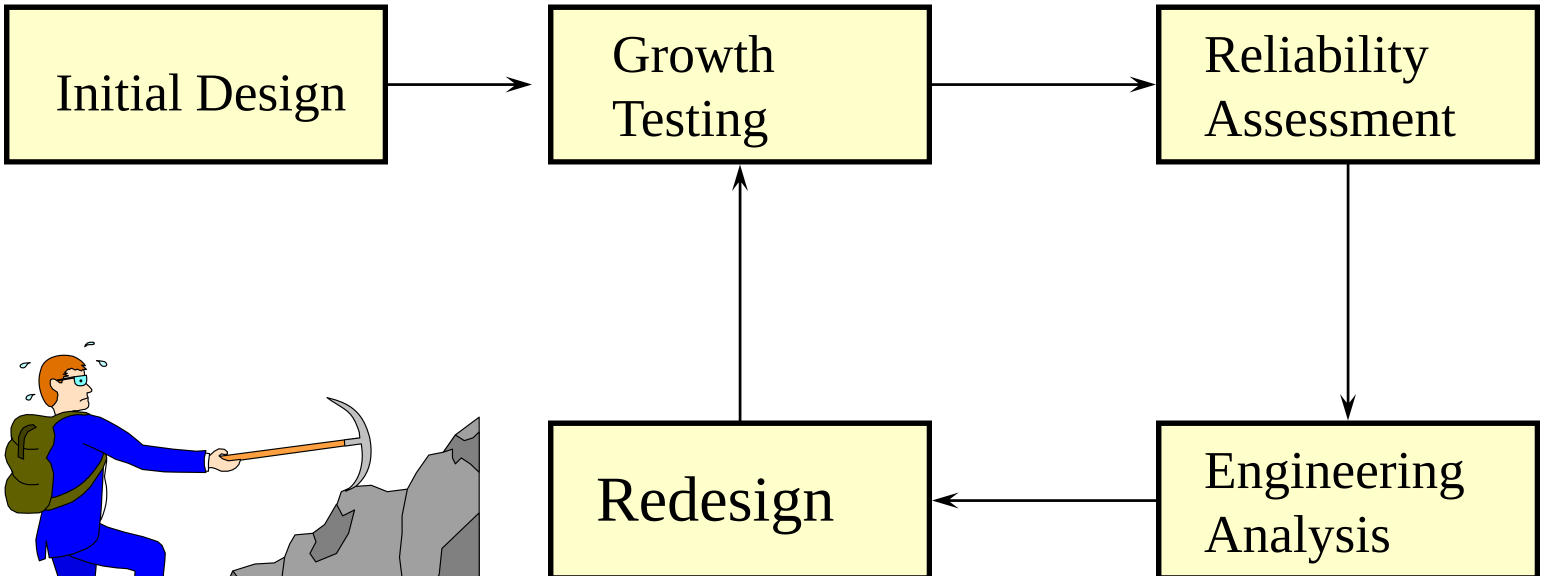
Chapter 14
Reliability Growth Testing

Reliability Growth Process

Duane Growth Model

AMSAA Model

The Reliability Growth Cycle



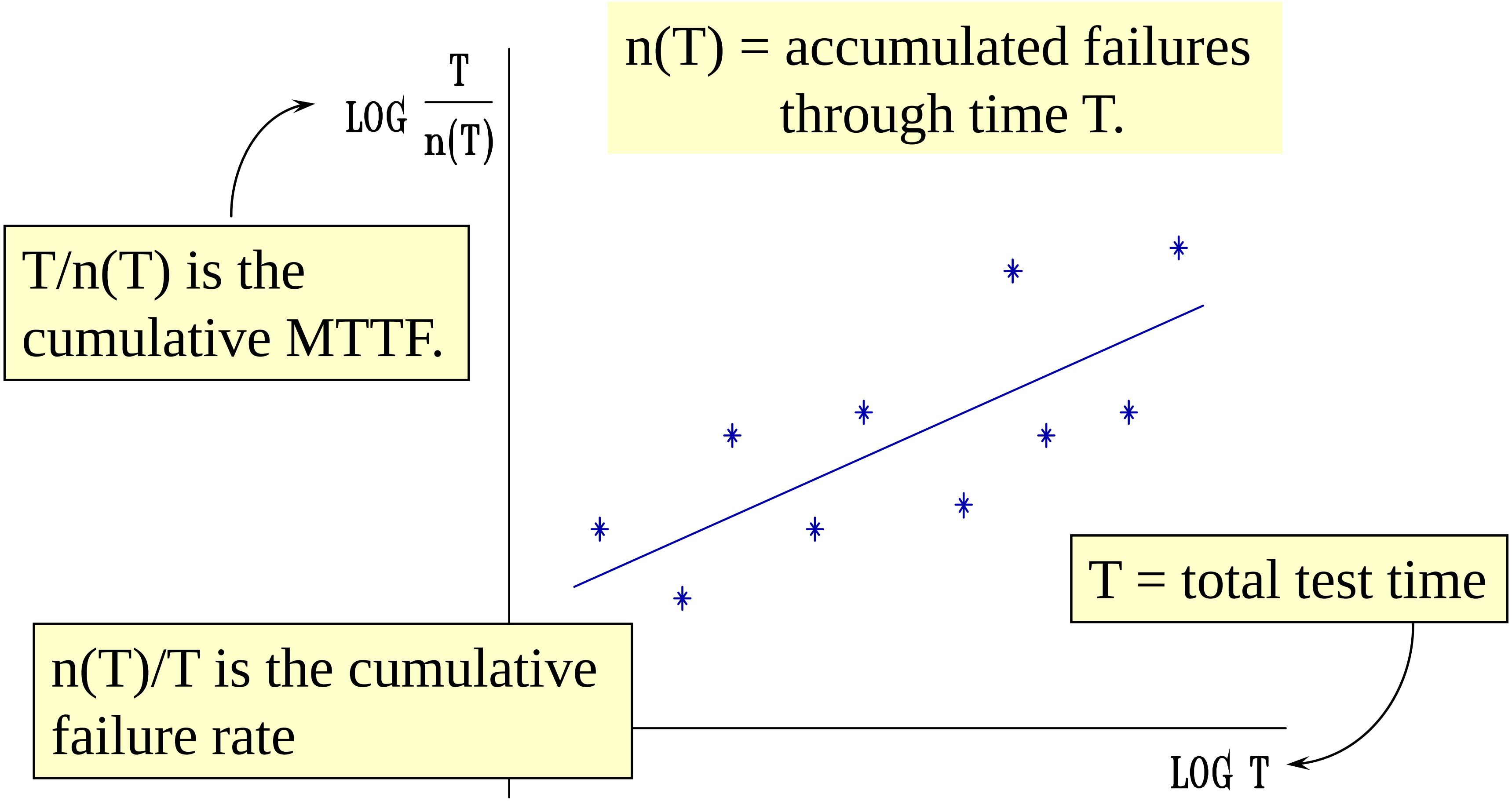
The Beginnings

In 1962, J. T. Duane while with GE Jet Engines published a report in which he presented failure data of different systems during their development programs.

While analyzing the data, it was observed that the cumulative MTBF versus cumulative operating time followed a straight line when plotted on log-log paper

Duane, J. T., "Learning Curve Approach To Reliability Monitoring," *IEEE Transactions on Aerospace*, Vol. 2, pp. 563-566, 1964.

Duane Growth Model



Duane Growth Model

$$\ln[T/n(T)] = a + b \ln T$$

$$MTTF_c = T/n(T) = e^{a+b \ln T} = e^a T^b = kT^b$$

$$n(T) = (1/k) \times T^{1-b}$$

$$\frac{d\, n(T)}{dT} = \lambda(T) = \frac{(1-b)}{k} T^{-b}$$

$$MTTF_i = k \frac{T^b}{1-b} = \frac{MTTF_{cum}}{1-b}$$

Least-Squares Curve Fit

let $x_i = \ln(t_i)$, $y_i = \ln[t_i/n(t_i)]$;

and $$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}, \quad \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

Least-Squares Curve Fit

Then

$$\hat{b} = \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$$

$$\hat{a} = \bar{y} - b\bar{x}$$

$$r^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{a} - \hat{b}x_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

coefficient of determination

L-S Estimates

$$k = e^{\hat{a}}$$

$$MTTF_i = \frac{k\,T^{\hat{b}}}{1-\hat{b}}$$

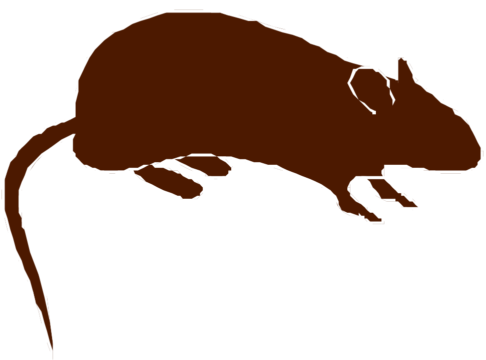
solve for T

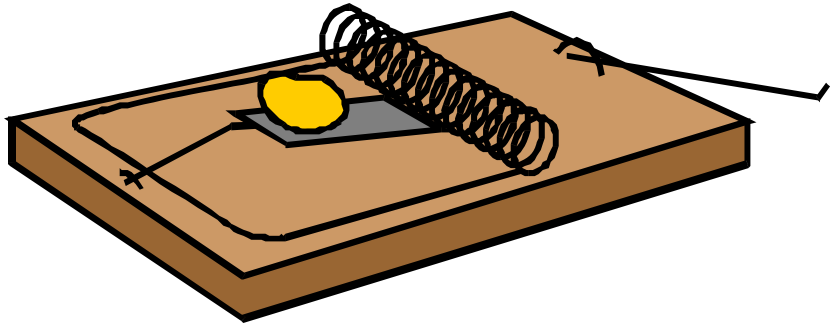
$$T_m = \left[\frac{(1-\hat{b})\,\hat{M}_f}{K}\right]^{1/\hat{b}}$$

Example 14.2

A new product still in the development stage undergoes reliability growth testing. Each test-fix cycle consists of 50 hours of testing with the following failures per cycle observed in order:
24, 17, 9, 5, 3, 2, 1.

Estimate the current MTTF and the additional test time required to obtain an MTTF goal of 20 hours.





Example 14.2

T	N(T)	T/N(T)	LOG T	LOG T/N(T)	x(i)y(i)	$x(i)^2$
50.0	24.0	2.0833	3.9120	0.7340	2.8713	15.3039
100.0	41.0	2.4390	4.6052	0.8916	4.1060	21.2076
150.0	50.0	3.0000	5.0106	1.0986	5.5047	25.1065
200.0	55.0	3.6364	5.2983	1.2910	6.8400	28.0722
250.0	58.0	4.3103	5.5215	1.4610	8.0670	30.4865
300.0	60.0	5.0000	5.7038	1.6094	9.1799	32.5331
350.0	61.0	5.7377	5.8579	1.7471	10.2342	34.3154
		TOTALS=	35.90933	8.832679	46.80305	187.025

Example 14.2

xbar = 5.1299 and ybar = 1.261811

b = [46.803 - 5.1299 (8.832679)] / [187.0252-7(5.1299)^2 = .53,
a = 1.261811 - .53 (5.1299) = -1.457
therefore K = $e^{-1.457}$ = .233 .

$MTTF_c$ = .233 $(350)^{.53}$ = 5.196
$MTTF_i$ = 5.196/(1-.53) = 11.0.
r = .97

$$T_{20} = \left[\frac{(1-.53)20}{.233}\right]^{\frac{1}{.53}} = 1071$$

1071 - 350 = 721 additional hr.

More about the Duane Model

- The Duane model has been used throughout the1960s and 1970s.
- It models reliability growth well enough to predict the future reliability
- It is strictly an empirical model
- Provides no insight into the cause of any growth (or deterioration)
- It assumes that having a reliability growth program will result in a growth curve that is log-linear

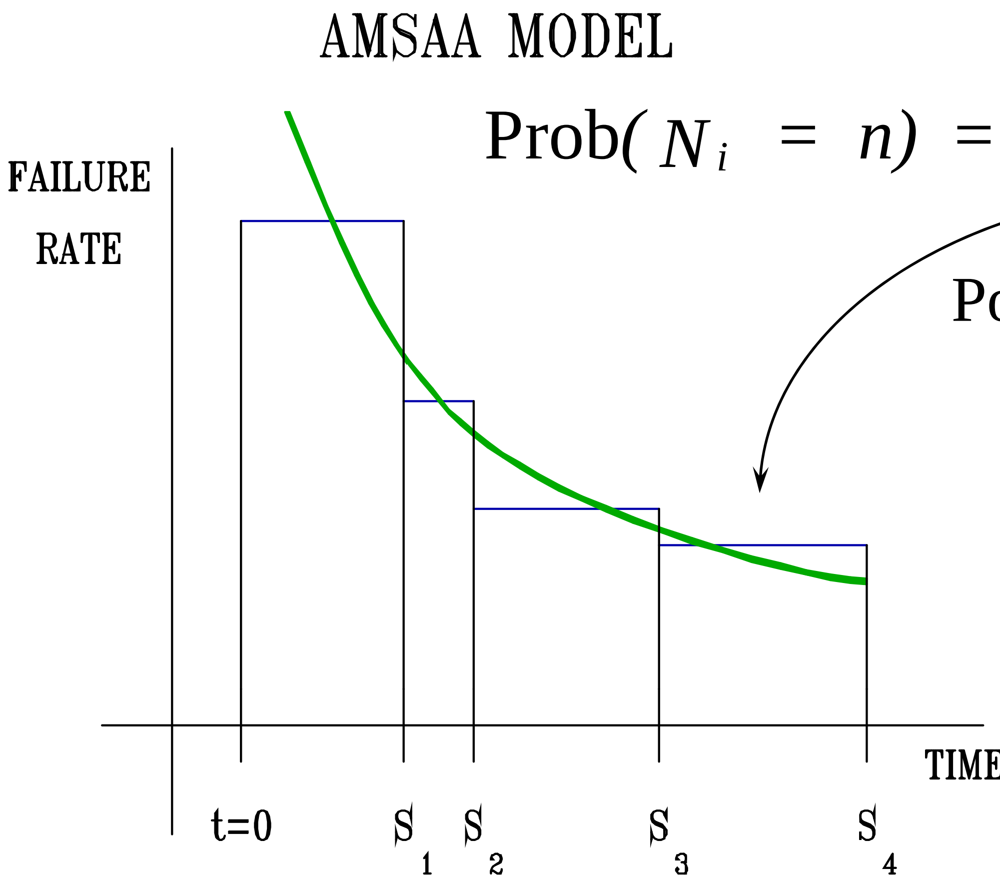
Along Came Crow

- L. H. Crow derived the maximum likelihood estimates to the Duane model
    - "Reliability Analysis for Complex, Repairable Systems" (1974),
- This provided for statistical analysis such as hypothesis testing and confidence interval estimation.
- This statistical based parameterization is referred to as the AMSAA (N.H.P.P.) model
- The AMSAA model is also the power-law process model

More to do with the AMSAA Model

- The AMSAA model has the same mathematical form as the Duane model
  - i.e. the cumulative number of failures is linear when plotted on ln-ln scale.
- However the AMSAA model is statistically based and the Duane model is empirically based
  - The Duane model: failure rate is linear on ln-ln scale.
  - The AMSAA model: failure intensity of the underlying non-homogeneous Poisson process (NHPP) is linear on ln-ln scale.

AMSAA Model



$$\text{Prob}(N_i = n) = \frac{[\lambda_i(s_i - s_{i-1})]^n e^{-\lambda_i(s_i - s_{i-1})}}{n!}$$

Poisson process

$$\text{mean} = \lambda_i(s_i - s_{i-1}).$$

N_i = number of failures during i^{th} test period

Nonhomogeneous Poisson Process (NHPP)

$$\text{Prob}[\, n(t) = n] = \frac{\lambda(t)^n e^{-\lambda(t)}}{n!}$$

$$\lambda(t) = \begin{cases} \lambda_1 t \ \text{ for } \ 0 \le t < s_1 \\ \lambda_1 s_1 + \lambda_2 (t - s_1) \ \text{ for } \ s_1 \le t < s_2 \\ \lambda_1 s_1 + \lambda_2 s_2 + \lambda_3 (t - s_2) \ \text{ for } \ s_2 \le t < s_3 \\ etc. \end{cases}$$

intensity function: $\rho(t) = \lambda_i \quad \text{for} \quad s_{i-1} < t < s_i$

Power Law Process

approximate intensity function with:

$$\rho(t) = abt^{b-1} \; ; \; t > 0 \; ; \; a, b > 0$$

expected cumulative number of failures:

$$m(t) = \int_0^t abx^{b-1}dx = at^b$$

$$MTTF =_i \left[abt_0^{b\text{-}1}\right]^{-1}$$

MLE - Parameter Estimation

Type I data:

$$\hat{b} = \frac{n}{n \ln T - \sum_{i=1}^{n} \ln t_i}$$

$$\hat{a} = \frac{n}{T^{\hat{b}}}$$

Type II data:

$$\hat{b} = \frac{n}{(n-1)\ln t_n - \sum_{i=1}^{n-1} \ln t_i}$$

$$\hat{\rho}(T) = \hat{a}\,\hat{b}\,T^{\hat{b}-1}$$

$$\hat{MTTF}_i = \frac{1}{\hat{\rho}(T)}$$

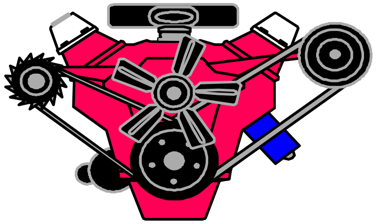
Confidence Intervals

$$\frac{L}{\hat{\rho}(T)} \leq MTTF \leq \frac{U}{\hat{\rho}(T)}$$

Type I Testing

degree of confidence

sample size N	0.90	0.90	0.95	0.95
	L	U	L	U
8	.436	2.981	.382	3.609
9	.457	2.750	.403	3.285
10	.476	2.575	.421	3.042
11	.492	2.436	.438	2.852



Example 14.3 - Type I test

Two prototype engines are tested concurrently with Type I testing with T=500 hours. The first engine accumulates a total of 200 hours and the second engine accumulates 300 hours. Times of failures (*) on each engine are identified below:

Engine #1 hours	Engine #2 hours	Cumulative hours
5.6*	0	5.6
10.2	8.6*	18.8
20.4*	18.1	38.5
41.8*	36.0	77.8
72.3	61.5*	133.8
88.5*	75.0	163.5
120.0*	105.4	225.4
170.7	152.8*	323.5
190.2	181.3*	371.5
200.0	256.6*	456.6
200.0	300.0	500.0

Example 14.3 - Type I test

Failure Time	(natural)Log Failure Time
5.6	1.722767
18.8	2.933857
38.5	3.650658
77.8	4.354141
133.8	4.896346
163.5	5.096813
225.4	5.417876
323.5	5.779199
371.5	5.917549
456.6	6.123808
total	45.89302

solution:

$$\hat{b} = \frac{10}{10 \ln(500) - 45.89302}$$

$$= .615268$$

$$\hat{a} = \frac{10}{500^{.615268}} = .218479$$

$$\rho(T) = .218479 x .615268\, t^{.615268-1}$$

$$= .134423\, t^{-.384732}$$

Example 14.3 - Type I test

at the conclusion of the testing: solution:

$$\rho(500) = .134423\,(500)^{-.384732} = .012305$$

$$MTTF_i = \frac{1}{\rho(500)} = 81.265\ hrs$$

A 90 percent confidence interval for the MTTF:

(.476 x 81.26, 2.575 x 81.26) = (38.68, 209.24)

Example 14.4 - Type II test

Estimate the AMSAA parameters from the following failure times:
(test terminated after 15 failures.

Failure Time		(natural) Log Failure Time
3		1.098612
15		2.70805
35		3.555348
58		4.060443
113		4.727388
187		5.231109
225		5.4161
465		6.142038
732		6.595781
1123		7.023759
1587		7.369601
2166		7.680638
5423		8.598404
8423		9.038721
	Total	79.24599

$$\hat{b} = \frac{15}{14\ \ln(12035) - 79.24599} = .28685$$

$$\hat{a} = \frac{15}{12035^{.28685}} = 1.013$$

$$\hat{\rho}(t) = 1.013 \times .28685\, t^{-(.28685-1)} = .29058\, t^{-.71315}$$

Example 14.4 - Type II test

$$MTTF = \frac{1}{\hat{\rho}(12035)} = 2797$$

A 90% confidence interval for the MTTF is given by:

(.6299 x 2797, 2.182 x 2797) = (1762, 6103)

Example 14.4

$$\hat{\rho}(t) = 1.013 \times .28685\, t^{-(.28685-1)} = .29058\, t^{-.71315}$$

Estimated $MTTF_i$ at the conclusion of the test program (25,000 cumulative hours):

$MTTF_i = [\ .29058\ (25{,}000)^{-.71315}\ ]^{-1} = 4{,}711$

Total test hrs to reach goal (5,000 hr.):

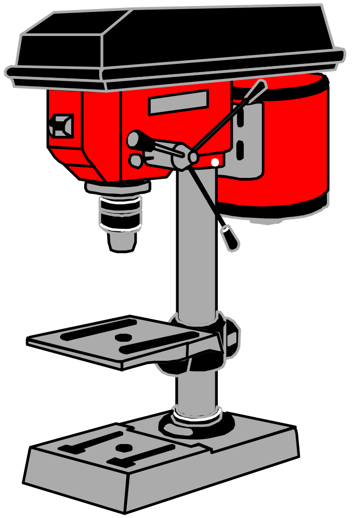
$t = [\ (5{,}000 \times .29058)^{-1}\ ]^{-1/\ .71315} = 27{,}176$

Additional test hrs required = 27,176 – 25,000 = 2176.

Another Example - Machine Failures under Minimal Repair

A drill press manufactured by the Kanot Faile Company and costing $ 1200 has experienced **minimal** repairable failures on the following days since it was first installed 2 1/4 years ago (820 days):

150 280 378 490 552 601 641
690 726 770 790



Management Displeasure

Management is displeased with the recent increase in the number of failures. It cost them $160 a failure for labor and parts. Should it be replaced?

A company manager upon receipt of the most recent machine reliability report.

Machine Failures
Power Law Intensity Function

AMSAA Reliability Growth Model & Power Law Process (NHPP)

CUM FAILURE TIME	LOG CUM FAILURE TIME
150	5.010635
280	5.634789
378	5.934894
490	6.194406
552	6.313548
601	6.398595
641	6.463029
690	6.536692
726	6.58755
770	6.64639
790	6.672033
TOTAL	68.39256

(Nbr data pts) N = 11

LOG T OR LOG T(N) = 6.709304

$$\hat{b} = \frac{11}{11\ln(820) - 68.392} = 2.033$$

$$\hat{a} = \frac{11}{820^{2.033}} = 1.308\,x10^{-5}$$

Analysis of the situation

1. Current (instantaneous) MTBF = 36 2/3 days
2. Since b = 2.03 > 1, drill is deteriorating
3. The expected number of failures over the next 6 months (days 820 - 1000) is:

Time	Cum nbr failures	MTBF	Reliability
820	11.0	36.6 inst	1.647E-05
1000	16.5	29.8 inst	6.900E-08
Interval	5.5	32.9 cum	.0042 (no failures)

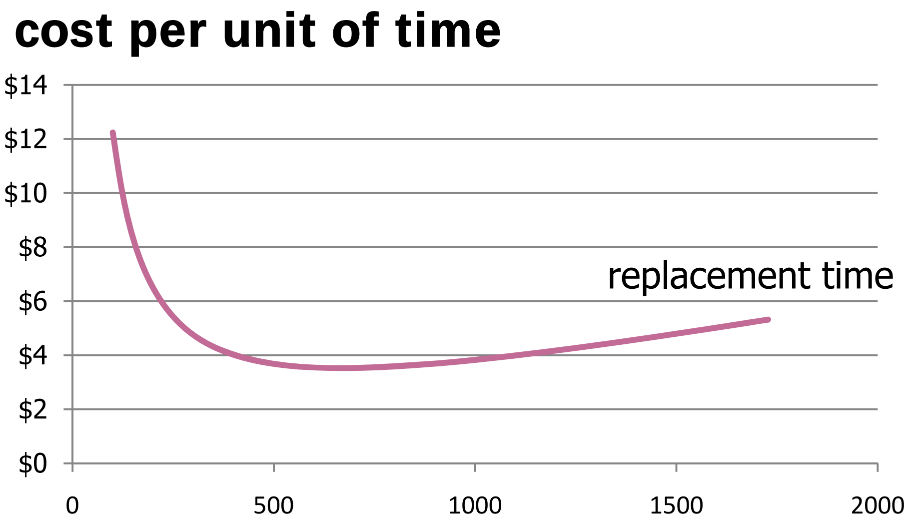
90% CI: $18.0 < MTBF_I < 89.3$

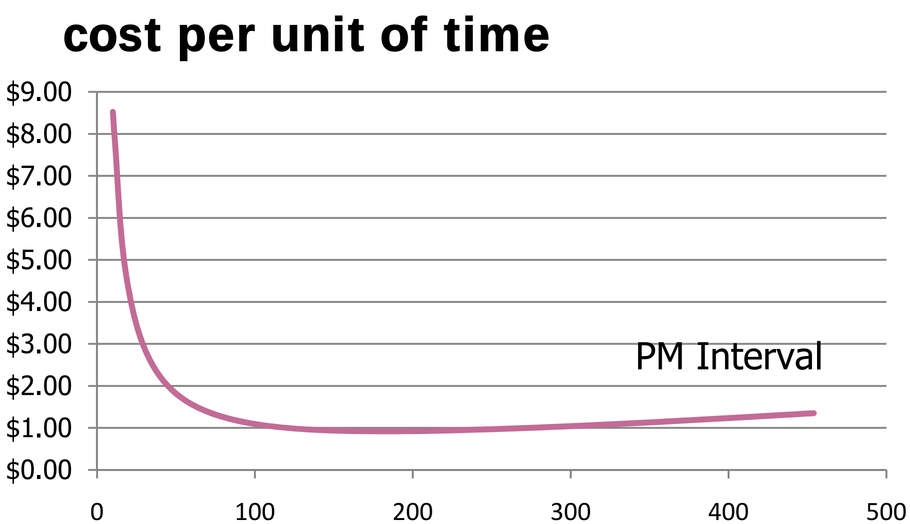
Replacement & PM Model

Power law process

unit of time:	days	
a	=0.00001308	
b	=2.033	
replacement cost		
Cu	=$1,200	
cost of a failure		
Cf	=$160	
cost of a PM	=$85	
time to replace	=669.2	days
min cost per day	$3.53	
time between PM		
	=182.0	days
min cost per day	$0.92	

Graphical Analysis





Parameter Estimation with Grouped Data

Let n_i be the number of failures that occurred in the interval (t_{i-1}, t_i) where t_i is the accumulated test time at the end of the i^{th} test cycle, $i = 1,\ldots,N$. Assuming a NHPP with the power law intensity function $\rho(t) = abt^{b-1}$, the maximum likelihood estimates are found from:

$$\hat{a} = \frac{\sum_{i=1}^{N} n_i}{t_N^{\hat{b}}} \qquad \sum_{i=1}^{N} n_i \left[\frac{t_i^{\hat{b}} \ln t_i - t_{i-1}^{\hat{b}} \ln t_{i-1}}{t_i^{\hat{b}} - t_{i-1}^{\hat{b}}} - \ln t_N \right] = 0$$

solve numerically for $\hat{b}$.

Example 14.5

Using the data in Example 14.2, Eq. (14.27) was solved numerically for $\hat{b} = .4185$

thereby giving $\hat{a} = \dfrac{61}{350^{.4185}} = 5.2558$

The intensity function is $\rho(t) = (5.2558)(.4185)t^{-.5815}$

The instantaneous MTBF at the end of the last test cycle is

$$\left[(5.2558)(.4185)(350)^{-.5815}\right]^{-1} = 13.71$$

Compared to the estimated MTTF_i from the Duane model of 11.0.

Chapter 14
Reliability Growth Testing

Reliability Growth Process

Duane Growth Model

AMSAA Model