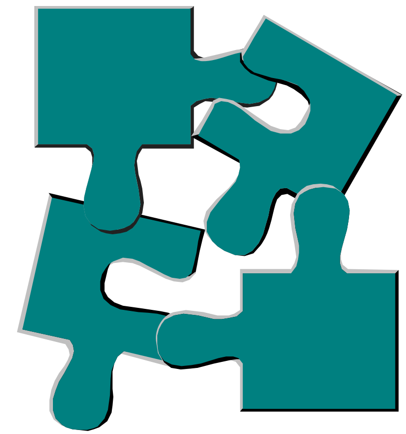


Chapter 16
Statistical Tests

Chi-Square Test

Bartlett's Test

Mann's test

Kolmogorov-Smirnov Test

Tests for the Power-Law Process Model

Hypothesis Testing

H_0: The failure times came from the specified distribution.

H_1: The failure times did not come from the specified distribution.

	H_0 true	H_1 true
Accept H_0	correct decision	Type II error
Accept H_1	Type I error	correct decision

Chi-Square GOF Test

$$x^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

with df = k - 1 - number of estimated parameters

where k = number of classes

O_i = observed number of failures in the i^{th} class

E_i = expected number of failures in the i^{th} class

$= n\, p_i$

n = total number at risk (sample size)

$p_i = F(a_i) - F(a_{i-1}) = R(a_{i-1}) - R(a_i)$

probability of a failure occurring in the i^{th} class if H_0 is true

Hypothesized distribution

i^{th} class is defined by $[a_{i-1}, a_i)$ with $a_0 = 0$

Chi-Square GOF Test - Repair

$$x^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

with df = k - 1 - number of estimated parameters

where k = number of classes

O_i = observed number of repairs in the i^{th} class

E_i = expected number of repairs in the i^{th} class

$= n\, p_i$

n = total number at risk (sample size)

$p_i = H(a_i) - H(a_{i-1})$

Hypothesized distribution

probability of a failure occurring in the i^{th} class if H_0 is true

Example - Exponential Distribution

35 failure times are grouped into the 6 cells as shown:

Cell	Upper bound	Count
1	354	18
2	688	10
3	1022	2
4	1356	2
5	1690	2
6	2026	1

$$\left.\begin{matrix}2\\2\\2\\1\end{matrix}\right\} 7$$

Expected cell counts >=5

MLE: $$\hat{\lambda} = \frac{1}{\widehat{MTTF}} = \frac{1}{485.4} = 0.00206.$$

Example - Exponential Distribution

H_0: Failure times are exponential with $\lambda = 0.00206$

H_1: Failure times are not exponential with $\lambda = 0.00206$

$$E_1 = 35\ P_1 = 35\ [\ 1 - e^{-354/485.4}\ ] = 18.120$$

$$E_1 = 35\ P_2 = 35\ [\ 1 - e^{-688/485.4} - P_1\ ] = 8.396$$

$$E_3 = 35\ P_3 = 35\ [\ 1 - P1 - P2\ ] = 8.483$$

Example - Exponential Distribution

UPPER BND	OBSERVED	PROB	EXPECTED	$(O-E)^2/E$
354	18	.5177247	18.12036	7.994791E-04
688	10	.239903	8.396606	.3061798
INFINITY	7	.2423723	8.483031	.2592684

X^2 = .5662476
df = 3 - 1 - 1 = 1

$$\chi^2 = 0.5662 < \chi^2_{CRIT,.10,1} = 2.706$$

Cannot reject H_0 at 10% level

Alternate Approach

$$F(a_i) = 1 - e^{-\lambda a_i} = \frac{i}{k} \; ; \; i = 1,2,\ldots,k-1$$

$$a_i = \frac{-\ln(1 - \frac{i}{k})}{\lambda} \; ; \; i = 1,2,\ldots,k-1$$

letting k = 5, then p_i = 0.2 and E_i = 35 (0.2) = 7 and

$$a_i = \frac{-\ln(1 - i/5)}{.00206}$$

Alternate Approach

CELL	LOWER	UPPER	OBSERVED	EXPECTED	(O-E)^2/E
1	0.00	108.3	5.00	7.00	.57
2	108.36	247.8	9.00	7.00	.57
3	247.8	444.6	9.00	7.00	.57
4	444.6	780.8	6.00	7.00	.14
5	780.8	inf	6.00	7.00	.14

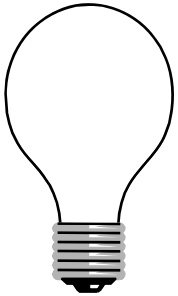
$X^2 = 1.99$

Since $X^2_{\text{crit},.10,3} = 6.25$, we accept H_0.

Weibull Example

The following 35 failure times in days were observed from 50 light bulbs placed on test. The test was terminated at the 35th failure (Type II Censoring). The failures are believed to follow a Weibull distribution.

1.3	7.3	7.8	13.3	13.9
19.4	19.7	22.3	22.8	26.7
29.7	30.2	31.9	32.2	33
36.8	37	41.7	46.7	50.4
51.4	60	61.3	61.4	65.6
65.8	72.6	78.4	100.4	110.6
111.4	118.2	119.4	132.1	139.7



The MLE's were computed using Equation (15-11) and (15-12) with estimated beta = 1.032 and theta = 112.9 days.

Weibull Example

The failure times are then grouped into 5 classes of width 28 [(139.7 - 1.3)/5 = 27.68]. Therefore a_1=28, a_2=56, a_3=84, a_4=112, and a_5=140. The remaining failure times, are placed in the 6th class. The expected cell counts are computed in the following manner:

$$E_i = 50\,P_i = 50\left[e^{-\left(\frac{a_{i-1}}{112.9}\right)^{1.032}} - e^{-\left(\frac{a_i}{112.9}\right)^{1.032}}\right] \text{ for } i = 1,2,3,4,5$$

$$E_6 = 50\left[1 - P_1 - P_2 - P_3 - P_4 - P_5\right]$$

H_0: Failure times are Weibull with B =1.03, θ =112.9 days

H_1: Failure times are not Weibull with B =1.03, θ =112.9

Weibull Example

UPPER BND	OBSERVED		PROB	EXPECTED	(O-E)^2/E
28	10		.2116768	10.58384	.0322
56	11		.1730505	8.652523	.6368834
84	7		.1369408	6.847042	.0034
112	3	combine	.1074198	5.370988	1.046657
140	4		.0838523	4.192615	.0088
INFINITY	15		.2870598	14.35299	.0292

$X^2 = 1.7572$

degrees of freedom = 6 - 1 - 2 = 3

Weibull Example

UPPER BND	OBSERVED	PROB	EXPECTED	(O-E)^2/E
28	10	.2116768	10.58384	.0322
56	11	.1730505	8.652523	.6368834
84	7	.1369408	6.847042	.0034
140	7	.1912721	9.563602	.6871948
INFINITY	15	.2870598	14.35299	.0292

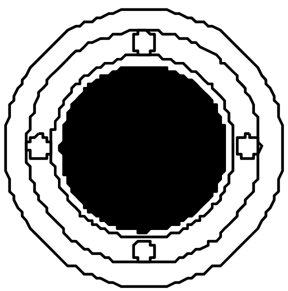
X^2 = 1.388868

degrees of freedom = 5 - 1 - 2 = 2

$$\chi^2 = 1.389 < \chi^2_{CRIT,0.10,2} = 4.605, \quad \text{cannot reject } H_0$$

A Normal Example

Fifty bearings were placed on an accelerated stress test until wear out failure was observed (complete data). It is believed wear out is normally distributed. Failure times are in (accelerated) operating hr.



278.2	320.2	361.8	346.5	387.7
331.7	295.3	355.4	386.1	287.1
333.7	332.5	391.5	335.2	297.3
346.2	376.4	446.7	313.3	314.8
340.3	273.3	361.6	361.5	389.2
391.2	372.8	336.8	357.6	331.7
342.6	305.7	272.6	359.1	399.9
443.1	375.2	364.7	300.5	359.4
298.8	276.0	339.3	447.5	350.6
397.0	301.8	282.5	357.2	346.5

The sample mean (MLE) is 345.5 and the sample std dev (MLE) is 43.6.

A Normal Example

272.6	272.8	273.3	276.0	278.2
282.5	287.1	295.3	297.3	298.8
300.5	301.8	305.7	313.3	314.8
320.2	331.7	331.7	332.5	333.7
335.2	336.8	339.3	340.3	342.6
346.2	346.6	346.5	350.6	355.4
357.2	357.6	359.1	359.4	361.5
361.6	361.8	364.7	375.2	376.4
386.1	387.7	389.2	391.2	391.5
397.0	399.9	443.1	446.7	447.5

H_0: Failures are normal with $\mu = 345.5$, $\sigma = 43.6$
H_1: Failures are not normal with $\mu = 345.5$, $\sigma = 43.6$

A Normal Example

$$E_i = 50\,P_i = 50\left[\Phi\left(\frac{a_i\text{-}345.5}{43.6}\right) - \Phi\left(\frac{a_{i\text{-}1}\text{-}345.5}{43.6}\right)\right]$$

UPPER BND	OBSERVED		PROB	EXPECTED	(O-E)^2/E
298	9		.1378	6.8915	.6451
322	7		.1567	7.8370	.0894
347	12		.2174	10.8685	.11878
372	10		.2171	10.855	.0673
397	8		.1519	7.5965	.0214
422	1	} combine	.0789	3.947	2.2004
448	3		.0307	1.50335	1.4024
INFINITY	0		.0094 .	.4700	.4700

X^2 = 5.33509

A Normal Example

UPPER BND	OBSERVED	PROB	EXPECTED	(O-E)^2/E
298	9	.1378	6.8915	.6451
322	7	.1567	7.8370	.0894
347	12	.2174	10.8685	.11878
372	10	.2171	10.855	.0673
397	8	.1519	7.5965	.0214
INFINITY	4	.1190	5.9500	.6391

df = 6 - 1 - 2 = 3

$X^2 = 1.58$

$$\chi^2 = 1.58 < \chi^2_{CRIT,0.10,3} = 6.25$$

Lognormal

Seventy-five repair times (in minutes) were observed for removing and replacing a failed component. Repair times are believed to have a lognormal distribution.

50.4	56.2	72.6	73.3	76.1
78.5	80.6	83.4	84.6	89.0
92.2	96.1	99.7	102.5	103.7
104.8	105.0	106.8	107.3	109.2
115.3	122.7	128.3	131.1	141.3
166.0	166.1	168.0	170.6	174.4
178.4	184.5	187.2	189.7	193.4
203.5	204.1	204.4	215.3	215.8
216.4	222.6	231.0	231.4	237.3
238.6	243.7	244.7	252.1	252.2
253.2	263.6	273.3	295.1	305.2
310.4	340.7	349.4	355.8	363.6
371.4	382.1	383.5	385.0	414.0
420.5	426.5	431.0	457.4	462.9
559.1	643.8	789.3	830.7	840.2

H_0: Repair times are lognormal with t_{med}=199.36 and s=0.654

H_1: Repair times are *not* lognormal with t_{med}=199.36 and s=0.654

Lognormal

$$E_i = 75p_i = 75\left\{\Phi\left[\frac{1}{.654}\ln\frac{a_i}{199.36}\right] - \Phi\left[\frac{1}{.654}\ln\frac{a_{i-1}}{199.36}\right]\right\}$$

UPPER BND	OBSERVED	PROB	EXPECTED	(O-E)^2/E
100	13	.1469	11.0123	.3588
200	22	.3731	27.9810	1.2785
300	19	.2125	15.9323	.5907
400	10	.12341	9.2295	.0643
INFINITY	11	.1446	10.8427	.0023

X^2 = 2.29

degrees of freedom = 5 - 1 - 2 = 2

$$\chi^2 = 2.29 < \chi^2_{CRIT,0.10,2} = 4.61$$

Chi-Square Test for Singly Censored Data

EXAMPLE 16.5. The following failure times in cycles resulted from submitting 35 mechanical switches to an accelerated life test terminating at failure or at 6000 cycles:

450	1479	1581	1750	1825	2116	2441	2545
	2609	2724	2732	3442	3624	3745	3831
	3839	3879	4641	4940	4989	5050	5217
	5596	5601	5654	5736	5851	5869	5911

The Weibull distribution with MLE's $\widehat{\beta} = 2.287$ and $\hat{\theta} = 4949.76$ was subjected to the chi-square test with the data grouped into 7 cells. The 6 censored units were counted in the last cell.

Chi-Square Test for Singly Censored Data

Cell	Upper Bound	Observed	Probability	Expected	$(O-E)^2/E$
1	1000	1	0.0255	0.8912	0.0133
2	2000	4	0.0928	3.2485	0.1739
3	3000	6	0.1542	5.3986	0.0670
4	4000	6	0.1865	6.5265	0.0425
5	5000	3	0.1816	6.3568	1.7726
6	6000	9	0.1477	5.1706	2.8361
7	∞	**6**	0.2117	7.4078	0.2676

The computed $X^2 = 5.1729 <$ critical $X^2 = 7.78$ with 4 degrees of freedom at the 10% level.
Therefore, the Weibull distribution cannot be rejected.

Goodness of Fit Tests for Specific Distributions

Bartlett's Test for exponential

Mann's Test for the Weibull

Kolmogorov-Smirnov Test for normal/lognormal

Trend & GOF for power law process model

Bartlett's Test for Exponential

H_0: Failures times are exponential
H_1: Failure times are not exponential

$$B = \frac{2r\left[\ln\left(\frac{\sum_{i=1}^{r} t_i}{r}\right) - \frac{1}{r}\sum_{i=1}^{r}\ln t_i\right]}{1 + \frac{(r+1)}{6r}}$$

where: t_i = time of failure of ith unit
r = number of failures

The test statistic, B, under the null hypothesis, has a chi-squared distribution with r-1 degrees of freedom.

Bartlett's Test for Exponential

If $\chi^2_{1-\frac{\alpha}{2},r-1} < B < \chi^2_{\frac{\alpha}{2},r-1}$ do not reject H_0

where $P\left\{\chi^2 < \chi^2_{1-\frac{\alpha}{2},r-1}\right\} = P\left\{\chi^2 > \chi^2_{\frac{\alpha}{2},r-1}\right\} = \frac{\alpha}{2}$

Thirty units were placed on test until 20 failures were observed. The following failure times were obtained in accelerated test hrs:

50.1	20.9	31.1	96.5	36.3	99.1	42.6	84.9	6.2	32.0
30.4	87.7	14.2	4.6	2.5	1.8	11.5	84.6	88.6	10.7

Bartlett's Test for Exponential

$$\sum_{i=1}^{20} t_i = 836.3 \quad ; \quad \sum_{i=1}^{20} \ln t_i = 63.93848$$

with r = 20:

$$B = \frac{2(20)\left[\ln(836.3/20) - \dfrac{63.93848}{20}\right]}{1 + \dfrac{20+1}{6(20)}} = 18.258$$

since

$$\chi^2_{.95,19} = 10.117 < B = 18.258 < \chi^2_{.05,19} = 30.144$$

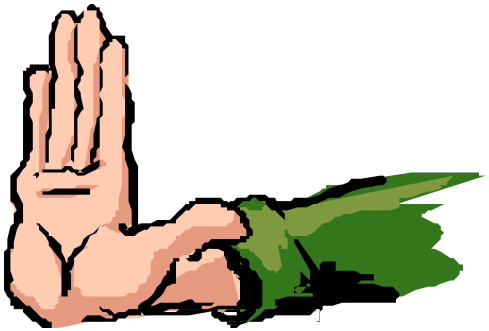
cannot reject H_0

Bartlett's Test (normal data)

$$B = \frac{100\left[\ln(17273.6/50) - \dfrac{291.8577}{50}\right]}{1 + \dfrac{50+1}{6x50}} = .663$$

$$B = 0.6630 < \chi^2_{.95,49} = 34.7$$

Reject the exponential distribution!



Mann's Test for the Weibull

H_0: The failure times are Weibull
H_1: The failure times are not Weibull

$$M = \frac{k_1 \sum_{i=k_1+1}^{r-1} \left[\frac{\ln t_{i+1} - \ln t_i}{M_i} \right]}{k_2 \sum_{i=1}^{k_1} \left[\frac{\ln t_{i+1} - \ln t_i}{M_i} \right]} \quad k_1 = int\left(\frac{r}{2} \right), \; k_2 = int\left(\frac{r-1}{2} \right)$$

$$M_i = Z_{i+1} - Z_i \quad \text{with} \quad Z_i = \ln\left[-\ln\left(1 - \frac{i-.5}{n+.25} \right) \right]$$

If $M > F_{crit}$, then H_1 is accepted.
Values for F_{crit} may be obtained from tables of the F-distribution where:
df for the numerator = $2k_2$,
df for the denominator = $2k_1$.

from Ex. 16.2:

Mann’s Test for the Weibull

t	ln t_i	M_i	n t_{i+1} - ln t_i	(ln t_{i+1}-ln t_i)/M_i
1.3	.2623642	1.108726	1.72551	1.5563
7.3	1.987874	.5211189	6.624937E-02	.1271291
7.8	2.054124	.3469455	.5336404	1.53811
13.3	2.587764	.2619765	4.412461E-02	.1684296
13.9	2.631889	.2115278	.3333843	1.576078
19.4	2.965273	.1781142	.01534557	.086155828
19.7	2.980619	.1543733	.1239684	.8030428
22.3	3.104587	.1366559	2.217364E-02	.1622589
22.8	3.126761	.1229487	.157903	1.2843
26.7	3.284664	.1120471	.1064837	.9503478
29.7	3.391147	.1031873	1.669478E-02	.161791
30.2	3.407842	9.586036E-02	5.476403E-02	.5712897

Therefore n = 50, r = 35, $k_1 = k_2 = 17$ and numerator = 352.3682 and denominator = 211.7246. M = 1.664 with 34 df for both the numerator and denominator.

Since M=1.664 < $F_{CRIT, .05, 34, 34}$, then H_0 is accepted.

Kolmogorov-Smirnov Test

Complete Samples Only!

H_0: The failure times are normal
H_1: The failure times are not normal
The test statistic is $D_n = \max\{ D_1, D_2 \}$, where

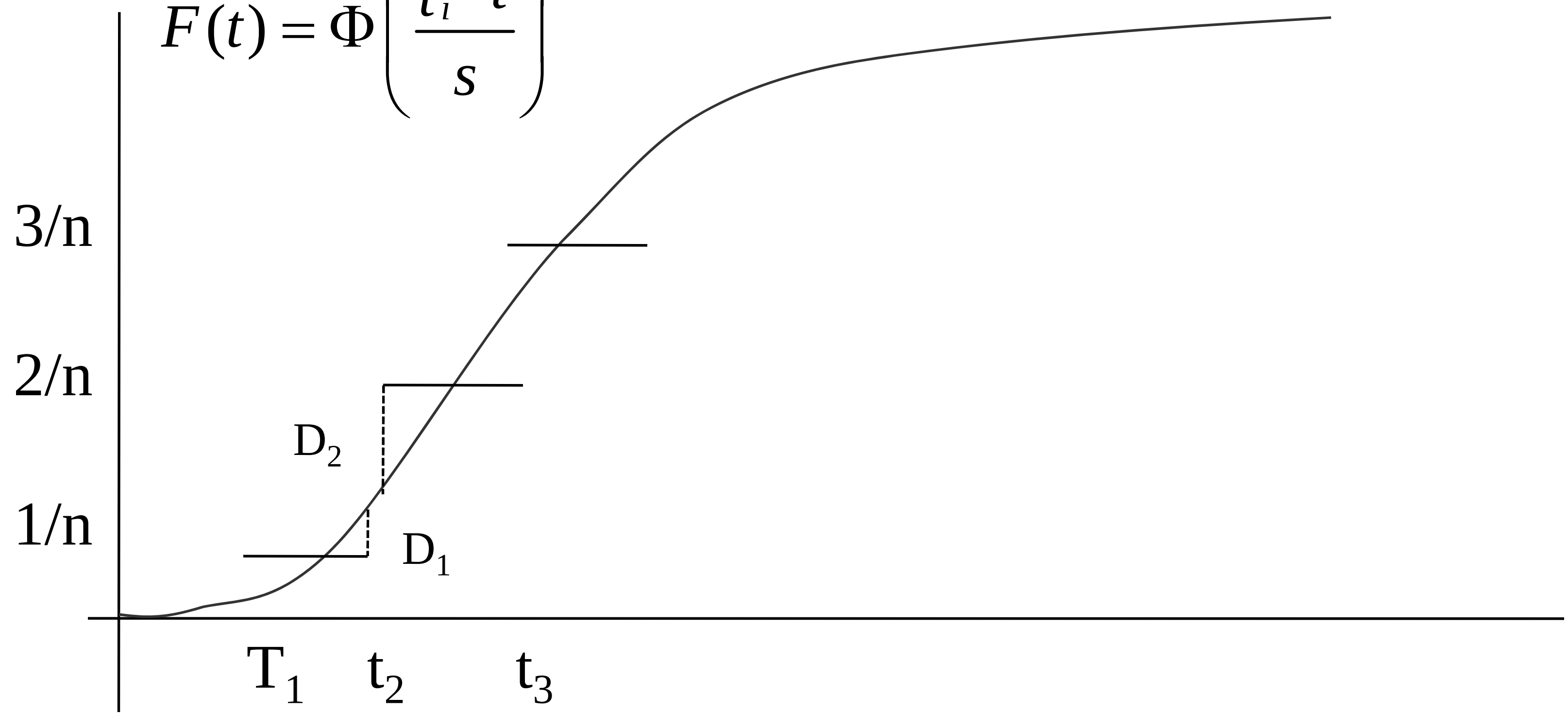
$$D_1 = \underset{1 \le i \le n}{\text{MAX}} \left\{ \Phi\left(\frac{t_i - \bar{t}}{s} \right) - \frac{i-1}{n} \right\} \qquad D_2 = \underset{1 \le i \le n}{\text{MAX}} \left\{ \frac{i}{n} - \Phi\left(\frac{t_i - \bar{t}}{s} \right) \right\}$$

$$\bar{t} = \sum_{i=1}^{n} \frac{t_i}{n} \qquad s^2 = \frac{\sum_{i=1}^{n} (t_i - \bar{t})^2}{n-1}$$

If $D_n < D_{crit}$, then accept H_0
If $D_n >= D_{crit}$, then accept H_1
The values for D_{crit} may be found in the Appendix

The Geometry of the K-S Test

$$F(t) = \Phi\left(\frac{t_i - \bar{t}}{s}\right)$$



$$D_1 = \underset{1 \le i \le n}{\mathrm{MAX}} \left\{ \Phi\left(\frac{t_i - \bar{t}}{s}\right) - \frac{i-1}{n} \right\} \qquad D_2 = \underset{1 \le i \le n}{\mathrm{MAX}} \left\{ \frac{i}{n} - \Phi\left(\frac{t_i - \bar{t}}{s}\right) \right\}$$

Kolmogorov-Smirnov Test

The following fifteen observations represent a sample of the repair times in hours of a complex piece of machinery. Test the hypothesis that the repair time is normal.

61.6	70.0	78.4	75.3	83.5
72.3	65.1	77.1	83.2	63.4
72.7	72.5	84.3	73.0	65.5

Rank ordering the data and computing the MLEs:

61.6	63.4	65.1	65.5	70.0
72.3	72.5	72.7	73.0	75.3
77.1	78.4	83.2	83.5	84.3

SAMPLE MEAN = 73.2 and SAMPLE STD DEV = 7.041221

H_0: Repair time is normal with mean = 73.2 and std dev =7.041
H_1: Repair time is not normal with mean =73.2 and std dev =7.041

(I-1)/N	I/N	CUM. PROB	D1(I)	D2(I)
0	6.666667E-02	.0495	.0495	0172
6.666667E-02	.1333333	.0823	.0156	.0511
.1333333	.2	.1251	-.0083	.0749
.2	.2666667	.1379	-.0621	.1288
.2666667	.3333334	.3264	.0597	.0070
.3333334	.4	.4483	.1149	-.0483
.4	.4666667	.4602	.0602	.0065
.4666667	.5333334	.4721	.0054	.0612
.5333334	.6	.4880	-.0453	.1120
.6	.6666667	.6179	.0179	.0488
.6666667	.7333334	.7088	.0422	.0245
.7333334	.8	.7704	.0370	.0297
.8	.8666667	.9222	.12222	-.0555
.8666667	.9333333	.9279	.0612	.0055
.9333333	1	.9430	.0096	.0570

MAX D_1 .1183821
MAX D_2 .1208173
K-S TEST STAT .1208173
SAMPLE SIZE 15

Since $D_{15} = 0.1208 < D_{CRIT,10} = .201$, then H_0 is accepted.

Kolmogorov-Smirnov Test - Lognormal

Given failure times $t'_1, t'_2, \ldots, t'_n$, then set $t_i = \ln t'_i$ and use Equations (16-4) where tbar and s are the sample mean and sample standard deviation of $t_1, t_2, \ldots, t_n$ respectively.

The time to failure of hose assemblies, due to structural fatigue and chemical breakdown, is believed to have a lognormal distribution. The following 25 failure times were obtained from environmental stress testing (complete data).

240.5	511.8	1083.4	821.3	1725.4	629.4	326.9	964.8	1677.8	282.3
652.3	639.2	1847.8	670.8	338.8	818.1	1407.5	4991.0	452.0	464.9
734.9	220.2	1078.1	1077.3	1773.0					

H_0: Failure times are lognormal with t_{MED}=765.426 and s=0.725

H_1: Failure times are not lognormal with t_{MED}=765.426 and s=0.725

Since $D_{25}=0.0756 < {}_{DCRIT, .10} = 0.165$, H_0 is accepted

Power-Law Process Model (AMSAA)

$\rho(t) = abt^{b-1}$, the hypotheses tested are:

H_0: The intensity function is constant (b = 1)

H_1: The intensity function is not constant ($b \neq 1$)

If b < 1, then system is improving (reliability growth)
if b > 1, then system is deteriorating (minimal repair)

Test statistic: $\chi^2 = \dfrac{2n}{\hat{b}}$ where $\hat{b}$ is the MLE for the AMSAA model

Under the null hypothesis, the test statistic has a chi-square distribution with 2N degrees of freedom (df) for Type I testing and 2(N-1) df for Type II testing.

The null hypothesis is rejected if $\chi^2 < \chi^2_{crit,1-a/2}$ or $\chi^2 > \chi^2_{crit,a/2}$

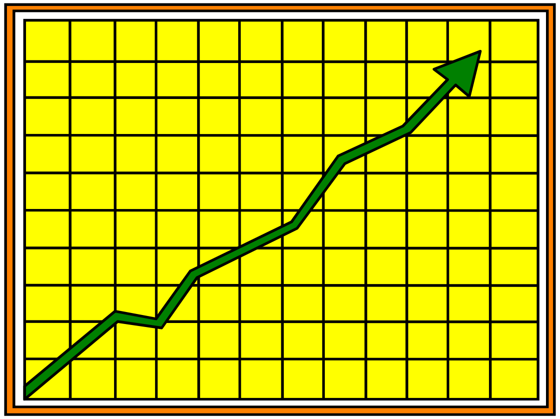
Trend Test - Example 14-4

N = 15 and the MLE for b = .28685.

Therefore, $\chi^2 = 30 / .28685 = 104.58$ has a chi-square distribution with 28 degrees of freedom.

$\chi^2_{crit,.95} = 16.928$ and $\chi^2_{crit,.05} = 41.337$.

Since $\chi^2 = 104.58 > 41.337$, then there is a significant trend present. Since the estimate for b < 1, there is significant growth.



Goodness-of-Fit Test

H_0: A nonhomogeneous Poisson process with intensity abt^{b-1}
H_1: The above process does not describe the data.

find an unbiased estimate for b:

$$\bar{b} = \begin{cases} \dfrac{n-1}{n}\hat{b} & \text{for time terminated data} \\ \dfrac{n-2}{n}\hat{b} & \text{for failure terminated data (or complete)} \end{cases}$$

test statistic:

$$C_M = \frac{1}{12M} + \sum_{i=1}^{M}\left[\left(\frac{t_i}{t_k}\right)^{\tilde{b}} - \frac{2i-1}{2M}\right]^2$$

$$M = \begin{cases} n & \textit{for TIME terminated data} \\ n-1 & \textit{for FAILURE terminated data} \end{cases} \qquad t_k = \begin{cases} T & \textit{for TIME terminated data} \\ t_n & \textit{for FAILURE terminated data} \end{cases}$$

T = total cumulative test time or total system observed time under time terminated data. Critical values are found in the Appendix.

GOF Test - Example 16.11 (14.3)

$$\tilde{b} = \frac{9}{10}(.6152685) = .5537416 \qquad M = 10$$

$$C_M = \frac{1}{12(10)} + \left[\left(\frac{5.6}{500}\right)^{.5537} - \frac{2-1}{2(10)} \right]^2 + \left[\left(\frac{18.8}{500}\right)^{.5537} - \frac{4-1}{2(10)} \right]^2$$

$$+ \ . \ . \ . \ + \left[\left(\frac{456.6}{500}\right)^{.5537} - \frac{20-1}{2(10)} \right]^2 = .01218$$

$C_{crit,\ .10} = .167.$

Since $C_M < .167$ then H_0 is accepted.

GOF Test - Example 16.12 (14.4)

$$\tilde{b} = \frac{13}{15}(.28685) = .24586 \qquad M = 14$$

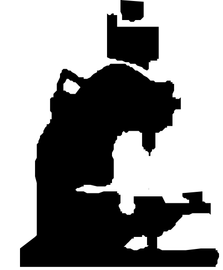
$$C_M = \frac{1}{12(14)} + \left[\left(\frac{3}{12035} \right)^{.2486} - \frac{2-1}{2(14)} \right]^2$$

$$+ \ . \ . \ . \ + \left[\left(\frac{8423}{12035} \right)^{.2486} - \frac{28-1}{2(14)} \right]^2 = 0.12714$$

$C_{crit,\ .10} = .169.$

Since $C_M < .169$, accept H_0.

Power-Law Process - Minimal Repair Example 16.13



The following failure times in working days were recorded on a numerical control (NC) machine (that has been operating for 916 days):

211, 287, 345, 456, 567, 631, 705, 784, 817, 856, 893, 916

$$\hat{a} = 8.51x10^{-}6, \; \hat{b} = 2.076, \; \rho(t) = 1.767x10^{-5}t^{1.076}$$

The Chi-square statistic for the trend test,

χ^2 = 24 / 2.076 = 11.56 < $\chi^2_{crit,.95}$ = 12.338 based on 22 df.

Therefore the hypothesis of a significant trend is accepted. Since the estimate of b > 1, the machine is deteriorating.

Power-Law Process - Minimal Repair Example 16.13

The goodness-of-fit test provided $C_M = .0239 < .172$ - the critical value at 10 percent level of significance.

As a result, the computed intensity function was accepted. After four years of usage (approximately 1000 working days), the MTBF of the machine is estimated to be:

$$MTBF = \frac{1}{\rho(1000)} = \frac{1}{1.767 x 10^{-5} (1000)^{1.076}} = 33.5 \text{ days}$$

GOF for grouped data

With grouped data as described in 14.4.2, the chi-square goodness-of-fit test can also be used to test the following hypotheses:

H_0: The nonhomogeneous Poisson Process with intensity ρ(t) describes the data

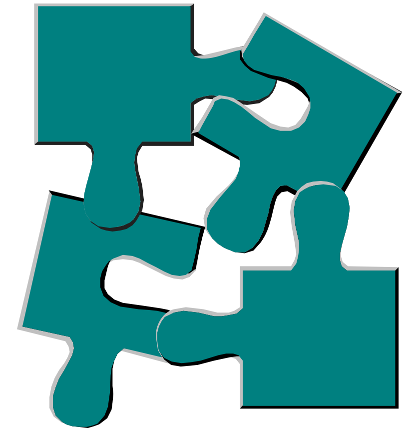
H_1: The above process does not describes the data

Letting $O_i = n_i$ = the observed count in the interval (t_{-i-1}, t_i) and

$$E_i = m(t_{i-1},\ t_i) = \int_{t_{i-1}}^{t_i} \rho(t)dt$$

Then $$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$ has a chi-square distribution with

k-2 degrees of freedom



Chapter 16
Statistical Tests

Chi-Square Test

Bartlett's Test

Mann's test

Kolmogorov-Smirnov Test

Tests for the Power-Law Process Model