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Specialist Masters Programme

Please list all names of group members: (Surname, first name) 1.Ibrahim, Basil 2.Remani, Shaan Ali 3.Santos, José	4.So, Wing Sze 5. **GROUP NUMBER:** 1
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Fixed Income Report

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Declaration

This report has been written collaboratively by Basil Ibrahim, Shaan Ali Remani, Wincy So, and José Santos. All contributors participated equally in the research and writing of this project.

Contents

1 Introduction

This report analyses the *Variable Rate Bond 2033* issued by *BNP Paribas* (ISIN: XS2392609181). Our valuation framework is developed in Python using the `QuantLib` library, permitting smooth interest rate curve construction, cap/floor pricing, and credit risk adjustments. The report begins with a description of the bond's characteristics and a replication strategy for its coupon payoff. We then present our pricing methodology, historical coupon analysis, and forward-looking scenarios. A hedging strategy against both interest rate and credit risk is developed through vanilla Interest Rate Swaps (IRS) and Credit Default Swaps (CDS). Finally, we compute the risk via Monte Carlo simulation and analytical methods to compute Value-at-Risk (VaR) and Expected Shortfall (ES). Based on our model, the bond is currently *undervalued by 1.52%* relative to its quoted clean price. We therefore consider it a **STRONG BUY**.

2 Product Description and Market Data

Bond Structure and Replication

We analyse the *Variable Rate Bond 2033* issued by *BNP Paribas*, a 5-year capped and floored floating rate note (FRN) with quarterly coupon payments. The bond pays a coupon equal to the 3-month Euribor rate, subject to a floor of *1.60%* p.a. and a cap of *3.70%* p.a., with interest computed on a 30/360 basis. Coupons are paid on the 29th of January, April, July, and October each year. The bond was issued on 29 July 2022 and matures on 29 July 2027. We value the bond as of 18 November 2024, with a trade date assumed to be 24 November 2024, and a 2-day settlement lag.

This bond can be decomposed into a standard FRN combined with an embedded long interest rate floor and a short interest rate cap. This decomposition allows us to interpret the bond as a synthetic position involving vanilla interest rate derivatives. Table 1 outlines the net payoff under different rate regimes and Figure 4 (Left) demonstrates the flattening effect of the embedded options at the extremes.

Rate Regime	FRN	Floor	Cap	Net Payoff
$r < 1.60\%$	r	$1.60 - r$	0	1.60%
$1.60\% < r < 3.70\%$	r	0	0	r
$r > 3.70\%$	r	0	$-(r - 3.70)$	3.70%

Table 1: Payoff structure under different rate regimes

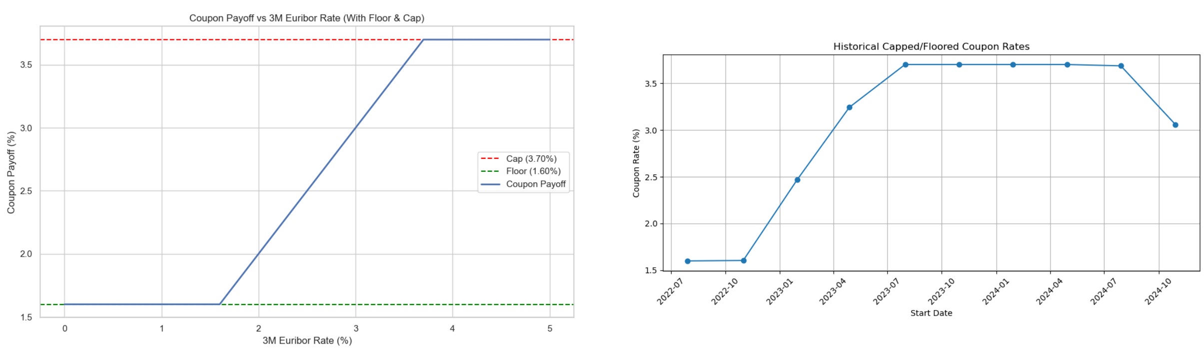

Figure 1: Theoretical Coupon Payoff (Left) vs Historical Coupon Payoff (Right)

Historical Coupons

We retrieve 3-month EURIBOR rates from `Suomen Pankki` and compute historical coupon payments based on reset dates, following the Modified Following convention. These realised rates are visualised in Figure 4. In particular, note the cap was hit multiple times between late 2023 and mid-2024, suggesting periods of rising short-term rates and illustrating the dampening effect of the cap on coupon variability.

Market Data and Curve Construction

Market data was sourced from the provided `MarketData.xlsx` file. Short-term deposit rates help build the short end of the curve, and interest rate swaps (IRS) are used for longer maturities. Since both deposit and swap rates are quoted in terms of par rates, we bootstrap to derive discount factors, spot curves and forward curves

We constructed four types of curves using QuantLib: linear, flat, cubic, and log-cubic. Among these, the log-cubic interpolation on the discount factor curve yielded the smoothest and most economically plausible forward rate curve (Figure 2). Smoothness is critical: forward rates directly affect projected cash flows, discounting, and pricing of the option components.

The log-cubic interpolation works by interpolating the logarithm of the discount factor, ensuring that the derived forward rate curve is continuously differentiable and free from 'kinks' or 'jumps' [Hagan and and, 2006]. The interpolation between two maturities t_i and t_{i+1} takes the form: $\log D(t) = a_i + b_i(t - t_i) + c_i(t - t_i)^2 + d_i(t - t_i)^3$, where the coefficients a_i, b_i, c_i, d_i are chosen to ensure C^2 continuity across intervals. The instantaneous forward rate is then: $f(t) = -\frac{d}{dt} \log D(t)$, ensuring forward rates evolve as smoothly as possible.

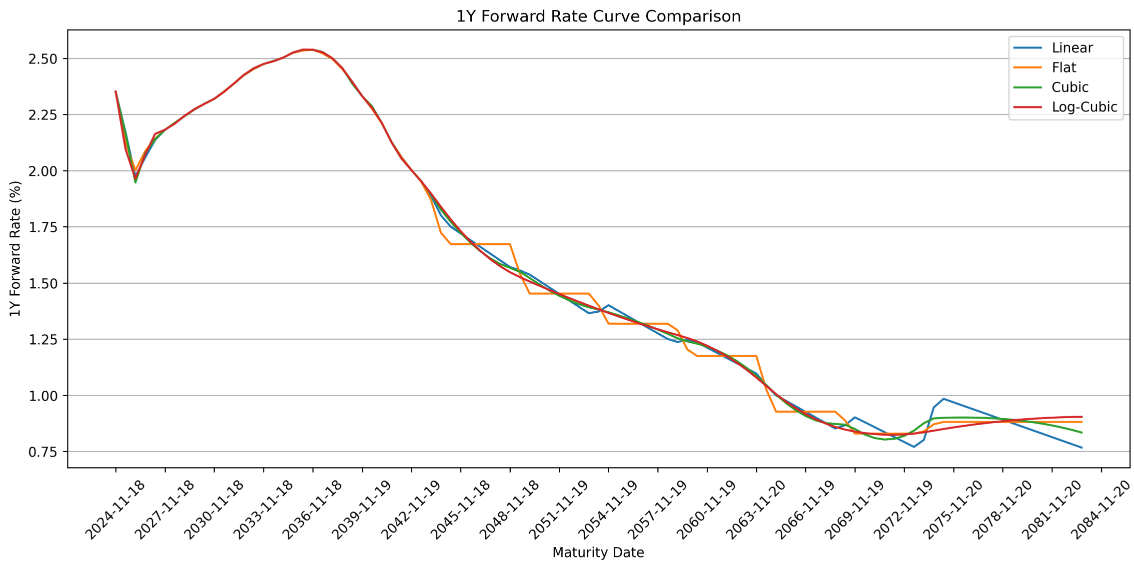

Figure 2: 1-Year Forward Rate Curve Comparison Across Interpolation Methods

3 Pricing Procedures and Model Justification

Decomposition of the Bond into Option Components

As per Section 2, the bond can be viewed as a combination of a FRN, a long interest rate floor, and a short interest rate cap. This decomposition builds a replicating portfolio to help price the structured bond. The FRN is valued using forward rates derived from the bootstrapped curve, while the cap and floor are priced as portfolios of caplets and floorlets respectively; thus, the choice of interest rate model is crucial.

Choice of Interest Rate Model: Shifted Black

We use the market-standard *shifted Black model*, implemented using `QuantLib`'s `BlackCapFloorEngine`. This model assumes a lognormal distribution of forward rates, which is consistent with positive interest rate environments, as seen in `MarketData.xlsx`. The price of one caplet under this model is given by:

$$\text{Caplet Price} = P(t,T) \cdot [F \cdot N(d_1) - K \cdot N(d_2)],$$

where:

$$d_1 = \frac{\log(F/K) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t},$$

with F being the forward rate, K the strike, σ the implied volatility from the market, and $P(t,T)$ the discount factor to maturity.

The shifted Black model is picked over alternatives such as *Bachelier* and *Hull-White* models. Bachelier is better suited to negative or near-zero rate environments, and Hull-White, although more flexible, increases calibration complexity beyond what is required for the simple instruments involved here. Given the current positive-rate regime and the limited optionality complexity, i.e. vanilla caplets/floorlets, the Black model is superior in both simplicity and efficiency.

Calibration to Market Cap Volatility Surface

We construct a volatility surface from the cap volatilities provided in `MarketData.xlsx`, spanning various maturities and strikes to calibrate our model for accurate valuations. Since cap instruments are quoted with maturity-level volatilities, we interpolate across both dimensions, maturity and strike, to extract caplet volatilities. Figure 3 visualises the implied volatility surface, highlighting the presence of a volatility smile that must be calibrated in our pricing model [Brigo and Mercurio, 2006].

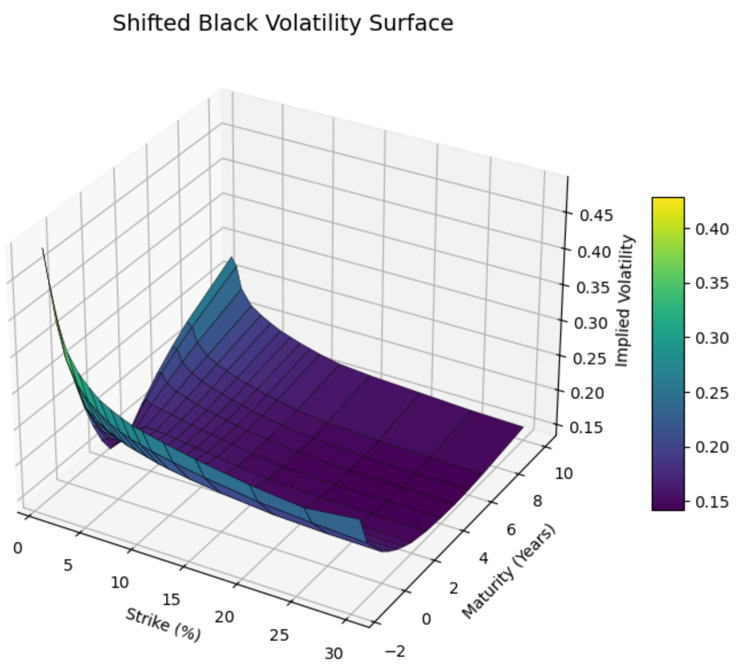

Figure 3: Interpolated Cap Volatility Surface (Strike vs Maturity)

Credit Risk Adjustment: CVA

The above valuation, albeit risk-free, is not complete, since have not accounted for the possibility of BNP Paribas defaulting on payments. We compute a Credit Valuation Adjustment (CVA) via the approximation:

$$\mathrm{CVA}(t,T) = (1-R)\cdot \mathrm{NPV}_{\text{risk-free}} \cdot (1-Q(t,T))\,, \tag{1}$$

where the survival probability approximation $Q(t,T)$ is:

$$Q(t,T) = \frac{e^{-\mathrm{CDS}(t,T)(T-t)} - R}{1-R}, \tag{2}$$

with $R = 0.4$ (recovery rate) and $\mathrm{CDS}(t,T)$ the market CDS level for BNP Paribas extracted from `Investing.com`.[1] This approximation allows us to price in the market-implied credit risk. Note, however, that the CDS level quoted applies to a 5-year term, so is likely an over-approximation. We return to this in Section 4.

Rationale for Modelling Choices

The following assumptions were made to support our bond pricing framework: The bond coupon accruals are calculated on 30/360 basis, which we assumed the same day-count convention is applied uniformly to both historical and forward coupon calculations. We have boststrapped the curves from historial rate and the boostrapped curve is applied for both forecasting future cash flows and discounting, however, in real practice OIS discounting might be preferred. The embedded cap and floor options are valued using the Black model, implemented via QuantLib's. A constant shifted Black volatility is applied, derived from a representative Black volatility surface.

The CVA adjustment, although simplified, allows us to account for counterparty risk in a way that aligns with how credit spreads are reflected in real-world bond prices. The combined approach yields a risk-adjusted valuation that reflects both market expectations and product-specific optionality.

4 Implementation of Pricing Procedure

Scenario Analysis

To understand the range of potential outcomes for bondholders, we analysed the best-case and worst-case scenario: EURIBOR rates remain above the cap and below the floor. For each scenario, coupon payments were computed as: $C_i = \text{Notional} \times \text{Coupon Rate}_{\text{cap/floor}} \times$ Year Fraction Cash flows were then discounted using the aforementioned log-cubic curve to determine present value (PV): $\mathrm{PV}_i = C_i \times DF(\text{Payment Date}_i)$, where $DF(\text{Payment Date}_i)$ is the discount factor on the corresponding payment date.

Effective coupon rates were determined by the formula:

$$\text{Effective Rate} = \min(\max(\text{Forward Rate}, \text{Floor}), \text{Cap})$$

Coupon payments based on these were then discounted as before, and added to the notional to determine the gross price. To obtain the clean price, i.e. the market quoted price, accrued interest was subtracted: Accrued Interest = Notional $\times$ Effective Rate $\times$ Elapsed Year Fraction.

We compare the clean price with the quoted price on EuroTLX, and compute the market-implied CDS spread to determine the market-implied credit risk of BNP Paribas.

[1] https://www.investing.com/rates-bonds/bnp-paribas-cds-5-year-eur-historical-data

Comparison of Computed Fair Value and Derivation of Market-Implied CDS

Recall that to adjust for credit risk, we compute a Credit Valuation Adjustment (CVA) (see Equation 1) using the survival probability approximation (see Equation 2). The marginal CVA for each cash flow is given by

$$\text{Marginal CVA} = \text{Exposure} \times (1 - Q(t, T)) \times DF(t).$$

Defining a price difference function,

$$f(\text{CDS}) = \text{Model Clean Price} - \text{CVA(CDS)} - \text{Market Clean Price},$$

we use Brent's root-finding method to solve for the CDS spread that equates the credit-adjusted model price with the market quote. The resulting CDS spread, CDS^*, represents the market-implied CDS spread.

Sensitivity and Risk Measures

Given the bond suffers interest rate risk, we generated several shifted yield curves by applying 10 bps shifts:

- **Parallel Shifts:** Uniform adjustment across all maturities.
- **Slope Shifts:** Linear adjustments that vary with maturity.
- **Curvature Shifts:** Non-linear adjustments affecting primarily the mid-curve rates.

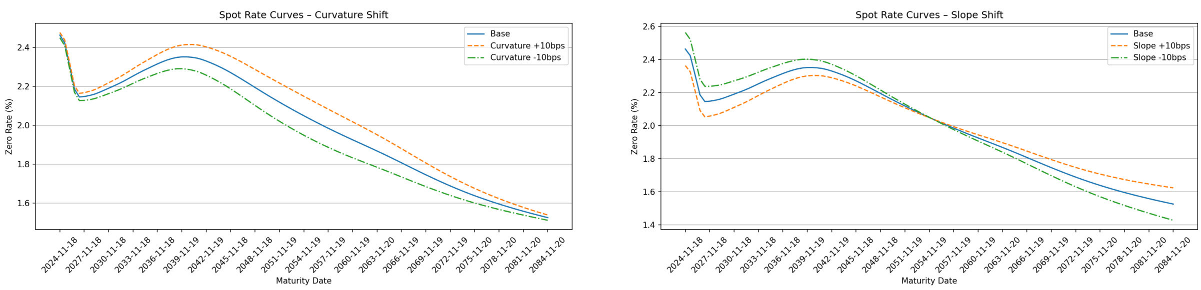

Figure 4: Example of curvature shift by 10bps (Left) and slope shift by 10bps (Right)

For each shifted curve, the bond's gross and clean price is calculated.. The sensitivity is summarised by the DV01 *(Dollar Value of a 1 bp change)*, computed as:

$$\text{DV01} = \frac{P_{\text{down}} - P_{\text{up}}}{2},$$

where P_{down} and P_{up} are the bond prices under downward and upward shifts, respectively.

Hedging Strategy Implementation

Our hedging strategy addresses two key risks:

1. **Interest Rate (IR) Risk:** Mitigated using a plain vanilla payer swap with 3 years maturity quoted on `EURIBOR3M`. For the fixed leg, the present value is computed as

$$\text{Fixed Leg PV} = \sum_{i=1}^{n} \left(\text{Notional} \times r_{\text{fixed}} \times \Delta t_i \times DF(t_i) \right),$$

while the floating leg is approximated by

$$\text{Floating Leg PV} \approx \text{Notional} \times (1 - DF(t_n)) \, .$$

The swap DV01 is calculated as before and the hedge ratio is calculated as:

$$\text{Hedge Ratio} = \frac{|\text{DV01}_{\text{bond}}|}{|\text{DV01}_{\text{swap}}|}.$$

2. **Default (Credit) Risk:** Mitigated using CDS contracts by providing bondholders with protection against an issuer's default. Essentially an insurance product, a CDS requires the protection buyer (bondholder) to pay periodic premiums, known as CDS spreads, to the protection seller. In return, the seller compensates the buyer if a defined credit event occurs, such as bankruptcy or payment failure. If a credit event occurs, the buyer is compensated through either physical settlement, exchanging the defaulted bond for its notional amount, or cash settlement, where the seller pays the buyer the difference between the bond's notional value and its post-default market price. The market-implied CDS spread obtained earlier not only calibrates the credit-adjusted bond price but also provides the basis for the CDS hedge.

Graphical and Tabular Representations:

- A plot illustrating the convergence of the price difference function as the CDS spread varies, highlighting the market-implied CDS spread.
- A table and accompanying graph displaying bond prices under various yield curve shift scenarios, along with the computed DV01 values.
- A comparative table of bond and swap DV01s, from which the hedge ratios are derived.

Scenario	**Gross Price**	**Accrued Interest**	**Clean Price**
Base	1000.6061	1.8501	998.7561
Parallel +10bps	1000.6176	1.9184	998.6992
Parallel -10bps	1000.5757	1.7677	998.8079
Slope +10bps	1000.5733	1.7683	998.8051
Slope -10bps	1000.6199	1.9178	998.7021
Curvature +10bps	1000.5980	1.8535	998.7444
Curvature -10bps	1000.5952	1.8325	998.7627

Table 2: Bond Price Table (from q11)

Hedging against these risks allows us to benefit from any mispricing in the bond, while hedging away any unwanted exposure to interest rate and credit risk.

Estimation of VaR and Expect Shortfall by Monte Carlo Simulations

In financial risk management, Value at Risk (VaR) and Expected Shortfall (ES) are key metrics for measuring potential portfolio losses. While analytical methods like Delta or Delta-Gamma offer speed, they may overlook complexities from multiple or non-linear risk factors. In such cases, Monte Carlo simulation provides a more robust and flexible alternative.

We considered the changes in bond gross price are given by a factor model:

$$\Delta GP(t, T_i) = \beta_{l,i}\Delta l(t) + \beta_{s,i}\Delta s(t) + \beta_{c,i}\Delta c(t) + \beta_{cds,i}\Delta cds(t) + \epsilon(t)$$

Interpolation Method	**Gross Price**	**Accrued Interest**	**Clean Price**
Base	997.5269	1.2233	996.3036
Parallel +1bps	997.8439	1.2233	996.6206
Parallel -1bps	997.3117	1.2233	996.0883
Slope +1bps	997.3351	1.2233	996.1118
Slope -1bps	997.8205	1.2233	996.5972
Curvature +1bps	997.6284	1.2233	996.4051
Curvature -1bps	997.5272	1.2233	996.3039

Table 3: Swap Price Table

Interpolation Method	**Bond DV01**	**Swap DV01**	**Hedge Ratio**
Parallel +10bps	-0.020950	-0.266100	0.078730
Slope +10bps	0.023300	0.242700	0.096003
Curvature +10bps	-0.001400	-0.050600	0.027668

Table 4: DV01 and Hedge Ratio under Various Interpolation Methods

Betas were taken from the sensitivities to four key risk factors: parallel rate shifts (DV01), slope, curvature, and CDS spread changes, which we have obtained from previous sections.

Following the Monte Carlo method, we:

1. Simulated a large number of scenarios for the factor changes, based on their assumed distributions (that is normal distribution with zero mean and their corresponding variances)
2. Computed the corresponding bond price changes using the given factor model;
3. Constructed the distribution of profit and loss (P&L) across all simulated scenarios;
4. Estimated VaR as the empirical quantile of the simulated P&L distribution;
5. Estimated ES as the conditional mean loss given that the loss exceeds the VaR.

For comparison, we also derived analytical (exact) VaR and ES by computing the total portfolio variance and applying the normal distribution quantiles. This comparison allows us to assess the accuracy and effectiveness of the Monte Carlo simulation relative to the exact formula.

Marginal and Component VaR

Decomposing the total VaR of the portfolio into contributions of the individual risk factor provides a clearer understanding of how much each factor (parallel shifts, slope, curvature and CDS spread) contributes to the overall risk. This decomposition enables more targeted risk management.

The Marginal VaR for each factor was calculated as the ratio of that factor's weighted contribution ($beta \times \sigma$) to the total standard deviation of the portfolio. It measures how sensitive the overall VaR is to a marginal increase in exposure to a specific factor. Then, the Component VaR was obtained by multiplying each Marginal VaR by its corresponding beta and scaling with the 99% quantile.

5 Results and Discussion

Our results span pricing, scenario analysis, sensitivity testing, credit valuation adjustment, hedging strategy, and risk decomposition.

The model-derived clean price of the bond is **€999.31**, while the market quote on EuroTLX is €984.30, implying the bond is undervalued by approximately €15.01 or **1.52%**. After adjusting for credit risk via CVA, which we estimate at €20.14, the credit-adjusted value is €980.47, and the bond remains attractively priced. This is likely due to BNP Paribas' A+ credit rating on Fitch.

Decomposing the bond into its replicating portfolio, FRN and option-like components, (Table 5) shows that the embedded short cap and long floor options have a net value of €3.70, and the FRN is valued close to par at €1000.61.

Component	Fair Value (€)	DV01 (per 1 bp)	Credit Sensitivity
Floating Rate Note	1000.6080	0.0	N/A
Cap Option (Short)	-1.1992	-0.05	0.02
Floor Option (Long)	4.8997	0.03	0.01
Total Structured Bond	**999.3075**	**0.23**	**0.02**

Table 5: Valuation and Sensitivities of Structured Bond Components

Forward-looking coupon projections (Table 6) reflect a softening Euribor curve, and the bondholder is likely to receive moderate coupons between €4.8 and €6.2. The discounted notional value is €943.13.

Payment Date	Forward Rate (%)	Effective Rate (%)	Coupon Amount	Discount Factor	Present Value
29 Jan 2025	2.4641	2.4641	6.1603	0.995096	6.1301
29 Apr 2025	2.4342	2.4342	6.0855	0.989077	6.0190
29 Jul 2025	2.3612	2.3612	5.9030	0.983209	5.8039
29 Oct 2025	2.2460	2.2460	5.6149	0.977597	5.4891
29 Jan 2026	2.0961	2.0961	5.2403	0.972389	5.0956
29 Apr 2026	1.9801	1.9801	4.9502	0.967599	4.7898
29 Jul 2026	1.9297	1.9297	4.8241	0.962902	4.6452
29 Oct 2026	1.9444	1.9444	4.8610	0.958141	4.6576
29 Jan 2027	2.0191	2.0191	5.0476	0.953223	4.8115
29 Apr 2027	2.0962	2.0962	5.2404	0.948253	4.9692
29 Jul 2027	2.1485	2.1485	5.3713	0.943131	5.0659
Redemption	–	–	1000.0000	0.943131	943.1311

Table 6: Forward Rate-Based Coupon Table (Including Redemption)

Sensitivity analysis shows limited interest rate exposure, as the collar structure neutralises much of the interest volatility (Table 7).

Risk Factor	Sensitivity (per 1 bp)
Parallel DV01	0.005435
Slope Sensitivity	-0.005150
Curvature Sensitivity	0.000915
CDS Sensitivity	-0.437277

Table 7: Bond Price Sensitivities to Risk Factors

VaR and Expected Shortfall from Monte Carlo simulations aligns with analytical approximations (Table 8), suggesting the assumptions of a linear sensitivity analysis and Gaussian distributions are sound.

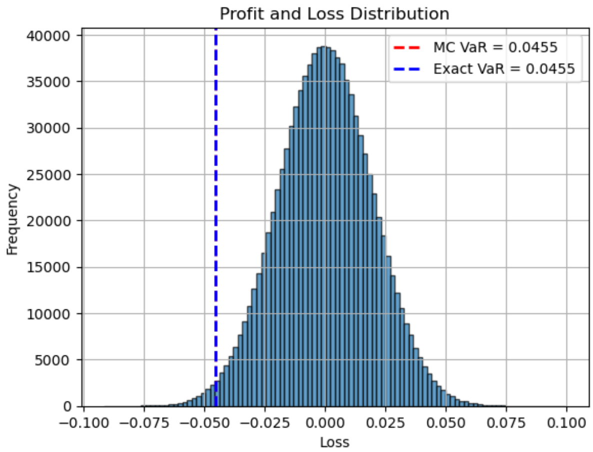

Figure 5: MC PnL Distribution

	99% VaR	**99% ES**
Monte Carlo	0.0455	0.0521
Exact (Analytical)	0.0455	0.0522

Table 8: 99% VaR and ES Comparison: Monte Carlo vs. Exact

In Table 9, note that over 99% of the portfolio's risk originates from credit spreads. The CDS spread dominates the Marginal and Component VaR. The remaining risk factors (parallel, slope, curvature) provide negligible contributions, and, in fact, diversification benefits. Very clearly, the main risk facing bondholders is credit risk, and not interest rate risk.

Table 9: Marginal and Component VaR Contributions

Risk Factor	**Marginal VaR**	**Component VaR**
l	0.041184	0.001875
s	-0.014411	-0.000656
c	0.001478	0.000067
cds	-0.999047	-0.045493

That is, the collared bond 'naturally hedges' against changes in interest rates, protecting the bondholder from sharp rate increases and decreases. In contrast, credit risk cannot be neutralised through the bond and must be actively managed. The hedge using a payer swap effectively neutralises the minimal rate exposure (see Section 4), but a CDS hedge is essential to address the main source of risk. The implied CDS spread of 34.06 bps unites our model price and market quote, and allows us to calibrate this hedge effectively. For completeness, the PnL distribution is included in Figure 5.

Instrument	**Fair Value (€)**	**DV01 (per 1 bp)**	**Hedge Ratio**
Vanilla Swap	997.5269	-0.266100	0.078730
CDS	999.3075	-0.437180	0.047921

Table 10: Valuation and Sensitivities of the Hedging Instruments

Table 10 summarises hedging instruments. The vanilla swap has hedge ratio of just 7.87% allowing us to neutralise the bond's interest rate exposure with a hedge ratio of just 7.87%. CDS requires only 4.79% notional to hedge the bond's credit spread sensitivity. Even though the credit risk exposure is higher, low hedge ratios confirm that both risks are not only hedgeable in theory but also hedgeable in practice.

Despite our modelling assumptions, including a fixed recovery rate, the use of static yield curves, and simplified forward rate model, our approach offers a highly robust valuation. The bond's modest underpricing, dominant credit exposure, and the reliable risk mitigation strategy together support a confident investment thesis. We reaffirm our recommendation: the BNP Paribas capped/floored FRN is a **STRONG BUY**.

Conclusions and Final Recommendations

A model-derived clean price of €999.31 compared to a market quote of €984.30 suggests market participants are mispricing this bond. The source of mispricing lies in market-implied credit risk; the implied CDS spread of 34.06 bps is higher than quoted levels.

The bond's exposure to interest rate risk is limited by the interest rate collar. Sensitivity analyses confirm that curve shocks contribute very little to VaR. The cap and floor effectively bound the coupon profile, while a plain vanilla payer swap neutralises what little DV01 remains. In contrast, credit spreads are the dominant risk factor and require active hedging with CDS.

For investors seeking a structured bond with limited interest rate exposure and an attractive credit spread pickup, this instrument offers a compelling opportunity. We therefore recommend this bond as a **STRONG BUY**, supported by sound valuation, quantified risks, and manageable hedging.

References

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