



Chapter 7

Physical Reliability Models

Periodic Loads

Random Loads

Physics of Failure



Reliability just has to be
my favorite subject!



Periodic Loads

X_i , Y_i = the load and strength observed on the i th cycle

After n cycles, the reliability, R_n is found from

$$R_n = P(X_1 < Y_1, X_2 < Y_2, \dots, X_n < Y_n)$$

$$= P(X_1 < Y_1) P(X_2 < Y_2) \dots P(X_n < Y_n)$$

assuming independent load and strength applications each cycle.

Since $\Pr\{X_i < Y_i\} = R$, then $R_n = R^n$

Building in Baltimore surviving 10 years: $R(10) = .98214^{10} = .8351$



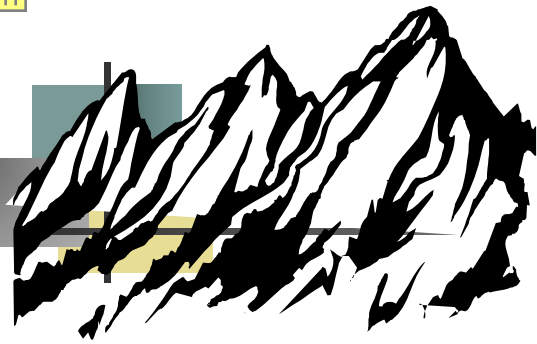
Periodic Loads

$$R(t) = R^n \quad \text{for } t_n \leq t < t_{n+1}$$

With $t_0 = 0$ and uniformly spaced load times,

$$R(t) = R^{\lfloor t / \Delta t \rfloor} \quad \text{where } \lfloor z \rfloor \text{ is the integer portion of } z$$

and $\Delta t = t_{i+1} - t_i$



Dynamic Model – Example 1

A aerial tramway takes sightseers to the top of a scenic mountain twice a day (two round trips). The tramway has a capacity of 10 people, and it is usually full during the tourist season. The cabling system on the tramway has a design strength of 1900 pounds.

If the average weight of an individual is 170 pounds with a standard deviation of 20 pounds, determine the reliability of the tramway over the 90 day tourist season.



Dynamic Model – Example 1- solution

Assume the sum of the weights are normally distributed with a mean of 1700 pounds (10×170) and a variance of 4,000 pounds (10×20^2).

$$R = \Phi\left(\frac{k - \mu_x}{\sigma_x}\right) = \Phi\left(\frac{1900 - 1700}{63.24}\right) = \Phi(3.16) \approx .99921$$

With t in days: $R(t) = R^{2t} = (.99921)^{2t}$

$$R(90) = R^{2(90)} = (.99921)^{2(90)} = .8674$$



Random Loads

Let N = a RV, the number of loads per unit of time.

Then the probability of n loads in time t is given by:

a Poisson process \longrightarrow
$$P_n(t) = \frac{(\alpha t)^n e^{-\alpha t}}{n!}$$

Where α is the mean number of loads per unit of time.



Random Loads

$$R(t) = \sum_{n=0}^{\infty} R^n P_n(t) = \sum_{n=0}^{\infty} R^n \left[\frac{(\alpha t)^n e^{-\alpha t}}{n!} \right]$$

$$= e^{-\alpha t} \sum_{n=0}^{\infty} \frac{(\alpha t R)^n}{n!}$$

$$= e^{-(1-R)\alpha t}$$

$$\text{using } \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$



MTTF for Random Loads

$$\int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-(1-R)\alpha t} dt = \frac{1}{(1-R)\alpha}$$

Exponential time to failure with $\lambda = (1-R)\alpha$



Dynamic Example #2

The maximum wind velocity during spring thunderstorms in Dayton has a lognormal distribution with a median velocity of 34 mph and a shape parameter of 1.2. A canvas tent to be used by the Pi Mu fraternity for their long anticipated St. Patrick's Day weekend celebration has a design strength against high winds which is lognormal with a median value of 40 mph and a shape parameter of .9. The number of thunderstorms in Dayton during the month of March has a Poisson distribution with a mean rate of 6.2. If the "big bash" begins at 4:00 pm on Friday and ends at midnight on Sunday, what is the reliability of the tent? What is the MTTF in days (Pi Mu may decide to keep partying)?



Dynamic Example #2 - solution

$$R = \Phi \left(\frac{\ln(40/34)}{\sqrt{.9^2 + 1.2^2}} \right) = \Phi \left(\frac{.1625}{1.5} \right) = \Phi(.108) = .5438$$

$$\alpha = 6.2/31 = .2 \text{ per day} = .008333 \text{ per hr.}$$

$$R(t) = \exp(- [1-.5438] \times .008333 \times t)$$

$$R(56 \text{ hr.}) = \exp(-.0038 \times 56) = \exp(-.2129) = .8082$$

$$\text{MTTF} = 1/.0038 = 263.16 \text{ hrs} = 10.96 \text{ or } 11 \text{ days}$$



Thin-walled Containers

The internal (tangential) stress (s) on the walls of a thin-walled cylinder which tend to pull the wall apart is given by

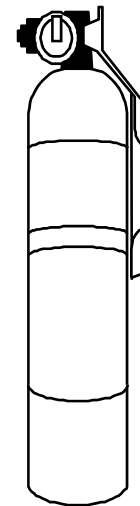
$$s = \frac{p D}{2 t}$$

s is in psi

p = internal pressure (psi)

D = mean diameter of the cylinder (in.)

t = thickness of the cylinder wall (in.)





Thin-walled Containers - Example part A

A cylindrical tank is being designed to hold nitrogen at an internal pressure which is normally distributed with a mean of 290 psi and a standard deviation of 50 psi. If the cylinder having a mean diameter of 25 in. and a wall thickness of 1/16 of an inch is made of steel having an ultimate tensile strength (bursting strength) of 86,200 psi, what is the static reliability of the tank?

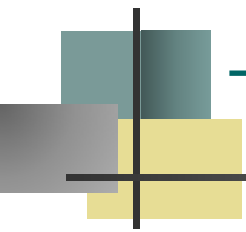
$$s = pD/2t$$

$$E(s) = (25 \times 290)/(2 \times 1/16) = 58,000 \text{ psi and}$$

$$\text{STD}(s) = (25 \times 50)/(2 \times 1/16) = 10,000 \text{ psi}$$

$$R = \Pr\{s < 86,200\} = \Phi [(86,200 - 58,000)/10,000]$$

$$= \Phi[2.82] = .9976$$



Thin-walled Containers - Example part B

If the tensile strength of the steel is also normally distributed with a mean of 86,200 psi and a standard deviation of 3,900 psi, what is the static reliability of the tank?

$$R = \Phi\left(\frac{86,200 - 58,000}{\sqrt{3900^2 + 10^8}}\right) = \Phi(2.63) = .99573$$



Thin-walled Containers - Example part C

Based upon the static reliability found in part (a), if the tank is emptied and refilled randomly at the mean rate of 10 times a year, what is the dynamic reliability of the tank over a 5 year period?

$$\alpha = 10 \text{ per yr}; \quad 1-R = .0024;$$

$$\text{therefore } R(t) = \text{Exp}[-.0024 (10) t]$$

$$\text{and } R(5) = e^{-.12} = .887$$



Thin-walled Containers - Example part D

If in part (b) a static reliability of .999 is desired, determine the thickness of the cylinder wall.

Since $\Phi(3.09) = .999$, then

$$\left(\frac{86,200 - \frac{(.5)(25)(290)}{t}}{\sqrt{3900^2 + [(.5)(25)(50)]^2 / t^2}} \right) = 3.09$$

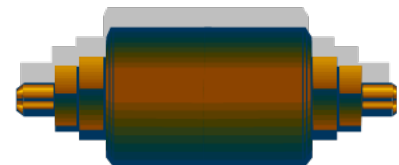
$$86,200 - \frac{(.5)(25)(290)}{t} = 3.09 \sqrt{3900^2 + [(.5)(25)(50)]^2 / t^2}$$

squaring both sides, multiplying through by t^2 , and using the quadratic formula results in $t = .019484$ and $.0663$ with $t = .0663$ inches as the desired solution.



Example 3

- A filtration device is designed to capture contaminants from entering a mechanical pump whenever an overflow occurs in a distribution system for crude oil. The filter system must be replaced after each use.
- Laboratory tests on the filtration device has determined that its design strength is gamma with $\alpha = 45$ psi and $\gamma = 2.2$.
$$\mu = \alpha \gamma = 99 \text{ psi}$$
- The force of the overflow on the filtration device has been measured to be exponential with a $\mu = 21$ psi.
- Safety Factor = $\mu_y / \mu_x = 99/21 = 4.714$





More Example 3

The static reliability is:

$$R = 1 - \left(\frac{\mu}{\mu + \alpha} \right)^{\gamma} = 1 - \left(\frac{21}{21 + 45} \right)^{2.2} = .9195$$

If overflows occur periodically four times a year when at the beginning of each quarter, excess oil is pumped through the system, then

$$R(t) = (.9195)^{\text{integer}(4t)}$$

$$R(2 \text{ years}) = (.9195)^8 = .511$$



More Example 3

The static reliability is:

$$R = 1 - \left(\frac{\mu}{\mu + \alpha} \right)^{\gamma} = 1 - \left(\frac{21}{21 + 45} \right)^{2.2} = .9195$$

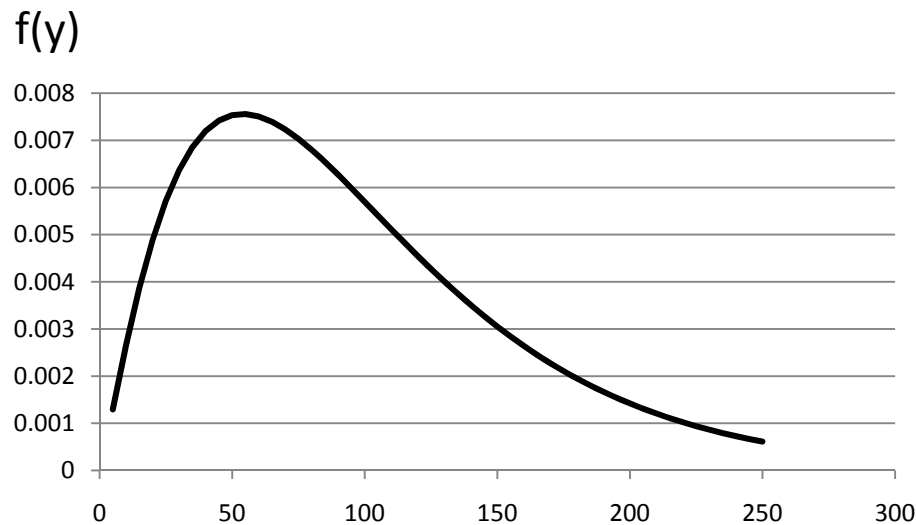
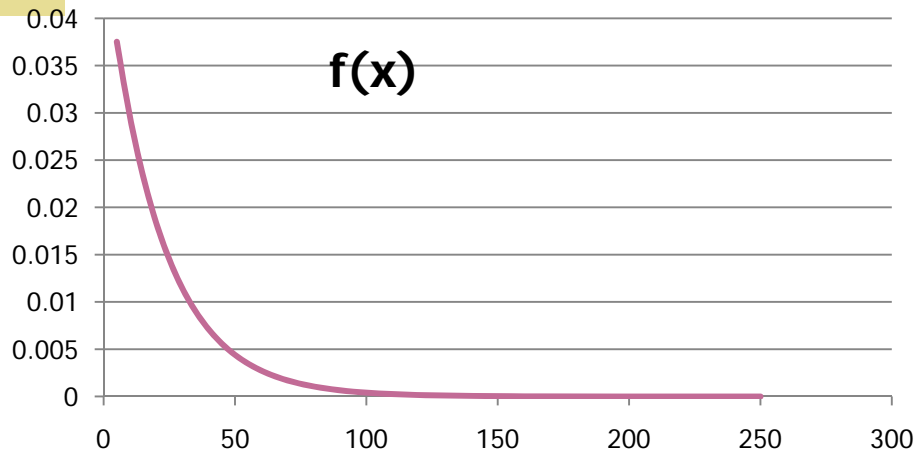
If overflows occur randomly (Poisson) at a mean rate of four times a year, then

$$R(t) = e^{-(1 - .9195)(4)t}$$

$$R(2) = e^{-(1 - .9195)(4)(2)} = .5252$$

$$MTTF = \frac{1}{(1 - .9195)(4)} = 3.106$$

The very useful Chapter 7 spreadsheet



load frequency is once every	0.25	time periods
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Exponential	μ
value	21

Gamma	γ	α
value	2.2	45

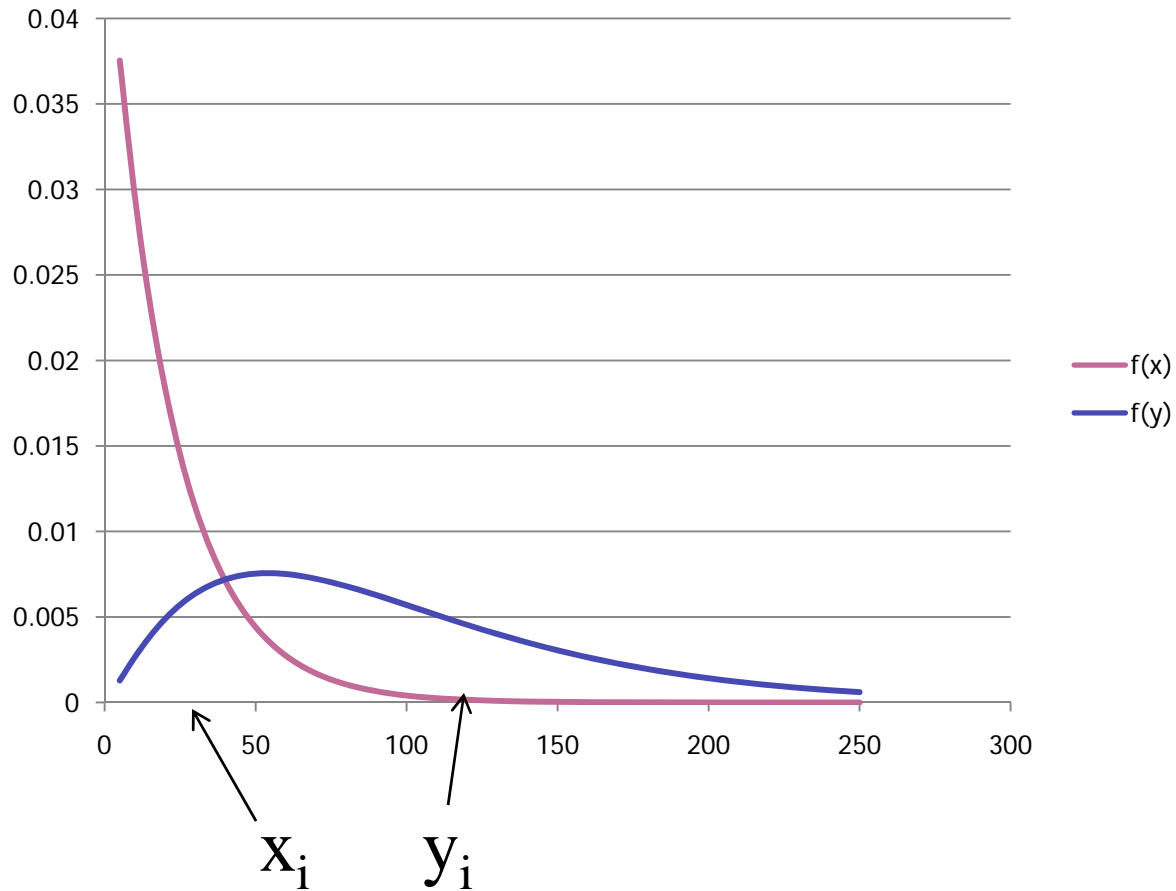
0.91948

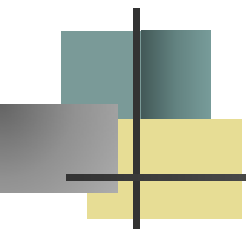
periodic loads	random loads	time periods
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MTTF = 3.10

0.714785	0.72465
0.510918	0.52512
0.365196	0.38053
0.261037	0.27575
0.186585	0.19982

Interference





Physics of Failure (POF)

An approach to reliability which utilizes knowledge of the failure mechanisms to prevent failures through robust design and manufacturing practices.

POF models are derived for each failure mode from design information, experimental data, engineering analysis, and physical and chemical laws.

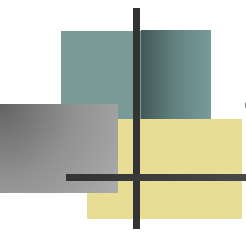
Basic premise: to eliminate failures, it is necessary to find and eliminate their root causes. To do this, the underlying failure mechanisms as a function of time, stresses, environment, and material properties of components must be found.



More Physics of Failure (POF)

Assumption - failure processes follow physical (or chemical or metallurgical) laws that can be modeled and the amount of life available for exposure to a given stress condition can then be calculated. From this a failure free operating period is determined.

POF are physically based rather than statistically based!



Even More Physics of Failure (POF)

The Process

1. Identify failure mechanisms, failure sites, and failure modes.
2. Determine appropriate failure models and their input parameters.
Material characteristics
damage properties
failure site geometry
manufacturing flaws and defects
environment and operating loads
3. Determine variability of each design parameter.
4. Compute an effective reliability function



Failures can be defined by the failure mode, failure site, and failure mechanism

Failure mode is an observable change caused by a failure mechanism. Examples are opens, shorts, leaks, and excessive noise or vibration.

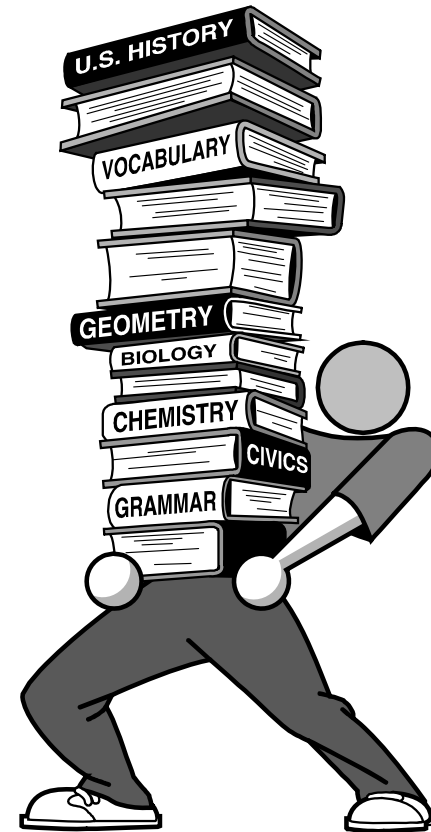
Failure sites are the specific locations and geometry at which the failure mechanism interacts with the material.

Failure mechanisms are physical (or chemical) processes by which stresses damage the material at the failure site.



Stresses trigger the failure mechanism

Mechanical
thermal
electrical
radiation
chemical
biological
combinations





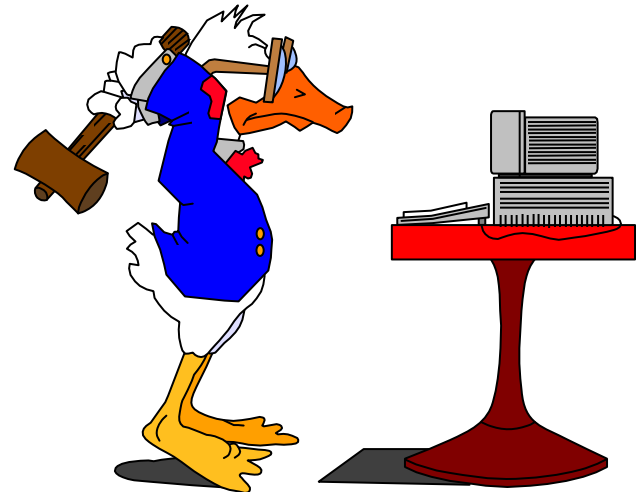
Examples of Failure Mechanisms

Wearout

- fatigue
- friction
- corrosion
- creep
- electromigration
- contamination
- molecular migration
- temperature cycling

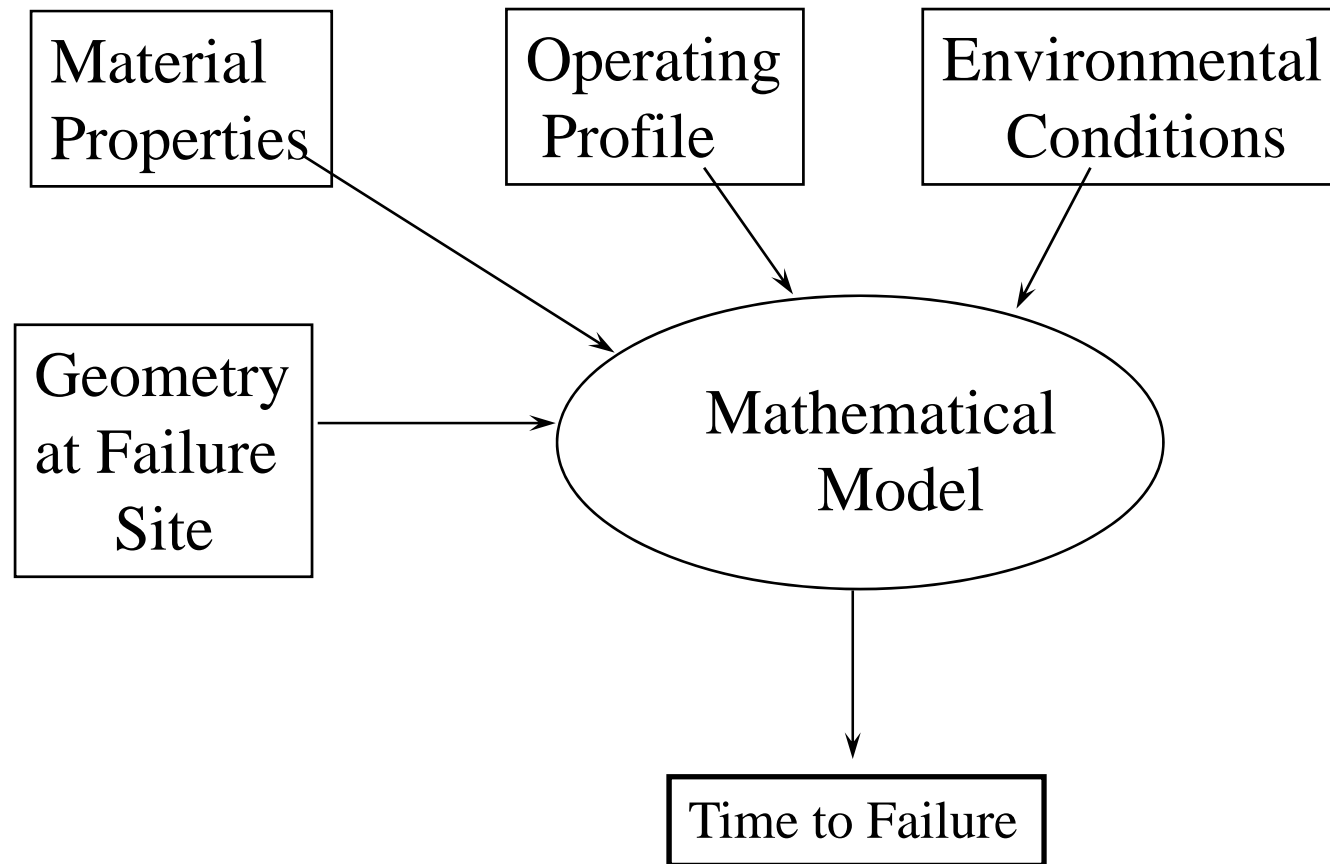
Overstress

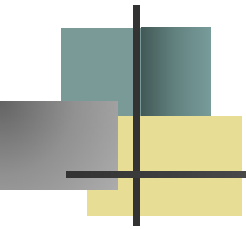
- dielectric breakdown
- fracture
- buckling
- thermal breakdown
- electrical overstress





Physics of Failure

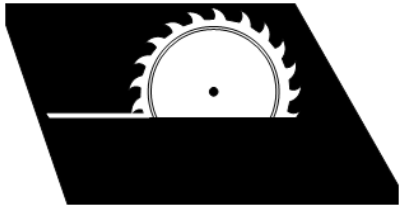




Model Accuracy

- accuracy of the material failure or damage model
- accuracy of the corresponding stress analysis (environment)
- the accuracy of the empirical data (geometry at the failure site and material property characteristics)

Example 7.19



$$t = \frac{c(B_{hn})^m}{v^\alpha f^\beta d^\gamma}$$

where t = the life of a tool in minutes,

B_{hn} = the Brinell hardness number of the work material

v = cutting speed in feet per minute

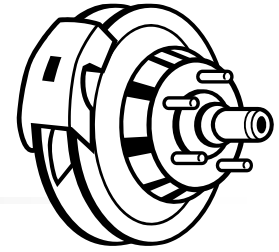
f = the feed in inches per revolution or inches per saw tooth

d = depth of the cut in inches

$c, m, \alpha, \beta, \text{ and } \gamma$ and are empirical constants.



Example 7.20



$$W = \left(\frac{10^4 W_b}{2 A} \right) \left(\frac{W t (\Delta v^2 N) y}{4 g} \right)$$

W = pad wear per mile in inches

W_b = specific wear rate of friction material (in³ /ft-lbf)

A = lining area, in²

Δv = average change in velocity per brake action, ft/sec

Wt = weight of vehicle

g = acceleration due to gravity, 32.2 ft/sec²

N = frequency of brake applications per mile

y = proportion of total braking effort transmitted through the lining

$$\text{Pad Life} = d \text{ (pad thickness)} / W \text{ in miles}$$



Example 7.21

$$\varepsilon = \varepsilon_0 (1 + \beta t^{1/3}) e^{kt}$$

ε = the strain at time t ,

ε_0 = the initial elastic strain, and

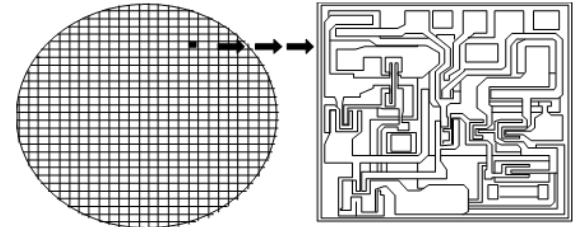
β , k are constants depending upon the particular material.

If ε_{\max} is the fracture stress of the material, then setting $\varepsilon = \varepsilon_{\max}$ in the above formula and solving for t provides the design life with respect to creep (progressive deformation of the material).



Example 7.22- Electromigration failures

$$t = \frac{bA}{J^2} e^{\frac{E_a}{kT}}$$



t = mean time to failure in hours

b = empirically derived constant

(2.85×10^{15} for an aluminum conductor)

A = cross-sectional area of the conductor in cm^2

J = current density in amp/cm^2

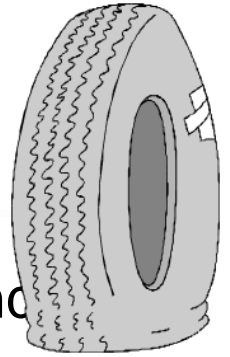
E_a = activation energy in eV

k = Boltzman's constant (8.62×10^{-5} eV/degree K.)

T = temperature in degrees Kelvin



Wear Due to Friction



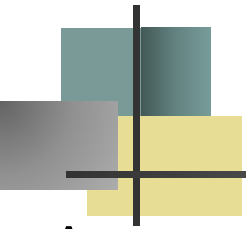
adhesive wear - fragments are pulled from one surface and adhere to the other. Loose particles may also be produced.

abrasive wear - occurs when a hard surface slides over a softer surface generating a series of grooves in the softer material along with loose particles.

corrosive wear - sliding surfaces chemically react with the the partner surface or the environment, or both.

surface fatigue - is generated when cycling loading produces cracks which break up the surface. Ball bearings, wheels on rails, and gears are susceptible to surface fatigue.

Wear due to friction can be reduced with the proper choice of materials and lubrication



Law of adhesive wear

Amount of wear is directly proportional to the load, L and distance slid x , and inversely proportional to the hardness, p , of the surface being worn away. That is $V = k L x / p$

where V = volume worn away in mm^3

k = nondimensional wear coefficient determined experimentally and dependent upon the materials in contact and the lubricant if present,

L = load in kg

x = sliding distance in mm

p = hardness in kg/mm^2 (an indenter force divided by the projected area of the indent)


If A = surface area in contact in mm^2 then
$$h = \frac{V}{A} = \frac{k L x}{p A}$$

is the average depth of the material worn away in mm.




Chapter 7 Summary

- Covariate Models
- Static & Dynamic Models
- Physics of Failure Models



These are some really great models.



They sure are!
However, I am anxious to begin the next chapter to learn about other exciting reliability concepts.