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Simplified versions for the Penman evaporation equation using routine weather data

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Summary At standard weather stations the routine weather records usually available are air temperature, T (°C), solar radiation R_S (MJ/m^2/d), relative humidity, RH (%), and wind velocity, u (m/s). A simple algebraic formula, equivalent in accuracy to the Penman equation is derived for computing evaporation from readily available measured data. The derivation of the formula is based on simplifications made to the ''standardized'' form of the Penman equation. The two components of the standard Penman equation (radiation and aerodynamic) were computed indirectly from the available routine weather data using the standardized calculation procedure recommended by Shuttleworth [Shuttleworth, W.J., 1993. In: Maidment, D.R. (Ed.), Evaporation, McGraw-Hill, New York, pp. 4.1–4.53 (Chapter 4)] and Allen et al. [Allen, R.G., Pereira, L.S., Raes, D., Smith, M., 1998. Crop evapotranspiration: guidelines for computing crop water requirements. FAO Irrigation and Drainage Paper, 56, Rome, 300 pp]. In addition, another, more simplified formula, easy to use for routine hydrologic applications, is also developed. On the other hand, at many places reliable wind speed data are rarely available. For such cases, an expression, which does not require wind speed data, is proposed. The simplified formula for the Penman equation proposed to estimate potential evaporation from open-water without recourse to wind data is $E_{PEN} \approx 0.047 R_S \sqrt{T + 9.5} - 2.4 (R_S/R_A)^2 + 0.09 (T + 20)(1 - RH/100)$ where R_A (MJ/m^2/d) is the extraterrestrial radiation. A simplified expression for the calculation of R_A is also given. The simplicity of the formulas is demonstrated in a computational example applied for monthly climatic data. The performance of the new derived formulas was tested under various climatic conditions using a global climatic data set including monthly data as well as daily data obtained from a weather station. The new open water evaporation formulas were also adapted for calculating reference crop evapotranspiration.

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Introduction

Evaporation is a major component of the hydrologic cycle and its estimate is widely used in hydrologic and irrigation engineering applications. Its accurate estimation is required for many studies, such as hydrologic water balance, water resources planning and management, and irrigation scheduling.

Evaporation direct measurement techniques are not recommended for routine hydrologic engineering applications because they imply a time consuming procedure requiring expensive equipment in order to obtain precise and carefully designed experiments.

On the other hand, evaporation estimation methods based on climatic data are very common in the case of hydrologic and irrigation applications. Such methods vary from simple empirical formulations to complex methods such as the physically based combination method of Penman (1948). Penman (1948) published the radiation-aerodynamic combination equation to predict evaporation from open water, bare soil, and grass. The original Penman (1948) equation is widely used as the standard method in hydrologic engineering applications to estimate potential evaporation from open water under varying locations and climatic conditions (Shuttleworth, 1993; Dingman, 1994).

The original Penman (1948, 1963) equation as well as its various modifications have also been widely used to estimate reference crop evapotranspiration.

The main disadvantage of the Penman equation is that the main weather variables appearing directly in the equation are air temperature, T, net radiation, R_n, vapor pressure deficit, D, and wind velocity, u. Although there are specific instruments to measure net radiation and vapor pressure deficit, the usually available weather records in standard weather stations over the world are air temperature, T, relative humidity, RH, solar radiation, R_S (or, more frequently, bright sunshine hours, n, from which R_S is indirectly estimated) and wind speed, u, data (Shuttleworth, 1993). Then net radiation and vapor deficit are usually estimated indirectly from the available weather variables measured. Furthermore, a plethora of other parameters appear in the application of the Penman equation, such as the latent heat of vaporization, the saturation vapor pressure, the actual vapor pressure, the psychrometric coefficient, the slope vapor pressure curve, the atmospheric pressure, the effective emissivity of the surface, the clear-sky solar radiation, the Stephan–Boltzman constant, the cloudiness factor, and many others. The complexity of calculations increases as each of these parameters could be expressed by a variety of units. The use of all these parameters could create confusion in the calculation steps during the application of the Penman equation, thus resulting in significant errors should the appearing parameters not be expressed in the appropriate units.

Linacre (1992, 1993) has also developed simplified formulas for the Penman (1948) equation to estimate open water evaporation and crop evapotranspiration.

In this paper, an algebraic formula, using the variables T, R_S (or n), RH, and u, equivalent in accuracy to the Penman equation, is developed. The estimation of R_n and D is based on the ''standard'' computation sequence recommended by Shuttleworth (1993) and Allen et al. (1998). According to this procedure, R_n depends on R_S, but also on T and RH. The only additional parameter appearing in the suggested formula is the extraterrestrial radiation, R_A. Furthermore, another more simplified (but less accurate) formula, easy to use for routine hydrologic applications, is also developed. In many places, wind speed data are either unavailable or of questionable integrity. For such cases, an expression, which does not require wind speed data, is proposed. Lastly, simplified expressions to estimate the extraterrestrial radiation and the daylight hours (for monthly evaporation estimates) are also given. A computational application estimating potential evaporation from a hypothetical open water surface is presented. A modification of the new evaporation formula adjusted for the difference between albedos of open water and a grass surface corresponding to the Penman (1963) – for grass – model is also proposed.

The performance of the derived formulas for estimating open water evaporation and reference crop evapotransira-tion is also tested.

The Penman equation

The classical form for the Penman (1948, 1963) equation to estimate potential evaporation or evapotranspiration (Shuttleworth, 1993) is

$$E_{PEN} = \frac{\Delta}{\Delta+\gamma}\cdot\frac{(R_n)}{\lambda} + \frac{\gamma}{\Delta+\gamma}\cdot\frac{6.43(f_U)D}{\lambda}, \quad (1)$$

where E_{PEN} is potential – open water – evaporation or evapotranspiration (mm/d); R_n is net radiation at the surface (MJ/m^2/d); Δ is the slope of the saturation vapor pressure curve (kPa/°C); γ is psychrometric coefficient (kPa/°C); λ is latent heat of vaporization (MJ/kg); f_U is wind function,

$$f_U = a_U + b_U u, \quad (1a)$$

where a_U and b_U are wind function coefficients; and u is wind speed at 2 m height (m/s), for the original Penman (1948, 1963) equation $a_U = 1$, $b_U = 0.536$; $D = (e_S - e_a)$ is vapor pressure deficit (kPa); e_S is saturation vapor pressure (kPa); e_a is actual vapor pressure (kPa).

The Penman's development, Eq. (1), did not include heat exchange with the ground, or water advected energy, or change in heat storage. This assumption is acceptable for monthly or daily estimations in practical hydrological applications (Shuttleworth, 1993; Linacre, 1993; Allen et al., 1998).

The usually available weather records in standard climatological stations over the world are T, R_S (or n), RH, and u. If apart from the site location the available data are T, R_S (or n), RH, and u for daily, 10-day or monthly calculation, then the Penman equation for routine hydrologic engineering applications (Shuttleworth, 1993; Allen et al., 1998) can be formulated as follows.

The standardized Penman equation

The standardized calculation procedure for estimating R_n and D from readily available data as recommended by Shuttleworth (1993) and Allen et al. (1998) is presented.

The net radiation, R_n, is computed as the difference between the incoming net short wave radiation, R_{nS}, and the outgoing net long wave radiation, R_{nL}:

$$R_n = R_{nS} - R_{nL}. \tag{2}$$

The R_{nL} is computed as

$$R_{nL} = f\varepsilon'\sigma \cdot (T + 273.2)^4, \tag{3}$$

where R_{nL} is outgoing net long wave radiation (MJ/m^2/d); f is adjustment for cloud cover; ε' is net emissivity between the atmosphere and the ground; σ is Stephan-Boltzman constant = 4.903×10^{-9} (MJ/m^2/K^4/d); T is mean air temperature for the examined time interval (°C), conventionally calculated as $T = 0.5(T_{max} + T_{min})$ where T_{max} and T_{min} are mean maximum and mean minimum temperature, respectively (°C).

The R_{nS} is calculated as

$$R_{nS} = (1 - \alpha) \cdot R_S, \tag{4}$$

where R_S is measured or estimated incoming solar radiation (MJ/m^2/d); α is reflection coefficient or albedo. A typical value of the albedo for a grass cover is $\alpha = 0.23$, whereas for open water surfaces $\alpha = 0.08$ (Shuttleworth, 1993; Allen et al., 1998).

Since e_S is not a linear function of temperature, Shuttleworth (1993) recommended the following procedure to compute the mean vapor pressure deficit, $D_{(av)}$:

$$D_{(av)} = e_{S(av)}\left(1 - \frac{RH}{100}\right), \tag{5}$$

where $e_{S(av)}$ is mean saturation vapor pressure for the examined time interval estimated as

$$e_{S(av)} = 0.5[e_S(T_{max}) + e_S(T_{min})], \tag{6}$$

$e_S(T_{max})$ and $e_S(T_{min})$ are saturated vapor pressures corresponding to the temperature values of T_{max} and T_{min}, respectively, and computed by Eq. (44) in the Appendix; and RH (%) is mean relative humidity for the examined time interval.

The computation of the parameters f and ε' according to the standard procedure as well as the calculation of other meteorological parameters appearing in the Penman equation, Eq. (1), are given in the Appendix.

New simplified versions

For the development of the simplified versions it is initially assumed that $Z = 0$, where Z is elevation of the site (m).

For $Z = 0$ the coefficient γ takes the single constant value of $\gamma_0 = 0.0671$ kPa. Furthermore, as the value of λ varies only slightly over a normal temperatures range, a single constant value (for $T = 20$ °C) is considered $\lambda_0 = 2.45$ MJ/kg. Then, the standardized form of the Penman equation, taking into account the formulation presented in Appendix, can be written as follows:

$$E_{PEN} = E_{radS} - E_{radL} + E_{aero}, \tag{7}$$

where the terms E_{radS}, E_{radL} and E_{aero} are the terms of the Penman combination equation corresponding to the incoming net short wave radiation component, the outgoing net long wave radiation component and the aerodynamic component, respectively. These terms are given by the following equations (according to the standardized equations given in the Appendix):

$$E_{radS} = \frac{1}{\lambda_0}(1 - \alpha)R_S\frac{\Delta}{\Delta + \gamma_0}, \tag{8}$$

$$E_{radL} = \frac{1}{\lambda_0}\left(1.35\frac{R_S}{0.75R_A} - 0.35\right)\left(0.34 - 0.14\sqrt{\frac{RH}{100}e_{S(av)}}\right) \times \sigma(T + 273.2)^4\frac{\Delta}{\Delta + \gamma_0}, \tag{9}$$

$$E_{aero} = \frac{1}{\lambda_0}\frac{\gamma_0}{\Delta + \gamma_0}6.43 f_U D_{(av)}, \tag{10}$$

where R_A is extraterrestrial solar radiation (MJ/m^2/d); for a given latitude of the site and a Julian number day (or the month) the value of R_A is obtained from tables or can be calculated using a set of equations (Shuttleworth, 1993; Allen et al., 1998).

In the following sections, a series of purely empirical simplified expressions were developed to approximate the standardized components of the Penman equation, Eqs. (8)–(10). The selection of the mathematical form of these expressions is mainly based on two criteria: the simplicity of the suggested formulas and their accuracy to approximate the standardized components.

In the approximation procedure that follows, it is initially – for instance – assumed that the mean saturation vapor pressure can be approximated as

$$e^{(0)}_{S(av)} = e_S(T). \tag{11}$$

Based on the assumption of Eq. (11), initial approximate formulas for estimating the long wave and aerodynamic components, $E^{(0)}_{radL}$ and $E^{(0)}_{aero}$, are proposed. Next a correction is made for the above components using the more accurate estimation of $e_{S(av)}$ from Eq. (6).

Approximation of the E_{radS}

The term $\Delta/(\Delta + \gamma_0)$ (dimensionless) appearing in Eq. (8) is a function only of the mean temperature T. A preliminary analysis of the behavior of $\Delta/(\Delta + \gamma_0)$ as a function of T indicated that this term is well approximated by a simple empirical formula of the form $C_1\sqrt{T + C_2}$, where C_1 and C_2 are regression parameters to be determined. Using the least squares regression procedure, the two regression parameters C_1 and C_2 were determined. Then, the term $\Delta/(\Delta + \gamma_0)$ of the component E_{radS} can be approximated by the following simple empirical formula:

$$\frac{\Delta}{\Delta + \gamma_0} \approx 0.125\sqrt{T + 9.5}, \tag{12}$$

where T is expressed in °C.

In order to test the accuracy of Eq. (12), numerical simulations over a typical temperatures range, T, varying between 2 and 37 °C were performed, for $n_0 = 500$ different values of T. Approximate values obtained from Eq. (12) are compared with the exact values of $\Delta/(\Delta + \gamma_0)$ in Figure 1a. Results indicated a very good agreement between approximate and exact values. Regression yields $Y = 1.001X$, and $R^2 = 0.998$ where Y is approximate estimation, X is exact value, and R^2 is coefficient of determination. Substituting Eq. (12) into Eq. (8), E_{radS} can be approximated by the following empirical equation:

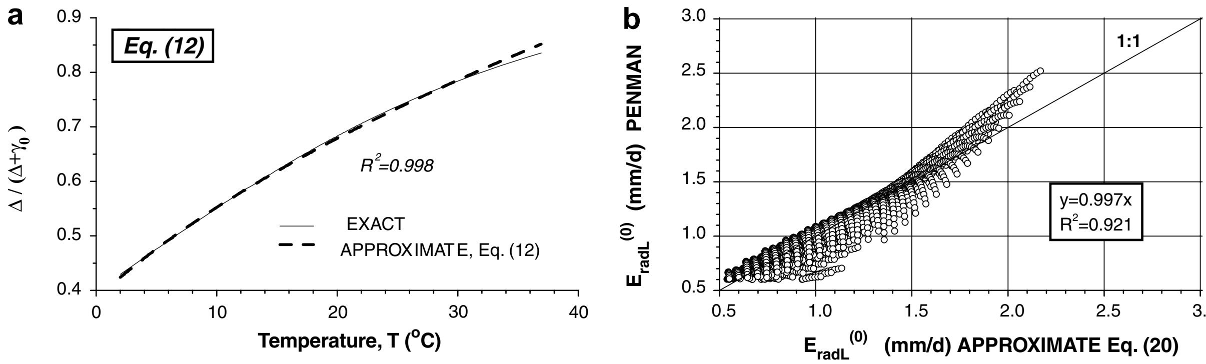

Figure 1 (a) Comparison of approximate, Eq. (12), with exact values of the term $\Delta/(\Delta+\gamma_0)$, over a typical range of temperature. (b) Comparison of approximate, Eq. (20), with exact values of the Penman component $E^{(0)}_{\mathrm{radL}}$, for series of computations over typical range of input meteorological variables.

$$E_{\mathrm{radS}} \approx 0.051(1-\alpha)R_{\mathrm{S}}\sqrt{T+9.5}, \tag{13}$$

where R_{S} should be expressed in MJ/m^2/d.

Approximation of the $E^{(0)}_{\mathrm{radL}}$

Assuming for instance the validity of Eq. (11), a similar procedure of simulations is applied for the terms of $E^{(0)}_{\mathrm{radL}}$ affected only by the temperature, T. The dependent on the variable, T, terms of the equation can be approximated as follows:

$$\frac{\Delta}{\Delta+\gamma_0}(T+273.2)^4 \approx 1.534\times10^8(T+13), \tag{14}$$

$$\sqrt{e_{\mathrm{S}}(T)} \approx 3.6\times10^{-4}(T+45)^2. \tag{15}$$

Regression statistics, for Eqs. (14) and (15) yields $Y = 0.996X$, $R^2 = 0.9992$ and $Y = 0.985X$, $R^2 = 0.999$, respectively, indicating a perfect agreement between approximate and exact values for both equations. Then the component $E^{(0)}_{\mathrm{radL}}$ can be approximated as

$$E^{(0)}_{\mathrm{radL}} \approx 0.188(T+13)\left(\frac{R_{\mathrm{S}}}{R_{\mathrm{A}}}-0.194\right) \times\left(1-0.00015(T+45)^2\sqrt{\frac{RH}{100}}\right). \tag{16}$$

Estimation of the $E^{(0)}_{\mathrm{aero}}$

It is initially assumed that the mean vapor pressure deficit can be approximated by the following equation:

$$D^{(0)}_{(\mathrm{av})} = e_{\mathrm{S}}(T)\left(1-\frac{RH}{100}\right). \tag{17}$$

Assuming for instance that Eq. (17) is valid, a procedure similar to the previously developed approximation procedures is followed. It is observed that the $E^{(0)}_{\mathrm{aero}}$ component obtained from Eq. (10) using Eq. (17) can be written as $E^{(0)}_{\mathrm{aero}} = G^{(0)}_{\mathrm{u}}(1-RH/100)(a_{\mathrm{U}}+b_{\mathrm{U}}u)$, where the term $G^{(0)}_{\mathrm{u}} = 6.43e_{\mathrm{S}}(T)\gamma_0/(\Delta+\gamma_0)/\lambda_0$ is a new auxiliary function introduced to facilitate the computations in the approximation procedure. The function $G^{(0)}_{\mathrm{u}}$ depends only on the variable T, and can be approximated by the following simple equation:

$$G^{(0)}_{\mathrm{u}} = \frac{6.43e_{\mathrm{S}}(T)\gamma_0}{(\Delta+\gamma_0)\lambda_0} \approx 0.048(T+20), \tag{18}$$

where T is expressed in °C.

To demonstrate Eq. (18) numerical simulations of Eq. (18) where T varies between 2 and 37 °C were also performed. The approximate results were plotted versus the ''exact'' values of $G^{(0)}_{\mathrm{u}}$, Eq. (18), for $n_0 = 500$ simulations (figure not shown). Regression results indicated that Eq. (18) is a very good approximation ($Y = X$; $R^2 = 0.999$). Then, the initial approximation of the aerodynamic component, $E^{(0)}_{\mathrm{aero}}$ (calculated using $D^{(0)}_{(\mathrm{av})}$ from Eq. (17)), can be approached by the following simplified equation:

$$E^{(0)}_{\mathrm{aero}} \approx 0.048(T+20)\left(1-\frac{RH}{100}\right)(a_{\mathrm{U}}+b_{\mathrm{U}}u). \tag{19}$$

Further simplified approximation for $E^{(0)}_{\mathrm{radL}}$

A more simplified form for the $E^{(0)}_{\mathrm{radL}}$ component could be obtained as follows:

A series of numerical simulations were carried out to generate ''exact'' E_{radL} values according to Eq. (9), $e_{\mathrm{S(av)}}$ is assumed to be calculated by Eq. (11). Combination of the required input meteorological variables, as $[R_{\mathrm{S}}/(0.75R_{\mathrm{A}})]$, T and RH covering typical ranges of variables were used to calculate the ''exact'' values of $E^{(0)}_{\mathrm{radL}}$. The ratio of $[R_{\mathrm{S}}/(0.75R_{\mathrm{A}})]$ varied between 0.2 and 1.0, the mean relative humidity, RH, between 25% and 97%, and the mean temperature, T, between 12 and 27 °C. In a first stage of analysis, a narrower (but still typical) range of temperatures than that used in the development of the previous formulas is examined. In a second step of analysis, a broader range of temperatures is also examined. The ''exact'' $E^{(0)}_{\mathrm{radL}}$ data sets were generated by combining in all possible ways the previous typical input values of meteorological variables. An empirical formula of the form $E^{(0)}_{\mathrm{radL}} \approx w_1(R_{\mathrm{S}}/0.75/R_{\mathrm{A}})^{w_3} + w_2(T+20)(1-RH/100)$ is examined where w_1, w_2 and w_3 are empirical parameters to be determined. A preliminary analysis of the behavior of the $E^{(0)}_{\mathrm{radL}}$ function indicated that the exponent w_3 can be roughly approximated as $w_3 \approx 2.0$. Then, using the least squares regression procedure on the above equation the two regression parameters w_1 and w_2 were determined. Finally, the component $E^{(0)}_{\mathrm{radL}}$ can be approximated by the following empirical formula:

$$E^{(0)}_{\mathrm{radL}} \approx 2.4\left(\frac{R_S}{R_A}\right)^2 + 0.024(T+20)\left(1-\frac{RH}{100}\right). \tag{20}$$

Note that the second term of the right hand side of Eq. (20) is selected to have the same form as the aerodynamic term in Eq. (19) omitting the wind function. Figure 1b presents the variation of the approximate values of $E^{(0)}_{\mathrm{radL}}$ [Eq. (20)] versus the ''exact'' values [Eq. (9) in conjunction with Eq. (11)] for $n_0 = 2383$ simulation data. The rather unusual values of E_{radL}, 0.4 mm/d > E_{radL} > 3.0 mm/d were excluded. Results presented in Figure 1b indicated that Eq. (20) is a relatively good approximation of Eq. (9) ($Y = 0.997X$; $R^2 = 0.921$). For a broader range of temperatures, T, varying from 5 to 30 °C, the regression results for Eq. (20) yield $Y = 0.97X$, and $R^2 = 0.891$. The scatter of the approximate values in Figure 1b can be considered acceptable as the component $E^{(0)}_{\mathrm{radL}}$ is in average a relatively small portion of the sum of the other two components ($E_{\mathrm{radS}} + E_{\mathrm{aero}}$) (about 19% on average). A further reduction of the scatter of values is expected as the intercorrelation between the input weather variables used in simulations is not taken under consideration.

Simplified Version for the Penman equation with $e_{S(av)} = e_S(T)$

According to Jensen et al. (1990), the Penman (1963) equation applied for $e_{S(av)} = e_S(T)$, with the value of albedo, α, adjusted for grass, produced relatively good results for estimating reference crop evapotranspiration. Systematic comparisons with lysimeter-measured evapotranspiration ranked the Penman (1963) method fourth best overall method. Furthermore, the performance of the method is insignificantly lower than the other two Penman modifications ranked as second and third methods. Substituting Eqs. (13), (20) and (19) into Eq. (7), the following simplified formula for the Penman (1963), $E^{(0)}_{\mathrm{PEN}}$, method calculated with $e_{S(av)} = e_S(T)$ $a_U = 1$, $b_U = 0.536$ is obtained:

$$E^{(0)}_{\mathrm{PEN}} \approx 0.051(1-\alpha)R_S\sqrt{T+9.5} - 2.4\left(\frac{R_S}{R_A}\right)^2 + 0.048(T+20)\left(1-\frac{RH}{100}\right)(0.5+0.536u). \tag{21}$$

Correction for $e_{S(av)}$ and $D_{(av)}$

The formulas developed above will be corrected for the E_{radL} and E_{aero} components using the more ''accurate'' values of $e_{S(av)}$ and $D_{(av)}$ obtained from Eqs. (6) and (5), respectively. It is initially observed that Eq. (10), yielding the ''accurate'' E_{aero} values, can be written as

$$E_{\mathrm{aero}} = G_u\left(1-\frac{RH}{100}\right)(a_U + b_U u), \tag{10a}$$

where

$$G_u = 6.43\frac{e_{S(av)}\gamma_0}{\lambda_0(\Delta+\gamma_0)} \tag{22}$$

is a term affected only by the values of temperature $T_{\max}$ and $T_{\min}$. Similarly, the term $\sqrt{e_{S(av)}}$ appearing in the calculation of E_{radL}, from Eq. (9), depends only on $T_{\max}$ and $T_{\min}$. To investigate the behavior of these terms, a series of numerical simulations were carried out to generate the ''accurate'' values of G_u and $\sqrt{e_{S(av)}}$.Combinations of the required input temperature parameters $T_{\max}$, and $T_{\min}$, covering a typical range of variation, were used to calculate G_u and $\sqrt{e_{S(av)}}$ from Eqs. (22) and (6), respectively. The temperature values $T_{\max}$ and $T_{\min}$ were implicitly generated by varying the mean temperature T and the difference of temperatures ($T_{\max} - T_{\min}$) over a typical range of variations.Then the values of $T_{\max} = T + (T_{\max} - T_{\min})/2$, and $T_{\min} = T - (T_{\max} - T_{\min})/2$ were provided. Mean temperature T varied between 2 and 37 °C, and ($T_{\max} - T_{\min}$) between 3 and 24 °C. The temperature data sets were generated by combining all possible ways the previous ''typical'' input values of T, and ($T_{\max} - T_{\min}$). For a given set of data the values of G_u and $\sqrt{e_{S(av)}}$ were computed. Data sets for which $T_{\max} > 46$ °C or $T_{\min} < -5$ °C were excluded from simulations. Afterwards, the ''accurate'' values of G_u and $\sqrt{e_{S(av)}}$ obtained from simulations were plotted against the values of mean temperature T. The results from $n_0 = 875$ simulations are reported in Figure 2a and b, respectively. It is shown that there is a relatively high dependence of the computed G_u and $\sqrt{e_{S(av)}}$ terms on the T variable. Using the least square procedure the variation of the G_u and $\sqrt{e_{S(av)}}$ values with T are given by the following approximate relationships, respectively,

$$G_u \approx 0.052 \cdot (T+20), \tag{23}$$

$$\sqrt{e_{S(av)}} \approx 3.8 \times 10^{-4}(T+45)^2. \tag{24}$$

Statistical regression results for Eq. (23) (shown in Fig. 2a) are $Y = 1.0X$ and $R^2 = 0.930$, whereas for Eq. (24) (shown in Fig. 2b) the results are $Y = 1.0X$ and $R^2 = 0.989$. The coefficient of determination is not particularly high for Eq. (23). Substituting Eq. (23) into Eq. (10a), the following simplified approximate formula for E_{aero} is obtained:

$$E_{\mathrm{aero}} \approx 0.052(T+20)\left(1-\frac{RH}{100}\right)(a_U + b_U u). \tag{25}$$

Afterwards, the same procedure of simulations as the one providing the simplified version of $E^{(0)}_{\mathrm{radL}}$, Eq. (20), is applied. The only difference in simulations is that the values of $\sqrt{e_{S(av)}}$ appearing in the calculation of the ''exact'' E_{radL} generated data obtained from the Penman equation, Eq. (9), were approximated by Eq. (24). The results indicated that the factor 0.024 appearing in the second term of Eq. (20) (obtained when $e_{S(av)} = e_S(T)$) should be changed to 0.02.

More accurate, but rather complex versions for the E_{aero} and E_{radL} components were obtained by plotting the ''exact'' G_u and $\sqrt{e_{S(av)}}$ values generated by the $n_0 = 875$ simulations versus the selected variables of ($T_{\max}$) and ($0.7T_{\max} + 0.3T_{\min}$), respectively. The results are presented in Figure 3a and b. The results presented in Figure 3a and b indicate that the scatter of estimations is significantly reduced when the variables G_u and $\sqrt{e_{S(av)}}$ are plotted versus the new selected temperature variables. Applying a regression procedure the following formulas for approximating the G_u and $\sqrt{e_{S(av)}}$ terms are obtained

$$G_u \approx 0.049 * (T_{\max} + 16.3), \tag{26}$$

$$\sqrt{e_{S(av)}} \approx 3.43 \times 10^{-4}(0.7T_{\max} + 0.3T_{\min} + 46)^2. \tag{27}$$

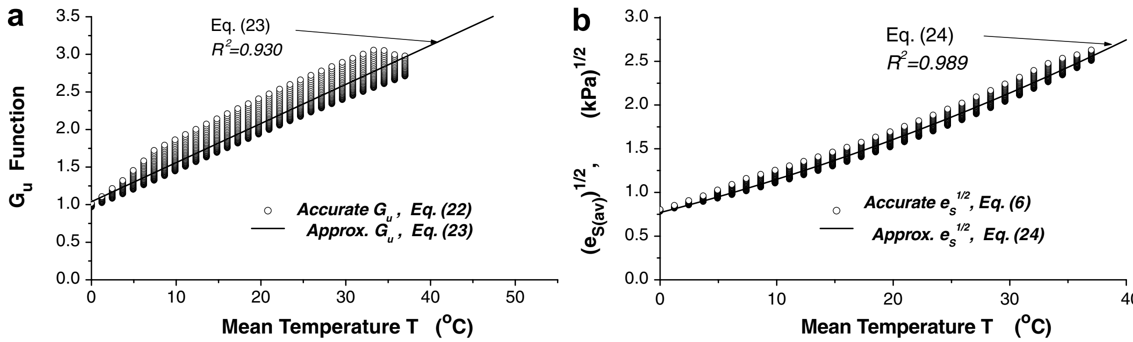

Figure 2 (a) Variation of approximate and accurate values of the function G_{u} with mean temperature, T, for series of computations over typical range of input meteorological variables; (b) Variation of approximate and accurate values of $(e_{\mathrm{S(av)}})^{1/2}$ with mean temperature, T, for series of computations over typical range of input meteorological variables.

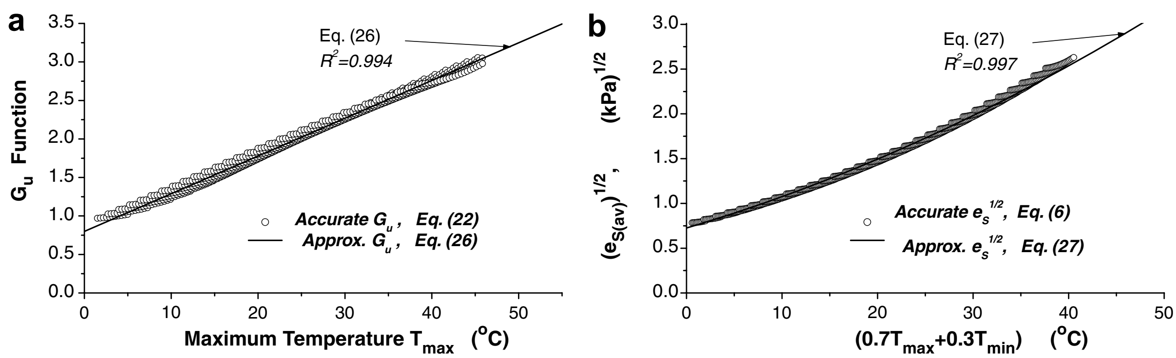

Figure 3 (a) Variation of approximate and accurate values of the function G_{u} with maximum temperature, $T_{\max}$, for series of computations over typical range of input meteorological variables; (b) Variation of approximate and accurate values of $(e_{\mathrm{S(av)}})^{1/2}$ with $(0.7T_{\max} + 0.3T_{\min})$, for series of computations over typical range of input meteorological variables.

The statistical regression parameters presented in Figure 3a and b indicated that the approximate formulas, Eqs. (26) and (27), yield more accurate results than those obtained when G_{u} and $\sqrt{e_{\mathrm{S(av)}}}$ were plotted against the mean temperature values, T. The coefficient of determination is very high for both Eqs. (26) and (27), $R^2 = 0.994$ and $R^2 = 0.997$, respectively. Accurate approximations for the E_{aero} and E_{radL} components can be obtained by substituting Eq. (26) into Eq. (10a) and Eqs. (27) and (14) into Eq. (9), respectively, yielding with $\lambda_0 = 2.45$ MJ/kg:

$$E_{\mathrm{aero}} \approx 0.049(T_{\max} + 16.3)\left(1 - \frac{RH}{100}\right)(a_{\mathrm{U}} + b_{\mathrm{U}}u), \tag{28}$$

$$E_{\mathrm{radL}} \approx 0.188(T + 13)\left(\frac{R_{\mathrm{S}}}{R_{\mathrm{A}}} - 0.194\right) \times \left(1 - 0.00014(0.7T_{\max} + 0.3T_{\min} + 46)^2\sqrt{\frac{RH}{100}}\right). \tag{29}$$

Wind function for evaporation from open water

Penman (1948, 1963) originally suggested a wind function of the form: $f_{\mathrm{U}}^{(1)} = 1 + 0.536u$. The original function of Penman (1948) is also recommended by Shuttleworth (1993) for estimating potential evaporation from open water. Later, Penman (1956) suggested a reduction of the original function by proposing $f_{\mathrm{U}}^{(2)} = 0.5 + 0.536u$. In hydrological applications both forms of the wind function are used to estimate open water evaporation, the first one more frequently than the second one (Brutsaert, 1982). Cohen et al. (2002) considered that the original wind function is unrealistically high for open water evaporation and suggested the second one. Linacre (1993) reduced further the value of the wind function proposing $f_{\mathrm{U}}^{(3)} = 0.54u$.

A global climatic data set including monthly data (FAO-CLIMWAT) was applied to compare the results of the Penman model provided using each of the three previously reported wind functions. Regression of the results yields

$$E_{\mathrm{PEN}}|_{f_{\mathrm{U}}=1+0.54u} \approx 1.06E_{\mathrm{PEN}}|_{f_{\mathrm{U}}=0.5+0.54u} \approx 1.12E_{\mathrm{PEN}}|_{f_{\mathrm{U}}=0.54u}. \tag{30}$$

The coefficient of determination obtained from regression of Eq. (30) is relatively high about $R^2 = 0.991$. Shuttleworth (1993), reported that the standardized Penman formulation in conjunction with the original Penman (1948) wind function yields estimates of sufficient accuracy for evaporation from open water surfaces with reasonable small surface area as from shallow, ground-level evaporation pans or small ponds or lakes. The effective value of the aerodynamic resistance (which is inversely proportional to wind speed function) for much larger expanses of water is larger and the evaporation rate is therefore reduced. The Penman equation with the original wind function might therefore systematically overestimate the evaporation for very large lakes by approximately 10–15% (Shuttleworth, 1993). It is therefore expected (taking into consideration Eq. (30)) that the use of the wind function suggested by Linacre, $f_U = 0.54u$, is more appropriate for estimating evaporation from such large open water surfaces (i.e., large lakes).

In this paper, the first original wind function as recommended by Shuttleworth (1993), in conjuction with the ''standardized'' Penman computational procedure is considered, as the reference method to estimate potential evaporation from open water surface. However, all the three wind functions can be applied in the suggested formulas.

Simplified versions of the standardized Penman equation

Substituting the approximate Eqs. (13), (28) and (29) into Eq. (7), an accurate approximate version for the Penman (1948) equation is obtained:

$$E_{PEN} \approx 0.051(1-\alpha)R_S\sqrt{T+9.5} - 0.188(T+13)\left(\frac{R_S}{R_A} - 0.194\right) \times \left(1 - 0.00014(0.7T_{max} + 0.3T_{min} + 46)^2\sqrt{\frac{RH}{100}}\right) + 0.049(T_{max} + 16.3)\left(1 - \frac{RH}{100}\right)(a_U + 0.536u) \quad (31)$$

where $a_U = 1$: when the original Penman (1948) wind function is to be used; $a_U = 0.5$: when the reduced Penman (1956) wind function is to be used; and $a_U = 0.0$: when the Linacre (1993) wind function is to be used. R_S and R_A should be expressed in MJ/m^2/d (1 MJ/m^2/d = 23.88 cal/cm^2/d = 0.408 mm/d (equivalent evaporation) = 11.57 W/m^2), T, T_{max} and T_{min} in °C, u in m/s, and RH (%). Note that the above formula depends on T but also on T_{max} and T_{min} values.

A more simplified form for the Penman (1948) equation is obtained by substituting Eqs. (13), (20) (with the factor ''0.024'' replaced by ''0.020'') and (25) into Eq. (7)

$$E_{PEN} \approx 0.051(1-\alpha)R_S\sqrt{T+9.5} - 2.4\left(\frac{R_S}{R_A}\right)^2 + 0.052(T+20)\left(1 - \frac{RH}{100}\right)(a_U - 0.38 + 0.54u). \quad (32)$$

Note that the only temperature parameter that appears in the above simplified formula, Eq. (32), is T.

The solar radiation data, R_S, may not be available from many agro-meteorological stations. It may then be estimated from measured sunshine hours, n, if available. In this case the value of R_S calculated by Eq. (49) is substituted into the previously developed formulas Eq. (31) or Eq. (32).

Expression not requiring wind speed data

The Penman equation requires as input the commonly measured variables, i.e., solar radiation, R_S, wind speed, u, air temperature, T, and relative humidity, RH to calculate evaporation. However, these variables are not always available in weather stations. Although temperature and humidity are routinely measured and solar radiation can be estimated with sufficient accuracy, wind speed data are rarely available or of questionable precision. In such cases, simplified empirical temperature or radiation type equations not requiring wind speed data can be used. Where no wind data are available the average value of $u = 2$ m/s of 2000 stations over the globe (Allen et al., 1998) can be used in the Penman equation. Substituting the value of $u = 2$ m/s in the previously developed formula, Eq. (32), considering the original Penman (1948, 1963) value for the wind speed function coefficients, $a_U = 1$ and $b_U = 0.536$, and using the value of albedo $\alpha = 0.08$ for open water, the following approximate formula is proposed for evaporation estimation, when wind speed data are missing

$$E_{PEN} \approx 0.047R_S\sqrt{T+9.5} - 2.4\left(\frac{R_S}{R_A}\right)^2 + 0.09(T+20)\left(1 - \frac{RH}{100}\right). \quad (33)$$

When the Penman (1956) wind function is applied in Eq. (33) the value of ''0.09'' in the second term should be replaced by ''0.06''. When the Linacre (1992) wind function is applied the value of ''0.09'' should be replaced by ''0.04''.

Simplified expressions for R_A and N (monthly estimations)

The extraterrestrial radiation, R_A, and the daylight hours, N, are input parameters required in the application of the Penman equation and the simplified formulas, Eqs. (31)–(33).

Simplified approximate empirical expressions are presented in this paper to calculate R_A, and, N, for monthly estimations of evaporation. These simplified formulas are almost of the same accuracy as the rather complex set of equations suggested by the standardized Penman model given in Appendix (Eqs. (50)–(55)).

The proposed empirical equations are

$$N \approx 4\phi\sin(0.53i - 1.65) + 12, \quad (34)$$

$$R_A \approx 3N\sin(0.131N - 0.95\phi) \quad \text{for } |\phi| > \frac{23.5\pi}{180}, \quad (35a)$$

$$R_A \approx 118N^{0.2}\sin(0.131N - 0.2\phi) \quad \text{for } |\phi| < \frac{23.5\pi}{180}, \quad (35b)$$

where ϕ is latitude of the site (radians), positive for the Northern hemisphere and negative for the Southern; i is rank of the month (first month is January); N is calculated in hours and R_A in MJ/m^2/d. Eq. (35a) is valid for the temperate zone, whereas Eq. (35b) is valid for the tropical zone. To demonstrate the use of Eqs. (34) and (35), a series of numerical simulations were carried out to generate ''exact'' N and R_A values according to the accurate set of Eqs. (50)–(55) (Shuttleworth, 1993 given in Appendix). The latitude varied from 0° to 53° whereas i varied from 1 to 12. The approximate values of N and R_A obtained from Eqs. (34) and (35) were plotted against the ''exact'' results in Figure 4a and b, respectively, for $n_0 = 12{,}012$ simulations. Regression for Eqs. (34) and (35) yields $R^2 = 0.991$ and $R^2 = 0.990$, respectively, indicating a perfect agreement between approximate and exact values for both equations. The standard error for the daylight hours estimations, N, is SEE = 0.065 h and SEE = 0.33 mm/d for the R_A estimations.

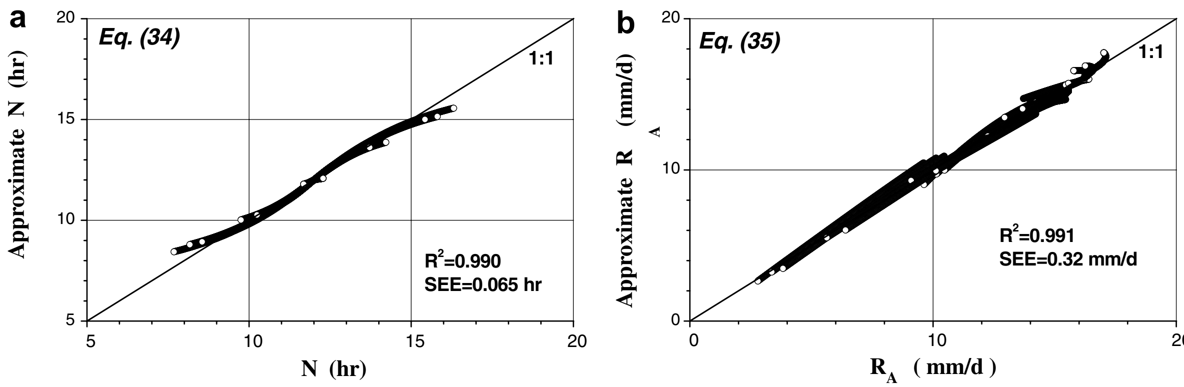

Figure 4 (a) Comparison of accurate with approximate maximum possible daylight values, N, for monthly estimations of evaporation. (b) Comparison of accurate with approximate extraterrestrial radiation values, R_A, for monthly estimations of evaporation.

Effect of the elevation Z

The potential evaporation estimated by the approximate formula, Eq. (31) (with an albedo of $\alpha = 0.08$ — for open water — and wind functions coefficients $a_U = 1$ and $b_U = 0.536$) is compared with the ''standard'' method of Penman (1948) Eq. (1), for monthly data obtained from 535 stations included in a global climatic data set (FAO-CLIMWAT). Comparison of the formula, Eq. (31), with the standardized Penman model applied for the specific value of $Z = 0$ shows an almost perfect agreement between the two methods. This is clearly shown in Figure 5a presenting the variation of the differences of the Penman model results (obtained for $Z = 0$) from the estimations of Eq. (31). The results indicate that the maximum error does not generally exceed ±0.1 mm/d.

A comparison of Eq. (31) with the standardized Penman model, Eq. (1), obtained for various Z is presented in Figure 5b. Figure 5b shows the variation of the differences, $\Delta E = E_{PEN} - E_{Eq.\ (31)}$, with the elevation Z, where E_{PEN} are the results of the Penman model (obtained with variable Z) and $E_{Eq.\ (31)}$ are the estimations of Eq. (31). The results presented in Figure 5b indicate that the difference ΔE between the two models varies almost linearly with Z. Linear regression yields (see Fig. 5b) that

$$E_{PEN} = E_{Eq.\ (31)} + 0.00012Z \tag{36}$$

Although the effect of Z on the Penman model is insignificant, Eqs. (31) and (32) can be corrected to take into account the effect of Z by adding to the formulas the relatively small correction term $0.00012Z$.

Testing formulas for estimating open water evaporation

Application example

In the site of Patrai in Greece (latitude $\phi = 38.15° = 38.15\pi/180 = 0.665$ rad), at elevation $Z = 1$ m the measured long-term average weather data (FAO-CLIMWAT dataset) for the month of June ($i = 6$) are $T_{max} = 27.5$ °C, $T_{min} = 15.8$ °C, mean temperature, $T = (T_{max} + T_{min})/2 = 21.65$ °C, relative humidity $RH = 67.0\%$, wind speed at 2 m, $u = 1.51$ m/s, and bright sunshine hours, $n = 9.5$ h. The daylight hours, N, and the extraterrestrial radiation, R_A are calculated from the suggested empirical formulas Eqs. (34) and (35a), respectively, as $N \approx 4 \times 0.665 \times \sin(0.53 \times 6 - 1.65) + 12 = 14.66$ h $R_A \approx 3 \times 14.66 \times \sin(0.131 \times 14.66 - 0.95 \times 0.665) = 42.2$ MJ/m²/d; R_S is indirectly calculated from n using Eq. (49) as $R_S = 42.2 \times (0.5 + 0.25 \times 9.5/14.66) = 24.2$ MJ/m²/d. The potential open water evaporation, E_{OW}, is calculated by the proposed formula, Eq. (32) as

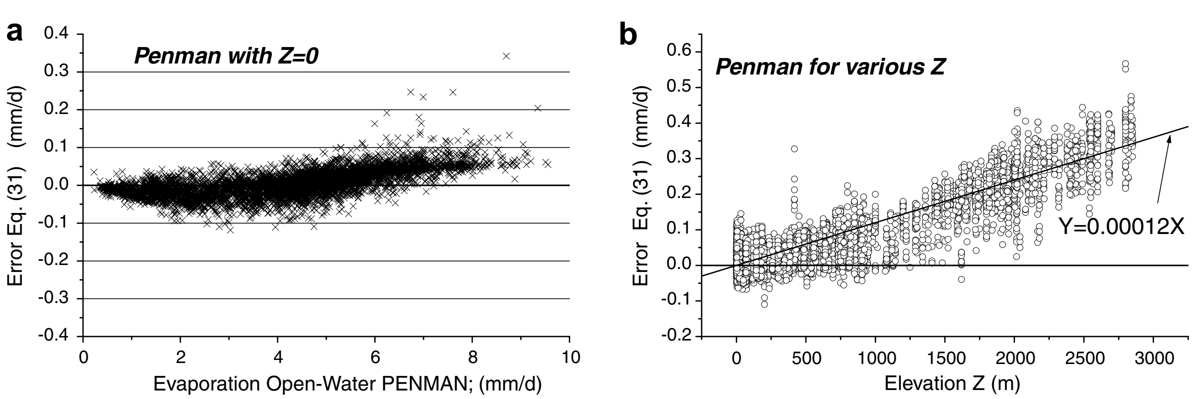

Figure 5 (a) Error of the formula, Eq. (31) with the standardized Penman model applied for the specific value of site elevation, $Z = 0$. (b) Error of the formula, Eq. (31) with the standardized Penman model applied for various values of site elevation, Z.

$$E_{OW} \approx 0.051 \times (1 - 0.08) \times 24.2 \times \sqrt{21.65 + 9.5} - 2.4 \times (24.2/42.2)^2 + 0.052 \times (21.65 + 20)(1 - 67.0/100) \times (0.62 + 0.54 \times 1.51) = 6.58\ \text{mm/d}.$$

By following the standardized sequence of computations for the Penman equation (Shuttleworth, 1993) the potential evaporation is computed as E_{PEN} = 6.51 mm/d (the relative error of the estimate obtained from Eq. (32) is only 1%).

Applying the expression, which does not require wind speed data, Eq. (33) yields

$$E_{PEN} \approx 0.047 \times 24.2 \times \sqrt{21.65 + 9.5} - 2.4 \times (24.2/42.2)^2 + 0.09 \times (21.65 + 20)(1 - 67.0/100) = 6.77\ \text{mm/d}.$$

The relative error of the estimate obtained from Eq. (33) is 4%.

Comparisons

Monthly and daily data sets were used to test the performance of the derived formulas. A first series of data sets used for evaluation was obtained from the United Nations Food and Agriculture Organization (UN-FAO) database, known as CLIMWAT (Smith, 1993), which includes long-term monthly average data. Thirteen countries with relatively high quality records of climatic data (Temesgen et al., 1999) were selected: Spain, France, Italy, Greece, and Cyprus in Europe; Pakistan, Libanon and some stations of India in Asia; Egypt, Tunisia, Algeria, Ethiopia, and Sudan in Africa. The total number of the selected stations is 535. These data were selected for the evaluation of the derived formulas because they practically cover the entire typical range of variation of the input weather variables T, R_S, RH, and u. Some of the countries were selected to represent humid and semi-humid temperate climates, while others represent dry arid and semi-arid tropical climates (Temesgen et al., 1999). From the full set of data, the n_0 = 4461 monthly estimates corresponding to well watered conditions (Temesgen et al., 1999) were retained for comparisons.

The standardized Penman (1948) computational scheme as recommended by Shuttleworth (1993) was used as the reference method to estimate potential evaporation, E_{OW}, from hypothetical open water surface at various locations of the globe.

The potential evaporation from hypothetical open water surfaces was estimated by the formula, Eq. (31) with a_U = 1, corresponding to the original Penman (1948) wind function. The estimation of the formula, Eq. (31), was compared with the results of the standard method of Penman (1948), Eq. (1) with a_U = 1, for the 535 stations over the globe. The scatter plot in Figure 6(a) shows a perfect agreement between

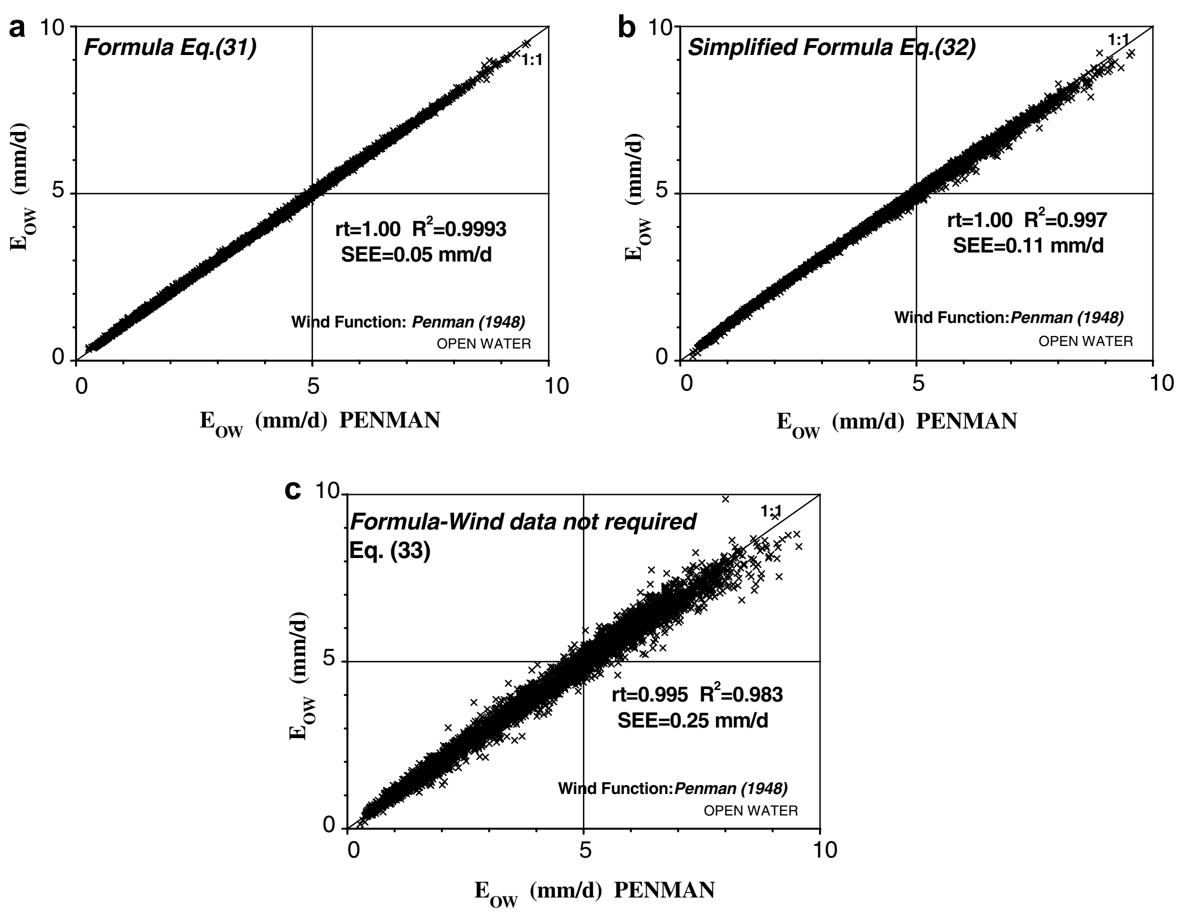

Figure 6 Monthly values of potential open-water evaporation estimated by the suggested expressions versus the standardized Penman scheme corresponding to the original Penman (1948) wind function, for a global climatic data set.

the E_{OW}-formula, Eq. (31), and the standardized Penman model. For all the sites, the points spread around the perfect fit line (1:1) with the following statistical results: The coefficient of determination has a very high value of $R^2 = 0.9993$, the systematic error is practically null, rt = 1.00, where rt is the long-term average ratio of ''approximate'' to ''reference'' estimates, and the value of the standard error, SEE, is only 0.05 mm/d corresponding to 1.1% only of the long-term average value of E_{OW}. If the even more simplified version, Eq. (32), is applied – instead of Eq. (31) – then the accuracy of estimations is slightly decreased (Fig. 6b). However, the agreement between the simplified version, Eq. (32) and the standardized Penman method remains excellent. The value of the coefficient of determination remains high $R^2 = 0.997$, the systematic error is practically null, rt = 1.00 and the value of SEE = 0.11 mm/d is relatively low corresponding to 2.5% only of the long term average value of E_{OW}. If R_A and N are estimated by the simplified expressions, Eqs. (34) and (35), then the accuracy of estimations is further slightly decreased ($R^2 = 0.991$).

Lastly, the suggested expression, Eq. (33), which does not require wind data, produced remarkably good estimates close to the standardized Penman method E_{OW} (Fig. 6c). The systematic error of the suggested Eq. (33) is insignificant, rt = 0.995, the coefficient of determination is relatively high $R^2 = 0.983$, and the value of SEE = 0.25 mm/d.

The Linacre (1993) wind function, $f_U \approx 0.54u$, is perhaps more appropriate to describe evaporation from large extension open water surfaces (i.e., large lakes). Therefore, a series of comparisons were performed using as basis the Linacre (1993) wind function. The standardized Penman procedure applied using the Linacre (1993) wind function was considered as the standard method for estimating evaporation from large open water surfaces (Eq. (1) with $a_U = 0$). The suggested formulas Eqs. (31)–(33) were applied for a wind coefficient value of $a_U = 0$ and compared with the reference Penman method (applied with $a_U = 0$) for the $n_0 = 4461$ data from the 535 stations over the globe. In this case too, the results indicate a perfect agreement of the suggested formulas, Eqs. (31) and (32) with the Penman model (with $R^2 = 0.9992$ and $R^2 = 0.997$, respectively). The suggested expression, Eq. (33), which does not require wind data, produced also remarkably good estimates with $R^2 = 0.980$.

Daily weather data obtained at the region of Davis CA characterized by a semi-arid climate were also used. Daily minimum and maximum temperatures, minimum and maximum relative humidity values, solar radiation and wind speed collected by the California Irrigation Management Information System (CIMIS) at the weather station located at Davis CA (elevation 18 m, latitude 38°32′N) were used for comparisons. The station is located on a large well-irrigated grassed tract. The data used concern a six-year period from January 1, 1994 to December 31, 1999.

Assuming the original Penman-1948 wind function, $f_U = a_U + 0.536u$ with $a_U = 1$, the potential evaporation from a hypothetical open water surface at the site of Davis was estimated by the following expressions (with $a_U = 1$): the formula, Eq. (31), the even more simplified version, Eq. (32), and the expression which does not require the wind data, Eq. (33). All these expressions were compared with the standardized Penman method (Shuttleworth, 1993), Eq. (1) with $a_U = 1$, for the $n_0 = 2177$ daily weather data obtained from the Davis station. The correlation results are presented in Figure 7a–c.

The results presented in Figure 7 for daily data lead to conclusions similar to those obtained from the comparison of monthly estimations in the previous section. Formulas (31) and (32) show a perfect agreement with the standard method. The suggested E_{OW}-formula, which does not require wind data, Eq. (31), has a relatively good performance.

Evapotranspiration — Simplified expressions

The FAO-56 Penman–Monteith scheme (Allen et al., 1998; Monteith, 1965) is considered as the ''standard'' method for estimating reference crop evapotranspiration, ET_0

$$ET_0 = \frac{0.408\Delta(R_n - G)}{\Delta + \gamma(1 + 0.34u)} + \frac{900\gamma}{[\Delta + \gamma(1 + 0.34u)]} \cdot \frac{uD}{(T + 273)}, \tag{37}$$

where G is soil heat flux density (MJ/m^2/d). The weather data required for the application of the FAO-56 Penman–Monteith computational sequence scheme are T_{max}, T_{min}, R_S, RH_{max}, RH_{min} and u, where RH_{max}, RH_{min} are maximum and minimum relative humidity, respectively.

Simplified ET-Formula

The simplified ET-formula that approximates the Penman (1963) – for grass – scheme for estimating the reference crop evapotranspiration, ET_0, is (see Eq. (21)):

$$\begin{aligned} ET_0 \approx\ & 0.051(1-\alpha)R_S\sqrt{T+9.5} - 2.4\left(\frac{R_S}{R_A}\right)^2 \\ & + 0.048(T+20)\left(1-\frac{RH}{100}\right)(0.5+0.536u) \\ & + 0.00012Z. \end{aligned} \tag{38}$$

The formula, Eq. (38), was applied for two different values of albedo (grass being the reference crop), $\alpha = 0.23$, and $\alpha = 0.25$. Systematic comparisons of the estimations of Eq. (38) with the FAO-56 Penman–Monteith estimations have shown that the formula, Eq. (38), using an albedo value of $\alpha = 0.25$ performed better than Eq. (38) with $\alpha = 0.23$. Therefore, in this paper, the value of albedo, $\alpha = 0.25$, is suggested when the derived formula, Eq. (38), is used to estimate ET_0.

For places where wind speed data are not available, a simplified expression, which does not require wind data corresponding to the Penman-1963 scheme, is obtained by substituting the value of u = 2 m/s (average wind speed value over the globe) into Eq. (38). The ET-formula, which does not require wind data, is the following:

$$ET_0 \approx 0.038R_S\sqrt{T+9.5} - 2.4\left(\frac{R_S}{R_A}\right)^2 + 0.075(T+20)\left(1-\frac{RH}{100}\right). \tag{39}$$

The simplified ET-formula, Eq. (38) with $\alpha = 0.25$, was applied to estimate evapotranspiration from hypothetical reference crop (grass) surfaces, ET_0, for the $n_0 = 4461$ data over the globe, corresponding to the 535 stations of the

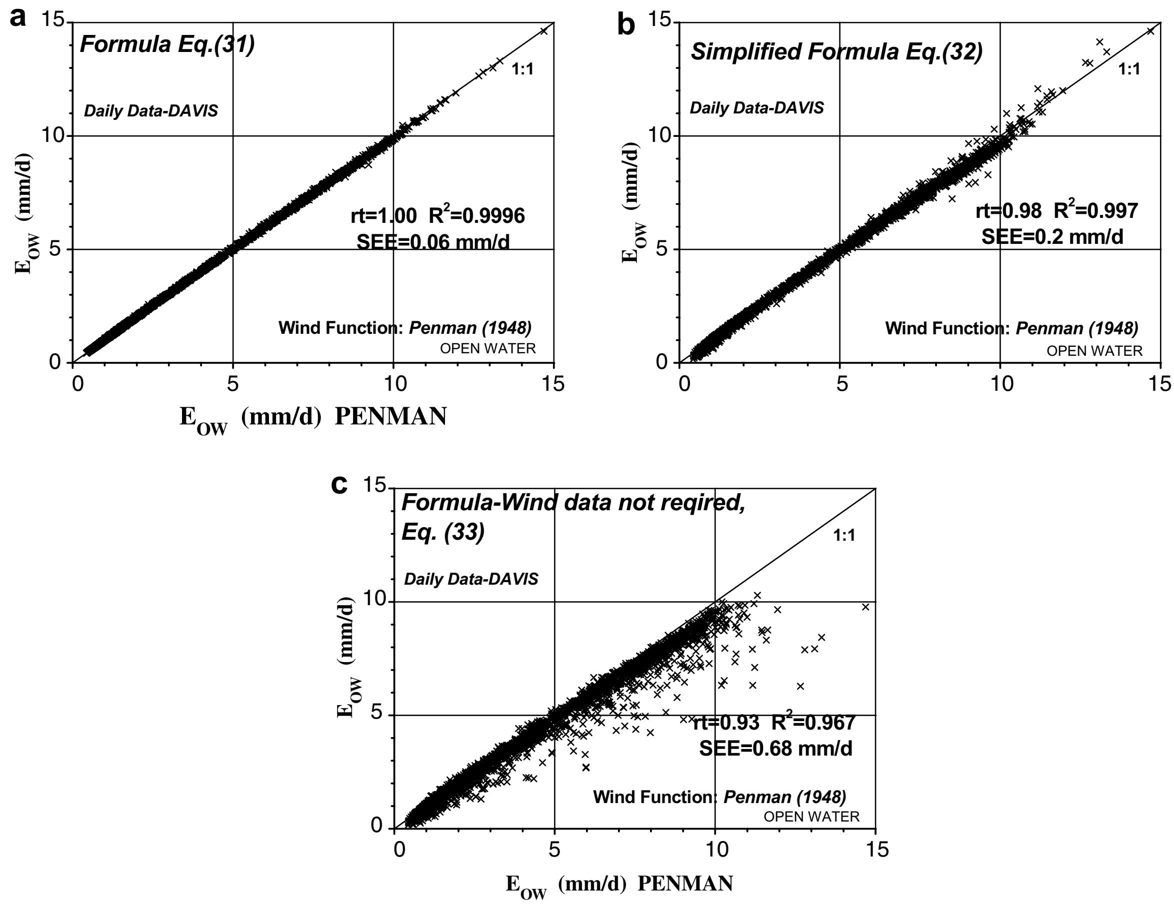

Figure 7 Daily values of potential open-water evaporation estimated by the suggested expressions versus the standardized Penman scheme, for Davis.

selected 13 countries obtained from the CLIMWAT data set. The estimations of the formula were compared with the FAO-56 Penman—Monteith ''reference'' method (FAO P-M). The statistical results of the correlation between the approximate formula Eq. (38) and FAO-56 Penman—Monteith scheme monthly ET_0 for the $n_0 = 4461$ data are $rt = 1.05$, $R^2 = 0.977$ and SEE = 0.27 mm/d (Fig. 8a). The approximate formula, Eq. (38) equivalent to the original 1963 Penman model, although tending to overestimate ET_0, resulted in relatively good estimations.

The expression, Eq. (39), which requires only the variables T, R_S, and RH as input data for estimating ET_0, was also compared with the ''standard'' FAO P-M method. Figure 8b presents the correlation results between Eq. (39)

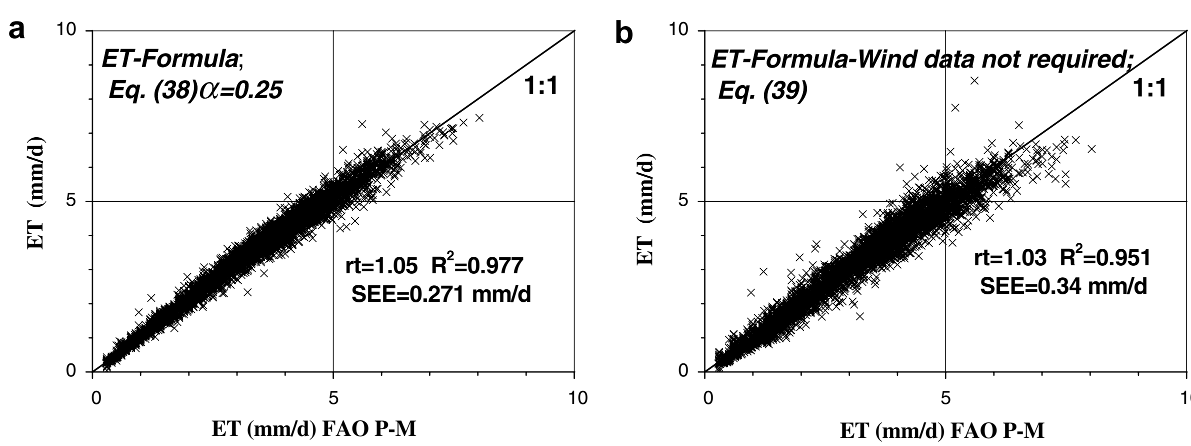

Figure 8 Monthly values of reference crop evapotranspiration estimated by the suggested expressions versus the standardized FAO-56 Penman—Monteith scheme, for a global climatic data set.

and the ''standard'' FAO P-M method. Eq. (39), produced reasonably good estimates of ET_0 and resulted in a low systematic error (3% of systematic overprediction, $rt = 1.03$) a relatively low value of SEE (0.34 mm/d) and the R^2 value of 0.951.

Summary and conclusion

Two simplified formulas, easy to use for routine hydrologic applications, were derived to approximate the Penman equation. The first one depends on the commonly measured weather data, solar radiation, maximum and minimum temperature, relative humidity, and wind velocity. The second formula is a simplified version of the first one. For places where wind speed data are not readily available an expression, which does not require wind speed data, is proposed. The standardized Penman equation is used as reference equation. The standardized computational procedure proposed by Shuttleworth (1993) was followed to evaluate the two components of the Penman equation when solar radiation, temperature, relative humidity, and wind speed are the available data. For a zero elevation site, a first set of approximate formulas is derived assuming simplified approximations for the two weather variables, the mean saturation vapor pressure, and the vapor deficit. Then it follows a correction of the formulas using more a accurate set of approximations for the two weather variables. The effect of the site elevation is also taken into consideration. Simplified expressions were proposed to evaluate the extraterrestrial radiation. The simplicity of the formulas is demonstrated in a computational example.

The suggested new formulas for estimating open water evaporation were tested using the measured long-term monthly data of a global climatic data set (FAO CLIMWAT) as well as the measured daily data from a weather station. The open water evaporation formulas were compared with the standardized Penman equation. Comparisons indicate the excellent performance of the first two formulas that depend on the commonly measured weather data. The suggested expression, which does not require wind data, resulted also in relatively good estimates.

A modification of the new evaporation formula adjusted for the difference between albedos of open water and a grass surface, corresponding to the Penman (1963) — for grass — model is also proposed. Comparisons indicated that the formulas suggested for estimating reference crop evapotranspiration have a reasonably good performance.

Appendix. Computation of the standardized Penman equation parameters

The parameters f and ε', for average climatic conditions, are calculated as (Doorenbos and Pruitt, 1977; Shuttleworth, 1993; Brunt, 1952)

$$f = \left(1.35\frac{R_S}{R_{SO}} - 0.35\right), \tag{40}$$

$$\varepsilon' = (0.34 - 0.14\sqrt{e_a}), \tag{41}$$

where R_{SO} is clear sky radiation calculated as

$$R_{SO} = (0.75 + 2\times 10^{-5}\cdot Z)\cdot R_A, \tag{42}$$

$$e_a = \frac{RH}{100} e_s(T) \tag{43}$$

and

$$e_S(t) = 0.611\cdot\exp\left(\frac{17.27\cdot t}{t + 237.3}\right), \tag{44}$$

where t is temperature in °C.

Brutsaert (1982), Croley (1989) and Dingman (1994) have suggested more accurate formulations for the estimation of long wave energy exchange. However, their approaches require additional measurements of temperature at the soil or the water. Since these data are rarely available, therefore the original simplified form of the Penman equation for the R_{nL} estimation (previously developed) is applied in routine hydrological applications.

The other meteorological parameters of the Penman equation (Eq. (1)), are calculated as follows:

$$\lambda = 2.501 - (2.361\times 10^{-3})\cdot T, \tag{45}$$

$$\gamma = 0.0016286\cdot\frac{P}{\lambda}, \tag{46}$$

where P is atmospheric pressure (kPa) estimated by the following equation:

$$P = 101.3\cdot\left(\frac{293 - 0.0065\cdot Z}{293}\right)^{5.26}, \tag{47}$$

$$\Delta = \frac{4098\cdot e_S}{(T + 237.3)^2}, \tag{48}$$

where Δ is the slope of the saturation vapor pressure curve (kPa/°C).

Measured solar radiation data may be unavailable at many standard meteorological stations. In such case they can be estimated from measured sunshine hours, if available, by the following empirical relationship for average climates

$$R_S = R_A\cdot\left(0.5 + 0.25\cdot\frac{n}{N}\right), \tag{49}$$

where n is measured bright sunshine hours per day (h); N is the maximum possible duration of daylight (h) that can be estimated from tables or by the following set of equations when the latitude of the site and the number of Julian day (or the month) are given (Shuttleworth, 1993; Allen et al., 1994)

$$N = \frac{24}{\pi}\omega_S, \tag{50}$$

where ω_S is the sunset hour angle (radians) given by

$$\omega_S = \arccos(-\tan(\phi)\tan(\delta)), \tag{51}$$

where δ is solar declination (radians) given by

$$\delta = 0.409\sin\left(\frac{2\pi}{365}J - 1.39\right) \tag{52}$$

where J is Julian day number. For monthly estimations the Julian day corresponding to ith month is calculated as

$$J = \mathrm{INT}(30.5i - 14.6). \tag{53}$$

Finally R_A in MJ/m^2/d is calculated as

$$R_A = 37.59 d_r[\omega_S\sin(\phi)\sin(\delta) + \sin(\omega_S)\cos(\phi)\cos(\delta)], \tag{54}$$

where d_r is relative distance between the earth and the sun given by

$$d_r = 1 + 0.033 \cos\left(\frac{2\pi}{365} J\right). \quad (55)$$

In summary the standardized Penman sequence of computations to calculate potential evaporation is as follows: Applying Eq. (53) yields J, Eq. (52) yields δ, Eq. (51) yields ω_S, Eq. (50) yields N, Eq. (55) yields d_r, and finally substituting the previously calculated variables into Eq. (54) yields R_A.

Applying Eq. (49) yields R_S, Eq. (4) yields R_{nS}, Eq. (44) for $t = T_{max}$ yields $e_S(T_{max})$, Eq. (44) for $t = T_{min}$ yields $e_S(T_{min})$, then Eq. (6) yields $e_{S(av)}$, Eq. (43) yields e_a, Eq. (45) yields λ, Eq. (47) yields P, Eq. (46) yields γ, Eq. (42) yields R_{S0}, Eq. (41) yields ε', Eq. (40) yields f, and consequently Eq. (3) yields R_{nL}, Eq. (44) for $t = T$ yields $e_S(T)$, Eq. (48) yields Δ, Eq. (2) yields R_n, and finally substituting the previously calculated variables into the original Penman Eq. (1) yields E_{PEN}.

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