LOGM 634 - Homework Set #5

Due 29 September 2017

From the Ebeling text - Exercise 12.3

Three hundred AC motors were originally installed in 1984 as part of a fan assembly. They all have failed. The following data were collected over their operating history.

Year	Number of Failures
1985	15
1986	20
1987	18
1988	27
1989	35
1990	31
1991	45
1992	43
1993	66

Derive an empirical reliability function, density function, and hazard rate function for this motor. Estimate the MTTF and the standard deviation of the failure times. Would you conclude that the failure rate is decreasing, constant, or increasing? Which would you expect it to be if the dominant failure mode were due to mechanical wearout.

For this dataset we don't when each of the 300 motors actually failed, but are only told in which year it failed. Because the are no censored observations, the data are grouped and complete. For this type of data the empirical reliability function, density function, and hazard rate function may be expressed as shown below.

$$\widehat{R}(t_i) = \frac{n_i}{300}$$

$$\widehat{f}(t_i) = \frac{n_i - n_{i+1}}{(t_{i+1} - t_i)(300)}$$

$$\widehat{\lambda}(t_i) = \frac{n_i - n_{i+1}}{(t_{i+1} - t_i)(n_i)}$$

An estimate of $MTTF$ can be computed as

$$\widehat{MTTF} = \sum_{i=1}^{k-1} \bar{t}_i \frac{n_i - n_{i+1}}{n} = \frac{1681}{300} = 5.6033333$$

And an estimate of standard deviation σ can be computed as

$$\widehat{s} = \left[\sum_{i=1}^{k-1} \bar{t}_i^2 \frac{n_i - n_{i+1}}{n} - \widehat{MTTF}^2\right]^{1/2}$$

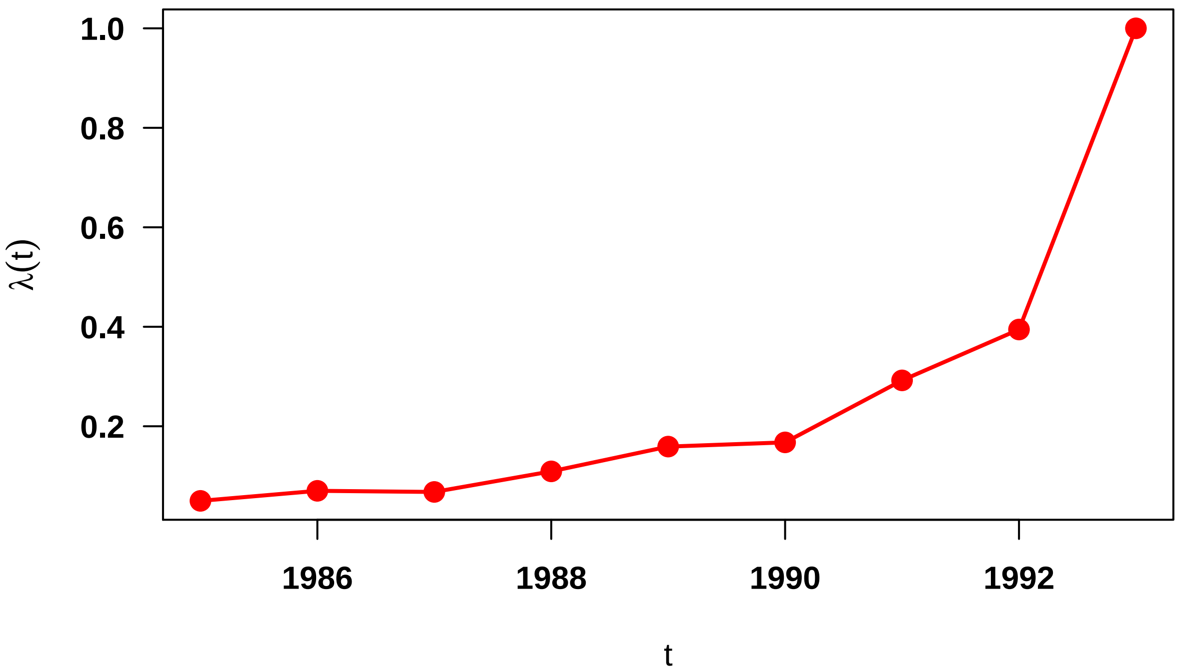
$$= \left[\frac{1.1227 \times 10^4}{300} - 5.6033333^2\right]^{1/2}$$

$$= \left[6.0259889\right]^{1/2}$$

$$= 2.454789$$

Finally, we can check if $\lambda(t)$ is decreasing, constant, or increasing by plotting $\lambda(t)$ against t for each time observed. The table below shows the values to be plotted.

$\lambda(t_i)$	t_i
0.0500000	1985
0.0701754	1986
0.0679245	1987
0.1093117	1988
0.1590909	1989
0.1675676	1990
0.2922078	1991
0.3944954	1992
1.0000000	1993



From the plot it is apparent that the data exhibits an increasing failure rate, which indicates that the failures are due to a wearout type of failure mode.

From the Ebeling text - Exercise 12.21

In order to estimate warranty costs, 70 prototypes of a new product are tested with the test terminating after 53 failures. The following failure times were recorded in 100's of operating hours. Failure mode B is failures due to manufacturing defects and failure mode C is failures due to premature wear-out of parts. All other failures are identified as failure mode A. The product is under warranty for 10,000 hours. Using the product limit estimator, estimate the probability of failure under warranty if onl failure modes B and C fall under the warranty program.

Failure Time	Failure Mode	Failure Time	Failure Mode	Failure Time	Failure Mode
1	A	204	A	457	C
3	A	204	B	478	C
23	A	211	B	494	A
38	A	214	C	522	C
66	C	222	B	532	C
95	B	260	B	547	A
100	C	262	C	593	C
130	C	267	B	667	C
136	A	273	B	822	C
138	A	283	C	849	A
146	A	310	B	862	A
155	A	319	C	935	C
169	A	330	B	1180	A
173	A	333	B	1198	C
174	A	348	B	1391	A
187	A	349	B	1443	C
192	B	381	B	1466	C
197	A	391	B		

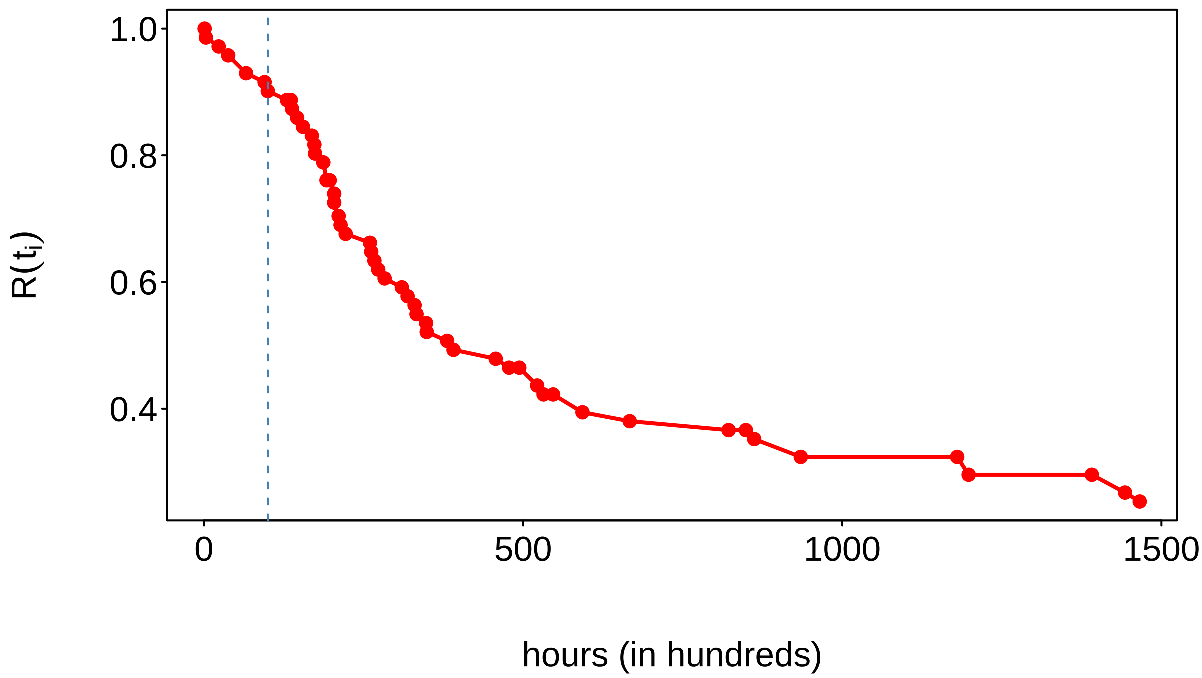
The product limit estimator is a nonparametric estimator for the reliability function. The product limit estimator is expressed in Equation 12.18 as

$$\widehat{R}(t_i) = \left(\frac{n+1-i}{n+2-i}\right)^{\delta_i} \times \frac{n+2-i}{n+1}$$

where $n = 53$, i is the ordered rank of thefailure observation and

$$\delta_i = \begin{cases} 1 \text{ if a failure occurs at time } t_i \\ 0 \text{ if censring occurs at time } t_i \ . \end{cases}$$

Using the product limite estimator gives the estimates for $R(t_i)$ as shown in the plot below. The dashed line represent $t = 10,000$ hours. Observing where the estimate for $R(t)$ crosses this dashed line we see that $\widehat{R}(10000) \approx 0.90$.



1.0
0.8
0.6
0.4
$R(t_i)$
0
500
1000
1500
hours (in hundreds)

From the Ebeling text - Exercise 13.4

Determine the burn-in test time for a new product. The product after reliability growth testing has a Weibull failure distribution with $\beta = 0.3$ and $\theta = 3,750,000$ hr. Contract specification require a 0.95 reliability at 1000 operating hours.

If no burn-in testing were performed, the reliability of the 'as-is' system at 1000 operating hours would be below the required value of 0.95.

$$
\begin{aligned}
R(1000) &= \exp\left[-\left(\frac{1000}{3750000}\right)^{0.3}\right] \\
&= 0.9188045
\end{aligned}
$$

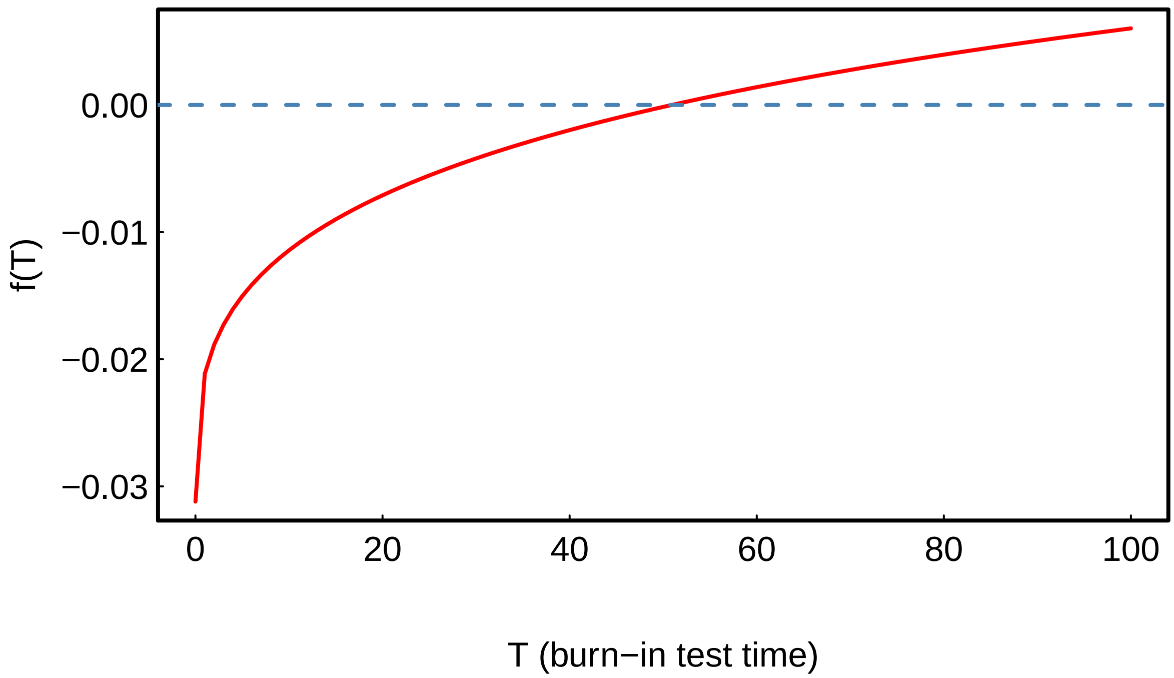
Thus, we need to determine the appropriate amount of burn-in testing T such that the conditional reliability

$$
R(1000|T) = \frac{R(1000+T)}{R(T)} = 0.9
$$

To find the desired value for T we use Equation 13.6 (shown below) and solve for T either numerically or graphically.

$$
\exp\left[-\left(\frac{1000+T}{3750000}\right)^{0.3}\right] - R_o \exp\left[-\left(\frac{T}{3750000}\right)^{0.3}\right] = 0 \tag{13.6}
$$

The graph below plots Equation 13.6 for various values of T. The desired value corresponds to where the function equals zero.



The plot seems to show that the preferred length of burn-in testing is around 50 hours. We can find a more exact value for T using a numerical solver. using a solver in R gives the following result.

```
fun <- function(t) {
  exp(-((1000+t)/3750000)^0.3)-0.95*exp(-(t/3750000)^0.3)
}
uniroot(fun, interval = c(40,60))$root
## [1] 50.90621
```

From the Ebeling text - Exercise 13.17

Twenty (20) units are placed on test for 200 hr (Type I Testing). If the units are believed to have a lognormal distribution with $s = 1.21$ and $t_{med} = 480$ hours, what is the expected number of failures?

The expected number of failures depends on the length of the test and the $MTTF$ of the units being tested. To determine the expected number of failures for this exercise we use Equation 13.4, shown below.

$$
\begin{aligned}
E\Big[r\Big] &= n\Big[1 - R(200)\Big] \\
&= 20 \times \Phi\left(\frac{1}{1.21} \ln\left[-\frac{200}{480}\right]\right) \\
&= 20 \times \Phi\Big(-0.7235279\Big) \\
&= 20 \times 0.2346778 = \mathbf{4.6935563}
\end{aligned}
$$