Chapter 12 – Part I
Data Collection and Empirical Methods

- Data collection
- Ungrouped Complete Data
- Grouped Complete Data

Data Collection

Basic problem: Obtain a sample of failure or repair times:

$$t_1 \, , \, t_2 \, , \, t_3 \, , \, . \, . \, . \, t_n \; ; \text{ and}$$

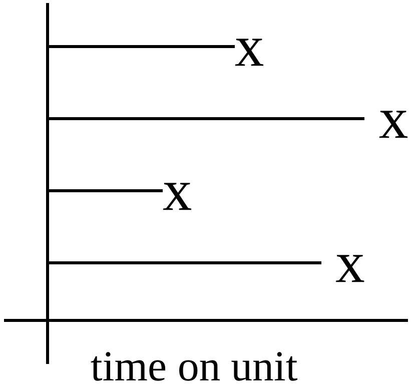
determine the most appropriate reliability or maintainability model (i.e. find R(t) or f(t)).

t_i represents the failure time of the ith unit or the ith observed repair time. The t_i are assumed to be an independent sample from the same population.

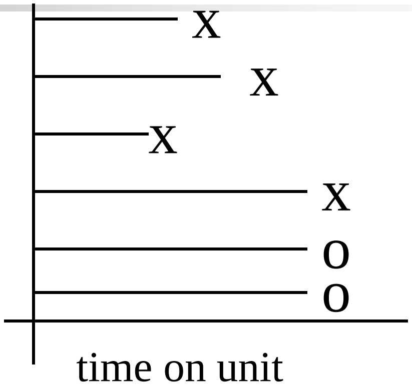
Taxonomy of Failure Data

- Operational vs. test-generated failures
- grouped vs. ungrouped data
- large vs. small samples
- complete vs. censored data
  - singly censored - operating times the same
    - Type I censored - time terminated test
    - Type II censored - terminated at r failures
  - multiply censored - operating times differ

Censoring

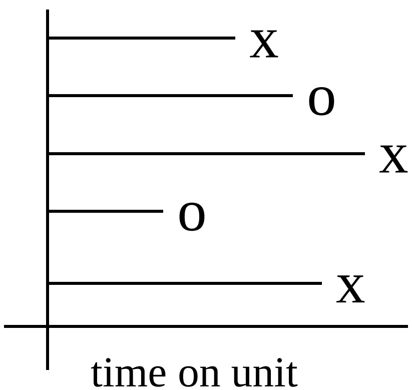


(a) Complete Data



(b) Singly Censored

x - failure
o - censor



(c) Multiply Censored

Ungrouped Complete Data

Given t_1 , t_2 , t_3 , . . . t_n are ordered failure times

i.e. $t_i <= t_{i+1}$. Then n- i is the fraction surviving at time t_i

"hat symbol means it is an estimate"

$$\hat{R}(t_i) = \frac{n-i}{n} = 1 - \frac{i}{n}$$

Ungrouped Complete Data

$$\hat{F}(t_i) = 1 - \hat{R}(t_i) = \frac{i}{n}$$

$$note: \quad \hat{F}(t_n) = \frac{n}{n} = 100\%$$

This effect is undesirable, therefore let:

$$\hat{F}(t_i) = \frac{i}{n+1}$$

$$\hat{R}(t_i) = 1 - \frac{i}{n+1} = \frac{n+1-i}{n+1}$$

Ungrouped Complete Data

Sample Size	Cumulative Probabilities					
1	0		.50			1
2	0	.33		.67		1
3	0	.25	.50	.75		1
4	0	.20	.40	.60	.80	1
4 Failure Times	t_0	t_1	t_2	t_3	t_4	t_∞

Plotting Positions

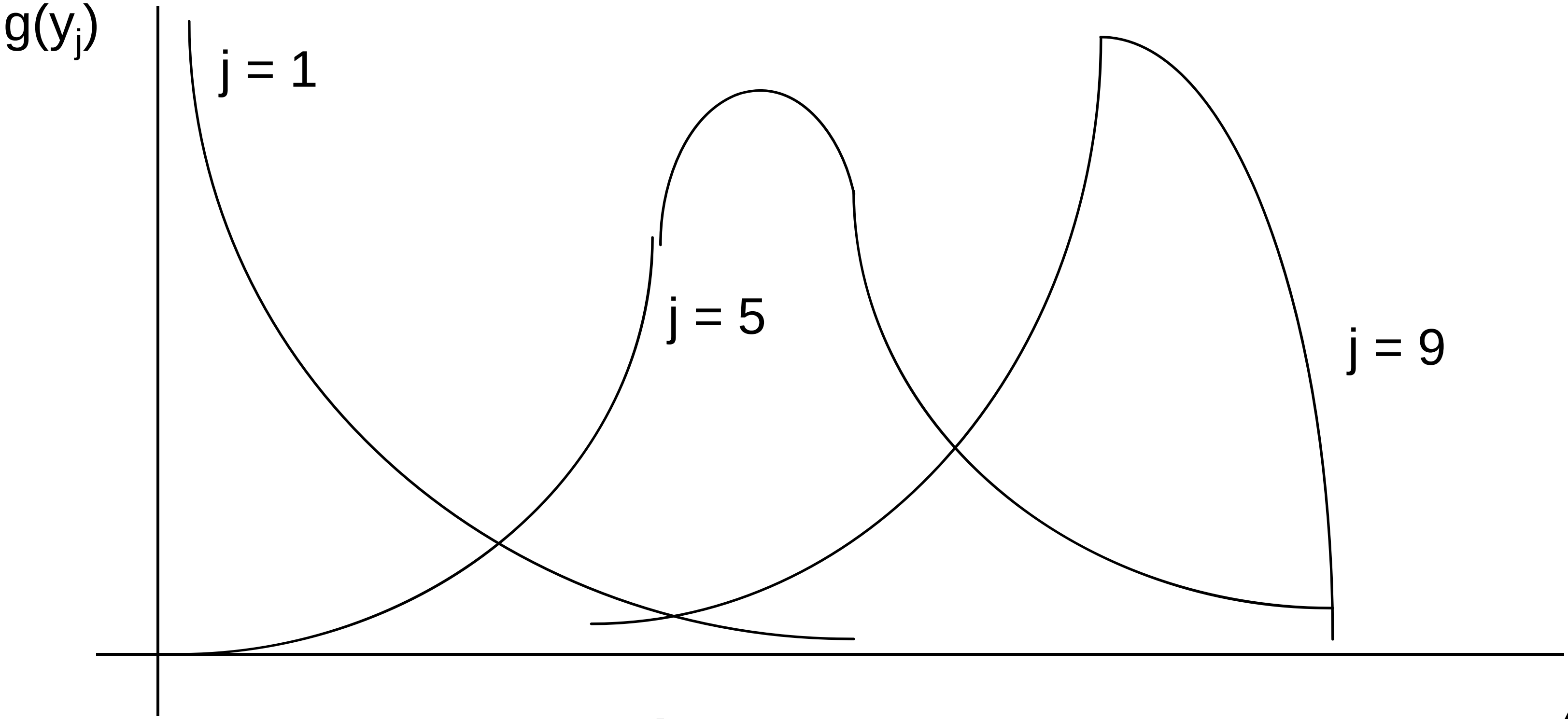
$$\left(t_i , \hat{F}(t_i) \right)$$

$\hat{F}(t_i)$ is the fraction of observations below the ith sample observation where

$$E\left[\hat{F}(t_i) \right] = \frac{i}{n+1}$$

Rank Order Distribution

n = 10



$$g(y_j) = \frac{n!}{(j-1)!(n-j)!} y_j^{\,j-1}(1-y_j)^{n-j}$$

$y_j = \hat{F}(t_j)$

Median Plotting Position

Must be computed numerically

approximated by: $\hat{F}(t_i) = \dfrac{i-.3}{n+.4}$

i	i/n	i/(n+1)	median	(i-.3)/(n+.4)
1	.125	.111	.083	.083
2	.250	.222	.201	.202
3	.375	.333	.321	.321
4	.500	.444	.440	.440
5	.625	.555	.560	.560
6	.750	.666	.680	.679
7	.875	.777	.799	.798
8	1.000	.888	.917	.917

Table A.5, p. 465

Empirical PDF & Hazard Rate

$$\hat{f}(t) = -\frac{\hat{R}(t_{i+1}) - \hat{R}(t_i)}{t_{i+1} - t_i} = \frac{1}{(t_{i+1} - t_i)(n+1)} \quad \text{for} \quad t_i < t < t_{i+1}$$

$$\hat{\lambda}(t) = \frac{\hat{f}(t)}{\hat{R}(t)} = \frac{1}{(t_{i+1} - t_i)(n+1-i)} \quad \text{for} \quad t_i < t < t_{i+1}$$

$$\hat{R}(t_i) = \frac{n+1-i}{n+1}$$

Sample Mean & Variance

$$\hat{MTTF} = \sum_{i=1}^{n} \frac{t_i}{n}$$

$$s^2 = \sum_{i=1}^{n} \frac{(t_i - \hat{MTTF})^2}{n-1}$$

$$s^2 = \frac{\sum_{i=1}^{n} t_i^2 - n \, \hat{MTTF}^2}{n-1}$$

Confidence Interval for the Mean

$$\hat{MTTF} \pm t_{\frac{\alpha}{2},\, n-1} \frac{s}{\sqrt{n}}$$

where $$\text{Prob}\{T > t_{\frac{\alpha}{2},\, n-1}\} = \frac{\alpha}{2}$$

Table A.2, p. 462

Example 12.2

Given the following 10 failure times in hours, estimate R(t), F(t), f(t), λ(t) and compute a 90 percent confidence interval for the MTTF:

24.5, 18.9, 54.7, 48.2, 20.1, 29.3, 15.4, 33.9, 72.0, 86.1

TIME	RELIABILITY	DENSITY	HAZARD RATE
0.0	1.00	.0059	.0059
15.4	.9090	.0260	.0286
18.9	.8182	.0757	.0926
20.1	.7273	.0207	.0284
24.5	.6364	.0189	.0298
29.3	.5455	.0198	.0362
33.9	.4546	.0064	.0140
48.2	.3636	.0140	.0385
54.7	.2727	.0053	.0193
72.0	.1818	.0064	.0355
86.1	.0909		

Example 12.2

$$\hat{R}(15.4) = \frac{10+1-1}{11} = 0.9090$$

$$\hat{f}(t) = \frac{1}{(18.9-15.4)\bullet 11} = 0.0260 \quad for \;\; 15.4 < t < 18.9$$

$$\hat{\lambda}(t) = \frac{1}{(18.9-15.4)\bullet 10} = 0.0286 \quad for \;\; 15.4 < t < 18.9$$

Example 12.2

$$\hat{MTTF} = \frac{15.4 + 18.9 + 20.1 + \ldots + 86.1}{10} = 40.31$$

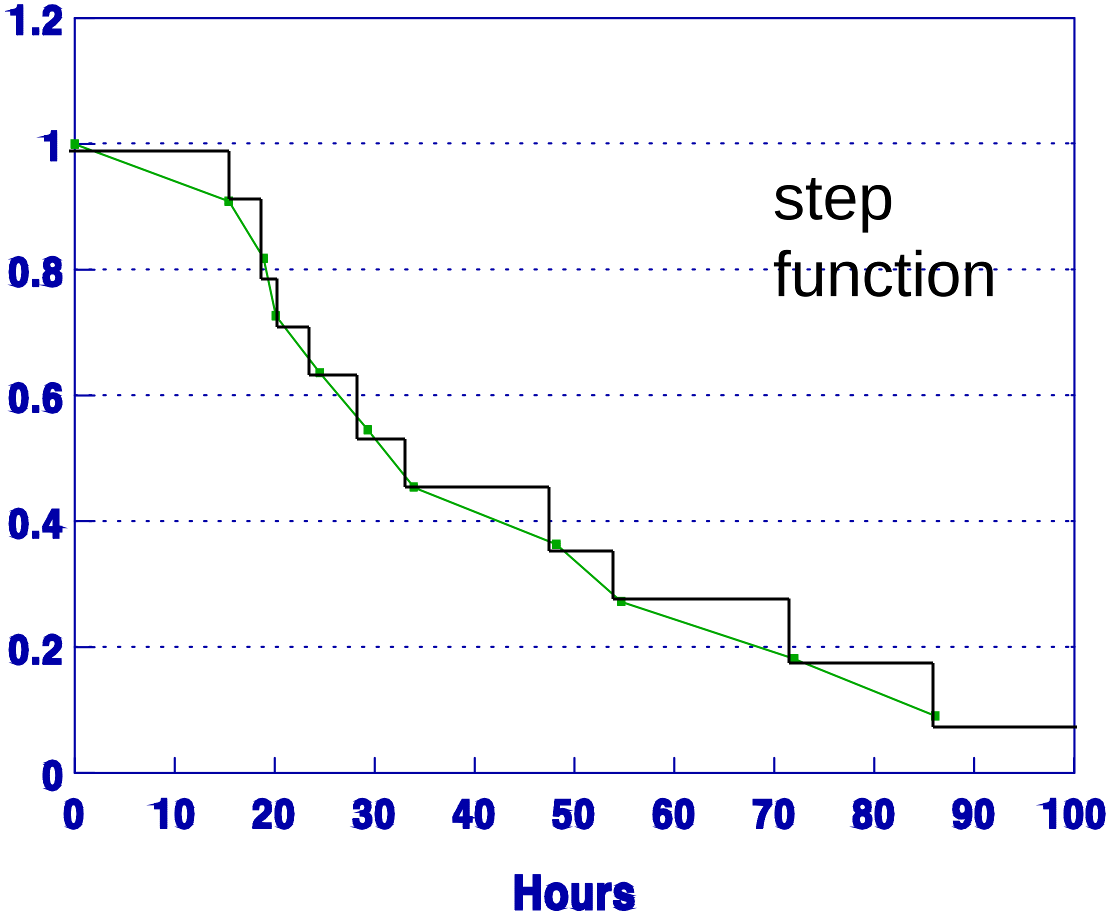
$$s^2 = \frac{15.4^2 + 18.9^2 + \ldots + 86.1^2 - 10 \bullet 40.31^2}{9} = 585.5454$$

$$or \;\; s = 24.198$$

40.31 ± 1.833 x 24.198/ √10 = [26.284, 54.34] is a 90 percent confidence interval where $t_{.05,9}$ = 1.833 from Appendix 12-B.

Example 12.2

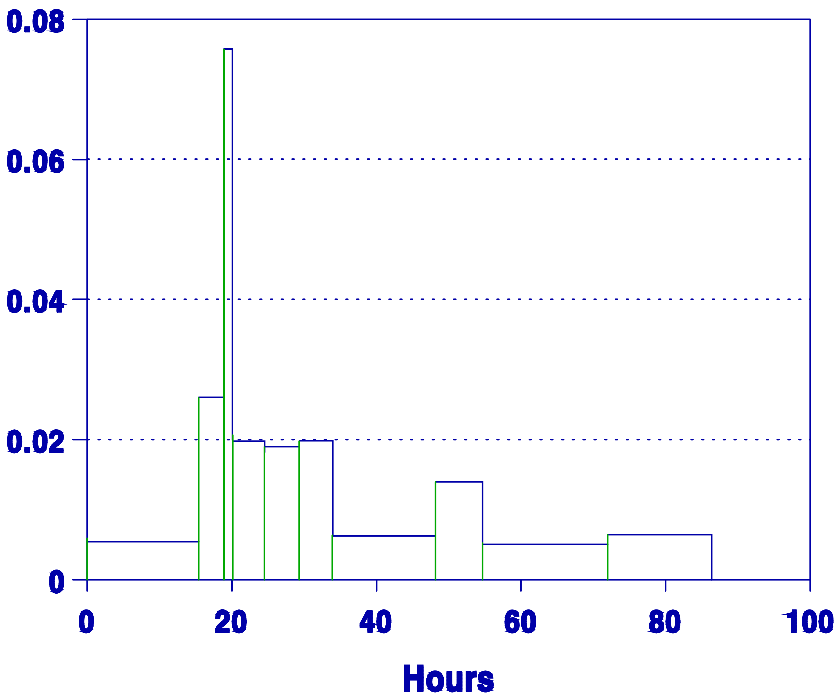
reliability



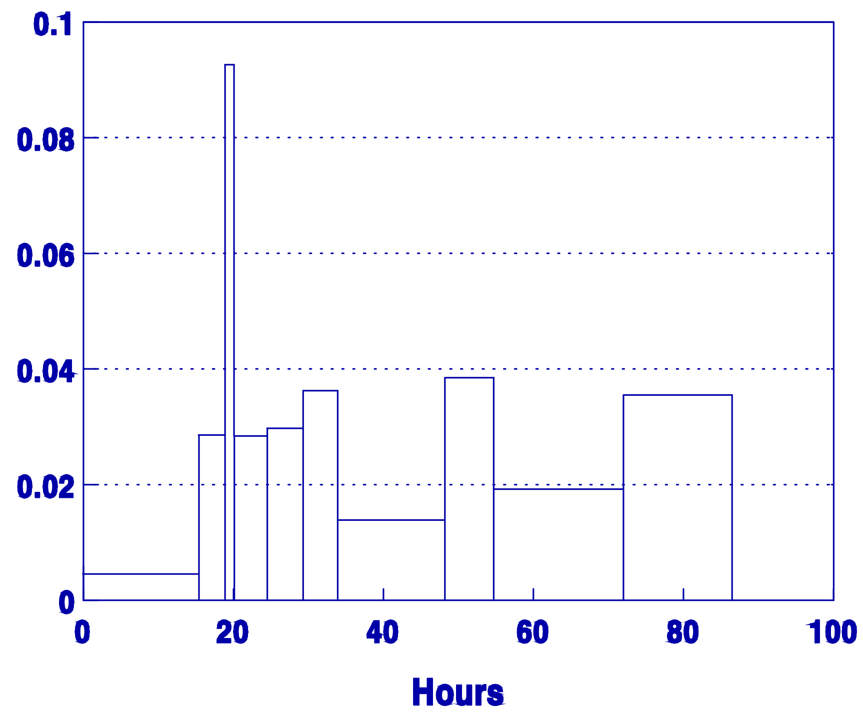
step function

Example 12.2

Failure Density



Hazard Rate



Example 12.3

The following repair times in hours were observed as part of a maintainability demonstration on a new packaging machine:

5, 6.2, 2.3, 3.5, 2.7, 8.9, 5.4, 4.6.

After rank ordering the data:

i	Repair Time	i / (8+1)
1	2.3	.111
2	2.7	.222
3	3.5	.333
4	4.6	.444
5	5.0	.556
6	5.4	.667
7	6.2	.777
8	8.9	.889

$$\hat{H}(t) = \frac{i}{n+1}$$

fewer than 90% repaired in 8 hrs

$$MTTR \pm t_{.05,7}\frac{s}{\sqrt{n}} = 4.825 \pm 1.894\frac{2.123}{\sqrt{8}} = (3.4, 6.2)$$

Chapter 12
Data Collection and Empirical Methods

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- Grouped Complete Data

Grouped Complete Data

Let $n_1, n_2, \ldots, n_k$ be the number of units having survived at ordered times $t_1, t_2, \ldots, t_k$ respectively.

$$\hat{R}(t_i) = \frac{n_i}{n}, \quad i = 1, 2, \ldots, k$$

$$\hat{f}(t) = -\frac{\hat{R}(t_{i+1}) - \hat{R}(t_i)}{(t_{i+1} - t_i)} \quad \text{for} \quad t_i < t < t_{i+1}$$

$$= \frac{n_i - n_{i+1}}{(t_{i+1} - t_i)n}$$

Grouped Complete Data

$$\hat{\lambda}(t) = \frac{\hat{f}(t)}{\hat{R}(t)} = \frac{n_i - n_{i+1}}{(t_{i+1} - t_i)n_i} \quad \text{for} \quad t_i < t < t_{i+1}$$

$$\hat{MTTF} = \sum_{i=0}^{k-1} \bar{t}_i \frac{(n_i - n_{i+1})}{n} \quad \text{where} \quad \bar{t}_i = \frac{(t_i + t_{i+1})}{2}$$

$$s^2 = \sum_{i=0}^{k-1} \bar{t}_i^2 \frac{(n_i - n_{i+1})}{n} - \hat{MTTF}^2$$

Example 12.4

Seventy compressors are observed at 5 month intervals with the following number of failures: 3, 7, 8, 9, 13, 18, and 12.

UPPER BND	NBR FAIL	NUMBER SURVIVE	RELI-ABILITY	FAILURE DENSITY	HAZARD RATE
0	0	70	1.000	.0086	.0086
5	3	67	.957	.0200	.0209
10	7	60	.857	.0229	.0267
15	8	52	.743	.0257	.0346
20	9	43	.614	.0371	.0605
25	13	30	.429	.0514	.1200
30	18	12	.171	.0343	.2000
35	12	0	0.000		

Example 12.4

$$\hat{R}(5) = \frac{67}{70} = 0.957$$

$$\hat{f}(t) = \frac{67 - 60}{(10 - 5)70} = 0.0200 \quad for\ 5 < t < 10$$

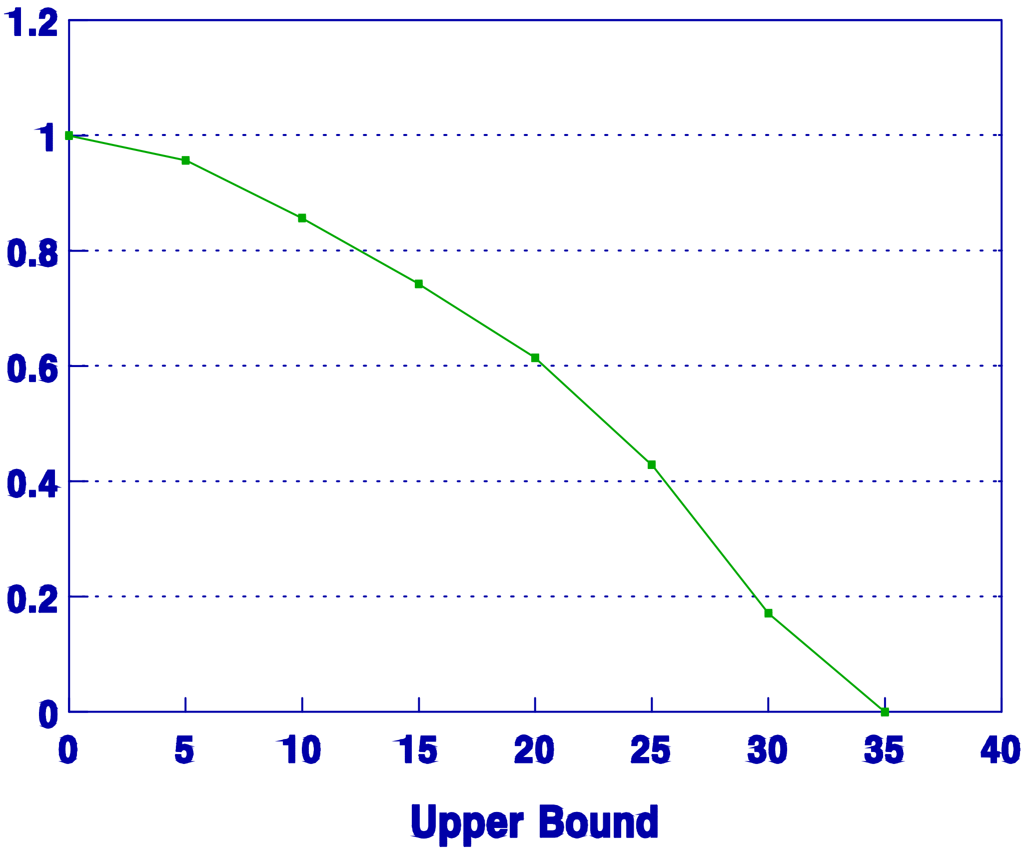
$$\hat{\lambda}(t) = \frac{67 - 60}{(10 - 5)67} = 0.0209 \quad for\ 5 < t < 10$$

$$\widehat{MTTF} = \frac{[2.5(3) + 7.5(7) + \ldots + 32.5(12)]}{70} = 21.357$$

$$s^2 = \frac{[2.5^2(3) + 7.5^2(7) + \ldots + 32.5^2(12)]}{70} - 21.357^2 = 76.551$$

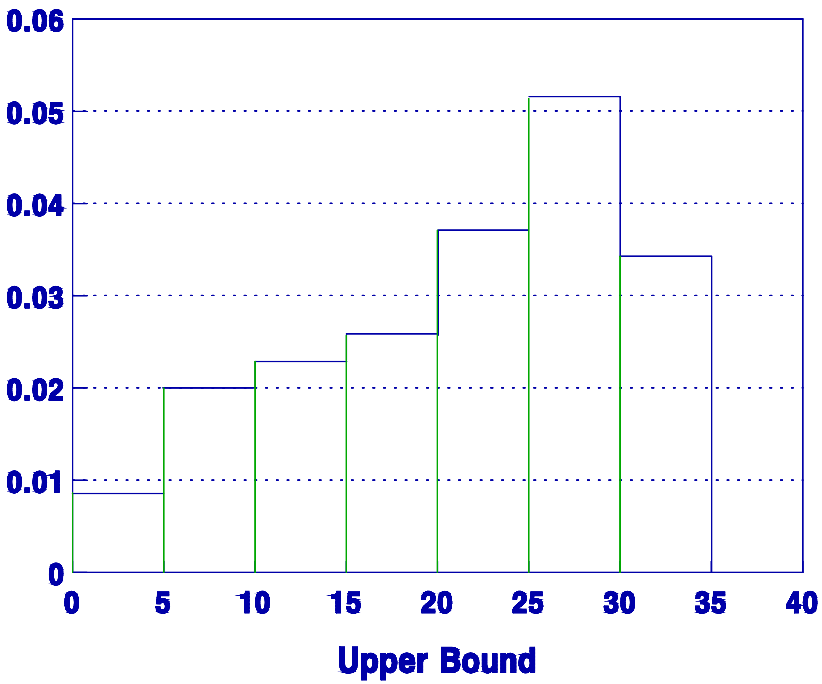
$$or\ s = 8.75$$

Example 12.4

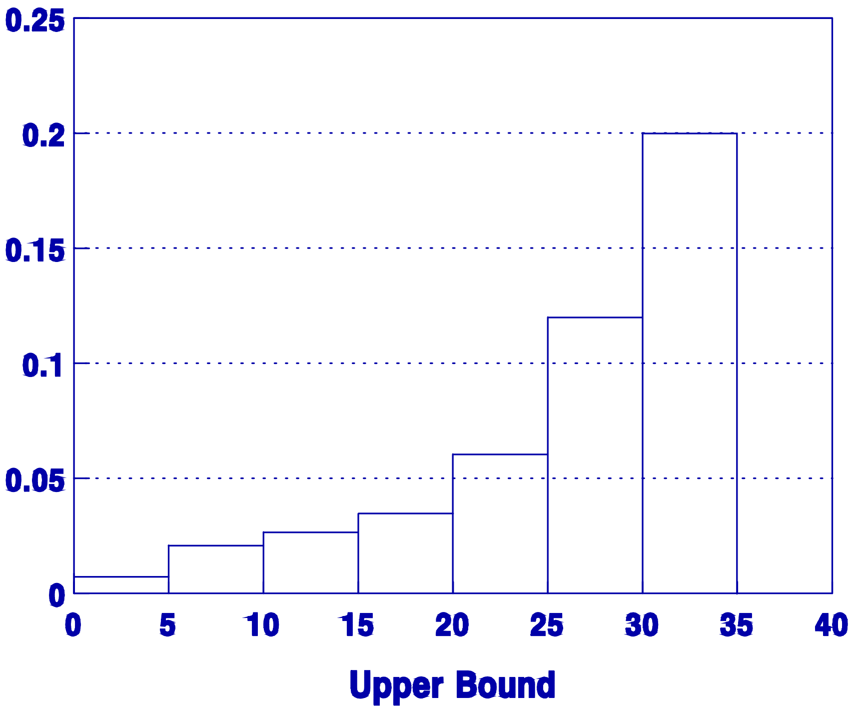


Example 12.4

Failure Density



Hazard Rate



Example 12.5

The following aircraft repair data reported by the maintenance organization shows the number of days an aircraft was out of service because of unscheduled maintenance.

Days	Number of Aircraft
1-2	4
3-4	7
5-6	9
7-8	6
9-10	4
total	30 repairs

Example 12.5

$$\hat{H}(t) = 1 - \frac{n_i}{n}$$

where n_i is the number of repairs exceeding time t_i

i	Upper Bnd (t_i)	n_i	$1 - n_i / 30$
1	2 days	26	.133
2	4 days	19	.367
3	6 days	10	.667
4	8 days	4	.867
5	10 days	0	1.00

estimated MTTR = 4.9 days with s = 2.44