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**APEROL from Networks:
Analyzing Pipeline and Embedding Representations
for Optimized Learning (from Networks)**

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1 Motivation

Graphs have emerged as a natural model for the representation and analysis of data and complex systems in various domains. For instance, link prediction in social networks can help to understand the spreading process of rumors or epidemics [1], and in biological networks, it is commonly used for predicting novel interactions between proteins [2].

However, most traditional machine learning algorithms are not designed to work directly with graph-structured data, as they require numeric vectors or matrices as inputs. To address this challenge, embedding techniques have been developed to learn low-dimensional vector representations of nodes, edges, or entire graphs while preserving their topological properties.

Consequently, in order to design an effective embedding technique, it is necessary to consider several criteria [3]. Two of these are adaptability and scalability. An embedding is considered adaptable if it can be utilized for various data and tasks without the need for retraining. The scalability of an embedding is determined by its capacity to process large-scale networks within a reasonable amount of time.

The objective of this study is to evaluate known embeddings from the perspective of these two criteria.

From the perspective of adaptability, the objective is to determine the potential impact of the graph domain on the precision of an embedding. In other words, the objective is to ascertain whether there exist approaches that are better suited for a particular field, whether there is one embedding that consistently outperforms the others, or if the embeddings are comparable. Eventually, we would like to assess the adaptability of these embeddings when they are applied to different tasks. In the context of scalability, the objective is to identify the embeddings that are better suited to process large graphs. The objective of this study is to evaluate the tradeoff between the accuracy of the predictions and the time required for training and inference.

2 Datasets

The datasets that will be used in this project are nine and they are divided into three categories: Road Networks, Social Networks, and Biological Networks. For each of these categories, three datasets of increasing size were chosen: one small, one medium, and one large.

The datasets, with their main properties (number of nodes, number of edges, and type of graph), are reported in Table 1.

| Network | $ V $ | $ E $ | Type |
|--|-----------|-----------|------------|
| Pennsylvania [4] | 1.088.092 | 1.541.898 | Undirected |
| Padua (province) [5] | 122.680 | 304.184 | Directed |
| Hong Kong (city) [6] | 43.620 | 91.542 | Directed |
| Italian Covid-19 Retweet Network [7] | N/A | 800.000 | Directed |
| Twitch Gamers [8] | 168.114 | 6.797.557 | Undirected |
| GitHub Developers [9] | 37.700 | 289.003 | Undirected |
| Mus Musculus Protein Interactions [10] | N/A | 800.000 | Undirected |
| Saccharomyces cerevisiae Protein Interactions [10] | N/A | 100.000 | Undirected |
| Bio-grid-fission-yeast [4] | 2.000 | 25.300 | Undirected |

Table 1: Datasets used in the project

3 Methods

Node embedding is the task of mapping nodes into an embedding space, where each node v is represented by a vector $\mathbf{z}_v \in \mathbb{R}^d$ using the mapping function $ENC : v \rightarrow \mathbf{z}_v$. The encoding of nodes into embeddings has to be such that similarity between each pair of nodes is kept in the correspondent embeddings. In order to achieve this, a similarity score $S(u, v)$ is computed between pairwise nodes and a decoder function $DEC : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$ is used for computing a similarity score for the embeddings. The discrepancy between the two scores is then compared and minimized by means of a loss function, such as the empirical loss:

$$\mathcal{L} = \sum_{i,j} \ell(DEC(\mathbf{z}_u, \mathbf{z}_v), S(u, v))$$

Based on this context, the specifics of which encoder and decoder is used is what really differentiates the node embedding methods. In this project, four of these methods will be employed:

- Node2Vec [11], a shallow encoding method and essentially a tunable version of the DeepWalk algorithm, which is often used in the scientific literature as the baseline for comparing node embedding methods;
- LINE [12], a shallow encoding which is an highly-scalable approximation algorithm;
- Deep Variational Network Embedding in Wasserstein Space [13], a deep encoding method using a Variational Autoencoder. This metod has a particular approach of encoding each node to a probability distribution, so in this case the similarity score between embeddings is a distance between probability distributions, which can be measured using the Wasserstein distance;
- GraphSage [14], a deep encoding method making use of a GNN that aggregates the features of each node's neighborhood and learns to generate node embeddings from that.

Given the node embeddings, we use them as input to ML models to make predictions, depending on the task to solve. For this project the main task is link prediction, where the goal is to predict the existence of a link between two nodes in a network. Since the quality of the embeddings could depend on the model used, we'll try models of different complexity:

- Support Vector Machines [15], where eventually the kernel trick could be used to handle non-linear relations between embeddings;
- Random Forest [16], for its really fast computation;
- Multilayer Perceptron [17], for accurate predictions.

4 Experiments

To compare the embedding methods, we first make a sample of the edges of the graph and then partition it into training, validation, test sets. We then apply the embedding methods and use the models described in Sec3 to predict new links, and finally compare the predictions with the edges which are not included in the sample. We employ each model for all embeddings, so that we can compare the impact of each one of them on the prediction of each model.

The implementations used are: [<empty citation>] for

Hardware used:

To evaluate the methods used in this project, two different evaluation measures will be used: the Area Under the Receiver Operating Characteristic curve (AUROC), and the Area Under the Precision-Recall curve (AUPR). AUROC is historically considered the primary performance metric used to evaluate the performances of Link Prediction methods [18]. However, AUROC favors accurate classification of positive examples, at the cost of misclassifying the negative ones. In a scenario like Link Prediction problem, which is inherently skewed towards the negative class, it may not be suitable and can overestimate the performance of the methods. For this reason, the AUPR curve was also selected, as it can provide better evaluation of Link Prediction in the presence of class imbalance.

Finally, the metrics will be used to get relevant information about the quality of the embeddings. The results will be analyzed to assess how each embedding scales and adapts with different categories and sizes.

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Contribution of Authors and AI Usage

The contributions for this project are:

- Motivation: Marco Annunziata (research), Silvia Mondin (research and writing), Sveva Turola (research);
- Datasets: Marco Annunziata (research), Silvia Mondin (research), Sveva Turola (research and writing);
- Methods: Marco Annunziata (research and writing), Silvia Mondin (research), Sveva Turola (research);
- Experiments: Marco Annunziata (research and writing), Silvia Mondin (research), Sveva Turola (research and writing).

It was used AI tools (DeepL) for correcting the grammar and syntax of the text.