$$\frac{uptime}{uptime + downtime}$$

Chapter 11 Availability

Concepts and Definitions (11.1)

Exponential Model (11.2)

System Availability (11.3)

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11.1 Concepts and Definitions

Definition: *Availability is the probability that a system or component is performing its required function at a given point in time or over a stated period of time when operated and maintained in a prescribed manner.*

Availabilities

1. A(t) = the availability at time t - referred to as the point availability

2. $$A(T) = \frac{1}{T}\int_0^T A(t)\,dt$$ is the average availability over [0,T]

generalize: $$A_{t_2\text{-}t_1} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} A(t)\,dt$$

3. $$A = \lim_{T\to\infty} A(T)$$ is the steady-state availability

Steady-State Availability

To have steady-state, a **renewal process** is necessary

1. Inherent $$A_{inh} = \lim_{T\to\infty} A(T) = \frac{MTBF}{MTBF + MTTR}$$

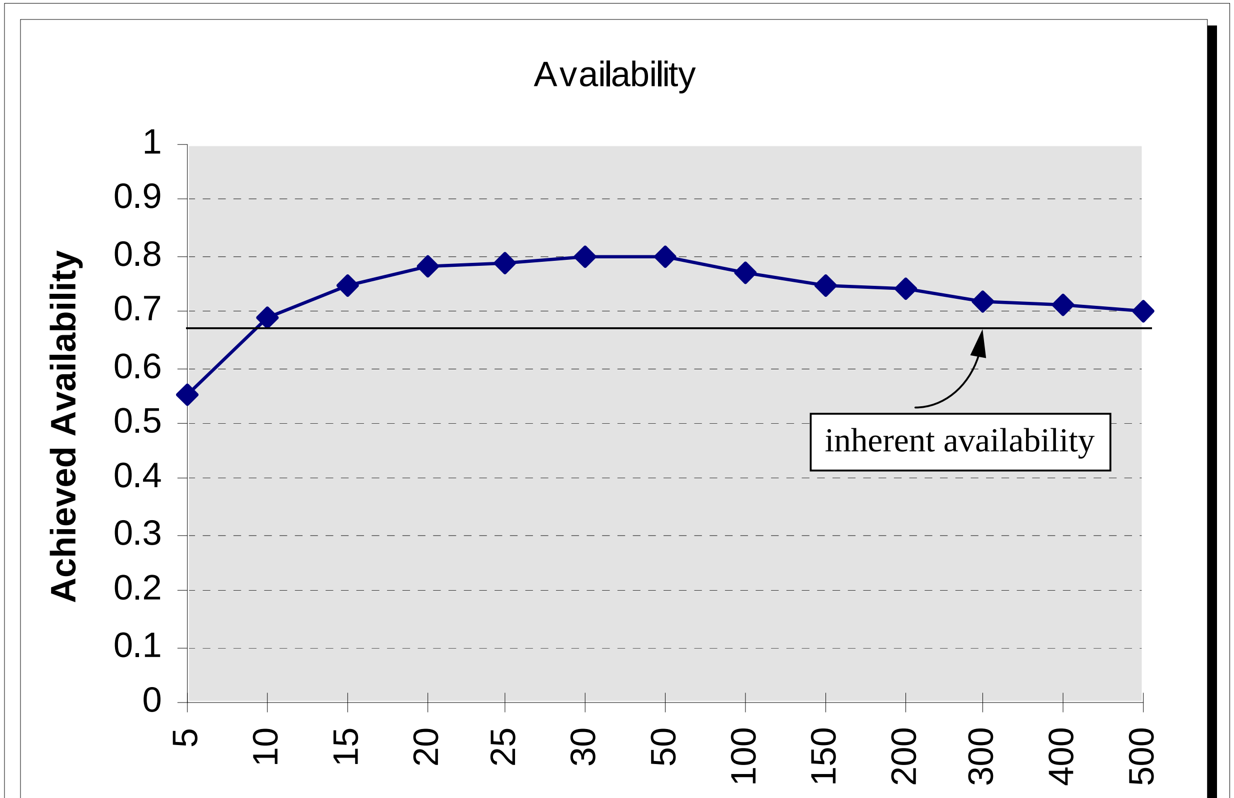
2. Achieved

$$A_a = \frac{MTBM}{MTBM + \overline{M}}$$

$$MTBM = \frac{t_d}{m(t_d) + \frac{t_d}{T_p}} \qquad \overline{M} = \frac{m(t_d)\,MTTR + \frac{t_d}{T_p}\,MPMT}{m(t_d) + t_d / T_p}$$

For a renewal process: $m(t_d) = t_d / MTBF$

Achieved Availability



$$MTBF = a + b/T_{pm}$$

Operational Availability

$$A_o = \frac{MTBM}{MTBM + \overline{M'}}$$

$$\overline{M'} = \frac{m(t_d)\,MTR + \dfrac{t_d}{T_p}\,MPMT}{m(t_d) + t_d / T_p}$$ and MTR = MTTR+SDT+MDT

$$A_G = \frac{MTBM + ready\ time}{MTBM + ready\ time + \overline{M}}$$

Availability with Minimal Repair

Inherent Availability

$$A_{t_2-t_1} = \frac{t_2 - t_1}{t_2 - t_1 + m(t_1,t_2)\cdot MTTR} \text{ where } m(t_1,t_2) = \int_{t_1}^{t_2} \rho(t)\,dt$$

Achieved Availability

$$A_{t_2-t_1} = \frac{t_2 - t_1}{t_2 - t_1 + m(t_1,t_2)\cdot MTTR + \left(\frac{t_2 - t_1}{T_{pm}}\right) MPMT}$$

Availability with Minimal Repair

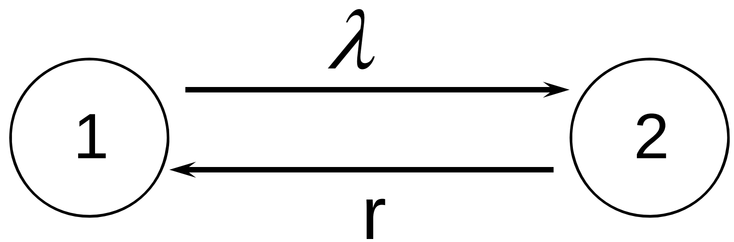
A machine has minimal repair upon failure with an intensity function of $\rho(t) = .00017t^{.7}$, t measured in days. Repair times are lognormal with a median repair of 20.3 hours and s=1.2. Estimate its availability over its first 5 operating years assuming 7/24.

$m(t) = .0001t^{1.7}$ and $m(5 \times 365) = 35.0$

MTTR = $20.3 \exp[1.2^2 /2] = 41.7$ hrs = $41.7/24 = 1.74$ days

A(5 oper yrs) = $5 \times 365 / [5 \times 365 + 35 \times 1.74] = .9677$

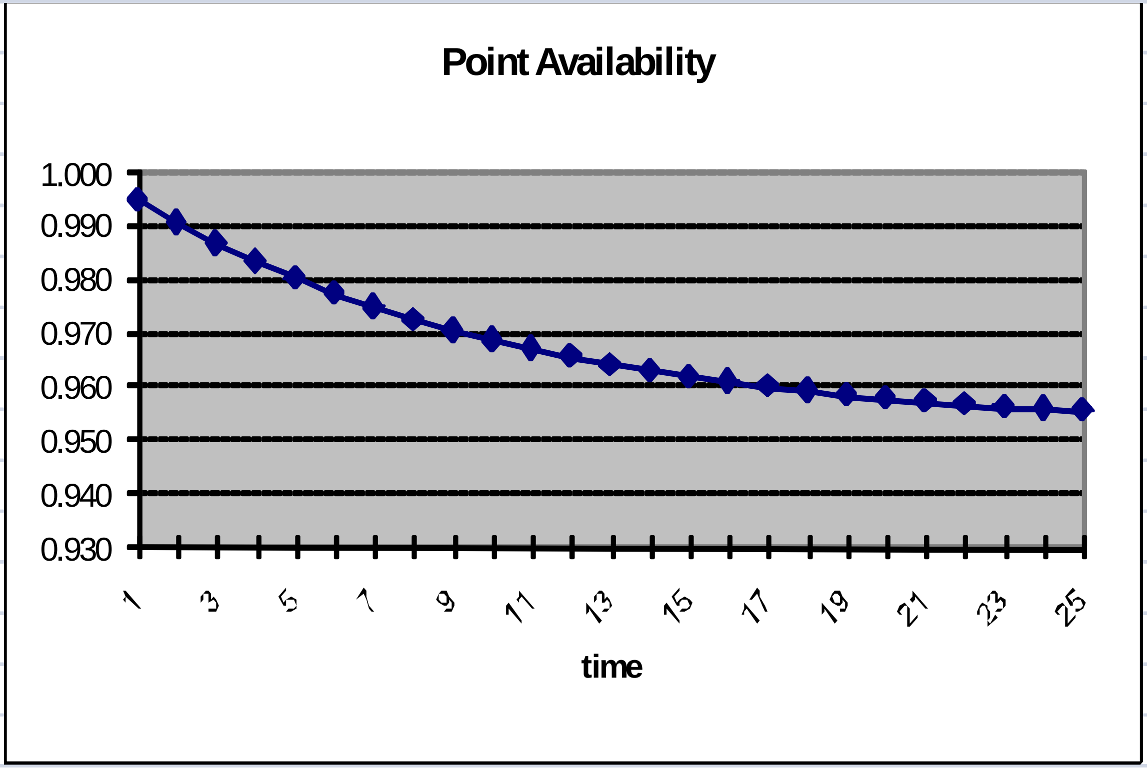
11.2 Exponential Model



$$\frac{d\,P_1(t)}{dt} = -\lambda P_1(t) + r\,P_2(t)$$

$$P_1(t) + P_2(t) = 1$$

Exponential Model



$$A(t) = P_1(t) = \frac{r}{\lambda + r} + \frac{\lambda}{\lambda + r} e^{-(\lambda + r)t}$$

Exponential Model - Interval Availability

$$A_{t_2\text{-}t_1} = \frac{1}{t_2\text{-}t_1} \int_{t_1}^{t_2} \left(\frac{r}{r+\lambda} + \frac{\lambda}{r+\lambda} e^{-(\lambda+r)t} \right) dt$$

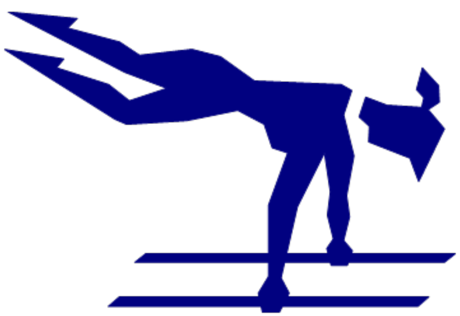
$$= \frac{r}{r+\lambda} + \frac{\lambda}{(r+\lambda)^2 (t_2 - t_1)} \left[e^{-(\lambda+r)t_1} - e^{-(\lambda+r)t_2} \right]$$

steady-state $$A_{inh} = \lim_{t\to\infty} A_{t-o}$$

$$= \frac{r}{r+\lambda} = \frac{MTBF}{MTBF + MTTR}$$

11.3 System Availability Series versus Parallel

$$A_s(t) = \prod_{i=1}^{n} A_i(t)$$



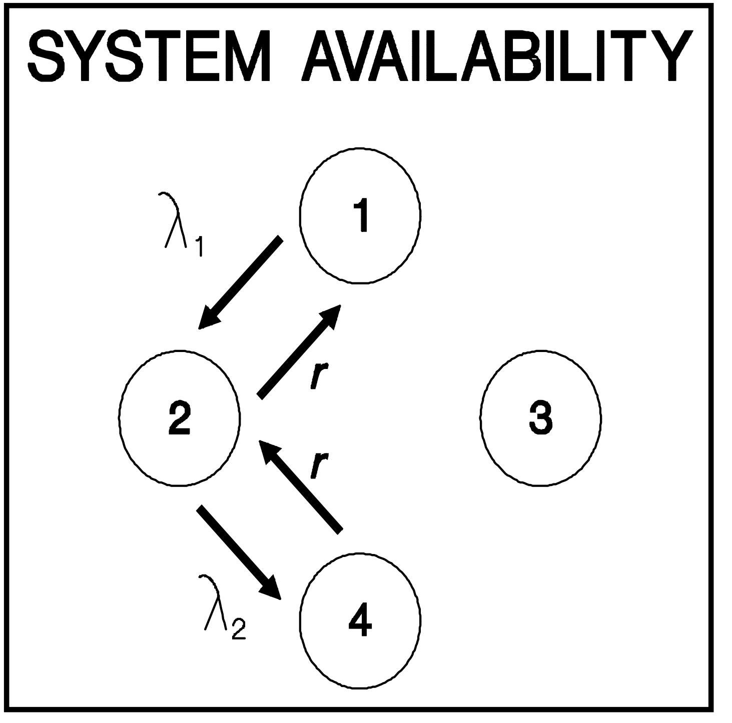
$$A_s(t) = 1 - \prod_{i=1}^{n} (1 - A_i(t))$$

Standby Systems

$$\frac{dP_1(t)}{dt} = -\lambda_1 P_1(t) + rP_2(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) + rP_4(t) - (\lambda_2 + r) P_2(t)$$

$$P_1(t) + P_2(t) + P_4(t) = 1$$



for steady-state: $\lim_{t \to \infty} \frac{d\,P_i(t)}{dt} = 0$ and set $P_i(t) = P_i$

Steady-State Standby Systems

$$-\lambda_1 P_1 + r P_2 = 0$$

$$\lambda_1 P_1 + r P_4 - (\lambda_2 + r) P_2 = 0$$

$$P_1 + P_2 + P_4 = 1$$

$$P_1 = \left[1 + \frac{\lambda_1}{r} + \frac{\lambda_1 \lambda_2}{r^2} \right]^{-1}$$

$$P_2 = \frac{\lambda_1}{r} P_1$$

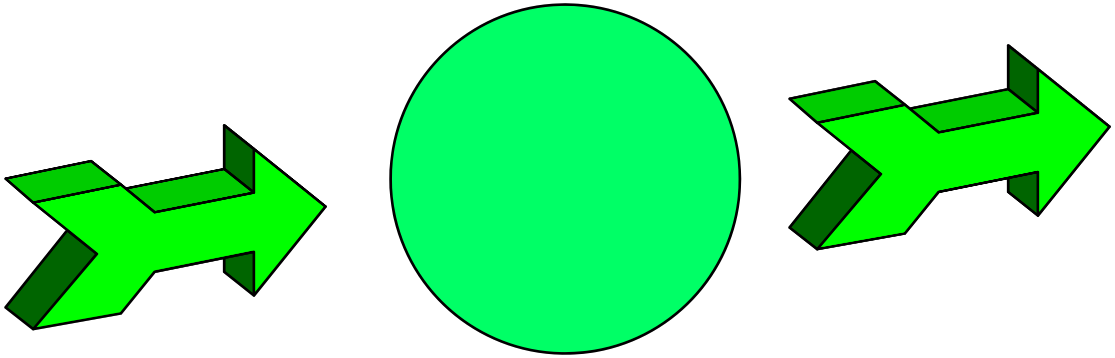
$$P_4 = \frac{\lambda_1 \lambda_2}{r^2} P_1$$

$$\boxed{A = P_1 + P_2}$$

Generalize - steady-state availability

$$\sum_j \text{Rate into state i from state j} \times P_j$$

$$= \text{Rate out of state i} \times P_i$$

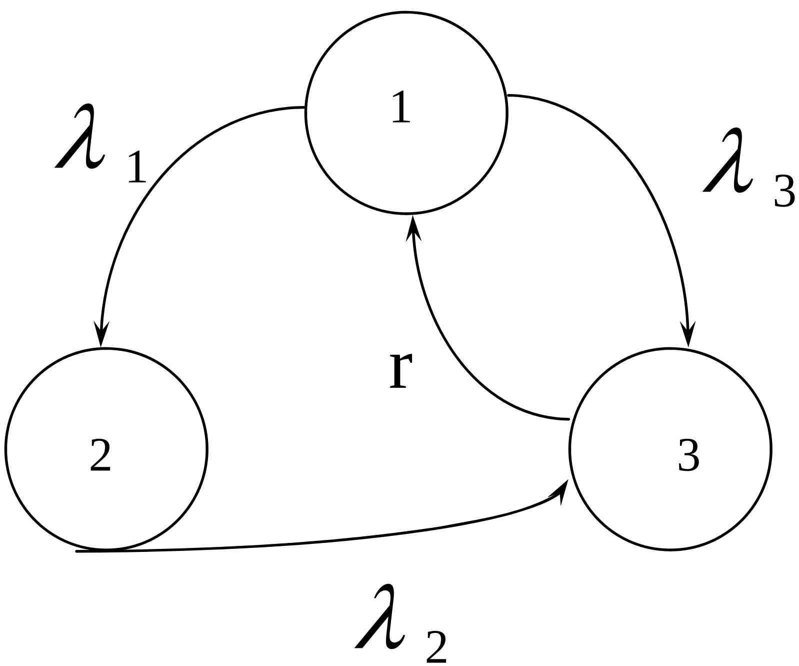


Example #1

$$-\lambda_1 P_1 - \lambda_3 P_1 + rP_3 = 0$$

$$\lambda_1 P_1 - \lambda_2 P_2 = 0$$

$$P_1 + P_2 + P_3 = 1$$



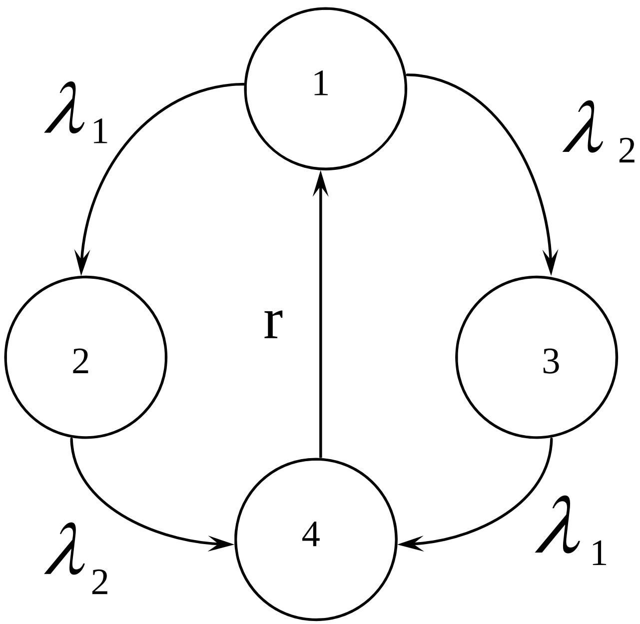
$$\boxed{A = P_1 + P_2}$$

$$P_1 = \left(1 + \frac{\lambda_1}{\lambda_2} + \frac{\lambda_1 + \lambda_3}{r}\right)^{-1}$$

$$P_2 = \frac{\lambda_1}{\lambda_2} P_1$$

$$P_3 = \frac{\lambda_1 + \lambda_3}{r} P_1$$

Example #2



$$(\lambda_1 + \lambda_2)\, P1 = r\, P_4$$
$$\lambda_2 P_2 = \lambda_1 P_1$$
$$\lambda_1 P_3 = \lambda_2 P_1$$

$$r\, P_4 = \lambda_2 P_2 + \lambda_1 P_3$$
$$P_1 + P_2 + P_3 + P_4 = 1$$

11.4 Inspect and Repair Model

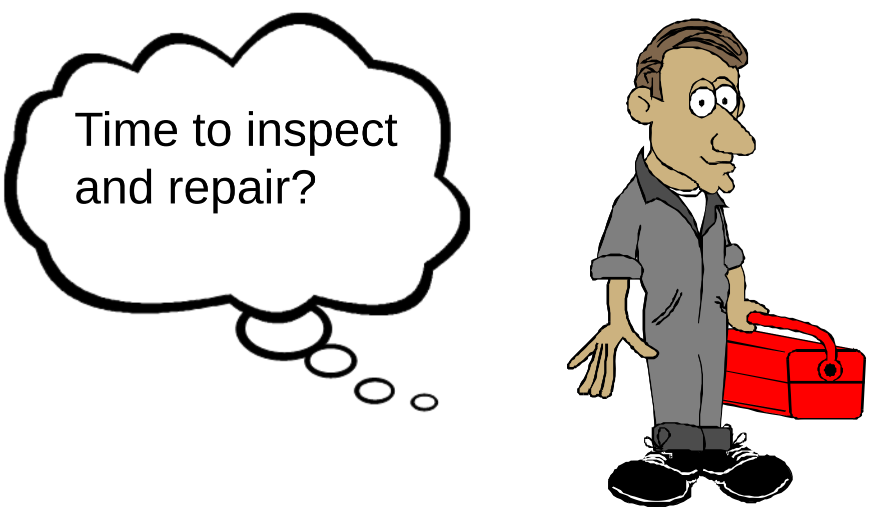
Let $R(t)$ be the dormant failure distribution,

t_1 = inspection time,

t_2 = repair time if necessary

T = time between inspections

Then $T + t_1 + t_2 [1 - R(T)]$ is the expected cycle time



Inspect and Repair Model

Expected uptime = $\int_{o}^{T} R(t)dt = R(T)T + \int_{0}^{T} tf(t)dt$

$$\int_{0}^{T} t\ f(t)dt = -tR(t)\Big|_{0}^{T} + \int_{0}^{T} R(t)dt$$

$$A(T) = \frac{\int_{0}^{T} R(t)dt}{T + t_1 + t_2[1 - R(T)]} = \frac{R(T)T + \int_{o}^{T} tf(t)dt}{T + t_1 + t_2[1 - R(T)]}$$

Exponential Failure Time

$$\int_0^T R(t)dt = \int_0^T e^{-\lambda t} dt = \left. \frac{e^{-\lambda t}}{-\lambda} \right|_0^T = \frac{1}{-\lambda}\left(e^{-\lambda T} - 1\right) = \frac{1}{\lambda}\left(1 - e^{-\lambda T}\right)$$

Inspect and Repair Model Exponential Case

$$A(T) = \frac{1 - e^{-\lambda T}}{\lambda[T + t_1 + t_2(1 - e^{-\lambda T})]}$$

If t_1 and t_2 are negligible

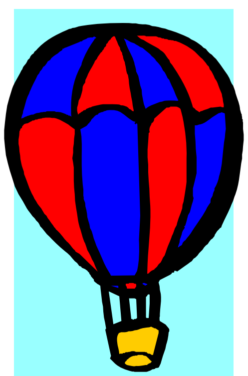
$$A(T) \approx \frac{1 - e^{-\lambda T}}{\lambda T}$$

Example

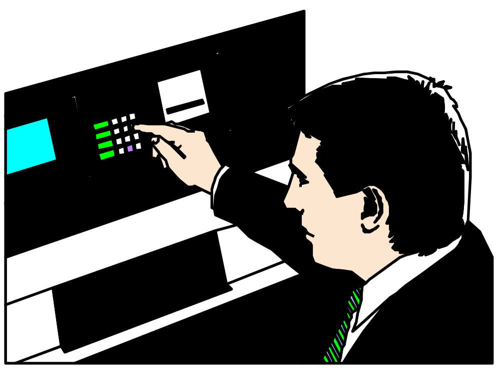
$$A(T) = \frac{1 - e^{-.0002T}}{.0002[T + 16 + 48(1 - e^{-.0002T})]}$$

T	100	200	300	400	500	600	700	800	900	1000
Avail	.847	.900	.913	.916	.914	.910	.904	.898	.891	.884

CFR=.0002



resupply time = 48 hr



inspect time = 16 hr.