LOGM 634 - Homework Set #2

Due 20 September 2017

From the Ebeling text - Exercise 4.1

For a system having a Weibull failure distribution with a shape parameter of 1.4 and a scale parameter of 550 days, find the following

a. $R(100\text{days})$

Using the Weibull reliability function presented in the Ebeling, and the given parameters, we can compute $R(100)$ as

$$R(t = 100; \theta = 550, \beta = 1.4) = \exp\left[-\left(\frac{100}{550}\right)^{1.4}\right] = \mathbf{0.9121622}$$

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b. The $B1$ life

The B1 life is the time at which 1% of the poulation of units has failed. This is also the same as the design life for a reliability of 0.99. This value can be computed as

$$\begin{aligned}
\exp\left[-\left(\frac{t_{0.99}}{550}\right)^{1.4}\right] &= 0.99 \\
\left(\frac{t_{0.99}}{550}\right)^{1.4} &= -\ln[0.99] \\
\left(\frac{t_{0.99}}{550}\right)^{1.4} &= 0.0100503 \\
\frac{t_{0.99}}{550} &= (0.0100503)^{1/1.4} \\
t_{0.99} &= 550 \times 0.0374099 = \mathbf{20.575425} \text{ days}
\end{aligned}$$

c. *MTTF*

Using the expression for the mean of a Weibull random variable (Equation 4.4), we get

$$\begin{aligned}\text{MTTF} &= \theta \times \Gamma(1 + 1/\beta) \\ &= 550 \times \Gamma(1 + 1/1.4) \\ &= 550 \times \Gamma(1.714) \\ &= 550 \times 0.9113661 = \mathbf{501.2513706} \text{ days}\end{aligned}$$

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d. The standard deviation

Using the expression for the variance of a Weibull random variable (Equation 4.5), we get

$$\begin{aligned}\sigma &= \left[\theta^2 \left\{\Gamma(1 + 2/\beta) - \left[\Gamma(1 + 1/\beta)\right]^2\right\}\right]^{0.5} \\ &= \left[550^2 \left\{\Gamma(1 + 2/1.4) - \left[\Gamma(1 + 1/1.4)\right]^2\right\}\right]^{0.5} \\ &= \left[550^2 \left\{1.2658235 - \left[0.9114233\right]^2\right\}\right]^{0.5} \\ &= \left[550^2 \left\{0.435131\right\}\right]^{0.5} = \mathbf{362.8045315}\end{aligned}$$

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e. t_{med}

The median is the same as the design life for a reliability of 0.50. Using the expression for the design life of a Weibull random variable (Equation 4.6), we get

$$t_{0.5} = \theta \left(-\ln[0.5]\right)^{1/\beta} = 550 \left(-\ln[0.5]\right)^{1/1.4} = \mathbf{423.3178583} \text{ days}$$

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f. t_{mode}

Using the expression for the mode of a Weibull random variable (Equation 4.8), we get

$$t_{mode} = \theta(1 - 1/\beta)^{1/\beta} = 550(1 - 1/1.4)^{1/1.4} = \mathbf{224.7722318} \text{ days}$$

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g. The design life for a reliability of 0.90

One again Using the expression for the design life of a Weibull random variable (Equation 4.6), we get

$$t_{0.9} = \theta \left(-\ln[0.9]\right)^{1/\beta} = 550 \left(-\ln[0.9]\right)^{1/1.4} = \mathbf{110.2238221} \text{ days}$$

From the Ebeling text - Exercise 4.20

A rotor used in an AC motor manufactured by the Toole N. Di Company has a time failure that is lognormal with an $MTTF$ found to be 3600 operating hours and a shape parameter s equal to 2.

a. Current preventative maintenance practices require the rotor to be replaced every 100 operating hours. Determine the probability that a rotor will survive the 100 hr.

   This problem requires us to find $R(100)$. The Ebeling text defines the lognormal distribution using the standard deviation s and the median t_{med} where the reliability function is expressed as

$$R(t) = 1 - \Phi\left(\frac{1}{s}\ln\left[\frac{t}{t_{med}}\right]\right).$$

   For this problem we are told that $s = 2$, but still need to compute t_{med}. Table 4.2 shows the following relationship between μ and t_{med} lognormal distribution

$$\mu = t_{med} \times \exp\left(\frac{s^2}{2}\right).$$

   After inverting this relationship to solve for t_med and substituting the value $\mu = 3600$ that was provided gives the following value for the median.

$$t_{med} = 3600 \times \exp\left(-\frac{2^2}{2}\right) = 3600 \times 0.1353353 = 487.2070197$$

   Using this value we can compute $R(100)$ as

$$R(100) = 1 - \Phi\left(\frac{1}{2}\ln\left[\frac{100}{487.2070197}\right]\right) = \mathbf{0.7857495}$$

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b. If at the end of 100 operating hours, the maintenance department neglects to replace the rotor, what is the probability that it will survive until the next scheduled replacement (assume that that it has not failed at 100 hr).

   This problem requires that we compute the conditional reliability $R(T_o + t)/R(T_o)$. Performing this computation shows that the probability of failure in the second interval is actually less than in the first interval.

$$R\left(\frac{T_o + t}{T_o}\right) = R\left(\frac{100}{100}\right)$$

$$= \frac{1 - \Phi\left(\frac{1}{2}\ln\left[\frac{200}{487.2070197}\right]\right)}{1 - \Phi\left(\frac{1}{2}\ln\left[\frac{100}{487.2070197}\right]\right)} = \frac{0.6719073}{0.7857495} = \mathbf{0.8551164}$$

c. If the rotor is operating after 200 hr, should it be replaced?

Once again, we need to compute the conditional reliability $R(T_o + t)/R(T_o)$. Performing this computation shows that the probability of failure is again decreased.

$$R\left(\frac{T_o + t}{T_o}\right) = R\left(\frac{100}{200}\right)$$

$$= \frac{1 - \Phi\left(\frac{1}{2}\ln\left[\frac{300}{487.2070197}\right]\right)}{1 - \Phi\left(\frac{1}{2}\ln\left[\frac{200}{487.2070197}\right]\right)} = \frac{0.5957855}{0.6719073} = \mathbf{0.886708}$$

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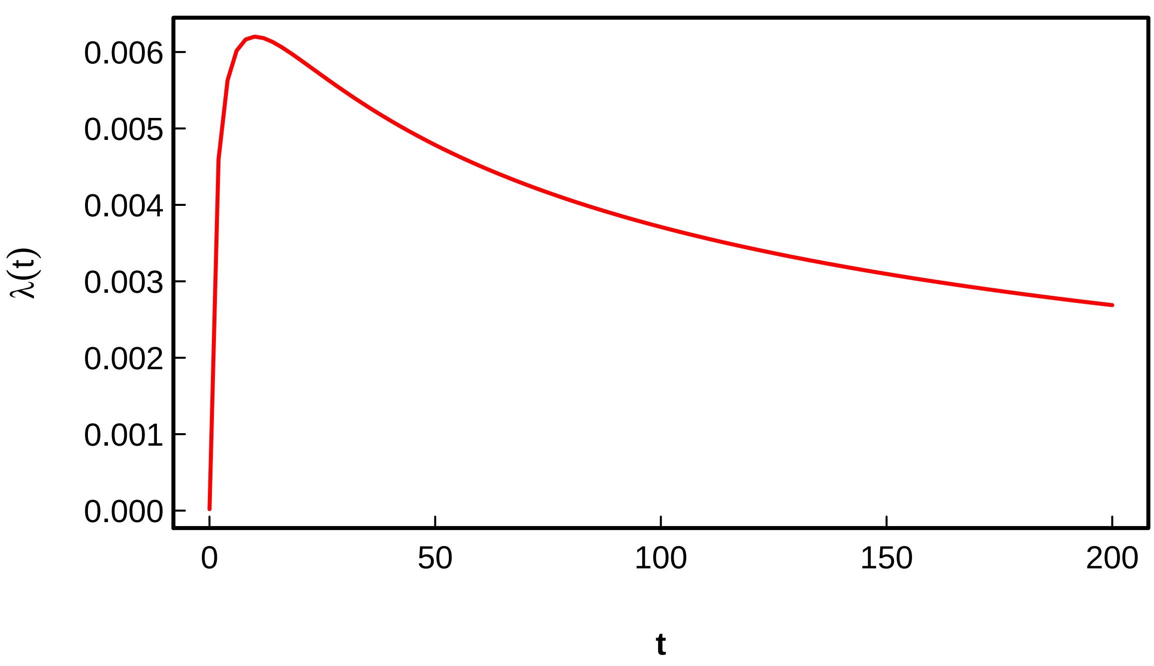
d. From the above analysis, what can you say about the hazard rate and the preventative maintenance replacement policy?

The hazard rate is decreasing after 100 hours of operation. Any unit that is still operating after 100 hours should be allowed to continue operating until failure.

The figure below shows that the hazard function steadily decreases after about 25 hours of operation.



Summary Exercise

Use the system relational block diagram shown in the figure below and the component distributions to answer the following questions:

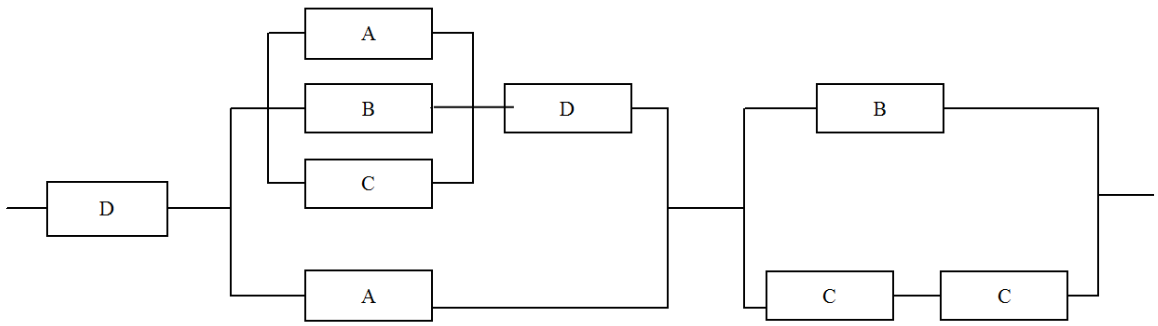


Figure 1: Reliability Block Diagram

$$\text{Component A} \sim EXP(\theta = 1/\lambda = 1250 \text{ hrs})$$
$$\text{Component B} \sim LOGNOR(t_{med} = 1500 \text{ hrs}, \quad s = 0.25)$$
$$\text{Component C} \sim WEIBULL(\theta = 1300 \text{ hrs}, \quad \beta = 3.5)$$
$$\text{Component D} \sim EXP(\theta = 1/\lambda = 1500 \text{ hrs})$$

a. What is the probability that the system will fail in it's first 1200 hrs of operation?

First, let's find each component's reliability at 1200 hours. These probabilities are computed from the respective CDF's of each distribution listed above. The table below shows these probabilities

Component	Distribution	$Pr(T > 1200)$
A	$EXP(\lambda = 1/1250)$	0.3829
B	$LOGNOR(t_{med} = 1500, s = 0.25)$	0.8140
C	$WEIBULL(\theta = 1300, \beta = 3.5)$	0.4697
D	$EXP(\lambda = 1/1500)$	0.4493

With these values in hand we can compute the probability of failure for the system in it's first 1200 hours of operation. We'll treat the system as three 'sub-systems' in series.

Since subsystem 1 only contains component D, we've already determined its reliability as **0.5507**.

For subsystem 2, the relibility is expressed as follows.

$$R_{SS_2} = 1 - \Bigg[1 - \Big(1 - [1 - R_A][1 - R_B][1 - R_C]\Big)R_D\Bigg]\Bigg[1 - R_A\Bigg]$$

$$= 1 - \Bigg[1 - \Big(1 - [1 - 0.3829][1 - 0.814][1 - 0.4697]\Big)0.4493\Bigg]\Bigg[1 - 0.3829\Bigg]$$

$$= 1 - \Bigg[1 - \Big(1 - [0.6171][0.186][0.5303]\Big)0.4493\Bigg]\Bigg[1 - 0.3829\Bigg]$$

$$= 1 - \Bigg[1 - 0.0608682 \times 0.4493\Bigg]\Bigg[1 - 0.3829\Bigg]$$

$$= 1 - 0.5780481 \times 0.6171$$

$$= 1 - 0.3567135 = \mathbf{0.6432865}$$

For subsystem 3, the reliability may be computed as follows.

$$R_{SS_3} = 1 - (1 - R_B)(1 - R_C^2)$$

$$= 1 - (1 - 0.814)(1 - 0.4697^2)$$

$$= 1 - (0.186)(0.7793819)$$

$$= 1 - 0.144965 = \mathbf{0.855035}$$

As these subsystems are arranged in series, the overall system reliability may be computed as follows.

$$R_{SYS} = R_{SS_1} \times R_{SS_2} \times R_{SS_3}$$

$$= 0.4493 \times 0.6432865 \times 0.855035 = \mathbf{0.2471296}$$

Finally, we compute the overall system failure probability as

$$1 - R_{SYS} = \mathbf{0.7528704}$$

b. What is the probability that the system will fail in the next 200 hrs, given it has already been operating, without failure, for 1000 hours?

As we did for part (a) we'll first find each component's conditional relibility $R(200|1000)$. These reliabilities are computed by taking the compliment of the CDF $1 - F(t)$ for each of the respective distributions listed above. The table below shows these reliabilities

Component	Distribution	$R(1200 \vert 1000)$
A	$EXP(\lambda = 1/1250)$	0.8521
B	$LOGNOR(t_{med} = 1500, s = 0.25)$	0.8590
C	$WEIBULL(\theta = 1300, \beta = 3.5)$	0.7001
D	$EXP(\lambda = 1/1500)$	0.8752

With these values in hand we can compute the reliability of the system in it's first 1200 hours of operation. We'll treat the system as three 'sub-systems' in series.

Since subsystem 1 only contains component D, we've already determined its reliability as **0.8752**.

For subsystem 2, the relibility is expressed as follows.

$$R_{SS_2} = 1 - \left[1 - \left(1 - [1 - R_A][1 - R_B][1 - R_C]\right)R_D\right]\left[1 - R_A\right]$$

$$= 1 - \left[1 - \left(1 - [1 - 0.8521][1 - 0.859][1 - 0.7001]\right)0.8752\right]\left[1 - 0.8521\right]$$

$$= 1 - \left[1 - \left(1 - [0.1479][0.141][0.2999]\right)0.8752\right]\left[1 - 0.8521\right]$$

$$= 1 - \left[1 - 0.0062541 \times 0.8752\right]\left[1 - 0.8521\right]$$

$$= 1 - 0.1302736 \times 0.1479$$

$$= 1 - 0.0192675 = \mathbf{0.9807325}$$

For subsystem 3, the reliability may be computed as follows.

$$R_{SS_3} = 1 - (1 - R_B)(1 - R_C^2)$$

$$= 1 - (1 - 0.859)(1 - 0.7001^2)$$

$$= 1 - (0.141)(0.50986)$$

$$= 1 - 0.0718903 = \mathbf{0.9281097}$$

As these subsystems are arranged in series, the overall system reliability may be computed as follows.

$$R_{SYS} = R_{SS_1} \times R_{SS_2} \times R_{SS_3}$$

$$= 0.8752 \times 0.9807325 \times 0.9281097 = \mathbf{0.796631}$$

Finally, we compute the overall system failure probability as

$$1 - R_{SYS} = \mathbf{0.203369}$$