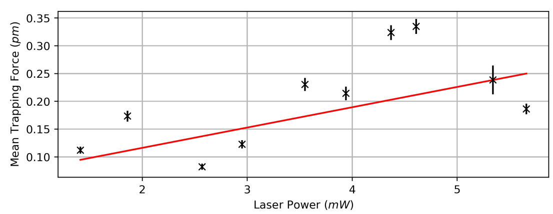

FIG. 3: Force against power gradient of laser, graph created by author

From Equation 3 and an expectation that $\langle \dot{x} \rangle$ should be of the order 1 μ m/s, the laser should provide a force on the order of at least 1 pN. From Fig 3 , the laser is providing $\sim$ 0.2 pN to overcome thermal motion at powers which trap the particle efficiently, however the force does increase with power as expected. Likewise, the gradient of force against power in Figure 3 is on the order of $10^{-11} N/W$, whereas the estimate from Equation 7 indicates that (in water) the laser should provide a gradient of $\sim 10^{-9} N/W$. Therefore, there is clearly a discrepancy that has not been accounted for, likely to be at least one of the following: the recording time being too short (leading to noisy data), that the standard deviation underestimates the displacement of the particle, or the effective area of the laser increases as power decreases (as the peak of the Gaussian lowers). These could be rectified by using 3σ to estimate $\langle x \rangle$ (corresponding to a >99% expectation that the bead will be in that region) and sampling data over a longer time,. However, the bead is shown to be sufficiently trapped over long timesteps, as shown by Fig 4.

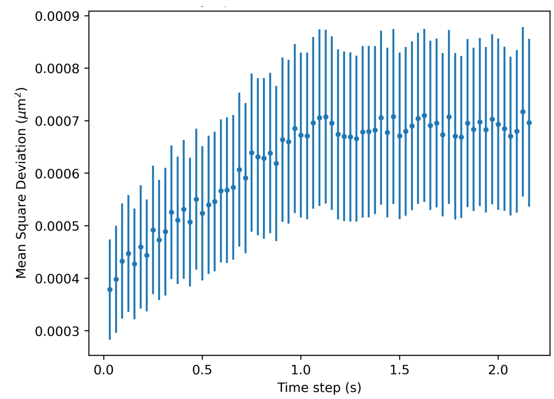

FIG. 4: MSD of bead in a 6mW laser beam, graph created by author

IV. CONCLUSION

Overall, this experiment was able to use the basics of the Einstein experiment to find a two mean values of the viscosity of water, one of which is close to other experimental values. However, some statistical failings have meant that the more accurate value has an incorrect error and the other has an incorrect value, which could be improved by taking the errors due to the resolving limit of the lens and correcting the data for integer frame rates. In addition, a bead was successfully trapped by the laser and the force exerted by the laser is near the scale of those expected from drag. Despite this, the gradient of force against laser power is less than the theoretical maximum, which may be due to three factors: $\langle x \rangle$ should be estimated to be 3σ instead of σ when fitting, the area of the beam increasing with decreases in power due to a shallower gradient (hence less force is exerted) and because the observations are too short to find a highly accurate mean value.

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[5] J. Swindells *et al.*, Absolute viscosity of water at 20°c, Journal of Research of the National Bureau of Standards **48**, 1 (1952).