CSC477: Comparative Analysis of Unscented and Extended Kalman Filters in Nonlinear System

Haike Yu
University of Toronto Mississauga
haike.yu@mail.utoronto.ca

Feiran Huang
University of Toronto Mississauga
philipp.huang@mail.utoronto.ca

Zhongqi He
University of Toronto Mississauga
zhongqi.he@mail.utoronto.ca

Abstract: This project proposes a comprehensive evaluation of the Unscented Kalman Filter (UKF) and the Extended Kalman Filter (EKF) in the context of estimating the states of nonlinear systems. Given the inherent challenges posed by nonlinear dynamics in various engineering and data assimilation applications, there is an urgent need to deploy estimation filters that provide accurate and efficient performance. By applying UKF and EKF to some nonlinear functions, we intend to compare their performance thoroughly, focusing on accuracy and computational speed.

Keywords: Unscented Kalman Filter (UKF), Extended Kalman Filter (EKF), Nonlinear systems, Lorenz system, Van der Pol oscillator, Duffing equation

1 Introduction

Our study aims to implement UKF from scratch and conduct a comprehensive evaluation of UKF and EKF for a series of well-known nonlinear functions commonly found in robotics. This research will include nonlinear functions such as Lorentz systems that model chaotic weather patterns; van der Pol oscillators, which represent non-conservative oscillations of systems with nonlinear damping; and the Duffing equation, which describes the More complex nonlinear restoring forces for damped oscillator motion. These features are emblematic of the complex nonlinear behaviours observed in robotic navigation and control systems, including trajectory planning and autonomous vehicle behaviour in uncertain environments. To comprehensively illustrate our findings, we will include an analysis and graphs to show the results, providing a visual and quantitative comparison of the performance of UKF and EKF across these nonlinear scenarios. This analysis is designed to highlight each filter's operational strengths and potential limitations within the realm of robotic applications.

2 Problem

The Kalman filter has become a cornerstone of the field of linear system state estimation, praised for its elegance and efficiency. However, the leap from linear systems to nonlinear systems reveals a complex situation, and traditional Kalman filter technology faces inherent limitations. Nonlinear dynamics are everywhere, from the erratic dance of chaotic weather systems to the complex movements of autonomous vehicles navigating unpredictable terrain. To solve these nonlinear problems, the extended Kalman filter (EKF) emerged as a groundbreaking improvement that cleverly linearizes the nonlinear function to fit the linear framework of the Kalman filter. This approach, while innovative, introduces an approximation that may distort reality, distorting the accuracy of the state estimate. Converting nonlinear functions to linear functions, while practical, inherently carries the risk of bias and error, subtly but significantly affecting filter performance.

Faced with these challenges, the unscented Kalman filter (UKF) offers a promising alternative. UKF operates by generating a set of points (called sigma points) around the current state estimate. These points are strategically chosen to accurately capture the mean and covariance of the state probability distribution. Once selected, these sigma points will propagate through true nonlinear system dynamics rather than linear approximations. This process produces a new set of sigma points that reflect the distribution of predicted states. The genius of this approach is its ability to preserve the integrity of the state probability distribution, thereby more faithfully representing the true nonlinear behaviour of the system. By employing deterministic sampling techniques, UKF handles nonlinearity directly, avoiding the pitfalls of linearization. The method carefully selects points around the average state estimate and propagates them to the actual nonlinear function to predict the new state. This strategy preserves the nature of the system's nonlinear behaviour and potentially provides more accurate estimates.

By employing this deterministic sampling technique, UKF can produce more accurate estimates of systems. However, this accuracy comes with its computational requirements. The process of selecting and propagating sigma points, although systematic, requires more computing power than the EKF's linearization method. This raises a key question as we explore state estimation for nonlinear systems: Can we find a harmonious balance between computational overhead and accuracy? Specifically, does UKF or EKF offer a more efficient compromise, providing the best combination of time efficiency and accuracy when applied to the well-known nonlinear functions that embody robotics challenges? Our research delves into this problem, comparing the performance of these filters in nonlinear dynamic tests to find the best method for practical applications.

3 Related Work

The theoretical foundations and practical applications of the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF) are well documented in the literature, particularly in the fields of Bayesian filtering and robotics. Two notable contributions are Simo Särkka's "Bayesian Filtering and Smoothing", and Sebastian Thrun, Wolfram Burgard and Dieter Fox's "Probabilistic Robotics".

Julier and Uhlmann's contribution is particularly relevant to our comparative analysis of the Unscented Kalman Filter (UKF) and Extended Kalman Filter (EKF). In "Unscented Filtering and Nonlinear Estimation," they introduce the UKF as a robust alternative to the EKF for nonlinear estimation problems.

4 Proposed Method

4.1 Implementation

Our first step involves the implementation of both the UKF and EKF from scratch. This ensures a deep understanding of each filter's mechanics and allows for modifications tailored to our specific test cases. The implementation will be done in Python, utilizing libraries such as NumPy for efficient mathematical operations, ensuring that our filters are computationally optimized for the tasks at hand.

4.2 Selection of Nonlinear Systems

We will select a series of well-known nonlinear functions and systems for our study, including but not limited to the Lorenz system, Van der Pol oscillator, and Duffing equation. These systems are chosen for their relevance to real-world applications in robotics, such as chaotic behaviour modelling, non-conservative oscillations, and complex nonlinear restoring forces.

4.3 Evaluation Criteria

Our comparison will focus on two primary metrics: accuracy and computational speed. Accuracy will be measured by comparing the estimated states provided by each filter against the true states of the systems, using root mean square error (RMSE) as a quantitative metric. Computational speed will be assessed based on the time taken to perform the state estimations, providing insight into each filter's efficiency and practicality in real-time applications.

4.4 Analysis and Graphical Representation

The results of our simulations will be analyzed to identify trends, strengths, and weaknesses in the performance of the UKF and EKF. Graphical representations, including error plots and computational time charts, will be used to visualize the differences between the filters. This visual and quantitative analysis will form the basis of our conclusions on the comparative advantages of the UKF and EKF in nonlinear system estimation.

5 Proposed Evaluation

5.1 Implementation Evaluation

The basic requirement for these two filters is to encapsulate a bug-free implementation of the required functionality. Each filter must accurately represent its theoretical model, ensuring that all implemented features work as expected without unnecessary additions. The focus will be on validating the integrity of the core algorithms and ensuring that UKF and EKF effectively perform their intended state estimation tasks within the confines of our test environment.

5.2 Accuracy Evaluation

Accuracy will be quantified using the root mean square error (RMSE) between the estimated state of the filter and the true state of our chosen nonlinear system. It is crucial that the estimation accuracy remains consistent even if the predictions vary, and the outputs of the filter are expected to closely follow the nonlinear behaviour of Lorentz systems, van der Pol oscillators, and the Duffing equation.

5.3 Computational efficiency Evaluation

We will carefully record the execution time of each filter in all test scenarios to evaluate computational efficiency. The goal is to ensure that processing times remain within reasonable limits and to avoid excessively long durations that would make the filter impractical for real-time or computationally constrained applications.

5.4 Visualization Evaluation

The results of our comparative analysis are presented in clear and informative charts. These visualizations will clearly illustrate the performance differences between UKF and EKF for each nonlinear function, focusing on accuracy and computational efficiency. Additionally, explanatory diagrams will be included to demystify the operational logic behind both filters, providing insights into their respective methods of handling nonlinear state estimation.

6 References

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