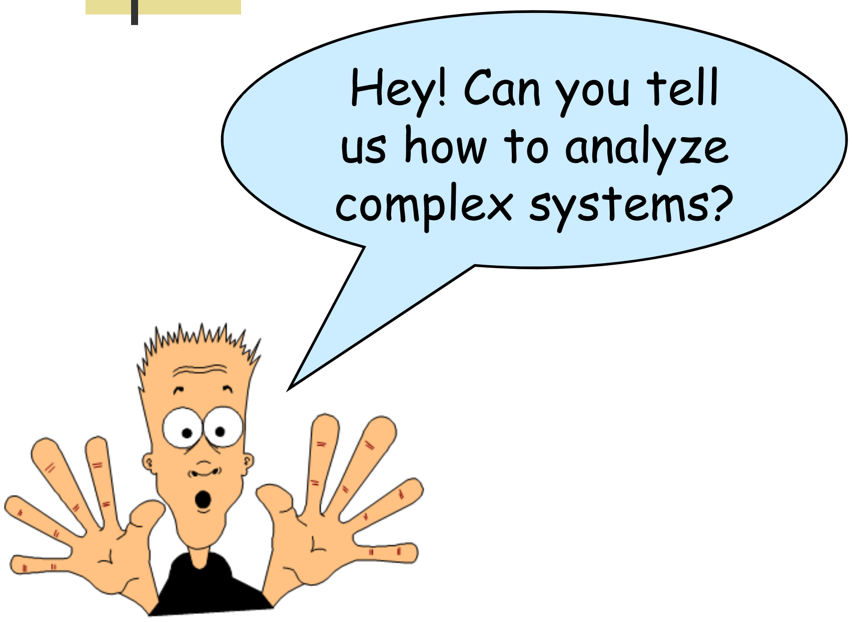
Chapter 5
Reliability of Systems



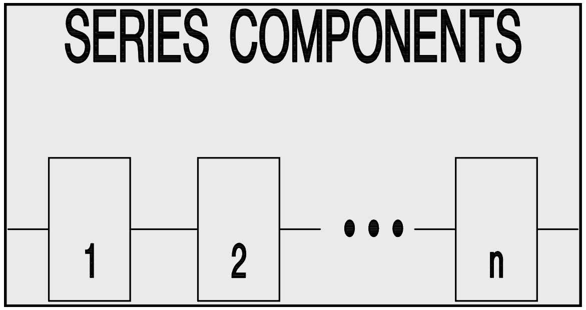
Serial Configuration

Parallel Configuration

Combined Series-Parallel

Serial Configuration

Reliability
Block Diagram



E_1 = the event, component 1 does not fail, and
E_2 = the event, component 2 does not fail, then
$P\{E_1\} = R_1$ and $P\{E_2\} = R_2$ where
R_1 = the reliability of component 1, and
R_2 = the reliability of component 2.
Therefore assuming independence:
$R_s = P\{E_1 \cap E_2\} = P\{E_1\}\, P\{E_2\} = R_1\, R_2$

Multiple Components in Series

Generalizing to n mutually independent components in series;

$$R_s(t) = R_1(t) \times R_2(t) \times \ldots \times R_n(t)$$

and $$R_s(t) \leq \min\{R_1(t), R_2(t), \ldots, R_n(t)\}$$

Component Count vs. System Reliability

Component Reliability	Number of Components 10	100	1000
.900	.3487	$.266 \times 10^{-4}$	$.1748 \times 10^{-45}$
.950	.5987	.00592	$.5292 \times 10^{-22}$
.990	.9044	.3660	$.443 \times 10^{-4}$
.999	.9900	.9048	.3677

System Reliability

Constant Failure Rate Components

$$R_s(t) = \prod_{i=1}^{n} R_i(t) = \prod_{i=1}^{n} e^{-\lambda_i t} = e^{-\sum_{i=1}^{n} \lambda_i t} = e^{-\lambda_s t}$$

$$\text{where } \lambda_s = \sum_{i=1}^{n} \lambda_i$$

Weibull Components

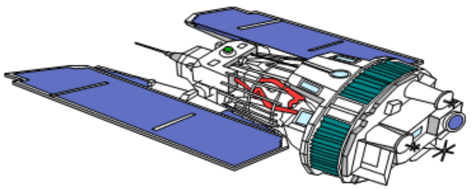
$$R_s(t) = \prod_{i=1}^{n} e^{-\left(\frac{t}{\theta_i}\right)^{\beta_i}} = e^{-\sum_{i=1}^{n}\left(\frac{t}{\theta_i}\right)^{\beta_i}}$$

$$\lambda(t) = \frac{e^{-\sum_{i=1}^{n}\left(\frac{t}{\theta_i}\right)^{\beta_i}}\left[\sum_{i=1}^{n}\frac{\beta_i}{\theta_i}\left(\frac{t}{\theta_i}\right)^{\beta_i-1}\right]}{e^{-\sum_{i=1}^{n}\left(\frac{t}{\theta_i}\right)^{\beta_i}}} = \sum_{i=1}^{n}\frac{\beta_i}{\theta_i}\left(\frac{t}{\theta_i}\right)^{\beta_i-1}$$

Components in Series - Example

A communications satellite consists of the following components:

Component	Probability Distribution	Shape Parameter	Characteristic life
Power unit	Weibull	2.7	43,800 hr.
Receiver	Weibull	1.4	75,000 hr.
Transmitter	Weibull	1.8	68,000 hr.
Antennae	Exponential	MTTF = 100,000 hr.	

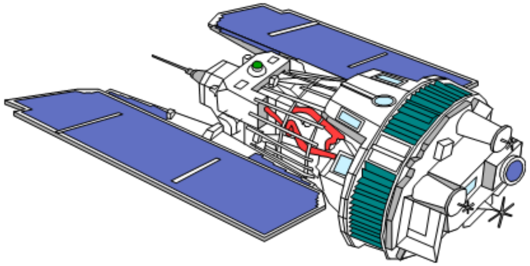


Components in Series - Example

$$\lambda_S(t) = \frac{2.7}{43{,}800}\left(\frac{t}{43{,}800}\right)^{1.7} + \frac{1.4}{75{,}000}\left(\frac{t}{75{,}000}\right)^{.4} + \frac{1.8}{68{,}000}\left(\frac{t}{68{,}000}\right)^{.8} + \frac{1}{100{,}000}$$

$$R_s(t) = e^{-\left(\frac{t}{43{,}800}\right)^{2.7}} e^{-\left(\frac{t}{75{,}000}\right)^{1.4}} e^{-\left(\frac{t}{68{,}000}\right)^{1.8}} e^{-\frac{t}{100{,}000}}$$

$$R_s(17{,}520) = e^{-\left(\frac{17{,}520}{43{,}800}\right)^{2.7}} e^{-\left(\frac{17{,}520}{75{,}000}\right)^{1.4}} e^{-\left(\frac{17{,}520}{68{,}000}\right)^{1.8}} e^{-\frac{17{,}520}{100{,}000}} = .62$$

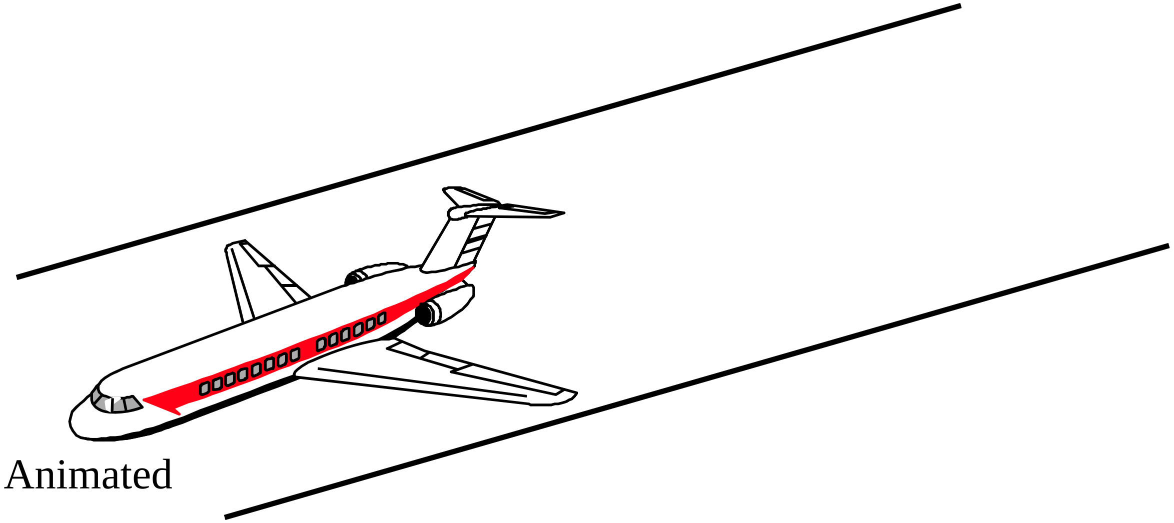


Class Exercise

The failure distribution of the main landing gear of a commercial airliner is Weibull with a shape parameter of 1.6 and a characteristic life of 10,000 landings. The nose gear also has a Weibull distribution with a shape parameter of .90 and a characteristic life of 15000 landings. What is the reliability of the landing gear system if the system is to be overhauled after 1000 landings?

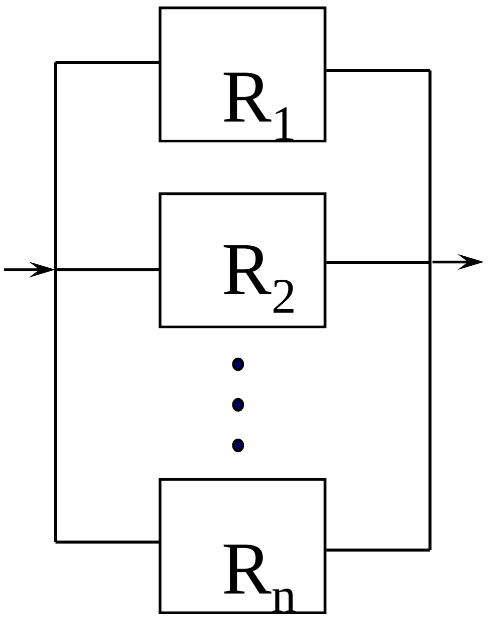
Class Exercise - solution

$$R(1000) = e^{-\left(\frac{1000}{10,000}\right)^{1.6}} e^{-\left(\frac{1000}{15,000}\right)^{.9}} = (.975)\ (.916) = .893$$



Animated

Parallel Configuration



$$R_s = P\{E1 \cup E2\} = 1 - P\{(E1 \cup E2)^c\}$$

$$= 1 - P\{E1^c \cap E2^c\}$$

$$= 1 - P\{E1^c\}\, P\{E2^c\} = 1 - (1-R_1)\ (1-R_2)$$

where E1 = event, component 1 does not fail

E2 = event, component 2 does not fail, and

$Pr\{E1\} = R_1$ and $Pr\{E2\} = R_2$

The probability at least one component does not fail!

Parallel Configuration - Generalization

$$R_s(t) = 1 - \prod_{i=1}^{n} \left[1 - R_i(t) \right]$$

$$R_s\ (t) >= \max\ \{R_1(t), R_2(t), \ldots, R_n(t)\}$$

Animated

Parallel Configuration - CFR Model

$$R_s(t) = 1 - \prod_{i=1}^{n} \left[1 - e^{-\lambda_i t} \right]$$

Parallel Configuration – 2-component CFR Model

$$R_s(t) = 1 - \prod_{i=1}^{n}\left[1 - e^{-\lambda_i t}\right]$$

For n = 2:

$$R_s(t) = 1 - (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t})$$

$$= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

$$MTTF = \int_0^{\infty} R_s(t)dt = \int_0^{\infty} e^{-\lambda_1 t}dt + \int_0^{\infty} e^{-\lambda_2 t}dt - \int_0^{\infty} e^{-(\lambda_1+\lambda_2)t}dt$$

$$= \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2}$$

Parallel Configuration - Weibull Model

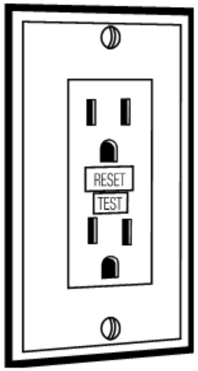
$$R_s(t) = 1 - \prod_{i=1}^{n}\left[1 - e^{-\left(\frac{t}{\theta_i}\right)^{B_i}}\right]$$

If n identical Weibull components are in parallel:

$$R_s(t) = 1 - \left[1 - e^{-\left(\frac{t}{\theta}\right)^{B}}\right]^n$$

Parallel Configuration - Example

A circuit breaker has a Weibull failure distribution (against a power surge) with beta equal to .75 and a characteristic life of 12 years.



$$R(1) = e^{-\left(\frac{1}{12}\right)^{.75}} = .856$$

Determine the one year reliability if two identical circuit breakers are redundant.

Parallel Configuration – Example Solution

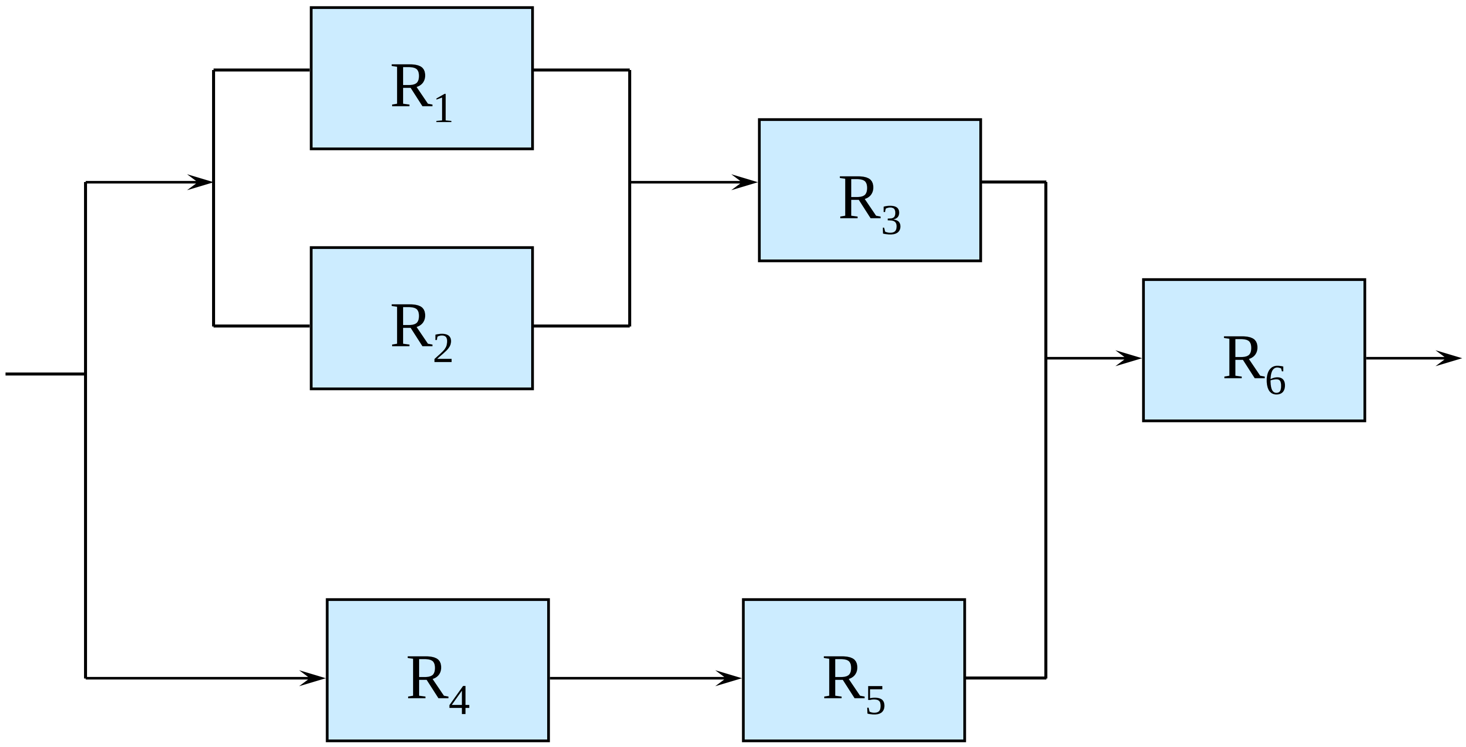
Two redundant breakers have a reliability function of:

$$R(t) = 1 - \left[1 - e^{-\left(\frac{t}{12}\right)^{.75}}\right]^2$$

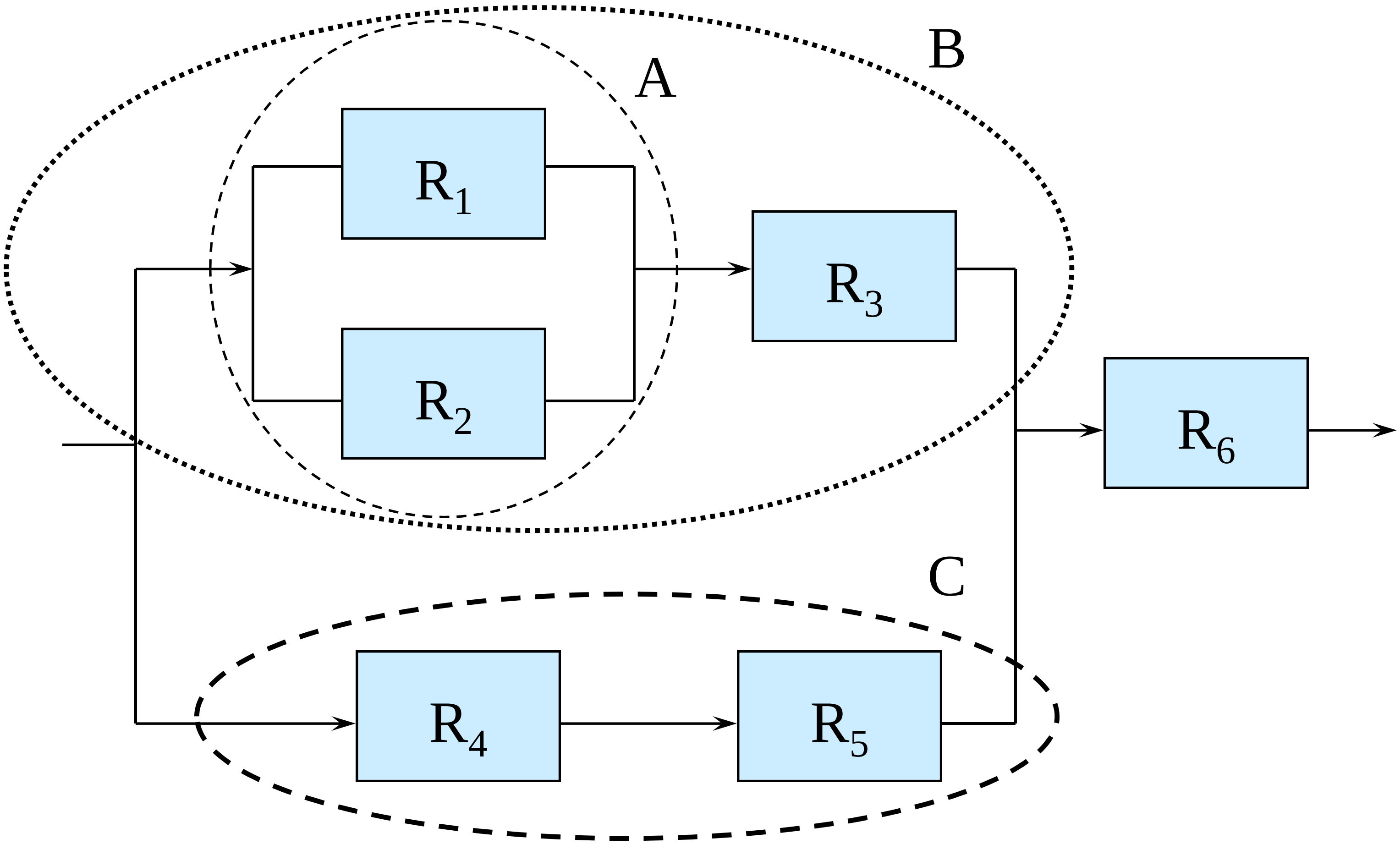
$$R(1) = 1 - \left[1 - e^{-\left(\frac{1}{12}\right)^{.75}}\right]^2 = 1 - (1 - .856)^2 = .979$$

Question: how are two redundant circuit breakers physically configured?

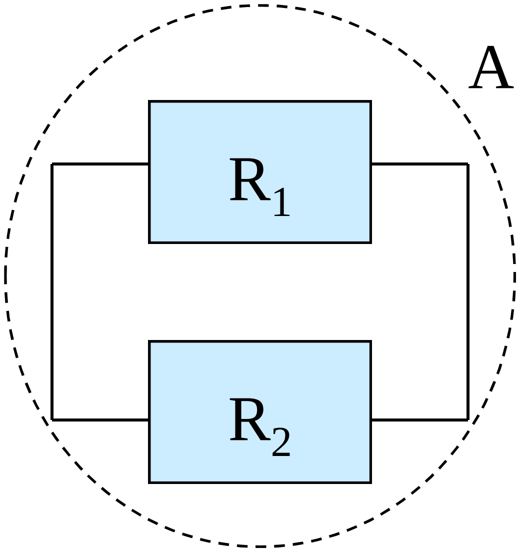
Combined Series - Parallel Systems



Combined Series - Parallel Systems

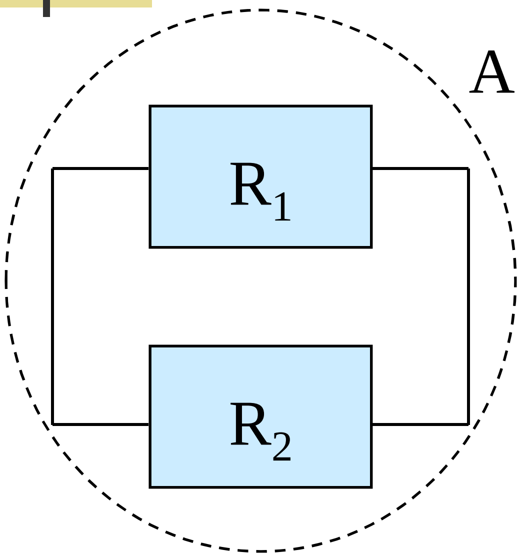


Combined Series - Parallel Systems



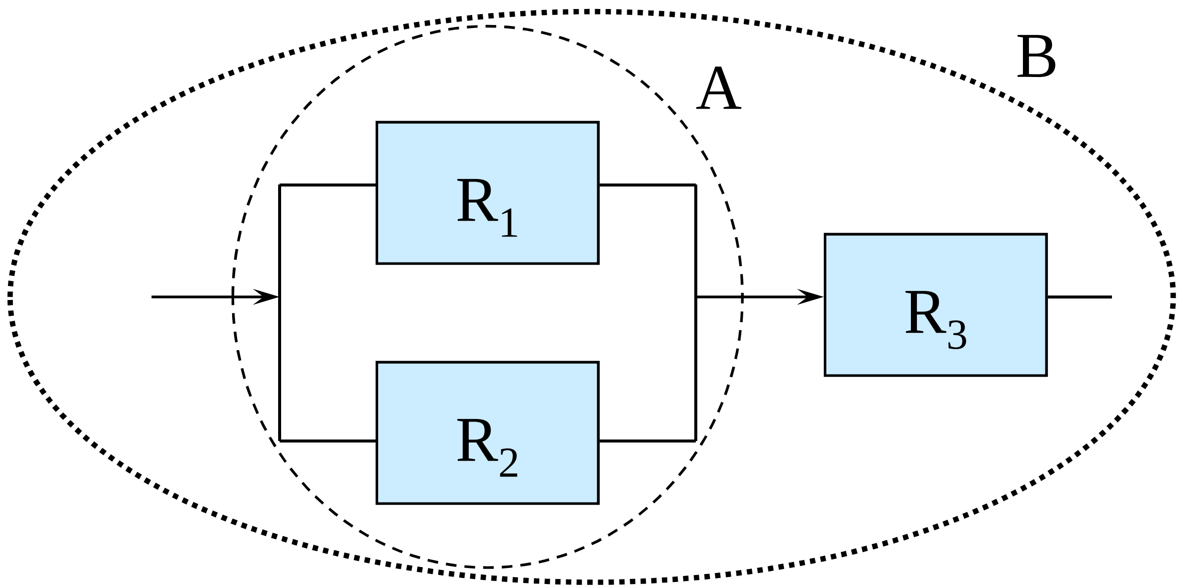
$$R_A = [\,1 - (1 - R_1)\,(1 - R_2\,)]$$

Combined Series - Parallel Systems

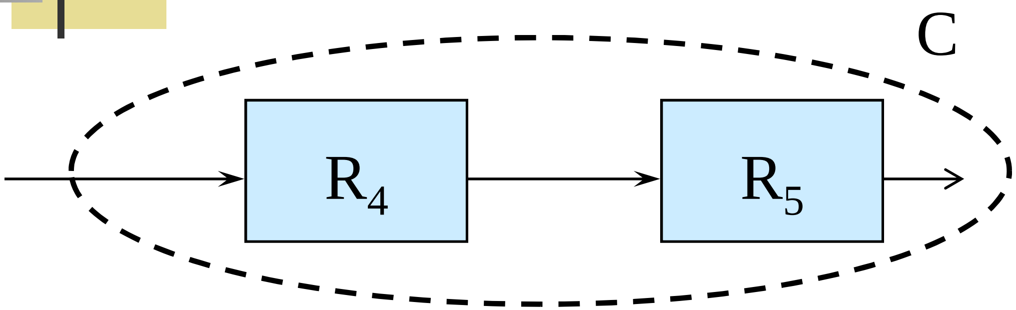


$$R_A = [\,1 - (1 - R_1)\,(1 - R_2\,)]$$

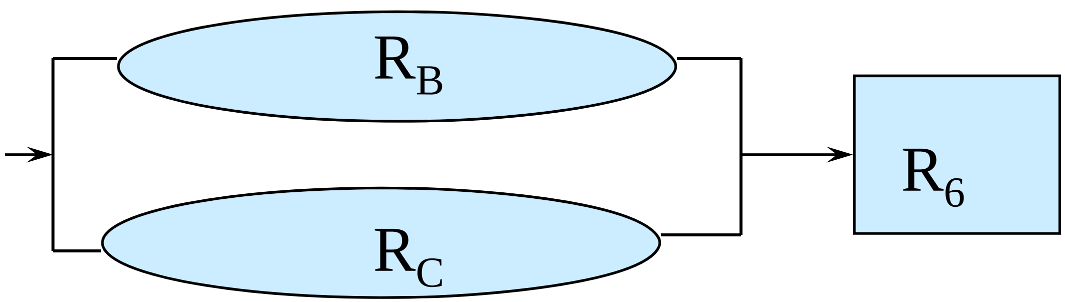
$$R_B = R_A\,R_3$$



Combined Series - Parallel Systems

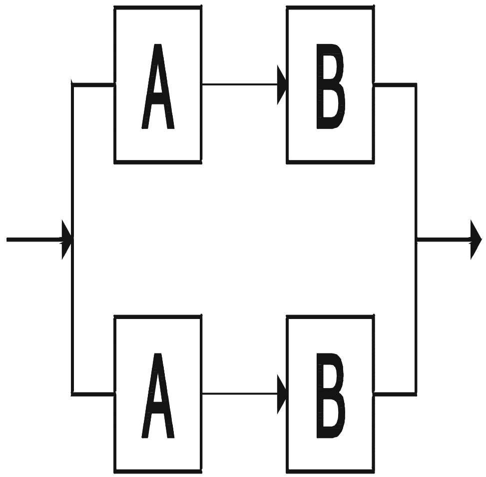


$$R_C = R_4 \, R_5$$



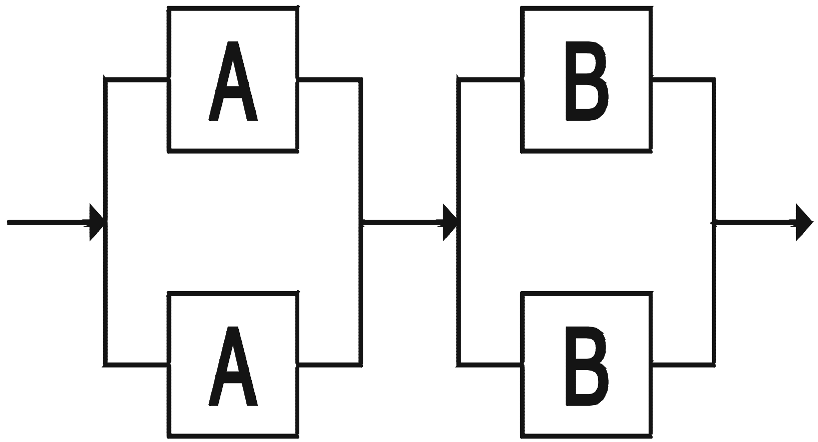
$$R_s = [1 - (1 - R_B)\,(1 - R_c)\,]\,R_6$$

High Level Redundancy



$$R_{high} = 1 - (1-R^2)^2 = 1 - [1-2R^2 + R^4] = 2R^2 - R^4$$

Low Level Redundancy



$$R_{LOW} = [1-(1-R)^2]^2 = [1-(1-2R+R^2)]^2 = (2R-R^2)^2$$

High vs Low Level Redundancy

$R_{low} - R_{high} = (2R - R^2)^2 - (2R^2 - R^4)$

$= R^2 (2-R)^2 - R^2 (2 - R^2)$

$= R^2 [4 - 4R + R^2 - 2 + R^2]$

$= 2 R^2 [R^2 - 2R + 1] = 2 R^2 (R - 1)^2 \geq 0$

k-out-of-n Redundancy

Let n = the number of redundant, identical and independent components each having a reliability of R.

Let X = a random variable, the number of components (out of n components) operating. Then

$$\Pr\{X = x\} = P(x) = \binom{n}{x} R^x (1 - R)^{n-x}$$

If k (<= n) components must operate for the system to operate:

$$R_s = \sum_{x=K}^{N} P(x)$$

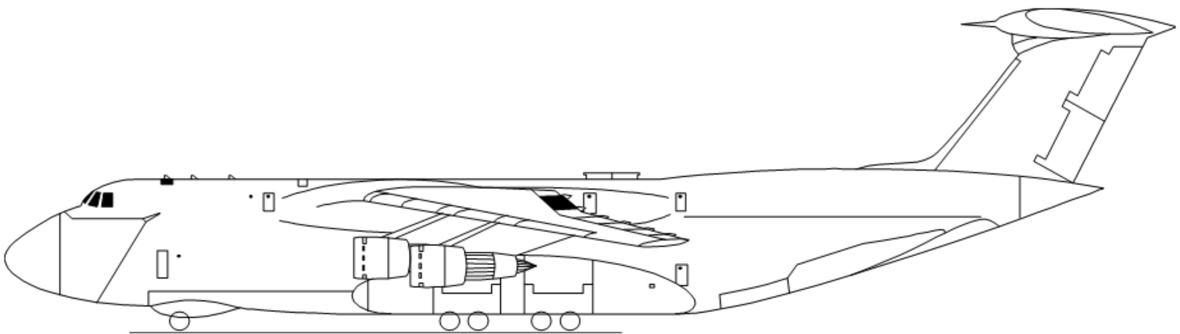
k-out-of-n Redundancy Exponential Distribution

$$R_s(t) = \sum_{x=k}^{N} \binom{N}{x} e^{-\lambda x t} \left[1 - e^{-\lambda t}\right]^{N-x}$$

$$MTTF = \int_0^\infty R_s(t)dt = \frac{1}{\lambda} \sum_{x=k}^{N} \frac{1}{x}$$

A Very Good Example

Out of the 12 identical AC generators on the C-5 aircraft, at least 9 of them must be operating in order for the aircraft to complete its mission. Failures are known to follow an exponential distribution with a mean of 100 operating hours. What is the reliability of the generator system over a 10 hour mission? Find the MTTF.



A Very Good Solution

Let T_i = time to failure of the i^{th} generator

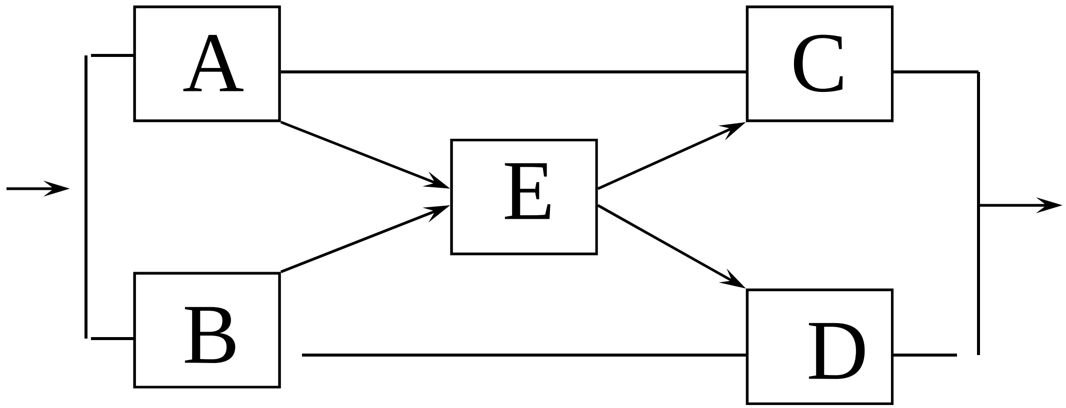
$$\Pr\{T_i \geq 10\} = e^{-10/100} = .9048$$

$$R_s(t) = \sum_{x=9}^{12} \binom{12}{x} .9048^x \left[1 - .9048\right]^{12-x} = .9782$$

$$MTTF = 100 \sum_{x=9}^{12} \frac{1}{x} = 38.53 \text{ hours}$$

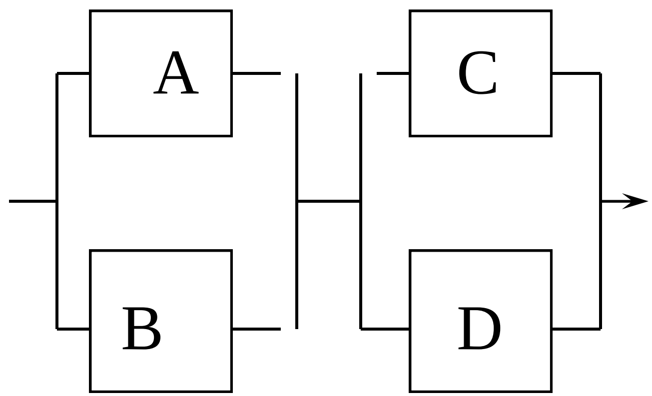
Complex Configurations

a. linked network:



Decomposition Approach

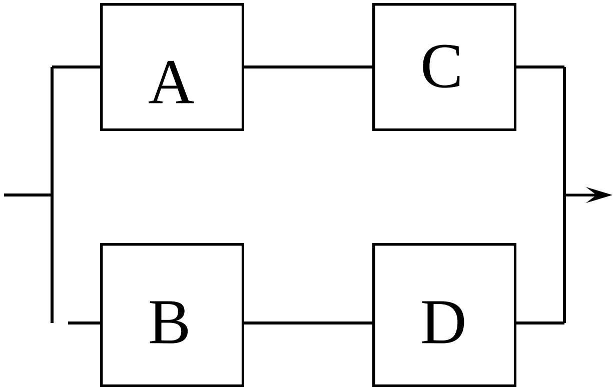
(b) Component E does not fail, R_E:



$$R_{(b)} = [1-(1-R_A)(1-R_B)][1-(1-R_C)(1-R_D)]$$

Decomposition Approach

(c) Component E fails ($1-R_E$):



$$R_{(c)} = 1-(1-R_A R_C)\,(1-R_B\,R_D)$$

Decomposition Approach

$$R_S = R_E\, R_{(b)} + (1 - R_E)\, R_{(c)}$$

if $R_A = R_B = .9$, $R_C = R_D = .95$, and $R_E = .80$, then

$R_{(b)} = [1-(1-.9)^2]\,[1-(1-.95)^2] = .99 \times .9975 = .9875$

$R_{(c)} = 1 - [1-(.9)\,(.95)]^2 = .978975$

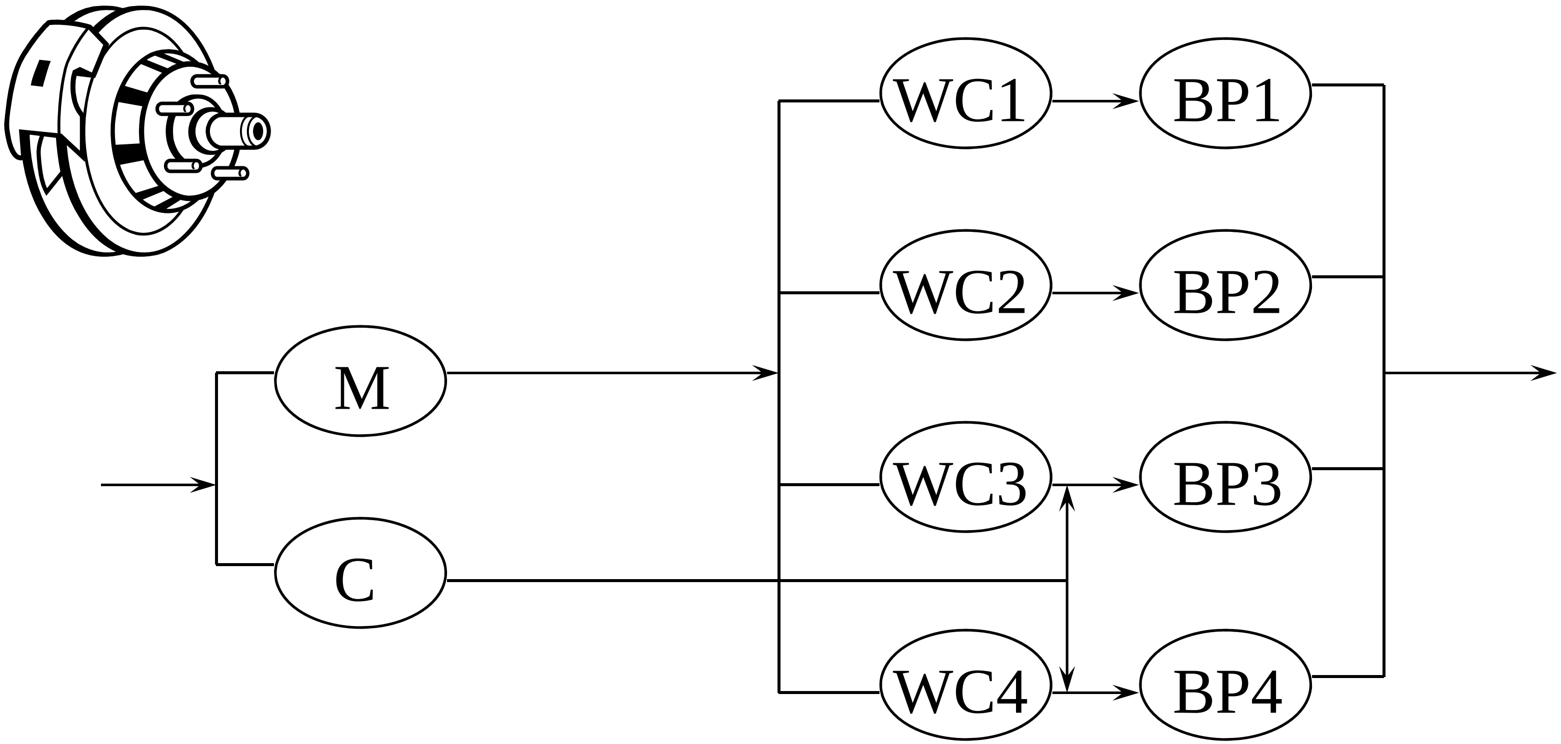
$R_s = .8\,(.9875) + (1-.8)\,(.978975) = .9858$

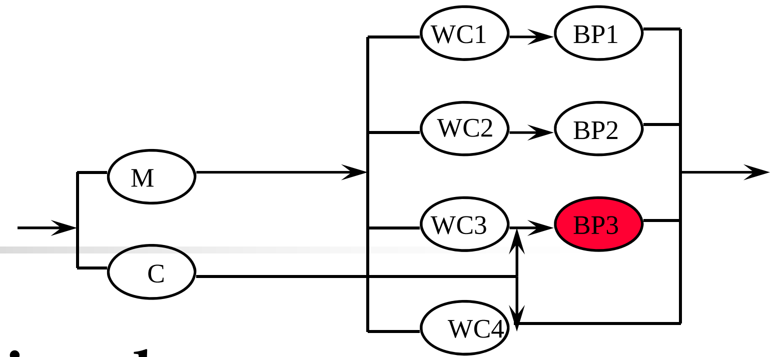
Enumeration Method

S = success; F = failure

A	B	C	D	E	System	Probability
S	S	S	S	S	S	.58482
F	S	S	S	S	S	.06498
S	F	S	S	S	S	.06498
S	S	F	S	S	S	.03078
S	S	S	F	S	S	.03078
S	S	S	S	F	S	.146205
F	F	S	S	S	F	
S	F	F	S	S	S	.00342
S	S	F	F	S	F	
S	S	S	F	F	S	.007695
					TOTAL	.9858
	•	•	•	•		⋮

Example – Automotive Braking System





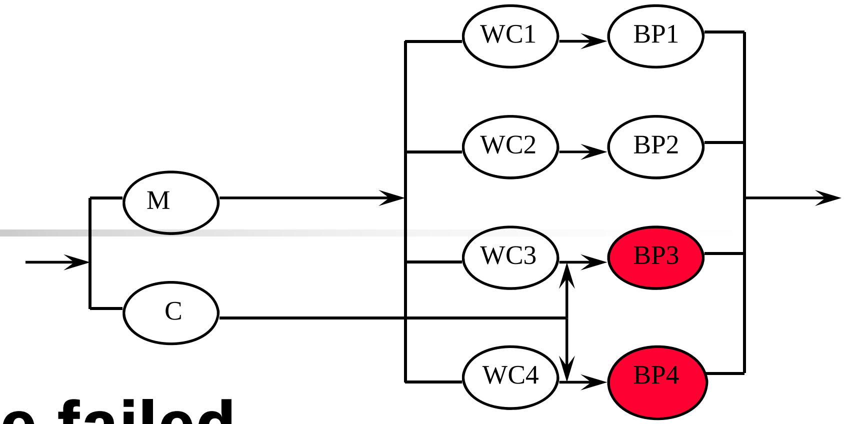
Case I. BP3 fails and BP4 is operational.

1. $P_I = [1 - R(BP)]\, R(BP)$
2. $R_f = R(M)\{1 - [1 - R(WC)\, R(BP)]^2\, [1 - R(WC)]\}.$
3. $R\{mechanical\} = R(C)$
4. These two subsystems operate in parallel, therefore,

$$R_I = 1 - [1 - R_f\,]\,[1 - R(C)].$$

Case II. BP4 fails and BP3 is operational.

Then $P_{II} = P_I$ and $R_{II} = R_I$ (by symmetry of the network and assuming identical reliabilities)

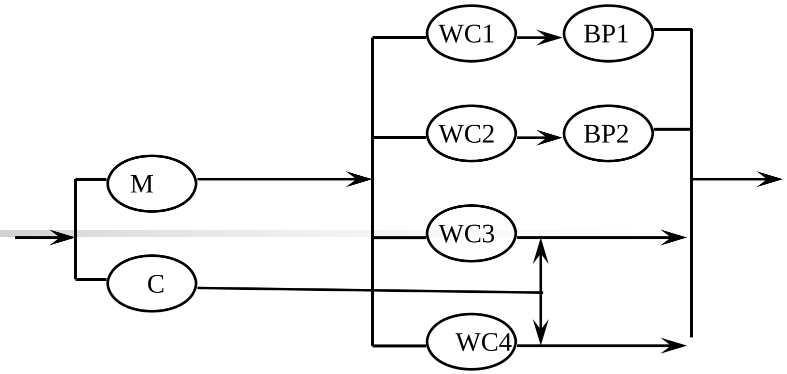


Case III. BP3 and BP4 have failed.

1. $P_{III} = [1 - R(BP)]^2$

2. Cable system has failed.

3. $R_{III} = R(M) \{1 - [1 - R(WC) R(BP)]^2 \}$



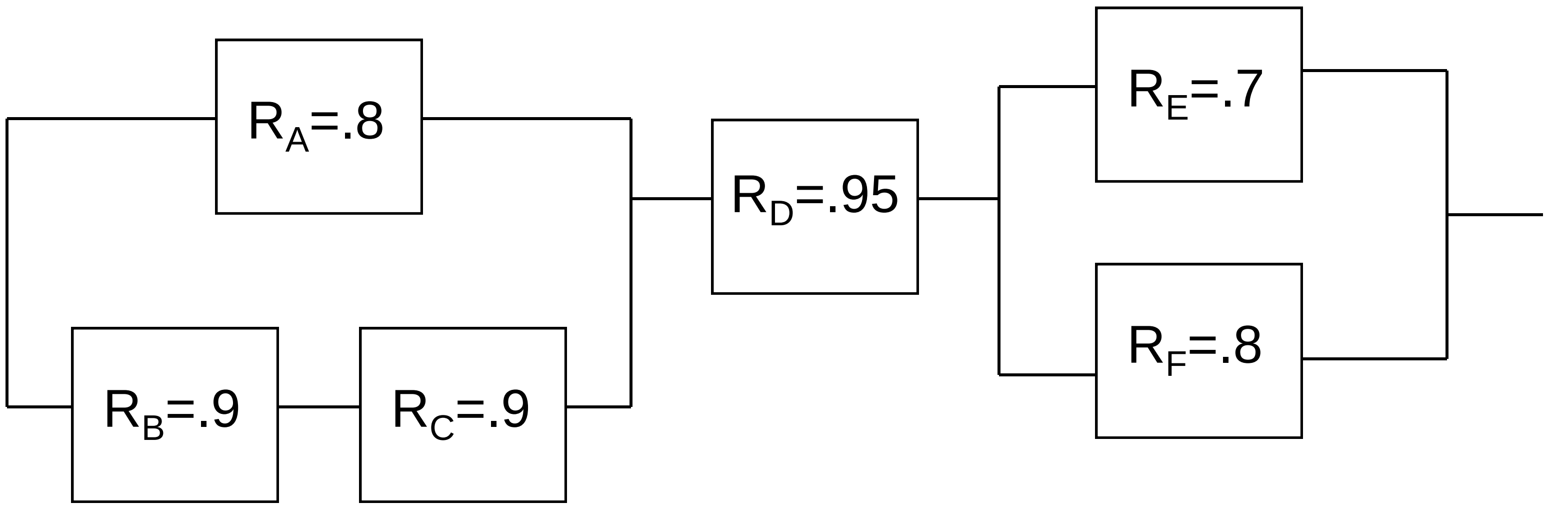
Case IV. Both BP3 and BP4 are operational.

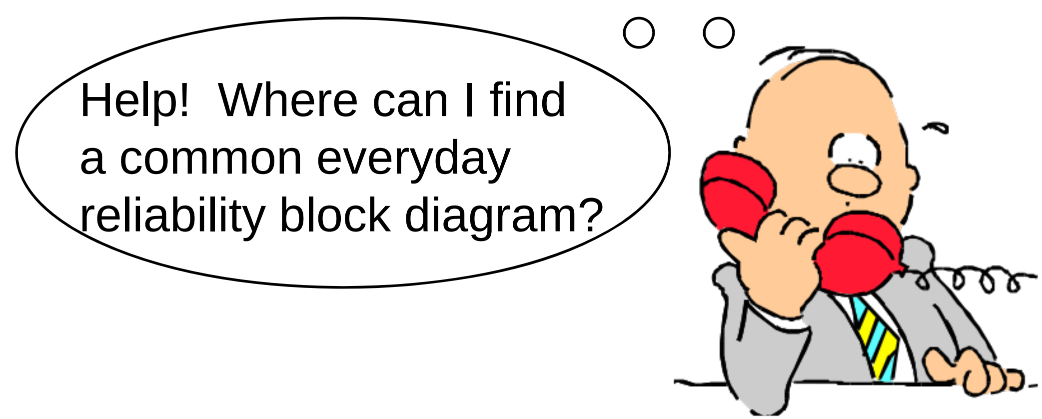
1. $P_{IV} = [R(BP)]^2$
2. $R_f = R(M)\{1 - [1 - R(WC)\ R(BP)]2\ [1 - R(WC)]2\}$.
3. R{cable system} = R(C)
4. $R_{IV} = 1 - [1 - Rf]\ [1 - R(C)]$

The overall system reliability can now be found from:

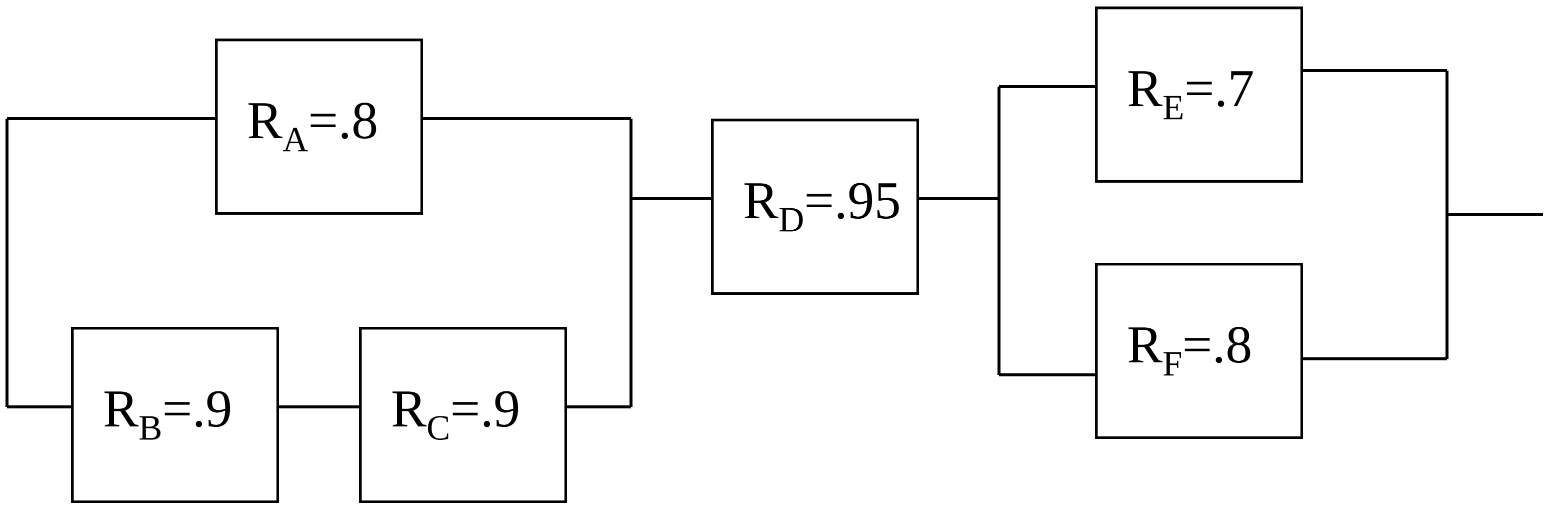
$$R_S = P_I\ R_I + P_{II}\ R_{II} + P_{III}\ R_{III} + P_{IV}\ R_{IV}$$

A Common Everyday Reliability Block Diagram to Solve





A Common Everyday Reliability Block Diagram – solved



.81

1-(1-.7)(1-.8) = .94

1-(1-.8)(1-.81) = .962

R_s = (.962) (.95) (.94) = .859066

Summary

- Series Configuration
- Parallel Configuration
- Combined Series-Parallel Configuration
- High / Low Level Redundancy
- K out-of-n Redundancy
- Complex Configurations – linked networks