On souhaite trouver la solution de l'équation $u_{xx} - (x^6 + 3x^2)u = 0$.

Avec conditions initiales $u(-1) = u(1) = 1$.

On suppose une solution de la forme: $u_2(x) = 1 + (1 - x^2)(a_0 + a_1 x + a_2 x^2)$

```
syms x a0 a1 a2

u = 1 + (1-x^2)*(a0+a1*x+a2*x^2);
uxx = diff(u,x,2);
V = (x^6+3*x^2);
```

On parvient à l'équation résiduelle:

```
R = uxx-u*V;
%L'equation residuelle
R = collect(R,x)
```

R = $a_2\, x^{10} + a_1\, x^9 + (a_0 - a_2)\, x^8 + (-a_1)\, x^7 + (3\, a_2 - a_0 - 1)\, x^6 + (3\, a_1)\, x^5 + (3\, a_0 - 3\, a_2)\, x^4 + (-3\, a_1)\, x^3 + (-3$

On choici 3 points de collocations $\{-0.5, 0, 0.5\}$ où l'on remplace x en assumant qu'il s'agit d'une solution exacte.

Ainsi on parvient à 3 équations du type $R(x_i; a_0, a_1, a_2) = 0$ qui engendre un système d'équation pour les coefficients.

```
xi = [-0.5,0,0.5];

R1 = subs(R,x,xi(1))
```

R1 =

$$\frac{1683\, a_1}{512} - \frac{659\, a_0}{256} - \frac{1171\, a_2}{1024} - \frac{49}{64}$$

```
R2 = subs(R,x,xi(2))
```

R2 = $2\, a_2 - 2\, a_0$

```
R3 = subs(R,x,xi(3))
```

R3 =

$$-\frac{659\, a_0}{256} - \frac{1683\, a_1}{512} - \frac{1171\, a_2}{1024} - \frac{49}{64}$$

```
solve([R1,R2,R3],[a0,a1,a2])
```

ans = struct with fields:

```
a0: -784/3807
a1: 0
a2: -784/3807
```

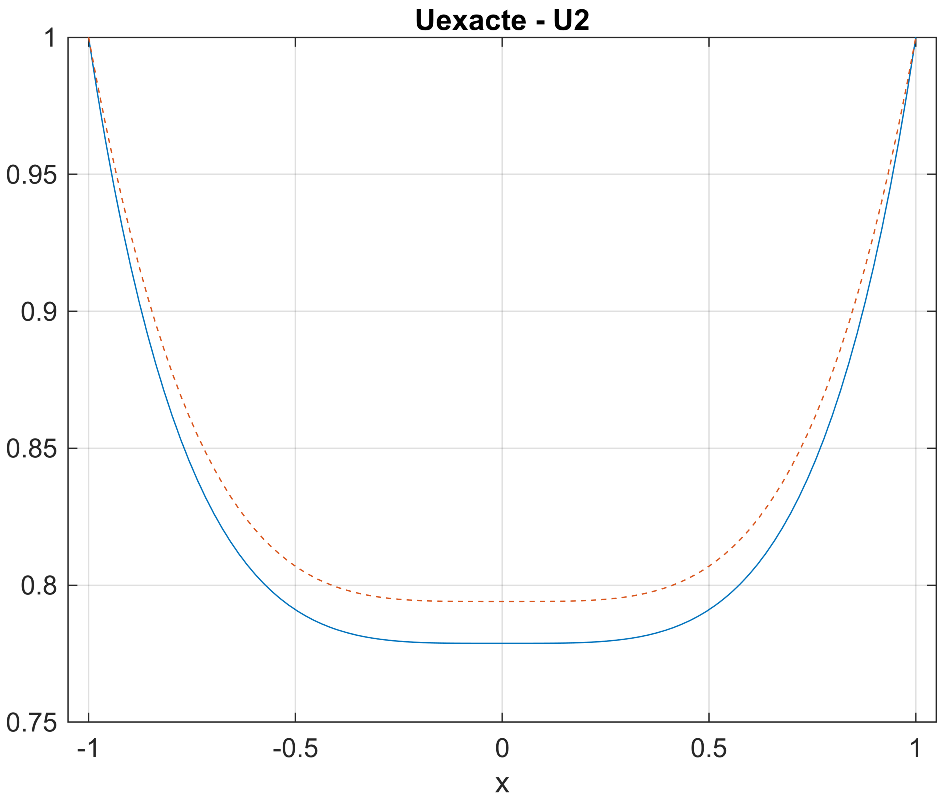
En sachant que la vraie solution est $u(x) = e^{\frac{x^4-1}{4}}$. On peut calculer l'erreur:

```
xx = linspace(-1,1);

uexact = exp(0.25*(x^4-1));
uexact = matlabFunction(uexact);
u2 = subs(u,[a0,a1,a2],[-784/3807,0,-784/3807])
```

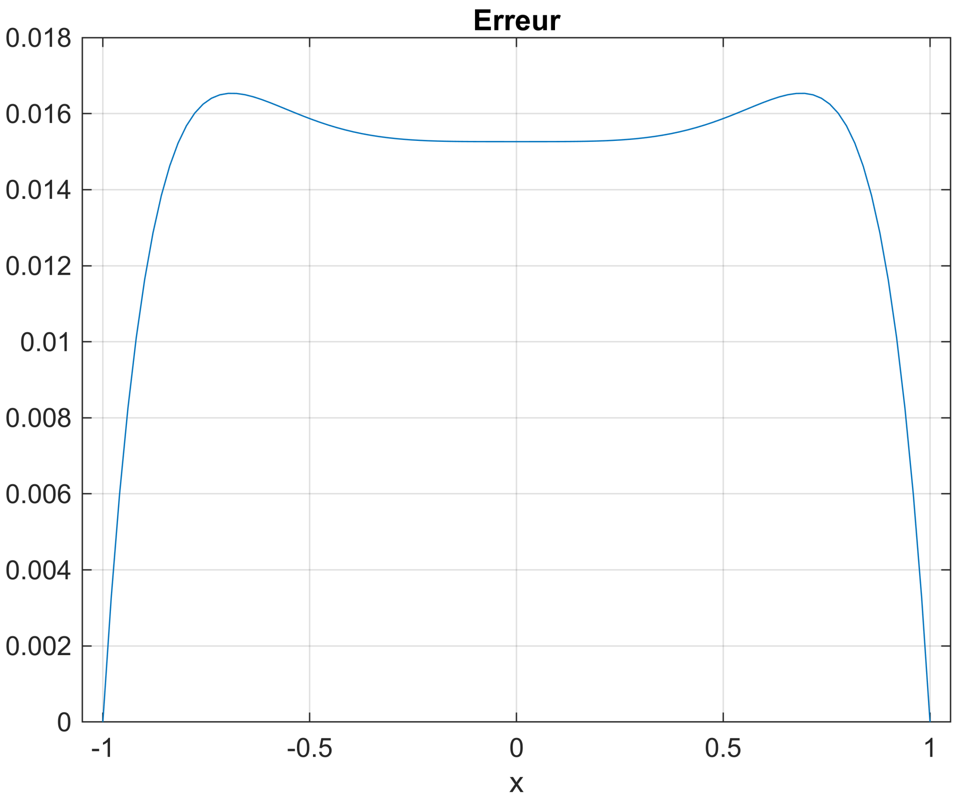
```
u2 =
```

$$(x^2 - 1)\left(\frac{784\,x^2}{3807} + \frac{784}{3807}\right) + 1$$

```
u2 = matlabFunction(u2);

plot(xx,uexact(xx))
hold on
plot(xx,u2(xx),'--')
hold off
title('Uexacte - U2')
xlabel('x')
xlim([-1.05,1.05])
grid on
```




```
plot(xx, abs(u2(xx)-uexact(xx)))
title('Erreur')
xlabel('x')
xlim([-1.05,1.05])
grid on
```


Erreur
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
-1
-0.5
0
0.5
1
x