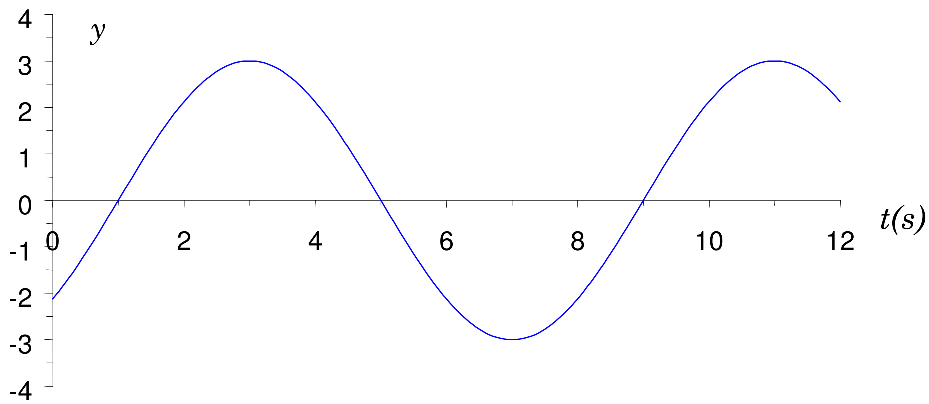
OBTAINING THE TRIGONOMETRIC EXPRESSION OF A SINUSOID FROM ITS GRAPH

Once you have a graph, information on A, ω and θ can be obtained directly from that graph. Once this information is obtained, the expressions for the waveform can be written.



We can describe this waveform as either a sine or a cosine

A. Describe the waveform with an expression in the form $y = A\sin(\omega t + \theta)$.

** The magnitude A is the maximum height of the waveform, in this case, 3.

** To find ω, we find T first: the graph crosses the t-axis at 1, 5 and at 9, so the period T = 9 – 1 = 8 seconds. The frequency is $\frac{1}{T} = \frac{1}{8}$ Hz. The angular frequency $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$.

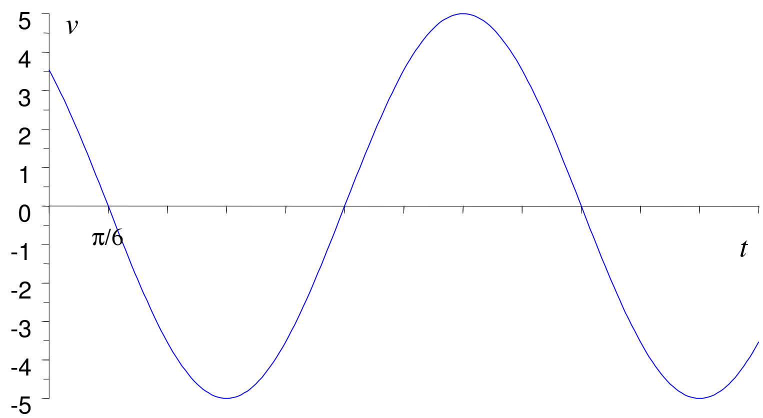
**For a sine wave, the displacement, d, is the distance to the y axis from where the rising wave crosses the t-axis. In this example, d = 1. As $d = \frac{-\theta}{\omega}$, we conclude $\theta = -\omega d$, therefore, that $\theta = \frac{-\pi}{4}\times 1 = \frac{-\pi}{4}$.

Thus, the above waveform is described by the expression $y = 3\sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$

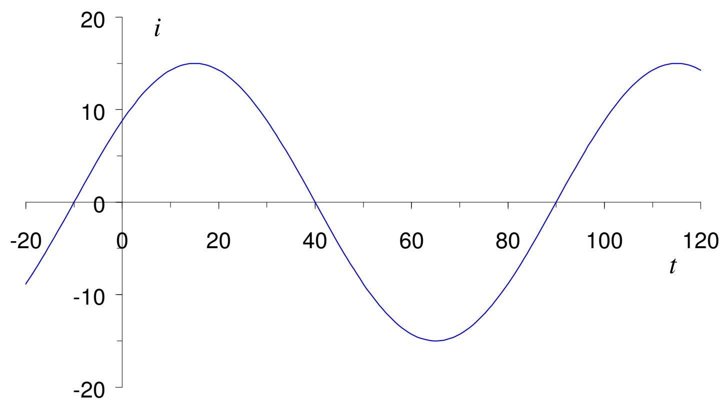
B. Describe the waveform with an expression in the form $y = A\cos(\omega t + \theta)$
Notice that the only thing that changes is the displacement. For cosine waves, the phase displacement is the distance from the top of the waveform to the y-axis, in this example, d = 3. Therefore, $\theta = -\omega d = \frac{-3\pi}{4}$ and the cosine representation of the above waveform is $y = 3\cos\left(\frac{\pi}{4}t - \frac{3\pi}{4}\right)$.

<u>Exercise 1</u>. Find a **sine** and a **cosine** representation for the following waveforms.

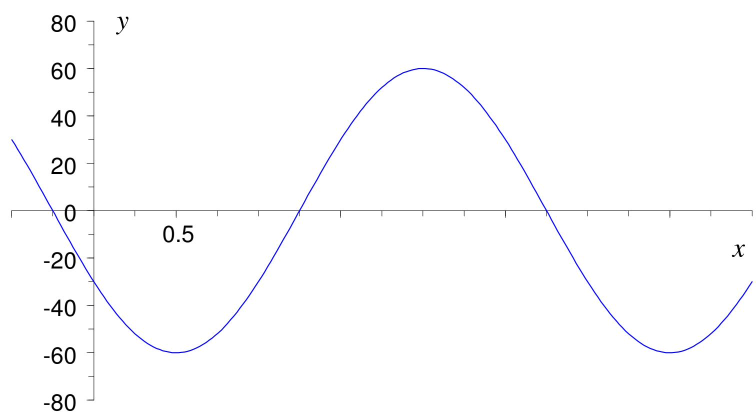
a)



b)



c)



<u>Answers</u>

a) $v = 5V\,\sin\left(\frac{3}{2}t - \frac{5\pi}{4}\right)$ $\qquad$ $v = 5V\,\cos\left(\frac{3}{2}t - \frac{7\pi}{4}\right)$

b) $i = 15A\sin\left(\frac{\pi}{50}t + \frac{\pi}{5}\right)$ $\qquad$ $i = 15A\cos\left(\frac{\pi}{50}t - \frac{3\pi}{10}\right)$

c) $y = 60\sin\left(\frac{2\pi}{3}x - \frac{5\pi}{6}\right)$ $\qquad$ $y = 60\cos\left(\frac{2\pi}{3}t - \frac{4\pi}{3}\right)$

Pg 1

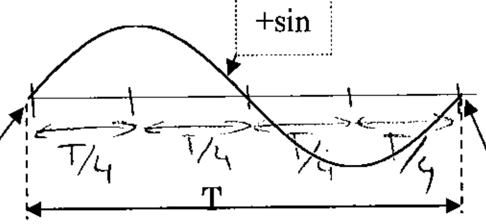
Obtaining a Trig Equation from a sinusoidal graph – pages 2,3,4

Recall:

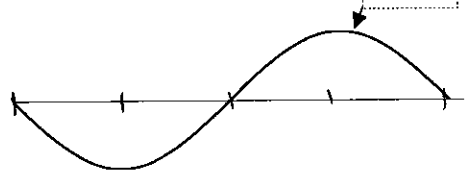
When graphing $y = a\sin(bx + c)$ and $y = a\cos(bx + c)$ we do one cycle first:

Main Cycle for a SINE curve:

+sin



-sin



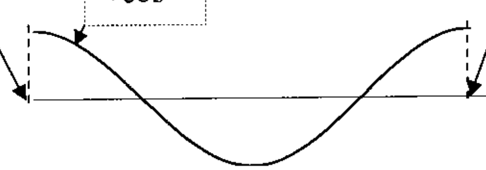
displacement

$\boxed{d = -\frac{c}{b}}$

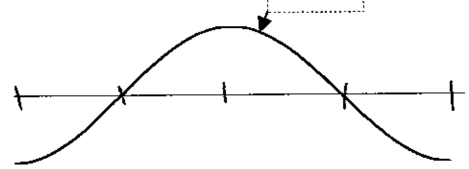
End point

Main Cycle for a COSINE curve:

+cos



-cos



(consider $bx + c = 0$ and solve for x)

Amplitude (max positive y-value) = $|a|$

Displacement or 1st Critical point (the beginning of the main cycle): $\boxed{d = -\frac{c}{b}}$

so, when you solve for c, use $\boxed{c = -bd}$

Period (length of one cycle): $\boxed{T = \frac{2\pi}{b}}$ (in seconds) (so, when you solve for b use $\boxed{b = \frac{2\pi}{T}}$)

Frequency (number of cycles per second) $\boxed{f = \frac{1}{T}}$ (Hz)

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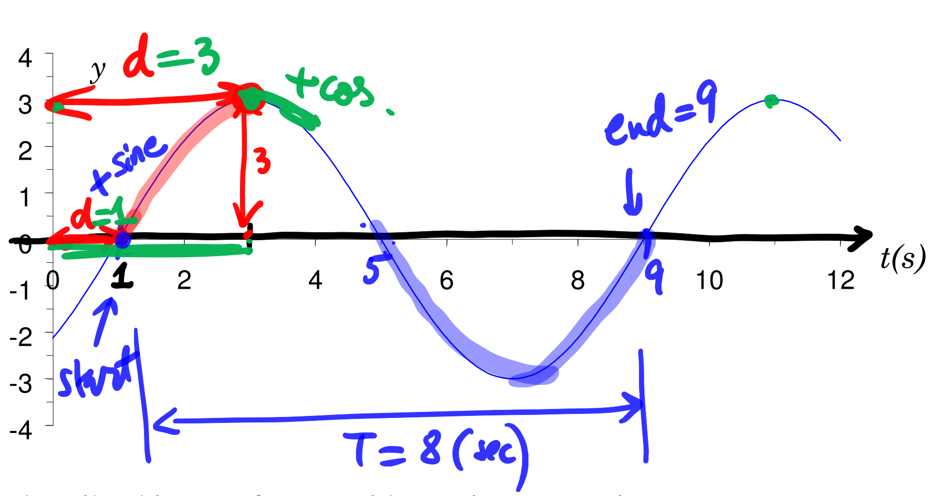
$$\frac{3+(-3)}{2} = \boxed{0}$$



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Given the graph write the equation either as a sine/cosine

$$\boxed{y = a\cdot\sin(\omega\cdot t + \theta)} \quad \text{OR} \quad \boxed{y = a\cdot\cos(\omega\cdot t + \theta)}$$

From the graph (Read) → Amplitude $A = 3 \Rightarrow \boxed{a = \pm A}$ 1

→ Period : $\boxed{T = \text{end} - \text{start}}$ = #units start/end

$T = 9 - 1 = \boxed{8 \text{ sec}}$

$\Rightarrow$ Find $\boxed{\omega}$: $T = \frac{2\pi}{\omega} \Rightarrow \boxed{\omega = \frac{2\pi}{T}}$ 2

Ex: $\omega = \frac{2\pi}{8} = \boxed{\frac{\pi}{4} \text{ rad/sec}}$

3

To find phase angle θ → use eq. $\boxed{\omega\cdot d + \theta = 0} \Rightarrow \boxed{\theta = -\omega\cdot d}$

To find start of a cycle/displacement/phase shift d

4

For a sine wave : the displacement $\boxed{d}$ = the distance to the y-axis from where the wave rises crossing t-axis

Sine wave : $d = 1 \Rightarrow \theta = -\omega\cdot d = -\frac{\pi}{4}\cdot(1) = -\frac{\pi}{4} \Rightarrow \boxed{\theta = -\frac{\pi}{4}}$

Sine wave eq : $\boxed{y = 3\sin\left(\frac{\pi}{4}\cdot t - \frac{\pi}{4}\right)}$ & $\boxed{y = 3\cos\left(\frac{\pi}{4}\cdot t - \frac{3\pi}{4}\right)}$

For a cosine wave : the displacement $\boxed{d}$ the distance to the y-axis to the top of waveform.

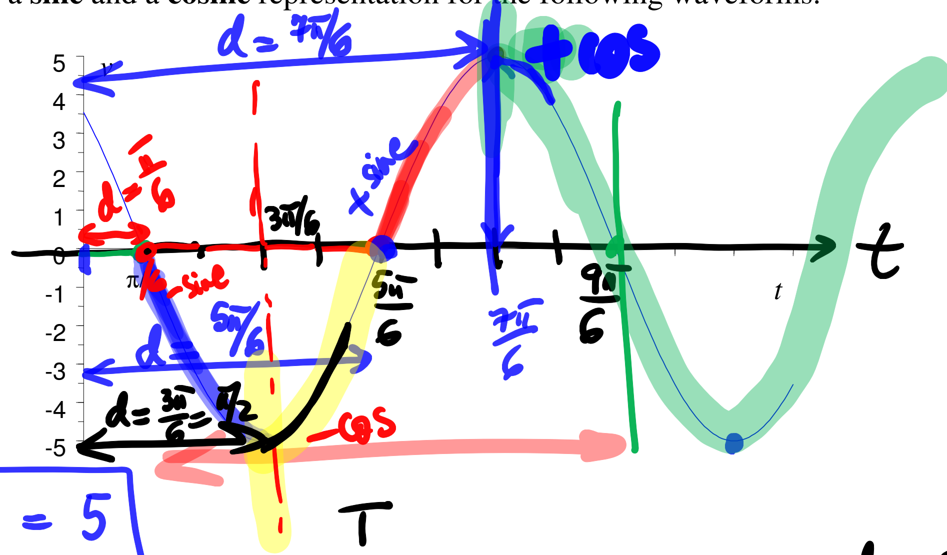
Cosine wave $\boxed{d = 3}$

$\theta = -\omega\cdot d = -\frac{\pi}{4}\cdot 3 = -\frac{3\pi}{4}$

$y = a\cdot\sin(\omega\cdot t+\theta)$ or $y = a\cdot\cos(\omega\cdot t+\theta)$

Exercise 1. Find a **sine** and a **cosine** representation for the following waveforms.

a)



① Ampl. $\boxed{A = 5}$

② Period $T = \frac{4\pi}{\cancel{6}_3}(2) = \boxed{\frac{4\pi}{3}}$ or $T = \text{end} - \text{start}$

$T = \frac{9\pi}{6} - \frac{\pi}{6} = \frac{8\pi}{6} = \boxed{\frac{4\pi}{3}}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{4\pi/3} = \cancel{2\pi}\cdot\frac{3}{\cancel{4\pi}_2} = \boxed{\frac{3}{2}} \Rightarrow \boxed{\omega = \frac{3}{2}\ \text{rad/sec}}$

③ Phase angle: $\omega\cdot \cancel{t}_d + \theta = 0 \Rightarrow \boxed{\theta = -\omega\cdot d}$

displacement from y-axis to +sine, +cos

+sine: $\boxed{d = \frac{5\pi}{6}}$

$\theta = -\omega\cdot d = -\frac{\cancel{3}}{2}\cdot\frac{5\pi}{\cancel{6}_2} = \boxed{-\frac{5\pi}{4}}$

$\Rightarrow \boxed{y = 5\sin\left(\frac{3}{2}t - \frac{5\pi}{4}\right)}$

Answers

a) $v = 5V\sin\left(\frac{3}{2}t - \frac{5\pi}{4}\right)$ $\qquad v = 5V\cos\left(\frac{3}{2}t - \frac{7\pi}{4}\right)$

$\boxed{-\text{sine}}$ $\boxed{d=\frac{\pi}{6}}$ $\Rightarrow$ $\theta=-\omega\cdot d=-\frac{\cancel{3}}{2}\cdot\frac{\pi}{\cancel{6}_{2}}=-\frac{\pi}{4}$

$$\boxed{\theta=-\frac{\pi}{4}}$$

$$\boxed{y=-5\sin\left(\frac{3}{2}\cdot t-\frac{\pi}{4}\right)}$$

+cosine: $\boxed{d=\frac{7\pi}{6}}$ $\Rightarrow$ $\theta=-\omega\cdot d$

$$\theta=-\frac{\cancel{3}}{2}\cdot\left(\frac{7\pi}{\cancel{6}_{2}}\right)=\boxed{-\frac{7\pi}{4}}$$

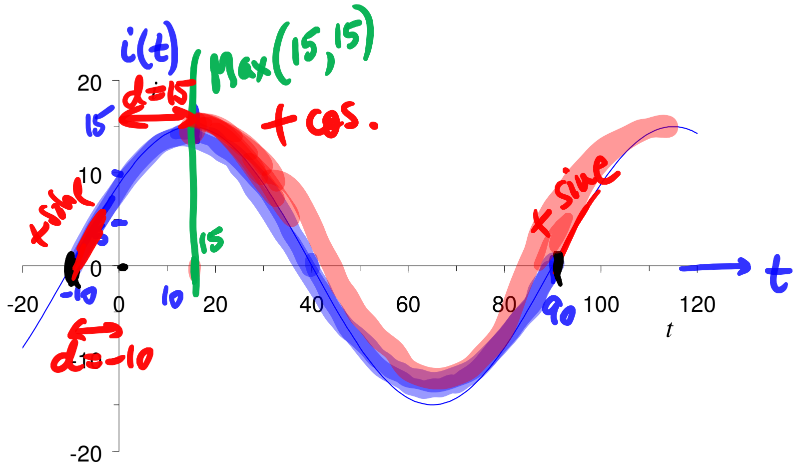
$$\boxed{y=5\cos\left(\frac{3}{2}\cdot t-\frac{7\pi}{4}\right)}$$

−cosine: $d=\frac{3\pi}{6}=\boxed{\frac{\pi}{2}}$ $\Rightarrow$ $\theta=-\omega\cdot d$

$$\theta=-\frac{3}{2}\cdot\frac{\pi}{2}=\boxed{-\frac{3\pi}{4}}$$

$$\Rightarrow\boxed{y=-5\cos\left(\frac{3}{2}\cdot t-\frac{3\pi}{4}\right)}$$

b)



+Sine, +cosine

Ampl = 15 A

Period : $T = 90-(-10) = \boxed{100 \text{ sec}}$ ⇒ ang. velocity

$$\omega = \frac{2\pi}{T} = \frac{\cancel{2}\pi}{\cancel{100}_{50}} = \boxed{\frac{\pi}{50}} \frac{\text{rad}}{\text{sec}}$$

Phase angle : $\boxed{\theta = -\omega \cdot d}$ → $d = ?$

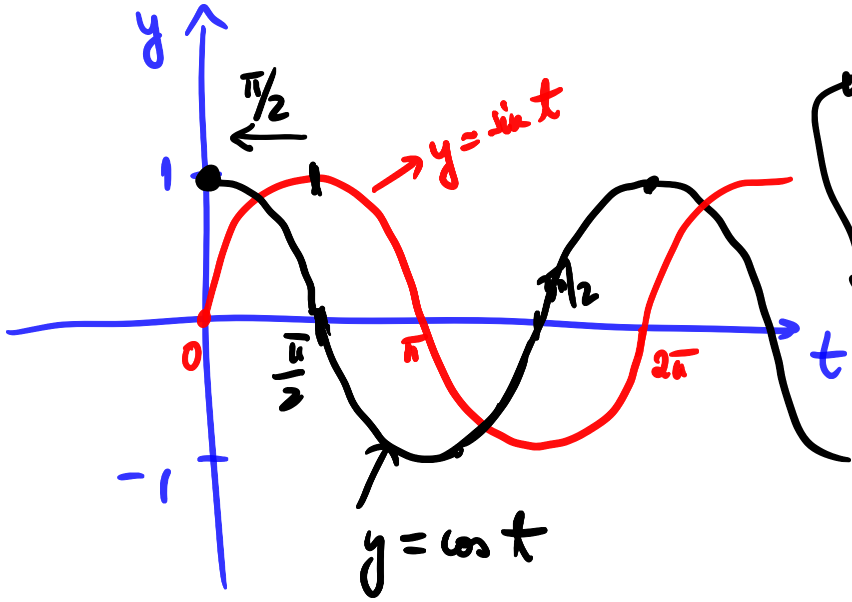
+Sine $\boxed{d=-10}$ (sec) ⇒ $\theta = -\frac{\pi}{\cancel{50}_{5}} \cdot (-\cancel{10}) = \boxed{\frac{\pi}{5}}$

$$\boxed{y = 15\sin\left(\frac{\pi}{50}\cdot t + \frac{\pi}{5}\right)}$$ (Amps)

+cosine $\boxed{d=15}$ (sec) ⇒ $\theta = -\omega \cdot d = -\frac{\pi}{\cancel{50}_{10}} \cdot \cancel{15}^{3} = \boxed{-\frac{3\pi}{10}}$

$$\Rightarrow \boxed{y = 15\cos\left(\frac{\pi}{50}t - \frac{3\pi}{10}\right)}$$

b) $i = 15A\sin\left(\frac{\pi}{50}t + \frac{\pi}{5}\right)$ $\qquad$ $i = 15A\cos\left(\frac{\pi}{50}t - \frac{3\pi}{10}\right)$



shift $\frac{\pi}{2}$ left the graph of sine

$\Rightarrow$ graph cosine

shift $\frac{\pi}{2}$ right the graph of cos

$\Rightarrow$ graph sine

Trig. identities: $\boxed{\sin(A) = \cos\left(A - \frac{\pi}{2}\right)}$ (*)

Recall prev. ex b) $\boxed{i(t) = 15\sin\left(\frac{\pi}{50}t + \frac{\pi}{5}\right)}$

Use trig identity (*) to convert $\boxed{\text{sine}} \rightarrow \boxed{\text{cosine}}$

$$\rightarrow i(t) = 15\sin\Big(\underbrace{\frac{\pi}{50}t + \frac{\pi}{5}}_{A}\Big) = 15\cos\left(A - \frac{\pi}{2}\right)$$

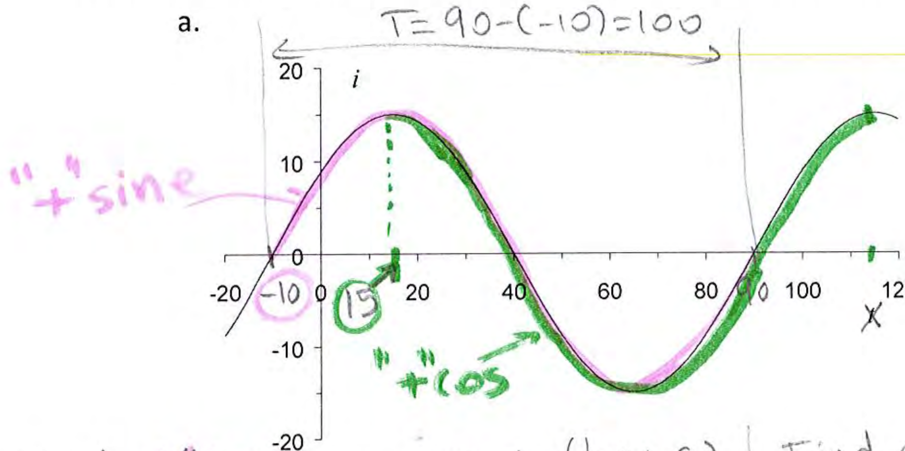
$$= 15\cos\Big(\frac{\pi}{50}t + \underbrace{\frac{\pi}{5} - \frac{\pi}{2}}\Big)$$

2/ 5/

$\rightarrow$ cosine waveform $\boxed{i(t) = 15\cos\left(\frac{\pi}{50}t - \frac{3\pi}{10}\right)}$

1. Find a sine and a cosine representation for the following waveforms.

a. $T = 90-(-10) = 100$



Find a "+" sine : $y = 15\sin(bx+c)$

Find $b = \frac{2\pi}{T} = \frac{2\pi}{100} = \frac{\pi}{50}$

Find $c = -d\,b$

$d = -10$ for a "+" sine cycle

$\Rightarrow c = -(-10)\cdot\frac{\pi}{50} = \frac{\pi}{5}$

so $\boxed{y = 15\sin\left(\frac{\pi}{50}x + \frac{\pi}{5}\right)}$

Find a "+" cos: $y = 15\cos(bx+c)$

Find $b = \frac{\pi}{50}$ - same as for sin

Find $c = -d\,b$

$d = 15$ for a "+" cos cycle

$\Rightarrow c = -15\cdot\frac{\pi}{50} = -\frac{3\pi}{10}$

$\Rightarrow \boxed{y = 15\cos\left(\frac{\pi}{50}x - \frac{3\pi}{10}\right)}$

Another method for obtaining the cos equation:

$$y = 15\sin\left(\boxed{\frac{\pi}{50}x + \frac{\pi}{5}}\right) = 15\cos\left(\boxed{\frac{\pi}{50}x + \frac{\pi}{5}} - \frac{\pi}{2}\right)$$

$$\boxed{\sin(\boxed{A}) = \cos\left(\boxed{A} - \frac{\pi}{2}\right)}$$

$$\boxed{y = 15\cos\left(\frac{\pi}{50}x - \frac{3\pi}{10}\right)}$$

Another method for getting these equations:

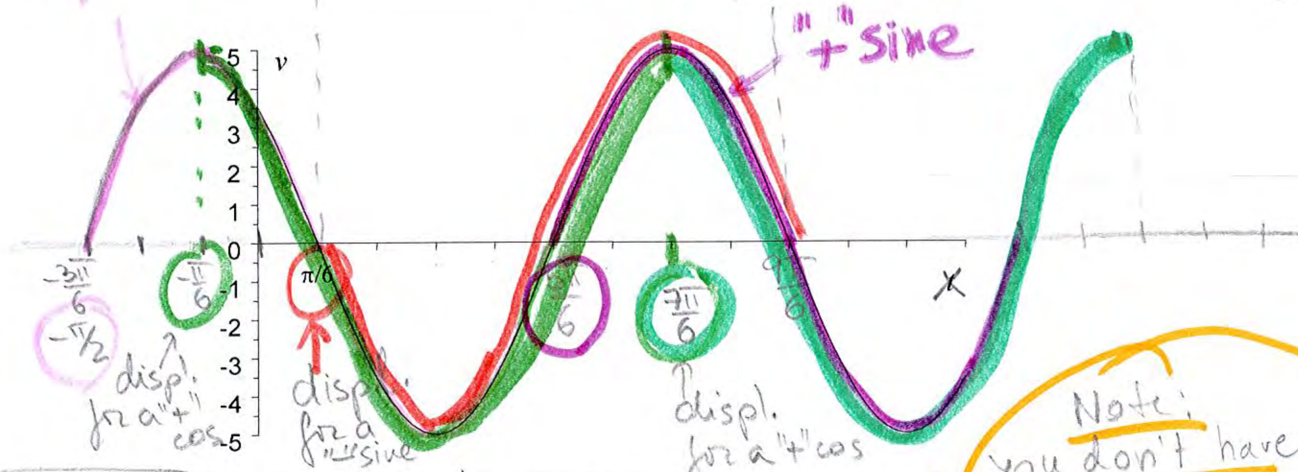
Find a "+" sine $y = 15\sin\left(\frac{\pi}{50}(x+10)\right) = 15\sin\left(\frac{\pi}{50}x + \frac{\pi}{5}\right)$

this becomes "0" if $x = -10$ - displ. for a "+" sine

OR $y = 15\cos\left(\frac{\pi}{50}(x-15)\right) = 15\cos\left(\frac{\pi}{50}x - \frac{3\pi}{10}\right)$

this becomes "0" if $x = 15$ - displ. for a "+" cos b.

$T = \frac{9\pi}{6} - \frac{\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$



Note: you don't have to find ALL these equations!

Find a "+" sine $y = 5\sin(bx + c)$

Find $b = \frac{2\pi}{T} = \frac{2\pi}{4\pi/3} = \frac{3}{2}$

Find $c = -db$

$d = \frac{5\pi}{6}$ OR $-\frac{\pi}{2}$ for a "+" sine

$\Rightarrow c = -\left(\frac{5\pi}{6}\right) \cdot \frac{3}{2} = -\frac{5\pi}{4} \Rightarrow \boxed{y = 5\sin\left(\frac{3}{2}x - \frac{5\pi}{4}\right)}$

OR

$c = -\left(-\frac{\pi}{2}\right) \cdot \frac{3}{2} = \frac{3\pi}{4} \Rightarrow \boxed{y = 5\sin\left(\frac{3}{2}x + \frac{3\pi}{4}\right)}$

these represent the same eq. since $\boxed{\sin(\theta) = \sin(\theta + 2\pi)}$ and there is a 2π phase angle difference:

$-\frac{5\pi}{4} + 2\pi = \frac{3\pi}{4}$

Find a "-" sine $y = -5\sin(bx + c)$

$b = \frac{3}{2}$ (same)

Find $c = -db$

$d = \frac{\pi}{6}$ for a "-" sine

$\Rightarrow c = -\frac{\pi}{6} \cdot \frac{3}{2} = -\frac{\pi}{4} \Rightarrow \boxed{y = -5\sin\left(\frac{3}{2}x - \frac{\pi}{4}\right)}$

these represent the same eq. since $\boxed{\sin(\theta) = -\sin(\theta \pm \pi)}$ and there is a "π" or "$-\pi$" phase angle difference:

$\frac{3\pi}{4} - \pi = -\frac{\pi}{4}$

Find a "+" cos: $y = 5\cos(bx + c)$

$b = \frac{3}{2}$ (same)

$-\frac{\pi}{2}$

Find $c = -db$

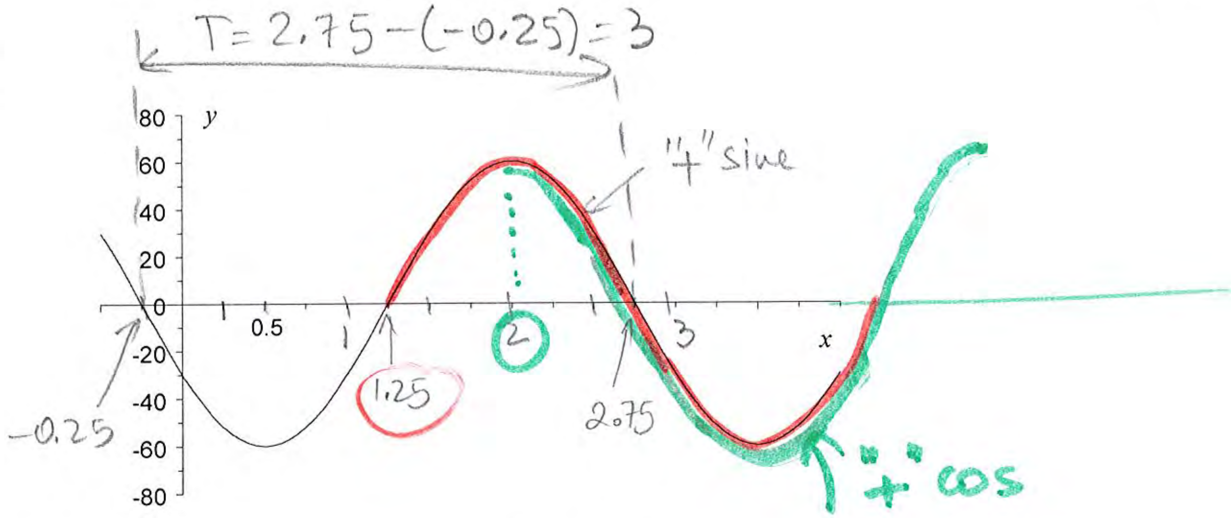
$d = -\frac{\pi}{6}$ or $\frac{7\pi}{6}$ for a "+" cos

$\Rightarrow c = -\left(-\frac{\pi}{6}\right)\left(\frac{3}{2}\right) = \frac{\pi}{4} \Rightarrow \boxed{y = 5\cos\left(\frac{3}{2}x + \frac{\pi}{4}\right)}$

OR $c = -\left(\frac{7\pi}{6}\right)\left(\frac{3}{2}\right) = -\frac{7\pi}{4} \Rightarrow \boxed{y = 5\cos\left(\frac{3}{2}x - \frac{7\pi}{4}\right)}$

same; 2π phase angle difference

c.



Find a "+" sine $\quad y = 60\sin(bx + c)$

Find $b = \frac{2\pi}{T} = \frac{2\pi}{3}$

Find $c = -db$

$d = 1.25$ for a "+" sine

$\Rightarrow c = -1.25\left(\frac{2\pi}{3}\right) = -\frac{5}{4}\cdot\frac{2\pi}{3} = -\frac{5\pi}{6} \Rightarrow \boxed{y = 60\sin\left(\frac{2\pi}{3}x - \frac{5\pi}{6}\right)}$

Find a "+" cosine $\quad y = 60\cos(bx + c)$

$b = \frac{2\pi}{3}$ (same)

Find $c = -db$

$d = 2$ for a "+" cos

$\Rightarrow c = -2\cdot\frac{2\pi}{3} = -\frac{4\pi}{3} \Rightarrow \boxed{y = 60\cos\left(\frac{2\pi}{3}x - \frac{4\pi}{3}\right)}$

OR $\quad y = 60\sin\left(\frac{2\pi}{3}x - \frac{5\pi}{6}\right) = 60\cos\left(\frac{2\pi}{3}x - \frac{5\pi}{6} - \frac{\pi}{2}\right)$

$\left(\sin(A) = \cos\left(A - \frac{\pi}{2}\right)\right)$

$-\frac{5\pi}{6} - \frac{\pi}{2} =$

$= -\frac{5\pi}{6} - \frac{3\pi}{6}$

$= -\frac{8\pi}{6} = -\frac{4\pi}{3}$