SMM272 Risk Analysis

Group Coursework

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- Question 1 is mandatory. Afterward, select two additional questions based on your preference.
- Submission deadline and group composition as set by the course officer.
- It is your responsibility to manage your own time constraints and any potential problem which might arise within your group. No extensions will be granted for this kind of issues.
- Extensions on deadline will not be granted.
- The coursework has to be submitted electronically via Moodle.
- Usual rules apply in case of plagiarism.
- The report must be typed in a suitable word processor (Word, laTex ...) and then submitted in pdf format. No other format will be accepted.
- Ensure that your report adheres to the specified formatting guidelines, comprising a minimum of 15 and a maximum of 30 pages, excluding the cover page with the author's name. Follow the A4 format, single-spacing, and use an 11-point font. Include a cover page featuring the report's title and the authors' names. Maintain clarity in your responses, avoiding unnecessary repetition of well-known material or proofs.
- Questions pertaining to the coursework should be raised at the commencement of the lecture in the presence of the entire class, either at the beginning or immediately following the break. Please note that I will not entertain inquiries sent via email or personal messages regarding the coursework.
- Every question contributes to the overall coursework mark. The specific weight or proportion of each question towards the total coursework mark will not be disclosed.
- The presentation and clarity of design in your reports are equally significant as the accuracy of your computations and the quality of output discussions where necessary. It is essential that your reports are neatly written and easily readable to prevent any unintended loss of marks.
- All plots and tables summarizing your results must be incorporated into the written report. When you include a figure or a table, it is imperative to provide an explanation of its relevance and significance in the context of the analysis.
- Please do not refer to tables or figures that are not reproduced in the document.
- Number and caption all figures and tables appropriately, providing concise descriptions of their content.
- If you reference books or articles in your report, include a bibliography. Ensure that the bibliography exclusively includes references to books or articles explicitly mentioned in your report. Adhere to a book-style format for your references.
- For the practical aspects involving Excel/Matlab in the coursework, alongside your hardcopy submission, kindly upload the corresponding workbook on Moodle. Ensure you submit only ONE Excel file or ONE zipped Matlab folder. For Excel submissions, dedicate a separate sheet in the file for each question, and name each sheet according to the question's number and part. The Excel file submission should be handled by a single group member.
- In summary, please submit: (a) A printed report encompassing your responses to the questions, including tables, plots, and discussions. (b) An Excel/Matlab file on Moodle, showcasing your workings where necessary. Ensure that the Excel file contains separate sheets for each question, labeled according to the question's number and part.

Q1. VaR Modelling

Download the adjusted closing prices for the following five stocks: AAPL, MSFT, IBM, Nvidia, Alphabet, Amazon for the period spanning January 1, 2014, to December 31, 2024. Then build an equally weighted portfolio and perform the following analysis discussing carefully the adopted procedure and your findings

1. Perform a statistical analysis of the portfolio returns, describing your main findings.
2. Using a rolling window of 6 months and starting on July 1st 2014, you have to estimate for each day the VaR at 90% and 99% confidence levels at one day horizon using at least four different methods.
3. Given your VaR forecasts, compute the number of VaR violations for each model and each confidence level.
4. Using the Kupiec, the conditional coverage test and the distributional tests described in the Christoffersen book, evaluate the backtesting performance of these models.
5. Discuss your results.

Q2. The Risk Parity portfolio

The **risk parity portfolio** is a portfolio in which each asset contributes equally to the Component Value at Risk (VaR). This portfolio is constructed by selecting weights that minimize the dispersion (or standard deviation) of the individual Conditional VaRs (CVaRs).

Using the dataset from **Question 1**, compute the Component VaR using both the parametric approach, which relies on the sample covariance matrix, and the non-parametric approach.

Another portfolio to consider is the **maximum diversification portfolio**, which aims to maximize the following ratio:

$$\max_{\mathbf{w}} \frac{\mathbf{w}'\sigma}{\sqrt{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}}$$

subject to: $\mathbf{w}'\mathbf{1} = 1$ and $\mathbf{w} \geq \mathbf{0}$, where $\mathbf{w}$ represents the portfolio weights, σ is the vector of asset volatilities, and $\boldsymbol{\Sigma}$ is the covariance matrix.

A third portfolio is the **equally-weighted portfolio**, where all assets are assigned equal weights.

Procedure

1. Split the dataset into two parts:
   - Use the first half of the sample to determine the portfolio compositions.
   - Use the second half of the sample to compute the daily log-returns for each portfolio.
2. Evaluate the performance of the portfolios using the following metrics:
   - **Sharpe Ratio:** Measures risk-adjusted returns, assuming a zero risk-free rate.
   - **Maximum Drawdown:** Quantifies the largest peak-to-trough decline in portfolio value.
   - **Number of VaR Violations:** The count of instances where portfolio losses exceed the Value at Risk (VaR) at a 95% confidence level.
   - Other measures you believe to be relevant for your analysis

Analysis

After calculating these metrics, analyze and interpret the results, comparing the performance of the different portfolio strategies. Discuss the trade-offs between risk parity, maximum diversification, and equally-weighted portfolios, focusing on their relative strengths and weaknesses in terms of risk-adjusted returns, drawdowns, and turnover.

Q3 VaR of a bond

Consider a bond with a 10-year maturity and annual coupon payments. This bond has a face value of 100 and a coupon rate of 5%. Currently, the gross price of the bond is 99. Assume a year consists of 360 days.

Compute the yield to maturity (YTM) for this bond, under the assumption that the daily fluctuations in YTM follow independent and identically distributed (i.i.d.) Gaussian random variables with a mean of 0 and a standard deviation of 0.006.

- Estimate the probability of a 10% decline in the bond price within a 30-day period.
- Compute the Value at Risk (VaR) for your bond at a 99% confidence level across various horizons (1, 10, 20, 30,...,90 days) using the following methods:
  1. Exact formula (provide a detailed explanation of the procedure).
  2. Exact formula via delta approximation (i.e., exploiting Taylor's formula truncated to the first order).
  3. Exact formula via delta-gamma approximation (i.e., exploiting Taylor's formula truncated to the second order).
  4. Monte Carlo simulation with at least 10,000 simulations and delta approximation.
  5. Monte Carlo simulation with at least 10,000 simulations and delta-gamma approximation.
  6. Monte Carlo simulation with at least 10,000 simulations and full revaluation.
  7. Discuss and compare your results.
- Compute the Expected Shortfall of your bond at a 99% confidence level for different horizons (1, 10, 20, 30,...,90 days) using your preferred method. Provide a detailed explanation of the adopted procedure.

Monte Carlo simulations must be performed using at least 10,000 simulations. Greeks (i.e. Delta, Gamma and Theta) can be computed analytically or via finite differences.

Q4. VaR of a Portfolio with Options

Consider the following tickers: INTC, JPM, AA, and PG. Using two years of historical data, estimate the covariance matrix based on your preferred approach.

On February 21st, 2024, you hold a portfolio consisting of the following derivatives:

- INTC: Short 3 call options with a strike price at 90% of the stock price and a time to maturity of 9 months.
- JPM: Long 6 at-the-money put options with a time to maturity of 6 months.
- AA: Long 6 call options with a strike price at 105% of the underlying stock price and a time to maturity of 12 months.
- PG: Short 2 put options with a strike price at 110% of the underlying stock price and a time to maturity of 9 months.

The options are priced using the Black-Scholes model, with annualized historical volatility as an input. The annualized risk-free rate is set at 4.0%, and we assume that the stocks do not pay dividends.

Simulate stock returns using at least 10,000 simulations over a 10-day horizon. Compute the Value at Risk (VaR) and Expected Shortfall (ES) for the options portfolio at a 99% confidence level.

Next, calculate the Marginal and Component VaR/ES using the appropriate formula for the non-Gaussian case.

Provide a step-by-step explanation of the procedure, detailing your calculations and offering insights into the results.

Q5. ESTIMATING THE PROBABILITY OF LOOSING

Consider the portfolio in Question 1.

1. Explain how to use bootstrap simulation to estimate the probability of losing more than 5% at different horizons (eg. from 1 to 50 days).
2. Implement a Matlab code and provide a plot of the estimated probability at different horizons.
3. Now assume that portfolio log-returns have a Gaussian distribution. Estimate their mean and standard deviation and compute the theoretical probability of loosing more than 5% at different horizons. Overimpose the plot of the theoretical probability to the one you have generated via Monte Carlo simulation.
4. Comment your findings.

Q6. POWER OF THE KUPIEC TEST

Explain how to estimate the power of the Kupiec test assuming that under H_0 the true generating process that underlies market returns is Gaussian with zero mean and constant volatility and under H_1 the true generating process is the GARCH(1,1) model. Explain carefully your procedure and your results. Both models can be fitted to the return series of Question 1.