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Chaos control and schedule of shuttle buses

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Abstract

We study the dynamical behavior of a few shuttle buses when they pass each other freely and control the speed to retrieve the loading's delay. The dynamics of the buses is expressed in terms of the nonlinear maps. The tour times of buses and the time headway between buses exhibit the complex behavior with varying trips. The buses exhibit deterministic chaos even if there are no noises. Bus speeds up to retrieve the delay induced by loading the passengers on its bus. The bus chaos is controlled by varying the degree of speedup. The chaotic motion depends on both loading and speedup's parameters. The shuttle bus schedule is connected with the complex motions of shuttle buses. The region map (phase diagram) is shown to control the complex motions of buses.

Keywords: Chaos; Bus schedule; Traffic dynamics; Nonlinear map

1. Introduction

Recently, transportation problems have attracted much attention in the fields of physics [1–5]. The traffic flow, pedestrian flow, and bus-route problem have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–23]. The interesting dynamical phase transitions have been found in the transportation system. The jams and chaos are typical signatures of the complex behavior of traffic flow [24,25]. The shuttle bus system is closely connected to the traffic flow.

Frequently, one experiences an irregular arrival of shuttle buses when he waits the coming bus at a terminal. The irregularity will be induced not only by stochastic variation in passengers arriving at the terminal, but also by varying headway between a bus and the bus ahead of it. In managing the shuttle bus operation, the usual criterion is that one should be able to transport everyone from the starting point to his destination within some period and a passenger's waiting time should not exceed some specified value [26–28]. Another criterion is that buses shuttle on time between the origin and destination. Therefore, it is important and necessary to suppress the irregularity of tour times and operate the buses regularly and on time.

Until now, some models of the bus route system have been studied. In the bus route model with many buses, it has been found that the bunching transition between an inhomogeneous jammed and a homogeneous phases occurs with increasing bus density [17–22]. In the cyclic bus system not passing each other, it has been shown

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that the bus exhibits such complex behaviors as the periodic and chaotic motions [27]. Also, it has been found that the distinct chaotic motion is induced by passing each other freely in the system including a few shuttle buses [28]. The dynamical model of the shuttle buses has been extended to take into account the maximum capacity of buses. Fluctuation of the riding passengers has been connected to the chaotic motions of shuttle buses [29]. The chaos is attributable to such origin that the time headway varies abruptly when buses pass each other freely. With the increase of awaiting passengers at bus stop, a bus slows down because it takes more time for awaiting passengers to board the bus. The time headway of the bus increases more and more with trips. However, the time headway of the next coming bus decreases. In time, the bus is overtaken by the next coming bus. As the result, the bus motion becomes irregular. If the bus slows down, the bus will speed up in order to operate it on time. It will be expected that the speedup of a bus suppresses the irregularity.

In this paper, we study the chaos control by the speedup in the shuttle bus system. We present a dynamical model to describe the motions of buses taking into account the speedup. We investigate the dynamical behavior of buses induced by the interaction between buses through the passengers waiting at the starting point, when the buses shuttle between the starting point and destination repeatedly. We clarify the dynamical states of shuttle buses by varying both loading and speedup parameters. We show the phase diagram (region map) for the dynamical states.

2. Nonlinear map model

We consider the dynamical behavior of M buses shuttling between the starting point (origin) and the destination. Fig. 1 shows the schematic illustration of the shuttle bus system. The awaiting passengers at the origin board a bus just arrived, then the bus starts at the origin, moves toward the destination, all currently riding passengers leave the bus when the bus arrives at the destination, and the bus returns to the origin. When a bus reaches other buses, the bus passes those freely. A bus is stopping at the origin during the period that all the passengers waiting at the origin board the bus. Similarly, a bus is stopping at the destination during the period that all boarding passengers leave the bus. The bus slows down proportionally to the stopping periods. Also, the bus speeds up to retrieve the delay of stopping.

We derive the equations of bus motion to describe the above dynamics. Define the number of passengers boarding bus i at trip m by $B_i(m)$. The parameter γ is the time it takes one passenger to board the bus, so $\gamma B_i(m)$ is the amount of time needed to board all the passengers at the origin. The parameter η is the time it takes for one passenger to leave the bus, so $\eta B_i(m)$ is the amount of time needed to leave all the passengers at the destination. The moving time of bus i at trip m is $2L/V_i(m)$ where L is the length between the origin and the

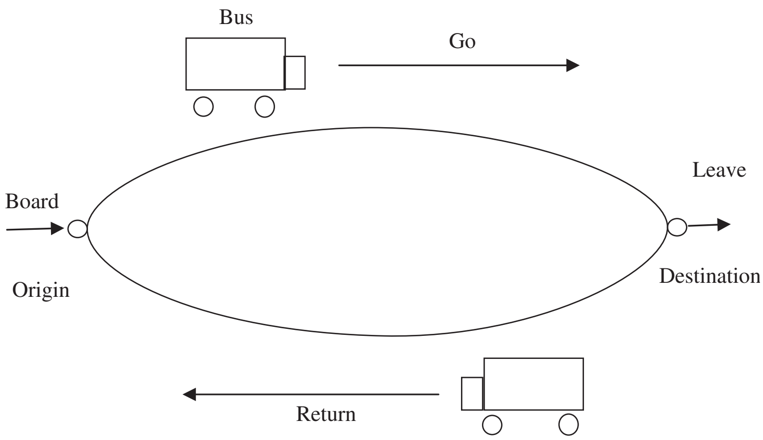


Fig. 1. Schematic illustration of the shuttle bus system. The awaiting passengers at the origin board a bus just arrived, then the bus starts at the origin, moves toward the destination, all currently riding passengers leave the bus when the bus arrives at the destination, and the bus returns to the origin. When a bus reaches other buses, the bus passes those freely. A bus is stopping at the origin during the period that all the passengers waiting at the origin board the bus. Similarly, a bus is stopping at the destination during the period that all boarding passengers leave the bus. The bus slows down proportionally to the stopping periods. The bus speeds up to retrieve the delay of stopping.

destination, and $V_i(m)$ is the mean speed of bus i at trip m. The tour time equals the sum of these periods. Then, the arrival time $t_i(m+1)$ of bus i at the origin and trip $m+1$ is given by

$$t_i(m+1) = t_i(m) + (\gamma + \eta)B_i(m) + \frac{2L}{V_i(m)} \quad \text{for } i = 1, 2, \ldots, M. \tag{1}$$

If the maximum capacity of bus i is sufficiently large, the number $B_i(m)$ of boarding passengers equals the number $W_i(m)$ of passengers waiting at the origin just before bus i arrives at the origin and trip m. It is expressed by

$$W_i(m) = B_i(m) = \mu(t_i(m) - t_{i'}(m')), \tag{2}$$

where bus i' is that arrived at the origin just before bus i arrives at the origin and trip m, $t_{i'}(m')$ is the arrival time of bus i' at the origin. New passengers arrive at the origin at rate μ. So $\mu(t_i(m)-t_{i'}(m'))$ is the number of passengers that have arrived since the bus ahead i' leaves the origin.

A bus speeds up proportionally to the delay of the stopping for boarding and getting off. The bus velocity is expressed by

$$V_i(m) = V_0 + s_i(\gamma + \eta)B_i(m), \tag{3}$$

where parameter s_i represents the degree of speedup for bus i and V_0 is the mean speed of buses without speedup. By combining Eqs. (1)–(3), one obtains the equation of motion for bus i:

$$t_i(m+1) = t_i(m) + \mu(\gamma + \eta)(t_i(m) - t_{i'}(m')) + \frac{2L}{V_0 + s_i\mu(\gamma + \eta)(t_i(m) - t_{i'}(m'))} \quad \text{for } i = 1, 2, \ldots, M. \tag{4}$$

By dividing time by the characteristic time $2L/V_0$, one obtains the following equation for the dimensionless arrival time of bus i at the origin:

$$T_i(m+1) = T_i(m) + \Gamma(T_i(m) - T_{i'}(m')) + \frac{1}{1 + S_i(T_i(m) - T_{i'}(m'))}, \tag{5}$$

where $T_i(m) \equiv t_i(m)V_0/2L$, $\Gamma \equiv \mu(\gamma + \eta)$, and $S_i \equiv s_i\mu(\gamma + \eta)2L/V_0^2$.

Thus, the dynamics of the buses is described by the simplified nonlinear map (5). The map should be iterated simultaneously for M buses. The dynamical property of the map is controlled by two parameters: loading parameter Γ and speedup parameter S_i. When perspective passengers increase, the value of loading parameter becomes high.

For a few buses, the order of buses changes with trip m and the time headway between buses changes from trip to trip because a bus passes other buses or is outstripped by other buses. It will be expected that the buses exhibit a complex behavior. We study whether or not the irregular motion changes to the regular motion by varying the speed.

3. Simulation result

We investigate the dynamical behavior of shuttle buses by the use of iterate of map (5). We calculate the time headways and tour times with varying trips for the typical case of two buses. We study how time headway varies with both loading and speedup parameters. Fig. 2 shows the plots of the time headway $H_1(m)$ of bus 1 against loading parameter Γ from sufficiently large trip, $m = 900-1000$. Diagrams (a)–(d) are obtained, respectively, for (a) $S_1 = S_2 = 0$, (b) $S_1 = S_2 = 0.2$, (c) $S_1 = 0.3$ and $S_2 = 0.2$, and (d) $S_1 = 0.5$ and $S_2 = 0.2$. The enlargements of Fig. 2(a)–(d) are shown in Fig. 3(a)–(d) for $0<\Gamma<0.5$. Diagram (a) indicates the distribution of time headway of bus 1 for the case of no speedup. Diagram (b) represents the time headway distribution for the case that two buses speed up with the same degree. Diagrams (c) and (d) indicate the time-headway distributions for such cases that two buses speed up one another with different rates.

For case (a), when the loading parameter Γ is low, the time headway fluctuates around the three values and the distribution exhibits the three peaks around the values. With increasing loading parameter Γ, the localized distributions extend around the peaks and become the two extended distributions. When loading parameter Γ

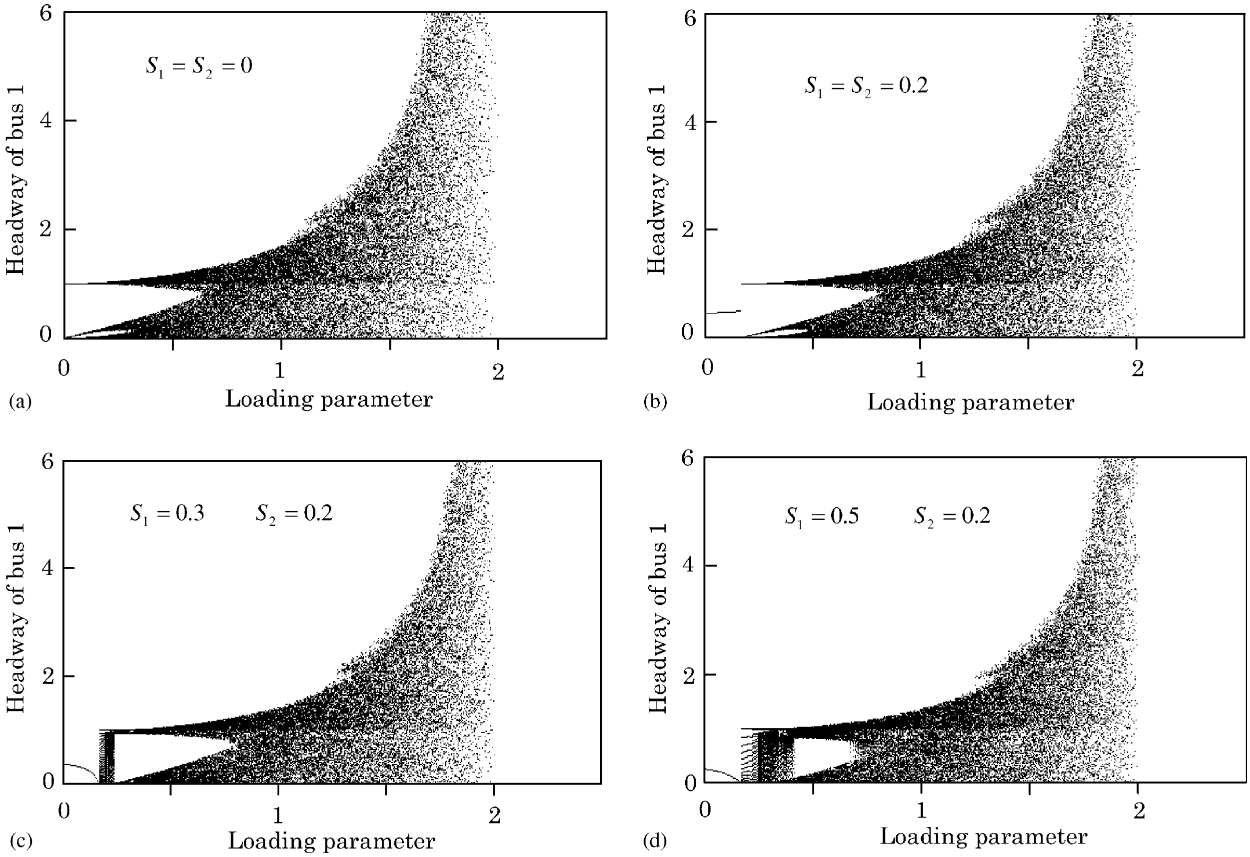


Fig. 2. Plots of the time headway $H_1(m)$ of bus 1 against loading parameter Γ from sufficiently large trip $m = 900-1000$. Diagrams (a)–(d) are obtained, respectively, for (a) $S_1 = S_2 = 0$, (b) $S_1 = S_2 = 0.2$, (c) $S_1 = 0.3$ and $S_2 = 0.2$, and (d) $S_1 = 0.5$ and $S_2 = 0.2$.

is high, the two extended distributions collapse and becomes a single extended distribution. If loading parameter Γ is higher than 2, the delay of buses increases with trip m and the time headway diverges. Thus, the chaotic motion occurs for $0 < \Gamma < 2$.

For the cases (b)–(d), two buses move at the constant speed and the bus motions do not fluctuate until $\Gamma = 0.167$. The fluctuation of bus motions is suppressed by the speedup. For case (b), when the loading parameter is higher than $\Gamma = 0.167$, the time headway fluctuates similarly to that of case (a). The time-headway distribution of case (b) for $0.167 < \Gamma < 0.5$ in Fig. 3(b) agrees with that of case (a) for $0 < \Gamma < 0.333$. For such cases (c) and (d) that the degree of speedup is different one another, when the loading parameter is higher than $\Gamma = 0.167$, the time headway fluctuates periodically and then exhibits the chaotic behavior. The dynamic behavior of time headway in Fig. 3(c) is distinctly different from that of Fig. 3(d). Thus, the difference between speedup parameters induces the very complex motions of buses. For case (d), bus 1 moves regularly at the constant speed until $\Gamma = 0.167$. At point 1 ($\Gamma = 0.167$), the motion of bus 1 changes from the regular to the periodic motions. After some period-adding bifurcations occur, the bus motion changes to the chaos at point 2 ($\Gamma = 0.248$). Furthermore, the bus motion exhibits the other dynamical transition at point 3 ($\Gamma = 0.407$).

Fig. 4(a) and (b) show the plots of tour times of buses 1 and 2 against loading parameter Γ from sufficiently large trip $m = 900-1000$ for case of $S_1 = 0.5$ and $S_2 = 0.2$. The plots of tour times correspond to the time headway of Fig. 2(d). The enlargements of Fig. 4(a) and (b) are shown in Fig. 5(a) and (b) for $0 < \Gamma < 0.5$. Buses 1 and 2 move regularly until $\Gamma = 0.167$. The tour times of buses 1 and 2 increases with loading parameter until $\Gamma = 0.167$. When the loading parameter is higher than value $\Gamma = 0.167$ of the transition point,

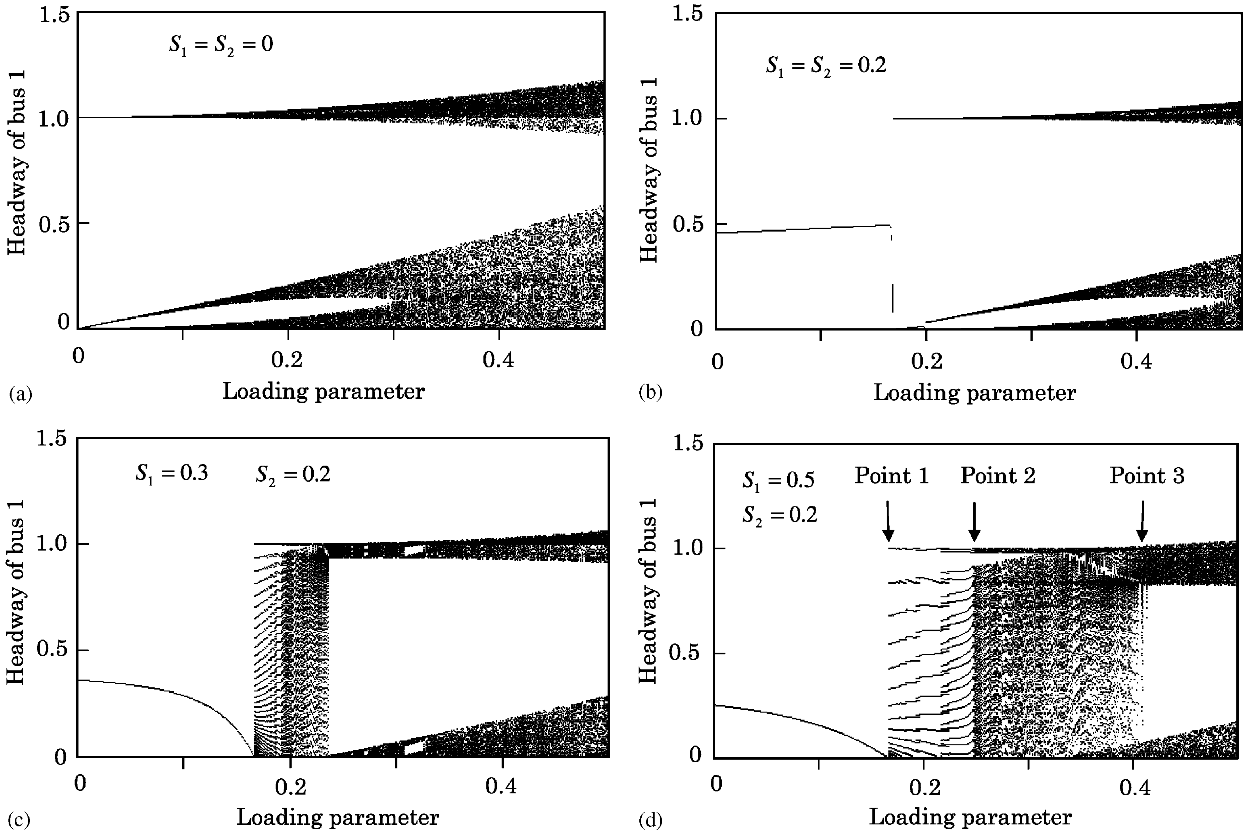


Fig. 3. Enlargements of Fig. 2(a)–(d) for $0 < \Gamma < 0.5$. Diagrams (a)–(d) correspond to those in Fig. 2(a)–(d).

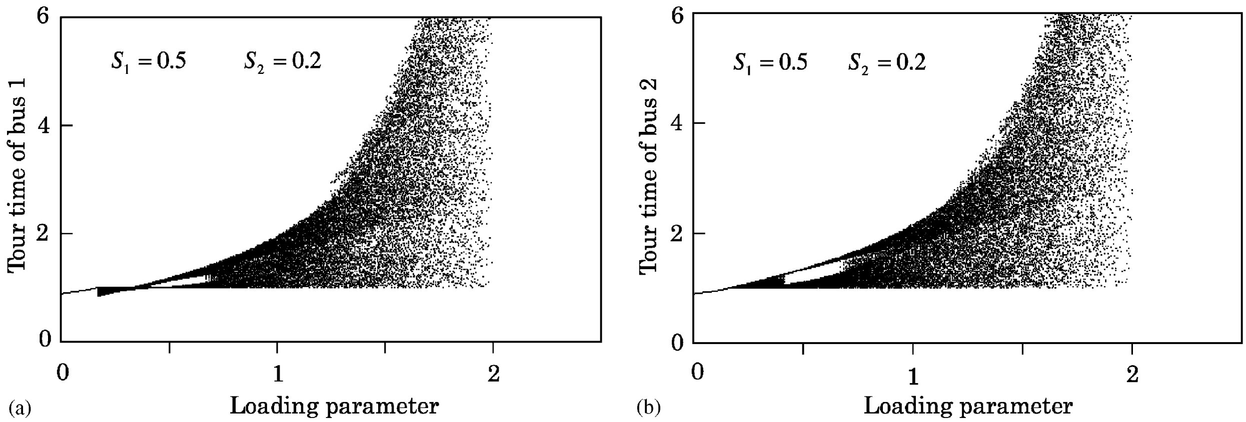
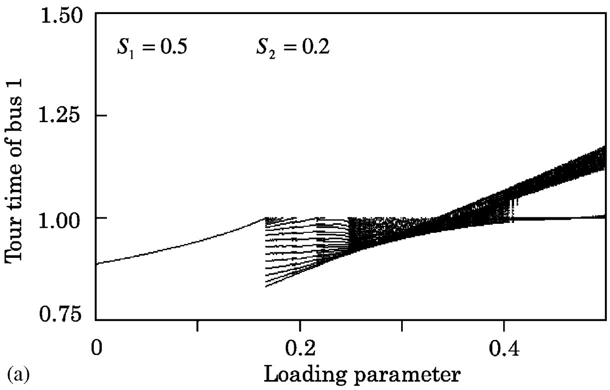


Fig. 4. Plots of tour times of buses 1 and 2 against loading parameter Γ from sufficiently large trip $m = 900-1000$ for case of $S_1 = 0.5$ and $S_2 = 0.2$. The plots of tour times correspond to the time headway of Fig. 2(d).

the tour time of bus 1 fluctuates highly and periodically, while the fluctuation of tour time of bus 2 is very small. The fluctuation of tour time of bus 1 becomes small with loading parameter until $\Gamma = 0.333$. When loading parameter is higher than $\Gamma = 0.333$, the fluctuation of bus 1 increases again with loading parameter. However, the fluctuation of bus 2 increases as the loading parameter increases. Thus, the behavior of bus 1 is definitely different from that of bus 2.



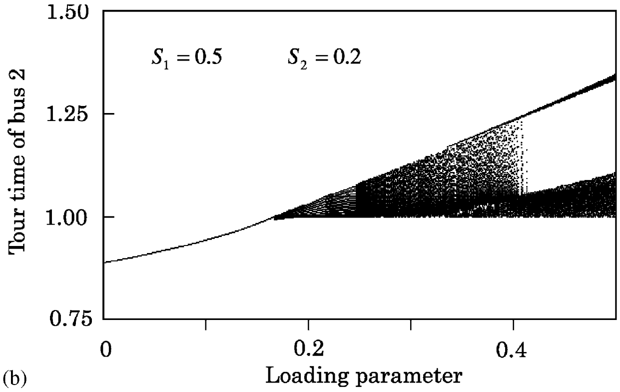


Fig. 5. Enlargements of Fig. 4(a) and (b) for $0<\Gamma<0.5$. Diagrams (a) and (b) correspond to those in Fig. 4(a) and (b).

We derive the return map of bus dynamics to study the periodic and chaotic motions of buses. Fig. 6 shows the plots of $H_1(m+1)$ against $H_1(m)$ from $m = 1000$ to 2000 for speedup parameters $S_1 = 0.5$ and $S_2 = 0.2$ in Figs. 2(d) and 3(d) where $H_1(m)$ is the time headway of bus 1 at trip m. Return maps (a)–(d) are obtained for loading parameters $\Gamma = 0.2$, 0.3, 0.5, and 0.8. Return map (a) of $\Gamma = 0.2$ consists of 11 points. It exhibits the periodic motion with period 11 (see Fig. 3(d)). Return map (b) of $\Gamma = 0.3$ is a piecewise map. It produces the chaotic motion with a single extended distribution (see Fig. 3(d)). Return map (c) of $\Gamma = 0.5$ is also a piecewise map. It induces the chaotic motion with two extended distributions (see Fig. 3(d)). Return map (c) of $\Gamma = 0.8$ is a very complex piecewise map which is multi-valued function. It produces the chaotic motion with a single extended distribution (see Fig. 2(d)). Thus, the bus dynamics exhibits various return maps, which are characteristic of the complex motions for various values of loading parameters. It is found that the bus dynamics is described in terms of the simple piecewise map in Fig. 6(b) for low value of loading parameter.

We study the mean values of time headways and tour times of buses 1 and 2. Fig. 7(a) shows the plots of mean headways H_{1a}, H_{2a} and tour times ΔT_{1a}, ΔT_{2a} against loading parameter Γ for speedup parameters $S_1 = 0.5$ and $S_2 = 0.2$ in Figs. 2(d) and 3(d). Here H_{1a} and H_{2a} are the mean values of time headways of buses 1 and 2. ΔT_{1a} and ΔT_{2a} are the mean values of tour times of buses 1 and 2. Fig. 7(b) is the enlargement of Fig. 7(a) for $0<\Gamma<0.5$. The transition points 1, 2, and 3 correspond, respectively, to those in Fig. 3(d). Buses 1 and 2 move with the same value of tour time but the time headway of bus 1 is different from that of bus 2 when the loading parameter is lower than the value at point 1. The time headway of bus 1 decreases with increasing loading parameter until transition point 1 and becomes zero at point 1, while the time headway of bus 2 increases with loading parameter until point 1 and agrees with the tour time at point 1. After the dynamic transition from the regular to the periodic motions occurs at point 1, the tour time of bus 1 is different from that of bus 2 until the transition point 3 and agrees again with that of bus 2 at point 3, while the time headway of bus 2 decreases accordingly as the loading parameter increases and the time headway of bus 1 increases with loading parameter. Before the transition point 2, the headway of bus 1 is higher than that of bus 2. With increasing loading parameter furthermore, the time headway of bus 1 equals that of bus 2 at point 5 and both headways increase with loading parameter. When the loading parameter is higher than the value at point 4, the tour time of bus 2 is higher than that of bus 1. At $\Gamma = 2.0$, the headways and tour times diverge. Thus, the mean values of tour times and headways exhibit the very complex behavior due to the regular, periodic and chaotic motions of buses.

We study the root-mean squares (rms) of time headways and tour times of buses 1 and 2. Fig. 7(c) shows the plots of rms of headways H_{1v}, H_{2v} and tour times ΔT_{1v}, ΔT_{2v} against loading parameter Γ for speedup parameters $S_1 = 0.5$ and $S_2 = 0.2$ in Figs. 2(d) and 3(d). Here H_{1v} and H_{2v} are the values of rms of time headways of buses 1 and 2. ΔT_{1v} and ΔT_{2v} are the values of rms of tour times of buses 1 and 2. Fig. 7(d) is the enlargement of Fig. 7(c) for $0<\Gamma<0.5$. The transition points 1, 2, and 3 correspond, respectively, to those in Fig. 3(d) and Fig. 7(b). The rms of tour times and headways are zero until transition point 1 because the buses move regularly. When the loading parameter is higher than the value of point 1, the rms value of headway of

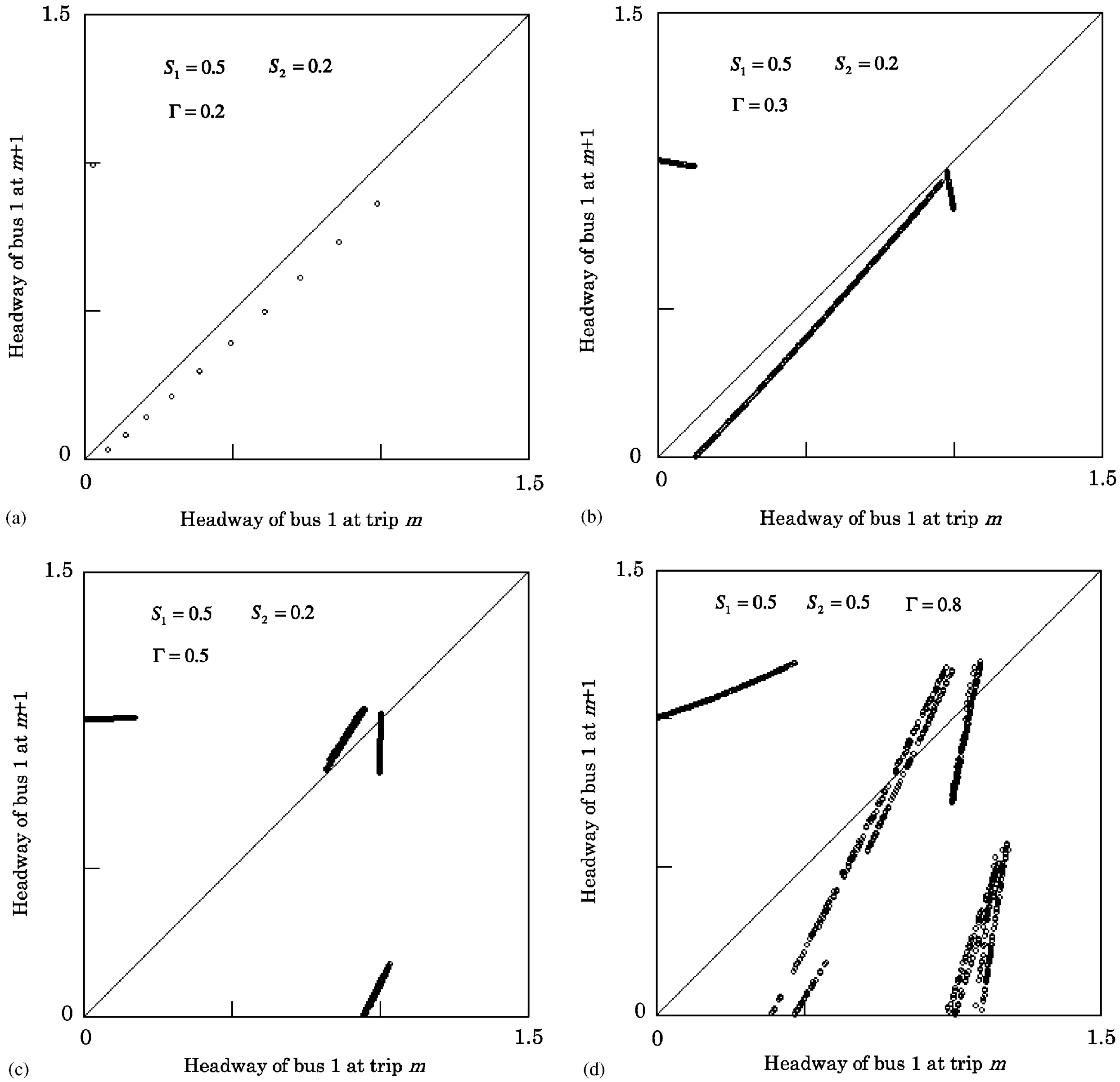


Fig. 6. Plots of $H_1(m+1)$ against $H_1(m)$ from $m = 1000$ to 2000 for speedup parameters $S_1 = 0.5$ and $S_2 = 0.2$ in Figs. 2(d) and 3(d) where $H_1(m)$ is the time headway of bus 1 at trip m. Return maps (a)–(d) are obtained for loading parameters $\Gamma = 0.2$, 0.3, 0.5, and 0.8.

bus 2 increases monotonically as the loading parameter increases, while the rms value of headway of bus 1 decreases until $\Gamma = 0.333$ and increases monotonically with loading parameter for $\Gamma > 0.333$. At point 2 where the transition to the chaos occurs, the rms of headway of bus 1 equals that of bus 2. At $\Gamma = 2.0$, the rms values of headways of buses 1 and 2 diverges. The rms values of tour times decreases a little bit until point 3 accordingly as the loading parameter increases. When the loading parameter is higher than the value of point 3, the rms values of tour times increases monotonically with loading parameter and diverge at $\Gamma = 2.0$. The rms of tour time of bus 2 is a little higher than that of bus 1. Thus, the rms's of tour times and headways exhibit the characteristic behavior similarly to the mean values.

We study the phase diagram (region map) for the bus control. Fig. 8 shows the plot of transition points between the regular and periodic motions in phase space (Γ, S_1) of loading and speedup parameters for

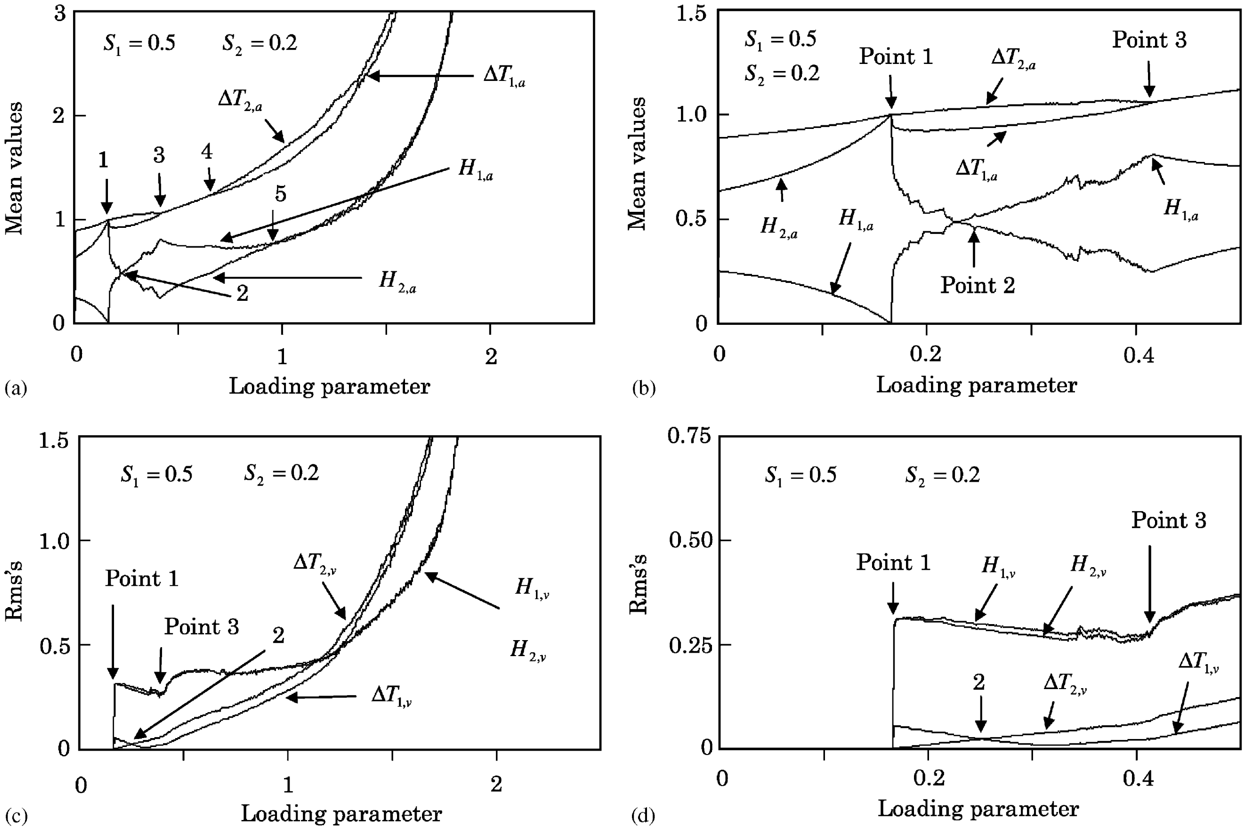


Fig. 7. (a) Plots of mean headways H_{1a}, H_{2a} and tour times ΔT_{1a}, ΔT_{2a} against loading parameter Γ for speedup parameters $S_1 = 0.5$ and $S_2 = 0.2$ in Figs. 2(d) and 3(d). Here H_{1a} and H_{2a} are the mean values of time headways of buses 1 and 2. ΔT_{1a} and ΔT_{2a} are the mean values of tour times of buses 1 and 2. (b) Enlargement of Fig. 7(a) for $0 < \Gamma < 0.5$. (c) Plots of rms of headways H_{1v}, H_{2v} and tour times ΔT_{1v}, ΔT_{2v} against loading parameter Γ for speedup parameters $S_1 = 0.5$ and $S_2 = 0.2$ in Figs. 2(d) and 3(d). Here H_{1v} and H_{2v} are the values of rms of time headways of buses 1 and 2. ΔT_{1v} and ΔT_{2v} are the values of rms of tour times of buses 1 and 2. (d) Enlargement of Fig. 7(c) for $0 < \Gamma < 0.5$.

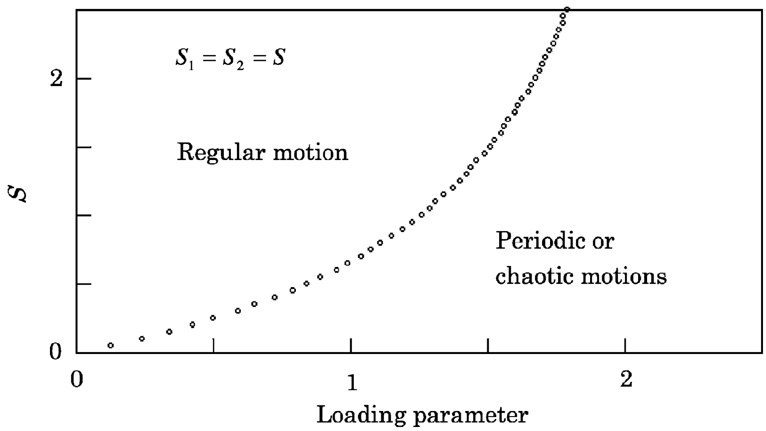


Fig. 8. Phase diagram (region map) for the regular and periodic (or chaotic) motions in phase space (Γ, S_1) of loading and speedup parameters for $S_1 = S_2$. The circles indicate the transition points obtained from simulation.

$S_1 = S_2$. The circles indicate the transition points obtained from simulation. The regular motion occurs in the region above the transition points, while the periodic or chaotic motions occur in the region below the transition points. Thus, the chaotic motion of buses is suppressed by the speedup. As the result, one is able to control the chaos and to obtain the bus schedule deterministically.

4. Summary

We have presented the dynamical model of the shuttle buses to take into account both loading's delay and speedup. We have expressed the dynamics of the buses in terms of the nonlinear maps. We have studied the dynamic behaviors of buses by varying both loading and speedup parameters. We have shown that the buses display the very complex behavior with trips and the behavior of buses exhibits a deterministic chaos. We have clarified whether or not the chaotic motion is controlled by the speedup. We have found that the fluctuation of arrival time is attributable to the deterministic chaos. We have shown that the chaotic motion depends on both loading and speedup parameters. We have found that when the loading parameter is higher than the threshold, the dynamical transition from the regular to periodic or chaotic motions occurs and the transition point is controlled by the speedup. We have clarified the dependence of the threshold (transition point) on the speedup parameter and the bus schedule is closely connected with the complex motion of buses.

References

[1] T. Nagatani, Rep. Prog. Phys. 65 (2002) 1331.
[2] D. Helbing, Rev. Mod. Phys. 73 (2001) 1067.
[3] D. Chowdhury, L. Santen, A. Schadscheider, Phys. Rep. 329 (2000) 199.
[4] B.S. Kerner, The Physics of Traffic, Springer, Heidelberg, 2004.
[5] D. Helbing, H.J. Herrmann, M. Schreckenberg, D.E. Wolf (Eds.), Traffic and Granular Flow '99, Springer, Heidelberg, 2000.
[6] K. Nagel, M. Schreckenberg, J. Phys. I France 2 (1992) 2221.
[7] E. Ben-Naim, P.L. Krapivsky, S. Redner, Phys. Rev. E 50 (1994) 822.
[8] E. Tomer, L. Safonov, S. Havlin, Phys. Rev. Lett. 84 (2000) 382.
[9] M. Treiber, A. Hennecke, D. Helbing, Phys. Rev. E 62 (2000) 1805.
[10] H.K. Lee, H.-W. Lee, D. Kim, Phys. Rev. E 64 (2001) 056126.
[11] I. Lubashevsky, S. Kalenkov, R. Mahnke, Phys. Rev. E 65 (2002) 036140.
[12] I. Lubashevsky, R. Mahnke, P. Wagner, S. Kalenkov, Phys. Rev. E 66 (2002) 016117.
[13] A. Kirchner, A. Schadschneider, Physica A 312 (2002) 260.
[14] K. Nishinari, D. Chowdhury, A. Schadschneider, Phys. Rev. E 67 (2002) 036120.
[15] F. Weifeng, Y. Lizhong, F. Weicheng, Physica A 321 (2003) 633.
[16] S. Maniccam, Physica A 321 (2003) 653.
[17] O.J. O'loan, M.R. Evans, M.E. Cates, Phys. Rev. E 58 (1998) 1404.
[18] D. Chowdhury, R.C. Desai, Eur. Phys. J. B 15 (2000) 375.
[19] T. Nagatani, Phys. Rev. E 63 (2001) 036116.
[20] H.J.C. Huijberts, Physica A 308 (2002) 489.
[21] S.A. Hill, Cond-Mat/0206008, 2002.
[22] T. Nagatani, Physica A 297 (2001) 260.
[23] T. Nagatani, Phys. Rev. E 66 (2002) 046103.
[24] T. Nagatani, Phys. Rev. E 60 (1999) 1535.
[25] L.A. Safonov, E. Tomer, V.V. Strygin, Y. Ashkenazy, S. Havlin, Chaos 12 (2002) 1006.
[26] G.F. Newell, Transp. Res. B 32 (1998) 583.
[27] T. Nagatani, Physica A 319 (2002) 568.
[28] T. Nagatani, Physica A 323 (2003) 686.
[29] T. Nagatani, Phys. Rev. E 68 (2003) 036107.