LOGM 634 - Homework Set #4

Due 27 September 2017

From the Ebeling text - Exercise 8.2

In the design of a space station, four major subsystems have been identified, each having a Weibull failure distribution with parameter values as given here

Subsystem	Scale Parameter	Shape Parameter
Computer	3.50	0.91
Avionics	4.00	0.80
Structures	5.00	1.80
Life Support	6.00	1.00

The reliability of the station must be 0.995 at the end of the first year. Determine the percentage increase in reliability for each major sybsystem needed to reach the system goal. Assign equal reliability goals to all subsystems.

While not clearly stated, these subsystems operate in series – where the failure of any subsystem results in system failure. Therefore, the system reliability will be expressed as

$$R_{sys}(1) = 0.995 = R_C(1) \times R_A(1) \times R_S(1) \times R_{LS}(1).$$

We are also asked to assign an equal reliability goal to every subsystem, meaning that

$$\begin{aligned} R^*_C(1) = R^*_A(1) = R^*_S(1) &= R^*_{LS}(1) \\ &= \sqrt[4]{R_{sys}(1)} \\ &= \sqrt[4]{0.995} = 0.9987476 \end{aligned}$$

The increase in reliability would then be the difference between the goal reliability 0.9987476 and the reliability of each subsystem at $t = 1$. These values, computed using the information in the table, are shown below

$$R_C(1) = \exp\left[-\left(\frac{1}{3.5}\right)^{0.91}\right] = 0.7263$$

$$R_A(1) = \exp\left[-\left(\frac{1}{4.0}\right)^{0.80}\right] = 0.7190$$

$$R_S(1) = \exp\left[-\left(\frac{1}{5.0}\right)^{1.80}\right] = 0.9463$$

$$R_{LS}(1) = \exp\left[-\left(\frac{1}{6.0}\right)^{1.00}\right] = 0.8465.$$

Thus, the increase in reliability for each subsystem is found as follows

$$\Delta R_C = \frac{R_C^*(1) - R_C(1)}{R_C(1)} = \frac{0.998748 - 0.7263}{0.7263} = \mathbf{.3751}$$

$$\Delta R_A = \frac{R_A^*(1) - R_A(1)}{R_A(1)} = \frac{0.998748 - 0.7190}{0.7190} = \mathbf{.3891}$$

$$\Delta R_S = \frac{R_S^*(1) - R_S(1)}{R_S(1)} = \frac{0.998748 - 0.9463}{0.9463} = \mathbf{.0554}$$

$$\Delta R_{LS} = \frac{R_{LS}^*(1) - R_{LS}(1)}{R_{LS}(1)} = \frac{0.998748 - 0.8465}{0.8465} = \mathbf{.1799}$$

From the Ebeling text - Exercise 8.14

Nether Fales, a reliability engineer, must decide wich of two components to use in the design of a new product. Acording to the supplier, component A has a unit cost of $225 and a Weibull failure distribution with a shape parameter of 1.7 and a characteristic life of 12 yr. Component B has a unit cost of $245 and a constant failure rate of 0.11 failures per year, according to the manufacturer's specifications. A failure of either component results primarily in the replacement of a comparable (same age) part. The average part cost of component A is $40 and that of component B is $35. In addition, if component A is selected a special test unit must be purchased at a cost of $4300 to be used in identifying the failed part. The product will have a 10-yr design life, at the end of which component A has a salvage value of $60 and component B has a salvage value of $40. Operating costs and other support costs are the same for either component. If 100 units of the product are to be manufactured, determine which component to use in the design of the product by comparing the life-cycle costs. Assume a 5% effective discount rate.

We are asked to choose between two component options on the basis of the lower total life cycle cost. This can be done by substituting the information provided in the problem into Equation 8.3. For Option A, the total life cycle cost, based on the information given, is computed as shown in the expression below.

$$\text{LCC} = C_u N + F_o + \left[P_A(i, t_d) C_f \frac{10}{MTTF} N\right] + F_s - \left[P_F(i, t_d SN)\right]$$

The values for each option are shown in the table below.

	Option A	Option B
C_u	225.0	245.0
N	100.0	100.0
F_o	0.0	0.0
$P_{_A}(i, t_d)$	7.7	7.7
C_o	0.0	0.0
C_f	40.0	35.0
t_o	8760.0	8760.0
$MTTF$	93732.0	79716.0
F_s	4300.0	0.0
C_s	0.0	0.0
$P_{_F}(i, t_d)$	0.6	0.6
S	60.0	40.0

Substituting the values from the shown above into the life cycle cost expression gives the following results for each option. Observing these values we see that option B is perferred on the basis of life cycle cost.

$$LCC_{_A} = \mathbf{\$26078.5}$$

$$LCC_{_B} = \mathbf{\$25061.54}$$

From the Ebeling text - Exercise 9.1

The time to repair a power generator is best described by the following probability density function

$$f(t) = \frac{t^2}{333} \qquad 1 \leq t \leq 10$$

Determine the probability that a repair with be completed in 6 **hr. What is the** $MTTR$**? What is the median time to repair.**

The probability that a repair can be accomplished in 6 hr is $P(T \leq 6) = F(6)$, which is expressed as

$$F(t) = \int_1^t \frac{t^2}{333} dt = \frac{t^3}{3(333)}\Bigg|_1^t = \frac{t^3 - 1}{999}.$$

The requested value is then

$$F(6) = \frac{6^3 - 1}{999} = \mathbf{0.215}.$$

The MTTR is computed as

$$MTTR = \int_1^{10} t \frac{t^2}{333} dt = 7.5068.$$

The median time to failure is expressed as

$$F(t_{med}) = 0.5$$

$$\frac{t_{med}^3 - 1}{999} = 0.5$$

$$t_{med} = \sqrt[3]{(0.5 * 999) + 1} = \mathbf{7.93965}$$

From the Ebeling text - Exercise 10.4

An aircraft consists of the following subsystems having the reliability parameters shown

Subsystem	Failure Distribution	Parameters
Propulsion	Weibull	$\theta = 1000, \beta = 1.7$
Avionics	Exponential	$\lambda = 0.003$
Structures	Weibull	$\theta = 2000, \beta = 2.1$
Electrical	Weibull	$\theta = 870, \beta = 1.8$
Environmental	Exponential	$\lambda = 0.001$

A system $MTTR$ of 5 hr based on the average number of failures over $50,000$ hr of operation is desired. Allocate the $MTTR$ to the above subsystems. Assume that a repair will restore the system to as good as new.

Based on the assumptions of this problem we can allocate the $MTTR$ across each subsystem using Equation 9.24.

$$MTTR_S \frac{\sum_{i=1}^{n} q_i f_i MTTR_i}{\sum_{i=1}^{n} q_i f_i} \tag{9.24}$$

We are told that the desired $MTTR_S = 5$, rearranging Equation 9.24 and substituting this value gives an expression for the allocated reliability of each subsystem.

$$MTTR_i = \frac{MTTR_S \sum_{i=1}^{n} q_i f_i}{n q_i f_i} \tag{9.24}$$

To find $MTTR_i \forall i$ we still need to find q_i and f_i for each subsystem. However, note that only one component exists for each subsystem (the subsystem itself) - therefore $q_i = 1$. This leaves us the the task of finding $\sum f_i$. For renewal processess we have that

$$f_i = \frac{t_{oi}}{MTTF_i}$$

where $t_{oi} = 50000$ for every subsystem. Using the expressions from chapters 3 and 4 we can compute $MTTF_i$ for each subsystem, these values are shown in the table below.

Subsystem	MTTF	f_i
Propulsion	892.24	$50000/892.24 = 56.04$
Avionics	333.33	$50000/333.33 = 150$
Structures	1771.39	$50000/1771.39 = 28.23$
Electrical	773.68	$50000/773.68 = 64.63$
Environmental	1000.00	$50000/1000 = 50$

Then the allocated MTTR values for each system are

$$MTTR_i = \frac{5 \times \sum_{i=1}^{n} f_i}{5 \times f_i}$$

$$MTTR_{_{pro}} = \frac{5 \times 348.9}{5 \times 56.04} = \mathbf{6.2259101} \text{ hours}$$

$$MTTR_{_{avi}} = \frac{5 \times 348.9}{5 \times 150} = \mathbf{2.326} \text{ hours}$$

$$MTTR_{_{str}} = \frac{5 \times 348.9}{5 \times 28.23} = \mathbf{12.3591923} \text{ hours}$$

$$MTTR_{_{ele}} = \frac{5 \times 348.9}{5 \times 64.63} = \mathbf{5.3984218} \text{ hours}$$

$$MTTR_{_{env}} = \frac{5 \times 348.9}{5 \times 50} = \mathbf{6.978} \text{ hours}$$