CSE 101 Homework 3

Fall 2018

This homework is due on gradescope Friday November 2nd at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Verifying Shortest Path Lengths, 30 points)**.** *Let G be a directed, weighted graph with potentially negative edge weights, but no negative weight cycles. Let s be a vertex of G. Using Bellman-Ford to compute the shortest path lengths from s to the other vertices is somewhat slow, however verifying that the answer you have is correct is somewhat faster. Provide an algorithm that given values $d(v)$ for each vertex $v \in V$, determines whether or not $d(v)$ is the length of the shortest path from s to v for all v (your algorithm should return a single TRUE/FALSE value, returning FALSE if there is* any *vertex v so that $d(v)$ is not the correct shortest path length from s). For full credit your algorithm should run in linear time or better.*

Question 2 (The Skyline Problem [1], 30 points)**.** *A city's skyline is traced out by n rectangular buildings. In particular, each building has a height h_i and a base given by an interval $[x_i, y_i]$. For our purposes we will assume that the x_i and y_i used by the n buildings are exactly the integers from 1 to $2n$ without any repeats. The height of the skyline at location ℓ_i at a position i is given by $\max_{i:x_i \leq \ell \leq y_i} h_i$. Give an algorithm to compute $\ell_1, \ell_2, \ldots, \ell_{2n}$. For full credit, your algorithm should run in time $O(n \log(n))$ or better. Hint: use divide an conquer. Your subproblems may need to look slightly more complicated than your original problem.*

Question 3 (Densest Interval, 30 points)**.** *Given a set S of n real numbers and a positive real number L, give an algorithm to compute an interval of length at most L that contains as many elements of S as possible. For full credit, your algorithm should run in time $O(n \log(n))$ or better.*

Question 4 (Divide an Conquer Runtimes, 10 points)**.** *For each of the following, give the asymptotic runtime of the following divide and conquer algorithms:*

(a) An algorithm that splits a problem of size n into five problems of size $n/3$ and does $O(n^{3/2})$ work combining the answers.

(b) An algorithm that splits a problem of size n into eight problems of size $n/2$ and does $O(n^3)$ work combining the answers.

(c) An algorithm that splits a problem of size n into three problems of size $2n/3$ and does $O(n^3)$ work combining the answers.

(d) An algorithm that splits a problem of size n into one problem of size $99n/100$ and does $O(1)$ work combining the answers.

(e) An algorithm that splits a problem of size n into two problems of size $2n$ and does $O(n)$ work combining the answers.

Question 5 (Extra credit, 1 point)**.** *Approximately how much time did you spend on this homework?*

[1] s d c t f{N 1 c e _ d 0 r K i N G _ C @ p t 4 1 N}
This string is not related to the assignment. If you don't know what it is, don't worry about it. If you are curious about it, see https://sdc.tf/.