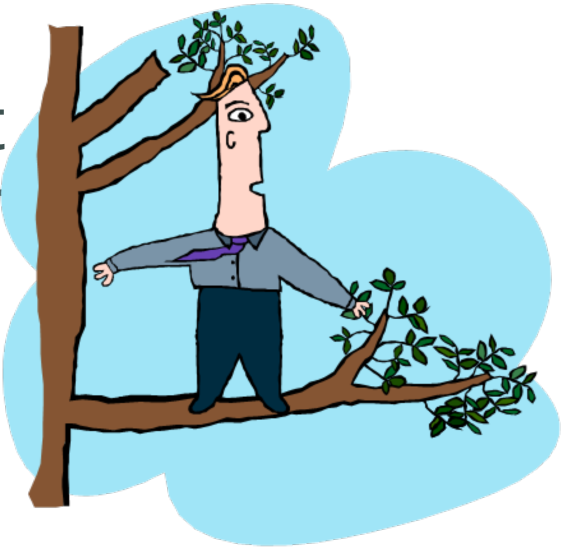
Chapter 18 Section 8.5
Fault Trees Analysis (FTA)

Don't get caught out
on a limb of your
fault tree.



C. Ebeling, *Intro to Reliability & Maintainability Engineering, 2nd ed.* Waveland Press*, Inc.* Copyright © 2010

Characteristics

- Graphical design technique
- Alternative to reliability block diagrams
- Broader in scope
- Perspective on faults rather than reliability
- Model events rather than components
- Faults include failures
- Focus on a catastrophic event (top event)
- Top-down deductive analysis

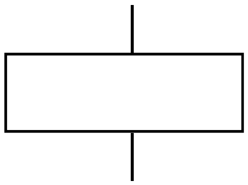
The Four Steps to a FTA

(1) Define the system, its boundaries, and the top event,
(2) Construct the fault tree representing symbolically the system and its relevant events,
(3) Perform a qualitative evaluation by identifying those combinations of events which will cause the top event,
(4) Perform a quantitative evaluation by assigning failure probabilities or unavailabilities to the basic events and computing the probability of the top event.

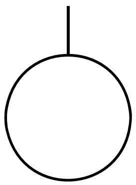
Fault Tree Symbols

AND gate - a logic gate where an output event occurs only when all the input events have occurred.

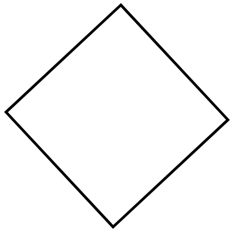
OR gate - a logic gate where an output event occurs if at least one of the input events have occurred.



Resultant event - a fault event resulting from the logical combination of other fault events and usually an output to a logic gate.

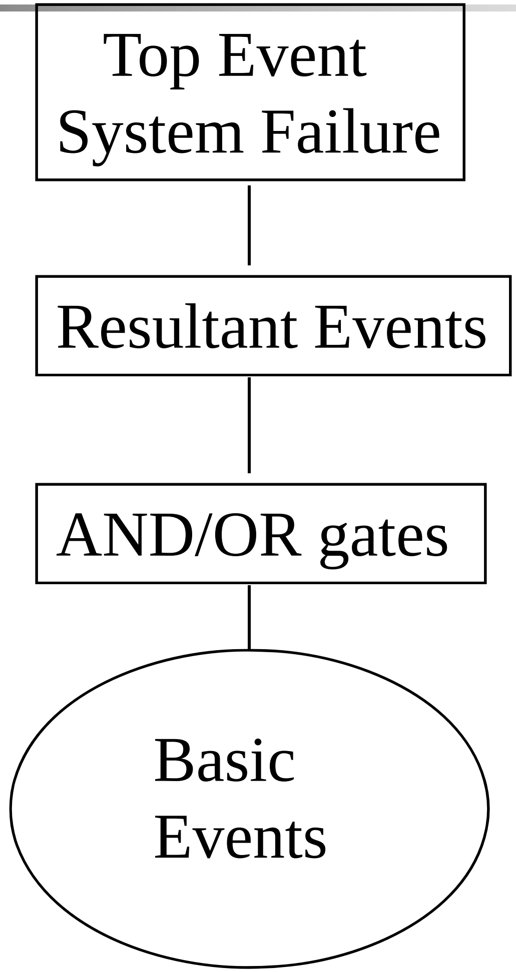


Basic event - an elementary event representing a basic fault or component failure.

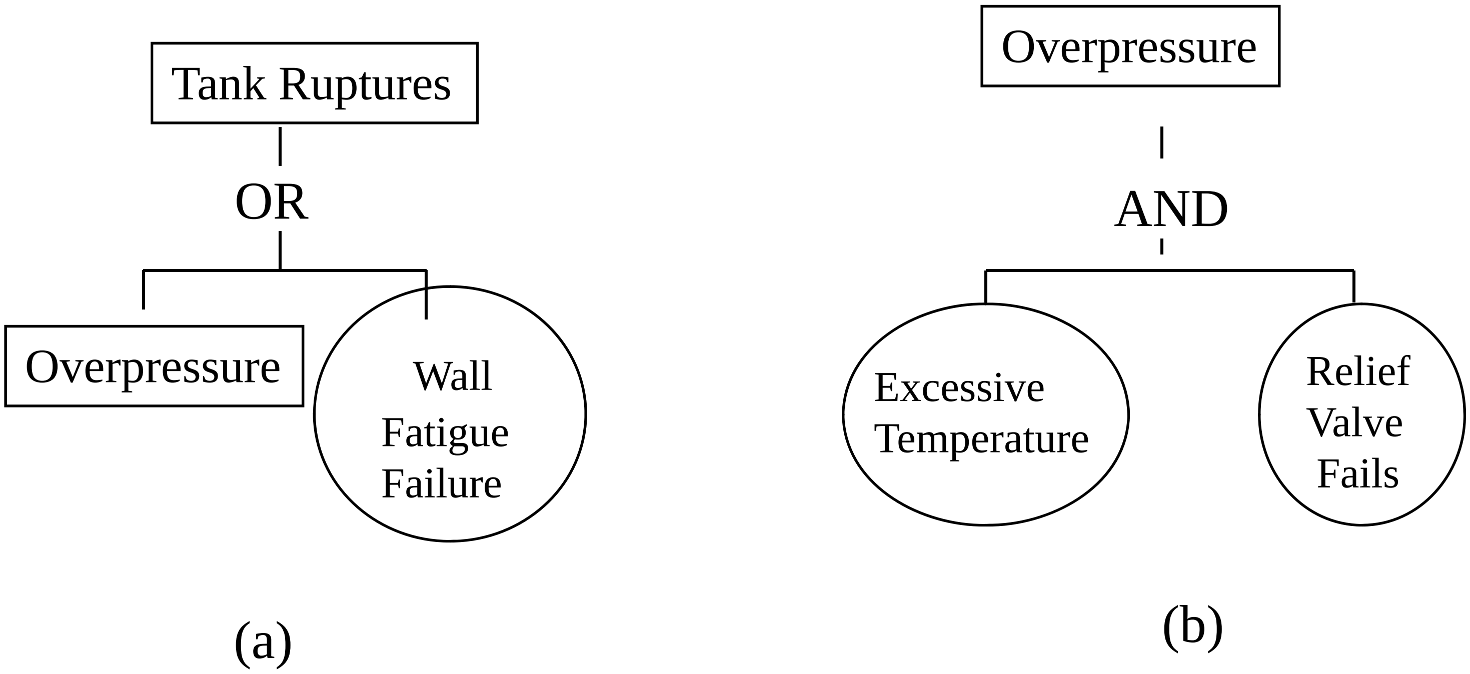


Incomplete event - an event that has not been fully developed because of lack of knowledge or its unimportance.

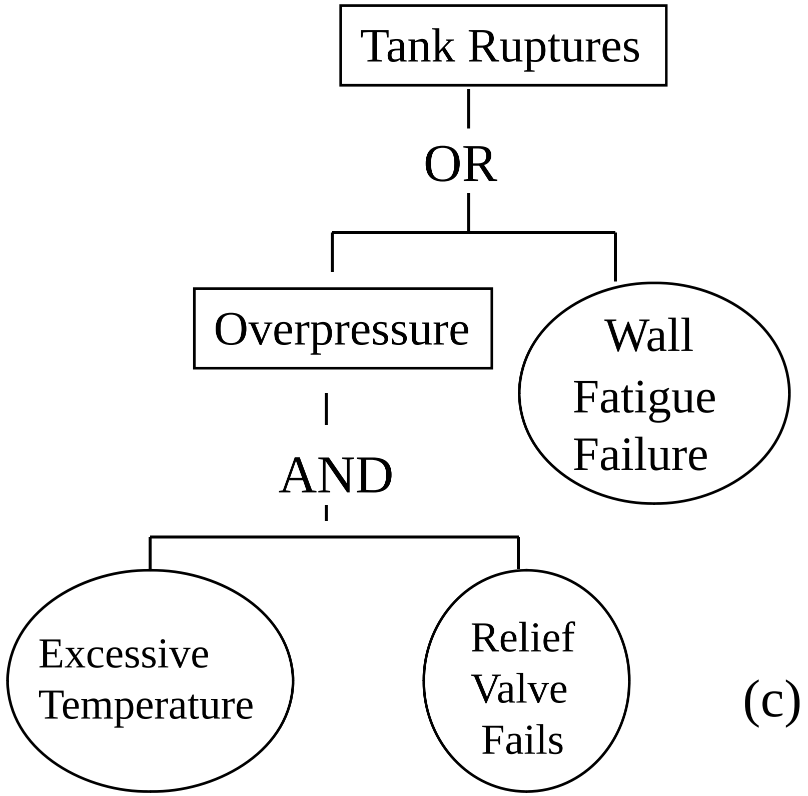
General Structure of a Fault Tree



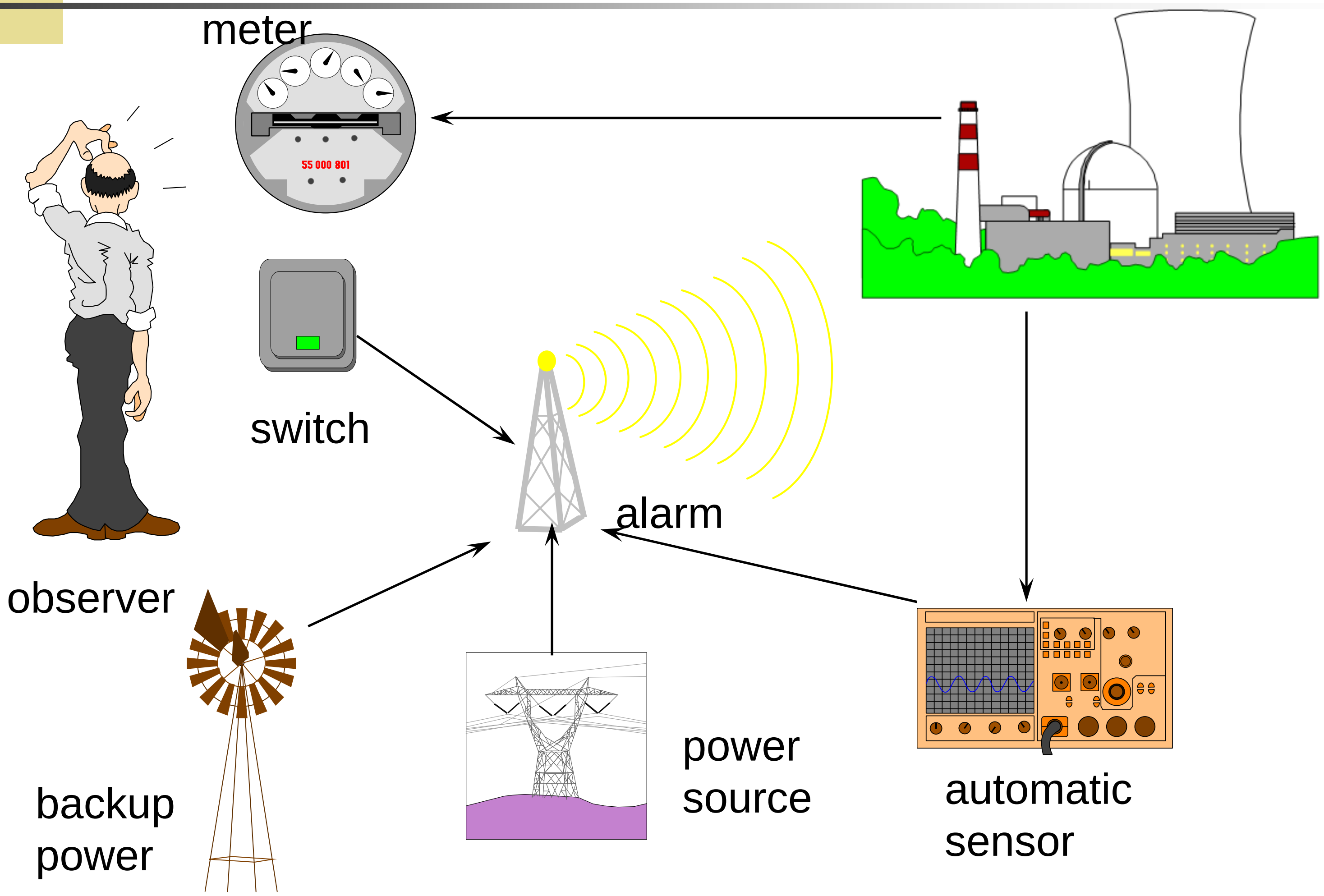
Example of AND / OR Gates



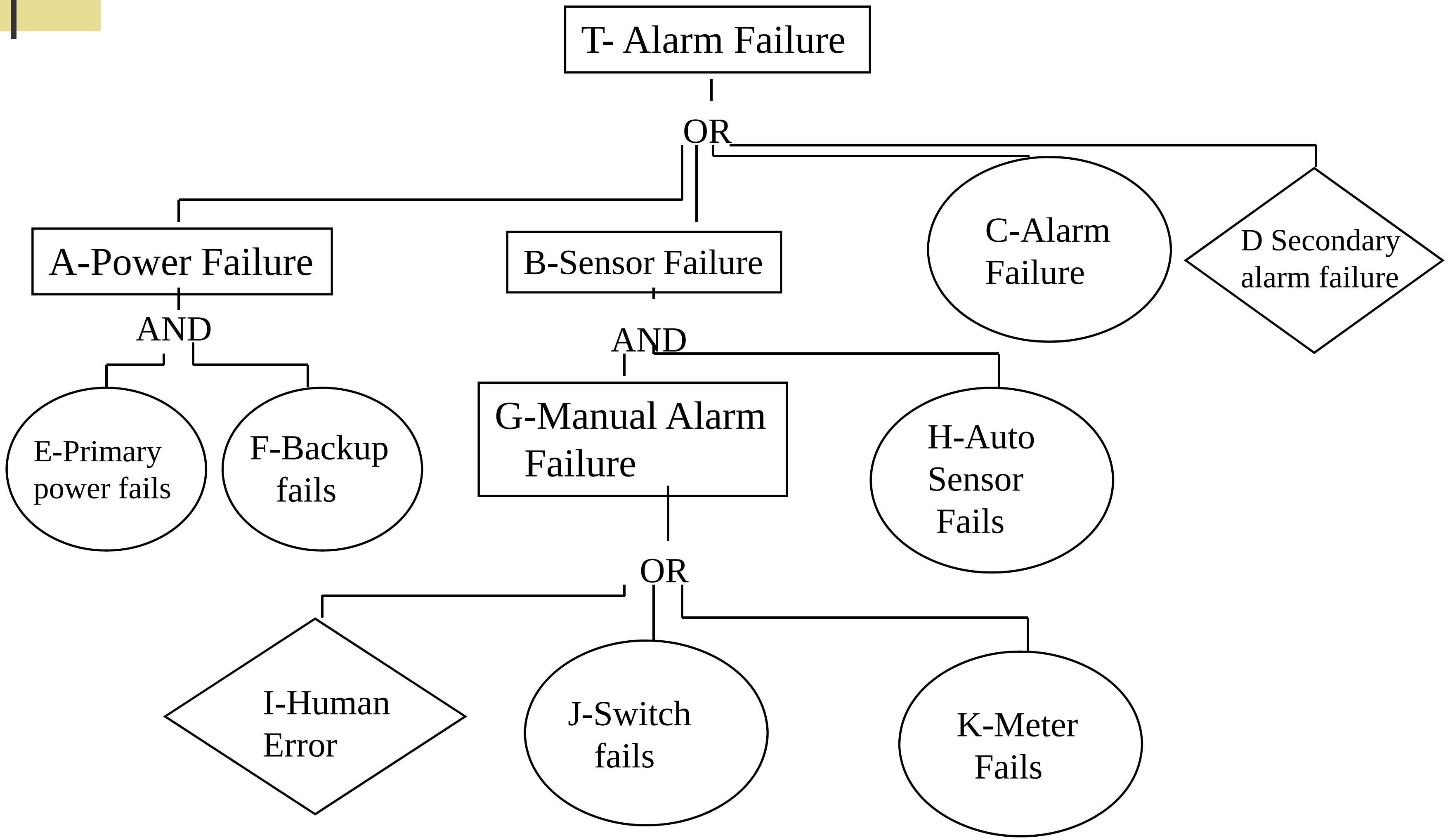
Example of AND / OR Gates



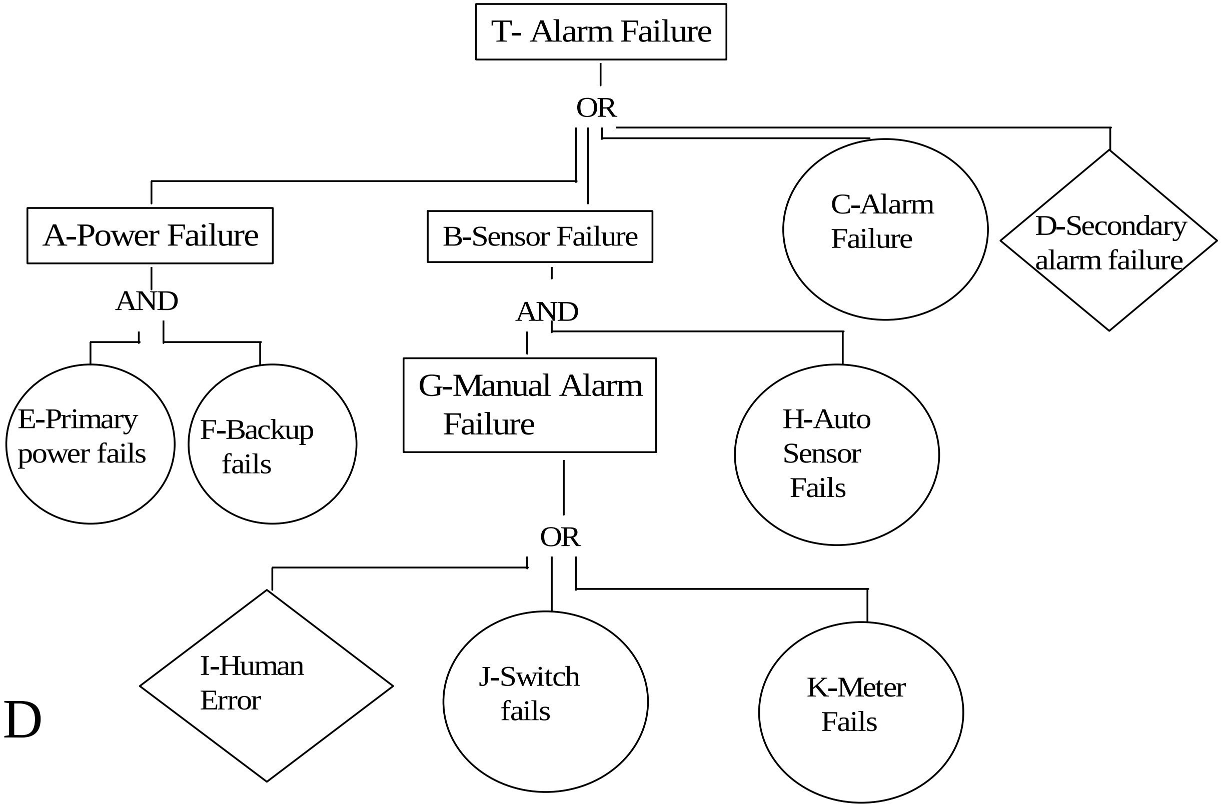
Alarm System Example



Alarm System Fault Tree



Boolean representation of Top Event

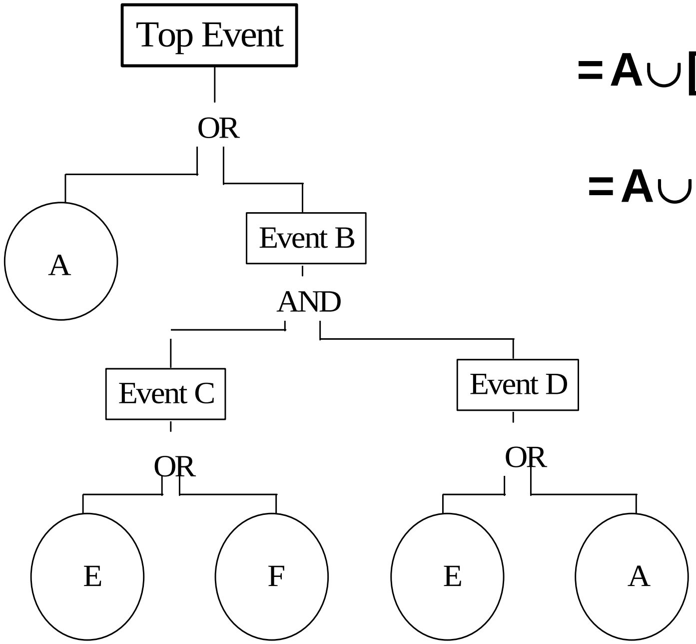


$$T = A \cup B \cup C \cup D$$

$$= (E \cap F) \cup (G \cap H) \cup C \cup D$$

$$= (E \cap F) \cup [(I \cup J \cup K) \cap H] \cup C \cup D$$

Example



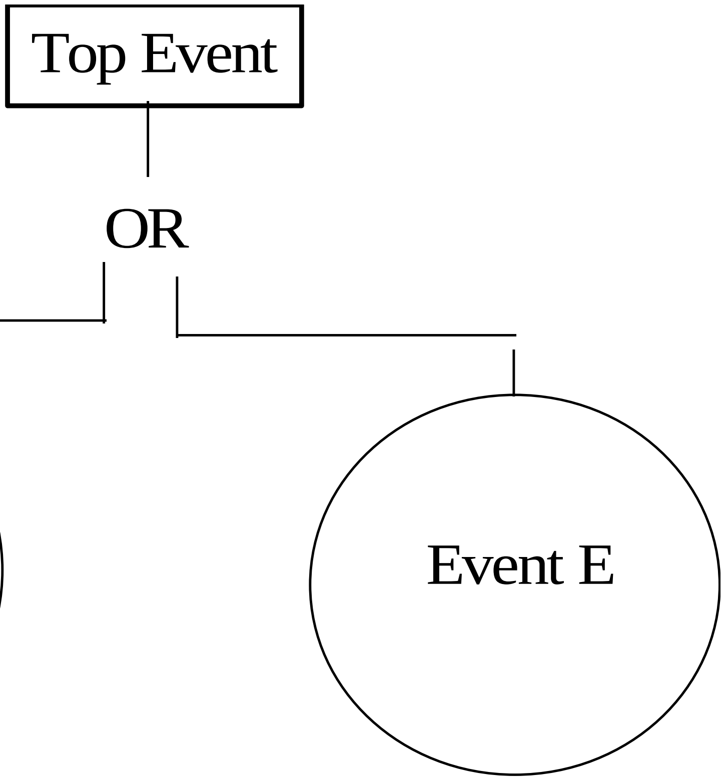
$$T = A \cup B = A \cup (C \cap D)$$

$$= A \cup [(E \cup F) \cap (E \cup A)]$$

$$= A \cup [E \cup (F \cap A)] = A \cup E \text{ since } A \cup (F \cap A) = A$$

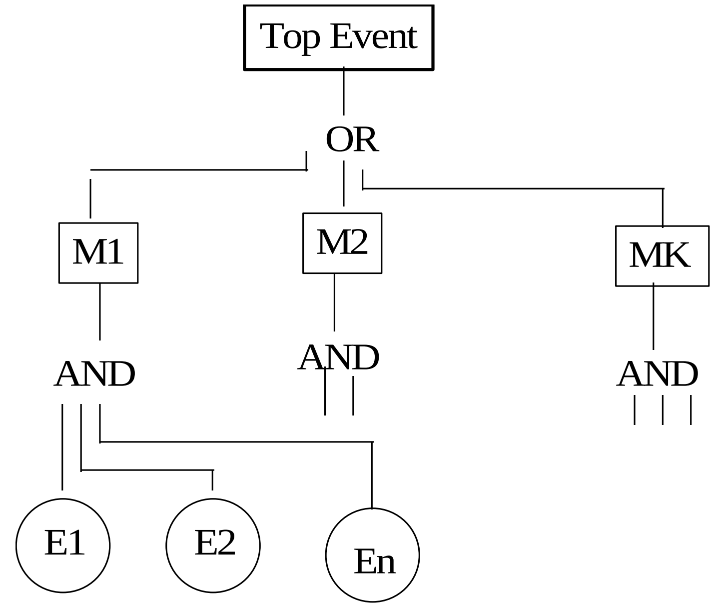
Example Equivalent Fault Tree

$T = A \cup E$



Minimal Cut Sets

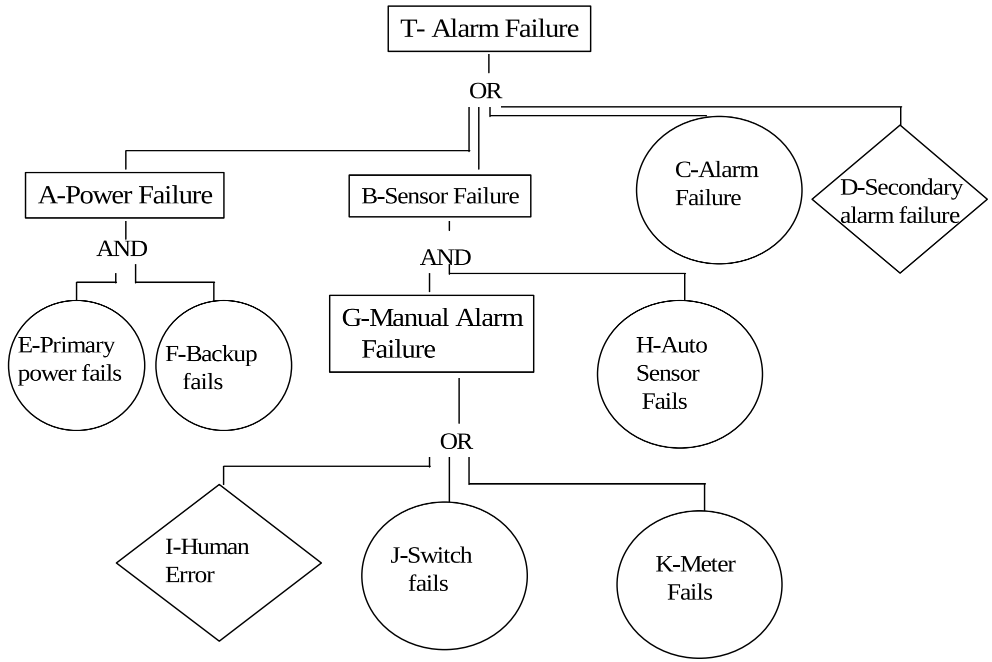
A cut set is a collection of basic events which will cause the top event. A minimal cut set is one with no unnecessary events. That is, all the events within the cut set must occur to cause the top event



$T = M_1 \cup M_2 \cup ... \cup M_k$ where $M_i = E_1 \cap E_2 \cap .... \cap E_{ni}$

and E_i are basic events.

Example

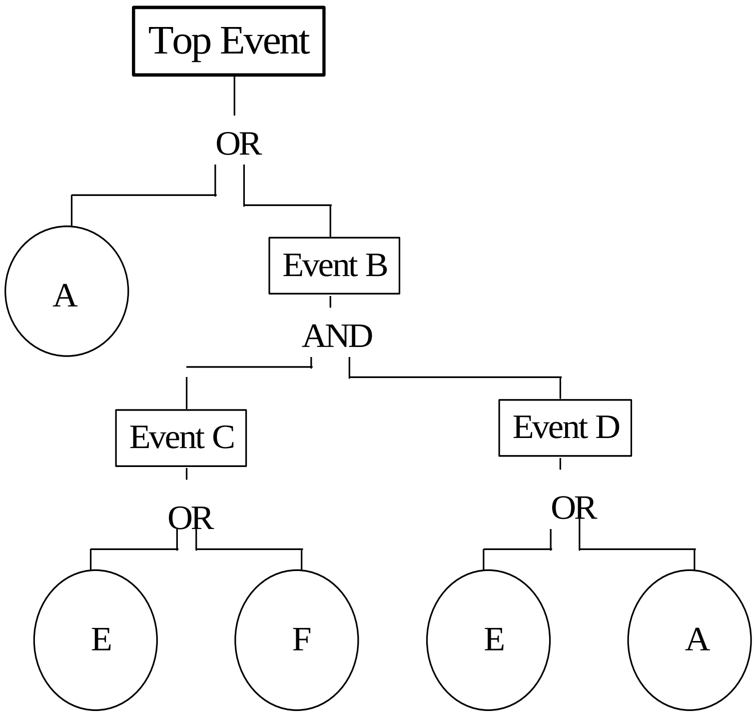


1	**2**	**3**
A	E,F	E,F
B	G,H	I,H J,H K,H
C	C	C
D	D	D

$$\mathbf{T = (E \cap F) \cup (I \cap H) \cup (J \cap H) \cup (K \cap H) \cup C \cup D}$$

Example Cut Sets

1	2	3	4	5
A	A	A	A	A
B	C, D	E, D	E, E	E
		F, D	E, A	
			F, E	
			F, A	



since $E \cap E = E$, $E \cap A \subseteq A$, $F \cap E \subseteq E$, and $F \cap A \subseteq A$. Therefore $T = A \cup E$

Quantitative Analysis

If cut sets are mutually exclusive:

If $P(M) < 10^{-3}$

$$P(T) = P(M_1 \cup M_2 \cup \ldots \cup M_k)$$

$$= P(M_1) + P(M_2) + \ldots + P(M_k)$$

If not:

$$P(T) = P(A \cup E) = P(A) + P(E) - P(A \cap E).$$

$$P(T) = P(M_1) + P(M_2) - P(M_1)\,P(M_2) + \text{etc.}$$

$$P(M_i) = P(E_1 \cap E_2 \cap \ldots \cap E_{ni}) = P(E_1)\,P(E_2)\ldots P(E_{n1})$$

if independent

Then $P(M_1)\,P(M_2) < 10^{-6}$

Example

$$P(T) = P\{ (E \cap F) \cup [(I \cup J \cup K) \cap H] \cup C \cup D \}$$

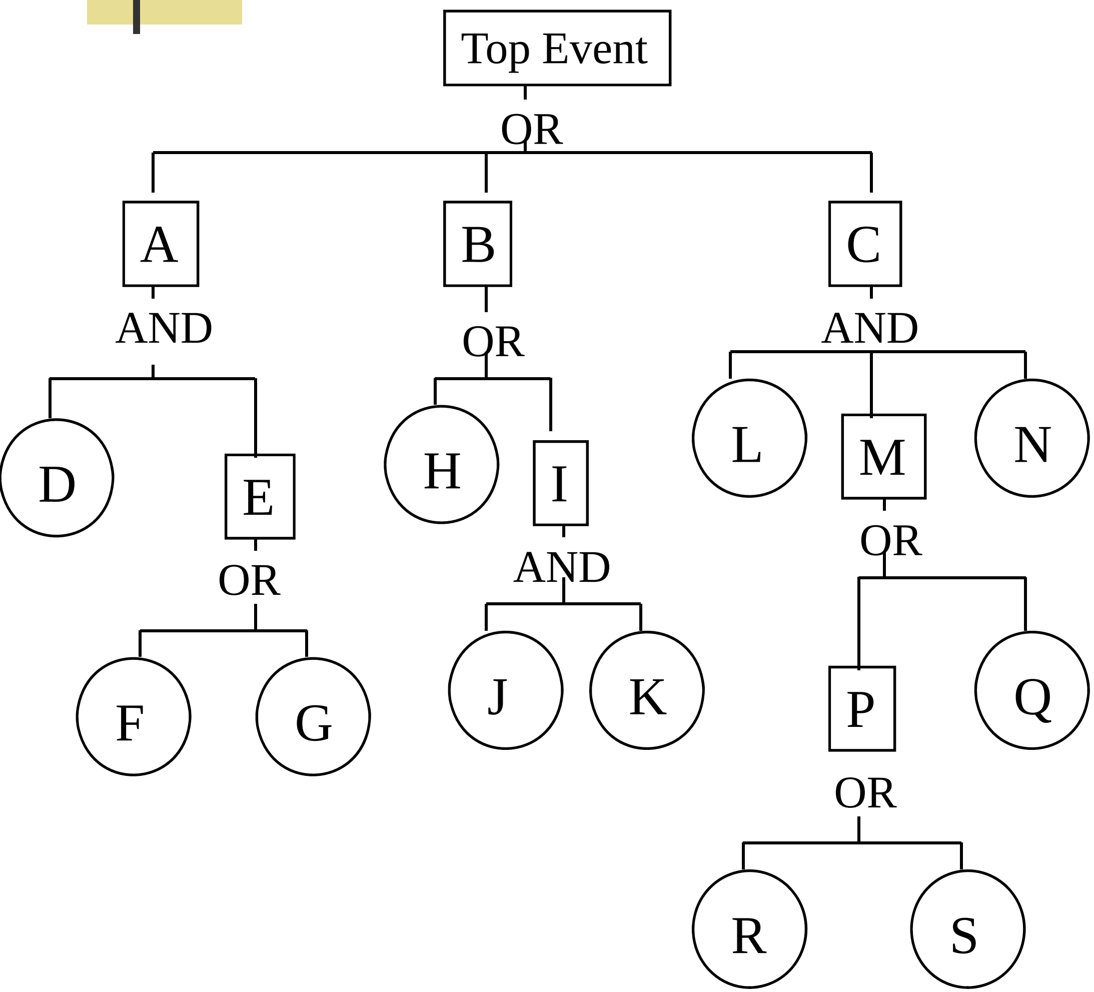
$$\approx P(E \cap F) + P[(I \cup J \cup K) \cap H] + P(C) + P(D)$$

$$\approx P(E)\,P(F) + [P(I) + P(J) + P(K)]\,P(H) + P(C) + P(D)$$

If each basic event has a probability of .01, then

$$P(T) \approx (.01)^2 + (.01+.01+.01)(.01) + .01 + .01 = .0204$$

One Last Example



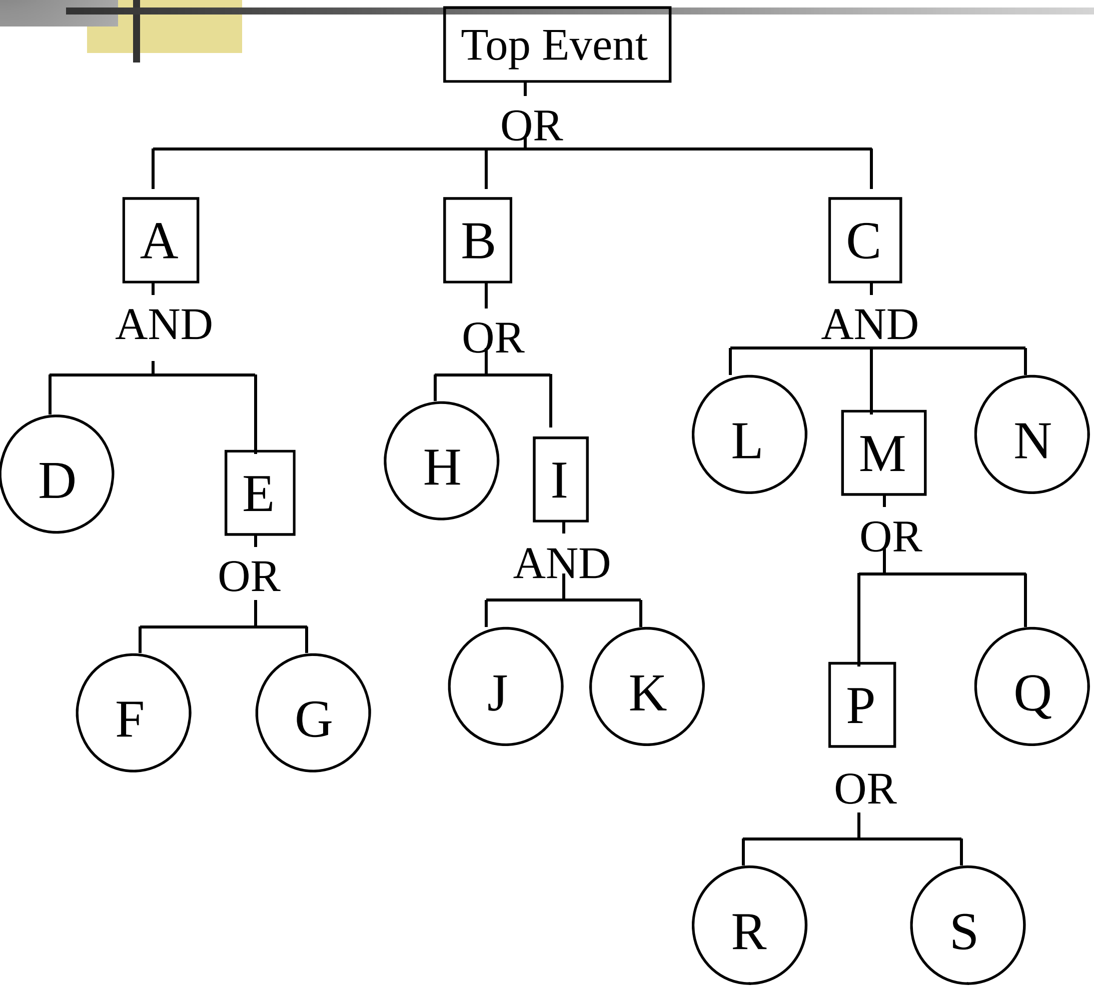
$T = A \cup B \cup C$
$= (D \cap E) \cup \ (H \cup I) \cup (L \cap M \cap N)$

$= [D \cap \ (F \cup G) \cap [H \cup (J \cap k)]$
$\cup [L \cap \ (P \cup Q) \cap N]$

$= [D \cap \ (F \cup G) \cap [H \cup (J \cap k)]$
$\cup [L \cap \ (R \cup S \cup Q) \cap N]$

$P(T) \approx P(D)\ [P(F) + P(G)] + P(H) + P(J)P(K) + P(L)\ [\ P(R) + P(S) + P(Q)]\ P(N)$

The Cut Sets



#1	#2	#3	#4
A	D,E	D,F	D,F D,G
B	H I	H J,K	H J,K
C	L,M,N	L,P,N L,Q,N	L,R,N L,S,N L,Q,N