LOGM 634 - Homework Set #3

Due 25 September 2017

From the Ebeling text - Exercise 6.3

A computerized airline reservation system has a main computer on-line and a secondary standby computer. The on-line computer fails at a constant rate of 0.001 failures per hour, and the standby units fails when on-line at a constant rate of 0.005 failures per hour. There are no failures while the unit is in the standby mode.

a. Determine the system reliability over a 72-hour period.

   We can compute this reliability using Equation 6.27 where $\lambda_2^- = 0$. Using this equation we have

$$R(72) = e^{-0.001(72)} + \frac{0.001}{0.001 - 0.005}\Big[e^{-0.005(72)} - e^{-0.001(72)}\Big]$$

$$= 0.9305309 + \Big[-0.25 \times -0.2328546\Big]$$

$$= \mathbf{0.9887445}$$

b. The airline desires a system MTTF of 2000 hours. Determine the (minimum) MTTF of the main computer to achieve this goal assuming that the standby computer MTTF does not change.

   We can determine the desired mean time to failure for the main computer $MTTF^* = 1/\lambda_1^*$ using Equation 6.28 where $\lambda_2^- = 0$. Using this equation we find the desired MTTF as

$$2000 = \frac{1}{\lambda_1^*} + \frac{1}{0.005}$$

$$2000 - 200 = \frac{1}{\lambda_1^*}$$

$$\mathbf{1800} = \frac{1}{\lambda_1^*}$$

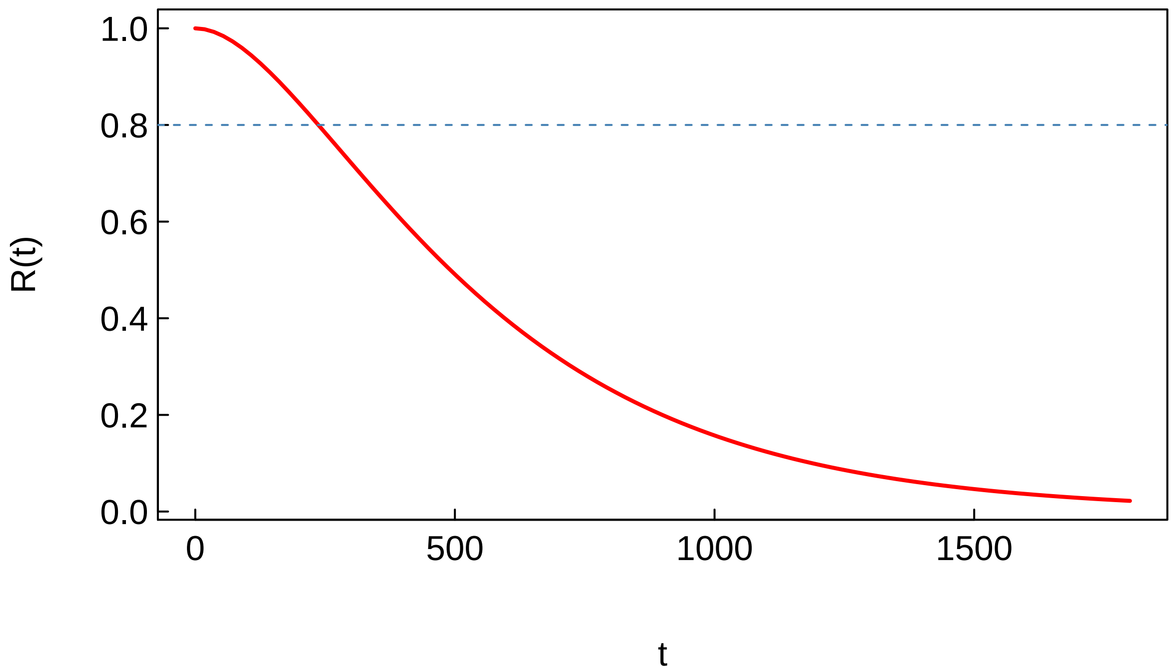
From the Ebeling text - Exercise 6.8

A contractor must decide between two different sump pump systems to be installed in a new housing development. The option is to install a single 1000 gallon per minute (gpm) system or two 500-gpm pumps. If the two-pump system is used, one pump can carry most of the load in the event the other pump fails. Both of the 500-gpm pumps have an $MTTF$ of 800 hr when working together. Their individual $MTTF$ is 200 hr. The 1000-gpm system has a rated $MTTF$ of 700 hr. Which system is preferred on the basis of system $MTTF$? Which system has the best design life for a reliability of 0.80?

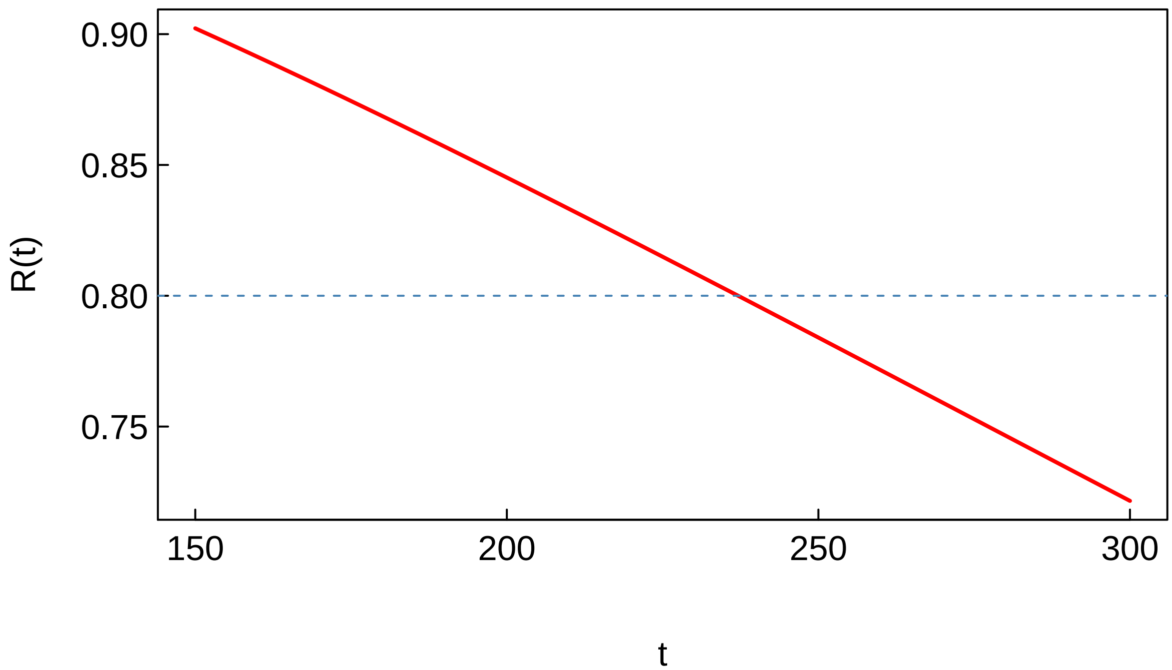
> This problem asks us to compare the reliability of a load sharing system against that of a single unit on the basis of the design life $t_{0.80}$.
>
> For the single-pump alternative the design life may be computed directly as
>
> $$t_{0.80} = -\frac{\ln[R]}{\lambda} = -\ln[0.8] \times 700 = \mathbf{156.2004859} \text{ hours.}$$
>
> For the two-pump (load sharing) system we can express the reliability function using Equation 6.19 where $\lambda_1 = \lambda_2 = \lambda = 1/800$ and $\lambda^+ = 1/200$. Substituting these values into Equation 6.19 gives the following expression
>
> $$R(t) = \exp[-2t/800] + \frac{2/800}{2/800 - 1/200}\Big[\exp[-t/200] - \exp[-2t/800]\Big]$$
>
> $$= \exp[-t/400] + \exp[-t/400] - \exp[-t/200]$$
>
> We can find $t_{0.8}$ by
>
> 1. Taking the derivative of $R(t)$ and setting it equal to 0 and solving for t
> 2. Entering $0.8 = \exp[-t/400] + \exp[-t/400] - \exp[-t/200]$ into a solver to find $t_{0.8}$
> 3. Graphing $R(t)$ over a wide interval to find the value of t such that $R(t) = 0.8$
>
> Using a solver in `R` give the following value for $t_{0.8}$

```r
f1 <- function(x) 0.8-(exp(-x/400)+exp(-x/400)-exp(-x/200))
uniroot(f = f1, interval = c(100,1000))$root
## [1] 237.1134
```

> Plotting $R(t)$ over a large interval gives the plot below



After observing the previous plot we can produce another plot of $R(t)$ over a narrower region of t to get a better estimate of $t_{0.8}$. The plot show $R(t)$ over the interval $t \in [150, 300]$ we can see that $t_{0.8}$ is approximately 230 hours.



Based on the results already shown, we conclude that the two-pump alternative is the better choice on the basis of the design life at a reliability of 0.8.

From the Ebeling text - Exercise 7.5 (Use Eqn 7.15)

A load is exponentially distributed with a mean of 25. The strength is also exponentially distributed. Determine the minimum value of the mean strength to achieve a reliability of 0.95.

> In this question, we are asked to determine the probability that the level of stress applied to an item X is less than the item's strength Y. The trick in this problem is that neither the stress or the strength are known. Regardless, knowledge of the four main probability distribution functions are more than enough to solve this. Let's break this down.
>
> At the base level, we want to know $P(X \leq y)$. But, this is just the CDF of the stress distribution evaluated at y. We denote this quantity as $F_X(y)$, which just means the CDF for the random variable X - evaluated at y, which is expressed as
>
> $$F_X(y) = 1 - \exp\left[-\frac{y}{\lambda_X}\right]$$
>
> We still don't know the value of y. True, so we have to determine $P(X \leq y)$ for each value of y. BUT WAIT!!! We also need to know the probability of observing y in the first place. Right, so that means that
>
> $$\begin{aligned} R &= P(X \leq y) \\ &= P(X \leq y | Y = y) \cdot P(Y = y) \qquad \forall y \\ &= \int_0^\infty F_X(y) \cdot f_Y(y) dy \\ &= \int_0^\infty \left(1 - \exp\left[-\frac{y}{25}\right]\right) \frac{1}{\lambda_X} \exp\left[-\frac{y}{\lambda_X}\right] dy \end{aligned}$$
>
> Now, all we have to do is solve this integral...set that equal to 0.95...and then solve for λ_X. Okay, that would suck - thankfully Ebeling provides the following equation to make this easier
>
> $$R = \frac{1}{1 + \lambda_X / \lambda_Y}$$
>
> We can find λ_X by setting the above equation equal to 0.95 and solving. In this case we have

$$0.95 = \frac{1}{1 + \lambda_X/\lambda_Y}$$

$$0.95(1 + \lambda_X/\lambda_Y) = 1$$

$$1 + \lambda_X/\lambda_Y = \frac{1}{0.95}$$

$$\frac{\lambda_X}{\lambda_Y} = 1.052632 - 1$$

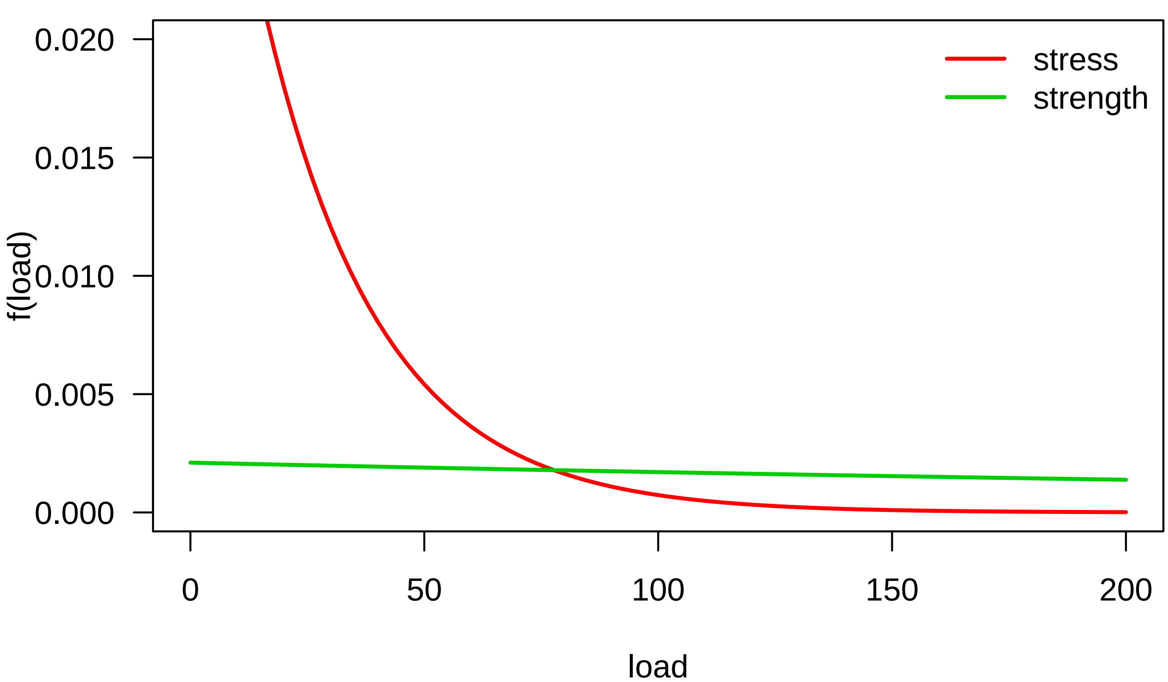
$$\frac{\lambda_X}{\lambda_Y} = .052632$$

$$\lambda_X = \lambda_Y \cdot .052632$$

$$\lambda_X = \frac{.052632}{25} = 0.002105263$$

And the mean strength is $\mathbf{1/\boldsymbol{\lambda}_x = 475}$

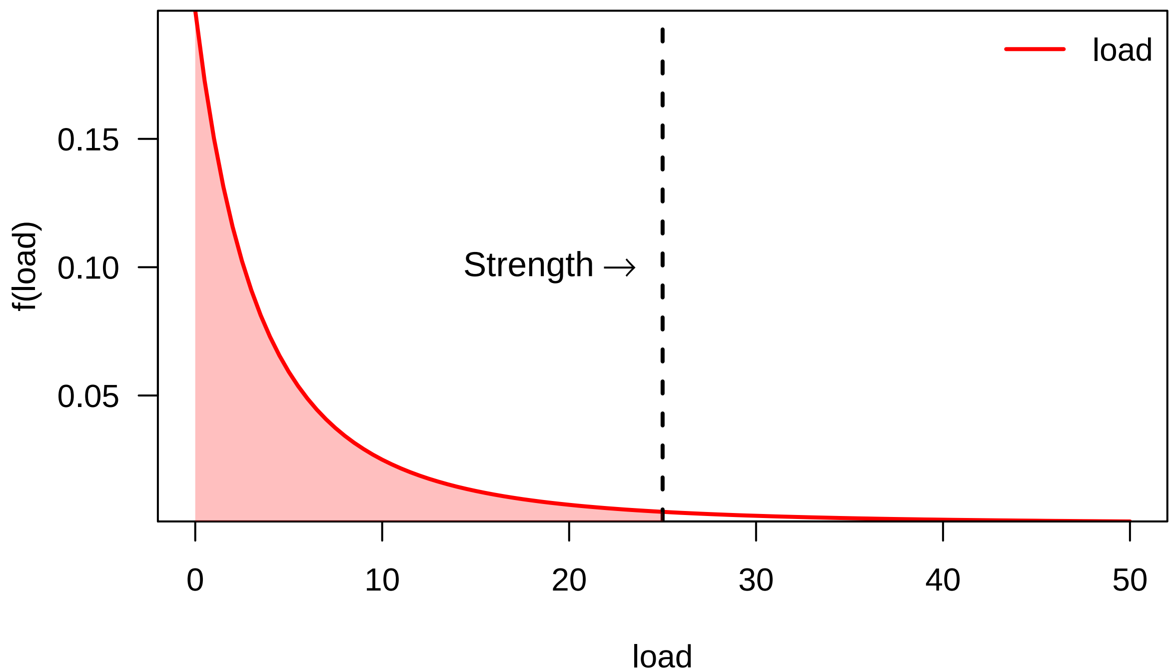
The figure below shows a plot of the density functions where load $\sim EXP(\lambda = 1/25)$ and load $\sim EXP(\lambda = 1/475)$.



From the Ebeling text - Exercise 7.15 (Use Eqn. 7.11)

The breaking strength of a cutting tool is a constant 25 lb. If the load being placed on the tool has the following probability density function, compute the tool's static reliability.

$$f_X(x) = \frac{200}{(x+10)^3} \qquad x \geq 0$$



The static reliability for this cutting tool is mathematically expressed as

$$R = P(\text{load} \leq \text{strength})$$

$$= P(\text{load} \leq 25)$$

This value is equal to the area under the load curve in the interval $[0, 25]$, which can be found by taking the following integral

$$\int_0^{25} \frac{200}{(x+10)^3} dx = \mathbf{0.9183673}.$$