

# A SIMPLE ORDERING SOLUTION FOR EFFICIENT GEOMETRICAL LEVEL OF DETAILS ON LIDAR POINT CLOUD

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## 1. Abstract

We introduce a new ordering for point cloud based on octree, which allows, after a pre-computing phase, to effortlessly get a portable representative geometric Level Of Details (LOD). This LOD have then multiple applications : point cloud size reduction for visualisation (point cloud streaming) or speeding of slow algorithm, fast density peak detection and correction as well as safeguard for methods that may be sensible to density variations. The LOD method is also used as a crude dimensionality descriptor, enabling fast classification and on-the-fly filtering with simple classes.

## 2. Introduction

### 2.1. Problem

Point cloud data is becoming more and more common. Following the same trend, the acquisition frequency and precision are also increasing. Thus point cloud processing is clocking on the Big Data door.

Yet the usage of point cloud data is also spreading and going out of the traditional user communities. Lidar are now commonly used by non-specialized users.

For many usage, having the raw, complete point cloud is unnecessary, or even damageable. Thus we deal with a simpler version of a problem that the G.I.S community has faced for a long time : how to generalize point cloud, with data sets that are several order of magnitude bigger than usual vector data set ?

It is after all a problematic very common in data processing. Having a big data set, how to reduce its size while preserving its characteristics. It is the essence of compression for instance.

Generalization is also more difficult when mixing data set with varying densities. For instance an aerial Lidar map augmented at certain places by terrestrial scanners, or vehicle-based Lidar acquisition, where the density varies with speed and scene geometry. This illustration 6 shows the density variation in the two data set used for experiments in this article.

Here we deal with a simplified version : given a point cloud, how to efficiently generate Level Of Detail (LOD, cf figure 1) of this point cloud while preserving the geometric characteristic, without duplicating data ? The key to LOD approach is efficiency. Indeed LOD approaches sacrifices of a part of information in exchange of a massive reduction of data size. That's why a solution using LOD must by nature be efficient, or the information loss would be pointless.

### 2.2. Motivation

- Point cloud are becoming common. Point cloud are becoming common because sensors are smaller, cheaper, easier to use. Point cloud from image (using Stereo Vision) are also easy to get with several mature structure from motion solutions. Point cloud complements very well images, Lidar point cloud allowing to avoid the ill-posed problem of stereo-vision, and providing key data to virtual reality.
- Growing data set and multi sources. At such the size of data set are growing, as well as the number of dataset and their diversity.
- A now widely use data type. The point cloud data are now well established in a number of industries, like construction, architecture, robotics, archaeology, as well as all the traditional GIS fields (mapping, survey, cultural heritage).
- Much less focus on informatics/storing. The LIDAR research community is very active. The focus of Lidar researchers is much more on Lidar processing and Lidar data analysis, or the sensing device, than on methods to render the data size tractable.

### 2.3. State of the Art

One way to tackle data size is to use a Level Of Detail strategy. Octree methods have been common in computer graphics for several decades (Meagher, 1982). They are used in many methods to speed computing, or compress data, like in the work of Schnabel and Klein (2006); Huang et al. (2006) (which have not been designed to scale).

Elseberg et al. (2013) give a good overview of octree usage for point clouds. Their method proposes to directly store the points and octree in a file. They explore many applications that could benefit from our method (visual LOD, registration, processing). We share many objectives. It is an extremely effective approach that also enables visual LOD, but is very specialized on geometry. In particular, this method is not integrated with other GIS data (Geographical Information System), point cloud fast querying is not possible on attributes or metadata, and point cloud format is extremely specific.

Another orthogonal way is to use no file, but store point cloud in DBMS. van Oosterom (2014) implements such system at very big scale and discuss how it can answer to various need. This approach has gained recent interest (pgPointCloud, 2014) because it is generic, it scales naturally to very large data set and is easier to implement than starting from scratch. We also observe that most of

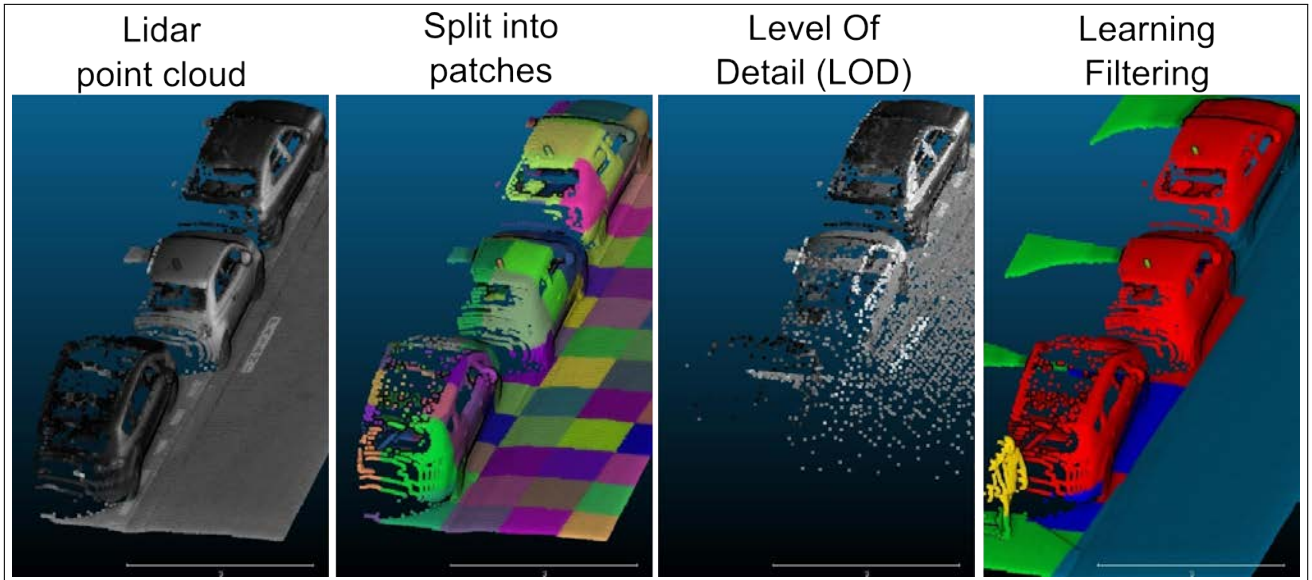


FIGURE 1: Data flow : starting from Lidar point cloud, we split it into patches, then reorder the patches to obtain free LOD (a gradient of LOD on this illustration). Lastly we use this ordering as feature for learning and efficient filtering

Big Data system use methods from the DBMS world.

[Demantké \(2014\)](#) introduces a sophisticated per-point dimensionality descriptor, which is used to find optimal neighbourhood size. A main difference is that this feature is computed for each point (thus is extremely costly to compute), and that dimensionality is influenced by density variation.

Random Forest method started with [Amit and Geman \(1997\)](#) and theorized by [Breiman \(2001\)](#) and has been very popular since then. They are for instance used by [Golovinskiy et al. \(2009\)](#) who perform object detection, segmentation and classification. They analyse separately each task on an urban data set, thus providing valuable comparison. Their method is uniquely dedicated to this task, like [Serna and Marcotegui \(2014\)](#) who provide another method and a state of the art of the segmentation/classification subject. Both of this methods are in fact 2D methods, working on an elevation image obtained by projecting the point cloud. However we observe that street point clouds are dominated by vertical surfaces, like building (about 70% in Paris data set). Our method is fully 3D and can then easily be used to detect vertical object details, like windows or doors on buildings.

## 2.4. Contribution

This paper re-uses and combines existing and well established methods with a focus on simplicity and efficiency. As such, all the methods are tested on billions scale point cloud, and are Open Source for sake of reproducibility test and improvements. We propose a simple method that enables portable, computation-free, geometrical Level Of Detail. Our first contribution is to propose to store the LOD information directly into the ordering of points rather than externally, avoiding any data duplication. Thus, the more we read points, the more precise of an approximation of

the point cloud we get. If we read all the points, we have the original point cloud.

The second contribution is a simple way to order points in order to have an increasingly better geometric approximation of the point cloud when following this order.

The third contribution is to use the ordering construction by-product as a simple and free dimensionality descriptor. We demonstrate the interest of this descriptor by performing a Random Forest classification that can then be used for very fast pre-filtering of points, and other applications.

## 2.5. Plan of the article

The rest of this article is organized as follows : in the next section [3](#) we present the methods. In the result section [4](#) we give the results. We discuss it and the possible limitations in section [5](#).

# 3. Method

## 3.1. Introduction

In this section we introduce a simple method which proposes free geometrical LOD features, at the price of a small preprocessing time. The method relies on ordering the points so that reading the points following this order is going to gradually increase the details of the point cloud. We use a by-product of this method to perform efficient training and filtering with Random Forest.

This method has been designed to work with Lidar data sets, but may be used on noisy Structure from Motion (SfM) point clouds after filtering. This method has been used inside our point cloud data management system which is centered on a Point Cloud Server (see [Cura \(2014\)](#) for a very detailed presentation of it).

We stress that Point Cloud Server works with patches,

which are groups of points obtained by cutting the original point cloud into regular pieces ( $1^3$  meter wide cube for Paris data set,  $50^3$  meter wide cube for Vosges dataset). A patch is technically a subset of a point cloud, so also a point cloud. For generality we will use the term point cloud to describe both patch or a global point cloud for the rest of this article, as our method can be used indifferently on patches and on the global point cloud. Lastly, computing this LOD gives interesting by-product that can be used as a crude local dimensionality descriptor. We use that to perform extremely fast rough-filtering of massive point cloud.

## 3.2. Exploiting the order of points

### 3.2.1. Principle

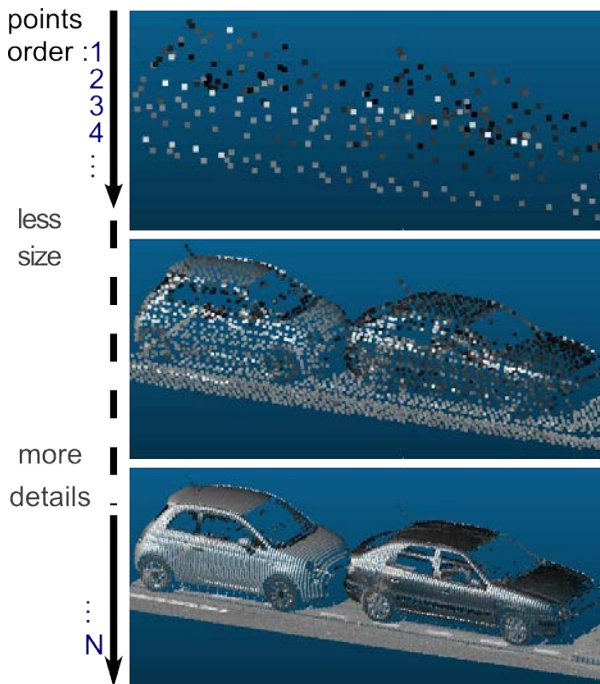


FIGURE 2: 3 geometrical Level Of Detail (LOD) for the same point cloud. Starting from very low detail by reading the first point of the patch, reading further points increases the details.

Starting from a patch (i.e. a piece of point cloud) we generate geometrical Levels Of Details on it. Point clouds are physically stored as a list of points (compressed or not). Once the points are written to the disk, the information contained by a list of point is not the sum of information for each point. The big difference is that a list is ordered. This fact is common for every point cloud format, they are inherently ordered. In fact it would be quite hard to store the points in an unordered fashion (true randomness is expensive).

The first contribution of this article is to propose to exploit this ordering to store information. The key idea is to order the points of the point cloud so that when reading the points from the beginning to the end, we get gradually a more accurate geometrical approximation of the point cloud. This is then a continuously better LOD.

We demonstrate it with an ordering presented in 3.3, but we could use any ordering methods.

### 3.2.2. Conceptual example

For a given list  $L$  of points ordered from 1 to  $N$ . Reading the points 1 to 5 is going to give a rough approximation of the point cloud, and reading another 16 points (points 1 to 21) is going to give a slightly better approximation. Reading points 1 to  $N$  is going to get the exact point cloud, so there is no data loss, nor data duplication.

### 3.2.3. Advantages

This method has 3 majors advantages.

*No on-line processing time.* Except a pre-processing step to write the point cloud following a given ordering, each time the user wants to get a Level Of Detail version of the point cloud, there is no computing at all (only data reading). This may not make a big difference for non-frequent reading, but in a context where the same point cloud is going to get used several times at several levels and by several users simultaneously (for instance Point Cloud as a Service), no processing makes a big difference. (See [this illustration](#) for an example with LOD visualisation.)

*No data duplication.* Another big advantage of using the order of points to store the LOD information rather than using external structure data is that it is free regarding the storage. This is an advantage on low level. First it saves disk space, it ensures that all the information is at the same place, which avoids to perform OS level commands like going through directories, opening the structure file, etc. Having no data duplication is also a security in concurrent use. In the scenario where a point cloud and the associated data structure are separated, and both are accessed by concurrent users, it is possible that one user change the point cloud. The associated data structure is then updated, but during this laps of time it is possible for another user to get updated point cloud and the wrong associated data structure. On the opposite, when the ordering gives the LOD information, a user either get the old points with the old ordering, or the new points with the new ordering, which guarantee that data is in a coherent state. Lastly we avoid all precision-related issues raised by traditional subsample methods because the order doesn't change a bit of the existing points and their attributes, but only there order in the point cloud.

*Portable.* The last advantage comes from the simplicity of using the ordering. Because it is already something that all point cloud tools can deal with (a list of points !), this way to create LOD is portable. Most softwares do not change the points order inside a cloud. Even if a tool were to change the order, it is easy to add the ordering number as an attribute, which increases a little bit storage size, but is totally portable and can be used with all existing tools. This simplicity also implies that adapting tools to use this ordering is very easy.



### 3.3. MidOc : an ordering for gradual geometrical approximation

#### 3.3.1. Principle

Ordering the points necessitate an efficient method, because using LOD is already a trade-off between data information loss and data size reduction.

We note that having usable LOD is only possible because Lidar point clouds have intrinsic structure we must exploit. Precisely because we do a trade-off, we must exploit the intrinsic structure of the point cloud.

For this, we make some assumption that are mostly verified on Lidar point cloud :

- Point cloud represents 3D surfaces sensed by a fix or mobile sensor (with the exception of multi-echo, which is correctly dealt with anyway).
- geometrical noise (outliers) is low.
- the density may vary, but we don't want to preserve it, nor does it give information about the nature of the object being sampled.

that is, depending on the sensing situation, we some parts of the cloud are more or less dense, but this has nothing to do with the nature of the object sensed, thus must not be preserved.

We rely on a classical middle of octree subsampling (called MidOc in this article for simplicity) to create an ordering. This is a re-use of well known and well proven existing methods (for instance, the octree subsampling is used in [Girardeau-Montaut \(2014\)](#)). We name this ordering for clarity of this article, nonetheless we don't think we are the first to use it.

Again the principle is very simple, and is to take for each level the points closest to centre of its octree voxel, if any.

**Data:** A list of  $N$  points

**Result:** the MidOc ordering of the points for an octree of  $L$  depth

**foreach** *level* in  $[0..L-1]$  **do**

**foreach** *cell* of *level* that is non empty **do**

        Compute *center* of the *cell*;

        Choose the point in the *cell* that is closest to the *center*, if it exists;

**end**

**end**

Order chosen points by level (coarse to fine),  
random (or hash or inverted Z curve);

**Algorithm 1:** MidOc principle

We illustrate this principle on [this figure](#) in 2D (quad tree) for graphical comfort. The layer are level 0 to 2. For each layer we compute the closest point (large green point) to the center of each cell (medium black point) using the squared euclidian distance (continuous red tone). The points picked on a level are not available for further level (no duplication of information). When reaching the desired max level, all the remaining points are ordered either randomly or deterministically (for instance, with an inverted Z Morton curve (read backward) , ). We use the same random or deterministic order for points picked on

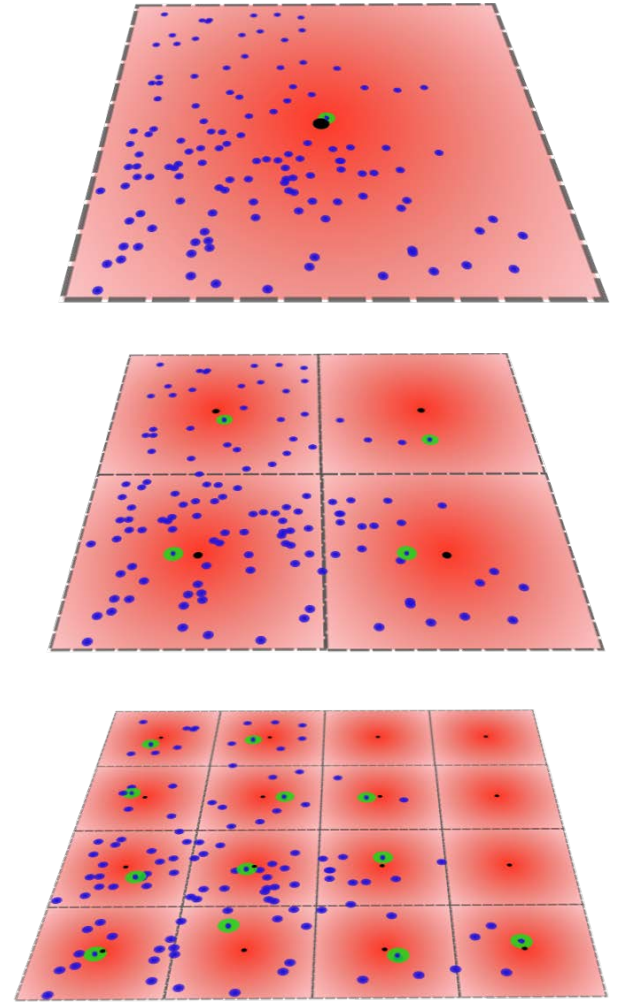


FIGURE 3: Principle of MidOc explained on a Quad tree for comfort. Blue dots are a 2D point cloud. Black dots are centers of each cell, green dots are the point selected per non empty cell per level that is the closest to the center of the cell (black dots). Red continuous tone represents the distance to center of cell

the same level. Overall, the ordering is then (tree depth ascending, random or inverted Morton).

#### 3.3.2. Applications

We see 3 type of application for this ordering :

- graphical LOD : service for point cloud visualisation.
- density correction : service for complex processing that may fail to deal with important density variation
- point cloud generalisation, service for processing that may use only a fraction of points.

We stress that the LOD are in fact almost continous (as in the third illustrations of [1](#)).

#### 3.3.3. Implementation

This method has the same complexity as an octree construction. Similar strategies can be followed, pending on available resources.

The most straightforward implementation is streamed : The closest point to the center is stored in each octree



FIGURE 4: All successive levels for common objects (car, window, 2 wheelers, light, tree, people, pole, ground), color is intensity for other points.

cell. At the beginning the octree is empty. When receiving a point, traverse the octree update/creating cells and updating the closest point for each cell traversed. When the

tree is computed, traverse it breadth-first to collect chosen points. The worst complexity is  $O(N \cdot T)$ , the entire octree must be stored in memory.

The simplest implementation use a recursive strategy. it only necessitate a method that given a cell and a list of points chose the point closest to the center of the cell, then split the cell and the list of points for the next level, and recursively calls itself on this subcells with the sublists.

We propose this kind of implementation in Python with extensive use of bit-level operations (cf 3.3.4) as proof of concept. The octree is not stored in memory, but it may prove difficult to parallelize (which is done at the data level anyways for our experiments).

### 3.3.4. Efficiency and performance

Octree construction may be avoided by simply reading coordinates bitwise in a correctly centred/scaled point cloud. We centre a point cloud so that the lowest point of all dimension is  $(0, 0, 0)$ , and scale it so that the biggest dimension is in  $[0, 1[$ . The point cloud is then quantized into  $[0..2^L - 1]$  for each coordinate. The coordinates are now integers, and for each point, reading its coordinates bitwise left to right gives the position of the point in the octree for level of the bit read. This means performing that this centring/scaling/quantization directly gives the octree. Moreover, further operations can be performed using bit arithmetic, which is extremely fast.

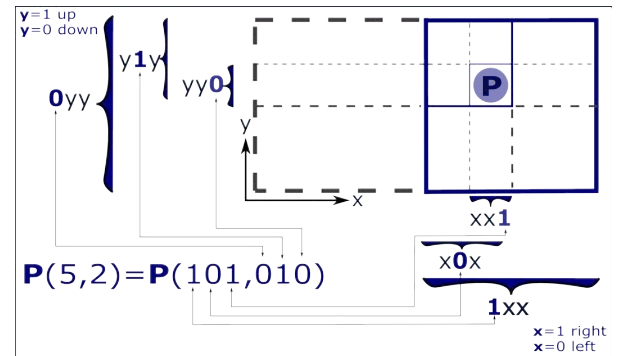


FIGURE 5: Principle of binary coordinates for a centered, scaled and quantized point cloud.

*Example.* On this illustration the point  $P$  has coordinates  $(5, 2)$  in a  $[0, 2^3 - 1]^2$  system. Reading the coordinates as binary gives  $(b'101', b'010')$ . Thus we now that on the first level of a quad tree,  $P$  will be in the right ( $x=b'1xx'$ ) bottom ( $y=b'0yy'$ ) cell. For the next level, we divide the previous cell in 2, and read the next binary coordinate.  $P$  will be in the left ( $x=b'x0x'$ ) up ( $y=b'y1y'$ ) cell. There is no computing required, only bit mask and data reading.

### 3.3.5. Advantages

*Common, Simple and efficient.* This method feels classical and is based on Octree. This makes it simple to implement, and possibly extremely memory and CPU efficient.

The complexity is  $O(n_{points} * Levels)$  or less. It is very simple to implement. Octree construction has been commonly done on GPU for more than a decade.

*Fixed density.* This method can be used to guarantee an almost constant density for a given levels, even when the acquisition process produced varying-density point cloud. Thus using the output of this method is a safeguard for most of the complex point cloud processing methods that may be badly affected by extrem variation in density ( real world case illustrated in [this illustration](#)).

*Generic.* Few hypothesis are made on the points properties. In particular, this method works well with 2.5D point cloud (aerial single echo Lidar) and full 3D point cloud (urban 3D cloud with multi echo).

*Construction Descriptors.* Lastly, the information given by this ordering can be used as a geometrical descriptor of the point cloud. The number of points per level for each level gives a crude dimensionality descriptor of the geometrical nature (at the patch scale) for the object contained by the point cloud.

### 3.4. Using the ordering by-product as a crude dimensionality descriptor

#### 3.4.1. Principle

During the ordering process the number of chosen point per each level can be stored. This number of chosen points per level gives an indication on the geometric nature of the object in the point cloud. We demonstrate the use of this crude descriptor along with other simple feature with a Random Forest classifier on a real world dataset publicly available. This patch classification can be used in other ways than pure classification.

#### 3.4.2. Conceptual example

In the [figure 4](#), we manually segmented typical parts of a street in the Paris dataset : a car, a wall with window, a 2 wheelers, a public light, a tree, a person, poles and piece of ground including curbs.

Due to the geometry of acquisition and sampling, the public light is almost a 3D line, resulting in the occupation of very few octree cells. A typical number of points chosen per level for the public light would then be (1, 2, 4, 8), which looks like a  $2^L$  function. A piece of ground is often extremely flat and very similar to a plan, which means that the points chosen per level could be (1, 4, 16, 64), a  $4^L$  function. Lastly a piece of tree foliage is going to be very volumetric in nature, due to the fact that a leaf is about the same size as point spacing and is partly transparent to laser (leading to several echo). Then a foliage patch would typically be (1, 8, 64, 512) (if enough points), so a  $8^L$  function.

#### 3.4.3. Patch classifier applications

The base method is to use Random Forest to classify patches (and not points directly).

Because patches may contains points belonging to several classes, transferring the patch classes to points naturally increases the errors.

We can foresee three type of application for patch classification.

*Speeding a complex point classifier.* The first application could be to speed up and/or improve a complex per point classifier. For a speed up, the patch classifier could perform a first basic classification extremely fast, thus eliminating a large number of points, before the complex classifier is used. If necessary, it is possible to artificially increase recall at the cost of a diminution of precision, like in [15](#).

*Improving a complex point classifier.* Patch classifier can also be used to improve result of a complex classifier by performing a first rough analysis which may determinate which complex classifier to use amongst several, like Cascaded classifier. For instance a patch classified as urban object would lead to chose a classifier specialized in urban object, and not the general classifier. This is especially precious for classes that are statistically very minority. If necessary it is possible to artificially increase precision at the cost of recall.

*Filtering.* Another application is filtering for applications that only require one class. When the learning is done, classifying is extremely fast.

Many applications only need one class, and do not require all the points in it, but only a subset with good confidence. For this it is possible to artificially boost the precision by accepting only high confidence prediction. For instance computing a Digital Terrain Model (DTM) only requires ground points. Moreover, the ground will have many parts missing due to objects, so using only a part of all the points will suffice anyway. The patch classifier allow to find the ground patch extremely fast. Another example is registration. A registration process typically require reliable points to perform mapping and registration. In this case there is no need to use all points, and the patch classification can provide patches from ground and facade with high accuracy (for point cloud to point cloud or point cloud to 3D model registration) , or patches of objects and trees (for points cloud to landmark registration). In other applications, finding only a part of the points may be sufficient, for instance when computing a building map from facade patches.

#### 3.4.4. Descriptors

*Crude dimensionality descriptor.* Again we work at the patch level ( $1^3$  or  $50^3 \text{ m}^3$ ). We order all patches following the MidOc ordering. For each ordered patch, we associate the number of points per level that where chosen. A Random Forest classifier is trained using the number of chosen points per level. We use the number of points for the level [1..4] included. For each level, the number of points is normalized by the maximum number of points possible ( $8^i$ ), so every feature is in  $[0, 1]$ .



*Other simple features.* We also use other very simple features that require almost no computing. Feature usage is then analysed afterwards. For the sake of simplicity and efficiency, all the feature use basic statistics per patch that need to be precomputed anyway by the storage compression mechanism. This free statistics are min, max, and average of any attribute.

Contextual features are avoided. They are more in the spirit of complex classification, would require computing, and could introduce a bias (in our favor) in the result. However using the context after the classification is a lever to significantly improve recall.

#### 3.4.5. Analyzing data set classes

*Analysing class hierarchy.* The Paris data set classes are organized in a hierarchy (100 classes). Because of the hierarchy and the unbalancing of classes, we first determine or similar the classes are for the simple descriptors. This information is extracted from the confusion matrix, which is used as an affinity matrix. The matrix is clustered by spectral clustering and the result are interpreted as a graph of classes. From this we choose the classes to use.

*Balancing the data set.* We tried two classical strategies to balance the data set regarding the number of observation per class. The first is undersampling : we randomly undersample the observations to get roughly the same number of observation in every class.

The second strategy is to compute a statistical weight for every observation based on the class prevalence. This weight is then used in the learning process.

#### 3.4.6. Patch classifier

To ensure significant results we follow a K-fold cross-validation method. We randomly split the observations into K parts, then for each part, we use the K-1 others to learn and predict on the part. All the evaluation are then performed on the total of predicted observations.

Contrary to classical classification method, we are not only interested in precision and recall per class, but also by the evolution of precision when prediction confidence varies.

In fact, for a filtering application, we can leverage the confidence information provided by the Random Forest method to artificially boost precision (at the cost of recall diminution). We can do this by limiting the minimal confidence allowed for every prediction. Similarly, it is possible for some classes to increase recall at the cost of precision by using the result of a first patch classification and then incorporate in the result the other neighbour patches.

We stress that if the goal is to detect objects (and not classify each point), this strategy can be extremely efficient. For instance if we are looking for objects that are big enough to be in several patches (e.g. a car). In this case we can perform the classification (which is very fast and efficient), then keep only highly confident predictions, and then use the position of predictions to perform a local search for car limits. The classical alternative solution would be to perform a per point classification on each point, which would be extremely slow.

#### 3.4.7. Advantages

*Dimensionality descriptor.*

- simple : Dimensionality feature for point clouds are already well researched, and can be more precisely computed (Demantké (2014)), with less sensibility to outliers (but more to density variation). However This kind of feature is generally designed at the point level, and is more complex. Using the result of the MidOc ordering has the advantage of being free and extremely simple.
- Efficient : Moreover, because  $x_0 \rightarrow (2^0)^x$ ,  $x_1 \rightarrow (2^1)^x$ ,  $x_2 \rightarrow (2^2)^x$  diverge very fast, we only need to use few levels to have a quite good descriptor. For instance, using  $L = 2$ , we have  $D = 4, 16$  or  $64$ , which are very distinguishable values, and don't require a density above 70 points /patch.
- Density and scale independent : As long as the patch contains a minimal number of points, the descriptors is density and scale invariant.
- Mixed result : Lastly a mixed result (following neither of the  $x_i \rightarrow (2^i)^x$  function) can be used as an indicator that the patch contains mixed geometry, either due to nature of the objects in the patch, or due to the way the patch is defined (sampling).

*Patch classification.*

- simple and fast When the Random Forest classifier is trained, prediction is extremely fast.
- good result Even if the classifier works on patch that may contain points from several classes, the global results for well represented classes are not far from state of the art.
- many applications teh classification is not necessarily interesting per see, but also for fast filtering or other applications.

## 4. Result

### 4.1. Introduction to all experiments

We design and execute several experiments in order to validate all points that have been introduced in the "method" part. First we prove that is it effectively possible to leverage points order, even using canonical open sources software out of the box. Second we perform MidOc ordering on very large point cloud and analyse the efficiency, quality and applications of the results. Third we use the number of points chosen in the MidOc ordering as a descriptors for a random forest classifier on two large data sets. We analyse the potential of this free descriptors, and what it brings when used in conjunction to other simple descriptors.

*Software stack.* The base DBMS is PostgreSQL (2014). The spatial layer PostGIS (2014) is added to benefits from generic geometric types and multidimensional indexes. The specific point cloud storage and function come from pgPointCloud (2014). The MidOc is either plpgsql or made in python with SciPy (2014). The classification is done with Scikit (2014), and the network clustering with Networkx (2014).

## 4.2. Point Cloud server introduction

### 4.2.1. Principle

*Server.* All the experiments are performed using a Point Cloud Server (cf [Cura \(2014\)](#)). The key idea are that point clouds are stored inside a DBMS (postgres), as patch. Patch are groups of points along with some basic statistics about points in the group. Patch are compressed using various strategies. This organisation is based on the observation that in typical point cloud processing workflow, a point is never needed alone, but almost always with its surrounding points.

*Fast filtering.* Each patch of points is then indexed in an R tree for most interesting attributes (obviously X,Y,Z but also time of acquisition, meta data, number of points, distance to source, etc.)

Having such a meta-type with powerful indexes allows use to find points based on various criteria extremely fast. (order of magnitude : ms). As an example we can find all points in a data set of 2 Billion points in a matter of milliseconds - between -1 and 3 meters high in reference to vehicle wheels - in a given 2D area defined by any polygon - close to a street called Rue Madame (according to IGN BDTopo) - between 3 and 5 meters to the sensor position - not in buildings according to Open Data Paris building layer - acquired in the second passage of the vehicle at this place - acquired between 8h and 8h10

*Parallelism friendly.* Cutting a point cloud into patches provides also a very easy parallel processing possibility by data partition, which we extensively use in our experiments.

*Point cloud splitting.* For our experiments we cut terrestrial Lidar point cloud into  $1 \text{ m}^3$  cubes oriented on (North, Est, Z) axis. We cut aerial lidar point cloud into  $50^3 \text{ m}^3$  cubes. The choice of size is a compromise between speed, index size, patch size, typical feature size, etc. In fact the patch can be cut arbitrary, we chose this splitting for simplicity.

### 4.3. Data set used

We use two data sets. There were chosen as different as possible to further evaluate how proposed methods can generalise on different data.

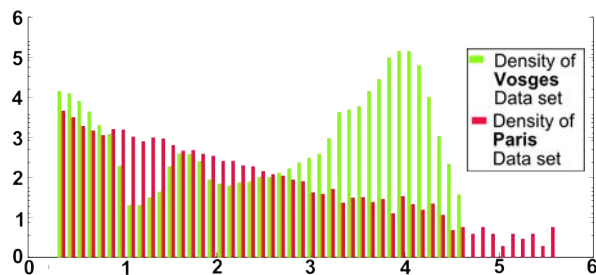


FIGURE 6: Histogram of number of points per patch, with a logarithmic scale for X and Y axis

### 4.3.1. IQmulus data set

First [IQmulus \(2014\)](#), an open source urban data set with varying density, singularities, and very challenging point cloud geometry. Data set is about 600 Millions points, over 12 kms of road. Points are typically spaced by 5cm to 0.2 cm. It is a multi echo laser (Riegl). We have access to a training set where every point is labeled in a hierarchy of 100 classes. The training set is only 12 millions points. Only 22 classes are represented. We will refer to this data set as Paris data set.

### 4.3.2. Vosges data set

We also use the Vosges data set, which is a very wide spread aerial data set of 6 billions points.

Density is much more constant to 10k pts/patch. We have access to a vector ground truth about surface occupation nature (type of forest), produced by the French Forest Agency. We will refer to this data set as the Vosges data set.

## 4.4. Exploiting the order of points

### 4.4.1. Experiment summary

In this first experiment we check that point cloud ordering is correctly preserved by common open source point cloud processing software. For this, we use a real point cloud, which we order by MidOc ordering. We export it as a text file as the reference file. Then for each processing software, we read the reference file and convert it into another format, then check that the conversion didn't change the order. The tree common open source software tested are CloudCompare, LasTools and Meshlab. This test point cloud is available in supplementary materials.

### 4.4.2. Results

All software passes test. We stress that even if software change order, it is still very easy to add the order as an attribute, thus making it fully portable.

## 4.5. MidOc : an ordering for gradual geometrical approximation

### 4.5.1. Experiment summary

In this experiment we first test ordering on typical street objects, then on terrestrial dataset to visually appreciate the fitness to use it for geometrical LOD. Then we compute MidOc for both our dataset and evaluate the trade-off between point cloud size and point cloud LOD. We briefly consider computing bottleneck.

We demonstrate an immediate application of LOD for fast abnormal density detection and correction. Lastly as a proof of concept we stream 3D point cloud with various LOD to a browser.

### 4.5.2. Visual evaluation

*Visual evaluation on typical objects (ground, facade, car, pole, vegetation).* The figure 4 illustrates LOD on common street objects, of various dimensionality.



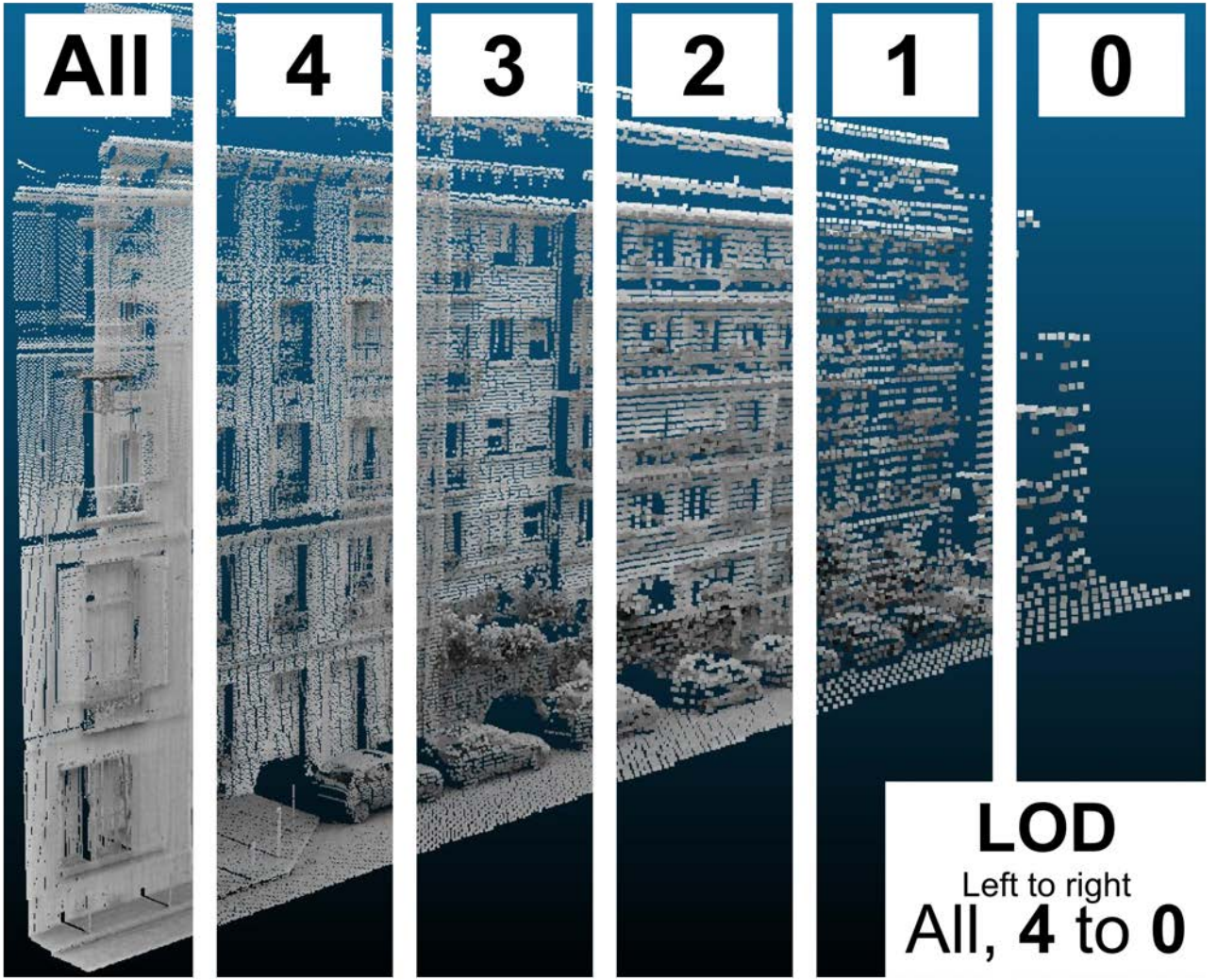


FIGURE 7: Schematic illustration of different LOD. Left to right, all points, then LOD 4 to 0. Visualized in cloud compare with ambient occlusion. Point size varies for better visual result.

#### 4.5.3. Size versus LOD trade-off

We compute the size and canonical transfer time associated for a representative street point cloud. For this order of magnitude, the size is estimated for 5\*4 Byte (5 floats) per point, and the internet transfer rate is estimated at 1 Mbyte/s.

TABLE 1: Number of points per LOD, plus estimated transfer time with modern internet connection.

Level	Typical spacing (cm)	Points number (k)	Percent of total size	Estimated time (s)
All	0.2 to 5	1600	100	60
0	100	3	0.2	0.1
1	50	11.6	0.7	0.4
2	25	41	2.6	1.5
3	12	134	8.4	5
4	6	372	23	14

#### 4.5.4. Large scale computing

*MidOc implementation.* We use 3 implementations of MidOc, two being pure plpgsql (postgreSQL script language), and one Python.

*Computing on very large dataset.* We successively order all the Paris and Vosges data sets with MidOc, using 20 parallel workers, with a plpgsql implementation. The ordering is successful on all patches, even in very challenging areas where there are big singularities in density, and many outliers. The total speed is about 100 millions points/hour, which we consider at least 10 times too slow. We briefly analyse performances, and conclude that data IO limits the number of efficient workers to 10, and that most of the time is actually not spend on computing, but on getting the points and writing them back.

#### 4.5.5. Fast abnormal density detection and correction

*Density abnormality (peak).* Important variation of density can be a serious issue for some processing methods, or simply performances (especially in parallel environment). The figure 8 shows a place where the density is 5 times over the normal value for this data set. In this context of terrestrial Lidar, this density peak is simply due to the fact that the acquisition vehicle stopped at this place, while continuing to sense data.

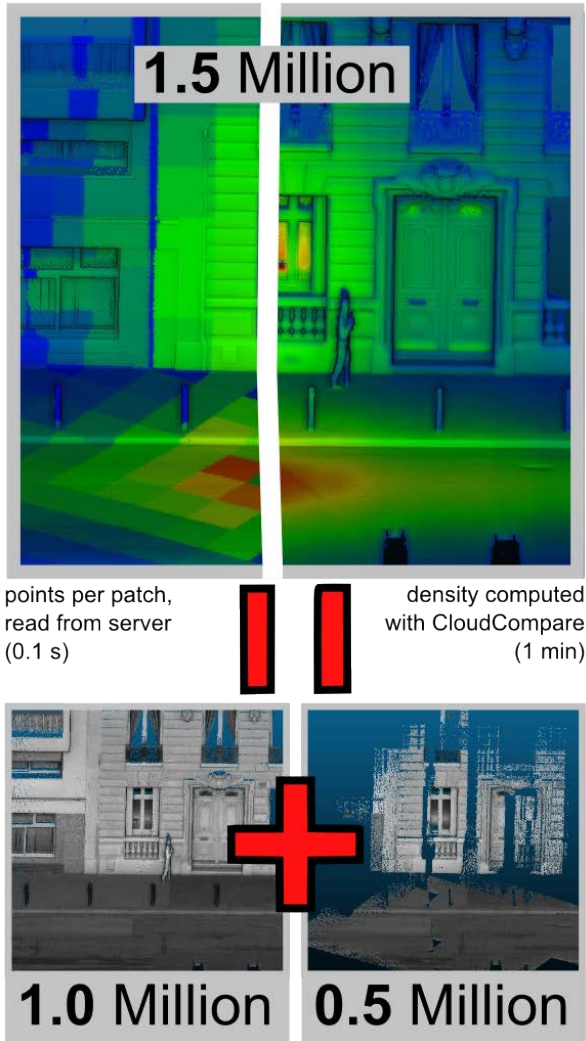


FIGURE 8: Abnormal density detection and correction. Top points per patch (left) or density (right), green-yellow-red. Bottom reflectance in grey.

**Fast detection.** We analyse the data set to find places of abnormally high density. It can be performed extremely fast at the patch level. (0.1 s for Vosges dataset, same for a 2 Billion points urban dataset). The Top left image shows the number of points per  $m^3$  directly read from database, increasing from blue to yellow to red) In comparison, computing the density per point with neighbourhood is extremely slow (only for this 1.5 Million extract, 1 minute with CloudCompare, 4x2.5GHz, 10cm rad) (top right illustration), and after the density is computed for each points, all the point cloud still need to be filtered to find abnormal density spot.

**Simple correction.** If the patch are ordered following MidOc, unneeded points are removed by simply putting a threshold on points per patch (bottom left, 1 to 5k points  $/m^3$ , bottom right, 5k to 24 k pts  $/m^3$ ). It considerably reduces the number of points (-33%).

#### 4.5.6. LOD stream

As a proof of concept we stream points from the data base to a browser [IGN \(2014\)](#). For this experiment we only stream a given number of points per patch, which allows to accelerate greatly data loading.

#### 4.6. Dimensionality descriptor and patch classification

##### 4.6.1. Experiment summary

For each patch, we store the associated number of points chosen per level (*ppl*) while computing MidOc ordering.

This dimensionality descriptor alone cannot be used to perform sophisticated classification, because many semantically different objects have similar dimension (for instance, a piece of wall and of ground are dimensionally very similar, yet semantically very different). An analysis of confusion matrix shows which meta classes are separable or not.

Extra descriptors are then needed : (P : for Paris , V : for Vosges : - *points\_per\_level* (*ppl*), level 1 to 4 (P+V) - average of intensity (P+V) - average of *number\_of\_echo* (P+V) - average of height regarding laser origin(P) - average Z (V) - patch height (P) - area of *patch\_bounding\_box* (P) :

##### 4.6.2. Separator power of *ppl* descriptor for Paris data set

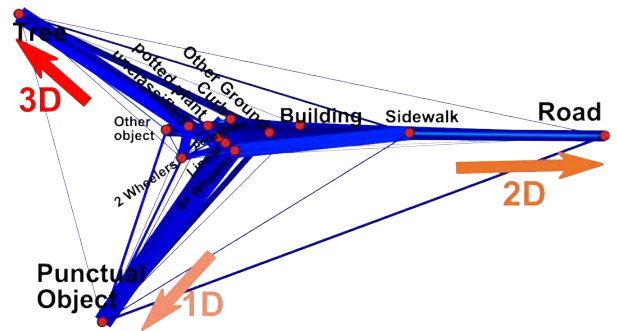


FIGURE 9: Spectral clustering of confusion matrix of Paris data set classification using only *ppl* descriptor. Edge width and colour are proportional to affinity. Node position is determined fully automatically. Red-ish arrows are manually placed to help understand

Using only the *ppl* descriptor, a classification is performed on paris data set, then a confusion matrix is computed. A spectral clustering of this matrix interpreted as a distance matrix between classes is performed. This clustering is used to place classes on the illustration. We manually add 1D, 2D and 3D arrows.

##### 4.6.3. Analyzing data set classes

**Balancing the data set.** Undersampling and weighting are used on the paris dataset. First Undersampling to reduce the over dominant building classe to a 100 factor of the smallest class support. Then weighting is used to compensate for differences in support.



For the Vosges data set only the weighting strategy is used.

The weighting approach is favoured over undersampling because it lessens variability of results when classes are very heterogeneous.

*Analysing class hierarchy.* Choosing which level of the class hierarchy uses depends on data set and applications. In a canonical classification perspective, we have to strongly reduce the number of classes if we want to have significant results. However reducing the number of class (i.e use a higher level in the classes hierarchy) also means that classes are more heterogeneous.

Both data set are extremely unbalanced (factor 100 or more). Thus our simple and direct Random Forest approach is ill suited for dealing with extremely small classes. (Cascading or one versus all framework would be needed).

For Vosges data set a short analysis convince us to use 3 classes : Forest, Land, and other, on this three classes, the Land class is statistically overweighted by the 2 others.

For the Paris data set, we analyse the confusion matrix by spectral clustering.

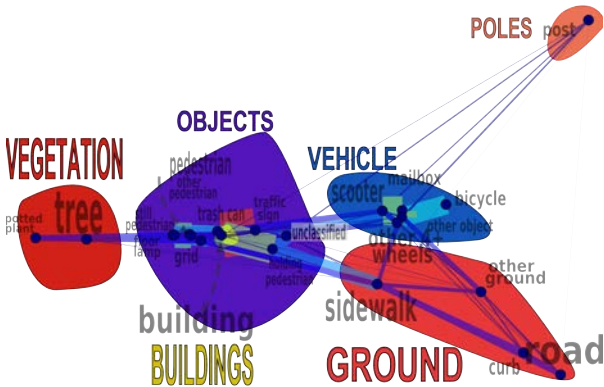


FIGURE 10: Result of automatic spectral clustering over confusion matrix for patch classification of Paris data set with all simple features. Edges width and colour are proportional to confusion. We manually draw clusters for easier understanding.

This analysis is useful to show the limit of the classification, because some class cannot be properly defined without context (e.g. the side-walk, which is by definition the space between building and road, hence is defined contextually).

#### 4.6.4. Patch classifier

*Vosges data set.* We perform a analysis of error on Vosges dataset and we remark that the error seems to be significantly correlated to distance ot borders.

The learning time is few minutes (monoprocess, python), the predicting time is few seconds (same).

*Paris data set.* The learning time is less than a minute, the predicting time is less than a second.

#### 4.6.5. Patch classifier applications

*Artificial increase of precision.* We demonstrate the use of artificial increase of precision. Initial results (blue) are mostly correct, but by only keeping patches with high confidence, it is possible to increase precision to 100%. Above 100%, we reduce the variability of the found building patches.



FIGURE 13: Plotting of patches classified as building, using confidence to increase precision. Ground truth from IGN and Open Data Paris

In this example the precision was already very good (most of the blue patches are in building), but increasing precision to reduce class heterogeneity provides a much better base for building reconstruction.

It is possible to use this method only when precision is a rising function of confidence given by random forest. This is the case for 4+wheeler class.

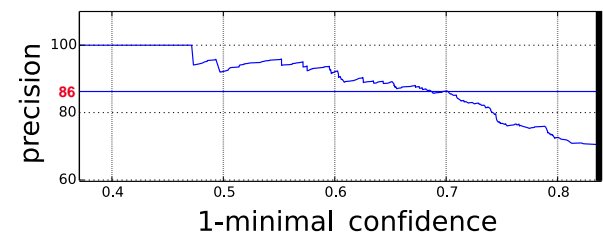
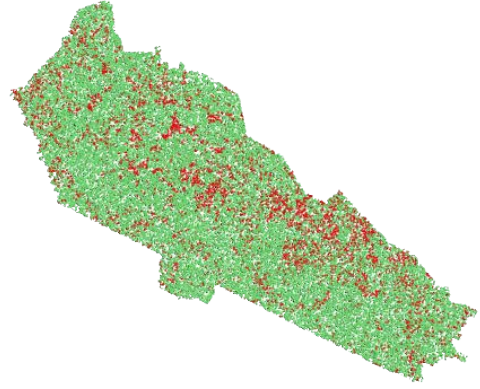


FIGURE 14: Precision of 4+wheelers class =  $f(1 - \text{random forest confidence score})$ . The horizontal line is the average confidence.

*Filtering : artificial increase of recall.* The patch classifier is used on paris data set. The goal is to find all ground patches very fast. We focus on an area for illustration purpose. This area contains 3086 patches, including 439 ground patches. Patch classification finds 421 ground patch, with a recall of 92.7% . Using the found patch, all the surrounding patches (X,Y : 2 m, Z : 0.5 m) are added to the result (few seconds). There are now 652 patches in the result, and the recall is 100%. This means that from a filtering point of view, a complex classifier that would try to find ground points can be used on  $652/3086 = 21\%$



	prec.	rec.	supp.	mix.
<b>Closed Forest</b>	<b>0.99</b>	<b>0.91</b>	<b>390k</b>	<b>0.883</b>
Moor	0.18	0.68	8.7k	0.741
<b>Not forest</b>	<b>0.86</b>	<b>0.89</b>	<b>128k</b>	<b>1</b>
avg/total	0.94	0.90	526k	0.901



ppl_1	ppl_2	ppl_3	ppl_4	intensity	nber of echo	mean Z	patch height
<b>0.57</b> (0.07+0.13+0.18+0.19)				0.09	0.06	<b>0.12</b>	<b>0.14</b>

FIGURE 11: Feature usage (first table). Precision(prec.), recall (rec.), support (supp.), and average percent of points of the class in the patches, for comparison with point based method (mix.).

of the data set, at the price of few seconds of computing, without any loss of information.

## 5. Discussion

In this section we discuss the results following the result section, and give limitations of the methods.

### 5.1. Point cloud server

The focus of this article is not the Point cloud server, thus we discuss only briefly it. The point cloud server has been proven to work easily with different type of point cloud, for many traditional application of point cloud. It is to the best of our knowledge the fastest and easiest way to filter very big point cloud using complex spatial and temporal criteria. It is also the only solution that is natively integrated with other GIS data (raster, vector).

*Point cloud server limitation.* The limitations are that the multi-acquisition process is not defined, the lack of facilities to easily add attributes to an existing point cloud, and more importantly, the fact that accessing only few points in a patch still requires to read the whole patch (all points and all dimensions of this patch). We note that these limitations are not fundamental but simply require more work.

### 5.2. Data set

*Vosges data set.* The figure 6 exposes important density variation, even for aerial lidar point cloud (green) that are supposed to be more homogeneous. The variation of density may be due to variation of plan speed, and

more importantly, variation of distance between plan and ground (due to height of plan or ground).

Some of this variation is also artificial and due to the way the acquisition is loaded into the point cloud server : the acquisition we use consist of thousand of las files.

Those files are separated into  $50^3 \text{ m}^3$  patches individually, which means that some patches on the border may be less filled.

*Paris data set.* Surprisingly the Paris data set density distribution seems to match a linear distribution (in the log /log space of the figure 6). For a short period of time, we assume building and ground form a U shape, the laser device being in the middle. The laser rotates on himself around the back-front of the vehicle axis, and acquire points at fixed angles (about 1/10 of degree). It creates points more and more spaced along the building height.

*Limitation of the data set.* The data set used are representative of urban and aerial Lidar point cloud. However the methods are not tested with indoor point cloud, or Structure from Motion point cloud.

### 5.3. Exploiting the order of points

*Limitation.* Schematically, the method proposes to exploit order to store LOD information. There is a strong and evident theoretical limitation : the order may already contain a useful information ! For instance with a static Lidar device the neighbourhood information can be reconstructed from the order. We note that for the Paris data set the point are initially ordered by time of acquisition. Because the time of acquisition is also an attribute, this ordering can be reconstructed if necessary.

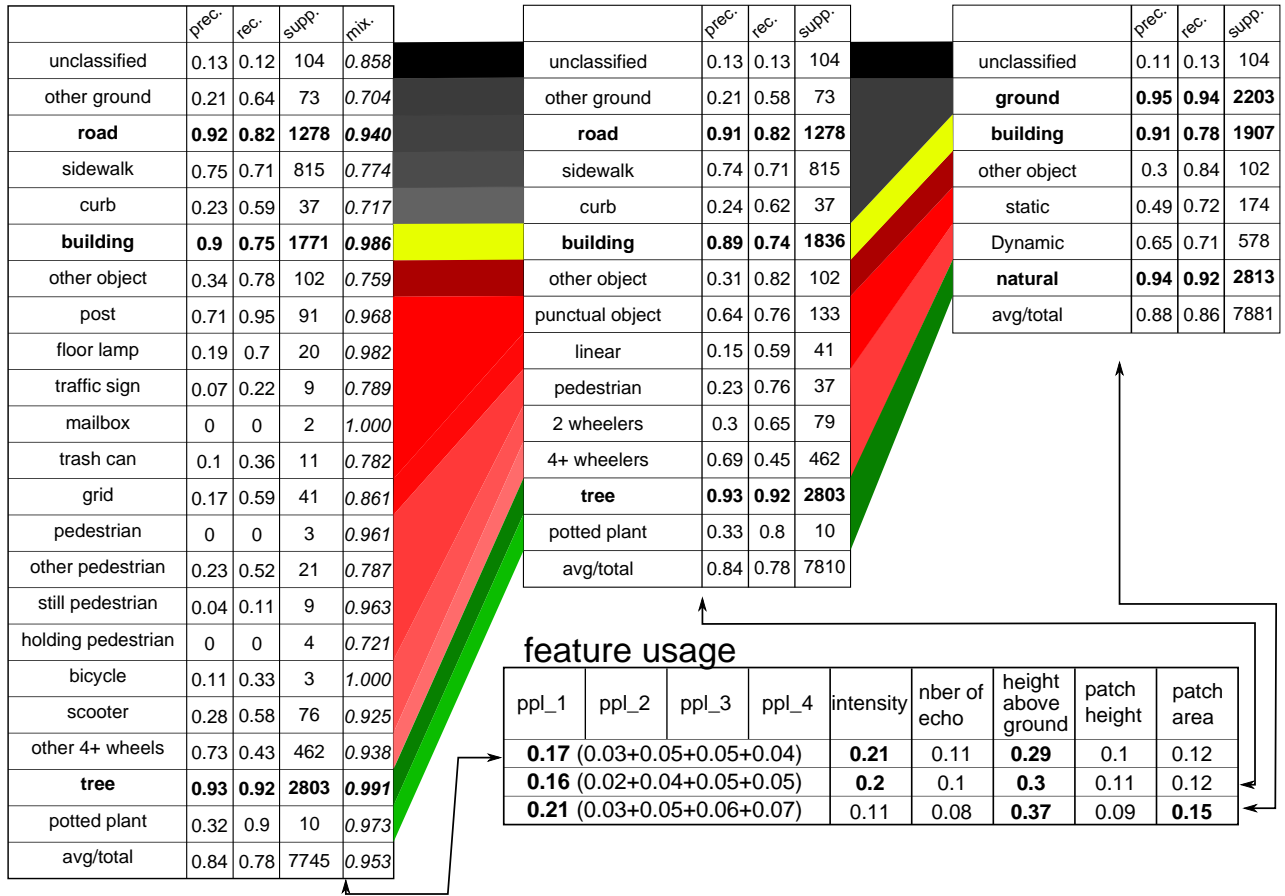


FIGURE 12: Results for Paris data set : at various level of class hierarchy. Precision(prec.), recall (rec.), support (sup.) and average percent of points of the class in the patches of the class, for comparison with point based method (mix.). Classes of the same type are in the same continuous tone. Feature usage is evaluated for each level in the class hierarchy.

#### 5.4. MidOc : an ordering for gradual geometrical approximation

##### 5.4.1. Visual evaluation

The illustration 7 gives visual example of LOD result and how it could be used to vary density depending on the distance to camera.

Figure 4 also gives visual examples for common objects of different dimensionality. It is visually clear that the rate of increase of points from LOD 0 to 4 for floor lamp (1D) window (2D) and tree (3D) is very different. Small details are also preserved like the poles or the antenna of the car. preserving those detail with random or distance based subsampling would be difficult.

##### 5.4.2. Size versus LOD trade-off

The table 1 shows that using the level 3 may speed the transfer time by a 10 factor. The point cloud server throughput is about 2-3 Mbyte /s(monoprocess), sufficient for an internet troughput, but not fast enough for a LAN 10 Mbyte /s. This relatively slow troughput is due to current point cloud server limitation (cf 5.1).

##### 5.4.3. Large scale computing

The relatively slow computing (100 Millions points /s) is a very strong limitation. About 2/3 of the computing

time is spend on data transformation because algorithm works and output a list of points, which must then be desagregated/aggregated from/to a patch. This could be avoided by a C implementation which can access raw patch, and would also be faster for ordering points.

##### 5.4.4. Fast abnormal density detection and correction

**Fast detection.** Density abnormality detection at the patch level offer the great advantage of avoiding to read points. This is the key to the speed of this method. We don't know any method that is as fast and easy.

The limitations stems from the aggregated nature of patch. the number of points per patch doesn't give the density per point, but a quantized version of this per patch. So it is not possible to have a fine per point density. It also introduces the classical sampling limits : only density abnormalities spatially bigger than 2 patches (at least 2 m wide for Paris) can be detected.

**Simple correction.** The correction of density peak we propose has the advantage of being instantaneous and not induce any data loss. It is also easy to use as safe-guard for an application that may be sensible to density peak : the application simply defines the highest number of points /m<sup>3</sup> it can handle, and the Point cloud server will



FIGURE 15: Map of patch clustering result for ground. The classical result finds few extra patches that are not ground (blue), and misses some ground patches (red). Recall is increased by adding to the ground class all patches that are less than 2 meters in X,Y and 0.5 meter in Z around the found patches. Extra patches are much more numerous, but all the ground patches are found.

always output less than that.

The most important limitation this method doesn't guarantee homogeneous density. For instance if an application requires 1000 points /  $\text{m}^3$  for ground patches, all the patches must have more than 1000 points, and patch must have been ordered with MidOc for level 0 to at least 5 ( $4^5 = 1024$ ). The homogeneous density may also be compromised when the patch are not only split spatially, but with other logics (in our case, points in patch can't be separated by more than 30 seconds, and all points in a patch must come from the same acquisition file).

#### 5.4.5. LOD stream

Streaming low level of detail patches greatly accelerate visualisation, which is very useful when the full point cloud is not needed. However all patches should not have the same LOD, but the required LOD should vary for each patch based on distance to camera.

As seen before (5.4.2), the point cloud server is fast enough for an internet connection, but is currently slower than a file-based points streaming. Thus for the moment LOD stream is interesting only when bandwidth is limited.

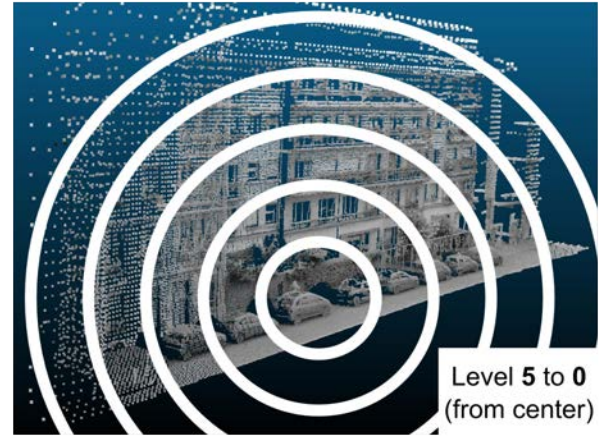


FIGURE 16: Schematic example of LOD depending on distance to camera

### 5.5. Dimensionality descriptor and patch classification

#### 5.5.1. *Ppl* as a crude dimensionality descriptor

Figure 9 is obtained fully automatically (excepting the red arrows), using only the *ppl* descriptor. Yet objects of pure dimensions 1D, 2D and 3D are neatly separated, with mixed dimensionality classes being in the center, which support the role of *ppl* as a crude dimensionality descriptor. Further evidence can be observed on the 2D axes. Sidewalk are not pure 2D because patches may contains parts of other objects. Building is also mostly 2D but balcony, building decoration and floors introduce lot's of object-like patches, which explain that building is closer to the object cluster.

*Limitations.* As expected, *ppl* descriptor are not sufficient to correctly separate complex objects, which is the main limitation.

There is a more fundamental limitation of *ppl* as crude dimensionality descriptor. Because *ppl* are per patch and not per point, dimensionality can only be estimated for objects that are bigger than a patch ( $1 \text{ m}^3$  for paris dataset). And for this estimation to be possible, a patch must also contains a sufficient number of points, which is not completely the case for distant tree patches for example.

#### 5.5.2. Analyzing data set classes

*Balancing the data sets.* Strategies to balance data sets have a big impact on result. We observe that some classes have simply not enough support and too much diversity to be correctly predicted. This is the limit of our approach where all classes are learned and predicted at once. For classes with very small support the one against all strategy should be tested.

*Analysing class hierarchy.* The figure 10 shows automatic class clustering using the confusion matrix between classes with all simple features. Interestingly, the layout preserve the separation between pure 1D, 2D, 3D, and mixed dimensions. The distinct meta classes that will be



possible to separate correctly are vegetation (3D), ground (2D), poles (1D), objects, vehicle and buildings. This illustration also shows the limit of a classification without contextual information. For instance the class grid and buildings are similar because in Paris buildings balcony are typically made of grids.

### 5.5.3. Patch classifier

*Vosges data set.* First the feature usage for vosges data set clearly shows that amongst all the simple descriptor, the *ppl* descriptor is largely favored. This may be explained by the fact that forest and bare land have very different dimensionality, which is conveyed by the *ppl* descriptor.

Second the patch classifier appears to have very good result to predict if a patch is forest or not. The precision is almost perfect for forest. Because most of the errors are on border area, the recall for forest can also be easily artificially increased. The percent of points in patch that are in the patch class allow to compare result with a point based method. For instance the average precision per point for closed forest would be  $0.99 * 0.883 = 0.874$ . We stress that this is averaged results, and better precision per point could be achieved because we may use random forest confidence to guess border area (with a separate learning for instance). For comparison with point methods, the patch classifier predict with very good precision and support over 6 billions points in few seconds (few minutes for training). We don't know other method that have similar result while being as fast and natively 3D.

The Moor class can't be separated without more specialised descriptor, because Moor and no forest classes are geometrically very similar.

The principal limitation is that for this kind of aerial Lidar data set the 2.5D approximation may be sufficient, which enables many raster based methods that may be more performant or faster.

*Paris data set.* The figure 12 gives full results for paris data set, at various class hierarchy level. Because the goal is filtering and not pure classification, we only comment the 7 classes result. The proposed methods appears very effective to find building, ground and trees. Even taking into consideration the fact that patch may contains mixed classes (column mix.), the result are in the range of state of the art point classifier, while being extremely fast. This result are sufficient to increase recall or precision to 1 if necessary. We stress that even results appearing less good (4+wheelers, 0.69 precision, 0.45 recall) are in fact sufficient to increase recall to 1 (by spatial dilatation of the result), which enables then to use more subtle methods on filtered patches.

*ppl* descriptor are less used than for the Vosges data set, but is still useful, particularly when there are few classes. It is interesting to note that the mean intensity descriptor seems to be used to distinguish between objects, which makes it less useful in the 7 classes case.

The patch classifier for Paris data set is clearly limited to separate simple classes. In particular, the performances for objects are clearly lower than the state of the

art. A dedicated approach should be used (cascaded or one versus all classifier).

### 5.5.4. Patch classifier Applications

Because the propose methods are preprocess of filtering step, it can be advantageous to increase precision or recall.

*Artificial increase of precision.* In the figure 13 gives a visual example where increasing precision and reducing class heterogeneity is advantageous. This illustrates that having a 1 precision or recall is not necessary the ultimate goal. In this case it would be much easier to perform line detection starting from red patches rather than blue patches.

The limitation of artificial precision increase is that it is only possible when precision is roughly a rising function of random forest confidence, as seen on the illustration 14. For this class, by accepting only prediction of random forest that have a confidence over 0.3 the precision goes from 0.68 to 0.86, at the cost of ignoring half the predictions for this class. This strategy is not possible for incoherent class, like unclassified class.

*Filtering : artificial increase of recall.* The method we present for artificial recall increase is only possible if at least one patch of each object is retrieved, and objects are spatially dense. This is because a spatial dilatation operation is used. This is the case for 4+wheelers objects in the paris data set for instance. The whole method is possible because spatial dilatation is very fast in point cloud server (because of index). Moreover, because the global goal is to find all patches of a class while leaving out some patches, it would be senseless to dilate with a very big distance. In this case recall would be 1, but all patches would be in the result, thus there would be no filtering, and no speeding.

The limitation is that this recall increase method is more like a deformation of already correct results rather than a magical method that will work with all classes.

## 6. Conclusion

We propose to store LOD in the order of points. For visualization application, the best ordering may not be geometrical (like MidOc), but may depends on visual saliency of the points. In this case the LOD of each patch should be adapted to the distance to camera.

Theoretically *ppl* descriptor could be decomposed on a pure dimensionality base. For all applications, it could be interesting to find methods to group patches (and merge *ppl*) to enable dimensionality analysis at several scales. Because some patches contain mixed classes, a bias is introduced which mechanically lower the average per points results. The random forest classifier should be in fact a random forest regressor, predicting for each patch not a class, but classes distribution.

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