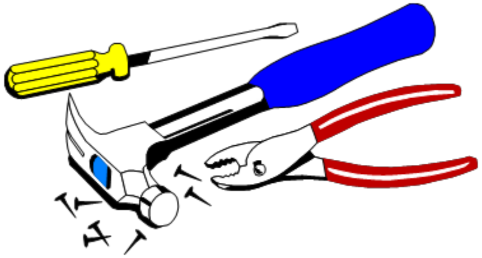
Chapter 9 Part II
Maintainability



9.4 System Repair Time

9.5 Reliability Under Preventive Maintenance

9.6 State-Dependent Systems with Repair

9.4 System Repair Time

$MTTR_i$ = the mean time to repair the ith unique subsystem,

f_i = the expected number of failures of the ith unique subsystem over the system design life,

q_i = the number of identical subsystems of type i,

$$MTTR_s = \frac{\sum_{i=1}^{n} q_i f_i \; MTTR_i}{\sum_{i=1}^{n} q_i f_i}$$

$$f_i = \begin{cases} \dfrac{t_{oi}}{MTTF_i} & \text{for renewal process} \\ \int_0^{t_{oi}} \rho(t)\,dt & \text{for minimal repair} \end{cases}$$

Redundant System Repair Time

A 2 out of 3 redundant system with each component having a constant repair rate equal to 1/MTTR.

	Repair one at a time	Repair simultaneously
Restore when one is repaired	MTTR	MTTR/2
Restore when both are repaired	2 MTTR	1.5 MTTR

system mean repair time

System MTTR Derivation Under Simultaneous Repair

Case: Restore when one is repaired and repair simultaneously

Assume: $r = 1/MTTR$ and independent repair times

T_1 = the time to repair component 1 and T_2 = the time to repair component 2

T = a random variable, the time of the first repair; T = minimum $\{T_1, T_2\}$

$$\Pr\{T \le t\} = \Pr\{T_1 \le t \text{ or } T_2 \le t\}$$
$$= \Pr\{T_1 \le t\} + \Pr\{T_2 \le t\} - \Pr\{T_1 \le t\}\Pr\{T_2 \le t\}$$

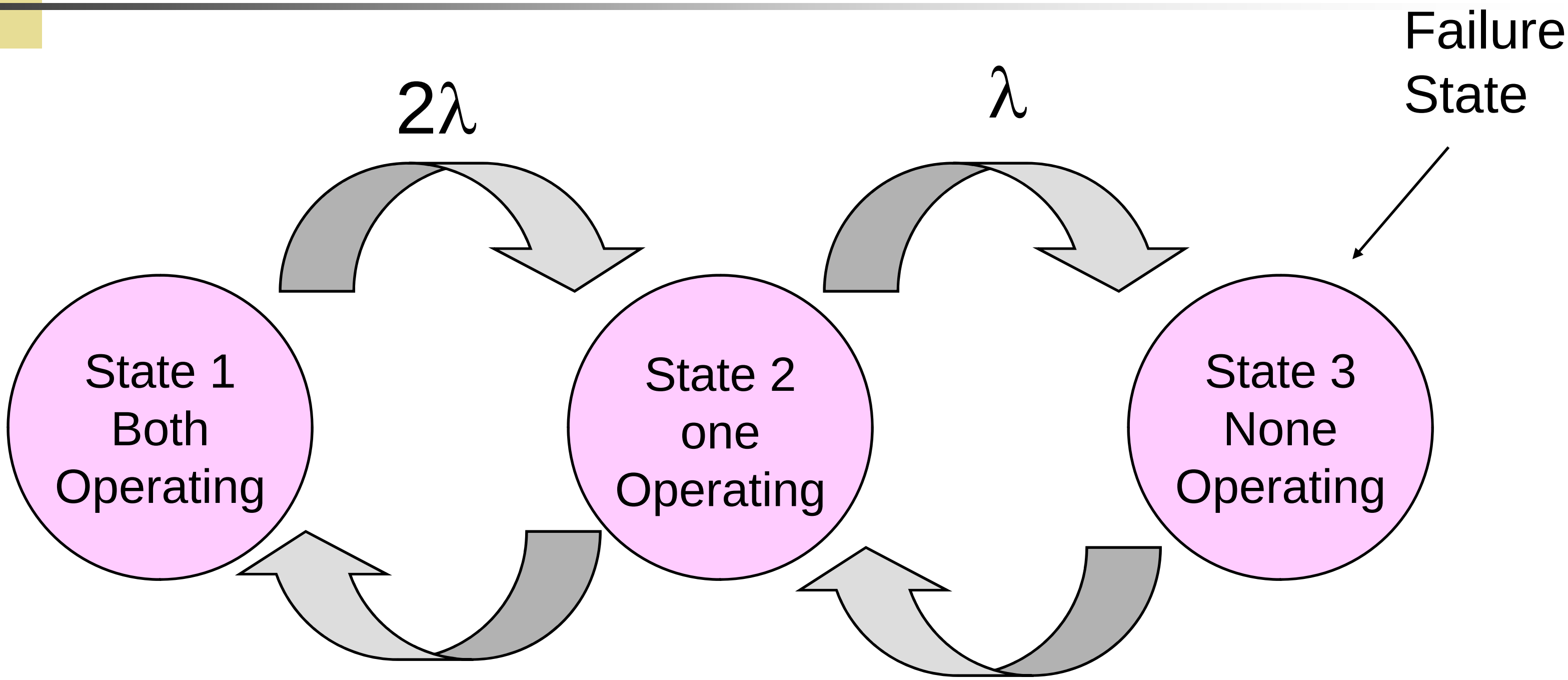
Under CFR, then $\Pr\{T_i \le t\} = 1 - Exp(-rt)$ and

$\Pr\{T \le t\} = 2\,[1 - Exp(-rt)] - [1 - Exp(-rt)]^2$

$= 2 - 2\exp(-rt) - [\,1 - 2Exp(-rt) + Exp(-2rt)\,] = 1 - Exp(-2rt)$

which is exponential with rate 2r and $MTTR_s = 1/(2r) = MTTR/2$

The Rate Diagram



r = 1/MTTR	r = 1/MTTR	one-at-a-time
r = 1/MTTR	r = 2/MTTR	simultaneously

9. 5 Reliability under Preventive Maintenance

$R(t)$ = system reliability without maintenance
T = interval of time between preventive maintenance
$R_m(t)$ = reliability of the system with preventive maintenance

$$R_m(t) = R(t) \text{ for } 0 \le t < T$$

$$R_m(t) = \underbrace{R(T)^n}_{\text{Prob of surviving n PM intervals of length T}} \; \underbrace{R(t - nT)}_{\text{Repair to "as good as new"}} \text{ for } nT \le t < (n+1)T$$

Reliability under Preventive Maintenance

$$MTTF = \int_0^\infty R_m(t)\,dt = \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} R_m(t)\,dt$$

$$= \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} R(T)^n \; R(t-nT)dt$$

$$= \sum_{n=0}^{\infty} R(T)^n \int_{nT}^{(n+1)T} R(t-nT)dt$$

$$= \sum_{n=0}^{\infty} R(T)^n \int_0^T R(t')d't \quad \text{where } t' = t-nT$$

$$\boxed{\text{Therefore } MTTF = \frac{\int_0^T R(t)dt}{1-R(T)}}$$

$\sum_{n=0}^{\infty} R(T)^n$ is an infinite geometric series having as its sum $\dfrac{1}{1-R(T)}$

CFR Model

$$R(t) = e^{-\lambda t}$$

$$R_m(t) = (e^{-\lambda T})^n e^{-\lambda(t-nT)}$$

$$= e^{-\lambda nT} \quad e^{-\lambda t} \quad e^{\lambda nT} = e^{-\lambda t} = R(t)$$

Another example of the memoryless property of the Exponential Distribution.

Weibull Example

$$R_m(t) = e^{-n\left(\frac{T}{\theta}\right)^{\beta}} \; e^{-\left(\frac{t-nT}{\theta}\right)^{\beta}} \quad , \quad nT \le t \le (n+1)T$$

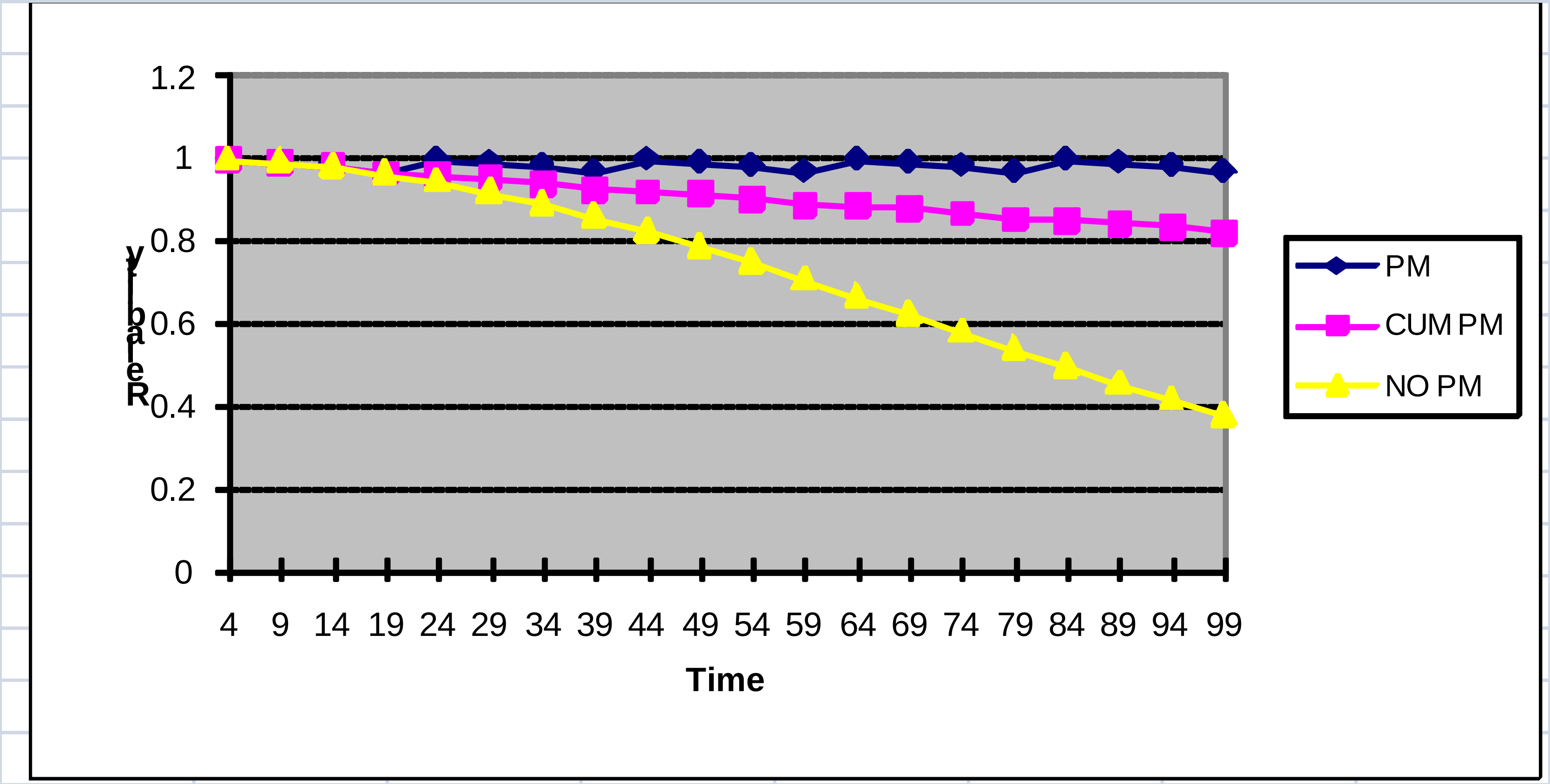
numerical example

$$R_m(t) = e^{-n\left(\frac{20}{100}\right)^{2}} \; e^{-\left(\frac{t-20n}{100}\right)^{2}} \quad , \quad 20n \le t \le 20(n+1)$$

To find $R_m(90)$, observe that n = 4. Then

$$R_m(90) = e^{-4\left(\frac{20}{100}\right)^{2}} \; e^{-\left(\frac{90-80}{100}\right)^{2}} = .8437$$

Weibull Example



Weibull Example

Find the .90 design life:

$$e^{-n\left(\frac{20}{100}\right)^{2}} \approx .90 \qquad n = \frac{(-\ln .90)}{\left(\frac{20}{100}\right)^{2}} = 2.63$$

$$R_m(t) = e^{-2\left(\frac{20}{100}\right)^{2}} e^{-\left(\frac{t-40}{100}\right)^{2}}, \quad 40 \le t < 60$$

$$= .9231\, e^{-\left(\frac{t-40}{100}\right)^{2}} = .90$$

$$t = 100\left[-\ln\left(\frac{.90}{.9231}\right)\right]^{\frac{1}{2}} + 40 = 55.9 \ days$$

Maintenance-induced Failures

$$R_m(t) = R(T)^n (1-p)^n R(t-nT), \quad nT \le t < (n+1)T$$

Maintenance-induced Failures - lognormal example

$$R(T)^n = \left[1 - \Phi\left(\frac{1}{s}\ \ln\frac{T}{t_{MED}}\right)\right]^n \qquad R(t - nT) = 1 - \Phi\left(\frac{1}{s}\ \ln\frac{t - nT}{t_{MED}}\right)$$

With t_{med} = 5,000 hr and s = 1.0:

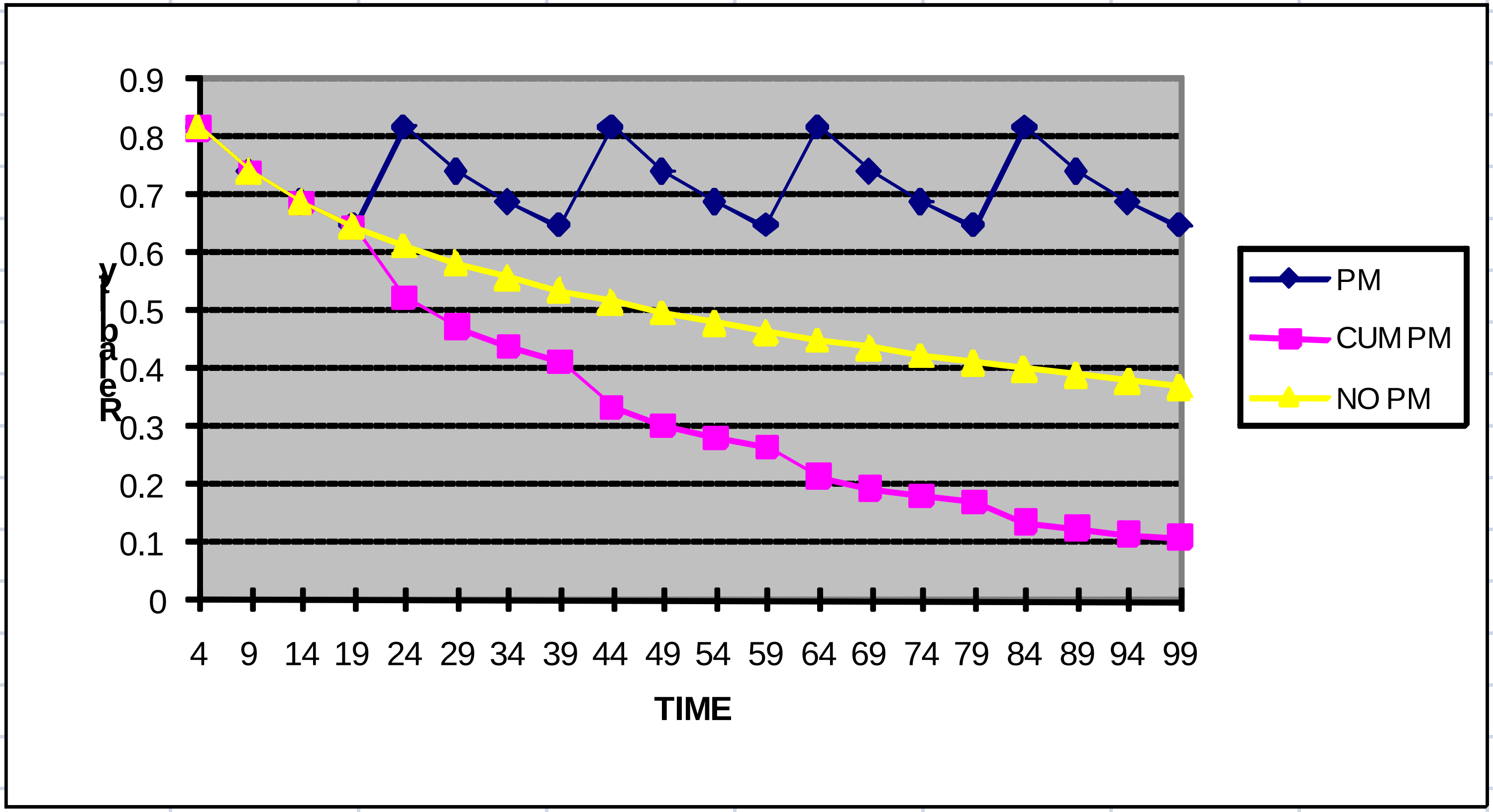
$$R(5000) = 1 - \Phi\left(\ln\frac{5000}{5000}\right) = 1 - .5 = .50$$

Assume p = .005 and T = 500 hr.

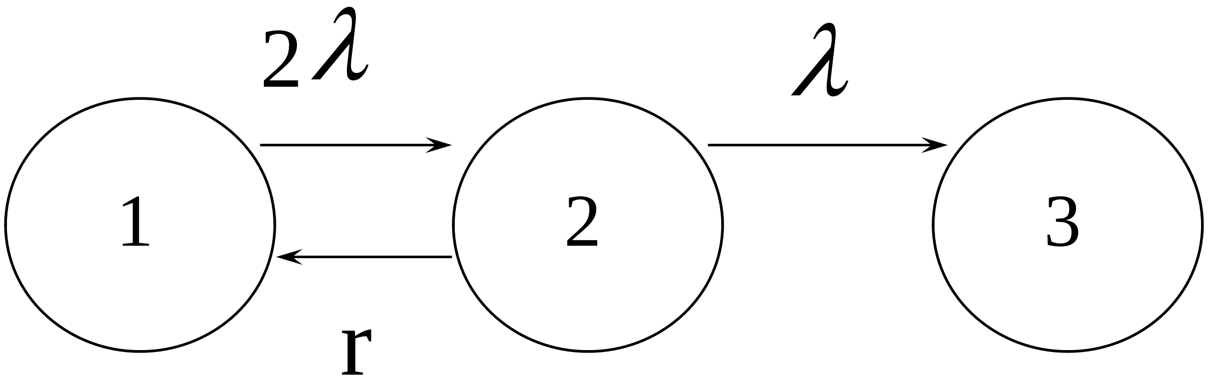
$$R_m(5000) = \left[1 - \Phi\left(\ln\frac{500}{5000}\right)\right]^{10} (1 - .005)^{10} = .854$$

DFR and PM

Weibull with beta = 0.5 and theta = 100 days



9.6 State Dependent Systems with Repair



$$\frac{dP_1(t)}{dt} = -2\lambda P_1(t) + rP_2(t)$$

$$\frac{dP_2(t)}{dt} = 2\lambda P_1(t) - (r+\lambda) P_2(t)$$

$$\frac{dP_3(t)}{dt} = \lambda P_2(t)$$

State Dependent Systems with Repair - solution

$$P_1(t) = \frac{\lambda + r + x_1}{x_1 - x_2} e^{x_1 t} - \frac{\lambda + r + x_2}{x_1 - x_2} e^{x_2 t}$$

$$P_2(t) = \frac{2\lambda}{x_1 - x_2} e^{x_1 t} - \frac{2\lambda}{x_1 - x_2} e^{x_2 t}$$

$$P_3(t) = 1 + \frac{x_2}{x_1 - x_2} e^{x_1 t} - \frac{x_1}{x_1 - x_2} e^{x_2 t}$$

where

$$x_1, x_2 = \frac{1}{2}\left[-(3\lambda + r) \pm \sqrt{\lambda^2 + 6\lambda r + r^2}\right]$$

State Dependent Systems with Repair - solution

$$R(t) = 1 - P_3(t) = \frac{x_1}{x_1 - x_2} e^{x_2 t} - \frac{x_2}{x_1 - x_2} e^{x_1 t}$$

MTTF =

$$\int_0^\infty \left(\frac{x_1}{x_1 - x_2} e^{x_2 t} - \frac{x_2}{x_1 - x_2} e^{x_1 t} \right) dt = \frac{1}{x_1 - x_2} \left[\frac{x_1}{x_2} - \frac{x_2}{x_1} \right] = \frac{x_1 + x_2}{x_1 x_2} = \frac{3\lambda + r}{2\lambda^2}$$

$$MTTF = \left(1.5 + .5 \frac{MTTF_c}{MTTR_c} \right) MTTF_c$$

State Dependent Systems with Repair - example

A computer system consists of two active parallel processors each having a constant failure rate of .5 failures per day. Repair of a failed processor requires an average of one half a day (exponential distribution).

MTTF = [3(.5) + 2] / [(2) (.25)] = 7 days.

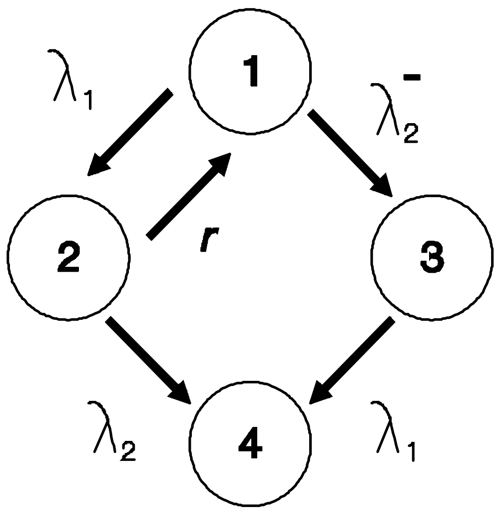
$$R(1) = \frac{-.149}{3.201} e^{-3.35} - \frac{-3.35}{3.201} e^{-.149} = .90$$

where $x_1 = -0.149$ and $x_2 = -3.35$

without repair, MTTF = 3 days and R(1) = .845

Standby System with Repair

PRIMARY SYSTEM REPAIR



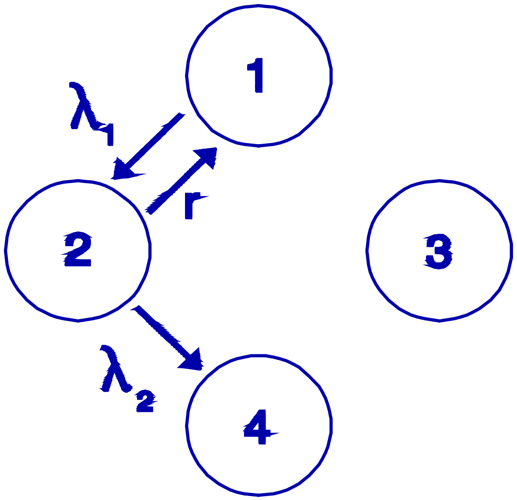
$$\frac{d\,P_1(t)}{dt} = -(\lambda_1 + \bar{\lambda}_2)\,P_1(t) + r\,P_2(t)$$

$$\frac{d\,P_2(t)}{dt} = \lambda_1\,P_1(t) - (\lambda_2 + r)\,P_2(t)$$

$$\frac{d\,P_3(t)}{dt} = \bar{\lambda}_2\,P_1(t) - \lambda_1\,P_3(t)$$

Standby System with Repair
No failures in standby

NO STANDBY FAILURE



$$\frac{d\,P_1(t)}{dt} = -\lambda_1 P_1(t) + r\,P_2(t)$$

$$\frac{d\,P_2(t)}{dt} = \lambda_1 P_1(t) - (\lambda_2 + r)\,P_2(t)$$

Standby System with Repair
No failures in standby - solution

$$P_1(t) = \frac{\lambda_2 + r + x_1}{x_1 - x_2} e^{x_1 t} + \frac{\lambda_2 + r + x_2}{x_2 - x_1} e^{x_2 t}$$

$$P_2(t) = \frac{\lambda_1}{x_1 - x_2} e^{x_1 t} + \frac{\lambda_1}{x_2 - x_1} e^{x_2 t}$$

$$R(t) = P_1(t) + P_2(t) = \frac{(k_1 + x_1) e^{x_1 t} - (k_1 + x_2) e^{x_2 t}}{x_1 - x_2}$$

$$x_1 \,,\; x_2 = \frac{-k_1 \pm \sqrt{k_1^2 - 4k_2}}{2}$$

$$k_1 = \lambda_1 + \lambda_2 + r$$

$$k_2 = \lambda_1 \lambda_2$$

Standby System with Repair
No failures in standby-example

An on-board computer system has, through the use of built-in-test equipment (BITE), the capability of being restored when a failure occurs. A standby computer is available for use whenever the primary fails.

λ_1 = .0005, r = .1, and λ_2 = .002 failures per hour.

$$K_1 = .0005 + .002 + .1 = .1025$$

$$K_2 = (.0005)(.002) = 10^{-6}$$

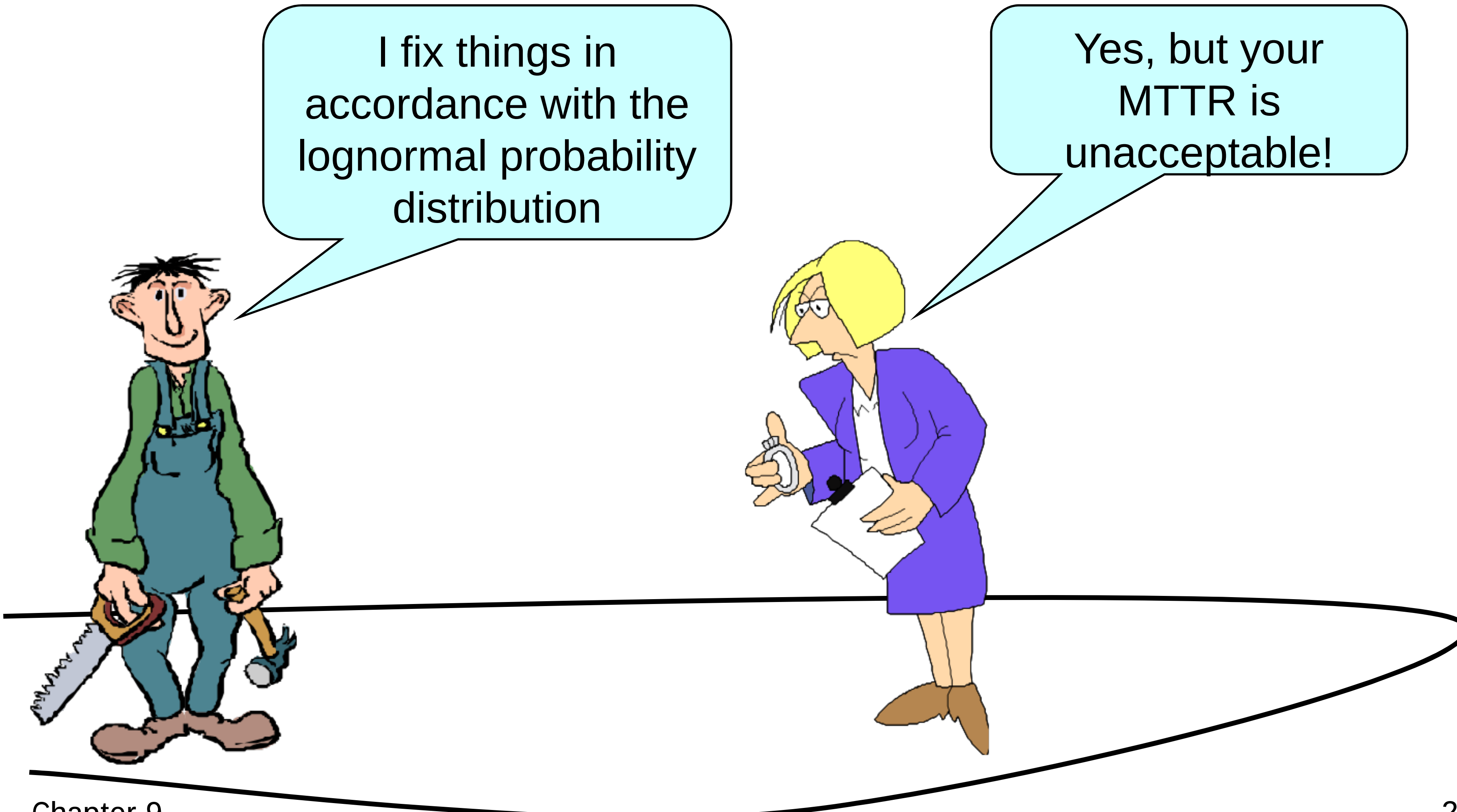
$$x_1, x_2 = \frac{-.1025 \pm \sqrt{(.1025)^2 - 4x10^{-6}}}{2} = -9.757 x 10^{-6}, -0.10249$$

Standby System with Repair
No failures in standby-example

$$R(t) = \frac{.10249\, e^{-9.757x10^{-6}t} - (9.757x\,10^{-6})\, e^{-.10249t}}{.1024852}$$

R(1000) = .99039
R(2000) = .98077
R(3000) = .97125
R(4000) = .96182
R(5000) = .95248

The End


I fix things in accordance with the lognormal probability distribution
Yes, but your MTTR is unacceptable!