Chapter 13
Reliability Testing

13.1 Product Testing

13.2 Reliability Life Testing

13.3 Test Time Calculations

13.4 Burn-in & screen testing

13.6 Accelerated life testing

Test Design

- Test objective
  - e.g. reliability demonstration, reliability improvement, screening
- Type of test
  - e.g. sequential, accelerated
- Operating & environmental conditions
- Number of units to be tested (sample size)
- Duration of test
  - failure terminated vs time terminated
- Definition of a failure

Cumulative Time on Test - CFR

n = nbr on test
r = nbr failures
k = nbr multiply censors
t_i = failure time
t_i^+ = censor time
t_* = test time (Type I)
t_r = test time (Type II)

MTTF = T / r

$$\text{Complete: } \sum_{i=1}^{n} t_i; \ r = n$$

$$\text{Type I: } \sum_{i=1}^{n} t_i + (n-r)t_*$$

$$\text{Type II: } \sum_{i=1}^{n} t_i + (n-r)t_r$$

$$\text{Type I multiply: } \sum_{i=1}^{n} t_i^+ + (n-r-k)t_*$$

$$\text{Type II multiply: } \sum_{i=1}^{n} t_i^+ + (n-r-k)t_r$$

$$\text{Type I replacement: } nt_*$$

$$\text{Type II replacement: } nt_r$$

Example 13.1

During a testing cycle, 20 units were tested for 50 hours with the following failure times and censor times observed:

10.8, 12.6+, 15.7, 28.1, 30.5, 36.0+, 42.1, 48.2

For Type I testing with $t^* = 50$ hours as the test termination time,

$$T = 10.8 + 12.6 + 15.7 + 28.1 + 30.5 + 36.0 + 42.1 + 48.2 + (20 - 6 - 2)\,50 = 824 \text{ hours}$$

Then MTTF = 824/6 = 137.3 hours.

Example 13.2

Ten units were placed on test with a failed unit immediately replaced. The test was terminated after the 8th failure which occurred at 20 hours.

This is type II testing with replacement. Therefore

$$T = (10)\ (20) = 200 \text{ hours}$$

$$MTTF = 200/8 = 25 \text{ hours}$$

Length of Test - Type II Testing - CFR

with replacement:

λ = failure rate of single unit

$n\lambda$ = system failure rate with n units operating

$1/(n\lambda)$ = MTTF/n = system MTTF

E(test time) = r x MTTF / n

Length of Test - Type II Testing - CFR

without replacement: generate r failures:

With n units on test: system MTTF = MTTF/n
With n-1 units on test: system MTTF = MTTF/(n-1)
With n-2 units on test: system MTTF = MTTF/(n-2)

With n-r+1 units on test: system MTTF = MTTF/(n-r+1)

$$E(test\ \ time) = MTTF\ \ x\ \ TTF_{r,n}\ \ =\ \ MTTF\left[\underbrace{\frac{1}{n} + \frac{1}{n-1} + \ldots + \frac{1}{n-r+1}}_{TTF_{r,n}}\right]$$

Example 13.3

To support the current cycle in a reliability growth testing program, a total of 8 failures need to be generated. The current estimate of the MTTF is 55 hours. The test department is scheduled to complete testing within 72 hours. How many units should be placed on test?

This is Type II testing. Since the length of the test is MTTF x TTF then the $TTF_{8,n}$ = 72/55 = 1.31. From the table,

TTF8,10 = 1.429
TTF8,11 = 1.187

n	r	TTFn,r
0	1	0.100
10	2	0.211
10	3	0.336
10	4	0.479
10	5	0.646
10	6	0.846
10	7	1.096
10	8	1.429
10	9	1.929
10	10	2.929
11	1	0.091
11	2	0.191
11	3	0.302
11	4	0.427
11	5	0.570
11	6	0.737
11	7	0.937
11	8	1.187
11	9	1.520
11	10	2.020
11	11	3.020

Example 13.4

For the problem in Example 13.3, the test department is told they must complete the testing within 48 hours. How many failures would they expect to generate?

$$E(r) = 11(1 - e^{-\frac{48}{55}}) = 6.4 \text{ units}$$

Burn-in Testing

- Goal is to increase the mean residual life
- Eliminate "early" customer failures by generating test hours in the factory
- There must be a dominate DFR failure mode
- Primary question is how long to test?
  - Specification model
  - cost model

Burn-in Testing Specification Model

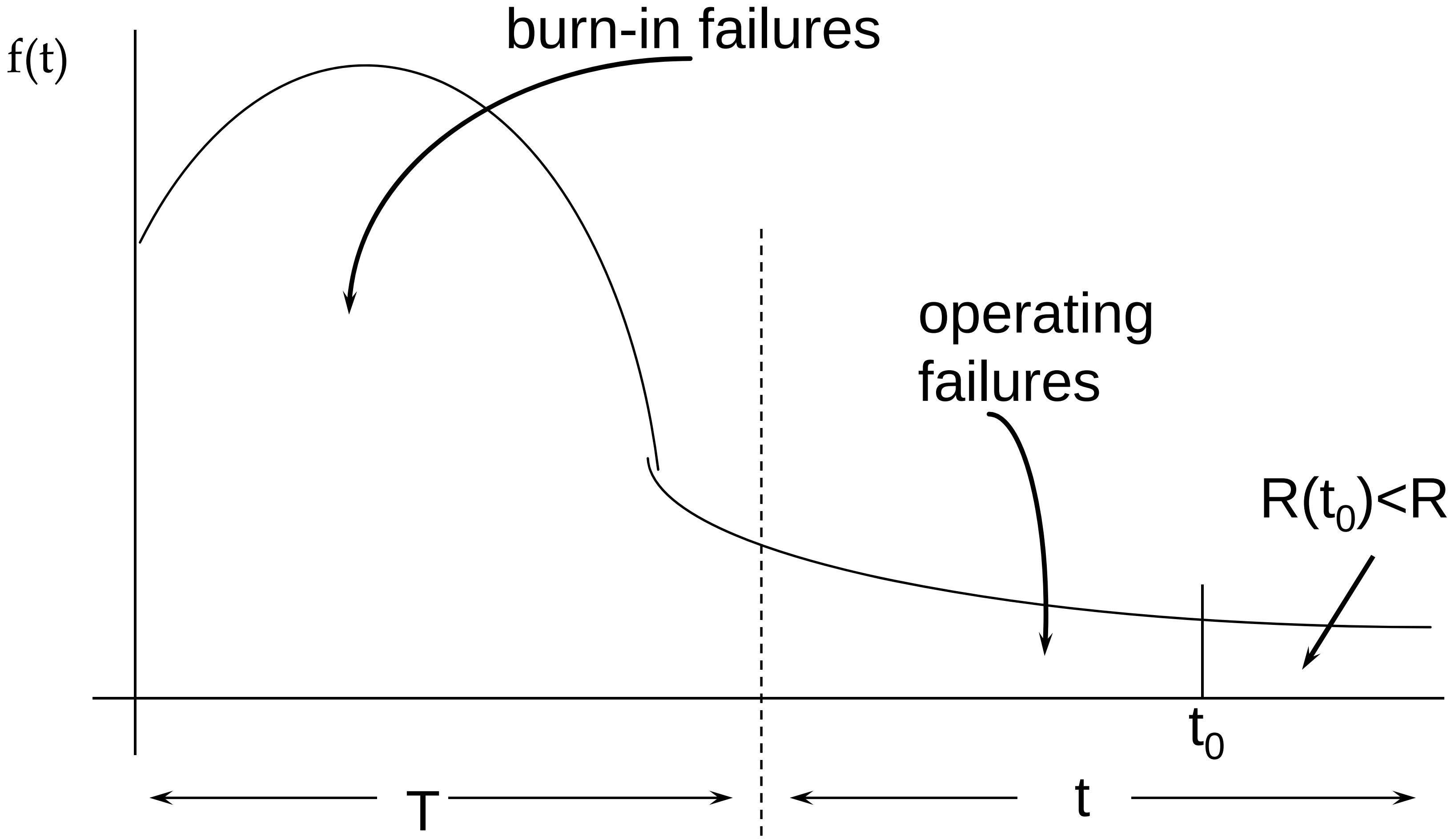
Given a reliability goal of R_0 where $R(t_0) < R_0$ and $R(t)$ has a DFR, then a burn-in period, T, is desired such that $R(t_0 \mid T) = R_0$.

For the Weibull distribution

$$R(t_0|T) = \frac{e^{-\left(\frac{t_0+T}{\theta}\right)^{\beta}}}{e^{-\left(\frac{T}{\theta}\right)^{\beta}}} = R_0$$

$$e^{-\left(\frac{t_0+T}{\theta}\right)^{\beta}} - R_0 e^{-\left(\frac{T}{\theta}\right)^{\beta}} = 0$$

Burn-in Testing



Example 13.5

Reliability testing has shown that a ground power unit used to supply DC power to aircraft has a Weibull distribution with beta = .5 and theta = 45,000 operating hours. Determine a burn-in period necessary to obtain a required reliability specification of R(1000) = .90.

Observe that R(1000) = .86 and beta< 1.

$$e^{-\left(\frac{1000+T}{45000}\right)^{.5}} - .90e^{-\left(\frac{T}{45000}\right)^{.5}} = 0$$

T^* = 126 hr. and R(1000|126)= 0.90

Burn-in Testing - Cost Model

C_b = cost per unit time for burn-in testing
C_f = cost per failure during burn-in
C_o = cost per failure when operational
T = length of burn-in testing
t = operational life of the units

$$E[C(T)] = C_b T + C_f [1 - R(T)] + C_o [R(T) - R(t+T)]$$

for the Weibull:

$$E[C(T)] = C_b T + C_f \left[1 - e^{-\left(\frac{T}{\theta}\right)^\beta}\right] + C_o \left[e^{-\left(\frac{T}{\theta}\right)^\beta} - e^{-\left(\frac{T+t}{\theta}\right)^\beta}\right]$$

Example 13.6

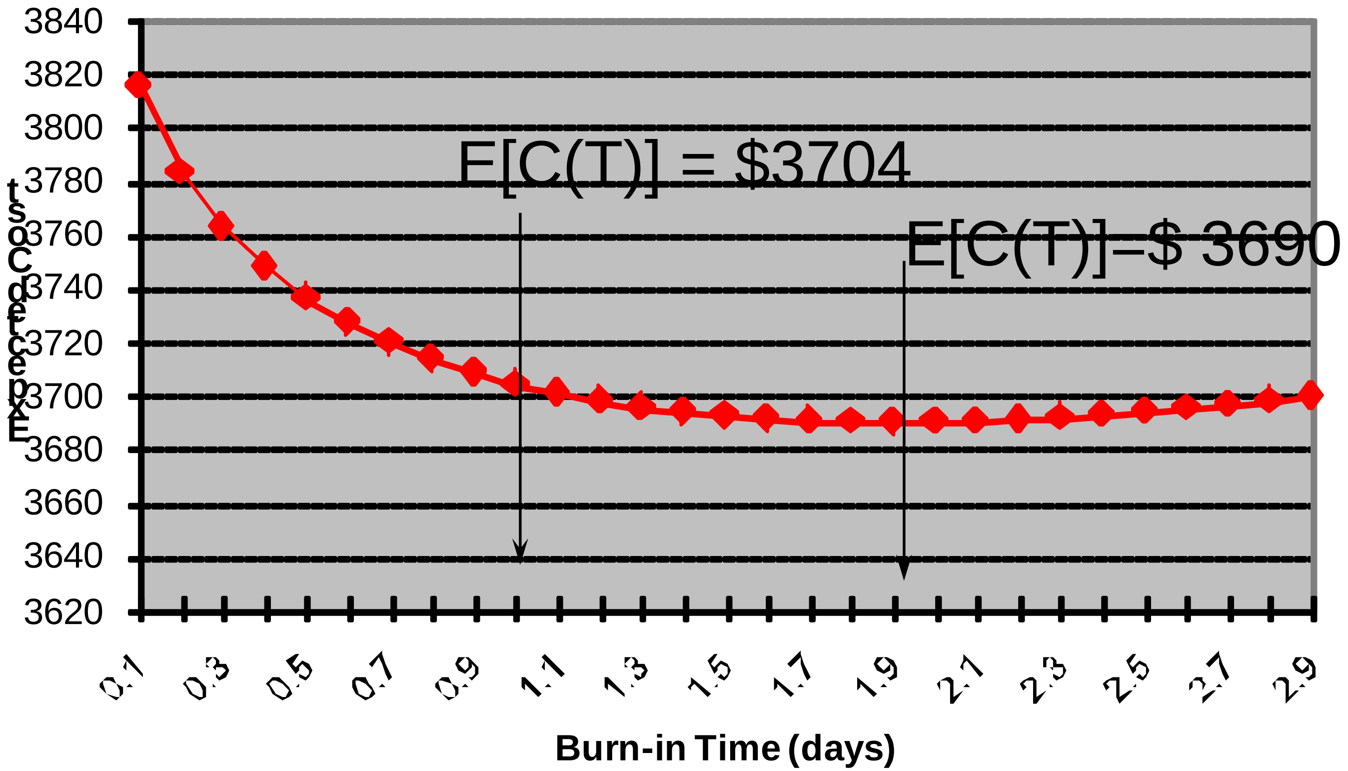
The replacement cost on a new product if it fails during its operational life of 10 years (3650 days) is $6200. It will cost the company $70 a day per unit tested to operate a burn-in program and any failures during burn-in will cost $500. Reliability testing has established the life distribution of the product to be Weibull with beta = .35, and theta = 3500 days. What is the minimum cost time period for the burn-in?

$$E[C(T)] = 70T + 500[1 - e^{-\left(\frac{T}{3500}\right)^{.35}}] + 6200\,[e^{-\left(\frac{T}{3500}\right)^{.35}} - e^{-\left(\frac{T+3650}{3500}\right)^{.35}}]$$

T^* = 1.9 days with E[C(T)] = $ 3690

Example 13.6

T = 0, E[C(T)] = $3952



Accelerated Life Testing

Problem: test time < expected lifetimes

Solution:

- Increase the number of units on test (compressed time)
- Accelerate the number of cycles per unit of time
- Increase the stresses that generate failures (accelerated stress testing)

Increase Number of Units on Test

For CFR: $$f_{r,n} = \frac{TTF_{n,r}}{TTF_{r,r}}$$ without replacement

$$f_{r,n} = r/n$$ with replacement

then

$f_{r,n}$ is the factor reduction in expected test time

$100(1-f_{r,n})$ is the percent savings in expected test time

Increase Number of Units on Test

for Weibull:

$$f_{r,n} = \left(\frac{TTF_{n,r}}{TTF_{r,r}} \right)^{1/\beta}$$

without replacement

$$f_{r,n} = \left(\frac{r}{n} \right)^{1/\beta}$$

with replacement

then

$f_{r,n}$ is the factor reduction in expected test time

$100(1-f_{r,n})$ is the percent savings in expected test time

Example 13.10

Given n = 15 and r = 8:

	without replacement	with replacement
CFR model:	$f_{8,15} = TTF_{8,15} / TTF_{8,8}$ $= .725/2.718 = .2667$	$f_{8,15} = 8/15 = .533$
Weibull (Beta = 2)	$f_{8,15} = (.725/2.718)^{1/2}$ $= .516$	$f_{8,15} = (8/15)1/2 = .730$

replacing vs not replacing failed units: $\dfrac{r}{nTTF_{r,n}} = \dfrac{8}{15(0.725)} = .7356$

Accelerated Cycling

for products that do not operate continuously or nearly continuously

x_n = the nbr of cycles per unit of time under normal cycling,
x_s = the nbr of cycles per unit of time under accelerated cycling,
t_n = time to failure under x_n cycles per unit of time
t_s = time to failure under x_s cycles per unit of time

$x_n\, t_n = x_s\, t_s$
e.g. 10 cycles/hr x 100 hrs = 50 cycles/hr x 20 hrs = 1000 cycles

$$R_n(t_n) = R_s(t_s) = R_s\left(\frac{x_n}{x_s} t_n\right)$$

An Observation

Let T_s = a random variable, the time to failure under accelerated cycling
T_n = a random variable, the time to failure under normal cycling

$$T_s = \frac{x_n}{x_s} T_n \quad and \quad E\left[T_s\right] = E\left[\frac{x_n}{x_s} T_n\right] = \frac{x_n}{x_s} E\left[T_n\right] = \frac{x_n}{x_s} \mu_n$$

$$Var\left[T_s\right] = Var\left[\frac{x_n}{x_s} T_n\right] = \left(\frac{x_n}{x_s}\right)^2 Var\left[T_n\right] = \left(\frac{x_n}{x_s}\right)^2 \sigma_n^2$$

Accelerated Cycling

for Weibull:

$$R_n(t_n) = e^{-\left(\frac{t_n}{\theta_n}\right)^{\beta n}} = e^{-\left(\frac{t_s}{\theta_s}\right)^{\beta s}} = e^{-\left(\frac{x_n t_n}{x_s \theta_s}\right)^{\beta s}}$$

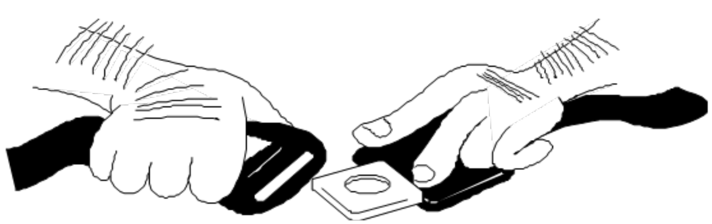
$$B_s = B_n = B \; and \;\; \theta_n = \frac{x_s}{x_n}\theta_s$$

Example 13.11

At automotive part was tested at an accelerated cycling level of 100 cycles per hour. The resulting failure data was found to have a Weibull distribution with beta = 2.5 and theta$_s$ = 1000 hours.

If the normal cycle time is 5 per hour, then

$$\theta_n = \frac{100}{5} 1000 = 20{,}000\, hr \quad and \quad R_n(t) = e^{-\left(\frac{t}{20000}\right)^{2.5}}$$



Accelerated Cycling and the Normal Distribution

$$R_n(t_n) = 1 - \Phi\left(\frac{t_n - \mu_n}{\sigma_n}\right) = 1 - \Phi\left(\frac{t_s - \mu_s}{\sigma_s}\right)$$

$$= 1 - \Phi\left(\frac{\frac{x_n}{x_s} t_n - \mu_s}{\sigma_s}\right) = 1 - \Phi\left(\frac{t_n - \frac{x_s}{x_n}\mu_s}{\frac{x_s}{x_n}\sigma_s}\right)$$

therefore: $\mu_n = \frac{x_s}{x_n}\mu_s$ and $\sigma_n = \frac{x_s}{x_n}\sigma_s$

Accelerated Cycling and the LogNormal Distribution

$$R_n(t_n) = 1 - \Phi\left(\frac{1}{s_n}\ln\frac{t_n}{t_{med,n}}\right) = 1 - \Phi\left(\frac{1}{s_s}\ln\frac{t_s}{t_{med,s}}\right)$$

$$= 1 - \Phi\left(\frac{1}{s_s}\ln\frac{\frac{x_n}{x_s}t_n}{t_{med,s}}\right) = 1 - \Phi\left(\frac{1}{s_s}\ln\frac{t_n}{\frac{x_s}{x_n}t_{med,s}}\right)$$

therefore: $t_{med,n} = \dfrac{x_s}{x_n}t_{med,s}$ and $s_n = s_s$

Constant Stress Models

Basic assumption: same failure mechanisms will be present at the higher stress levels and will act in the same manner as at normal stress levels.

t_n = time to failure under normal stress
t_s = time to failure at high stress level

then $t_n = AF \times t_s$ where AF is an acceleration factor

Therefore

$$\Pr\{T_n < t_n\} = F_n(t_n) = \Pr\{T_s < t_s\} = F_s(t_n / AF)$$

Failure Mechanisms and Stresses

Failure Mechanism	Accelerated Stresses
Thermal crack propagation	Temperature, dissipated power
Corrosion	Temperature and humidity
Cyclical (mechanical) fatigue	Cyclical vibration (stress amplitude and load frequency)
Thermal fatigue	Temperature cycling
Electro migration	Current density & temperature
Stress-strength	Critical load and load frequency
Adhesive or abrasive wear	Loading, surface area, duration

Test plans:

1. What type of stresses?
2. What stress levels?
3. What number to test at each stress level?

Example 13.12

For the CFR model, a component is tested at 120°C and found to have an MTTF = 500 hours. Normal use is at 25°C. Assuming AF = 15, determine the components MTTF and reliability function at normal stress levels.

$$F_n(t) = F_s\left(\frac{t}{AF}\right) = 1 - e^{-\lambda_s\left(\frac{t}{AF}\right)} = 1 - e^{-\frac{t}{500x15}}$$

$$R(t) = e^{-\frac{t}{7500}}$$ and MTTF = 7500 hr.

Weibull Case

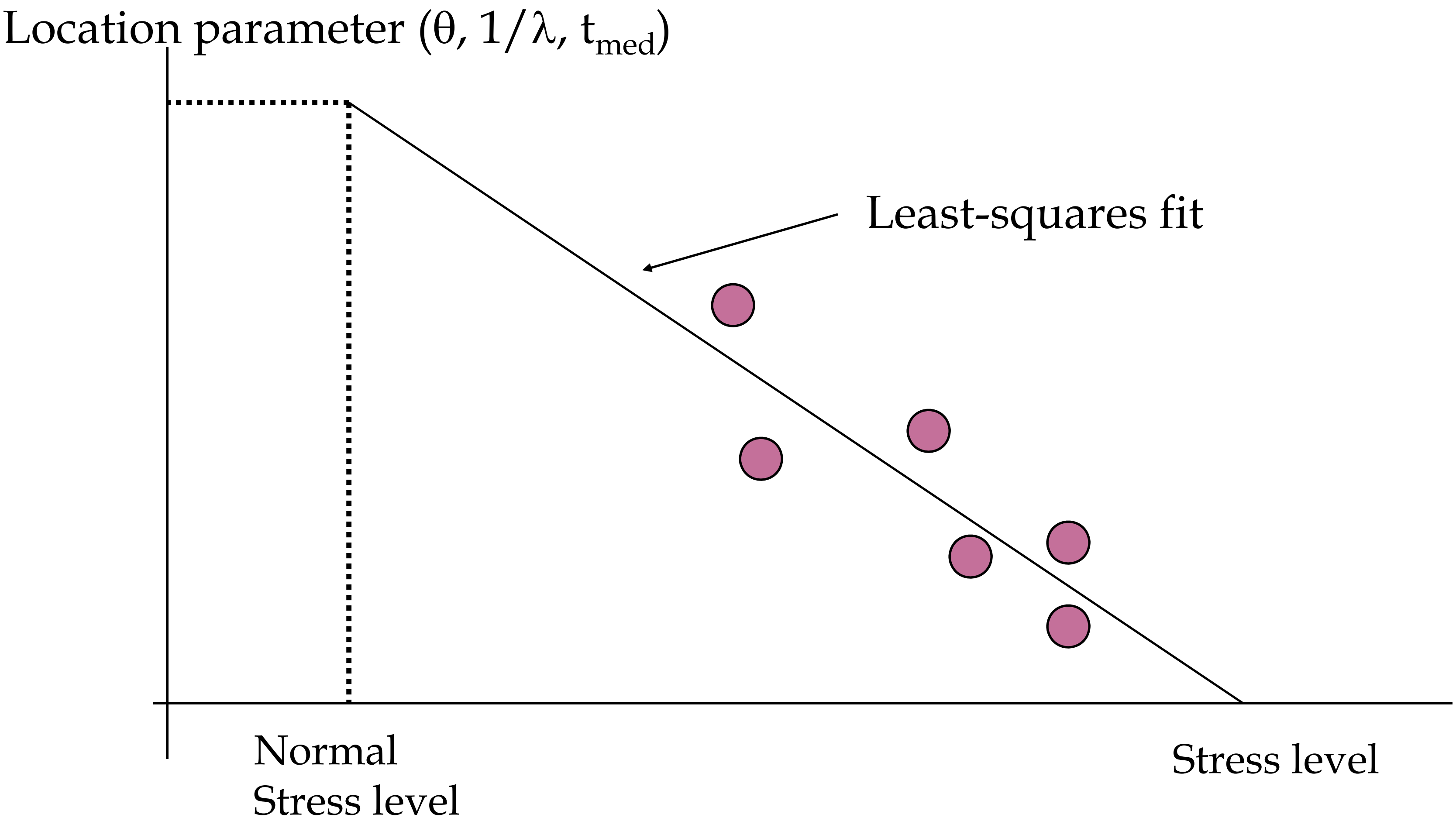
$$F_s(t) = 1 - e^{-\left(\frac{t}{\theta_s}\right)^{\beta_s}}$$

$$F_n(t) = 1 - e^{-\left(\frac{t}{AF\,\theta_s}\right)^{\beta_s}}$$

$$\theta_n = AF x\, \theta_s \quad \text{and} \quad \beta_n = \beta_s.$$

estimate AF by: $AF = \dfrac{\hat{\theta}_n}{\hat{\theta}_s}$

An Approach


Location parameter (θ, $1/\lambda$, t_{med})
Least-squares fit
Normal
Stress level
Stress level

Example 13.15

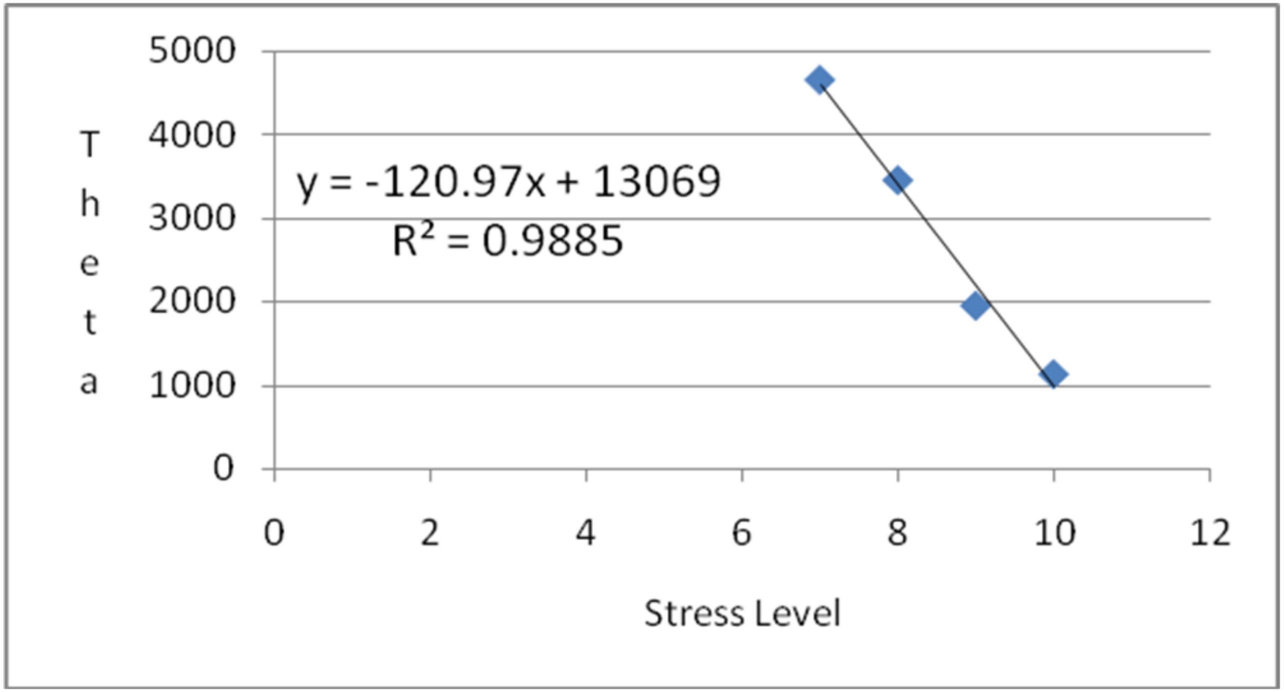
Accelerated life testing for fracture stress failures at high temperature was conducted on 40 units at four different accelerated stress levels with the following results in hours:

stress level*	7	8	9	10
sample	time to fail	time to fail	time to fail	time to fail
1.0	4355	1677	784	1067
2.0	4951	4707	1813	681
3.0	7503	2407	2240	1339
4.0	1475	1367	2731	1151
5.0	4703	5709	1533	1436
6.0	5002	2976	1639	507
7.0	4091	2343	1979	1174
8.0	2514	2879	1612	655
9.0	4775	3551	1233	1156
10.0	1898	2804	1705	773

*stress levels are measured in psi

Example 13.15

stress level	7	8	9	10
β	2.7	2.5	3.2	3.1
θ	4647	3445	1938	1117



Example 13.15

At the normal stress level of 2 psi, θ = 12,827 hr. Using the average of the four β values, the reliability model at the normal stress level (2 psi) becomes:

$$\theta = y = -120.97(2) + 13069 = 12,827$$

$$\beta = \frac{2.7 + 2.5 + 3.2 + 3.1}{4} = 2.875$$

$$R(t) = e^{-\left(\frac{t}{12,827}\right)^{2.875}}$$

Nonlinear stress effects

Assume $t_n = kt_s^m$, m ≠ 1.

$$F_\mathrm{n}\left(t_\mathrm{n}\right)=1-\exp\left[-\left(\frac{t_\mathrm{n}}{\theta_\mathrm{n}}\right)^{\beta\mathrm{n}}\right]=1-\exp\left[-\left(\frac{t_n^{1/m}}{k\cdot\theta_\mathrm{s}}\right)^{\beta_\mathrm{s}}\right]=1-\exp\left[-\left(\frac{t_n}{\left(k\cdot\theta_s\right)^m}\right)^{\beta_\mathrm{s}/m}\right]$$

both the scale and shape parameters change

Arrhenius Model

used when increased temperature is the applied stress

$$r = Ae^{-\frac{B}{T}}$$

where r is the reaction or process rate, A and B are constants, and T is temperature measured in degrees Kelvin

$$AF = \frac{Ae^{-\frac{B}{T_2}}}{Ae^{-\frac{B}{T_1}}} = e^{B\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}$$

$$B = \frac{\ln AF}{\left(\frac{1}{T_1}-\frac{1}{T_2}\right)} \quad where \quad AF = \frac{\theta_1}{\theta_2}$$

Example 13.14

An electronic component has a normal operating temperature of 294° K (about 21° C). Under stress testing at 430° K a Weibull distribution was obtained with theta = 254 hours, and at 450° K., a Weibull distribution was obtained with theta = 183 hours.
The shape parameter did not change with beta = 1.72.

$$B = \frac{\ln\left(\frac{254}{183}\right)}{\frac{1}{430} - \frac{1}{450}} = 3172$$

normal temperature

$$AF = e^{3172\left(\frac{1}{294} - \frac{1}{450}\right)} = 42.1$$

Weibull with a shape parameter of 1.72
and θ = 42.1 x 183 = 7704.3 hours.

Eyring Model

$$r = AT^{a}e^{-\frac{B}{T}}e^{CS}$$

A, α , B, and C are constants to be estimated

$$AF = \left(\frac{T_2}{T_1}\right)^{a} e^{B\left(\frac{1}{T_1}-\frac{1}{T_2}\right)} e^{C(S_2-S_1)}$$

Degradation Models

$y = a - b\,t$, where y is the performance measure, a and b are constants to be determined experimentally, and t is the amount of time the product is exposed at a constant stress level

time to failure, t_f :

$$t_f = \frac{a - y_f}{b}$$

where y_f is the performance level at which a failure occurs

Example 13.15

$$CPR = \frac{kw(t)}{\rho At}$$

t = exposure time in hours,
w(t) = weight loss due to corrosion after t hours exposure in mg
= density of the material in grams per cubic centimeter
A = exposed surface area in square centimeters
k = 87.6, a constant which converts CPR to millimeters per year

If l_f is the allowable loss in millimeters after which the material is no longer structurally sound, then the time to failure is projected to be

$$t_f = l_f / CPR$$

Example 13.16

$$p = e^{-rt}$$

where p = potency of the drug
r = rate of chemical reaction
t = drug exposure time

then t = - ln p / r

$$r = Ae^{-\frac{B}{T}} \textit{ then } t = \frac{-\ln p}{Ae^{-B/T}}$$

Cumulative Damage Models

Minor's rule:

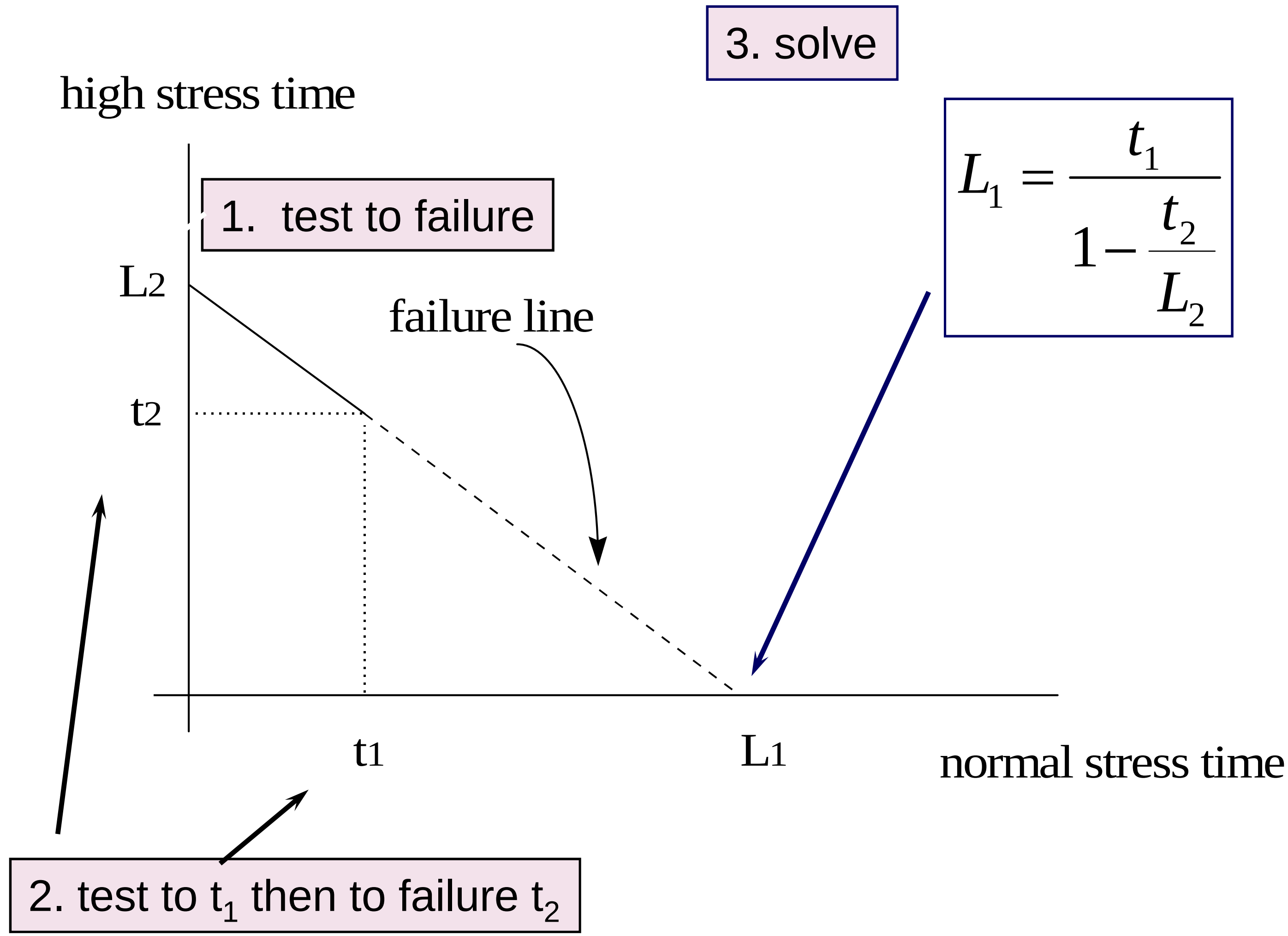
$$\sum_{i=1}^{n} \frac{t_i}{L_i} = 1$$

t_i = the amount of time at stress level i

L_i = the expected lifetime at stress level i

$$\frac{t_1}{L_1} + \frac{t_2}{L_2} = 1 \; or \; t_2 = L_2 - \frac{L_2}{L_1} t_1$$

Cumulative Damage Models



The End of Chapter 13