Optimal Investing:

Balancing Risk and Reward Over a Decade

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1. Introduction

This report analyses the performance of seven portfolios generated through *mean-variance optimisation* **(Section 4)** over the 10-year period from October 31st 2014 to October 31st 2024, "*Investment Period*", using historical data between 31st December 1995 and 30th September 2014, "*Pre-Investment Period*". Terminal wealth calculations show our "*Forecasting Model-Guided Portfolio*" delivers the best returns (3.72% CAGR), outperforming the FTSE All Share index, "*the market*" (+1.79% CAGR). We recommend this portfolio for investors wishing to build long-term wealth, avoid market uncertainty, and maximise risk-adjusted returns.

2. Forecasting Model: When Should You Invest?

An EGARCH model is used to forecast market volatility, capturing 51% of historic data **(Appendix 1)**.

Factor	Description
1-month lagged Nasdaq Composite Index Returns	Spillover effect caused by dominance of US stock markets in global market capitalisation
Interaction between M2 Monetary Supply and Interest Rates (UK)	High liquidity combined with low central bank rates supports investment in the market

Table 1: High-level overview of factors in EGARCH model

Decision Framework:

Investment decisions are made on a semi-annual basis. If the average forecasted market return over the next period is greater than the current UK 1-month government bond yield, we invest in the market for the following 6-month period. Otherwise, we invest in government bonds[1].

Date	Investment Decision	Date	Investment Decision
31/10/2014	Market	31/10/2019	Bonds
30/04/2015	Bonds	30/04/2020	Market
30/10/2015	Market	30/10/2020	Market
29/04/2016	Market	30/04/2021	Market
31/10/2016	Market	29/10/2021	Market
28/04/2017	Market	29/04/2022	Bonds
31/10/2017	Market	31/10/2022	Market
30/04/2018	Market	28/04/2023	Market
31/10/2018	Bonds	31/10/2023	Market
30/04/2019	Market	30/04/2024	Market

Table 2: Investment Decisions forecast a strong market with investment in 80% of periods.

3. Stock Selection

Our portfolio, tailored for risk-averse investors, consists of 5 UK Companies: AstraZeneca, British American Tobacco, Bunzl, National Grid, and Vodafone. These stocks optimise the risk-return trade-off with high Sharpe Ratios, and are diversified across industries to have low correlations, and minimise company-specific risk **(Appendix 2)**. Owing to this, the selected stocks have demonstrated robust performance during crises **(Figure 1)**.

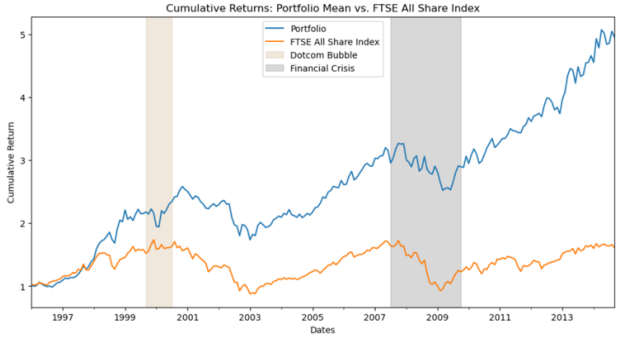


Figure 1: Portfolio performance during Dotcom Bubble 'Burst' and Financial Crisis

[1] UK 1-Month Government Bonds are considered 'risk-free' investments since the UK government has an almost-0 likelihood of defaulting on short-term debt obligations.

4. Efficient Frontier and Risk-Return Profiles

Clicking the efficient frontiers for each period directs to the risk-return profiles (**Appendix 3**). *Minimum-Variance Portfolio* offers stable growth with the lowest risk over the investment period – ideal for risk-averse investors. *Market Portfolio* provides moderate risk exposure, balancing risk and return. Points on CAL represent specific risk-return profiles, which can be tailored to clients' risk tolerance by combining investments in government bonds and the market portfolio.

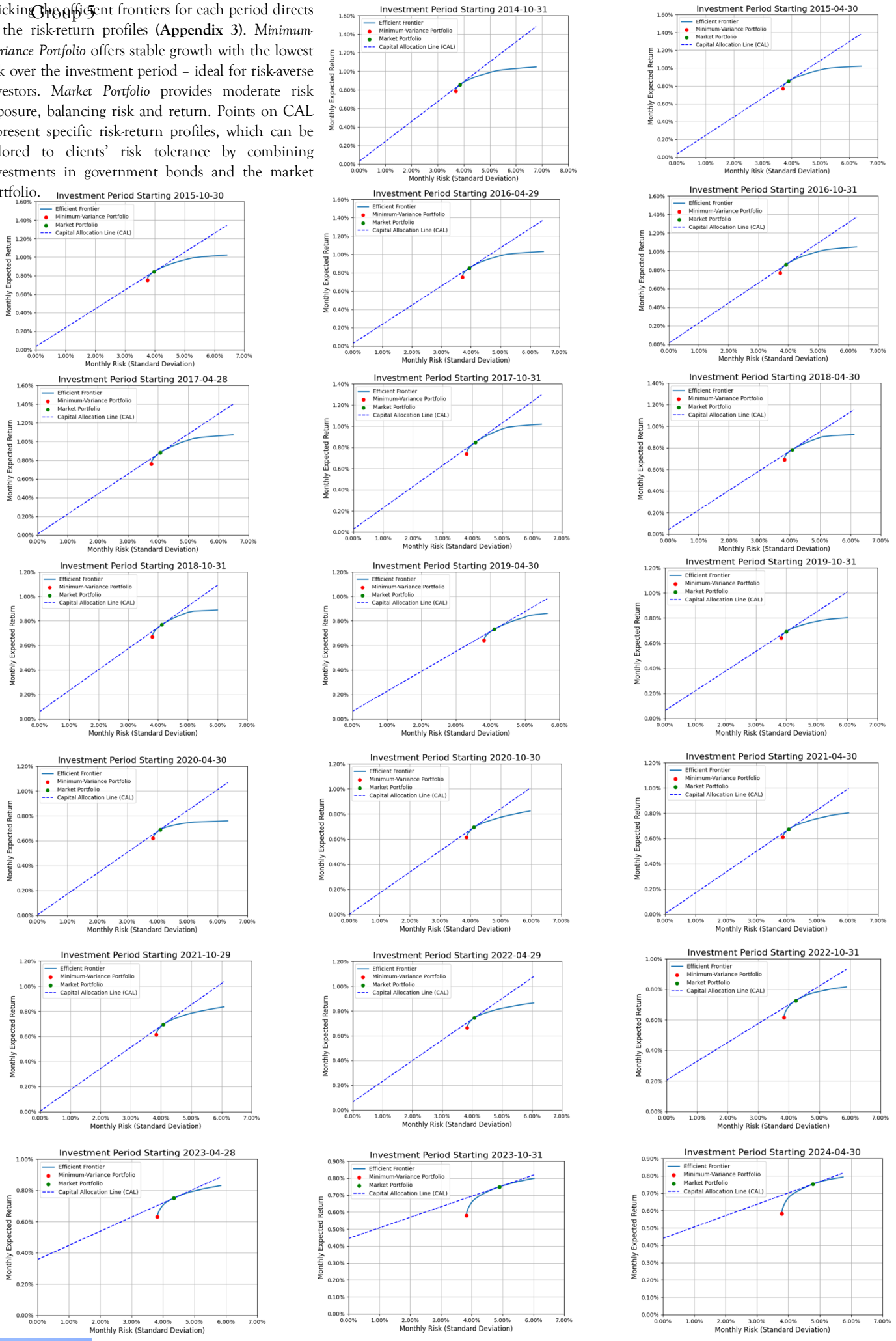


Figure 2: Efficient Frontiers (Semi-Annual), showing Minimum Variance Portfolios, Market Optimum Portfolios, and Capital Allocation Lines

5. Considerations for Replicating Mean-Variance Optimisation

Mean-variance optimisation is highly sensitive to its inputs, particularly calculations of expected returns and risk. Even with a diversified portfolio, a high expected return for a single asset can cause the model to heavily weight that asset. If this input is inaccurate, the portfolio can become overly concentrated and may result in significant losses. Companies are also assumed to be fairly valued (not underpriced or overpriced) and so optimisation relies solely on expected returns and risk. It does not account for potential reversion to true value (for overpricing), which may negatively impact performance.

This report uses historical mean returns and standard deviations as proxies to demonstrate the advantages of our forecasting-model-guided portfolio. The proprietary methodology used to evaluate company fundamentals is not disclosed in this analysis.

6. Terminal Wealth Calculation and Analysis

Portfolio	Initial Wealth	Terminal Wealth	CAGR
Market Portfolio (No Rebalancing)	£1,000.00	£1,285.74	2.56%
Market Portfolio (Semi-Annual Rebalancing)	£1,000.00	£1,322.73	2.81%
Minimum Variance Portfolio (No Rebalancing)	£1,000.00	£1,420.43	3.57%
Minimum Variance Portfolio (Semi-Annual Rebalancing)	£1,000.00	£1,355.02	3.14%
FTSE All Share Index	£1,000.00	£1,264.99	2.33%
UK 1-month Government Bonds	£1,000.00	£1,137.98	1.32%
Forecasting Model-Guided Portfolio	£1,000.00	£1,497.35	4.12%

Table 3: Terminal Wealth over the 10-Year Investment Period and CAGR for Portfolio Strategies

Rebalancing portfolio weights every 6 months has mixed results; improving *Market Portfolio's* performance (+0.25% CAGR) but reducing *Minimum Variance Portfolio's* performance (–0.43% CAGR). We suggest investors carefully consider rebalancing strategies, as outcomes depend heavily on the specific portfolio.

Minimum Variance Portfolio (No Rebalancing) beats *Market Portfolio (No Rebalancing)* (+1.01% CAGR), due to its lower exposure to the market downturn caused by COVID-19 pandemic **(Figure 3 Left).**

The best-performing strategy is the *Forecasting Model-Guided Portfolio* which enters positions in *Market Portfolio* based on forecasts in **Section 2** without rebalancing. Its lack of negative returns **(Figure 3 Right)** reflects consistent risk-adjusted growth, demonstrating the benefits of mean-variance optimisation combined with forecasting. The lower performances of the market and government bonds emphasise the importance of diversification for investors.

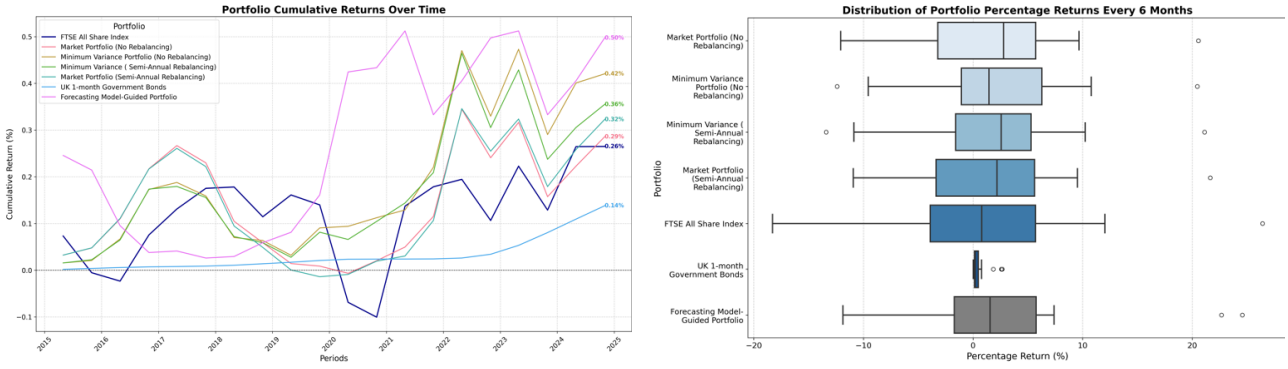


Figure 3: Forecasting Model-Guided Portfolio's Relative Performance

7. Recommendation

We strongly recommend *Forecasting Model-Guided Portfolio* for investors as it delivers greatest returns by adapting risk exposure to changing economic conditions. For steady wealth accumulation with lower risk, the Minimum Variance Portfolio remains a compelling choice.

Appendix 1: EGARCH Model

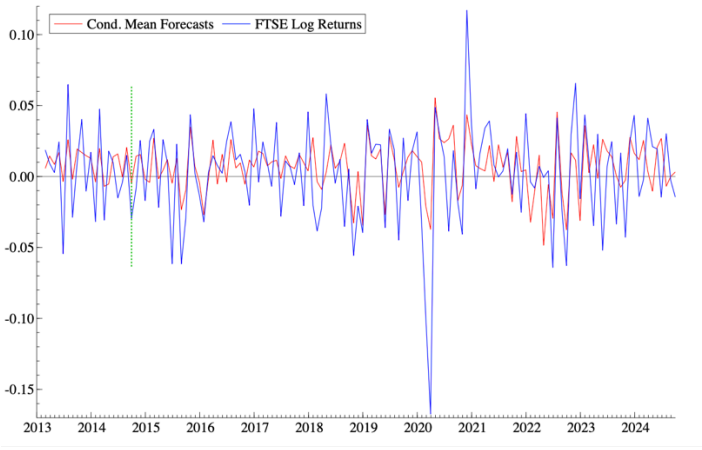


Figure 4: EGARCH Model Forecasts of FTSE All Share Log Returns vs. Actual FTSE All Share Log Returns

HAR-X - EGARCH Model Results

Dep. Variable:	Scaled Returns	R-squared:	0.506
Mean Model:	HAR-X	Adj. R-squared:	0.499
Vol Model:	EGARCH	Log-Likelihood:	-541.344
Distribution:	Normal	AIC:	1098.69
Method:	Maximum Likelihood	BIC:	1125.98
		No. Observations:	224
Date:	Mon, Nov 25 2024	Df Residuals:	220
Time:	15:12:08	Df Model:	4

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
Const	0.2936	0.192	1.532	0.126	[-8.207e-02, 0.669]
Scal...rns[0:1]	0.0210	5.490e-02	0.382	0.702	[-8.660e-02, 0.129]
NASDAQ Returns Lagged	37.6380	4.109	9.160	5.185e-20	[29.585, 45.691]
interest rate and m2 interaction	-6870.7962	1997.975	-3.439	5.841e-04	[-1.079e+04,-2.955e+03]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.2945	9.612e-02	3.064	2.181e-03	[0.106, 0.483]
alpha[1]	0.1093	9.136e-02	1.197	0.231	[-6.973e-02, 0.288]
gamma[1]	-0.2077	5.516e-02	-3.766	1.660e-04	[-0.316,-9.961e-02]
beta[1]	0.8524	4.588e-02	18.579	4.767e-77	[0.762, 0.942]

Figure 5: Model Performance Statistics

Stationarity Check (ADF Test):

- ADF Statistic: -18.0573
- p-value: 2.63×10^{-30}

Conclusion: Statistical properties of FTSE All Share index returns are consistent over time (null hypothesis of non-stationarity is rejected).

ARCH Effects Test:

- Statistic: 28.886
- p-value: 0.0013

Conclusion: *Significant ARCH effects* are present, indicating that there may be periods of high and low market uncertainty (volatility clustering). The time-series plot of scaled returns (**Figure 4**) illustrates periods of high volatility, which often coincide with economic crises, and low-volatility, suggesting market stability. The observed clustering indicates that a highly uncertain period is likely followed by another. The p-value is below 0.05, indicating significant ARCH effects; the patterns of market uncertainty are not random, but follow systematic patterns.

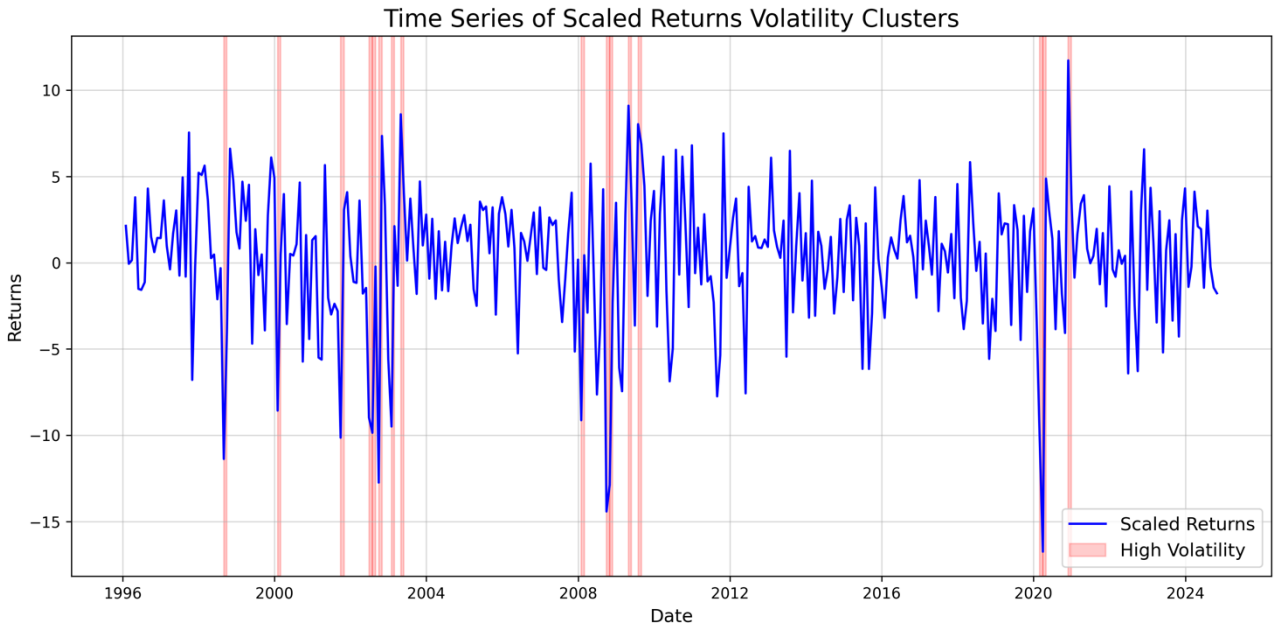


Figure 6: Periods of high volatility follow periods of high volatility (e.g. 2008 Financial Crash)

Autocorrelation of Squared Returns (ACF):

The autocorrelation function of squared returns supports the presence of volatility clustering. The ACF plot of squared returns (**Figure 5**) shows significant spikes at multiple lags, indicating that if the market experiences high volatility, it is likely to continue being volatile in the short term. This is often modelled using GARCH (Generalised Autoregressive Conditional Heteroskedasticity).

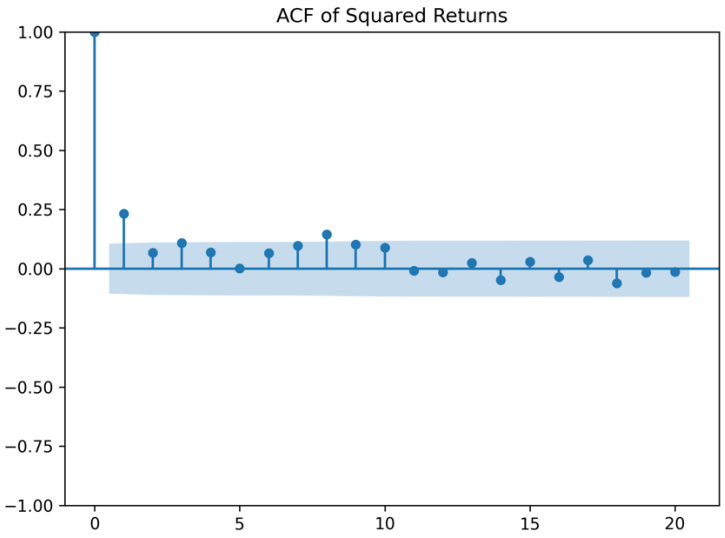
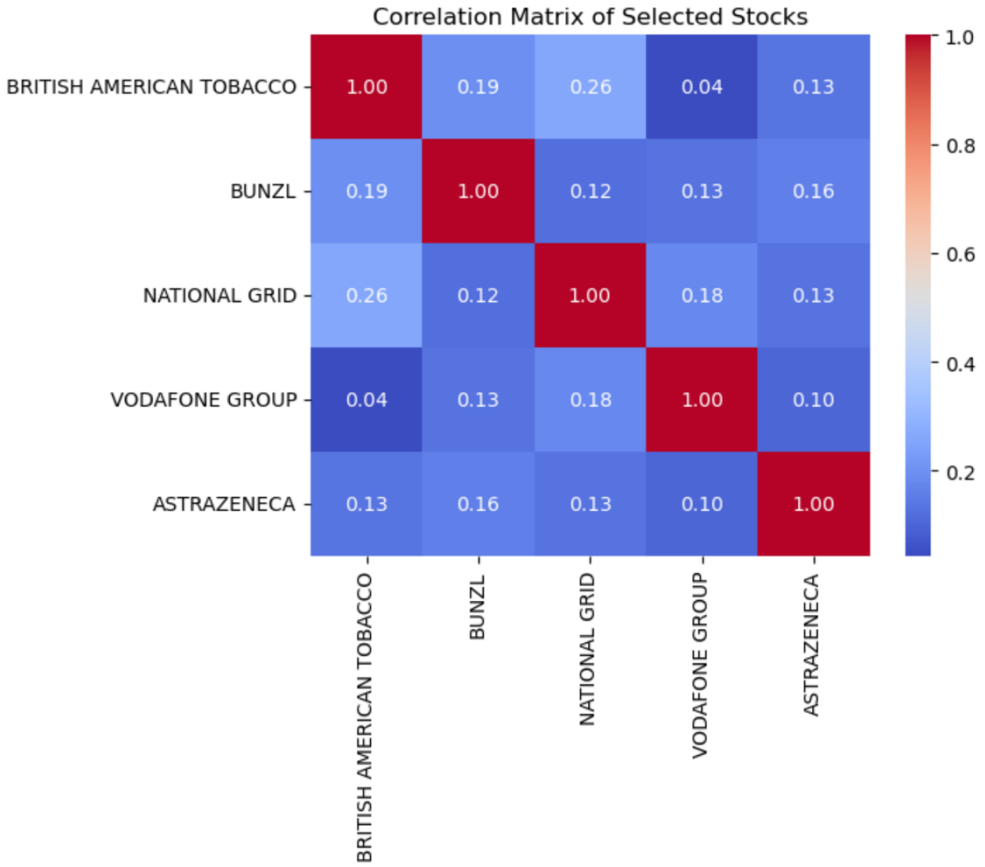
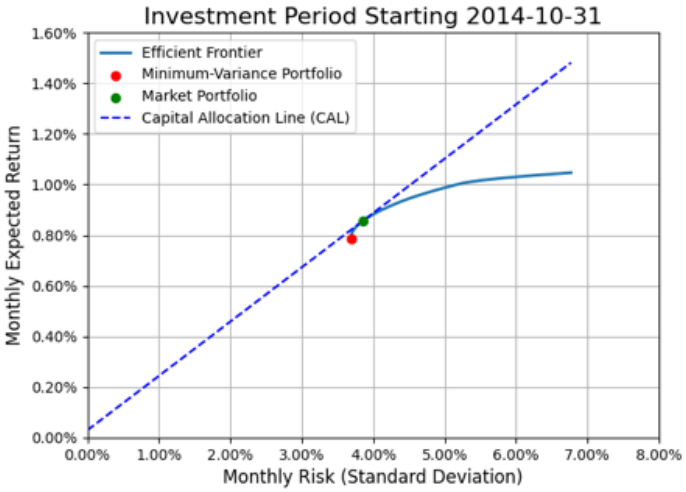


Figure 7: Autocorrelation of Squared Residuals (note Lag 1, 3, and 8).

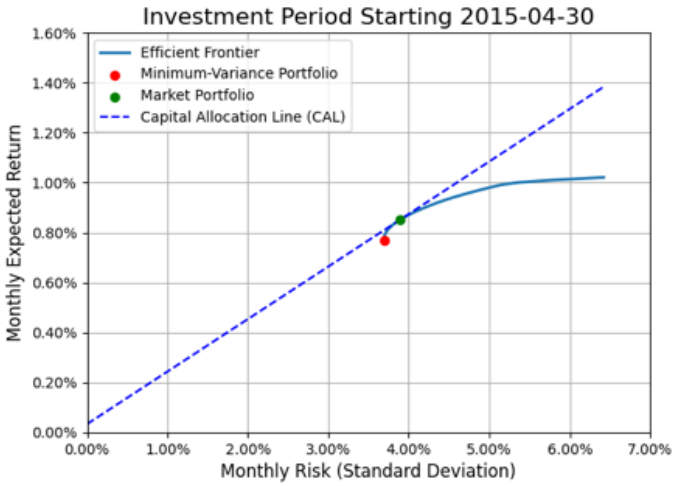
Appendix 2: Stock Correlation Matrix



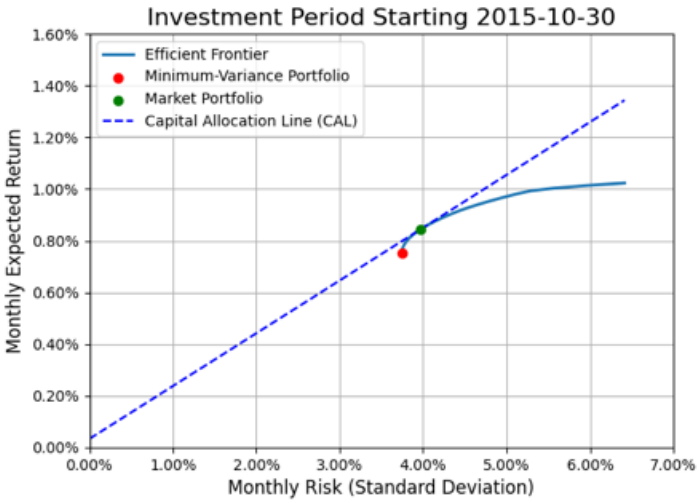
Appendix 3: Efficient Frontiers (In Depth)



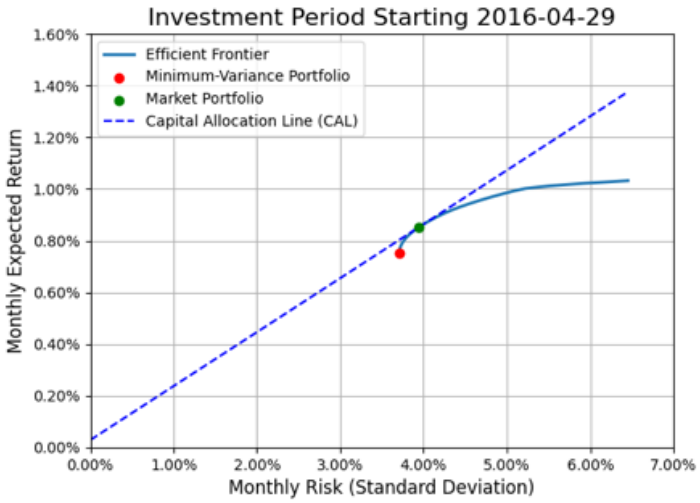
Monthly Expected Return Min. Variance Portfolio	0.79%
Monthly Risk of Min. Variance Portfolio	3.61%
Expected Return Optimal Portfolio	0.86%
Monthly Risk Optimal Portfolio	3.86%



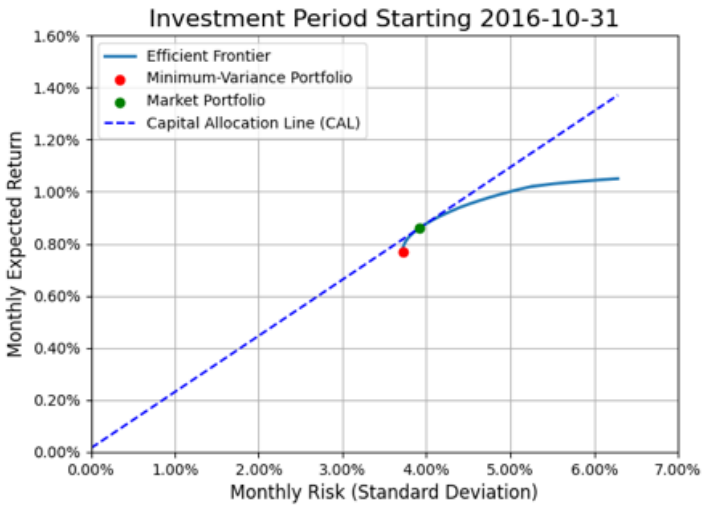
Monthly Expected Return Min. Variance Portfolio	0.77%
Monthly Risk of Min. Variance Portfolio	3.61%
Expected Return Optimal Portfolio	0.85%
Monthly Risk Optimal Portfolio	3.89%



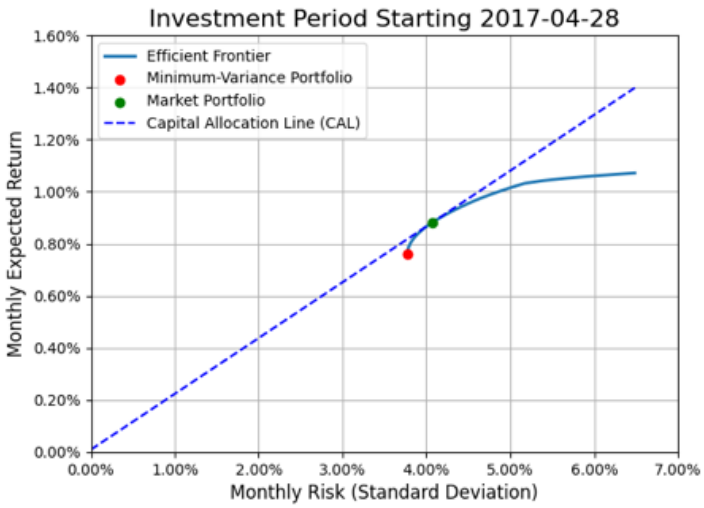
Monthly Expected Return Min. Variance Portfolio	0.75%
Monthly Risk of Min. Variance Portfolio	3.67%
Expected Return Optimal Portfolio	0.84%
Monthly Risk Optimal Portfolio	3.96%



Monthly Expected Return Min. Variance Portfolio	0.75%
Monthly Risk of Min. Variance Portfolio	3.63%
Expected Return Optimal Portfolio	0.85%
Monthly Risk Optimal Portfolio	3.94%

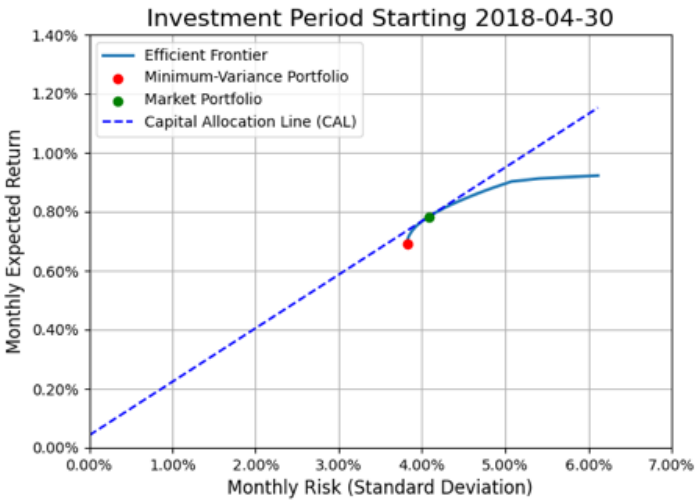


Monthly Expected Return Min. Variance Portfolio	0.77%
Monthly Risk of Min. Variance Portfolio	3.64%
Expected Return Optimal Portfolio	0.86%
Monthly Risk Optimal Portfolio	3.92%

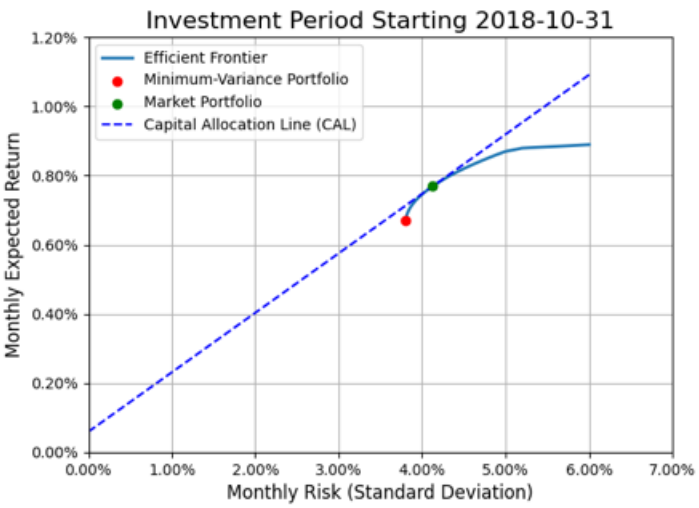


Monthly Expected Return Min. Variance Portfolio	0.76%
Monthly Risk of Min. Variance Portfolio	3.71%
Expected Return Optimal Portfolio	0.88%
Monthly Risk Optimal Portfolio	4.07%

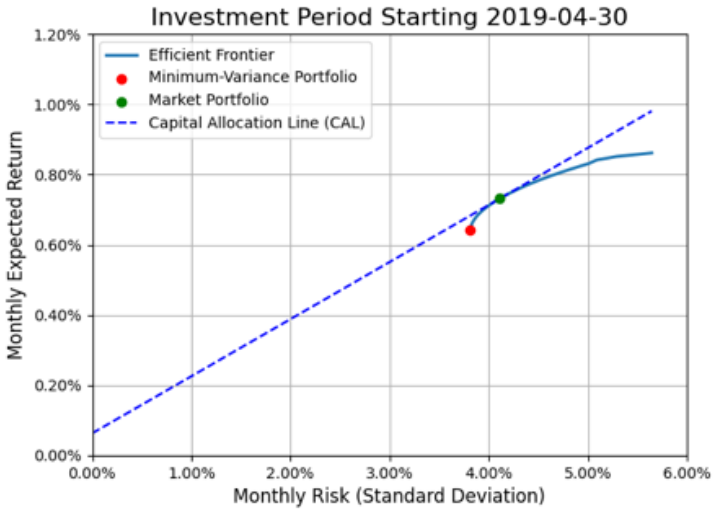
Monthly Expected Return Min. Variance Portfolio	0.74%
Monthly Risk of Min. Variance Portfolio	3.74%
Expected Return Optimal Portfolio	0.85%
Monthly Risk Optimal Portfolio	4.09%



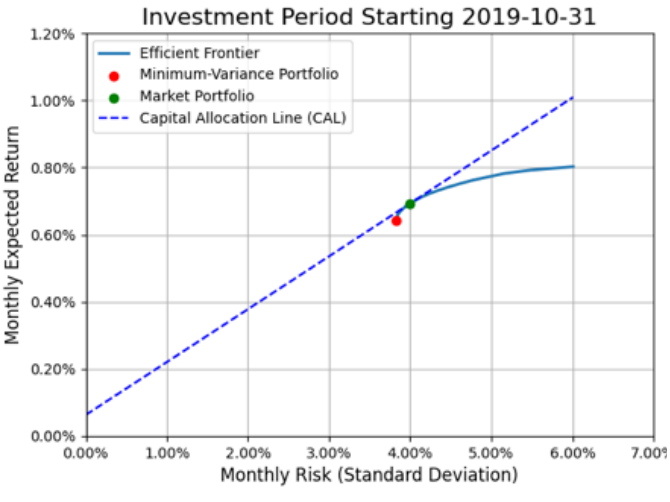
Monthly Expected Return Min. Variance Portfolio	0.69%
Monthly Risk of Min. Variance Portfolio	3.77%
Expected Return Optimal Portfolio	0.78%
Monthly Risk Optimal Portfolio	4.08%



Monthly Expected Return Min. Variance Portfolio	0.67%
Monthly Risk of Min. Variance Portfolio	3.74%
Expected Return Optimal Portfolio	0.77%
Monthly Risk Optimal Portfolio	4.13%



Monthly Expected Return Min. Variance Portfolio	0.64%
Monthly Risk of Min. Variance Portfolio	3.74%
Expected Return Optimal Portfolio	0.73%
Monthly Risk Optimal Portfolio	4.11%



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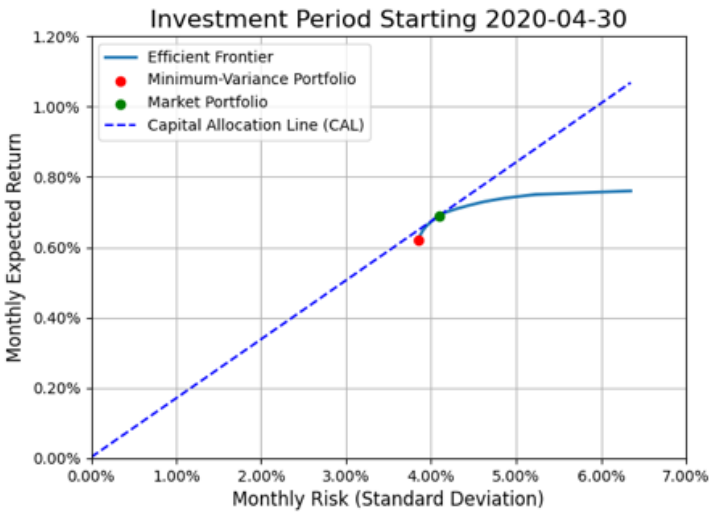
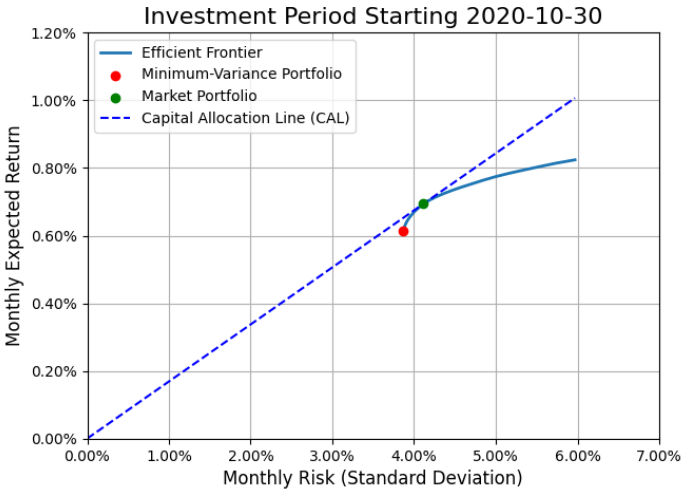


Monthly Expected Return Min. Variance Portfolio	0.62%
Monthly Risk of Min. Variance Portfolio	3.76%
Expected Return Optimal Portfolio	0.69%
Monthly Risk Optimal Portfolio	4.09%



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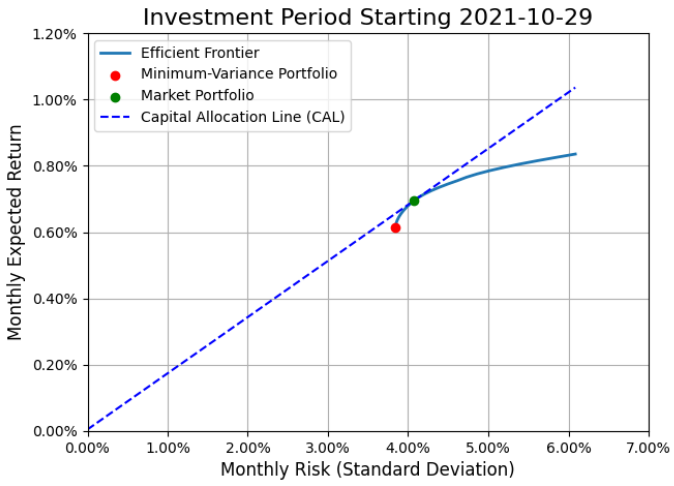
Expected Return Optimal Portfolio	0.67%
Monthly Risk Optimal Portfolio	4.06%



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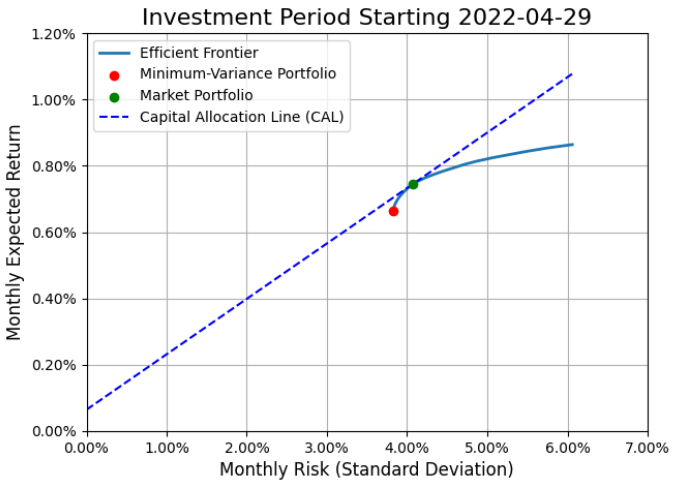

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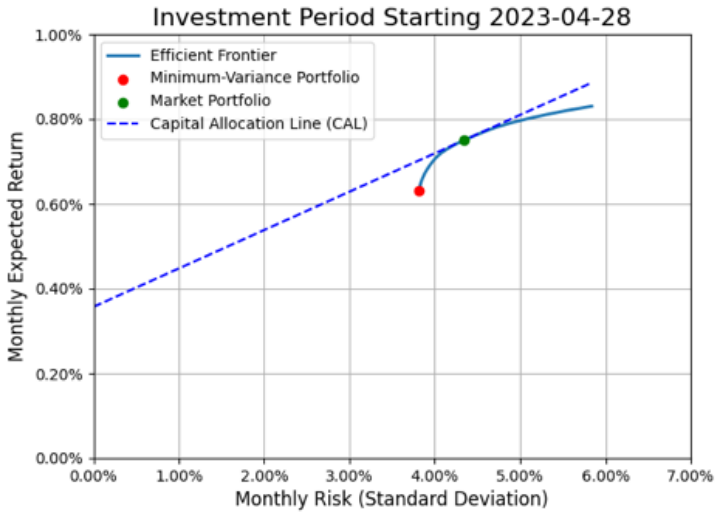
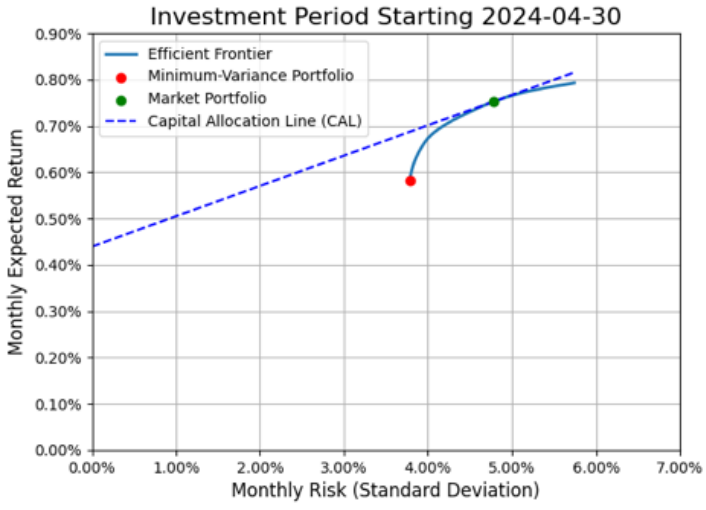


Monthly Expected Return Min. Variance Portfolio	0.63%
Monthly Risk of Min. Variance Portfolio	3.74%
Expected Return Optimal Portfolio	0.75%
Monthly Risk Optimal Portfolio	4.34%

Monthly Expected Return Min. Variance Portfolio	0.58%
Monthly Risk of Min. Variance Portfolio	3.75%
Expected Return Optimal Portfolio	0.75%
Monthly Risk Optimal Portfolio	4.90%



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Monthly Risk of Min. Variance Portfolio	3.72%
Expected Return Optimal Portfolio	0.75%
Monthly Risk Optimal Portfolio	4.79%

Appendix 4: Investment Period – Statistical Insight

Portfolio	Mean Return (%)	Min Return (%)	Max Return (%)	Final Cumulative Return (%)
Market Portfolio (No Rebalancing)	1.52	-12.08	20.57	28.57
Minimum Variance Portfolio (No Rebalancing)	2.03	-12.4	20.47	42.04
Minimum Variance (Rebalancing 6-Monthly)	1.8	-13.4	21.13	35.5
Market Portfolio (Rebalancing 6-Monthly)	1.66	-10.94	21.64	32.27
FTSE All Share Index	1.59	-18.3	26.42	26.5
UK 1-Month Government Bonds	0.65	0.01	2.66	13.8
Forecasting Model-Guided Portfolio	2.43	-11.87	24.57	49.73

Table 4: Summary Statistics for Portfolios during Investment Period.